Efficient Demonetization*

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Abstract

Traditional models of money assume that the marginal social cost of printing fiat currency is zero, justifying the optimality of the Friedman rule. However, in an environment where the degree of hidden income is alleviated by the dearth of cash, demonetization could be efficient. I isolate conditions under which state-contingent transfer limits are monotonic in reported endowments and promised values. A model calibrated to the Indian income process reveals that long-run gains in the surplus of the central bank upon switching to a state-contingent monetary policy from a non state-contingent one are 28.5% of aggregate income.

Keywords: Monetary policy, asymmetric information, dynamic contracting

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1 Introduction

On 8 November 2016, the Reserve Bank of India (RBI) discontinued 86% of its currency in circulation.[1] Prime Minister Narendra Modi's speech argued that this clandestine and draconian measure was taken to purge the economy of "black money," i.e. income illegally obtained or not declared for tax purposes.[2] The enforced swap of currency acted as a tax on illicit wealth. Keeping in mind the limited supply of new notes, in the first few days, there was a limit imposed on cash withdrawals: 10,000 rupees per day and 20,000 rupees per week. Therefore, when considering how much to demonetize the Indian Rupee, the RBI faced a tradeoff between alleviating hidden income frictions and avoiding a cash crunch.

I develop a dynamic contracting model in which the degree of hidden income is jointly and endogenously determined with transfer limits to evaluate efficient demonetization policies. In particular, I examine the efficient allocation of resources in an economy with the following characteristics. There are many households, all infinitely lived, and all with the same ex-ante preferences over time paths of the consumption of a single good, a proportion of which has to be financed by cash. Each of these households is subject to idiosyncratic income risk. To smooth their consumption over time, households can enter into long-term contractual agreements with a financial intermediary. Income shocks are independent from household to household, so with a large number of households risks can be pooled, with high income earners just balanced out by low income earners. Crucially, these income shocks are private information to households and unobserved by the financial intermediary. Thus, any allocation of resources must be based on household reports. To reduce this information asymmetry, the financial intermediary can access an auditing technology that is sensitive to the amount of cash in the economy.[3] This captures the idea that credit economies are easier to monitor.

Optimal monetary policy in this environment balances the insurance benefits of high transfer limits with the informational costs of weak income monitoring. If the marginal social cost of printing money is zero, then the Friedman rule holds. However, in a setting where flooding the economy with currency makes income monitoring less effective, demonetization could be efficient.

[1]India has previously witnessed demonetization twice in the 20th century. During the twilight of the British Raj, 11th January 1946, high denomination notes were demonetized to tackle tax evasion. The second instance was on 16th January 1978 where one, five and ten thousand rupee notes were discontinued. Modi's war on cash also has some international precedent: Singapore, for example, withdrew its largest denominated currency in 2014; the European Central Bank ceased issuance of the 500-euro bank note in 2016.

[2]Kar and Curcio (2009) estimate that the Indian economy lost 104 billion U.S. Dollars in black money between 2001 and 2008. Tax evasion is significant even in developed economies like the United States. The Internal Revenue Service estimates a tax gap–the difference between the calculated total tax yield and the actual receipts–of 458 billion U.S. Dollars annually over the period 2008-2010, amounting to 18.3% of tax liabilities.

[3]To this end, we need to take a stand on how to parametrize the degree of hidden income in a given economy. The approach I follow restricts the range of utilities to the positive orthant of the euclidean space and scales the value of misreporting by a function that ranges over the unit interval.

In effect, the RBI imposed non-discriminatory transfer limits. Such policies, which I refer to as *aggregate* demonetization, are too blunt to insure against idiosyncratic income risk. I propose a set of instruments that provide a better hedge against these shocks–transfer limits dependent on reported household income–which I refer to as *state-contingent* demonetization. Under mild assumptions on the shape of the income monitoring technology and transfer limits, I analytically show that the optimal state-contingent demonetization policy is weakly decreasing in reported income and strictly increasing in wealth.

In a quantitative exercise, I map the theory to the data by estimating a model (via simulated method of moments) to match salient facts regarding the wealth distribution in India. Using a variety of estimation techniques, I find that the model can account for 83-94% of the variation in India's Lorenz curve. Using this monetary model, it is estimated that long-run gains in the surplus of the central bank upon switching from an aggregate demonetization policy to a state-contingent one are 28.5% of aggregate income.

Depending on the instruments at disposal to the central bank, optimal monetary policy is set to either moderate or magnify income fluctuations. I find that the central bank tightens transfer limits when income increases in the state-contingent monetary economy, while the opposite is true in the aggregate monetary economy. Moreover, using the inverse of the covariance between money and transfers as a measure of binding informational frictions, I find that hidden income plays a larger role in shaping monetary policy in the state-contingent setting relative to the aggregate one.

The model also provides a novel link between monetary policy and wealth inequality. Quantitatively, I find that the aggregate monetary economy admits larger wealth inequality relative to the state-contingent setting. The coefficient of variation in the limiting distribution of wealth in the aggregate monetary economy is estimated to be 2.5 times larger than that in the state-contingent monetary economy. In a canonical principal-agent framework with asymmetric information that is devoid of transfer limits, the optimal contract spreads the wealth distribution over time in order to efficiently insure agents' while respecting incentive compatibility (Townsend (1982), Green (1987), Thomas and Worrall (1990), Green and Oh (1991), Atkeson and Lucas (1992), Aiyagari and Alvarez (1995), Wang (1995)). In my model, if transfer limits are sufficiently loose, a reduction in currency in circulation can enhance income monitoring and thereby reduce the spread in the limiting distribution of wealth. I show that the central bank finds it optimal to issue less notes in the state-contingent monetary economy relative to the aggregate one. This is because switching to a richer set of state-contingent instruments allows the central bank to lower transfer limits for high income earners without inhibiting risk sharing. As a result, the central bank needs to rely less on spreading the wealth distribution over time in order to distinguish between types.

Related Literature: This paper is related to three strands of research: (i) optimal monetary policy, (ii) contracting under hidden income, and (iii) costly state verification (CSV).

The monetary economics literature has traditionally focused on deriving the optimal inflation tax as the solution to a Ramsey (1927) tax problem in a representative agent economy. Chari et al. (1996), and Correia and Teles (1996) derive conditions on preferences or technology for the Friedman rule to be optimal within a Ramsey setting.[4] In particular, Chari et al. (1996) show that the Friedman rule is optimal in three standard models of money under simple homotheticity and separability assumptions on preferences, while Correia and Teles (1996) work within a shopping-time framework (introduced by Kimbrough (1986)) to show that the Friedman rule is optimal if the transactions technology is homogeneous. Some other papers that study the optimal inflation tax in a full information setting are Lucas and Stokey (1983), Correia and Teles (1999), Guidotti and Vegh (1993) and Mulligan and Sala-i Martin (1997). In the spirit of Mirrlees (1971), da Costa and Werning (2003) extend this analysis to an environment with heterogeneous idiosyncratic productivity which is private information.[5] These studies maintain the assumption that the marginal social cost of printing money is zero. I depart from this paradigm by assuming, in accordance with RBI policy, that higher levels of currency in circulation propagate black money, which I interpret as an increase in the severeness of hidden income constraints. Tax collection costs can justify a positive inflation tax as well. For instance, Végh (1989) considers a model where government expenditures are financed via the inflation tax and an alternative tax that is costly to collect. He finds inflation to be an increasing function of government spending.

The design of optimal incentive-compatible mechanisms in environments with privately observed income shocks has been studied by Green (1987), Thomas and Worrall (1990), and Phelan (1995) among others. In such environments, the principal extracts information from the agents about their income and uses this information to provide optimal incentive-compatible insurance. Following the ideas of Lucas (1990), Aiyagari and Williamson (2000) and Kim (2003) show how the theory of dynamic contracts under private information can be extended to address issues in monetary economics. In these models, random circumstances arise where it is costly for consumers to access financial intermediaries which would allow credit transactions. It may then be desirable to trade on the money market for consumption smoothing over time. In this paper, cash-in-advance constraints (Lucas and Stokey, 1985) play a similar role in that they create a transactions role for money. To study how monetary policy interacts with hidden income, I build on this framework by embedding an income monitoring technology that responds to changes in currency in circulation. In this respect, this paper is closely related to

[4] I refrain from restricting the choice set of the planner by competitive equilibrium allocations. Rather than probing a specific trading protocol, I focus on constrained-efficient mechanisms. This approach is pursued to highlight the fundamental trade-off between transfer limits and the degree of hidden income.

[5] At the heart of all these results lies the uniform-tax result of Atkinson and Stiglitz (1976).

the literature on CSV pioneered by Townsend (1979).[6] CSV is a mechanism which expands the space of incentive compatible, non-negative contracts. In this literature, a principal chooses an auditing technology directly given agent reports. This paper pursues a related but alternative approach where the principal takes as given an income monitoring technology that responds to changes in transfer limits via another choice variable–currency in circulation.

The next section analytically examines a dynamic mechanism design problem where a planner sets monetary policy to balance the tightness of incentive constraints that emerge due to the presence of hidden income and cash constraints. Sections 3 and 4 quantitatively examine optimal state-contingent and non state-contingent demonetization policies in a model calibrated to key moments of the Indian income process. Section 5 embeds demonetization in a model with heterogeneous labor income risk as in Mirrlees (1971) and da Costa and Werning (2003). Concluding remarks are offered in Section 6. All proofs are relegated to the Appendix.

2 Model

I augment Thomas and Worrall (1990) (henceforth TW) to evaluate the effects of demonetization. I consider an environment in which message-contingent transfers are constrained by a maximum limit. The principal has the ability to alleviate hidden income constraints in exchange for tightening transfer constraints.

Ex-ante identical agents receive a stochastic endowment $y_{t,s} > 0$ of a single good, where s is the state of nature drawn from a finite set $\mathcal{S} = \{1, ..., S\}$ with $S \geq 2$ and $t > 0$. Denote the ordered set of possible endowments by $\mathcal{H} \equiv \{y_1, ..., y_s\}$. Here $h^t = (y_t, ..., y_0) \in \mathcal{H}^t$ denotes an arbitrary history of reported income. The principal is faced with asymmetric information. Households have private information about their own income, and the insurer can see neither income nor consumption. It follows that any insurance payments between the planner and a household must be based on the household's own reports about income realizations. The agents' rate of time-preference is denoted by $\beta \in (0, 1]$.

Definition 1. *An allocation is a sequence of functions $\{c_t, \kappa_t\}_{t>0}$ where $c_t : \mathcal{H}^t \mapsto \mathbb{R}_+$ and $\kappa_t : \mathcal{H}^t \mapsto \mathbb{R}_+$.*

The sequence of functions assigns to the agent a history-dependent stream of consumption and demonetization policies.

Assumption 1. *Endowment shocks y_t are independent and identically distributed, i.e. $\pi(y_t \mid y_{t-1}) = \pi(y_t)$.*

[6] See Gale and Hellwig (1985) for an application to credit markets.

Assumption 2. *The utility function* $u : \mathbb{R}_+ \mapsto \mathbb{R}_+$ *is monotonically increasing, concave, twice-continuously differentiable, satisfies the Inada condition* $\lim_{c\to 0} u'(c) = \infty$, *and* $-u''/u'$ *is non-increasing.*

Consumption is limited by a cash-in-advance constraint,

$$c_t \leq M_t,$$

where $M_t \in \mathbb{R}_+$ denotes currency in circulation. I assume M_t is uniquely determined by the monetary policy instrument, $\kappa_t \in K$, and the endowment shock via the following condition:

$$M_t = F(\kappa_t) + y_t,$$

where the compact domain of demonetization policies is denoted by $K = [0, \kappa_{max}]$, $\kappa_{max} \in \mathbb{R}_+$.[7]

Assumption 3. $F : K \mapsto \mathbb{R}_+$ *is of class* $\mathcal{C}^2$ *such that* $F', -F'' > 0$.

By the Revelation Principle, any constrained efficient allocation must satisfy the modified one-shot deviation constraint $\forall h^{t-1}, y_t, \hat{y}_t$:

$$\begin{aligned} & u\big(c_t(h^{t-1}, y_t)\big) + \beta \lim_{T\to\infty} \sum_{s=1}^{T-t} \sum_{h^{t+s}\in\mathcal{H}^{t+s}} \beta^{s-1}\pi_{t+s}(y_{t+s}) u\big(c_{t+s}(h^{t-1}, y_t, h_{t+1}^{t+s})\big) \\ \geq \quad & \phi(\kappa_t(h^{t-1}, y_t))\big\{u\big(c_t(h^{t-1}, \hat{y}_t)\big) + \beta \lim_{T\to\infty} \sum_{s=1}^{T-t} \sum_{h^{t+s}\in\mathcal{H}^{t+s}} \beta^{s-1}\pi_{t+s}(y_{t+s}) u\big(c_{t+s}(h^{t-1}, \hat{y}_t, h_{t+1}^{t+s})\big)\big\}, \end{aligned}$$

where ϕ captures the income monitoring technology and controls the extent to which the severeness of hidden income constraints endogenously responds to currency in circulation.

Assumption 4. $\phi : K \mapsto [\underline{\phi}, 1]$, *where* $\phi' > 0$ $\forall \underline{\phi} \in [0, 1)$ *and* $\underline{\phi} \in [0, 1]$.

This approach of modeling income monitoring deserves qualification. A standard hidden income setting (with no monitoring) is obtained when $\phi(.) = 1$. The strict monotonicity of ϕ and the non-negative domain of the utility function imply that when κ_t is high (low) then $\phi(\kappa_t)$ is closer to 1 (0), corresponding to the economy where income is completely hidden (observable). Observe that assumptions 2 and 4 also imply:

$$\frac{\partial\phi(\kappa_i)\partial F(\kappa_j)}{\partial\kappa_i\partial\kappa_j} > 0 \ \forall(\kappa_i, \kappa_j) \in K^2 \ \forall\underline{\phi} \in (0, 1).$$

This captures the complementarity between the degree of hidden income and the slackness of the cash constraint.

[7] In the numerical exercise, I set κ_{max} the supremum of the feasible domain.

Assumption 5. *$C : K \mapsto \mathbb{R}_+$, where $C', C'' > 0$.*

I also assume that the central bank incurs a resource cost, C, of printing, distributing and managing currency in circulation.[8]

2.1 Recursive Representation of Planning Problem

It's convenient to study the planning problem recursively. We proceed by assuming endowment shocks are i.i.d. over time. Denote the message-contingent demonetization and consumption policies by $\kappa_s : V \mapsto K \subseteq \mathbb{R}_+$ and $c_s : V \mapsto \mathbb{R}_+$ where $V = [\inf u(c)/(1-\beta), \sup u(c)/(1-\beta)]$. Denote the transfer received by a policyholder who reports an income realization in state $s \in \mathcal{S}$ by $b_s \equiv c_s - y_s$. Instead of focusing on a welfare maximization problem, I consider minimizing the cost of providing a given level of utility to households. The RBI solves the following problem:

$$P(v) = \max_{\{b_s, w_s, \kappa_s\}_{s\in\mathcal{S}}} \sum_{s=1}^{S} \pi_s \big[-(C(\kappa_s) + b_s) + \beta P(w_s) \big]$$

s.t.

$$\sum_{s=1}^{S} \pi_s \big[u(y_s + b_s) + \beta w_s \big] \geq v, \tag{PK}$$

$$b_s \leq F(\kappa_s) \quad \forall s \in \mathcal{S}, \tag{CC}$$

$$\mathcal{D}_{s,\hat{s}} \equiv u(y_s + b_s) + \beta w_s - \phi(\kappa_s)\big[u(y_s + b_{\hat{s}}) + \beta w_{\hat{s}}\big] \geq 0 \quad \forall (s,\hat{s}) \in \mathcal{S}\times\mathcal{S}, \tag{IC}$$

$$b_s \in [c_{min} - y_s, c_{max} - y_s] \quad \forall s \in \mathcal{S},$$

$$w_s \in V \quad \forall s \in \mathcal{S},$$

$$\kappa_s \in K \quad \forall s \in \mathcal{S}.$$

The central bank's objective is to minimize the cost of delivering promised utility each period using state-contingent transfers, continuation utilities and demonetization policies that are restricted to compact sets. Constraint (PK) is a promise-keeping constraint.[9] Limits on cash transfers subject to demonetization policies are captured by state-contingent constraints (CC). Constraints (IC) are modified incentive constraints, which any constrained-efficient allocation must satisfy due to the Revelation Principle.[10] Notice that in this setting, local downward

[8] This function can also capture the opportunity cost of investing in an alternative asset.

[9] See Abreu et al. (1990).

[10] The planning problem subject to modified incentive constraints in which the income monitoring technology enters multiplicatively is isomorphic to one in which it enters additively. To see this define a map $g : K \times [c_{min}, c_{max}] \times V \mapsto \mathbb{R}_+$ by $g(x, y, z) = [u(y) + \beta z](1/\phi(x) - 1)$ and notice that (IC) can be re-written as $u(y_s + b_s) + \beta w_s + \bar{g}(s) \geq u(y_s + b_{\hat{s}}) + \beta w_{\hat{s}}$ $\forall (s,\hat{s}) \in (\mathcal{S}, \mathcal{S})$ where $\bar{g}(s) \equiv g(\kappa_s, y_s + b_s, w_s)$.

incentive compatibility does not guarantee global incentive compatibility. In addition, the lower bound on expected utilities is assumed to be non-negative to ensure the existence of a non-degenerate limiting distribution of wealth (Atkeson et al. (1995), Phelan (1995)). Figure 1 illustrates the effect of aggregate demonetization policy, where $\kappa_s = \bar{\kappa}\ \forall s$, on the constraint set. Observe that increasing $\bar{\kappa}$ has an ambiguous affect on the constraint set.[11] On one hand, transfer limits are larger as $\bar{\kappa}$ is increased due to the strict monotonicity of F; this is captured by $\text{CC}(v, \bar{\kappa}_1) \subset \text{CC}(v, \bar{\kappa}_2)\ \forall \bar{\kappa}_1 < \forall \bar{\kappa}_2\ \forall v$. On the other hand, incentive constraints are tighter as $\bar{\kappa}$ is increased due to the strict monotonicity of ϕ; this is captured by $\text{IC}(v, \bar{\kappa}_2) \subset \text{IC}(v, \bar{\kappa}_1)\ \forall \bar{\kappa}_1 < \forall \bar{\kappa}_2\ \forall v$. The tension implicit between cash and incentive constraints can generate an interior solution for monetary policy.

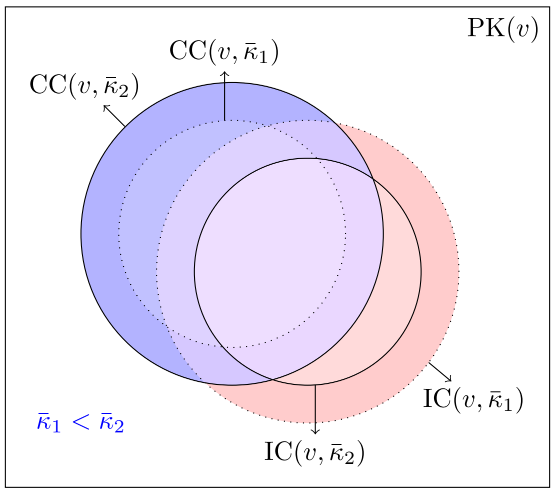


Figure 1: Illustration of Demonetization Effects on Constraint Set

2.2 Sufficient Conditions for Program Concavity

The following lemma isolates sufficient conditions for program concavity under full information when F is linear.[12]

Lemma 1. *If F is linear and $C^{(3)}, P^{(3)}, -u^{(3)} \geq 0$, then P is concave under full information.*

In the general formulation that features asymmetric information, the constraint set of the programming problem can be non-convex due to the way state-contingent demonetization enters into incentive constraints. This could potentially inhibit the first-order approach from isolating

[11]For illustrative purposes, cash and incentive constraints are drawn as strict subsets of the set of outcomes restricted by the promise keeping constraint.

[12]The linearity assumption, which is sufficient but not necessary for program concavity, is imposed here for analytical convenience.

global solutions. Nonetheless, if $\bar{\phi}$ is sufficiently close to unity, then the concavity of P follows from Assumption 2 and the concavity of F and $-C$.[13] I proceed under this assumption.

2.3 Characterization

Lemma 2. *The optimal allocation exists and is given by functions* $\{b_s, w_s, \kappa_s, \eta_s, \lambda_{s,\hat{s}}\}_{(s,\hat{s})\in\mathcal{S}\times\mathcal{S}}$ *where*

$$\begin{aligned}
&b_s : V \mapsto [c_{min} - \sup y_s, c_{max} - \inf y_s]\\
&w_s : V \mapsto V\\
&\kappa_s : V \mapsto [0, \kappa_{max}]\\
&\eta_s : V \mapsto \mathbb{R}_+\\
&\Lambda_{s,\hat{s}} : V \mapsto \mathbb{R}_+
\end{aligned}$$

such that[14]

$$\forall s \in \mathcal{S} \quad \pi_s[1 + P'(v)u'(c_s(v))] + \eta_s(v) = \sum_{\hat{s}\in\mathcal{S}\setminus s} \{\Lambda_{s,\hat{s}}(v)u'(c_s(v)) - \Lambda_{\hat{s},s}(v)\phi(\kappa_{\hat{s}}(v))u'(y_{\hat{s}} + b_s(v))\}, \tag{1}$$

$$\forall s \in \mathcal{S} \quad P'(w_s(v)) = P'(v) + \frac{1}{\pi_s}\sum_{\hat{s}\in\mathcal{S}\setminus s} \{\Lambda_{s,\hat{s}}(v) - \Lambda_{\hat{s},s}(v)\phi(\kappa_{\hat{s}}(v))\}, \tag{2}$$

$$\forall s \in \mathcal{S} \quad \pi_s C'(\kappa_s(v)) \geq \eta_s(v)F'(\kappa_s(v)) - \sum_{\hat{s}\in\mathcal{S}\setminus s} \Lambda_{s,\hat{s}}(v)\phi'(\kappa_s(v))[u(y_s + b_{\hat{s}}(v)) + \beta w_{\hat{s}}(v)] \ \ w.e.\ \kappa_s > 0, \tag{3}$$

$$\forall s \in \mathcal{S} \quad \eta_s(v)[b_s(v) - F(\kappa_s(v))] = 0, \tag{4}$$

$$P'(v)\left[\sum_{s=1}^{S} \pi_s[u(c_s(v)) + \beta w_s(v)] - v\right] = 0, \tag{5}$$

$$\forall (s,\hat{s}) \in \mathcal{S}\times\mathcal{S} \quad \Lambda_{s,\hat{s}}(v)\mathcal{D}_{s,\hat{s}}(v) = 0. \tag{6}$$

Equations (1) capture the wedge created between the state-contingent marginal rates of substitution, $u'(c_s)/\beta$, and their corresponding marginal rates of transformation, $-[\beta P'(w_s)]^{-1}$, due to binding transfer limits and incentive constraints. Conditions (2) reveal that the marginal return on continuation utilities should be equated with the marginal return in promised values when incentive constraints are slack. Equations (3) capture the key-trade off in monetary policy in this environment. In any interior equilibrium, if the expected marginal increase in the cost of printing fiat currency is negligible, then optimal monetary policy balances the shadow marginal gain in relaxing transfer limits with the shadow marginal cost of weakening the income monitoring technology. Conditions (4)–(6) are complementary slackness conditions corresponding to the transfers limits, the promise keeping constraint, and incentive constraints respectively.

Lemma 3. *If* $C(.) = 0$, $\underline{\phi} = 1$ *and* K *is complete, then demonetization is not optimal.*

[13] In the quantitative exercise, this is achieved using a modest level of $\psi > 0$.

[14] Here $w.e.$ stands for "with equality when."

Lemma 3 captures the perspective of traditional monetary models, which assume that the marginal social cost of printing fiat currency is zero. In this case the Friedman rule is optimal since printing money weakly relaxes cash constraints. However, in an environment where the degree of hidden income is alleviated by the paucity of cash, i.e. when $\underline{\phi} < 1$, demonetization may be optimal depending on which region of the state-space the economy lies.

I focus on two threshold levels of promised utility:

$$\underline{v} \equiv \sup_{x \in V} x : \eta_s(x) = 0 \ \forall s \in \mathcal{S},$$
$$\bar{v} \equiv \inf_{x \in V} x : \eta_s(x) > 0 \ \forall s \in \mathcal{S}.$$

Let $\mathcal{S}^b(v) \equiv \{s \in \mathcal{S} : \eta_s(v) > 0\}$, and $\mathcal{S}^c(v) \equiv \{s \in \mathcal{S} : \eta_s(v) = 0\}$. That is, for a given v, $\mathcal{S}^b(v) \subseteq \mathcal{S}$ denotes the set of states where cash constraints bind, while $\mathcal{S}^c(v) \subseteq \mathcal{S}$ denotes the set of states in which cash constraints are slack.

Lemma 4. *If $C'(0)/F'(0) = 0$, π has full support, and $\underline{\phi}$ is large enough then*

$$\kappa_s(v) < \kappa_{\hat{s}}(v) \quad \forall v \in [\bar{v}, \sup u(y_s)/(1-\beta)] \ \forall (s, \hat{s}) \in \mathcal{S} \times \mathcal{S} : y_s > y_{\hat{s}},$$
$$\kappa_s(v) < \kappa_{\hat{s}}(v) \quad \forall v \in [\underline{v}, \bar{v}] \ \forall (s, \hat{s}) \in \mathcal{S} \times \mathcal{S}^b(v) : y_s > y_{\hat{s}},$$
$$\kappa_s(v) = 0 \quad \forall v \in [\underline{v}, \bar{v}] \ \forall s \in \mathcal{S}^c(v),$$
$$\kappa_s(v) = 0 \quad \forall v \in [\inf u(y_s)/(1-\beta), \underline{v}] \ \forall s \in \mathcal{S}.$$

Lemma 4 shows that state-contingent monetary policy is weakly decreasing in reported endowment. The proof proceeds by segmenting the state-space into three regions, $[\inf u(y_s)/(1 - \beta), \underline{v}] \cup [\underline{v}, \bar{v}] \cup [\bar{v}, \sup u(y_s)/(1-\beta)] = V$.[15] I then show that transfers are decreasing in reported income when $\underline{\phi}$ is sufficiently high. In region $[\bar{v}, \sup u(y_s)/(1-\beta)]$, a strictly decreasing demonetization policy follows from the fact that currency in circulation and transfers co-move in cash constrained states when F is increasing. In the region $[\underline{v}, \bar{v}]$, a strictly decreasing demonetization policy follows from the co-movement of Lagrange multipliers on cash constraints, currency in circulation and transfers when $C'(0)/F'(0) = 0$ and $\pi_s > 0 \ \forall s \in \mathcal{S}$. In states where cash constraints are slack, transfer limits are necessarily zero due to the strict monotonicity of the cost function, C.

Lemma 5. *If π has full support, then $\frac{\partial \kappa_s}{\partial v} > 0 \ \forall s \in \mathcal{S}^b(v)$ in any interior equilibrium with full information.*

[15] An implicit assumption here is that $\bar{v} \geq \underline{v}$, which also holds in the baseline calibration. However, non-monotonic policy functions that stem from binding incentive constraints subject to CSV may upset this inequality in general.

Lemma 5 shows that state-contingent monetary policy is strictly increasing in promised values under full information in all states where transfer limits bind.[16] Intuitively, when the financial intermediary can perfectly observe income realizations of households, it is efficient to set higher transfer limits for wealthier households that are cash-constrained as it allows them to smooth their wealth over time. The quantitative exercises that follow show that this monotonic relationship between money and wealth is not robust to private information.

2.3.1 Aggregate Demonetization

I benchmark the outcomes obtained in the state-contingent demonetization economy against those obtained in an economy in which monetary policy is non state-contingent, i.e. where $\bar{\kappa} = \kappa_s \; \forall s \in \mathcal{S}$ is determined by the following condition in any interior equilibrium:[17]

$$C'(\bar{\kappa}(v)) = \sum_{s\in\mathcal{S}} \left\{ \bar{\eta}_s(v) F'(\bar{\kappa}(v)) - \sum_{\hat{s}\in\mathcal{S}\setminus s} \bar{\Lambda}_{s,\hat{s}}(v) \phi'(\bar{\kappa}(v)) [u(y_s + \bar{b}_{\hat{s}}(v)) + \beta \bar{w}_{\hat{s}}(v)] \right\}.$$

Since the instruments available to the central bank in the aggregate demonetization economy are a strict subset of the instruments available in the state-contingent economy, optimal allocations in the later case are weakly Pareto-dominant. Notate the aggregate and state-contingent demonetization economies as $\mathcal{E}_A$ and $\mathcal{E}_S$ respectively.

2.4 Invariant Distribution

Let $\mathcal{Z}(V, \mathcal{V})$ be the set of probability measures on $(V, \mathcal{V})$, and let $\mathcal{B}(V)$ denote the Borel set of V. Let $\mathcal{Q} : V \mapsto \mathcal{B}(V)$ be the transition function and $\mathcal{T}$ be the operators associated with $\mathcal{Q}$. In particular, define the transition

$$\mathcal{Q}(v, V) = \sum_{s=1}^{S} \pi_s \mathbb{1}_{\{w_s(v) \in V\}}$$

The associated operator on the set of all bounded measurable functions f is defined by

$$(\mathcal{T} f)(a) = \int f(a') \mathcal{Q}(a, da') \; \forall a \in V.$$

Lemma 6. *If π has full support, then there exists a unique invariant distribution of promised*

[16] Lemma 1 and Lemma 5 are codependent. The proof of Lemma 5 invokes the concavity of P, which can in turn be proved using the result that state-contingent monetary policy is increasing in promised values when transfer limits bind under full information. Therefore, the assumptions used to show either one of these lemmas are necessary for the other as well even though they are not explicitly stated.

[17] The corresponding aggregate policies and shadow prices are notated by $\{\bar{b}_s, \bar{w}_s, \bar{\eta}_s, \bar{\Lambda}_{s,\hat{s}}\}$.

values $\Psi^* \in \mathcal{Z}(V, \mathcal{V})$ such that for any initial measure Ψ_0, $\mathcal{T}^n(\Psi_0)$ converges to Ψ^* in the total variation norm.

3 Calibration

Let $\mathcal{S} = \{L, H\}$.

Functional Forms: I consider a log utility function:

$$u(x) = \log(x).$$

The transfer limit is determined by:

$$F(x) = zx^{\alpha}, z > 0, \ \alpha \in (0, 1).$$

The income monitoring technology, which responds to changes in the level of currency in circulation, is given by:

$$\phi(x) = \left(\frac{x}{\kappa_{max}}\right)^{\psi}, \ \psi > 0,$$

where $\kappa_{max} \equiv \sup y_s/(1-\beta) = y_H/(1-\beta)$. Lastly, I consider a quadratic cost function:

$$C(x) = x^2.$$

Data Generating Process: Endowment shocks are realized as per a standard AR(1) process with Gaussian errors. The AR(1) is discretized into a two-state Markov Chain with nodes $(\hat{y}_L, \hat{y}_H) \in \mathbb{R}_+^2$ using the Tauchen and Hussey (1991) quadrature method. The states are amplified using scaling parameters $\sigma_{\Delta y} \in [0, 1]$ and $\bar{y} \in \mathbb{R}_+$.

$$y_{i,t} = (1 + \sigma_{\Delta y} - 2\sigma_{\Delta y}\mathbb{1}_L(i))\mathbb{1}_{\hat{y}_i}(y_t)\bar{y} \ \ \forall i \in \{L, H\},$$
$$y_t = \rho y_{t-1} + \epsilon_t^y, \ \epsilon_t^y \sim \mathcal{N}(0, \sigma_y).$$

Parametrization: We need to assign values to 8 parameters that I classify into three sets: (i) independent parameters $\{\beta, \bar{y}\}$, (ii) exogenously estimated parameters $\{\rho, \sigma_y\}$, and (iii) endogenously estimated parameters $\{\sigma_{\Delta y}, \alpha, z, \psi\}$. The first two subsets are chosen independently of equilibrium conditions.

The income process is disciplined using Indian data. I estimate $\{\rho, \sigma_y\}$ using the cyclical component of HP-filtered Real GDP for India over the period 1996Q2:2016Q4. The autocorrelation and standard deviation in aggregate income is given by 0.78 and 0.012 respectively.

I use the set of endogenously estimated parameters to target the most salient and peculiar features of the data. The output differential between the idiosyncratic income states, $\sigma_{\Delta y}$, is calibrated to match the Gini coefficient for the 5th decile of the Indian wealth distribution, which is about 0.05 (AIDIS).[18] This captures the fact that wealth inequality *within* deciles is fairly subdued, especially near the median of the wealth distribution. Wealth inequality in India is mainly concentrated across deciles. This can be seen in the wealth percentile ratios. For instance, the p60–p40 wealth ratio is about 2 (Credit Suisse, 2014). To target this moment, I employ the parameter that controls the elasticity of the income monitoring technology with respect to currency in circulation, ψ. Moreover, wealth is concentrated at the top of the distribution. The wealth share of the bottom 20% of the population is only 1%. I use the parameter that determines the curvature of the transfer limit, α, to target this moment. The model is also particularly suited to capture the fact that consumption inequality is substantially lower than wealth inequality in India. The Gini coefficient for wealth is twice as large as the Gini coefficient for consumption, and this ratio has remained roughly constant post-liberalization (Anand and Thampi, 2016). I use the scalar coefficient on the transfer limit, z, to target this ratio. Table 1 reports the estimates I obtain from the calibration procedure. The output differential between states is equal to 0.077. The parameter that controls the elasticity of the income monitoring technology is set to .0001. The two parameters that discipline the tightness of transfer limits are both set to 0.5.

I assume that central bank policies are subjective in the sense that each state is perceived to be equally likely when policies are set. The simulations are carried out using a finite state Markov Chain discretized from the Indian Income process. This implies that perceived probabilities ($\pi_L = 0.5$) differ from realized probabilities ($\pi_L = 0.83$) and the central bank does not update its perception using the law of large numbers.[19]

Table 1: Parameterization

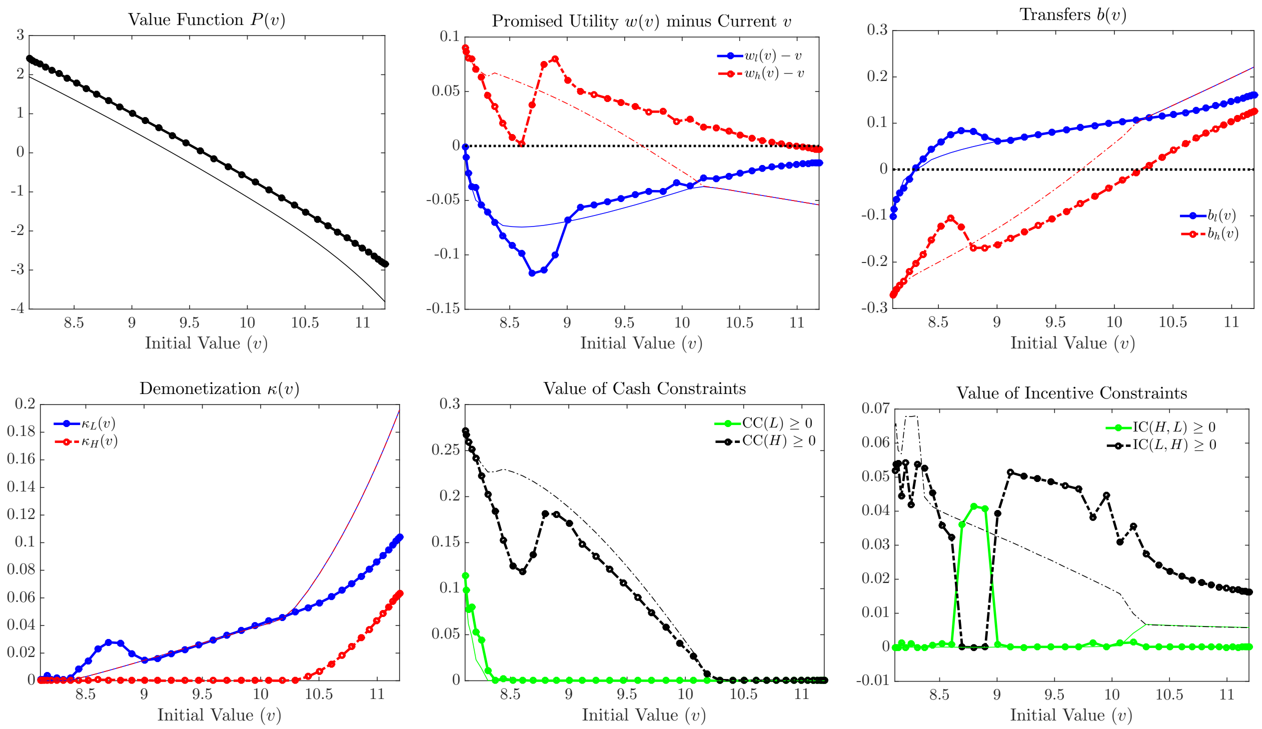
Parameters	Value	Description	Target
β	0.95	Rate of time-preference	Independent
$\bar{y}$	1.625	Mean Endowment	Independent
ρ	0.78	Autocorrelation in Income	India Income Process
σ_y	0.012	Standard Deviation in Output Innovations	India Income Process
$\sigma_{\Delta y}$	0.077	Output Differential	Wealth Gini in 5th Decile = 0.05
ψ	.0001	Income Monitoring Parameter	p60($\mathbf{w}$)–p40($\mathbf{w}$) ratio = 2
α	0.5	Curvature of Transfer Limit	Bottom 1/5th Wealth Share = 1%
z	0.5	Level of Transfer Limit	Wealth Gini/Consumption Gini = 2

[18] The wealth Gini coefficients for the 4th and 6th deciles also hover around this level.

[19] On restricting the central bank's perception to coincide with the realized probabilities from the data, I was not able to obtain a calibration in which incentive constraints for low types bind over a non-zero measure of the state-space. Without this property the model cannot generate non-monotone policy functions (see appendix).

4 Results

The model is solved using Value Function Iteration over 40 grid points for promised values that correspond to Chebyshev nodes over V. To avoid cases that feature a degenerate long-run distribution for promised values, I restrict the domain of promised values, V, to $[\inf_{s\in\mathcal{S}} u(c_s)/(1-\beta), \sup_{s\in\mathcal{S}} u(c_s)/(1-\beta)]$. This additional constraint can be rationalized by the presence of endogenously incomplete markets à la Kehoe and Levine (1993).



Note: The thin (thick) lines correspond to policies in the aggregate (state-contingent) demonetization economy. The constraints are manipulated such that positive values correspond to slackness.

Figure 2: Value Function, Policy Functions and Constraint Sets

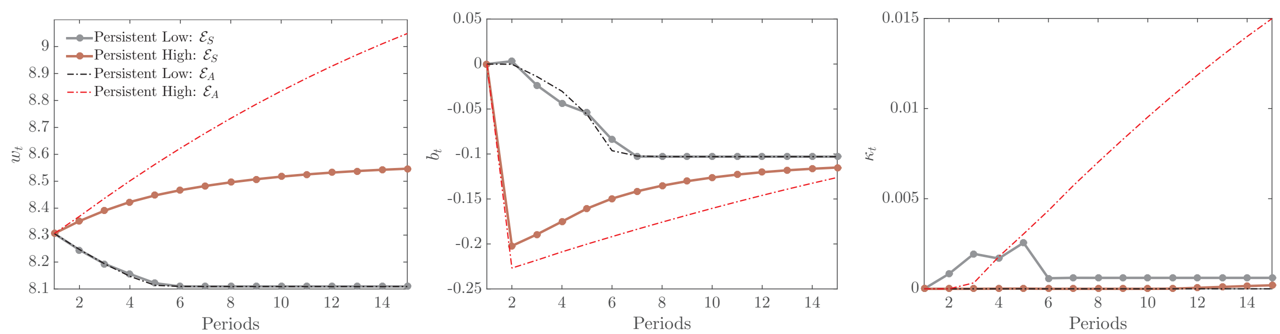
4.1 Value Function, Policy Functions and Constraint Sets

This section reports how optimal policies vary across income shocks, inherited wealth, and monetary regimes. The first four panels of Figure 2 depict the value and policy functions at the baseline parametrization. As is the case in an environment without demonetization, agents that report low income are provided with a larger transfer today. In exchange, the central bank is compelled to provide higher continuation utilities to agents who report high endowments to maintain incentive compatibility. Moreover, notice that the central bank's surplus is strictly decreasing in promised values. However, transfers and demonetization policy can be non-monotone in promised values.

Depending on the level of promised value inherited, state-contingent policies can be smaller, larger or equal to their aggregate counterparts. In contrast, irrespective of the level of promised value inherited, the central bank's surplus in $\mathcal{E}_S$ is larger than that in $\mathcal{E}_A$. The last two panels of Figure 2 depict the value of the inequality constraints across the endogenous grid of promised values. Depending on the promised value, any of the cash or incentive constraints can bind. Observe that the cash constraint for the agents who reported high (low) endowment are slacker (tighter) in $\mathcal{E}_A$ than in $\mathcal{E}_S$. With regard to informational constraints, notice that it's sufficient to restrict the constraint set to locally downward incentive compatibility in $\mathcal{E}_A$. However, downward or upward incentive compatibility constraints can bind in $\mathcal{E}_S$.

4.2 Transitional Dynamics

Figure 3 plots the optimal paths of promised values, transfers, and demonetization policies upon being exposed to persistently low and high endowment shocks at the long-run average of promised values. Transfers and demonetization policy at time 0 are set to zero for this exercise. In standard models with hidden income, the only way to provide insurance without creating incentives to misreport income is for the principal to increase (reduce) promised utilities over time in response to persistently high (low) reported endowment. This holds at the baseline calibration as well even though the inclusion of state-contingent demonetization can upset this pattern in general. Observe that responses of demonetization policy to income shocks can be non-linear due to non-monotonic policy functions in the state-contingent monetary economy. In contrast, the central bank alleviates transfer limits in response to persistently high income shocks in the aggregate monetary economy.



(a) Promised Values (b) Transfers (c) Demonetization

Figure 3: Responses to Persistent Income Shocks

4.3 Long-run Properties

This section compares the long-run properties of optimal allocations in the state-contigent and aggregate monetary economies. I show that wealth inequality can be considerably mitigated by swtiticing to the state-contingent setting.

4.3.1 Moments

Table 2 reports the first three moments of demonetization, transfers and promised values obtained from simulating the model over 100,000 periods in economies $\mathcal{E}_S$ and $\mathcal{E}_A$. All three moments of promised values in the aggregate demonetization economy are higher than their state-contingent counterparts. The differences in mean and standard deviation of promised values across the two economies are mirrored in demonetization and transfer policies; see columns (1) and (2) in Table 2. Currency in circulation is higher as well as more volatile in the aggregate monetary economy. Since transfers limits are sufficiently tight under the baseline calibration, the mean and standard deviation of simulated transfers in the aggregate monetary economy are also higher than their state-contingent counterparts.

Table 2: Long-run Moments: Mean, Standard Deviation and Skewness

	κ_t			b_t			w_t		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
$\mathcal{E}_S$	0.0013	0.0029	3.6991	-0.1101	0.0868	0.1215	8.2982	0.1471	0.1481
$\mathcal{E}_A$	0.0068	0.0074	0.8728	-0.0860	0.1030	0.0227	8.6000	0.3758	0.4102

Note: Columns (1), (2) and (3) denote the first three moments respectively. All moments are computed using simulations over 100,000 periods.

Table 3 reports the covariances among the planner's choices over time and endowment shocks. Observe that the central bank runs a *moderating* monetary policy in the state-contingent monetary economy in the sense that money and income share a negative relationship. This is the quantitative analog of Lemma 4. Contrastingly, the central bank runs a *magnifying* monetary policy in the aggregate monetary economy in the sense that money co-moves with income. Furthermore, promised values co-move with money in both economies. This echoes Lemma 5, which analytically shows that money holdings are increasing in wealth.

As in TW, promised values are increasing in income realizations. Another pattern observed in TW persists in the state-contingent economy, i.e. negative covariance between transfers and promised values. This relationship, however, does not hold in $\mathcal{E}_A$. Observe that the covariance between money and transfers is larger in $\mathcal{E}_A$, implying that incentive effects play a more prominent role in shaping monetary policy in $\mathcal{E}_S$. Moreover, the negative covariance between transfers and income verify that insurance is provided in the aggregate as well as the state-contingent monetary economy.

4.3.2 Wealth Inequality

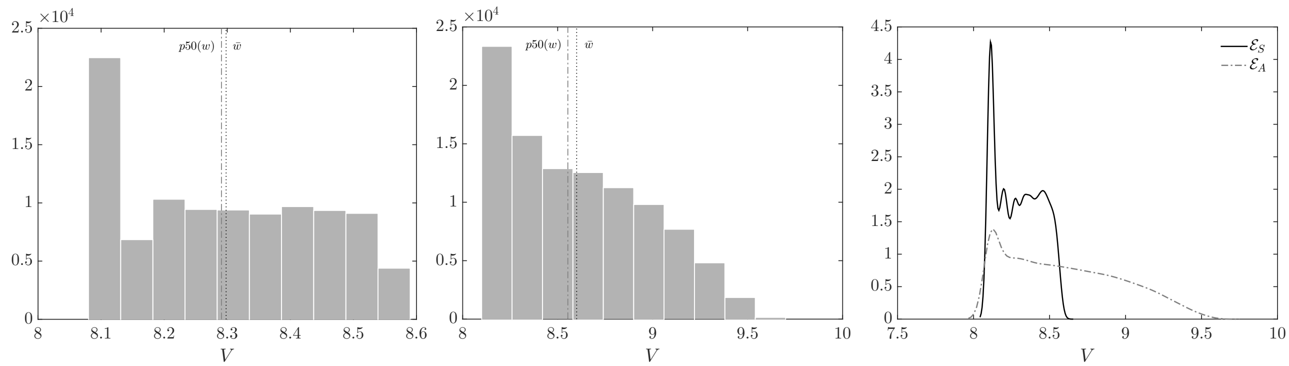
This section shows that the aggregate monetary economy features more wealth inequality than the state-contingent monetary economy in the long-run. Figure 4 compares the distribution of promised values over time in the two economies. Though mean levels of promised values

Table 3: Long-run Moments: Covariance

	$(\boldsymbol{\kappa}, \mathbf{y})$	$(\mathbf{w}, \mathbf{y})$	$(\boldsymbol{\kappa}, \mathbf{w})$	$(\boldsymbol{\kappa}, \mathbf{b})$	$(\mathbf{b}, \mathbf{w})$	$(\mathbf{b}, \mathbf{y})$
$\mathcal{E}_S$	-0.0001	0.0086	1.2914×10^{-5}	1.5644×10^{-4}	-0.0023	-0.0057
$\mathcal{E}_A$	0.0001	0.0118	0.0027	2.2415×10^{-4}	0.0060	-0.0069

Note: All moments are computed using simulations over T = 100,000 periods. Bold letters notate the time-series obtained from simulations over T periods.

are comparable across the two economies, the ergodic distribution of promised values is more dispersed in the aggregate monetary economy. Using the coefficient of variation in promised values as a measure, I find that wealth inequality is 2.5 times larger in the aggregate monetary economy relative to the state-contingent setting. Consistent with the data, the kernel density of the ergodic distribution of promised values in $\mathcal{E}_A$ also resembles a Pareto distribution.



(a) State-contingent (b) Aggregate (c) Kernel Densities

Notes: This figure compares histograms (using 10 bins) and associated kernel density functions of promised values obtained using 100,000 simulations in the aggregate and state-contingent monetary economies.

Figure 4: Ergodic Distribution of Promised Values

Differences in wealth inequality across the two settings stem from differences in respective monetary policies. The relationship between currency in circulation and wealth inequality is ambiguous in general. To illustrate this, I distinguish between two channels through which money can affect wealth inequality.

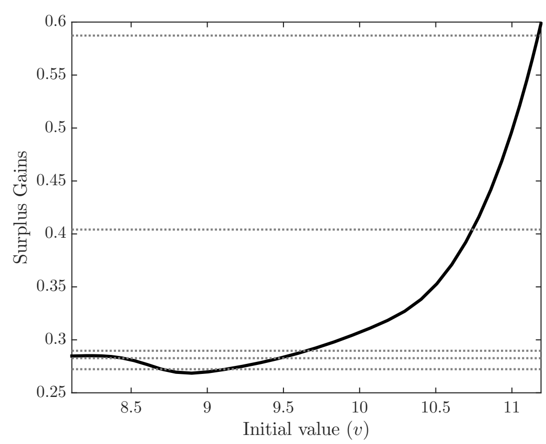
When cash constraints are slack, increasing currency in circulation (from its lower bound) increases wealth inequality. This is because increasing currency in circulation reduces income monitoring by assumption. As a result, the optimal contract spreads wealth over time to incentivize truthful reporting. This effect is more at play at lower levels of wealth. I refer to this as the *incentive channel* of monetary policy.

When cash constraints bind, an increase in currency in circulation can be consonant with a reduction in wealth inequality. Binding cash constraints tend to reduce the spread in transfer payments. As a result, truthful revelation of income can be supported by a lower spread in continuation values compared to an economy without cash constraints but with the same level of income monitoring. When all cash constraints bind in the aggregate monetary economy, for instance, transfers and continuation values are identical across types for any given level of promised value and there is no scope for wealth inequality. At the same time, the central bank may find it optimal to adopt a loose monetary policy stance to alleviate transfer limits. In such situations, wealth inequality and currency in circulation share a negative relationship. This effect is more prominent at higher levels of wealth. I refer to this as the *transfer channel* of monetary policy.

Table 2 shows that currency in circulation in the aggregate monetary economy is larger than its state-contingent counterpart. Moreover, the incentive channel outweighs the transfer channel for a majority of households since the wealth distribution is Pareto under the baseline calibration. This implies a positive relationship between currency in circulation and wealth inequality. Intuitively, switching to a state-contingent setting allows the central bank to reduce transfer limits for high types without inhibiting risk sharing across types. This not only makes monitoring income levels of high types more effective, but also has the byproduct of lowering printing costs. Better income monitoring implies that the central bank ultimately needs to rely less on spreading wealth over time to distinguish between income groups.

4.4 Welfare

I now compute the gain in surplus realized by the RBI upon switching from an aggregate demonetization policy to a state-contingent one in the long-run. In particular, $[P(v^{\infty}) - P(v^{\infty})\mid_{\kappa_L=\kappa_H}]/\mathbb{E}_{\pi}[y] = 28.5\%$ where $v^{\infty} \in V$ denotes the long-run average of simulated promised values in the state-contingent demonetization economy. Figure 5 depicts how the value of these gains varies over the ergodic region of promised values. Observe that surplus gains from state-contingent monetary policy relative to the aggregate benchmark are non-monotone in promised values. Over the region where cash constraints are sufficiently tight, surplus gains are increasing in promised values.



Notes: Surplus gains are in percent of expected income. Gridlines on the y-axis capture the 10th, 25th, 50th, 75th and 90th percentiles of surplus gains.

Figure 5: Gains from State-contingent (relative to Aggregate) Demonetization

5 Extension: Labor Income Risk

This section considers a setting conducive to computation while avoiding trivial solutions in order to evaluate the sensitivity of optimal allocations to changes in (i) the endowment spread, and (ii) the efficacy of the monitoring technology. I examine optimal demonetization policy in the presence of labor income risk.[20] The economy is populated with a continuum of agents of unit measure that differ in labor productivity $\theta \sim \Theta$. Agents privately learn their productivity θ and then produce $n(\theta)$ efficiency units of labor. This requires $n(\theta)/\theta$ units of work effort. Labor disutility is captured by a mapping $v : \mathbb{R}_+ \mapsto \mathbb{R}_-$ that is increasing, convex, and differentiable. I assume the mechanism designer inherits a pre-defined labor schedule. This reduces the choice set of the central bank to just the consumption schedule and monetary policy. I further restrict attention to aggregate demonetization. The planning problem with privately observed heterogenous productivity and exogenous labor schedules is given by:

$$\max_{\{c(\theta),\kappa\}\geq 0} \quad \sum_{\theta\in\Theta} \pi(\theta)\left\{u\big(c(\theta)\big) - v\left(\frac{n(\theta)}{\theta}\right)\right\} \tag{7}$$

$$\text{s.t.} \quad \sum_{\theta\in\Theta} \pi(\theta)\{n(\theta) - c(\theta)\} \leq C(\kappa), \tag{8}$$

$$c(\theta) \leq F(\kappa) + n(\theta) \ \forall\theta \in \Theta, \tag{9}$$

$$u\big(c(\theta)\big) - v\left(\frac{n(\theta)}{\theta}\right) \geq \phi(\kappa)\left[u\big(c(\theta)\big) - v\left(\frac{n(\theta)}{\theta'}\right)\right] \ \forall(\theta,\theta') \in \Theta\times\Theta. \tag{10}$$

In the numerical exercise I use an affine transformation of the transfer limit used in the dynamic model, i.e. $F(x) = zx^{\alpha} + \chi$ and employ a binary skill distribution $\Theta = \{\theta_L, \theta_H\}$. Labor disutility is an affine transformation of an exponential function:[21]

$$v(x) = \frac{\exp x}{\exp\left(\frac{\sup n(\theta)}{\inf\theta}\right)} - 1.$$

The baseline parametrization is given by:

$$\{n(\theta_H), \pi(L), \theta_L, \theta_H, z, \alpha, \chi, \psi, \kappa_{max}\} = \{10, 0.5, 0.05, 1, 0.5, 1, -5, 10^{-4}, \sup n(\theta)/\theta\}.$$

Since $\alpha = 1$, arc elasticities of demonetization policy, κ, will coincide with arc elasticities of aggregate currency in circulation, $F(\kappa)$. This is useful when interpreting the results. The economy subject to feasibility, transfer limits, and incentive constraints is denoted by CE, while CE-C denotes the economy subject to only feasibility and transfer limits. E denotes the efficient

[20] Observe that in a static model without labor disutility, full insurance necessarily features in every incentive compatible allocation.

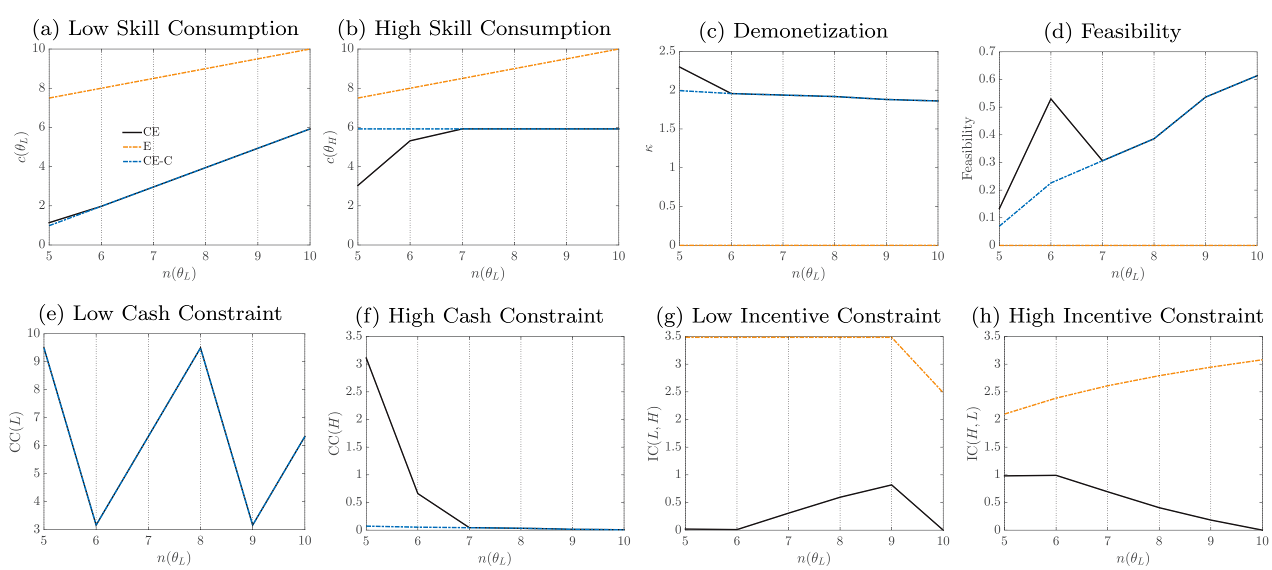
[21] The affine transformation is made to restrict the range of v to the negative orthant of the Euclidean space. This ensures that the income monitoring technology is well-defined.

economy without transfer limits.

A few patterns can be generalized irrespective of the comparative static exercises that follow. Incentive constraints are always slack in the efficient setting due to costless income monitoring; since C is monotonically increasing, setting $\kappa = 0$ weakly relaxes the constraint set without diverting any additional resources. Due to this full insurance is achieved in the efficient economy, $c(\theta_L) = c(\theta_H)$. This shows that incentive constraints have no bite in an economy without transfer limits. In contrast, the cash-constrained economies feature a positive spread between consumption allocations across types. This difference between allocations in CE and CE-C emerges due to binding incentive and cash constraints. Furthermore, resources are wasted in the cash-constrained economies, manifested by strictly slack feasibility conditions in CE and CE-C.

5.1 Sensitivity to Endowment Spread

Figure 6 depicts optimal allocations from the above program across a grid of the low type's efficiency units of labor, $n(\theta_L) \in [n(\theta_H)/2, n(\theta_H)]$. Observe that currency in circulation is strictly decreasing in $n(\theta_L)$. Moreover, consumption allocations in the efficient and cash-constrained economies are respectively strictly and weakly increasing in $n(\theta_L)$. This implies that welfare is monotonically increasing in the low type's efficiency units of labor when π has full support. As $n(\theta_L)$ is increased, monetary policy and welfare in the CE-C economy converge to the CE outcome.

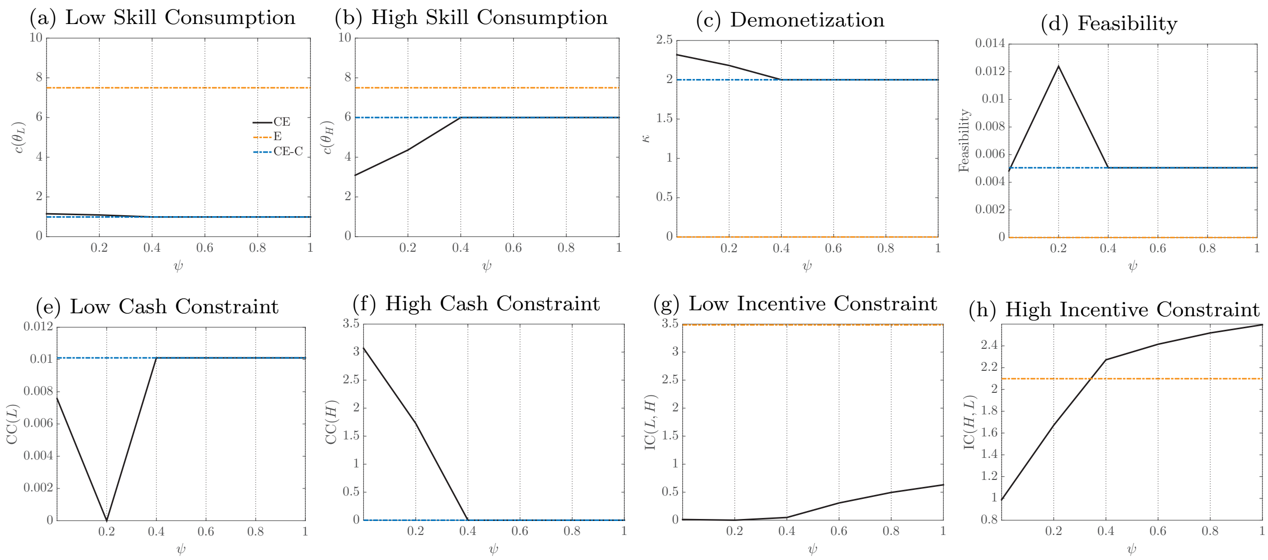


Notes: Gridlines on the x-axis signify the nodes at which optimal allocations are evaluated. The constraints are manipulated such that positive values correspond to slackness. The scale on the value of the cash constraint for the low type, CC(L), is 10^{-3}.

Figure 6: Optimal Allocation and Constraint Sets across Low Efficiency Labor Units

5.2 Sensitivity to Efficacy of Monitoring Technology

Figure 7 depicts optimal allocations across a grid of ψ, which is the parameter that controls the elasticity of the income monitoring technology with respect to currency in circulation. Notice that allocations for CE-C do not vary with ψ since the economy is devoid of informational frictions by assumption. When incentive constraints bind in CE, the central bank finds it optimal to reduce currency in circulation as the monitoring technology becomes more elastic. Over this region, the ψ-arc elasticity of currency in circulation is approximately $\frac{2.32-2}{2.16} \times \frac{0.2}{-0.4} = -0.07$. Moreover, consumption allocations in CE diverge as ψ increases. Thus, in the incentive-constrained region, welfare implications are ambiguous in general and depend on the skill distribution, $(\theta, \pi(\theta))$. An increase in ψ alleviates incentive constraints for both types, and beyond an endogenous threshold the CE and CE-C allocations coincide.



Notes: Gridlines on the x-axis signify the nodes at which optimal allocations are evaluated. The constraints are manipulated such that positive values correspond to slackness.

Figure 7: Optimal Allocation and Constraint Sets across Efficacy of Monitoring Technology

6 Conclusion

I develop a dynamic contracting model in which the degree of hidden income is jointly determined with transfer limits. Optimal monetary policy in this environment balances the insurance benefits of loose money with the informational costs of weak income monitoring. Under mild assumptions on the shape of the income monitoring technology and transfer limits, I show that state-contingent demonetization policy is weakly decreasing in reported endowment. Under complete information, I isolate conditions under which currency in circulation is increasing in promised values. A model calibrated to the Indian income process reveals that long-run gains in the surplus of the central bank upon switching to a state-contingent monetary policy from a non state-contingent one are 28.5% of aggregate income.

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Appendix

Proof of Lemma 1: Following steps similar to those outlined in the proof of Lemma 5, one can arrive at the following equation when F is linear $\forall s \in \mathcal{S}^b(v)$:

$$C''(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v} + \left[P''(v)u'(c_s(v)) + P'(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right]F'(\kappa_s(v)) = 0.$$

Differentiating w.r.t. v, we have $\forall s \in \mathcal{S}^b(v)$:

$$\begin{aligned}
&C^{(3)}(\kappa_s(v))\left[\frac{\partial \kappa_s(v)}{\partial v}\right]^2 + C''(\kappa_s(v))\frac{\partial^2 \kappa_s(v)}{\partial v^2} + F'(\kappa_s(v))\Bigg[P^{(3)}(v)u'(c_s(v))\\
&+2\left(P''(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right) + P'(v)u^{(3)}(c_s(v))\left(F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right)^2\\
&+P'(v)u''(c_s(v))F''(\kappa_s(v))\left(\frac{\partial \kappa_s(v)}{\partial v}\right)^2 + P'(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial^2 \kappa_s(v)}{\partial v^2}\Bigg]\\
&+\left[P''(v)u'(c_s(v)) + P'(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right]F''(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v} = 0.
\end{aligned}$$

Since F is linear by assumption, the above expression simplifies to:

$$\begin{aligned}
&C^{(3)}(\kappa_s(v))\left[\frac{\partial \kappa_s(v)}{\partial v}\right]^2 + C''(\kappa_s(v))\frac{\partial^2 \kappa_s(v)}{\partial v^2} + F'(\kappa_s(v))\Bigg[P^{(3)}(v)u'(c_s(v))\\
&+2\left(P''(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right) + P'(v)u^{(3)}(c_s(v))\left(F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right)^2\\
&+P'(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial^2 \kappa_s(v)}{\partial v^2}\Bigg] = 0.
\end{aligned}$$

Algebraic manipulation yields:

$$\begin{aligned}
\frac{\partial^2 \kappa_s(v)}{\partial v^2} = & -\{\overbrace{C''(\kappa_s(v))}^{>0 \text{ from A.5}} + \overbrace{P'(v)}^{=-\mu<0}\overbrace{u''(c_s(v))}^{<0 \text{ from A.2}}\overbrace{[F'(\kappa_s(v))]^2}^{>0}\}^{-1}\Bigg\{C^{(3)}(\kappa_s(v))\overbrace{\left[\frac{\partial \kappa_s(v)}{\partial v}\right]^2}^{>0}\\
&+\overbrace{F'(\kappa_s(v))}^{>0 \text{ from A.3}}\Bigg[P^{(3)}(v)\overbrace{u'(c_s(v))}^{>0 \text{ from A.2}} + 2\Bigg(P''(v)\overbrace{u''(c_s(v))}^{<0 \text{ from A.2}}\overbrace{F'(\kappa_s(v))}^{>0 \text{ from A.3}}\overbrace{\frac{\partial \kappa_s(v)}{\partial v}}^{>0 \text{ from Lemma 5}}\Bigg)\\
&+\overbrace{P'(v)}^{=-\mu<0}u^{(3)}(c_s(v))\overbrace{\left(F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right)^2}^{>0}\Bigg\}.
\end{aligned}$$

I now use this expression to guess and verify the strict concavity of P. Notice that if P is strictly concave, i.e. $P''(v) < 0$, then under the conditions $C^{(3)}, P^{(3)}, -u^{(3)} \geq 0$, we find that $\kappa_s(v)$ is strictly convex in v $\forall s \in \mathcal{S}^b(v)$. Moreover, $\frac{\partial^2 \kappa_s(v)}{\partial v^2} = \frac{\partial^2 b_s(v)}{\partial v^2}$ $\forall s \in \mathcal{S}^b(v)$. Now observe that under full information $\frac{\partial \kappa_s(v)}{\partial v} = \frac{\partial^2 \kappa_s(v)}{\partial v^2} = 0$ $\forall s \in \mathcal{S}^c(v)$ since C is increasing. Furthermore, since full insurance features in the first-best contract, we have $\frac{\partial b_s(v)}{\partial v} = \frac{\partial^2 b_s(v)}{\partial v^2} = 0 \; \forall s \in \mathcal{S}^c(v)$. Thus, the concavity of P follows as compositions of affine transformations of strictly convex and constant functionals are also strictly convex. $\square$

Proof of Lemma 2: A standard contraction mapping argument assures existence of equilibrium allocations as the Bellman operator corresponding to the functional equation satisfies Blackwell's sufficient conditions. Let the Lagrange multipliers on the promise keeping constraint and the cash constraint in state $s \in \mathcal{S}$ be denoted by $\mu \geq 0$ and $\eta_s \geq 0$ respectively. Denote the multiplier on incentive constraint $\mathcal{D}_{s,\hat{s}}$ by $\Lambda_{s,\hat{s}} \geq 0$. If the programming problem is strictly concave, the following first-order conditions are necessary and sufficient for global optimality:

$$b_s : \qquad \pi_s + \eta_s = \mu \pi_s u'(c_s) + \sum_{\hat{s} \in \mathcal{S} \setminus s} \left\{ \Lambda_{s,\hat{s}} u'(c_s) - \Lambda_{\hat{s},s} \phi(\kappa_{\hat{s}}) u'(y_{\hat{s}} + b_s) \right\} \tag{11}$$

$$w_s : \qquad \pi_s [P'(w_s) + \mu] = \sum_{\hat{s} \in \mathcal{S} \setminus s} \left\{ \Lambda_{s,\hat{s}} - \Lambda_{\hat{s},s} \phi(\kappa_{\hat{s}}) \right\} \tag{12}$$

$$\kappa_s : \qquad \pi_s C'(\kappa_s) = \eta_s F'(\kappa_s) - \sum_{\hat{s} \in \mathcal{S} \setminus s} \Lambda_{s,\hat{s}} \phi'(\kappa_s) [u(y_s + b_{\hat{s}}) + \beta w_{\hat{s}}] \tag{13}$$

The envelope condition is given by:

$$P'(v) = -\mu. \tag{14}$$

Substituting (14) in equations (11)–(12) we find that $\forall s \in \mathcal{S}$:

$$\pi_s [1 + P'(v) u'(c_s)] + \eta_s = \sum_{\hat{s} \in \mathcal{S} \setminus s} \left\{ \Lambda_{s,\hat{s}} u'(c_s) - \Lambda_{\hat{s},s} \phi(\kappa_{\hat{s}}) u'(y_{\hat{s}} + b_s) \right\} \tag{15}$$

$$P'(w_s) = P'(v) + \frac{1}{\pi_s} \sum_{\hat{s} \in \mathcal{S} \setminus s} \left\{ \Lambda_{s,\hat{s}} - \Lambda_{\hat{s},s} \phi(\kappa_{\hat{s}}) \right\} \tag{16}$$

The above conditions, coupled with the complementary slackness conditions corresponding to the inequality constraints, completely characterize the solution to the planner's problem. $\square$

Proof of Lemma 3: By way of contradiction, suppose in an optimal allocation $\exists s' \in \mathcal{S} : \kappa_{s'} < \kappa_{max}$. By the completeness of K, $\exists \epsilon > 0 : \kappa_{s'} + \epsilon < \kappa_{max}$. The perturbed optimal allocation where the demonetization policy in state s' is increased by ϵ weakly relaxes the constraint set without affecting the objective of the planner as $C(.) = 0$. This contradicts the optimality of the original contract. $\square$

Proof of Lemma 4: We want to show that

$$(y_s - y_{\hat{s}})(\kappa_s(v) - \kappa_{\hat{s}}(v)) \leq 0 \;\; \forall v \in V \; \forall (s, \hat{s}) \in \mathcal{S} \times \mathcal{S},$$

with strict inequality over the particular region of the state-space defined in the statement of the lemma. Adding $\mathcal{D}_{s,\hat{s}} \geq 0$ and $\mathcal{D}_{\hat{s},s} \geq 0$ we get:

$$\begin{aligned} u(y_s + b_s(v;\underline{\phi})) - \phi(\kappa_{\hat{s}}(v;\underline{\phi}))u(y_{\hat{s}} + b_s(v;\underline{\phi})) + [1-\phi(\kappa_{\hat{s}}(v;\underline{\phi}))]\beta w_s(v;\underline{\phi}) \\ \geq \phi(\kappa_s(v;\underline{\phi}))u(y_s + b_{\hat{s}}(v;\underline{\phi})) - u(y_{\hat{s}} + b_{\hat{s}}(v;\underline{\phi})) + [1-\phi(\kappa_s(v;\underline{\phi}))]\beta w_{\hat{s}}(v;\underline{\phi}). \end{aligned}$$

Observe

$$u(y_s + b_s(v;1)) - u(y_{\hat{s}} + b_s(v;1)) \geq u(y_s + b_{\hat{s}}(v;1)) - u(y_{\hat{s}} + b_{\hat{s}}(v;1)).$$

As u is strictly concave,

$$(y_s - y_{\hat{s}})(b_s(v;1) - b_{\hat{s}}(v;1)) < 0.$$

Since ϕ and u are continuous,

$$\exists \underline{\phi}' \in (0,1] : (y_s - y_{\hat{s}})(b_s(v;\underline{\phi}) - b_{\hat{s}}(v;\underline{\phi})) < 0 \ \forall \underline{\phi} \geq \underline{\phi}'.$$

For the remainder of the proof, we assume $\underline{\phi} \geq \underline{\phi}'$ and drop explicit reference of the same. The state-space V can be divided into three regions (i) promised values where all cash constraints bind: $[\bar{v}, \sup u(y_s)/(1-\beta)]$, (ii) promised values where some but not all cash constraints bind: $[\underline{v}, \bar{v}]$, and (iii) promised values where all cash constraints are slack: $[\inf u(y_s)/(1-\beta), \underline{v}]$.

Region 1: Suppose $v \in [\bar{v}, \sup u(y_s)/(1-\beta)]$. We now show that transfers and demonetization policies co-move in cash constrained states. Since $\eta_s(v), \eta_{\hat{s}}(v) > 0$, by differencing the corresponding complementary slackness conditions we have:

$$b_s(v) - b_{\hat{s}}(v) = F(\kappa_s(v)) - F(\kappa_{\hat{s}}(v))$$

Multiplying by $\kappa_s(v) - \kappa_{\hat{s}}(v)$ on both sides:

$$(b_s(v) - b_{\hat{s}}(v))(\kappa_s(v) - \kappa_{\hat{s}}(v)) = [F(\kappa_s(v)) - F(\kappa_{\hat{s}}(v))](\kappa_s(v) - \kappa_{\hat{s}}(v))$$

Notice that the R.H.S. of the above equation is strictly positive as F is increasing. Thus,

$$(b_s(v) - b_{\hat{s}}(v))(\kappa_s(v) - \kappa_{\hat{s}}(v)) > 0.$$

This shows that demonization policies are strictly decreasing in reported endowment over $[\bar{v}, \sup u(y_s)/(1-\beta)]$.

Region 2: Suppose $v \in [\underline{v}, \bar{v}]$. Using the first-order condition for demonetization policy

$$\begin{aligned} \pi_s C'(\kappa_s(v)) \ \geq \ -\sum_{\hat{s}\in\mathcal{S}\setminus s} \Lambda_{s,\hat{s}}(v)\phi'(\kappa_s(v))[u(y_s + b_{\hat{s}}(v)) + \beta w_{\hat{s}}(v)] \\ \forall s \in \mathcal{S}^c(v), \text{ w/ equality if } \kappa_s(v) > 0. \end{aligned}$$

As $C', \phi', \Lambda_{s,\hat{s}} > 0 \ \forall (s,\hat{s}) \in \mathcal{S} \times \mathcal{S}$, this necessarily implies:

$$\kappa_s(v) = 0 \ \forall s \in \mathcal{S}^c(v).$$

Similarly,

$$\pi_s C'(\kappa_s(v)) \;\geq\; \eta_s(s)F'(\kappa_s(v)) - \sum_{\hat{s}\in\mathcal{S}\setminus s} \Lambda_{s,\hat{s}}(v)\phi'(\kappa_s(v))[u(y_s+b_{\hat{s}}(v))+\beta w_{\hat{s}}(v)]$$
$$\forall s\in\mathcal{S}^b(v),\ \text{w/ equality if } \kappa_s(v)>0.$$

Notice that again for $\underline{\phi}$ large enough, $\phi' \approx 0$. Then, since $C'(0)/F'(0)=0$ and $\pi_s>0\ \forall s\in\mathcal{S}$, we have

$$\kappa_s(v)>0\ \forall s\in\mathcal{S}^b(v).$$

Therefore $\forall (s,\hat{s})\in\mathcal{S}^c(v)\times\mathcal{S}^b(v)$

$$(\eta_s(v)-\eta_{\hat{s}}(v))(\kappa_s(v)-\kappa_{\hat{s}}(v))>0.$$

Moreover,

$$(\eta_s(v)-\eta_{\hat{s}}(v))(b_s(v)-b_{\hat{s}}(v))>0.$$

since by complementary slackness we have

$$\begin{aligned}\forall s\in\mathcal{S}^c(v)\ \eta_s(v)=0 \implies b_s(v) &\leq 0,\\ \forall s\in\mathcal{S}^b(v)\ \eta_s(v)>0 \implies b_s(v) &> 0.\end{aligned}$$

Thus, $(y_s-y_{\hat{s}})(\eta_s(v)-\eta_{\hat{s}}(v))<0$ which implies

$$(y_s-y_{\hat{s}})(\kappa_s(v)-\kappa_{\hat{s}}(v))<0\ \ \forall(s,\hat{s})\in\mathcal{S}^c(v)\times\mathcal{S}^b(v).$$

To show that

$$(y_s-y_{\hat{s}})(\kappa_s(v)-\kappa_{\hat{s}}(v))<0\ \ \forall(s,\hat{s})\in\mathcal{S}\times\mathcal{S}^b(v)$$

we can apply the argument analogous to that used in Region 1 when comparing demonetization policies *within* the set of cash constrained states.

Region 3: Suppose $v\in[\inf u(y_s)/(1-\beta),\underline{v}]$. Since $\eta_s(v)=0\ \forall s\in\mathcal{S}$ in this region and $C'>0$ we have $\kappa_s(v)=0\ \forall s\in\mathcal{S}$. □

Proof of Lemma 5: Given $v\in V$, in every interior equilibrium under full information the first-order condition for consumption yields $\forall s\in\mathcal{S}$:

$$\pi_s[1+P'(v)u'(c_s(v))]+\eta_s(v)=0. \tag{17}$$

Moreover, the optimality condition for the monetary policy instrument yields $\forall s\in\mathcal{S}$:

$$\pi_s C'(\kappa_s(v))=\eta_s(v)F'(\kappa_s(v)). \tag{18}$$

From complementary slackness, if $\eta_s(v)>0\ \forall s\in\mathcal{S}^b(v)$, we have

$$b_s(v)=c_s(v)-y_s=F(\kappa_s(v))\ \forall s\in\mathcal{S}^b(v). \tag{19}$$

Equations (17), (18) and (19) can be combined to arrive at one equation in the monetary policy instrument $\forall s \in \mathcal{S}^b(v)$:

$$C'(\kappa_s(v)) + [1 + P'(v)u'(F(\kappa_s(v)) + y_s)]F'(\kappa_s(v)) = 0. \tag{20}$$

Differentiating w.r.t. v:

$$C''(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v} + \left[P''(v)u'(c_s(v)) + P'(v)u''(c_s(v))F'(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v}\right]F'(\kappa_s(v))$$
$$+[1 + P'(v)u'(c_s(v))]F''(\kappa_s(v))\frac{\partial \kappa_s(v)}{\partial v} = 0 \;\forall s \in \mathcal{S}^b(v).$$

Algebraic manipulation yields $\forall s \in \mathcal{S}^b(v)$:

$$\frac{\partial \kappa_s(v)}{\partial v} = \frac{-P''(v)\overbrace{u'(c_s(v))}^{>0 \text{ from A.2}}\overbrace{F'(\kappa_s(v))}^{>0 \text{ from A.3}}}{\underbrace{C''(\kappa_s(v))}_{>0 \text{ from A.5}} + \underbrace{P'(v)}_{=-\mu<0}\underbrace{u''(c_s(v))}_{<0 \text{ from A.2}}\underbrace{[F'(\kappa_s(v))]^2}_{>0} + \underbrace{[1 + P'(v)u'(c_s(v))]}_{=-\frac{\eta_s(v)}{\pi_s}<0 \text{ from (17)}}\underbrace{F''(\kappa_s(v))}_{<0 \text{ from A.3}}}.$$

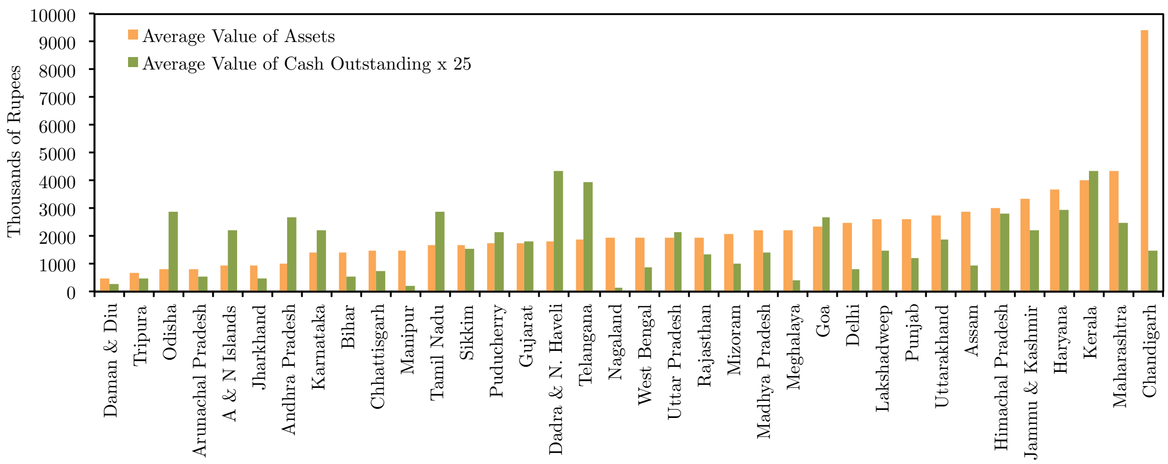
Observe that the above expression is strictly positive since P is concave, which completes the proof. □

Proof of Lemma 6: I first show that w_s is continuous. This follows from Theorem 9.6 in Lucas and Stokey (1989), which holds since the following conditions are satisfied: (i) the possible values of the endogenous state variables V is a convex Borel set in $\mathbb{R}$, (ii) the exogenous state space, S, is finite and hence a countable set, (iii) the incentive-feasible correspondence is non-empty (since the endowment shocks are strictly increasing), compact-valued and continuous, (iv) the function $\sum_{s=1}\pi_s(C(\kappa_s) + b_s)$ is bounded and continuous, and (v) $\beta \in (0, 1)$. Let $\epsilon \equiv \min_{s\in S}\pi_s$. Since π has full support, $\forall N \in \mathbb{Z}^+ \;\forall v \in V \;\; \mathcal{Q}^N(v, \inf u(c)/(1-\beta)) \geq \epsilon > 0$. Thus, the result follows from Theorem 11.12 in Lucas and Stokey (1989). □

Supplementary Appendix

Evidence on Non-Monotonicity of Money in Wealth

Without binding incentive constraints of low type agents over a non-zero measure of the state space, transfers would be perfectly (positively) correlated with wealth in the model (as in TW). In the data, however, cash transfers are non-monotonic in asset positions. Figure 8 compares the average value of total assets with the average value of cash outstanding across Indian States and Union Territories. The data extract comes from the National Sample Survey Organization's (NSSO) All India Debt and Investment Survey (AIDIS).[22] The data is presented in ascending order of total asset values. Observe that outstanding average cash balances are non-monotone in average asset holdings, which is consistent with the policy functions obtained from the model at the baseline calibration.



Source: NSS0 (70/18.2) Key Indicators of Debt and Investment in India. The data reflects estimates as on 30th June 2012.

Figure 8: Average Value of Assets and Cash Loan Outstanding (Rs.) in each State/UT: Urban

[22] AIDIS is a decennial survey conducted by the NSSO since 1971, with a roughly 0.01% sample of the Indian population, through a multi-stage design.

Lorenz Curve

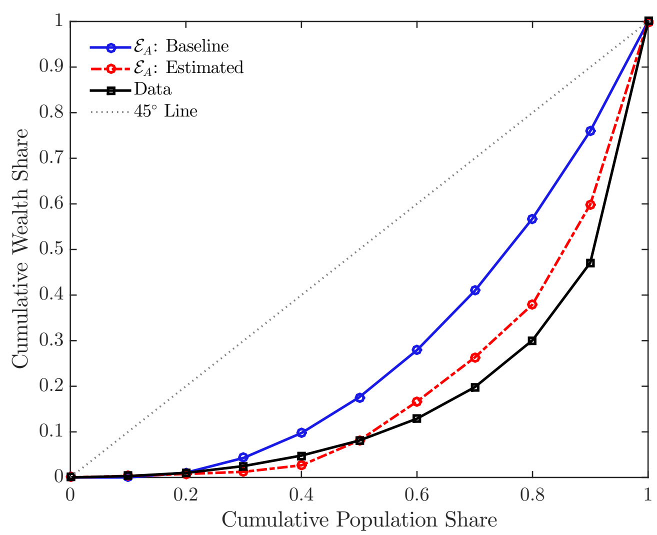
The baseline calibration targets only one observation on the Lorenz curve, i.e. the wealth share of the bottom 20% of the population. Alternatively, one could discipline the entire set of endogenously estimated parameters using additional information from the Lorenz curve. In particular, the AIDIS contains data on cumulative wealth shares for each decile, providing 9 sample moments. Since the number of parameters to be estimated is less than the number of moment conditions, I use GMM to estimate the unknown parameters (Hansen, 1982).

Let $\gamma \equiv (z, \alpha, \psi) \in \mathbb{R}^2_{++} \times [0,1] \equiv \Gamma$. Let Ω_i denote a 1×9 vector of moments comprising of the wealth shares of the bottom $n\%$ of the population $\forall n \in \{10, 20, ..., 90\}$ where $i \in \{\text{Model}, \text{Data}\}$. The GMM estimator is given by:

$$\hat{\gamma} \in \text{argmin}_{\gamma \in \Gamma} [\Omega_{\text{Model}}(\gamma) - \Omega_{\text{Data}}]' W [\Omega_{\text{Model}}(\gamma) - \Omega_{\text{Data}}],$$

where W is a 9×9 weighting matrix. Using an identity weighting matrix, this procedure delivers the following estimates for the unknown parameters: $\hat{\gamma} = (1.1, 0.8, 0.0008)$.

The model estimated using $\hat{\gamma}$ does a good job of matching the data. This can be seen vividly in figure 9, which compares the observed Lorenz curve for India with that simulated in two economies: (i) the aggregate monetary economy under the baseline parametrization, and (ii) the aggregate monetary economy under $\hat{\gamma}$. As observed in the data, both economies feature substantial wealth inequality, which can be graphically seen in their departure from the 45 degree line. However, outcomes under the baseline calibration underestimate the concentration of wealth at the top of the distribution. The aggregate monetary economy under the baseline parametrization can account for 83% of the variation observed in the data, whereas the model estimated using $\hat{\gamma}$ can account for 94% of the variation.



Notes: $\mathcal{E}_A$: Estimated employs the baseline calibration barring $(z, \alpha, \psi) = (1.1, 0.8, 0.0008)$. The data is extracted from the AIDIS.

Figure 9: Lorenz Curves