On the strategic use of quality scores in keyword auctions

Full extraction of advertisers’ surplus

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1 Introduction

• Keyword auctions are widely used for selling advertising slots on the Internet.

• In keyword auctions, advertisers bid for placement on a web page and are charged when users click on their ads.

• Keyword auctions use the PPC (pay-per-click) scheme, as opposed to the PPM or PPA scheme.

• Keyword auctions are the main source of revenue for Google, Yahoo!, and others.

• Keyword auctions are alternatively called sponsored search auctions, Internet ad-words auctions, or position auctions.

• In these auctions, several advertising slots or positions are simultaneously auctioned off using a payment scheme apparently similar to the second price auction.
• In particular, each advertiser or bidder pays not his/her own bid but the minimum price that would retain the current position.

• Edelman et al. (AER 2007) call the auction rule in practice as the generalized second-price (GSP) auction.

• One of the salient features of this auction is the use of quality scores: Advertisers are not ordered by their bid amounts but by the adjusted bids multiplied by the quality scores.

• Google initially used the click-through rates to determine the quality scores. It later switched to a less transparent system that incorporates such factors as the relevance of the keywords to its ad group, the landing page quality, the advertisers’ historical performance, and other relevant factors.

• Yahoo! initially used only the bids to determine the order, but began to use a ranking system similar to Google’s in 2007.
• Quality scores are designed to ensure that the most relevant ads are shown on the advertising slots. This will generate as many actual clicks as possible, and may help the advertisers and the users as well as Google and Yahoo!

• Quality scores can, in fact, do much a stronger job: This paper shows that, by strategically choosing the quality scores, it is possible to extract all the advertisers’ surplus.
2 The Model

- $K$ positions (advertising slots) and $I$ bidders (advertisers) with $K \leq I$.
- $c^k$: the (expected) number of views that an advertisement in position $k = 1, \ldots, K$ effectively receives.
- Without loss of generality, order the positions so that $c^1 > \cdots > c^K > 0$.
- Each bidder $i$ is characterized by two parameters:
  - $r_i$: the positive click-through rate (CTR for short) or the rate of clicks when viewed.
  - $v_i$: the positive valuation per click.
- When bidder $i$ is assigned to position $k$, the (expected) number of clicks is $c^k r_i$ and the total payoff that bidder $i$ obtains is $c^k r_i v_i$. 
The GSP auction rule

- Bidders submit non-negative bids $b_i$’s per click for their interested keywords.
- Bid $b_i$ is multiplied by the quality score $q_i > 0$, and these adjusted bids are arranged in a decreasing order.
- Let $\pi : I \rightarrow I$ be the permutation of bidders according to the order of adjusted bids $q_i b_i$ so that $\pi(k)$ is the bidder with the $k$-th highest adjusted bid.
- Then, we have $q_{\pi(1)} b_{\pi(1)} \geq \cdots \geq q_{\pi(I)} b_{\pi(I)}$.
- Bidder $\pi(k)$ for $k = 1, \ldots, K$ is assigned to position $k$, and pays $q_{\pi(k+1)} b_{\pi(k+1)}/q_{\pi(k)}$ per click.
- Each bidder is charged the minimum price to retain the current position.
- A bidder in position $k = 1, \ldots, K$ has a net payoff or surplus of $c^k r_{\pi(k)} (v_{\pi(k)} - q_{\pi(k+1)} b_{\pi(k+1)}/q_{\pi(k)})$.
- A bidder without a position does not pay and has a surplus of zero.
• When $q_i = 1$ for all $i$: The RBB rule.

• When $q_i = r_i$ for all $i$: The RBR rule.

• Following Edelman et al. (AER 2007) and Varian (IJIO 2007), we study the static environment with complete information.

**Definition 1.** A Nash equilibrium is a set of bids $\{b_1, \ldots, b_I\}$ that satisfies

$$c^k r_{\pi(k)}(v_{\pi(k)} - \frac{q_{\pi(k+1)}b_{\pi(k+1)}}{q_{\pi(k)}})$$

$$\geq c^j r_{\pi(k)}(v_{\pi(k)} - \frac{q_{\pi(j+1)}b_{\pi(j+1)}}{q_{\pi(k)}}) \quad \text{for} \quad j > k,$$

$$c^k r_{\pi(k)}(v_{\pi(k)} - \frac{q_{\pi(k+1)}b_{\pi(k+1)}}{q_{\pi(k)}})$$

$$\geq c^j r_{\pi(k)}(v_{\pi(k)} - \frac{q_{\pi(j)}b_{\pi(j)}}{q_{\pi(k)}}) \quad \text{for} \quad j < k.$$

• Hence, bidders do not have incentives to change their assigned positions.

• Note that this definition reflects the asymmetry.

• A refinement of the Nash equilibrium concept is proven to be extremely useful.
Definition 2. A symmetric Nash equilibrium (SNE) is a set of bids \( \{b_1, \ldots, b_I\} \) that satisfies

\[
c_k \pi(k) \left( v_k - \frac{q_{\pi(k+1)} b_{\pi(k+1)}}{q_{\pi(k)}} \right) \geq c_j \pi(k) \left( v_k - \frac{q_{\pi(j+1)} b_{\pi(j+1)}}{q_{\pi(k)}} \right)
\]

for \( k, j = 1, \ldots I \). Equivalently, an SNE set of bids satisfies

\[
c_k \left( q_{\pi(k)} v_k - q_{\pi(k+1)} b_{\pi(k+1)} \right) \geq c_j \left( q_{\pi(k)} v_k - q_{\pi(j+1)} b_{\pi(j+1)} \right)
\]

for \( k, j = 1, \ldots I \).
Some properties of SNE

**Fact 1.** Nonnegative surplus:

\[ c_k r_{\pi(k)} \left( v_{\pi(k)} - q_{\pi(k+1)} b_{\pi(k+1)} / q_{\pi(k)} \right) \geq 0. \]

**Fact 2.** Monotone values: \( q_{\pi(k)} v_{\pi(k)} \) is nonincreasing in \( k = 1, \ldots, K \), and \( q_{\pi(K)} v_{\pi(K)} \geq q_{\pi(j)} v_{\pi(j)} \) for all \( j = K + 1, \ldots, I \).

- The sum of advertisers’ total payoffs is \( \sum_{k=1}^{K} c_k r_{\pi(k)} v_{\pi(k)} \). Hence, it is easy to see from Fact 2 that this sum is maximized when \( q_i = r_i \) for all \( i \), or more generally, when \( (q_1, \ldots, q_I) \) are set to respect the order of \( r_i v_i \)’s.

**Fact 3.** Non-monotone payments: \( c_k r_{\pi(k)} q_{\pi(k+1)} b_{\pi(k+1)} / q_{\pi(k)} \) may not be monotone.

**Fact 4.** One step solution: If a set of bids satisfies the inequality in Definition 2 for two adjacent positions, then it satisfies the inequality for all positions. That is, if the inequality

\[ c_k \left( q_{\pi(k)} v_{\pi(k)} - q_{\pi(k+1)} b_{\pi(k+1)} \right) \geq c_j \left( q_{\pi(k)} v_{\pi(k)} - q_{\pi(j+1)} b_{\pi(j+1)} \right) \]

holds for each \( k = 1, \ldots, I \) and \( j = k - 1 \) and \( k + 1 \), then Definition 2 holds for all \( k, j = 1, \ldots, I \).
• Definition 2 of SNE gives the inequalities

\[ c^k (q_{\pi(k)} v_{\pi(k)} - q_{\pi(k+1)} b_{\pi(k+1)}) \]

\[ \geq c^{k+1} (q_{\pi(k)} v_{\pi(k)} - q_{\pi(k+2)} b_{\pi(k+2)}) \]

and

\[ c^{k+1} (q_{\pi(k+1)} v_{\pi(k+1)} - q_{\pi(k+2)} b_{\pi(k+2)}) \]

\[ \geq c^k (q_{\pi(k+1)} v_{\pi(k+1)} - q_{\pi(k+1)} b_{\pi(k+1)}) , \]

which can be combined to get

\[ (c^k - c^{k+1}) q_{\pi(k+1)} v_{\pi(k+1)} + c^{k+1} q_{\pi(k+2)} b_{\pi(k+2)} \]

\[ \leq c^k q_{\pi(k+1)} b_{\pi(k+1)} \]

\[ \leq (c^k - c^{k+1}) q_{\pi(k)} v_{\pi(k)} + c^{k+1} q_{\pi(k+2)} b_{\pi(k+2)} . \]

• The upper and lower boundary cases are recursively expressed as

\[ c^k q_{\pi(k+1)} b_{\pi(k+1)}^{U} = (c^k - c^{k+1}) q_{\pi(k)} v_{\pi(k)} + c^{k+1} q_{\pi(k+2)} b_{\pi(k+2)}^{U} \]

and

\[ c^k q_{\pi(k+1)} b_{\pi(k+1)}^{L} = (c^k - c^{k+1}) q_{\pi(k+1)} v_{\pi(k+1)} + c^{k+1} q_{\pi(k+2)} b_{\pi(k+2)}^{L} \]

from which we have

\[ c^k q_{\pi(k+1)} b_{\pi(k+1)}^{U} = \sum_{j=k}^{K} (c^j - c^{j+1}) q_{\pi(j)} v_{\pi(j)} , \]

\[ c^k q_{\pi(k+1)} b_{\pi(k+1)}^{L} = \sum_{j=k}^{K} (c^j - c^{j+1}) q_{\pi(j+1)} v_{\pi(j+1)} . \]
3 Main Results

• The lower bound of the auctioneer’s revenue is given by

\[
\sum_{k=1}^{K} \frac{r_{\pi(k)} c^k q_{\pi(k+1)} b^{L}_{\pi(k+1)}}{q_{\pi(k)}}
\]

\[
= \sum_{k=1}^{K} \frac{r_{\pi(k)}}{q_{\pi(k)}} \sum_{j=k}^{K} (c^j - c^{j+1}) q_{\pi(j+1)} v_{\pi(j+1)}
\]

\[
= \sum_{k=1}^{K} (c^k - c^{k+1}) q_{\pi(k+1)} v_{\pi(k+1)} \left( \sum_{j=1}^{k} \frac{r_{\pi(j)}}{q_{\pi(j)}} \right)
\]

• We will henceforth work with the lower bound. This bound is prominent in that it coincides with the Vickrey payment when the quality score \( q_i \) is set to the click-through rate \( r_i \).

• Moreover, as we show in Proposition 2 below, the optimal revenue even with the lower bound SNE extracts all the bidders’ surplus, hence all the other SNEs \textit{a fortiori} achieve the full extraction of surplus.
Let \( r_1 v_1 \geq r_2 v_2 \geq \cdots \geq r_I v_I \) without loss of generality. That is, rename the bidders in the order of \( r_i v_i \). We have:

**Proposition 1.** If the quality scores \((q_1, \ldots, q_I)\) maximize the lower bound of the auctioneer’s revenue, then \( q_1 v_1 \geq q_2 v_2 \geq \cdots \geq q_K v_K \) and \( q_K v_K \geq q_j v_j \) for all \( j = K+1, \ldots, I \).

- The following example illustrates the algorithm in the proof.

**Example 1.** There are three positions with \( c^1 = 3, c^2 = 2, \) and \( c^3 = 1, \) and three bidders with \( v_1 = 3, v_2 = 2, v_3 = 1, \) and \( r_1 = r_2 = r_3 = 1 \). Suppose that \( q_2 v_2 > q_3 v_3 > q_1 v_1, \) say \( q_1 = 1, q_2 = 3, q_3 = 4 \). Hence, \( \pi(1) = 2, \pi(2) = 3, \) and \( \pi(3) = 1 \). The auctioneer’s revenue is

\[
R = (c^1 - c^2)q_3 v_3 \frac{1}{q_2} + (c^2 - c^3)q_1 v_1 \left( \frac{1}{q_2} + \frac{1}{q_3} \right) = \frac{37}{12}.
\]

Now decrease \( q_2 \) and \( q_3 \) so that \( q_2^0 = \frac{3}{2} \) and \( q_3^0 = 3 \). This gives \( q_2^0 v_2 \geq q_1 v_1 \geq q_3^0 v_3 \). The auctioneer’s revenue becomes

\[
R^0 = (c^1 - c^2)q_1 v_1 \frac{1}{q_2^0} + (c^2 - c^3)q_3^0 v_3 \left( \frac{1}{q_2^0} + \frac{1}{q_1} \right) = 2 + 3\left( \frac{2}{3} + 1 \right) = 7.
\]
Proposition 1 shows that an outcome that maximizes the auctioneer’s revenue respects the order of the CTR-adjusted valuations $r_i v_i$.

**Proposition 2.** (Full extraction of advertisers’ surplus) Assume $K < I$. The optimal quality scores $(q_1^*, \ldots, q_I^*)$ that maximize the lower bound of the auctioneer’s revenue satisfy

$$q_1^* v_1 = q_2^* v_2 = \cdots = q_{K+1}^* v_{K+1} \geq q_j^* v_j \text{ for all } j = K + 2, \ldots, I$$

and the auctioneers’ optimal revenue is equal to the maximal sum of advertisers’ total payoffs, $\sum_{k=1}^{K} c_k r_k v_k$.

- What is the reason behind this result? As for the equilibrium bids and payments, we have:

**Corollary 3.** Assume $K < I$. With the optimal quality scores $(q_1^*, \ldots, q_I^*)$, we have $b_k = v_k$ for $i = 2, \ldots, K + 1$, and the payment per click for bidder $k$ is $v_k$ for $k = 1, \ldots, K$. 

13
• The optimal quality scores effectively set all the bidders’ val-
  uations equal to the highest valuation, which induces intense
  bidding competition.
• In the optimal auction design under incomplete information,
  Myerson (1981) has shown that the auctioneer can increase rev-
  enue by giving bid preferences to weak bidders whose expected
  willingness to pay is lower.
• Hence, optimal quality scores work similarly as bid prefer-
  ences in our complete information setting of the generalized
  second-price auction.
• By the way, full extraction is not possible when $K = I$ since
  there cannot be enough competition for positions.
4 Discussion

- The short-term incentive of exploitation may be checked by the long-term incentive of market cultivation. Simply put, keyword auctions cannot survive in the end unless advertisers find their surplus satisfactory.

- The following proposition shows that the advertising intermediaries such as Google and Yahoo! have the power to extract any portion of advertisers’ surplus.

**Proposition 4.** Assume $K < I$. For any $\alpha \in [0, 1]$, the auctioneer can choose the quality scores so that, for all $k = 1, \cdots, K$, the revenue from position $k$ is exactly $100 \alpha$ percent of the total advertiser’s surplus $c_k r_k v_k$ who occupies that position.

- Other models of keyword auctions, in particular incorporating search behavior of users, are emerging. (Athey and Ellison 2007, Aggarwal et al. 2008). It is worthwhile to study the implications of the quality scores in these alternative models.
THANK YOU!!!