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The authors disagree about which parts (and results) of the paper are more important or relevant than others, and about which versions of the models are interesting.
Outline of this Talk

I. Introduction, Motivation and Related Literature
II. The Model via an Example
III. The Results
IV. Conclusions
   *Don’t worry, each section is short!*
Questions

- Commuting is a ubiquitous feature of the urban economy.
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- Although the classic literature has answered the basic questions in the field, such as whether equilibrium commuting patterns are efficient, surprisingly some very important questions remain open.
- If cars can catch up with each other, what can we say about endogenous equilibrium congestion?
- Do models without an explicit time clock give us an accurate picture of traffic, in the sense that they can approximate equilibrium behavior in a truly dynamic model?
Finally, and most importantly, what happens to commuting when the situation is repeated daily?
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Does behavior differ dramatically from that observed in the simple context where the commuters know that they only have to commute once?

The latter is the exclusive focus of the extant literature. No connection between days in daily commuting.
(Some) Literature

- Beckmann et al. (1956)
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- Arnott et al. (1993) - 2 nodes, 1 link, exogenous bottleneck where commuters queue (latter typical)
- Fujita-sensei commutes to jobs in 3 different cities (but he doesn’t drive)
The Commuting Game

Introduction

Terminology

- Dynamic Game - morning commute on one day with arrival time. Route and departure time are choices. Penalty for early or late arrival.
The Commuting Game

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The Commuting Game

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- Dynamic Game - morning commute on one day with arrival time. Route and departure time are choices. Penalty for early or late arrival.

- Static Game - Only route is a choice. Something like a steady state of the dynamic model.

- Repeated Game - Either static or dynamic game is repeated daily.
In the static game, Nash equilibrium in pure strategies exists, Pareto optimum exists, they can be different.

In the dynamic game with a finite number of departure times, Nash equilibrium in pure strategies exists, Pareto optimum exists, they can be different.
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Preview of Results

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- In the repeated games with a sufficiently low discount rate, just about anything can be a Nash equilibrium. Even strategies that are not equilibria of the static or dynamic games. Even efficient strategies.
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This is the commuting folk theorem.
The Model

- For the purpose of a 20 minute talk, we will present the model through an example. The model as presented in the paper is rather general and uses lots of notation. We don’t have time.
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The Commuting Game

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- Continuum of commuters distributed uniformly on $[0, 1]$ with Lebesgue measure.
- Only morning rush hour is in model. Each commuter has inelastic demand for travel between nodes 1 and 2 once per day. There is no route choice.

\[ 1 \implies \implies \implies 2 \]
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- Time to travel link at speed limit is 1.
The Commuting Game

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The Static Model

- Link capacity is given by the positive real number $x$. 
The Commuting Game

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- For technical reasons discussed in the paper, it is best to allow only a finite number of departure times, say $\frac{1}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n}$, where $n$ can be chosen as large as desired. When a commuter chooses a departure time, their actual departure is distributed uniformly around their chosen departure time, but not larger than halfway to the next departure time nor smaller than halfway to the previous departure time. For technical reasons.
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- There is a required arrival time $\tau^A$ at work for all commuters. If a commuter arrives late to work, their utility is $-\infty$. General penalty functions for late and early arrival in paper.
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Simple Examples - Dynamic Model

- Road capacity $x \geq 1$. Arrival time $\tau^A = 2$. Commuter 0 departs at time 0, commuter 1 departs at time 1. The interval moves in complete synchrony to work. Time cost for each commuter = 1. Commuter 1 arrives at time 2.
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- **Road capacity** $x < 1$. Arrival time $\tau^A = \frac{1}{x} + 1$. Departure times as before. Traffic slows by a factor of $\frac{1}{x}$. It takes each commuter time $\frac{1}{x}$ to traverse the link. Commuter 1 arrives at $\frac{1}{x} + 1$. Call this the *congested commuting pattern*. 
The Commuting Game

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- Road capacity $x < 1$. Arrival time $\tau^A = \frac{1}{x} + 1$. Departure times: commuter 0 leaves at time 0, commuter 1 leaves at time $\frac{1}{x}$, arriving at $\frac{1}{x} + 1$. Traffic moves at the speed limit. It takes each commuter time 1 to traverse the link. Call this the uncongested commuting pattern.
There exists a Nash equilibrium in *pure strategies*. True even when the number of commuters is finite.
The Results - Static Model

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Due to the congestion externality, it is easy to construct examples where Nash equilibrium is not Pareto optimal (see paper).
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Due to the congestion externality, it is easy to construct examples where Nash equilibrium is not Pareto optimal (see paper, e.g. the congested commuting pattern).
Is the static model equilibrium the “steady state” of the dynamic model equilibrium? If not, then hard to justify use of the static model.

First problem: How to handle arrival time in the static model?

Second problem: Even if arrival time varied, might never attain steady state.

We address all of this by asking for something much weaker. We only ask that the average flows on links of the dynamic model match those of the static model. Alternatively, we ask that there be a time, and a distance on each link such that flows match the static model.

It is easy to construct examples where neither holds.
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- Alex wants the paper to END HERE.
It seems obvious that there are few other games better suited to the folk theorem than the commuting game.
The Repeated Commuting Game - I

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- It is not crazy to assume that people play the same game (but not necessarily the same strategy) every day.
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- It is not crazy to assume that people play the same game (but not necessarily the same strategy) every day.
- If commuters have discount factors sufficiently close to 1, in other words they are patient, then there is a huge variety of equilibria.
- Any individually rational, feasible strategy (not necessarily a Nash equilibrium in the one shot game) is an equilibrium strategy for the repeated game.
The equilibrium strategies are supported by various punishment strategies, that apply if the prescribed equilibrium is not followed by a player.
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Thus, one expects to see the one shot equilibrium played, perhaps, but also (for example) the efficient strategies.
The Repeated Commuting Game - III

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With a sufficiently fine departure grid, they will always arrive late, yielding payoff $-\infty$. So any strategy yielding payoff greater than $-\infty$ in the one shot game can be supported as a Nash equilibrium of the repeated dynamic game (with appropriate discounting).
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- The folk theorem is quite robust to all sorts of modifications, including the introduction of uncertainty.
Issues with the Commuting Folk Theorem

- The issue is observability by a positive measure of commuters of defections by one commuter so they can be punished.
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- For example, others departing home at the same time and using the same route can observe.
- We use a continuum of commuters for mathematical reasons.
- Conclusion of Anti-Folk Theorem: Only the Nash equilibria of the one shot game are equilibria of the repeated game.
### The Commuting Folk Theorem and the Commuting Anti-Folk Theorem

<table>
<thead>
<tr>
<th></th>
<th>Observable</th>
<th>Unobservable</th>
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<tr>
<td>Countable Commuters</td>
<td>Folk Theorem</td>
<td>Impossible</td>
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<tr>
<td>Continuum of Commuters</td>
<td>Folk Theorem</td>
<td>Anti-Folk Theorem</td>
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## Continuity

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In the model with a continuum of commuters, when an individual commuter changes their strategy, there will be no change in what is observed by other agents, say their commuting time, so there is no basis on which to punish deviators. Thus, the anti-folk theorem applies.
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But now consider the model with a finite number of commuters. Even if the number of commuters is large, deviations from a prescribed along-the-equilibrium-path-strategy can be detected (for instance by commuters on the same route using the same departure time on the equilibrium path) and therefore can be punished. This explains the contraction of the equilibrium set.
However, one can easily argue that as the number of agents gets large, these deviations become undetectable, as their effects are small and indistinguishable from noise.
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For example, an analog would be to assume perfect competition in the context of a finite number of agents, where the error from this assumption is small for large economies.
However, one can easily argue that as the number of agents gets large, these deviations become undetectable, *as their effects are small* and indistinguishable from noise.

For example, an analog would be to assume perfect competition in the context of a finite number of agents, where the error from this assumption is small for large economies.

If this were true, then there would be no substantial error in simply using the limit commuting game with a continuum of commuters and unobservability. The big problem here is that *in our commuting game, their effects are not small*.
Example

To see this, instead of using a continuum of commuters, consider a large but finite number. Consider either the uncongested or congested one shot Nash equilibrium pattern.
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- Pure commuter strategies are uniformly distributed over departure times that get the commuters to work by the given arrival time.
Snowball Effect

- Suppose that a commuter changes their strategy from the equilibrium strategy of the second departure time to the first. This will slow down the first cohort. The second cohort will quickly catch up, slowing down both cohorts.
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The third cohort will catch up to the first two, and so forth. No commuters will reach work by the designated arrival time.
Consequences of the Snowball Effect

- This “snowball effect” will not only be detectable (even if individual strategies aren’t), but it also substantially changes the behavior of the entire system due to one commuter’s deviation.
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- Such a “snowball effect” is simply not possible in the commuting game with a continuum of commuters.
- In other words, the behavior of the game with a continuum of commuters where individual commuter strategies are undetectable differs from the behavior of the commuting game with a large but finite number of commuters where individual strategies are unobserved.
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- Such a “snowball effect” is simply not possible in the commuting game with a continuum of commuters.
- In other words, the behavior of the game with a continuum of commuters where individual commuter strategies are undetectable differs from the behavior of the commuting game with a large but finite number of commuters where individual strategies are unobserved.
- For this reason, we view the repeated commuting game with a continuum of commuters where the individual strategies are undetectable, and the associated anti-folk theorem, as irrelevant.
The commuters who observe a defection are not necessarily those who punish. In the simple one route example, of course any commuter who observes a defection can punish. However, with many routes, this might not be possible.
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If individual strategies are observable to all, for example neighbors departing at the same time and along the same route, then of course we are back in the context of the folk theorem.
Crime and Punishment I

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- An alternative assumption is that any defection causes a snowball effect, in that a positive measure of commuters is affected. Then it is assumed that if a positive measure of commuters is affected, this is observable to all and the deviators can be punished.
The problem with this idea is that there is literally no snowball effect with a continuum of commuters, but only with a large but finite (or countable) number. In fact, this is the reason there is a discontinuity of the Nash equilibrium correspondence in the limit as the number of commuters goes to infinity.
The problem with this idea is that there is literally no snowball effect with a continuum of commuters, but only with a large but finite (or countable) number. In fact, this is the reason there is a discontinuity of the Nash equilibrium correspondence in the limit as the number of commuters goes to infinity.

A sufficient condition for a snowball effect in large but finite games close to the game with a continuum of commuters of interest is: for the given one shot strategy profile that is to be supported as a repeated game Nash equilibrium, at any time on any link with a positive local density of commuters, local density is above the capacity of the link. Under this condition on the strategy profile, whenever a commuter deviates, there is a snowball effect; this is detected and punished by everyone.
In summary, our conclusion is that although the snowball effect is not present in commuting games with a continuum of commuters, it is present in the large but finite games nearby.
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Thus, whether it be from observations of neighboring commuters or the snowball effect, the folk theorem in the model with a continuum of commuters seems relevant.
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Conclusions

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- *The commuting folk theorem poses a direct challenge to congestion pricing.* If commuters are already playing efficient equilibrium strategies, taxes are only going to mess this up.
- Can also apply commuting folk theorem to other models in the literature.
Conjectures

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- Apply the dynamic model to real world commuting. Cost-benefit analysis for infrastructure improvements.