The Role of Nonseparable Utility and Nontradables in International Portfolio Choice

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Outline

1. Motivation
2. Contribution (Result)
3. Literature Review
4. Model
5. Portfolio Solution
6. Model Property (if I have time)
7. Conclusion
Motivation of the Paper

- Can the presence of nontraded goods explain the home bias puzzle? Baxter, Jermann, and King (1998): “No” because the optimal holdings of traded good equities should be the well diversified world portfolio.

- Is this claim robust? Maybe not. “What is missing in their model?” Human Capital
Motivation of the Paper

• Can the presence of nontraded goods explain the home bias puzzle? Baxter, Jermann, and King (1998) : “No” because the optimal holdings of traded good equities should be the well diversified world portfolio.

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• Production and Nonseparability btw. L and C
  Lewis (1996) claims that nonseparability and some restrictions on asset trade might be able to explain risk sharing pattern.
  Jermann (2002) finds that nonseparability can change portfolio allocation to some degree in traded goods only model.
  Engel and Matsumoto (2009) show that human capital can potentially explain the home bias puzzle.

Main Contribution of the Paper

• I provide a closed form solution for optimal portfolios as functions of model parameters.

• In particular, I show that nontraded goods can potentially explain the home bias puzzle with nonseparable utility function, overturning the previous results.
**Main Contribution of the Paper**

- I provide a closed form solution for optimal portfolios as functions of model parameters.
- In particular, I show that nontraded goods can potentially explain the home bias puzzle with nonseparable utility function, overturning the previous results.
- I also provide a closed-form solution for real allocations to investigate international transmission mechanisms of technology shocks.
- I discuss the implications of the solution.
Literature Review

International Portfolio Allocation

Directly Related Papers:

• Cole and Obstfeld (1991), Engel and Matsumoto (2006) risk sharing though the terms of trade adjustment ⇒ Armington elasticity


• Jermann (2002), Lewis (1996) role of nonseparability ⇒ general utility function
Literature Review 2

Recent Model of International Portfolio Choice

- incomplete market:
  Devereux and Sutherland (2006), Evans and Hnatkovska (2006),
  Hnatkovska (2005), Tille and van Wincoop (2007), Ghironi, Lee
  and Rebuggi (2007)

- home bias in consumption:
  Kollmann (2006b), Heathcote and Perri (2004), Engel and
  Matsumoto (2008), Coeurdacier (2009)

International Business Cycle

- Backus and Smith (1993), Stockman and Tesar (1995), Tesar
  (1995), flexible price models with nontraded goods under
  complete asset markets
2-Country RBC Model w. Nontraded Goods w/o Capital Household

General Utility function: $U(C_t(j), L_t(j))$

\[ C_t(j) \equiv \left[ \eta^{1/\theta} C_N, t(j)^{\theta-1}/\theta + (1 - \eta)^{1/\theta} C_T, t(i)^{\theta-1}/\theta \right]^{\theta/(\theta-1)}, \]

\[ C_T, t(j) \equiv \left( n^{1/\omega} C_h, t(j)^{(\omega-1)/\omega} + (1 - n)^{1/\omega} C_f, t(j)^{(\omega-1)/\omega} \right)^{\omega/(\omega-1)}, \]

($\theta = 1$, and $\omega = 1$ corresponds to Cobb Douglas.)

\[ C_h, t(j) \equiv \left[ n^{-1/\lambda} \int_0^n C_h, t(j, i)^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)}, \]

\[ C_N, t(j) \equiv \left[ \int_0^1 C_N, t(j, i)^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)}, \]
Model 2

Asset Market

- Four mutual funds are traded. 
  \{\text{Home, Foreign}\} \times \{\text{Traded Good, Nontraded Good}\}
- Dividends – Profits of the firms.
- This asset market structure will replicate complete market allocations except for some combination of parameter values.
- I will use the following complete market condition instead of BC

\[
Q_t = \frac{S_t P_t^*}{P_t} = \kappa \frac{U_C(C_t^*, L_t^*)}{U_C(C_t, L_t)}
\]

Firms: production function

\[
Y_{T,t}(i) = A_{T,t} L_{T,t}(i), \quad Y_{N,t}(i) = A_{N,t} L_{N,t}(i),
\]

where \(A_{.,t}\) is technology level in each sector, and \(L_{.,t}(i)\) denotes labor hours used in each firm.

\[
\Pi_{.,t} = P_{.,t} Y_{.,t} - W_{.,t} L_{.,t} = \frac{1}{\lambda - 1} W_t L_{.,t}.
\]
Optimal Equity Portfolio (list of notations on p 17)

\[
\delta_T = \frac{1 - n}{1 - \zeta} \left\{ 1 + \frac{(1 - \eta)(\theta - 1) \frac{\phi C}{\rho}}{(1 - \eta)(\theta - 1) (1 + 1/\varepsilon_{l,w}) + \eta \left[ \frac{\psi + 1}{\rho} - (1 + 1/\varepsilon_{l,w}) \right]}\right\} \quad \text{for } \omega \neq 1
\]

\[
\delta_N = \frac{1 - n}{1 - \zeta} \left\{ 1 + \frac{(1 - \eta)(\theta - 1) \frac{\phi C}{\rho} - \frac{\psi + 1}{\rho} + (1 + 1/\varepsilon_{l,w})}{(1 - \eta)(\theta - 1) (1 + 1/\varepsilon_{l,w}) + \eta \left[ \frac{\psi + 1}{\rho} - (1 + 1/\varepsilon_{l,w}) \right]}\right\}
\]

- separable case \(\phi_c = 0, 1/\varepsilon_{l,w} = \psi\) or \(\theta = 1\) \(\Rightarrow\) traded goods equity portfolio “worse than you think”; \(\zeta = 0\), BJK.
- traded goods only \(\eta = 0\) (Jermann (2002)).
- \(\omega = 1\) traded goods equity is not determined. (Cole and Obstfeld (1991)) \(\Rightarrow\) ? show that if \(\omega\) is close to unity, then price stickiness matters.
- BJK show that the nontraded good equities can be home bias or anti home bias depending on the parameters. THIS PAPER shows that the claim is also true with traded goods equity portfolio. i.e. We should not dismiss the possibility that the existence of nontraded goods can explain home bias in equities.
Equity Portfolio

the share of foreign equity as a function of $\mu$, the elasticity of substitution btw. consumption and leisure.

![Graph showing portfolio share of foreign equity benchmark case](image)

- Discontinuity: when relative returns of nontraded/traded goods sector equities are collinear.
- Extreme Bias: lack of risk sharing may be due to small asset market frictions.

varying $\mu$ (other parameter values: p17 Table 1)
Real Allocation

\[ c_t^R = \frac{\kappa_{CN}}{K} \eta a_{N,t}^R + \frac{\kappa_{CT}}{K} (\omega - 1)(1 - \eta) a_{T,t}^R, \]

\[ l_t^R = \frac{\kappa_{LN}}{K} \eta a_{N,t}^R + \frac{\kappa_{LT}}{K} (\omega - 1)(1 - \eta) a_{T,t}^R, \]

\[ q_t^R = \frac{\kappa_{QN}}{K} \eta a_{N,t}^R + \frac{\kappa_{QT}}{K} (\omega - 1)(1 - \eta) a_{T,t}^R, \]

\[ \kappa_{CN} \equiv 1 + \phi_C (1 - \theta + \eta \theta) + [1 + (\omega - 1)(1 - \eta)] \psi, \]

\[ \kappa_{LN} \equiv \eta - \rho (1 - \theta + \eta \theta) - [1 + (\omega - 1)(1 - \eta)] \phi_L, \]

\[ \kappa_{CT} \equiv -(\eta \psi + \phi_C), \quad \kappa_{LT} \equiv \rho + \eta \phi_L, \]

\[ \kappa_{QN} \equiv \rho \kappa_{CN} + \phi_C \kappa_{LN}, \quad \kappa_{QT} \equiv \rho \kappa_{CT} + \phi_C \kappa_{LT} \]

\[ K \equiv \kappa_{LT} \kappa_{CN} - \kappa_{CT} \kappa_{LN} \]
Implications of Analytical Solution

- Cobb-Douglas aggregation of foreign goods and domestic goods in traded goods basket ($\omega = 1$) will eliminate transmission of relative productivity shocks in the traded good sector.
  - Stockman and Tesar (1995): “the model is missing some source of nation specific variation in consumption of traded goods.”
  - This is partly because they used $\omega = 1$.
- No Balassa-Samuelson effect if $\omega = 1$.
- (well know result) $\rho(c_t^R, q) = 1$ if utility function is separable.
- $\rho(c_t^R, q) = 1$ when $\omega = 1$ even if utility function is nonseparable.
## Calibration

based on Stockman and Tesar’s VCV for productivity shocks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data BP</th>
<th>Data FD</th>
<th>Model BP</th>
<th>Model FD</th>
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<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

Cross Country

| $(c, c^*)$  | 0.49 | 0.60 | 0.59 | 0.51 | 0.39 | 0.17 |
| $(y, y^*)$  | 0.79 | 0.68 | 0.36 | 0.06 | 0.47 | 0.11 |
| $(c_N, c_N^*)$ | 0.39 | 0.67 | 0.40 | 0.33 | 0.31 | 0.14 |
| $(c_T, c_T^*)$ | 0.46 | 0.49 | 0.89 | 0.81 | 0.58 | 0.30 |
| $(l, l^*)$  | 0.65 | 0.66 | 0.52 | 0.21 | 0.21 | -0.27 |
| $(w, w^*)$  | 0.11 | 0.21 | 0.65 | 0.65 | 0.53 | 0.54 |
| $(y_N, y_N^*)$ | 0.63 | 0.50 | 0.40 | 0.33 | 0.31 | 0.14 |
| $(y_T, y_T^*)$ | 0.75 | 0.71 | -0.34 | -0.70 | -0.30 | -0.74 |
| $(c_R, q)$  | 1.00 | 1.00 | 0.98 | 1.00 | 0.95 |

*a In the model section, other parameter assumptions are taken from Stockman and Tesar (1995). $\sigma = 5.17$, $\gamma = 0.24$, $\theta = 0.44$*
Calibration – based on alternative VCV

<table>
<thead>
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<th>Parameters(^a)</th>
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<th>(\eta)</th>
<th>(\omega)</th>
<th>(\theta)</th>
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<td>6.00</td>
<td>0.44</td>
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<tr>
<td>(\theta)</td>
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<td>0.50</td>
<td>2.00</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Domestic Corr.

| \(\rho(c, l)\) | -0.97   | -0.97   | -0.95   | 0.73    | 0.74    | -0.12 |

Cross Country

| \(\rho(c, c^*)\) | 0.86    | 0.87    | -0.06   | 0.95    | 0.95    | 0.65  |
| \(\rho(y, y^*)\) | 0.23    | 0.22    | -0.47   | 0.67    | 0.66    | 0.05  |
| \(\rho(c_N, c_N^*)\) | 0.61    | 0.65    | -0.53   | 0.89    | 0.68    | 0.44  |
| \(\rho(c_T, c_T^*)\) | 0.94    | 0.98    | 0.35    | 0.97    | 0.99    | 0.77  |
| \(\rho(l, l^*)\) | 0.58    | 0.60    | -0.64   | 0.25    | 0.24    | -0.83 |
| \(\rho(w, w^*)\) | 0.96    | 0.96    | 0.95    | 0.96    | 0.96    | 0.95  |
| \(\rho(y_N, y_N^*)\) | 0.61    | 0.65    | -0.53   | 0.89    | 0.68    | 0.44  |
| \(\rho(y_T, y_T^*)\) | -0.20   | 0.25    | -0.80   | 0.27    | 0.52    | -0.56 |
| \(\rho(c^R, q)\) | 0.52    | 0.44    | 0.64    | 0.86    | 0.84    | 0.71  |

\(^a\) \(\mu = 5\)
Conclusion

• Portfolio Allocation:

1. the existence of human capital affect portfolio choice significantly
2. especially with nonseparability
3. extreme position ⇒ lack of risk sharing may be due to asset market frictions (instead of missing assets)
4. sticky prices might affect portfolio if Armington elasticity is close to one.
Conclusion 2

- Business Cycle
  1. poor performance overall $\Rightarrow$ other shocks. (most natural addition is sticky price + monetary shocks.)
  2. Armington elasticity plays an important role.
  3. different utility function and parameters give very different moments even with simple model like this.
  Q: what is “realistic utility function?” (both form and parameter values.)
  4. VCV also affects business cycle properties significantly.