A Monetary Model of Banking Crises Based on the Lagos-Wright Framework

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Motivation

Analytical framework for the current financial crisis

- Standard business cycle (DSGE) models cannot analyze the financial crisis
- Need banking models compatible with DSGE models. (Need tractable distinction between money and goods.)
- Unified framework to evaluate the efficacy of the current policies (fiscal stimulus, monetary easing, and bank reforms).

Nature of banking services and banking crises:

- Liquidity insurance (Bryant, Diamond-Dybvig, Allen-Gale)
- Optimal contract under holdup problem (Diamond-Rajan)
- Payment services, or provision of media of exchange (Berentsen-Camera-Waller based on Lagos-Wright)
We construct a tractable monetary model of banking crises

- **Banking services** = Payment intermediation
  - **Cash economy** – Cash circulates only once:
    - Buyers ⇒ Sellers
  - **Banking economy** – Cash circulates $1/\rho = J$ times
    - Bank ⇒ Buyers $j$ ⇒ Sellers $j$ ⇒ Bank ⇒ Buyers $j + 1$ ⇒ · · · ($J$ times)

- **Banking crisis** — All sellers decide not to deposit money:
  - Expectations of Bank insolvency (exogenous; endogenous)
  - Bank ⇒ Buyers 1 ⇒ Sellers 1 ⇒ X
  - Banks run out of cash reserve in the first round.
  - Buyers 1 can withdraw cash and buy the goods.
  - Remaining buyers (Buyers 2, · · · , Buyers $J$) cannot buy the goods. (The sequential service constraint on depositors.)
  - Demand for the goods plunges.
  - Production of the goods decreases.
Social welfare of Banking economy is identical to Cash economy. This is because the banking service (= providing demand deposits) is only a substitute for cash.

Our simplistic model does not provide a new theory for raison d’être of banking sector.

Heterogeneous distribution of wealth is necessary for banking to improve social welfare in our model.

This paper describe banking crises as disruption of payment intermediation.

This paper focus on implications for recovery efforts from crisis.
Main Results (1/2)

Banking Crisis

- Basic model (loan enforcement and no bank insolvency shock)
  - Banking crisis never occurs.

- Model with bank insolvency shock
  - Banking crisis occurs.

- Model with incomplete loan enforcement and collateral constraint
  - Assume there is no bank insolvency shock.
  - Banking crisis can occur as a result of self-fulfilling coordination failure among depositors.

Main Results (2/2)

Policy implications

- Fiscal stimulus — The government purchase of the goods
  - Not good unless the government efficiently maintains the purchased goods.

- Monetary easing — The central bank lending to the banks
  - Not good if LLR lending is limited to solvent banks.

- Bank reforms to restore solvency of banks — bad asset disposals and capital injections
  - Good to restore confidence in bank deposits and restore transactions in the goods market.
  - Cost of policy implementation appears to be huge \( ex \ ante \), but it turns out to be small \( ex \ post \).
Related Literature

- Banking models with distinction between money and goods
  - Champ, Smith and Williamson (1996)
  - Allen and Gale (1998)
Plan

- Basic Model
  - Setup
  - Bank’s Problem
  - Night Market
  - Day Market
  - Equilibrium (No banking crisis)

- Model with Bank Insolvency Shock
  - Equilibrium (with banking crisis)

- Model with Incomplete Loan Enforcement and Collateral Constraints
  - Policy Implications
Basic Model – Setup (1/4)

- Closed Economy, Discrete time $t = 0, 1, \cdots, \infty$

- Two competitive markets open sequentially at each date $t$ — Day market and Night market

- Goods:
  - Consumption goods (numeraire) — Produced in the night market
  - Intermediate goods — Produced in the day market

- Assets:
  - Machines (productive, collateralizable, last for one period)
  - Cash — Injected by Central Bank in the night market
  - Bank deposits
  - Bank loans — Not tradable
Continuum of sellers, buyers, and banks

- Banks live for one period. Measure of banks: 1
- Sellers live for infinite periods. Measure of sellers: \( n \)
- Buyers live for infinite periods. Measure of buyers: \( 1 - n \)
- Discount factor: \( \beta \) (< 1) for sellers and buyers
Basic Model – Setup (3/4)

- **Previous Night Market** (date \( t - 1 \)): Sellers and buyers decide cash holdings, bank deposits, and bank loans that they carry over to date \( t \).

- **Day market**: Anonymous market (Trade credit is not available)
  - Sellers produce and sell the intermediate goods, \( q \), to buyers
  - Buyers have to pay cash to sellers. (Either they have cash in advance or they withdraw bank deposits)
  - After the goods trading, sellers and buyers decide cash holdings and bank deposits they carry to the night market.
Night market: Trade credit is available. Money is not needed as a medium of exchange, but is used as a store of value.

- Buyers are endowed with machines, $k$. Buyers repay bank loans.
- Sellers, buyers, (and banks) trade the intermediate goods $q$ and machines, $k$
- Buyers produce the consumption goods $y$ from $q$ and $k$ by $y = Ak^{1-\theta}q^\theta$. Consumption takes place.
- Bank deposits are paid out, and banks are liquidated.
- New banks are born. Cash is injected. Cash holdings, bank deposits, and bank loans carried over to date $t + 1$ are decided.
Bank’s Problem (1/4)

- Banks have record keeping technology for financial transactions of sellers and buyers.
- Banks can enforce loan repayment on the borrowers.

**Date-\((t - 1)\) night market**
- Banks make loans, \(L_t\), hold cash reserves, \(C_t\), and accept deposits, \(D_t\).

**Date-\(t\) day market**
- Deposits become \((1 + i_d)D_t\). Banks promise to exchange deposits to cash at anytime during the day market.

**Date-\(t\) night market**
- Banks collect loans, \((1 + i)L_t\), pay out deposits, \((1 + i_n)(1 + i_d)D_t\), and are liquidated.
Bank’s Problem (2/4)

Banks’ problem is

\[
\max_{L_t, C_t, D_t} [(1 + i)L_t + C_t - (1 + i_n)(1 + i_d)D_t]_+
\]

subject to

\[
L_t + C_t \leq D_t, \quad (1)
\]
\[
(1 + i_d)D_t \leq \frac{1}{\rho}C_t. \quad (2)
\]
The day market is divided into $J$ submarkets. $\rho = 1/J$.

Cash circulates $J$ times.

- Bank $\Rightarrow$ Buyers $j$ $\Rightarrow$ Sellers $j$ $\Rightarrow$ Bank $\Rightarrow$ Buyers $j + 1$ $\Rightarrow$ · · · (J times)

A buyer in Buyers $j$ withdraws all deposit: $(1 + i_d)d$.

Number of Buyers $j$ is $(1 - n)/J$.

Total withdrawal of Buyers $j$: $(1 + i_d)D/J$.

Total withdrawal must equal bank’s cash reserve: $C$

The reserve requirement:

$$(1 + i_d)D_t \leq \frac{1}{\rho}C_t.$$
Both (1) and (2) bind in equilibrium. The reduced form of bank’s problem is

$$\max_{C_t} \left[ (1 + i) \left\{ \frac{1}{(1 + i_d)\rho} - 1 \right\} + 1 - \frac{1 + i_n}{\rho} \right]_+ C_t. \quad (3)$$

Since $C_t$ cannot be infinite in equilibrium, it must be the case that

$$(1 + i_d)(1 + i_n) = 1 + \{1 - (1 + i_d)\rho\}i, \quad (4)$$

and the profit for the banks is zero.
Sequence of Decisions

**Date-\((t - 1)\) night market**
- Agent chooses \(m^d\) (cash), \(d^d\) (deposit), \(l\) (loan) to carry over to the date-\(t\) day market.

**Date-\(t\) day market**
- Deposit becomes \((1 + i_d)d^d\).
- Seller produces \(q^s\) (intermediate goods) with utility cost of \(c(q^s)\).
- Buyer buys \(q^b\) units and pays \(pq^b\).
- Agent chooses \(m^n\) (cash), \(d^n\) (deposit) to carry over to the date-\(t\) night market. (Loan, \(l\), does not change.)

**Date-\(t\) night market**
- Deposit becomes \((1 + i_n)d^n\). Loan becomes \((1 + i)l\).
- Production, trade, and consumption of the consumption goods take place.
- Agent chooses \(m^d_{+1}\), \(d^d_{+1}\), and \(l_{+1}\) to carry over to the date-\((t + 1)\) day market.
Bellman equation is

\[ W_s(m^n, d^n, l) = \max_{x, h, m_{n+1}, d_{n+1}, l_{n+1}} \left[ U(x) - h + \beta V^s_{n+1}(m_{n+1}, d_{n+1}, l_{n+1}) \right] \]  

subject to

\[ x + \phi(m_{n+1} + d_{n+1} - l_{n+1}) = h + \phi\{m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\}, \]  

where \( \phi \) is the real value of cash. This program can be rewritten as

\[ W_s(m^n, d^n, l) = \phi\{m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\} \]

\[ + \max_{x, m_{n+1}, d_{n+1}, l_{n+1}} \left[ U(x) - x - \phi(m_{n+1} + d_{n+1} - l_{n+1}) + \beta V^s_{n+1}(m_{n+1}, d_{n+1}, l_{n+1}) \right] \]
The first-order conditions (FOCs) are $U'(x) = 1$ and

$$
\phi \geq \beta V_{m}^{s}(+1), \quad \text{where if } >, \text{ then } m_{+1}^{d} = 0; \text{ if } =, \text{ then } m_{+1}^{d} \geq 0; \quad (7)
$$

$$
\phi \geq \beta V_{d}^{s}(+1), \quad \text{where if } >, \text{ then } d_{+1}^{d} = 0; \text{ if } =, \text{ then } d_{+1}^{d} \geq 0; \quad (8)
$$

$$
\phi \leq -\beta V_{l}^{s}(+1), \quad \text{where if } <, \text{ then } l_{+1}^{d} = 0; \text{ if } =, \text{ then } l_{+1}^{d} \geq 0, \quad (9)
$$

where $V_{x}^{s}(+1) \equiv \frac{\partial}{\partial x} V^{s}(m_{+1}^{d}, d_{+1}^{d}, l_{+1})$ for $x = m_{+1}^{d}, d_{+1}^{d}, l_{+1}$.

The envelope conditions imply that $W^{s}$ can be written as

$$
W^{s}(m^{n}, d^{n}, l) = \phi\{m^{n} + (1 + i_{n})d^{n} - (1 + i)l\} + \overline{W}_{l}^{s}, \quad (10)
$$

where $\overline{W}_{l}^{s}$ is independent from the state variables.
Bellman equation is

\[ W^b(q, m^n, d^n, l) = \max_{x, h, m+1, d+1, l+1} [U(x) - h + \beta V^b_{+1}(m^d_{+1}, d^d_{+1}, l+1)] \] (11)

subject to

\[ x + \phi(m^d_{+1} + d^d_{+1} - l+1) = h + \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\}, \] (12)

where \( k \) is the number of the machines, \( q \) is the quantity of the intermediate goods, and \( a \) and \( w \) are the market prices. This program can be rewritten as

\[ W^s(m^n, d^n, l) = \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l + (\gamma_t - 1)M_t\} \]
\[ + \max_{x, m+1, d+1, l+1} [U(x) - x - \phi(m^d_{+1} + d^d_{+1} - l+1) + \beta V^b_{+1}(m^d_{+1}, d^d_{+1}, l+1)] \]
The FOCs are $U'(x) = 1$ and

1. $\phi \geq \beta V_m^b(+1)$, where if $>, \text{ then } m_{+1}^d = 0$; if $=, \text{ then } m_{+1}^d \geq 0$; (13)
2. $\phi \geq \beta V_d^b(+1)$, where if $>, \text{ then } d_{+1}^d = 0$; if $=, \text{ then } d_{+1}^d \geq 0$; (14)
3. $\phi \leq -\beta V_l^b(+1)$, where if $<, \text{ then } l_{+1} = 0$; if $=, \text{ then } l_{+1} \geq 0$. (15)

The envelope conditions imply that $W_b^b$ can be written as

$$W_b^b(q^b, m^n, d^n, l) = \phi\{ak + wq + m^n + (1 + i_n)d^n - (1 + i)l\} + \bar{W}_t^b,$$ (16)

where $\bar{W}_t^b$ is independent from the state variables.
In the night market, the buyers produce the consumption goods with the Cobb-Douglas technology, \( y = A k^{1-\theta} q^\theta \).

Since \( k \) and \( q \) are competitively traded, the prices are determined by

\[
\phi a = (1 - \theta) A(q^b)^\theta, \\
\phi w = \theta A(q^b)^{\theta - 1},
\]

since \( k = 1 \) and \( q = q^b \) per buyer.
Bellman equation is

\[ V^s(m^d, d^d, l) = \max_{q, m^n, d^n} -c(q) + W^s(m^n, d^n, l) \]  \hspace{1cm} (19)

subject to

\[ m^n + d^n = pq + m^d + (1 + i_d)d^d, \] \hspace{1cm} (20)

\[ m^n \geq 0, \text{ and } d^n \geq 0. \] \hspace{1cm} (21)

This program can be rewritten as

\[ V^s(m^d, d^d, l) = \max_{q, d^n} \phi pq - c(q) + \phi\{m^d + (1 + i_d)d^d + i_n d^n - (1 + i)l\} + W^s_t \]

subject to \[ d^n \leq pq + m^d + (1 + i_d)d^d. \]
Given $i_n > 0$, the FOCs imply

\[
\phi p = \frac{c'(q^s)}{1 + i_n},
\]

\[
d^n = pq + m^d + (1 + i_d)d^d,
\]

\[
m^{ns} = 0.
\]

Sellers deposit all cash into their banks immediately.
The envelope conditions: $V^s_m = \phi (1 + i_n)$, $V^s_d = \phi (1 + i_d)(1 + i_n)$, and $V^s_l = -\phi (1 + i)$. These conditions and the FOCs for the night market imply that

\[
\phi \geq \beta \phi_{+1}(1 + i_{n,+1}), \quad \text{where if } >, \text{ then } m_{+1}^d = 0; \text{ if } =, \text{ then } m_{+1}^d \geq 0; \quad (25)
\]
\[
\phi \geq \beta \phi_{+1}(1 + i_{d,+1})(1 + i_{n,+1}), \quad \text{where if } >, \text{ then } d_{+1}^d = 0; \text{ if } =, \text{ then } d_{+1}^d \geq 0; \quad (26)
\]
\[
\phi \leq \beta \phi_{+1}(1 + i_{+1}), \quad \text{where if } <, \text{ then } l_{+1} = 0; \text{ if } =, \text{ then } l_{+1} \geq 0. \quad (27)
\]
Bellman equation is

\[ V^b(m^d, d^d, l) = \max_{q, m^n, d^n} \phi(ak_t + wq + m^n + (1 + i_n)d^n - (1 + i)l) + W^b_t, \]

subject to \( m^n + d^n + pq = m^d + (1 + i_d)d^d \).

In the case when \( i_n > 0 \),

\[ d^n = m^d + (1 + i_d)d^d - pq, \tag{28} \]

\[ m^{nb} = 0. \tag{29} \]

Buyers deposit all remaining money into the banks, and hold no cash.
The reduced form of the buyer’s program:

\[ V^b(m^d, d^d, l) = \max_q \phi\{ak + wq - (1 + i_n)pq + (1 + i_n)m^d + (1 + i_d)(1 + i_n)d^d - (1 + i)l\} + \bar{W}_t^b, \tag{30} \]

subject to

\[ pq \leq m^d + (1 + i_d)d^d. \tag{31} \]

The FOC is

\[ (1 + i_n + \lambda)p \geq w, \quad \text{where if} >, \text{then} q^b = 0; \text{if} = , \text{then} q^b \geq 0, \tag{32} \]

The envelope conditions are

\[ V^b_m = \phi(1 + i_n + \lambda), \quad V^b_d = \phi(1 + i_d)(1 + i_n + \lambda), \quad V^b_l = -\phi(1 + i), \quad \text{where} \ \lambda \ \text{is the Lagrange multiplier for (31)}. \]
The envelope conditions and the FOCs for the night market imply

\[
\phi \geq \beta \phi_{+1}(1 + i_{n,+1} + \lambda_{+1}),
\]

where if >, then \( m_{+1}^d = 0 \); if =, then \( m_{+1}^d \geq 0 \); (33)

\[
\phi \geq \beta \phi_{+1}(1 + i_{d,+1})(1 + i_{n,+1} + \lambda_{+1}),
\]

where if >, then \( d_{+1}^d = 0 \); if =, then \( d_{+1}^d \geq 0 \); (34)

\[
\phi \leq \beta \phi_{+1}(1 + i_{+1}),
\]

where if <, then \( l_{+1} = 0 \); if =, then \( l_{+1} \geq 0 \). (35)
The inflation rate $\gamma = \phi / \phi_{+1}$ is determined by

$$\frac{\gamma_{+1}}{\beta} = 1 + i_{+1}. \quad (36)$$

Since $0 < i_d < i$ and $0 < i_n < i$, sellers carry no cash nor deposit into the day market:

$$m^d = 0, \text{ and } d^d = 0. \quad (37)$$

Liquidity constraint, (31), binds:

$$\lambda_{+1} = \rho i_{+1} > 0, \quad (38)$$

Buyer carries no cash in the day market: $m^d = 0$. Buyer’s deposit is

$$(1 + i_d)d^d = pq^b. \quad (39)$$

$m^{nb} = d^{nb} = 0$, $d^{ns} = pq^s$, and $m^{ns} = 0$, where $q^s = (1 - n)q^b/n$. 
Equilibrium (2/2)

- Buyer’s purchase $q^b$:

$$\frac{\theta A(q^b)^{\theta-1}}{1 + i} = \frac{c'(q^s)}{(1 + i_d)(1 + i_n)}.$$  \hspace{1cm} (40)

- $\phi d^{db}$ is determined by

$$\phi d^{db} = \frac{\theta A(q^b)^{\theta}}{1 + i}.$$  \hspace{1cm} (41)

- The variables for banks are determined by

$$\phi D_t = (1 - n)\phi d^{db},$$

$$C_t = M_t,$$

$$\phi L_t = \phi n l^s + \phi (1 - n) l^b = (1 - n)(1 - \rho)\phi d^{db},$$

$$z \equiv \phi M_t = \phi C_t$$ is determined by

$$z = (1 + i_d)(1 - n)\rho \phi d^{db}.$$
Basic Model — No Bank Runs

If $i_n > 0$, there are no bank runs
- Bank insolvency never occurs. (Loan enforcement)
- No default on bank deposits
- Since $i_n > 0$, agents are strictly better-off by depositing their income into the banks rather than holding their income in the form of cash.

If $i_n = 0$, bank runs may occur as a herd behavior
- Agents are indifferent between bank deposit and cash. (The same returns)
- Herd behavior may induce bank runs.
- The Friedman rule is the first best. (But not so in the following model with banking crisis.)
Macroeconomic shock, $\tilde{\omega}$, hits the day market.

Banks have complete loan enforcement technology.

After loan repayments are made, $1 - \tilde{\omega}$ of bank assets are destroyed:

$$\tilde{\omega} = \begin{cases} 1 & \text{with probability } 1 - \delta, \\ \omega (< 1) & \text{with probability } \delta. \end{cases}$$ (42)

If $\tilde{\omega} = \omega$, agents expect bank insolvency.

Sellers decide to hold cash rather than deposits.

Circulation of cash stops in the first round. (Bank runs)

- Bank $\Rightarrow$ Buyers 1 $\Rightarrow$ Sellers 1 $\Rightarrow$ X
Each seller and buyer faces stochastic environment: \( \tilde{\Gamma} \) and \( \tilde{\Lambda} \).

Probability that a depositor can successfully withdraw the full amount of deposit in the day market, \( \tilde{\Gamma} \).

\[
\tilde{\Gamma} = \begin{cases} 
1 & \text{if } \tilde{\omega} = 1, \\
\Gamma (< 1) & \text{if } \tilde{\omega} = \omega,
\end{cases}
\]

(In Crisis, \( \Gamma \) is the Prob. for a buyer to be luckily in Buyers 1.)

A depositor who holds \( d_t \) units of deposits in the night market is ultimately paid \( \tilde{\Lambda}d_t \) units of cash:

\[
\tilde{\Lambda} = \begin{cases} 
1 & \text{if } \tilde{\omega} = 1, \\
\Lambda (< 1) & \text{if } \tilde{\omega} = \omega.
\end{cases}
\]
Night Market: Optimization problems are the same as Basic Model

Day Market: Seller’s Problem

- Bank runs do not affect (seriously) the Seller’s Problem.
- Sellers produce and sell $q$.
- If $\tilde{\omega} = \omega$, sellers hold cash and do not deposit their income in the banks, anticipating a lower return on deposits, i.e., $\tilde{\Lambda} = \Lambda (< 1)$. 
Day Market – Buyer’s Problem

- State of a buyer: $i = n, s, f$, which occurs with probability $\delta_i$.
  - State $n$: no bank run; $\delta_n = 1 - \delta$; $\tilde{\omega} = 1$
  - State $s$: Successful withdrawal during a bank run; $\delta_s = \delta \Gamma$; $\tilde{\omega} = \omega$.
  - State $f$: Failure to withdraw during a bank run; $\delta_f = \delta (1 - \Gamma)$; $\tilde{\omega} = \omega$.

- Buyer’s problem is

\[
V^b(m^d, d^d, l) = \sum_{i=n, s, f} \max_{q_i, m_i^n, d_i^n} \delta_i W^b(q_i, m_i^n, d_i^n, l; \Lambda_i), \quad (43)
\]

subject to budget and liquidity constraints for the respective states.
Equilibrium (1/2)

- Assume $\delta$, probability of bank insolvency, is sufficiently small.
- It is shown that $m^{db} = 0$ (Buyers do not carry cash in the day market.)
- Buyer’s problem becomes

$$
V^b(m^{db}, d^{db}, l) = \max_{q_n,q_s,q_f} E[\phi\{w_iq_i - (1 + \tilde{i}_n)p_iq_i\}] + \cdots
$$

$$
= \max_{q_n,q_s,q_f} (1 - \delta)\phi\{w_nq_n - (1 + i_n)p_nq_n\} + \delta\Gamma\phi\{w_\omega q_s - p_\omega q_s\}
$$

$$
+ \delta(1 - \Gamma)\phi\{w_\omega q_f - p_\omega q_f\} + \cdots,
$$

subject to

$$
p_n q_n^b \leq (1 + i_d)d^{db},
$$

$$
p_\omega q_s^b \leq (1 + i_d)d^{db},
$$

$$
p_\omega q_f^b \leq 0.
$$
Variables, $q_n$, $q_s$, and $\phi d^d$ are determined by

$$c' \left( \frac{1 - n}{n} q^b_n \right) q^b_n = (1 + i_d)\phi d^{db},$$

$$c' \left( \frac{1 - n}{n} \Gamma q^b_s \right) q^b_s = (1 + i_d)\phi d^{db},$$

$$\phi d^d \left( 1 - \delta (1 - \Gamma) \Lambda \frac{(1 + i_n)(1 + i_d)}{1 + i} \right) = \frac{(1 - \delta)\theta A(q^b_{n+1})^\theta + \delta \theta A(\Gamma q^b_{s+1})^\theta}{1 + i}.$$

The third eq. corresponds to $\phi d^{db} = \frac{\theta A(q^b)^\theta}{1+i}$ in the Basic Model.

$\Gamma = C/\{(1 + i_d)D\} = \rho$, if no cash injection.

$\Lambda = (1 + i)\omega L/(1 + i_n)\{(1 + i_d)D - C\} = (1 - \rho + i_n)\omega/\{(1 - \rho)(1 + i_n)\}$, if no government guarantee.
In a banking crisis, Buyers 1 can withdraw deposits, while the other buyers (Buyers $j$ for $j = 2, 3, \cdots, J$) cannot.

Only Buyers 1 can purchase the intermediate goods.

Production of the intermediate goods:

- $(1 - n)q_n^b$ in normal times
- $(1 - n)\rho q_s^b$ in the banking crisis
- It is shown that $(1 - n)\rho q_s^b < (1 - n)q_n^b$.

Production of the consumption goods:

- $Y_n = (1 - n)A(q_n^b)^\theta$ in normal times.
- $Y_\omega = (1 - n)A(\rho q_s^b)^\theta$ in the banking crisis.
- $(Y_n - Y_\omega)/Y_n = .42$, if $\theta = 1/2$, $\rho = 1/9$, and $c(q) = q^2$. 
Deflation

- Price in normal times: $\phi p_n = c' \left( \frac{1-n}{n} q^b_n \right)$
- Price in the banking crisis: $\phi p_\omega = c' \left( \frac{1-n}{n} \rho q^b_s \right)$

Since $\rho q^b_s < q^b_n$, it is shown that $p_\omega < p_n$

Price of the intermediate goods declines in the banking crisis.

Lower price does not increase the demand. (Cash is necessary to buy the goods and only Buyers 1 have cash.)
Incomplete Loan Enforcement Model – Setup

- Banks cannot enforce loan repayment on the borrowers.
- Banks need to secure loans by collateral, $k$.
- Only buyers are endowed with $k$.
- Collateral constraint is
  \[
  (1 + i)l_t^s = 0, \quad \text{for sellers}
  \]
  \[
  (1 + i)l_t^b \leq E_{t-1}[a_t k_t], \quad \text{for buyers}
  \]
- Macroeconomic sunspot shock, $\tilde{\omega}$, changes the depositors’ expectations on the other depositors’ withdrawal decision:
  \[
  \tilde{\omega} = \begin{cases} 
  1 & \text{with probability } 1 - \delta, \\
  \omega (< 1) & \text{with probability } \delta.
  \end{cases}
  \]
- If $\tilde{\omega} = \omega$, all agents believe that no sellers deposit their income in the banks.
Bank insolvency due to bank runs (1/2)

- Suppose that all agents have the expectations that all sellers never deposit their income in the banks, but hold it in the form of cash. (Bank runs)

- Agents expect that Buyers 1 can withdraw deposits and the other buyers (Buyers 2, ..., Buyers \( J \)) cannot.

- Agents expect that only Buyers 1 can buy the intermediate goods.

- Agents expect that the production of the intermediate goods decreases.
Agents expect that since the intermediate goods decrease, the marginal product of capital will decrease. \( Y = Ak^{1-\theta}q^\theta \).

Agents expect that the asset price (= MPK) will be low: 
\[ a_\omega = (1 - \theta)A(\rho q_s^b)^\theta. \]

- A bank cannot enforce loan repayment.
- When a borrower repudiates loan repayment, the bank can only seize the collateral, \( k (= 1) \), and sell it at the price of \( a \).
- If \( (1 + i)l^b > a_\omega \), the banks cannot collect the full amount of bank loans.
- Bank assets in the night market become \( (1 - n)a_\omega < (1 + i)L \).

Agents expect that the banks become insolvent once bank runs occur.
Coordination Failure

- If $\tilde{\omega} = 1$, agents expect the other agents deposit their income immediately in the banks (No bank runs)
  - Production and trading in the day market are normally done.
  - Asset price will be $a_n (> (1 + i)l^b)$.
  - Banks will be solvent.
  - Optimal decision for sellers and buyers is to hold bank deposits. (No-bank-run expectation is justified.)

- If $\tilde{\omega} = \omega$, agents expect the other agents to never deposit their income in the banks (Bank runs)
  - Production and trading in the day market are disrupted.
  - Asset price will be $a_\omega (< (1 + i)l^b)$.
  - Banks will be insolvent.
  - Optimal decision is to hold cash (Bank-run expectation is justified)
Equilibrium

- Equilibrium is calculated just like that of the Bank Insolvency Shock Model.

- Only difference is the endogeneity of $\Lambda$:
  - Bank asset in the night market becomes $(1 - n)a_\omega k$.
  - Bank liability becomes
    $$(1 + i_n)((1 + i_d)D - C) = (1 + i_n)(1 + i_d)(1 - \rho)(1 - n)d^{db}.$$  
  - The value of $\Lambda$ is determined by

$$\Lambda = \frac{(1 - n)a_\omega k}{(1 + i_n)(1 + i_d)(1 - \rho)(1 - n)d^{db}} = \frac{(1 - \theta)A\rho^\theta(q_s^b)^\theta}{(1 + i_n)(1 + i_d)(1 - \rho)\phi d^{db}}.$$
Policy Implications (1/4)

- **Monetary Policy (LLR Lending)**
  - Central Bank lends cash to the banks up to the value of the bank asset, \((1 - n)a_\omega\). Banks can increase cash reserve to \(C + (1 - n)a_\omega\), which can be withdrawn by depositors (= buyers).
  - This policy facilitates trading in the day market
  - Not sufficient to restore the normal production:
    - Cash needed: \((1 + i_d)D - C\)
    - LLR lending: \((1 - n)a_\omega\)
    - \((1 + i_d)D - C > (1 - n)a_\omega\)
  - Asset price after the LLR lending, \(a_L\), is higher than \(a_\omega\). But \((1 + i)L > (1 - n)a_L\).
  - Banks are still insolvent. Bank runs continue.
  - There is still welfare loss due to disruption in production and trading of the intermediate goods.
Policy Implications (2/4)

- Bank reforms to restore solvency of the banks
  - Government guarantee or subsidy to restore solvency of the banking system. (e.g., blanket guarantee; capital injection subsequent to stringent asset evaluations.)
  - Since banks restore solvency, all sellers deposit their income in the banks in the day market. (The rate of return on deposits is strictly greater than that on cash.)
  - Banks will not run out of cash reserves.
  - Bank runs cease and the normal production of the intermediate goods is restored.
  - Cost of the policy implementation is zero.
    - Asset price rises to \( a_n \), which satisfies \((1 - n)a_n > (1 + i)L\).
    - Banks restore the ability to collect full amount of loans. Bank asset becomes \((1 + i)L + C\).
    - Banks restore solvency without any resource injections from the government.
Fiscal Policy

Case 1: The government can maintain the intermediate goods properly.
- Government purchases the intermediate goods during a banking crisis, and sells them in the night market.
- Productions of the intermediate goods and the consumption goods are restored.
- Asset price rises to $a_n$. Banks restore solvency.
- Bank runs cease.
- Social welfare is improved and the cost of the policy implementation is zero.
- This is an optimal policy.
Policy Implications (4/4)

- Fiscal Policy

- Case 2: The government cannot maintain the intermediate goods properly.
  - Government purchases the intermediate goods during a banking crisis, while the purchased goods perish during the day market.
  - Production of the intermediate goods is restored, but not that of the consumption goods.
  - Asset price stays at $a_\omega$. Banks remain insolvent.
  - Bank runs do not cease.
  - Cost of the policy is large and must be financed by taxes.
  - Social welfare is not improved. Welfare is redistributed from buyers to sellers.

Implications for the current financial crisis

- Fiscal stimulus — The government purchase of the goods
  - Not good unless the government efficiently maintains and utilizes the purchased goods.
  - Maybe unable to stop the crisis from deteriorating further.

- Monetary easing — The central bank lending to the banks
  - Not good if LLR lending is limited to solvent banks.
  - Maybe unable to stop bank runs (or flight to quality).

- Bank reforms to restore solvency of banks — bad asset disposals and capital injections
  - Good to restore confidence in bank liabilities and restore transactions in the goods market.
  - Asset price responds positively to the policy. Banks may restore solvency without resource injections from the government.
  - Cost of policy appears to be huge *ex ante*, while it must ultimately turn out to be small *ex post*.
Miscellaneous issues

- Extension of the model: Incorporating productivity shocks and business cycles.
- Idiosyncratic shocks and individual bank runs
- Contagion of individual bank runs