Option Package Bundling

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Optional Goods and Services

- **Definition**: Valuable only if other goods or services are consumed together.

- **Examples**:
  - Leather seats for a car,
  - Caller ID for a basic phone service,
  - Soccer channels for a basic cable TV service
**Bundled Optional Goods**

- **Examples:**
  - A text editor software with an OS,
  - A microphone with a PC,
  - Short message service with a cell phone plan.

- **Bundling Strategies:**
  - Pure Bundling: Bundle only. No goods w/o options.
  - Mixed Bundling: Bundle, and goods w/o options.

- **Option Package Bundling Problem:**
  Firm’s optimization w.r.t. bundling strategies, and the associated prices.
Objectives and Scope

- **Multiproduct Monopolist’s Option Package Bundling Problem** with
  - **Two Goods**: a regular, an optional. No cost to produce.
  - Unit demand for each good. **Uniform distribution** of tastes.
  - **Parameter**: the dispersion (i.e., support size) of valuation for an optional good.
  - **Deterministic** mechanism.

- **Question**: *Which bundling scheme is optimal?*
Preview of Results

- **Pure vs. Mixed Bundling:**
  Based on cost and benefit analysis of screening, find a condition under which PB outperforms MB.

- **Comparison with Standard Bundling Model:**
  PB is never optimal.
  “Bunching” property arises in both models.
Related Literature

• Standard Bundling Problem:
  • Discrete type: Stigler (1963), Adams-Yellen (1976)

• Option Package Bundling Problem:
  • Discrete type: Pierce and Winter (1996): two types. Boils down to one-dimensional screening.
  • Continuous type: This work. Multidimensional screening.
**Model: Consumers**

- **Indivisible Goods:** 1 (regular) and 2 (optional).

- **Consumer’s Valuation:** \((v_1, v_2) \in [0,1] \times [0, \tau], \ 0 < \tau \leq 1\)

Uniform distribution with density \(1/\tau\).

Consume each good up to one unit.

**Willingness to pay:**

\[
\begin{cases}
  v_1 + v_2 : \text{if both goods are consumed} \\
  v_1 : \text{if a good 1 alone is consumed} \\
  0 : \text{otherwise.}
\end{cases}
\]
**Model: Monopolist**

Produces both goods w/o cost. Charges $p_1 \in [0,1], p_2 \in [0,\tau]$ for good 1 and 2.

- **Interpretation:** $p_1 + p_2$ is the bundle price, because no one buys good 2 alone.

**Equivalent Alternative:** charging $p_1$ for good 1 alone, and $b$ for a bundle.
Demand

\[ v_1 + v_2 = p_1 + p_2 \]

- Good 1 & 2
- No good
- Good 1 only

Diagram:
- Axes: \( \tau \) and \( v_1 \)
- Lines: \( p_2 \) and \( p_1 \)
Pure Bundling: $p_1 = 1$ or $p_2 = 0$

$$v_1 + v_2 = p_1 + p_2$$

Good 1 & 2

No good

$p_2 = 0$
Profit Maximization:

\[ p_1 < 1 \land p_2 > 0 \] is a solution
\[ \Rightarrow \] Mixed bundling is optimal.

\[ p_1 = 1 \lor p_2 = 0 \] is a solution
\[ \Rightarrow \] Pure bundling is optimal.
Preliminary Pure-Only Case

- Pure bundling price $b$.

Demand and profit functions:

$$D(b, \tau) = \begin{cases} 
(\tau - b^2/2)/\tau & \text{if } 0 \leq b \leq \tau \\
(2 + \tau - 2b)/2 & \text{if } \tau \leq b \leq 1 \\
(1 + \tau - b)^2/(2\tau) & \text{if } 1 \leq b \leq 1 + \tau, 
\end{cases}$$

$$\Pi(b, \tau) = bD(b, \tau).$$

\(\Pi\): Non-concave, but quasi-concave in $b$. 

Proposition 1: Optimal Pure Bundling

\[ \tau \geq \frac{2}{3} \]

\[ b^* = \frac{2\tau}{3} \]

\[ \tau \leq \frac{2}{3} \]

\[ b^* = \frac{\tau}{4} + \frac{1}{2} \]
Unrestricted Case

\[
\text{Max } \Pi_0(p_1, p_2, \tau)
\]

\[
= p_1 (1 - p_1) + p_2 (1 - p_1)(\tau - p_2) / \tau + (p_1 + p_2) n(p_1, p_2, \tau)
\]
Solving the Problem

Non-concavity in \((p_1, p_2)\):

- **Candidate Solutions**: FOCs, and points of boundaries or the regime change (for \(m(p_1, p_2, \tau)\)).

- **Points of boundaries and regime change**:
  - FOCs hold for optimal PB.
  - Other prices are not optimal (not hard to show but just tedious).

\(\Rightarrow\) **FOCs are necessary.**
Prices Satisfying FOCs

\[ \tau \geq \frac{2}{3} \]

\[ p_1 = \frac{2}{3}, \quad p_2 = \frac{\tau}{2} - \frac{1}{3} \]

\[ p_1 = \sqrt{\frac{2\tau}{3}}, \quad p_2 = 0 \]

\[ \tau \leq \frac{2}{3} \]

\[ p_2 = 0 \]

\[ p_1 = \frac{\tau}{4} + \frac{1}{2} \]
**SOC for Pure Bundling**

\[ p_1 \downarrow \text{by } \epsilon \text{ and } p_2 \uparrow \text{by } \epsilon. \]

Gain > Loss iff \( p_1 > 2/3 \)

\( \Rightarrow \) PB suboptimal for \( \tau > 2/3 \)

since optimal PB price is \( \sqrt{2\tau / 3} > 2/3. \)

New comer gain:

\[ (p_1 - \epsilon) \times \frac{\epsilon^2}{(2\tau)} = \frac{(p_1 - \epsilon) \times \epsilon^2}{(2\tau)} \]

Loss from separating buyers:

\[ \epsilon \times \frac{\epsilon \times (2 - 2p_1 + \epsilon)}{(2\tau)} = \epsilon \times (2 - 2p_1 + \epsilon) \frac{\epsilon}{(2\tau)} \]
Pure vs. Mixed Bundling

- **Role of Mixed Bundling:** Screening.

- **Screening benefit** from PB ↑ as PB price ↑.

- **Optimal PB price is higher for higher** $\tau$, since
  - average willingness to pay for optional goods is higher,
  - bundle price elasticity of demand is smaller.
Proposition 2: Optimal Bundling

\[ p_2 = \frac{\tau}{2} - \frac{1}{3} \]

\[ p_1 = \frac{2}{3} \]

\[ p_1 = \frac{\tau}{4} + \frac{1}{2} \]

\[ \tau \geq \frac{2}{3} \]

\[ \tau \leq \frac{2}{3} \]
Implication

- **Smaller** \( \tau \): optional good is less important, and the diversity of tastes is smaller
  \[ \Rightarrow \text{More pure bundling!} \]

- Looks consistent with the examples of option package bundling.
Two Regular Good Case

**Motivation:**
Wish to know how much the above result depends on one good being optional.

**Model:**
Adopt the same assumptions, except that both of two goods are regular.

A special case of McAfee, McMillan and Whinston (1989).
Demand

\[ v_1 + v_2 = b \]

<table>
<thead>
<tr>
<th>Region</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good 1 &amp; 2</td>
<td></td>
</tr>
<tr>
<td>Good 2 only</td>
<td></td>
</tr>
<tr>
<td>No good</td>
<td></td>
</tr>
<tr>
<td>Good 1 only</td>
<td></td>
</tr>
</tbody>
</table>

Diagram:
- Axes: \( p_1 \) and \( v_1 \)
- Axes: \( p_2 \) and \( v_2 \)
- \( v_1 + v_2 = b \) line
Pricing Strategies

- **No Bundling** (no discounting): \( p_1 + p_2 \leq b \).

- **Pure Bundling** (all buy bundles): \( \min \{ p_1, p_2 \} \geq b \).

- **Mixed Bundling** (some buy bundle, and others buy component): \( p_1 + p_2 > b \), and \( \min \{ p_1, p_2 \} < b \).

**Proposition 3**: MB dominates NB and PB. MB dominates NB (MMW), MB dominates PB by screening, using either good.
Proposition 4: Optimal Bundling Prices

\[ p_1 = \frac{2}{3} \]

\[ p_2 = \frac{2\tau}{3} \]

\[ b = \frac{2(1+\tau)}{3} - \frac{\sqrt{2\tau}}{3} \]

\[ \tau \geq 1/2 \]

\[ \tau \leq 1/2 \]
Comparison with Option Package Bundling

- **Difference:** MB is always optimal (or “semi-mixed” for lower $\tau$).

- **Similarity:** Bunching. For $\tau \leq \frac{1}{2}$, buyers for good 1 alone disappear. Same logic regarding costs and benefits of screening applies.

Bunching arises for a smaller domain of $\tau$’s. Screening buyers for good 2 induces higher bundling price, creating more profitable opportunity of screening those for good 1.
Robustness

- Other distributions? Correlations?
  Hard to obtain analytical solutions.
  The main logic seems to apply to more general cases. But careful investigation requires.

- More than two goods?
  Hard too. No attempt even in regular good cases, except for many good model relying on the Law of Large Numbers (e.g. Armstrong 1999).
Conclusion

- Monopolist's two-good option package bundling problem with the uniform distribution of buyers' valuation is studied.

- Mixed bundling is a screening device. For small $\tau$, screening is not profitable. PB outperforms MB iff $\tau \leq 2/3$. 
Conclusion

- PB can never be optimal in the comparable two regular good model.

- Both models share bunching property. For small $\tau$, screening buyers for a good alone is unprofitable.

- Shed light on why firms combine a potentially optional feature with the associated regular good.