The Dynamic Welfare Comparison of seignorage Tax and Consumption Tax

Lu, Chia-Hui
National Taipei University

2009.08
Abstract

- The welfare costs between a seignorage tax and a consumption tax.
- Compares equilibrium paths along both transitional dynamics and steady state.
- Consider the inelastic labor supply and the endogenous labor supply.
Literature Review

- Real balances as a consumption good and inflation as a tax on real balances.
- A zero nominal interest rate, known as the Friedman rule.
Phelps (1973) argued against the Friedman rule and showed that all goods, including real balances, would be taxed in a Ramsey-like fashion.

Chari, et al. (1991) supported Friedman rule in class of preference homogenous in the two consumption goods and weakly separable in leisure activities.

Braun (1994) against Friedman rule in favor of seignorage tax above the Friedman rule in more general preference.
Palivos and Yip (1995, JMCB), endogenous growth models, compared welfare cost between seignorage tax and income tax as alternative ways of financing exogenous public spending along a BGP, with C and fraction of I cash-in-advance constrained.

Results in support of Braun (1994) and against Chari, et al. (1991): optimal inflation tax higher than the Friedman rule when sufficiently large fraction of I is CIA constrained.
Purpose of this paper: Extend line of research in a public finance approach compare welfare cost between seignorage tax and consumption tax.

Framework: Standard growth model with CIA constraint, exogenous waste public spending, financed either by seignorage tax or by consumption tax.
Comparison of seignorage tax with consumption tax

- C is more close to M, medium of exchange; C is usually more cash constrained than other market activities. Seignorage tax amounts to taxing C at higher rate than other activities.

- Consumption tax is dynamically neutral and is less distortionary than an income tax. Recent tax reforms in advanced economies connote a trend toward the reliance on C taxes in place of other taxes.
Long-run steady state and transitional dynamics

- Consumption tax is neutral while seignorage tax hurts capital formation in the long run; thus, long-run welfare cost is higher in seignorage tax.

- Seignorage tax increases relative price of consumption (relative to shadow price of capital) less than what consumption tax does in early periods.

- If this adverse effect on welfare dominates the effect of higher consumption tax in the long run, then reverse welfare ranking is expected.

- Ho, et al (2007): welfare cost between seignorage tax and consumption tax; under some conditions, welfare cost in consumption tax is higher than in a seignorage tax.
- Our study adds value in:
  - Ho, et al. (2007), a MIUF model, M has store of value.
  - Ho, et al, result needs sufficiently large externality in production, assumption incompatible with empirical studies (Burnside, 1996; Basu and Fernald, 1997).
  - Ho, et al, steady-state comparison.
The Basic Model

\[ U = \int_{0}^{\infty} u(c)e^{-\rho t} dt. \] \hspace{2cm} (1)

\[ \dot{k} + \dot{m} = f(k) - (1 + \tau_c)c - \pi m - \delta k. \] \hspace{2cm} (2)

\[ \dot{k} = I - \delta k. \] \hspace{2cm} (3)

\[ (1 + \tau_c)c + \varphi I \leq m, \quad 0 \leq \varphi \leq 1. \] \hspace{2cm} (4)

\[ \mu m + \tau_c c = G = \beta f(k), \quad 0 \leq \beta \leq 1. \] \hspace{2cm} (5)

Government budget constraint: Cooley and Hansen, 1991; Palivos and Yip, 1995; Lucas, 2000 and Ho, et al., 2007
Different CIA constraints

\[(1 + \tau_c)c + \phi I \leq m, \ 0 \leq \phi \leq 1. \quad (4)\]

- \(\phi = 0\): Clower (1967) and Lucas (1980)
- \(\phi = 1\): Stockman (1981) and Abel (1985)
- \(0 \leq \phi \leq 1\): Wang and Yip (1992) and Palivos and Yip (1995).
The necessary conditions

\[ u'(c) = (1 + \tau_c)[\lambda_m + \zeta], \]  
(6a)

\[ \lambda_k = \lambda_m + \zeta \varphi, \]  
(6b)

\[ \dot{\lambda}_k = (\rho + \delta) \lambda_k - f'(k) \lambda_m, \]  
(6c)

\[ \dot{\lambda}_m = \rho \lambda_m - [-\lambda_m \pi + \zeta], \]  
(6d)

TVC’s: \[ \lim_{t \to \infty} e^{-\rho t} \lambda_k(t) k(t) = 0 \]

\[ \lim_{t \to \infty} e^{-\rho t} \lambda_m(t) m(t) = 0 \]
Equilibrium Conditions

- The goods market clearing condition
  \[ \dot{k} = (1 - \beta) f(k) - c - \delta k. \]
- The money market clearing condition
  \[ \dot{m} = (\mu - \pi)m. \]
- An exogenous change in the share of public expenditure \( \beta \) financed either by a change in the consumption tax rate \( \tau_c \) or a change in the rate of monetary growth \( \mu \) in order to balance the government budget.
Neoclassical Growth Model

- \( f'(k) > 0 > f''(k) \).
- Equilibrium is simplified to 3 equations in \( c, k, \) and \( \lambda_m \):
  - \( \dot{k} = (1 - \beta) f(k) - c - \delta k \),
  - \( \dot{c} = [(1 - \frac{1}{\varphi}) \lambda_m + \frac{\lambda_k(c, \lambda_m)}{\varphi}] \frac{\partial \tau_c}{\partial k} [(1 - \beta) f(k) - \delta k - c] - \frac{1 + \tau_c}{u''(c) - \frac{\partial \tau_c}{\partial c} \varphi} [f'(k) \lambda_m - (\varphi - 1) \lambda_m - (\rho + \delta) \lambda_k(c, \lambda_m)] \),
  - \( \dot{\lambda}_m = \lambda_m [\rho + \frac{1}{\varphi} + \pi(c, k, \lambda_m; \beta)] - \frac{\lambda_k(c, \lambda_m)}{\varphi} \).
Steady state

- $\dot{c} = \dot{k} = \dot{\lambda}_m = 0$.
- $\pi^* = \mu$
- locus $\dot{k} = 0$: $c^* = (1 - \beta) f(k^*) - \delta k^*$
- locus $\dot{c} = 0$: $f'(k^*) = (\rho + \delta)(1 + \phi \rho)$, if $\mu = 0$; and $f'(k^*) = (\rho + \delta)[1 + \phi(\rho + \mu)]$, if $\tau_c = 0$ so $\mu = \frac{\beta}{(1 - \beta)c^*/f(k^*) + \phi \delta k^*/f(k^*)}$. 
Steady state

- \( \lambda_m^* = \frac{u'(c^*)}{(1+\tau_c)(1+\rho+\mu)} \),

where \( \mu = \frac{\beta}{(1-\beta)c^*/f(k^*)+\varphi \delta k^*/f(k^*)} \) under \( \tau_c = 0; \)

and \( \tau_c = \frac{\beta f(k^*)}{c^*} \) under \( \mu = 0 \)

- \( m^* = (1 + \tau_c) c^* + \varphi \delta k^* \)

- \( U^* = \int_0^\infty u(c^*) e^{-\rho t} dt = \frac{u(c^*)}{\rho} \)
Steady state effect

- Permanent increase in share of government spending, \( d\beta > 0 \)
- Under balance of government budget and evaluated at initial conditions \( \beta = \mu = \tau_c = 0 \)
  - if financed only by \( \tau_c \): \( d\tau_c = \frac{f(k^*)}{c^*} d\beta \)
  - if financed only by \( \mu \): \( d\mu = \frac{f(k^*)}{m^*} d\beta \)
Steady state effect

\[ dU^*|_{d\tau_c>0} = -\frac{u'(c^*)}{\rho}f'(k^*)d\beta < 0. \]

\[ dU^*|_{d\mu>0} = -\frac{u'(c^*)}{\rho}f'(k^*)d\beta + \frac{u'(c^*)}{\rho}[(1 - \beta)f'(k^*) - \delta]dk^* < 0. \]

where \( dk^* = \phi \frac{\rho + \delta}{f''(k^*)} \frac{f(k^*)}{m^*}d\beta < 0 \)

In an optimal growth model when investment is cash constrained, in terms of the welfare criterion a consumption tax is always less costly than a seigniorage tax in the long run.
Steady state effect

- Public spending has crowding-out effect. No matter which way of finance, same negative effect on consumption in the long run, reducing agent’s welfare.
- Seignorage tax reduces capital formation in the long run when I is cash constrained while consumption tax does not.
- As a result, in terms of welfare, permanent consumption tax is better than permanent seignorage tax in the long run.
quantitative analysis

- \( u(c) = \frac{c^{1-\sigma}-1}{1-\sigma} \), \( \sigma > 0 \).
- \( f(k) = Ak^\alpha \), \( 0 < \alpha < 1 \), \( A > 0 \).
- \( \delta = 5\% \), \( \rho = 0.04 \), \( \sigma = 2.5 \), \( \varphi = 0.05 \), normalize \( A = 1 \), and \( \beta = \tau_c = \mu = 0 \) initially.
- Calibrating \( \alpha \) in consonance with the annual \( k/y \) at 3.32 (Cooley, 1995, p.21) obtains \( \alpha = 0.3 \).
- Unique long-run equilibrium:
  \( k^* = 5.5684 \), \( c^* = 1.3954 \), \( y^* = 1.6739 \) and \( U^* = 6.9805 \).
Steps of quantifying dynamic effects

- A linear Taylor’s expansion of the 3x3 equilibrium system around the unique steady state and obtain a Jacobean matrix;

- Assure 1 negative eigenvalue (stable root) and 2 positive eigenvalues for the Jacobean matrix (thus a unique dynamic equilibrium path);

- The coefficients of time paths $c(t), k(t), \lambda_m(t)$ are determined by boundary conditions, depending on types of policy changes.
(A) $\varphi=0.05$

Capital

Consumption

(B) $\varphi=0$

Capital

Consumption
A permanent policy

- A permanent seignorage tax now increases the shadow price of investment relative to consumption because a fraction of investment is constrained by tighter real balances.

- The level of consumption in a seignorage tax higher than the level of consumption under in a consumption tax in early periods.

- This effect dominates the effect in the long run, the welfare cost is higher in a consumption tax than in a seignorage tax.
### A permanent policy

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\rho$</th>
<th>$\Delta U% (\mu &gt; 0)$</th>
<th>$\Delta U% (\tau_c &gt; 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.1</td>
<td>-58.3880</td>
<td>-58.5203</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-32.5050</td>
<td>-32.5812</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.01</td>
<td>-28.8310</td>
<td>-28.9223</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.001</td>
<td>-30.0821</td>
<td>-30.1995</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0</td>
<td>0.04</td>
<td>-32.5407</td>
<td>-32.5407</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.01</td>
<td>0.04</td>
<td>-32.5326</td>
<td>-32.5490</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.001</td>
<td>0.04</td>
<td>-32.5397</td>
<td>-32.5427</td>
</tr>
<tr>
<td>0.2</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-78.0439</td>
<td>-78.5030</td>
</tr>
<tr>
<td>0.1</td>
<td>1.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-36.2976</td>
<td>-36.3921</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-40.1994</td>
<td>-40.3292</td>
</tr>
</tbody>
</table>
Lu, Chia-Hui  National Taipei University  The Dynamic Welfare Comparison

(A) $\varphi=0.05$

(B) $\varphi=0$

Capital

Consumption

Lu, Chia-Hui  National Taipei University  The Dynamic Welfare Comparison
Welfare in an optimal growth model–A temporary policy

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\rho$</th>
<th>$\Delta U% (\mu &gt; 0)$</th>
<th>$\Delta U% (\tau_c &gt; 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.1</td>
<td>-36.0603</td>
<td>-36.1926</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-10.1317</td>
<td>-10.1631</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.01</td>
<td>-2.5071</td>
<td>-2.5137</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0.5</td>
<td>0.001</td>
<td>-0.2691</td>
<td>-0.2699</td>
</tr>
<tr>
<td>0.1</td>
<td>2.5</td>
<td>0</td>
<td>0.04</td>
<td>-7.7670</td>
<td>-7.7760</td>
</tr>
<tr>
<td>0.01</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.9405</td>
<td>-0.9420</td>
</tr>
<tr>
<td>0.05</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-4.8595</td>
<td>-4.8685</td>
</tr>
<tr>
<td>0.2</td>
<td>2.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-22.1490</td>
<td>-22.2955</td>
</tr>
<tr>
<td>0.1</td>
<td>1.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-11.9382</td>
<td>-11.9957</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.04</td>
<td>-14.1550</td>
<td>-14.2924</td>
</tr>
</tbody>
</table>

Lu, Chia-Hui National Taipei University

The Dynamic Welfare Comparison
A temporary policy

\[ MRS_{t,t+1} : C_{t+1} = \left[ \frac{f'(k_t)+1-\delta}{1+\rho} \frac{(1+\tau_{ct})}{(1+\tau_{ct+1})} \right]^{1/\sigma} C_t \]

- \( C_{T+1} \) in proportion to \( C_{T+2} \), indep. of tax:
  \[ C_{T+2} = \left[ \frac{f'(k_{T+1})+1-\delta}{1+\rho} \right]^{1/\sigma} C_{T+1} \]

- \( C_T \) proportional to \( C_{T+1} \), affected by \( \tau_{cT} \), making
  \[ C_{T+1} > C_T : C_{T+1} = \left[ \frac{f'(k_T)+1-\delta}{1+\rho} (1 + \tau_{cT}) \right]^{1/\sigma} C_T \]

- \( C_{T-1} \) is proportional to \( C_T \) and is affected by \( \tau_{cT} \) and \( \tau_{cT-1} \), thus making
  \[ C_{T+2} = C_{T+1} > C_T > C_{T-1} > \ldots > C_1 > c_0 \]
A temporary policy

- Cash constraints on investment are not required in temporary tax changes because the policy change has a terminal period which creates a wedge in the marginal rate of substitution in consumption in periods before and after the policy is terminated.
Leisure Choice: Robustness Check

- The representative agent is endowed with $L$ units of time in which $l$ units are allocated to leisure and the remaining $L - l$ units to working.

- An agent’s lifetime utility:
  
  $$U = \int_0^\infty \left( \frac{c^{1-\sigma}-1}{1-\sigma} + b \frac{l^{1-\varepsilon}-1}{1-\varepsilon} \right) e^{-\rho t} dt.$$ 

- The per capita production function is
  
  $$y = f(k, L - l) \text{ with}$$

  $$f_1(k, L - l) > 0 > f_{11}(k, L - l) \text{ and}$$

  $$f_2(k, L - l) > 0 > f_{22}(k, L - l).$$
Leisure Choice: Optimal Growth Model

- The production technology: $y = Ak^\alpha (L - l)^{1-\alpha}$, where $0 < \alpha < 1$.
- Prescott (2006) pointed out that the labor supply elasticity is equivalent to an IES for leisure of 1.2 (so $\varepsilon = 0.83$) if the fraction of productive time allocated to market is 0.25, as it is for the United States (so $l = 0.75L$).
  - Set $L = 10$, $\delta = 5\%$, $\rho = 4\%$, $\sigma = 2.5$, $\phi = 0.05$
  - Same benchmark: $\beta = \mu = \tau_c = 0$
  - Calibrate $b = 0.2639$.
Welfare in an optimal growth model (endogenous labor)—A permanent policy

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\rho$</th>
<th>$\Delta U% (\mu &gt; 0)$</th>
<th>$\Delta U% (\tau_c &gt; 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>-1.8526</td>
<td>-1.8615</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.04</td>
<td>-1.6963</td>
<td>-1.7016</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.01</td>
<td>-1.6877</td>
<td>-1.6930</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.001</td>
<td>-1.7264</td>
<td>-1.7347</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.04</td>
<td>-3.8638</td>
<td>-3.8883</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.7999</td>
<td>-0.8012</td>
</tr>
<tr>
<td>0.01</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.1528</td>
<td>-0.1531</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.04</td>
<td>-1.7015</td>
<td>-1.7021</td>
</tr>
</tbody>
</table>
Welfare in an optimal growth model (endogenous labor)–A temporary policy

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\rho$</th>
<th>$\Delta U% (\mu &gt; 0)$</th>
<th>$\Delta U% (\tau_c &gt; 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>-1.1954</td>
<td>-1.2019</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.5657</td>
<td>-0.5683</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.01</td>
<td>-0.1591</td>
<td>-0.1599</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.001</td>
<td>-0.0168</td>
<td>-0.0169</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.04</td>
<td>-1.2380</td>
<td>-1.2494</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.2709</td>
<td>-0.2715</td>
</tr>
<tr>
<td>0.01</td>
<td>0.5</td>
<td>0.04</td>
<td>-0.0524</td>
<td>-0.0525</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.04</td>
<td>-0.5696</td>
<td>-0.5699</td>
</tr>
</tbody>
</table>
Concluding Remarks

- A permanent consumption tax always has a higher welfare cost than a permanent seignorage tax if investment is cash constrained;
- A temporary consumption tax always has a higher welfare cost than a temporary seignorage tax.
- In financing public spending, an inflation tax is preferred to a consumption tax and the inflation tax is higher than the Friedman rule.