Fertility and mortality changes in an overlapping-generations model with realistic demography

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1 Introduction

- relationship of demographic changes, capital accumulation and economic growth (Ehrlich and Lui, 1997).

- more formal approach:

  - Solow (1956): an exogenous increase in the population growth rate decreases steady-state output per worker (or output per unit of effective labor if there is technological progress)

  - capital dilution effect.

  - Ramsey-Cass-Koopmans model: this exogenous change has no effect on output per worker, since optimizing individuals respond by decreasing consumption and increasing saving
• Ramsey-Cass-Koopmans model can be used to examine the economic consequences and welfare implications of demographic changes

• but the overlapping-generations (OLG) model is, arguably, a better framework to use.

• finite life of mortal individuals

• and the interaction of different cohorts’ decisions

• However, in two most well-known OLG models (Diamond 1965; Blanchard 1985), the demographic assumptions are made in a tractable but not-so-realistic manner, in order to focus on other economic factors.

• less suited to analyze the economic consequences of demographic changes.
• Diamond (1965): uncertainty in lifetime not modelled

• Blanchard (1985): lifetime uncertainty is modelled, but the assumption of age-invariant mortality is quite different from the human experience.

• The types of mortality changes observed in industrial countries in recent years cannot be meaningfully analyzed in these models.

• more general demographic structure: OLG model with lifetime uncertainty captured by age-specific mortality rates.

• a general mortality pattern: Bommier and Lee (2003); d’Albis (2007); Lau (2009)

• d’Albis (2007) uses a general age-specific mortality schedule in an OLG model with neoclassical production function,

• and shows that the effect of a fertility increase on capital accumulation can be positive or negative (sharp contrast with the negative relation predicted in Diamond 1965; Blanchard 1985).

• Lau (2009) provides a quantitative assessment of these theoretical predictions,

• and finds that while the ambiguous effects predicted by d’Albis (2007) are possible, in practice the effects of fertility increases on capital accumulation are negative for industrial countries.
• two objectives in this paper.

• (a) First, extends the analysis of fertility changes in d’Albis (2007) and Lau (2009) to include mortality changes

• population changes can be caused by either fertility or mortality factors.

• fertility reduction in industrial countries has been very substantial in the last two centuries, and future fertility changes are likely to be less drastic.

• mortality changes in industrial countries continue to happen, especially for the old & the oldest old.

• One objective of this paper is to study the effect of mortality changes on capital accumulation.
• Interestingly, we find that a rise in fertility and a reduction in mortality, while both causing an increase in the population growth rate, lead to opposite effects on capital accumulation.

• (b) second objective: methodological (a model with more realistic demographic features)

• Many OLG models only assume one stage (working) or two stages (working and retirement) of the life cycle.

• discrete-time models: Diamond (1965); Abel (2003)

• continuous-time models: Blanchard (1985); d’Albis (2007); Lau (2009)
• Bommier and Lee (2003, p. 136), “Two age group models are not capable of representing the most basic feature of the human economic life cycle: that it begins and ends with periods of dependency, separated by a long intermediate period of consuming less than is produced.”

• Economic life cycle of a typical worker: Lee and Mason (2006)

• In studying the effects of demographic changes, an OLG model consisting of childhood, working and retirement stages is more appropriate than a two-stage model.

• particularly important for quantitative implications

• this paper contributes to the methodological aspects of OLG models with childhood, working and retirement stages in several aspects.
• (a) link mortality changes based on changes in life expectancy at birth, a concept familiar to most social scientists.

• relationship between mortality changes and changes in life expectancy is not one-to-one in general

• use the method based on Lee and Carter (1992) to achieve a one-to-one correspondence between changes in life expectancy and empirical changes in mortality pattern.

• (b) instead of relating fertility changes to changes in the crude birth rate, which is commonly used in the literature, we relate fertility changes to changes in the total fertility rate (TFR)

• TFR: average number of children who would be born per woman during her reproductive years, if she survives to the end of her reproductive years.
• crude birth rate: confounds behavioral and age-structure effects,

• TFR: not affected by the age composition of the population.

• (c) equilibrium of the two-stage OLG model (d’Albis 2007; Lau 2009) arises as a special case of the equilibrium of the three-stage OLG model

• organization

• Section 2: an OLG model with realistic demography

• demographic aspects: relatively complicated; economic aspects: standard
• Section 3: steady-state equilibrium, existence and uniqueness.

• Sections 4 & 5: study quantitatively the economic consequences of fertility & mortality changes

• Section 6: decomposition (capital dilution effect; aggregate saving effect); intuitions

• Section 7 concludes.

2 The model

• model modified from Blanchard (1985) & Yaari (1965).
• Blanchard (1985): time-invariant risk to mortality; constant population size

• assumption of constant population size can easily be relaxed

• Weil (1989) assumes no death rate and a positive birth rate in the Blanchard (1985) model

• Buiter (1988) assumes that the birth and death rates are non-negative and may be different

• when the assumption of age-invariant mortality rates is relaxed, analytical aggregation formula is in general not available

• but it is possible to obtain useful relationships among aggregate economic variables in the steady-state equilibrium, using the idea of the stable population theory (Lotka, 1939)
• Bommier and Lee, 2003; d’Albis, 2007; Lau, 2009

• use the actual life table data.

• the survival function \( l(x) \): probability that an individual survives to at least age \( x \), \( x \in [0, \Omega] \), \( \Omega \) is maximum possible age of an individual

• \( l(0) = 1; l(\Omega) = 0 \)

• Figure 1: survival function of USA (men and women combined) in 2005

• instantaneous mortality rate at age \( x \), \( \mu(x) \):

\[
\mu(x) = -\frac{1}{l(x)} \frac{dl(x)}{dx}.
\]  

\hspace{1cm} (1)

• Figure 2: mortality rates (in natural log)
• hugely inconsistent with the age-invariant mortality assumption.

• Economic model with demographic features:

• In general, at least three types of activities—consumption over time, fertility, and retirement—should be examined in studying the economic consequences of demographic changes.

• easily become intractable.

• Several features chosen to balance the tractability consideration and a reasonably accurate description of the economic and demographic characteristics of industrial countries.
• (a) the average growth rate of output per worker is roughly constant over a long period of time in industrial countries (Jones, 1995), despite a substantial change in demographic variables.

• consider a neoclassical production function with constant rate of exogenous technological progress.

• (b) average retirement age in industrial countries remains roughly unchanged (or even experiences a mild drop) in the last fifty years, despite the significant improvement in longevity.

• simply take retirement age to be exogenously fixed.

• (c) assume that aggregate output is endogenous, but fertility and mortality changes are exogenous (following Boucekkine et al., 2002; d’Albis, 2007)
• may not be appropriate for countries in the early stage of demographic transition,

• more reasonable for industrial countries (at a more advanced stage of demographic transition)

• Figure 3: time line of a cohort’s individual’s various activities (up to maximum possible age \( \Omega \))

• childhood stage: from the time of birth (at time \( s \)) to age \( T_w \) (at time \( s + T_w \)), an individual does not have to make economic decisions.

• levels of consumption during these years are determined by her parent.

• working stage:
• (i) works from age $T_w$ until retirement at age $T_r$.

• (ii) makes consumption decisions from age $T_w$ until the end of her life

• (iii) parental role: gives rise to (possibly fractional) $\beta$ babies at age $T_b$ if she survives to that age, and she is responsible for the children’s consumption from birth to age $T_w$ as well.

• assume $\Omega = 110$, $T_r = 65$, $T_b = 28$ and $T_w = 20$ (stylized facts of developed countries)

• (a) Population and labor supply:

• Using the commonly-used assumption that the male to female ratio at birth is 1.05,

$$\beta = \frac{TFR}{2.05},$$

(2)
• $B(t)$ is the number of births at time $t$:

$$B(t) = B(t - T_b) l(T_b) \beta,$$  

(3)

• Since age-specific mortality & fertility rates are time-invariant in this economy,

$$n = \frac{1}{T_b} \log \left[ \frac{l(T_b) \text{TFR}}{2.05} \right],$$  

(4)

where $n$ is the constant growth rate of the number of births (such that $B(t) = B(0) e^{nt}$)

• population size: aggregation formula

$$P(t) = \int_{t-\Omega}^{t} B(s) l(t - s) \, ds$$

$$= B(t) \int_{0}^{\Omega} e^{-nx} l(x) \, dx.$$  

(5)
• Labor supply at any age is assumed to be exogenous and either zero or full time (which is normalized to 1)

\[ N(s, s + x) = \begin{cases} 
1 & \text{if } T_w \leq x \leq T_r \\
0 & \text{otherwise} 
\end{cases}, \quad (6) \]

where \( N(s, v) \) is the labor supply of a cohort \( s \) individual at time \( v \).

• aggregate labor supply \( N(t) \) is given by

\[ N(t) = \int_{t-\Omega}^{t} B(s) l(t - s) N(s, t) ds \\
= B(t) \int_{T_w}^{T_r} e^{-nx} l(x) dx, \quad (7) \]

• (b) Individual intertemporal consumption decisions:

• standard assumption (as in Blanchard 1985): an individual begins her adult life without financial
assets or liabilities, and she cannot leave any debt at her death.

\[ Z(s, s + T_w) = 0; \quad Z(s, s + \Omega) \geq 0, \quad (8) \]

where \( Z(s, v) \) is the financial wealth of a cohort \( s \) individual at time \( v \).

- no bequest motive.

- An individual born at time \( s \) chooses \( \{C(s, v)\}_{t}^{s+\Omega} \), at time \( t \) (where \( s + T_w \leq t \leq s + \Omega \)), to maximize

\[
\int_{t}^{s+\Omega} e^{-\rho(v-t)} \frac{l(v-s)}{l(t-s)} \lambda(v-s) \left[ \frac{C(s, v)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \right] dv
\]

subject to the flow budget constraint

\[
\frac{\partial Z(s, v)}{\partial v} = \left[ r(v) + \mu(v - s) \right] Z(s, v)
\]

\[ + w(v) N(s, v) - C(s, v) \lambda(v - s), \quad (10) \]

and the terminal conditions in (8),
• $\rho =$ discount rate,

• $\sigma =$ intertemporal elasticity of substitution,

• $C(s, v)$ is consumption of a cohort $s$ individual at time $v$,

• $r(v) =$ (real) interest rate at $v$,

• $w(v) =$ (real) wage rate at $v$,

• $\lambda(x) =$ equivalent adult consumers in the household (when the parent is of age $x$):

$$\lambda(x) = \begin{cases} 
1 & \text{if } T_w \leq x \leq T_b \\
1 + \theta \beta \frac{l(x - T_b)l(T_b)}{l(x)} & \text{if } T_b \leq x \leq T_b + T_w \\
1 & \text{if } T_b + T_w \leq x \leq \Omega 
\end{cases}$$

(11)
• no social security system in this economy.

• interpretation of (9) to (11)

• budget constraint (10) assumes the presence of an actuarially-fair annuity (Yaari, 1965)

• even though there is uncertainty in lifetime at the individual level, the uncertainty has no effect at the aggregate level.

• preference:

• objective function in (9): weighted by conditional probability of survival at different ages, \( \frac{l(v-s)}{l(t-s)} \).

• instantaneous utility function: constant-intertemporal-elasticity-of-substitution (CIES) form
• $\lambda(x)$ captures the different consumption needs of the parent over the life cycle (Cutler et al., 1990).

• do not emphasize the consumption needs (such as health expenses) of the elderly

• emphasize that the consumption needs of children are less than those of adults (i.e., $\theta < 1$).

• $\frac{l(x-T_b)}{l(x)/l(T_b)}$ in (11): to ensure the accounting consistency of the economy (an extension of the Yaari’s idea)

• the ratio of the conditional probability of survival of a child from birth to age $x - T_b$ to the conditional probability of survival of her parent from age $T_b$ to $x$
• along the optimal consumption path,

$$\frac{\partial C(s,v)}{\partial v} = \sigma \left[ r(v) - \rho \right] C(s,v).$$  \hspace{1cm} (12)

• close the model:

• assuming the standard neoclassical production function with exogenous labor-augmenting technological progress in a closed economy

$$Y(t) = F(K(t), A(t)N(t)),$$  \hspace{1cm} (13)

where $Y(t)$, $K(t)$, $N(t)$ and $A(t)$ represent, respectively, output, capital input, labor input and technological level at time $t$.

• Technological progress is represented by

$$A(t) = A(0)e^{gt},$$  \hspace{1cm} (14)

where $g$ is the rate of technological progress.
• Capital accumulates according to

\[
\frac{dK(t)}{dt} = Y(t) - C(t) - \delta K(t),
\]

where \(\delta\) is the depreciation rate.

3 Steady-state equilibrium

• a variable per unit of effective labor = the variable divided by \(AN\) (denoted in lower case letter; e.g., \(y(t) = \frac{Y(t)}{A(t)N(t)}\)).

• production function in intensive form:

\[
y(t) = F\left(\frac{K(t)}{A(t)N(t)}, 1\right) \equiv f(k(t)),
\]

where \(f'(k) > 0, f''(k) < 0, \lim_{k \to 0} f'(k) = \infty\) and \(\lim_{k \to \infty} f'(k) = 0\).
• Moreover, (7), (14) (15) and (16) lead to
\[
\frac{dk(t)}{dt} = f(k(t)) - c(t) - (\delta + g + n)k(t). \tag{17}
\]

• steady-state (balanced-growth) equilibrium

• Denote a variable at the steady-state equilibrium with a *

• steady-state equilibrium is defined by \( \frac{dk(t)}{dt} = 0 \):
\[
f(k^*) - c^* - (\delta + g + n)k^* = 0. \tag{17a}
\]

• competitively-determined interest rate and wage rate at the steady-state equilibrium (SSE)
\[
r^*(t) = f'(k^*) - \delta \equiv r^*, \tag{18}
\]
\[
w^*(t) = A(t) \left[ f(k^*) - k^* f'(k^*) \right] \equiv A(t) w^*. \tag{19}
\]
\( c^* \) (consumption per unit of effective labor) in terms of \( k^* \) (capital per unit of effective labor):

\[
c^* = \frac{C^* (t)}{A(t) N(t)}
\]

\[
= w^* \left( \frac{\int_{T_w}^{T_r} e^{-(r^*-g)x} l(x) dx}{\int_{T_w}^{T_r} e^{-(r^*-g^*)x} l(x) \lambda (x) dx} \right) \left( \frac{\int_{T_w}^{\Omega} e^{-(g+n-g^*)x} l(x) \lambda (x) dx}{\int_{T_w}^{T_r} e^{-nx} l(x) dx} \right), \tag{20}
\]

where

\[
g^*_c = \sigma (r^* - \rho) \tag{21}
\]

is the steady-state growth rate of an individual cohort’s consumption.

- longitudinal constraints (from individual cohort’s intertemporal summation) in (20): the two integrals that involve \( r^* \)
• cross-sectional constraints (corresponding to summation across different cohorts at a particular time) in (20): the two integrals that involve $n$

• the equation that determines $k^*$ at the SSE:

$$f(k^*) - w^* \left( \frac{\int_{T_w}^{Tr} e^{-(r^*-g)x} l(x) \, dx}{\int_{T_w}^{\Omega} e^{-(r^*-g^*)x} l(x) \lambda(x) \, dx} \right)$$

$$\left( \frac{\int_{T_w}^{\Omega} e^{-(g+n-g^*)x} l(x) \lambda(x) \, dx}{\int_{T_w}^{Tr} e^{-nx} l(x) \, dx} \right) = (\delta + g + n) k^*. \quad (22)$$

• $r^*, w^*$ and $g^*_c$ depend on $k^*$ only

• interpretation of (22): similar to the neoclassical growth model (Solow, 1956; Cass, 1965):

• LHS = actual investment per unit of effective labor; RHS = break-even investment per unit of effective labor
• Once $k^*$ is determined, other variables (such as $y^*$ and $c^*$) at the SSE can be obtained.

• Proposition 1 gives the sufficient conditions for the existence and uniqueness of the SSE

Proposition 3.1 For the OLG model with CIES utility function, age-specific mortality rates, childhood and retirement years, and assumption $\delta + g + n > 0$, the steady-state equilibrium (with $k^* > 0$) exists and is unique when $\sigma \geq 1$, if

$$f(k) - kf'(k) + k^2 f''(k) \geq 0 \quad (23)$$

for all $k \in (0, \infty)$, and

$$\lim_{k \to 0} [-kf''(k)] > 0. \quad (24)$$

When $\sigma < 1$, the steady-state equilibrium (with $k^* > 0$) exists and is unique if (23) and (24) are satisfied, and

$$\sigma h(-\sigma u + g + n + \sigma \rho)$$
\[
+ (1 - \sigma) h ((1 - \sigma) u + \sigma \rho) - h_r (u - g) \\
- [h ((1 - \sigma) (g + n) + \sigma \rho) - h_r (n)] \leq 0 \text{ for all } u \in (-\delta, g + n) \text{ but } \geq 0 \text{ for all } u \in (g + n, \infty),
\]

\[
h(u) = \frac{\int_{T_w}^{\Omega} xe^{-ux}l(x)\lambda(x)dx}{\int_{T_w}^{\Omega} e^{-ux}l(x)\lambda(x)dx} \quad \& \quad h_r (u) = \frac{\int_{T_w}^{Tr} xe^{-ux}l(x)dx}{\int_{T_w}^{Tr} e^{-ux}l(x)dx}.
\]

- A special case: No childhood stage

- agree with Bommier and Lee (2003) that in general it is better to study the economic consequences of demographic changes in an OLG model with childhood, working and retirement stages.

- However, for some questions (such as how mortality changes affect the retirement age), the childhood stage may be less important, \& simpler to work with a model starting with the adult stage.
• our general result is related to the OLG model without the childhood stage in the literature.

• Specifically, the equilibrium of the OLG model used in d’Albis (2007) and Lau (2009) is a special case of the equilibrium of the model used in this paper when \( \theta = 0 \) (i.e., when the children’s consumption need is insignificant and goes to zero).

• When \( \theta = 0 \), for all \( T_w \leq x \leq \Omega \),

\[
\lambda (x) = 1 \quad (26)
\]

• adult age:

\[
\tilde{x} = x - T_w, \quad (27)
\]

• survival probability based on adult age:

\[
\tilde{l}(\tilde{x}) = \frac{l(\tilde{x} + T_w)}{l(T_w)}. \quad (28)
\]
• (20) is equivalent to:

\[ c^* = w^* \left( \frac{\int_0^{T_r-T_w} e^{-(r^*-g)(\tilde{x}+T_w)} l(T_w) \tilde{l}(\tilde{x}) d\tilde{x}}{\int_0^{\Omega-T_w} e^{-(r^*-g^*)(\tilde{x}+T_w)} l(T_w) \tilde{l}(\tilde{x}) d\tilde{x}} \right) \]

\[ = w^* \left( \frac{\int_0^{\tilde{T}_r} e^{-(r^*-g)\tilde{x}} \tilde{l}(\tilde{x}) d\tilde{x}}{\int_0^{\tilde{\Omega}} e^{-(r^*-g^*)\tilde{x}} \tilde{l}(\tilde{x}) d\tilde{x}} \right) \]

\[ \left( \frac{\int_0^{\tilde{\Omega}} e^{-(g+n-g^*)\tilde{x}} \tilde{l}(\tilde{x}) d\tilde{x}}{\int_0^{\tilde{T}_r} e^{-n\tilde{x}} \tilde{l}(\tilde{x}) d\tilde{x}} \right), \]  

where

\[ \tilde{T}_r = T_r - T_w, \]  

(27a)

\[ \tilde{\Omega} = \Omega - T_w. \]  

(27b)
• Note that (29) is essentially the same as (17) in d’Albis (2007) and (21) in Lau (2009)

• Even in the case with $\theta = 0$, it may be argued that our approach provides a slight advantage over the model in d’Albis (2007) and Lau (2009):

• while the population growth rate in d’Albis (2007) and Lau (2009) is related to the crude birth rate,

• we can express the population growth rate in terms of TFR

4 Fertility change and capital accumulation

• A major advantage:
• can distinguish between two major types of population changes: fertility and mortality changes.

• further assume a Cobb-Douglas production function in the quantitative exercises:

\[ y(t) = k(t)^\alpha, \quad (16a) \]

where \( 0 < \alpha < 1. \)

• (16a) satisfies conditions (23) and (24).

• have checked computationally that (25) is satisfied for all the experiments performed in this paper.

• \( \alpha = 0.3, \ \delta = 0.05, \ g = 0.02, \ \sigma = 0.5 \) and \( \rho = 0.02 \) (Barro et al., 1995)
• also, $\theta = 0.5$, $T_w = 20$, $T_b = 28$, $T_r = 65$, $\Omega = 110$

• population growth rate $n$ varies as a result of the fertility and mortality changes.

• benchmark case: use the fertility and mortality data of year 2005 in USA (Centers for Disease Control and Prevention, 2007; HMD, 2008).

• then perform experiments by varying fertility and mortality parameters around their 2005 values.

• USA: TFR increases from 1946 to 1964 (the baby-boom years) and then decreases in the next decade (the baby-bust years).

• After reaching a low TFR of 1.74 in 1976, it rebounded and fluctuated between 1.7 to 2.1 for the past three decades.
• benchmark TFR = 2.05 (in 2005); and consider the economic effects of fertility changes within the range of 1.7 to 2.1.

• Selected calculations of the demographic and economic variables are given in Table 1.

• behavior of the demographic variables are expected.

• When TFR increases, the proportion of children increases and the proportion of elderly decreases.

• support ratio = ratio of working-age people to equivalent adult consumers, which measures economic dependency

• combining these 2 effects, support ratio increases when there is an increase in TFR (Table 1)
• economic effects of an increase in TFR: aggregate saving rate increases

• Overall effect: when TFR increases, the steady-state capital per unit of effective labor decreases: Figure 4 (with $k^*$ at the benchmark case normalized to be 100)

• negative relationship between fertility increase and capital accumulation in the current model is similar to that in the OLG model without the childhood stage (Lau, 2009).

5 Mortality change and capital accumulation

• effects of mortality changes in industrial countries.
• many possible mortality change patterns:

• longevity improvement mainly occurs in infants and children during the early stage of demographic transition, but the improvement mainly falls on older ages during the maturing stage.

• focus on industrial countries, and use the survival function of USA in 2005 (see Figure 1) as the benchmark case.

• next step: to relate mortality changes to changes in life expectancy

• want to restrict our analysis to the mortality changes in which there is a one-to-one correspondence between change in mortality pattern and change in life expectancy.
• this one-to-one correspondence can be conveniently modelled by constant additive change or constant multiplicative change models.

• Unfortunately, neither form of these age-neutral mortality changes is empirically very accurate.

• a semi-parametric method: based on an interpolation in the spirit of the widely-applied Lee-Carter (1992) method in population forecasting.

• focuses on the logarithm of age-specific mortality rates, and assumes that

\[
\log [\mu(x, i)] = a(x) + ib(x), \quad (30)
\]

where \(a(x)\) is the natural log of age-specific mortality rate at age \(x\) at the most recent year (taken to be 2005 here),
• $b(x)$ is the difference of log age-specific mortality rate at age $x$ at the most recent year and that at an initial year (taken to be 1980 here),

• and $i$ is an index of the mortality level.

• The above method makes use of the life table information at two points of time (Figure 2)

• and assume that further changes in mortality can be well-approximated by a linear combination of log age-specific mortality rates at these two points of time.

• life expectancy at birth in USA increases monotonically during the post-war years, and it is expected to increase further in the coming years.
• the speed of improvement and whether there is a limit on life expectancy improvement is subject to debate (Oeppen and Vaupel, 2002).

• consider the possibility of longevity improvement up to age 85, and start the life expectancy at age 75 to allow for a possible longevity deterioration from its 2005 level (for the sake of completeness).

• different values of $i$ correspond to different values of the mortality schedule $\mu(x, i)$ in (30), & life expectancy at a particular value of $i$ is given by

$$LE(i) = \int_{0}^{\Omega} e^{-\int_{0}^{q} \mu(x,i)dx} dq.$$  (31)

• We start with an arbitrary value of life expectancy between 75 and 85, and then search for the value of index $i$ according to (30) and (31).
• then obtain the synthetic survival schedule and use it in the OLG model.

• Table 2: effects of a longevity improvement on demographic and economic variables

• When life expectancy rises, proportion of children decreases & proportion of elderly increases

• combined effect leads to a fall in the support ratio

• Moreover, aggregate saving rate increases.

• In contrast to an increase in TFR, the steady-state capital per unit of effective labor increases when life expectancy increases: Figure 5 (with $k^*$ at the benchmark case normalized to be 100
6 Economic impact of a demographic change: A decomposition

- we find that an increase in TFR and in life expectancy, while both contributing to an increase in the population growth rate, have opposite effects on capital accumulation.

- explanation:

- Guided by the interpretation of the equilibrium condition (22), find it useful to decompose the economic consequences of demographic changes into capital dilution & aggregate saving effects,

- and apply the graphical representation used in the Solow model to organize the above results
• benchmark case with TFR of 2.05 and life expectancy of 77.9 years (corresponding to the column in bold in either Table 1 or Table 2).

• LHS of (22) is represented by the concave curve $s^*f(k)$ in Figure 6 (equilibrium aggregate saving rate $s^* = 20.58\%$ for the benchmark case)

• RHS of (22) is represented by the straight line $(\delta + g + n)k$, where $n = -0.074\%$.

• The intersection of the two curves gives the SSE (denoted by point $E$ in Figure 6)

• consider a fertility change & a mortality change such that the resulting population growth rate is the same:
• a change of TFR from 2.05 to 2.064 (with life expectancy unchanged at 77.9) and a change of life expectancy from 77.9 to 81 (with TFR unchanged at 2.05).

• The first change is reported in the column in italics in Table 1, and the second change in the column in italics in Table 2

• In each case, the population growth rate increases from $n = -0.074\%$ to $n' = -0.050\%$ (an anti-clockwise rotation of the break-even investment line to $(\delta + g + n') k$ in Figure 6)

• the rise in population growth leads to a smaller capital per unit of effective labor (at the new equilibrium $E_s$) if there is no change in the aggregate saving rate: the capital dilution effect (Solow model)
• However, in general, aggregate saving does not remain unchanged with a demographic change.

• In terms of the OLG model used in this paper, both individual saving behavior and the age structure are affected, leading to the aggregate saving effect.

• Tables 1 and 2: aggregate saving rate increases slightly from 20.58% to 20.61% for the fertility change, but it rises more substantially from 20.58% to 21.58% for the mortality change.

• fertility change: the rather small increase in aggregate saving leads to the result that the steady-state capital per unit of effective labor decreases from $k^*$ to $k^*_F$ (at the new equilibrium $E_F$).

• capital dilution effect dominates aggregate saving effect
• mortality change: the much larger increase in aggregate saving leads to the result that the steady-state capital per unit of effective labor increases from $k^*$ to $k^*_M$ (at the new equilibrium $E_M$).

• aggregate saving effect dominates capital dilution effect

• Why does the aggregate saving rate increase more for the mortality change than for the fertility change in Figure 6?

• useful to understand these general equilibrium effects (with $r^*$, $w^*$ etc. changing in response to the exogenous shocks) by looking at the life-cycle saving behavior and demographic characteristics.

• (a) fertility increase (i.e., a rise in TFR): the direct effect is a decrease in individual saving (since each adult has more children and thus the household consumption expenditure increases).
• A further look at the demographic structure suggests that the support ratio increases.

• The net result of these opposing effects due to individual saving and age structure turns out to be a small increase in aggregate saving.

• (b) mortality reduction in industrial countries occurs mainly on older ages, instead of the young and child-bearing ages.

• As a result, the support ratio decreases.

• Individual saving increases in response to a higher proportion of the lifetime expected to be in the post-retirement years.

• Quantitatively, the increase in aggregate saving rate is very large,
• meaning that the individual saving effect is substantially higher than the decrease through the age-composition effect.

7 Conclusion

• (a) Objectives:

  • examine economic consequences of fertility and mortality changes in industrial countries

  • incorporate more realistic demographic features of age-specific mortality rates, childhood and retirement years into a standard OLG model.

• (b) Results:
• a fertility increase affects capital accumulation negatively, & a longevity increase affects it positively

• decompose the economic effect of a fertility or mortality change into capital dilution and aggregate saving effects (which is further decomposed into individual saving & age-structure effects)

• (c) Possible extensions:

• (i) One crucial ingredient of the current OLG model which leads to the above results is the exogenously fixed retirement age.

• While an unchanged retirement age seems to be a reasonable assumption for industrial countries in the last fifty years or so, it is quite puzzling why the retirement age remains relatively unchanged when there is a huge improvement in longevity.
• what factors affect the retirement decision?

• does the magnitude of the aggregate saving change in response to mortality reduction is substantially affected when retirement decision is endogenous?

• Gruber and Wise (1998): labor force participation at old age is strongly affected by the provisions of the publicly-funded pension scheme.

• interesting to study the effects of longevity improvement on individual retirement decision and aggregate saving under different institutional arrangements of the social security program.

• (ii) Another extension is to consider developing countries which are in an early stage of demographic transition.
• relax the assumption of exogenous fertility.

• One possibility is to consider fertility changes in response to mortality reduction (as in Kalemli-Ozcan, 2003; Soares, 2005) when they fall mainly on children and youth in developing countries.

• Endogenizing fertility choice not only may provide methodological contribution to the research on economic-demographic nexus,

• but also lead to a better understanding of many developing countries with relatively high but declining fertility rates.
tion or consumption, we find the effective numbers of producers and consumers; the ratio of producers to consumers is the “support ratio." During the dividend phase, the support ratio rises. A 1 percent increase in this support ratio allows consumption at each age to rise by 1 percent with no increase in the share of GDP consumed.

Among developing countries, the support ratio began to increase first in four regions of the world: East and Southeast Asia, Latin America, the Middle East and North Africa, and the Pacific Islands. The effect was to raise the annual rate of growth of output per effective consumer by about 0.5 to 0.6 percentage points annually between 1970 and 2000 (see table, first column). The first dividend phase did not begin in South Asia until the mid-1980s and in sub-Saharan Africa until 2000. Indeed, the first dividend was negative between 1970 and 2000 in sub-Saharan Africa because the survival of more children led to a decline in the support ratio.

The second dividend—increased capital accumulation—is larger (second column) than the first dividend, and the combined effects of the two (third column) range as high as 1.9 percent a year in East and Southeast Asia. In that region, the demographic dividends were equal to 44 percent of the actual growth in output per effective consumer (fourth column). Demographics can account for a major part of the rapid growth in East and Southeast Asia. Some regions, however, did not successfully exploit their demographic dividends. In Latin America, the dividends could have contributed economic growth of 1.7 percent a year, but actual growth fell well short of that opportunity.

Managing the dividends wisely

How much of the first dividend is realized during this demographic window of opportunity hinges on key features of the economic life cycle. The productivity of young adults depends on schooling decisions, employment practices, the timing and level of childbearing, and policies that make it easier for young parents to work. Productivity at older ages depends on health and disability, tax incentives and disincentives, and, particularly, the structure of pension programs and retirement policies. On the consumption side, some countries, like those in East Asia, place a high value on education expenditures for children. Other countries, like the United States, devote a large share of resources to health care for the elderly.

How much of the second dividend is realized depends on how a society supports its elderly. In the developing world, the elderly are supported by their families and the public sector, but, in addition, they depend on assets they have accumulated during their working years—housing, funded pensions, and personal savings, among other things. As populations age, the support burden placed on families and governments will increase relative to GDP, a matter of great concern in many countries. But through the second dividend, increased numbers of middle-aged workers may substantially raise capital relative to GDP if policies encourage workers to save for their retirement.

To the extent that countries meet the challenge of aging by expanding unfunded familial or public transfer programs, asset growth will be reduced, and the second dividend will be diminished. By contrast, if workers are encouraged to save and accumulate pension funds, population aging can boost capital per worker, productivity growth, and per capita income. Thus, policymakers, especially in developing countries, will need to focus on establishing financial systems that are sound, trusted, and accessible to the millions who wish to secure their financial futures. The time to do so is now so that, as a population ages, its growth-inducing potential will be realized.

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How big are the dividends?

The second has typically been even larger than the first.

<table>
<thead>
<tr>
<th>Demographic Dividends: contribution to growth in GDP/(N)²</th>
<th>Actual growth in GDP/(N)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>Second</td>
</tr>
<tr>
<td>Industrial economies</td>
<td>0.34</td>
</tr>
<tr>
<td>East and Southeast Asia</td>
<td>0.59</td>
</tr>
<tr>
<td>South Asia</td>
<td>0.10</td>
</tr>
<tr>
<td>Latin America</td>
<td>0.62</td>
</tr>
<tr>
<td>Sub-Saharan Africa</td>
<td>-0.09</td>
</tr>
<tr>
<td>Middle East and North Africa</td>
<td>0.51</td>
</tr>
<tr>
<td>Transition economies²</td>
<td>0.24</td>
</tr>
<tr>
<td>Pacific Islands</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Source: Andrew Mason, 2005, “Demographic Transition and Demographic Dividends in Developed and Developing Countries,” United Nations Expert Group Meeting on Social and Economic Implications of Changing Population Age Structures (Mexico City).

²Actual growth in GDP per effective consumer (GDP/N), 1970–2000, in percent a year. The effective number of consumers is the number of consumers weighted for age variation in consumption needs.

³Abkhazia, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Georgia, Hungary, Kazakhstan, Kyrgyz Republic, Latvia, Lithuania, FYR Macedonia, Moldova, Mongolia, Poland, Romania, Russian Federation, Serbia and Montenegro, Slovakia, Slovenia, Tajikistan, Turkmenistan, Ukraine, and Uzbekistan.

Credit: Ron Lee, Andrew Mason, and Amol Ahire

Economic life cycle of a typical Thai worker

Each individual has only 33 years to build the dividend.

(per capita labor income and consumption per year; 1998, thousand baht)

Figure 1: Percent Surviving (United States, 2005)

Figure 2: Age-Specific Mortality Rates (United States, 1980 and 2005)
Figure 3: Time Line of Various Activities

Life Time (Probably up to $\Omega$)  ←→  Consumption  ←→  Labor Supply
Table 1: Fertility Changes and Capital Accumulation

<table>
<thead>
<tr>
<th>Total Fertility Rate</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2</th>
<th>2.05</th>
<th>2.064</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ (%)</td>
<td>-0.743</td>
<td>-0.539</td>
<td>-0.346</td>
<td>-0.163</td>
<td><strong>0.074</strong></td>
<td>-0.050</td>
<td>0.012</td>
</tr>
<tr>
<td>Persons Aged 0-19 (%)</td>
<td>19.77</td>
<td>21.22</td>
<td>22.65</td>
<td>24.04</td>
<td><strong>24.72</strong></td>
<td>24.91</td>
<td>25.39</td>
</tr>
<tr>
<td>Support Ratio (%)</td>
<td>59.87</td>
<td>60.60</td>
<td>61.21</td>
<td>61.72</td>
<td><strong>61.93</strong></td>
<td>61.99</td>
<td>62.13</td>
</tr>
<tr>
<td>Aggregate Saving Rate (%)</td>
<td>19.63</td>
<td>19.93</td>
<td>20.21</td>
<td>20.46</td>
<td><strong>20.58</strong></td>
<td>20.61</td>
<td>20.70</td>
</tr>
<tr>
<td>$k^*$</td>
<td>5.123</td>
<td>4.999</td>
<td>4.888</td>
<td>4.786</td>
<td><strong>4.739</strong></td>
<td>4.727</td>
<td>4.694</td>
</tr>
</tbody>
</table>

Note: Support ratio = working-age population divided by equivalent adult consumers. The underlined cells (resp. cells not underlined) mean that the corresponding variable is decreasing (resp. increasing) in TFR.

Table 2: Mortality Changes and Capital Accumulation

<table>
<thead>
<tr>
<th>Life Expectancy</th>
<th>75</th>
<th>77</th>
<th><strong>77.9</strong></th>
<th>79</th>
<th>81</th>
<th>83</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ (%)</td>
<td>-0.107</td>
<td>-0.083</td>
<td><strong>0.074</strong></td>
<td>-0.065</td>
<td>-0.050</td>
<td>-0.038</td>
<td>-0.029</td>
</tr>
<tr>
<td>Persons Aged 0-19 (%)</td>
<td>25.30</td>
<td>24.90</td>
<td><strong>24.72</strong></td>
<td>24.48</td>
<td>24.03</td>
<td>23.57</td>
<td>23.11</td>
</tr>
<tr>
<td>Support Ratio (%)</td>
<td>63.62</td>
<td>62.46</td>
<td><strong>61.93</strong></td>
<td>61.27</td>
<td>60.07</td>
<td>58.87</td>
<td>57.67</td>
</tr>
<tr>
<td>Aggregate Saving Rate (%)</td>
<td>19.72</td>
<td>20.31</td>
<td><strong>20.58</strong></td>
<td>20.93</td>
<td>21.58</td>
<td>22.25</td>
<td>22.95</td>
</tr>
<tr>
<td>$k^*$</td>
<td>4.488</td>
<td>4.659</td>
<td><strong>4.739</strong></td>
<td>4.844</td>
<td>5.044</td>
<td>5.259</td>
<td>5.487</td>
</tr>
</tbody>
</table>

Note: Support ratio = working-age population divided by equivalent adult consumers. The underlined cells (resp. cells not underlined) mean that the corresponding variable is decreasing (resp. increasing) in life expectancy.
Figure 4: Fertility Change and Capital Accumulation

Total fertility rate

Figure 5: Life Expectancy and Capital Accumulation

Life expectancy at birth
Figure 6: Capital Dilution and Aggregate Saving Effects