# Limited Credit Records and Endogenous Credit Cycles[*]

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**Abstract**

In a dynamic credit market model with unknown borrowers' types a credit bureau reports positive and negative records about their past for a limited timespan.[1] A stationary equilibrium with lending exist only if negative records are kept sufficiently long while positive ones are erased soon enough. If these conditions are not satisfied and the average quality of borrowers is high, the credit market experiences endogenous and persistent lending cycles in equilibrium in the absence of any aggregate shocks. Cycles stem from the amount of information available

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[1]Retention limits vary across countries and records, and these limits have sizable short-term effects.

at the credit bureau, and corresponding endogenous changes in optimal lending standards for borrowers with no observable records: switches between phases of lending and no-lending to these borrowers generate large fluctuations in aggregate lending. We characterize cycles and their relation with records retention limits. When cycles and the stationary equilibrium co-exist, the latter delivers higher welfare.
*Keywords*: Credit records, lending standards, lending booms, credit bureaus, credit registers, data retention, endogenous cycles, deterministic cycles, financial stability, privacy rules.
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# Introduction

The recent financial crisis highlighted the importance of financial markets and financial frictions for modern economies. It has also once again suggested that financial markets may be the very origin of economic instability and fluctuations, and not only amplifiers of shocks in fundamentals as in most of the classic literature surveyed e.g. in Gertler et al. (2010).

In this paper we present a stylized continuous-time model of a credit market where an informational friction between lenders and borrowers combined with common institutional features of credit markets generates fully endogenous, deterministic and persistent credit cycles. The informational friction we focus on is incomplete information about borrowers' quality. The institutional feature that interacts with this friction and partially solves it is a standard information intermediary, like a credit bureau, that displays records of borrowers past performance for some time and then deletes them. This is an important feature in real world credit markets, as in all countries we are aware of where a private or public credit bureau or a public credit provide access to borrowers credit history, either for internal policies or for regulatory reasons this access is limited in time: after a certain time, each borrower record are removed. Retention limits vary wildly across countries, markets, and type of records, and a growing amount of empirical evidence indicates that they are binding; i.e. other information sources do not make them irrelevant, as the expiration or changes of these limits have economically large effects on credit market outcomes and beyond.[2]

In our model there are long-lived "good" and "bad" borrowers that each period may or may not have a project to finance. When they do, they are randomly matched with competitive lenders. If a loan is granted, the lender reports the outcome (default, repayment) to an information intermediary, e.g. a credit bureau. The information intermediary records and makes publicly available past borrowers' performance. After a

[2]Sizable effects on credit market outcomes of expiring credit records retention limits or exogenous changes in such limits are documented e.g. in Musto (2004), Bos and Nakamura (2014), Liberman (2016), Liberman et al. (2017) and Cahn et al. (2017). Bos et al. (forthcoming) also document large additional spillovers on labor markets: using a natural experiment in Sweden they estimate that one additional year of negative credit information reduces employment by 3% and wage earnings by $1,000. Dobbie et al. (2016) show that the removal of a Chapter 13 bankruptcy flag in the US leads to a large increase in credit scores and an economically significant increase in credit card balances and mortgage borrowing, but find instead no effects on labor market outcomes.

finite time past records are erased/removed from public access, and this time may differ for positive and negative records, as is the case for credit bureaus in all countries about which we have information,

We show that the model can have both stationary and non-stationary equilibria. When the average quality of borrowers in the population is relatively high and the memory of the credit bureau is sufficiently long, however, the stationary equilibrium with lending does not exist, and an endogenous, persistent, deterministic credit cycle is the only possible equilibrium.

The intuition is as follows. When lenders finance borrowers, after the investment new information is produced on these borrowers' quality: borrowers either default or repay and the corresponding record, negative or positive, appears in the credit bureau. Suppose the market starts with no information about borrowers, so that all borrowers are in the same pool of "unknown" borrowers. Then lending produces information about an increasing fraction of borrowers, which is recorded by the credit bureau and which the lenders have access to. Borrowers with negative past records are recognized to be "bad" and are excluded from further credit while lenders continue financing "unknown" and "good" borrowers. While this phase of partial separation unfolds, both "bad" and "good" borrowers are leaving the pool of "unknown" borrowers. Given that the information intermediary publishes past records for a finite period of time and that borrowers have projects to fund stochastically, eventually each borrower comes back to the pool of "unknown" borrowers when his last record is deleted. Suppose the credit bureau deletes positive past records after time $T^+$ and negative ones after time $T^-$. The longer good types are kept out of the "unknown" pool (the longer memory of positives records) and the faster bad types go back to the "unknown" pool (the shorter memory of negative records), the worse will be the average quality in the "unknown" pool.

With long enough memory for positive records - relative to that of negative ones - the "flight to quality" dominates the separation of "bad" borrowers so that the quality of the "unknown" pool will worsen with time, up to the point that lending to "unknown" borrowers becomes unprofitable. At this moment, a regime switch occurs from lending (boom, pooling phase) to no-lending (bust, separating phase) to "unknown" borrowers. After the regime switch only borrowers with good records can get funded, with a connected large drop in aggregate credit.

This no-lending (bust) phase lasts until a sufficient number of good borrowers get their past records deleted by the credit bureau, become "forgotten" and enter the "unknown" pool again. Then the average quality of the unknown pool increases, lending to that pool becomes profitable again, and the regime switches back to lending (boom, pooling phase). Since in the no lending phase only borrowers with positive credit records obtain credit, the negative phases of the cycle (busts) will be associated to tighter lending standards adopted by lenders, while loser lending standards are applied during positive phases of the credit cycles.

Our model is motivated by stylized facts about information intermediaries like credit bureaus in the real world. As already mentioned, in most modern credit markets credit registers collect and share data on borrowers' past behavior with other market participants, in an effort to reduce the well-known informational asymmetries that characterize lending, but erase them after a number of years Jappelli and Pagano (2002). Our model follows Kovbasyuk and Spagnolo (2016) in distinguishing positive and negative records, both because their effects on the economy turn out to be very different and because it is indeed a main dimension along which the information provided in reality by credit bureaus is classified and varies. Some registers record and make available only borrowers' past deficiencies - i.e. negative, or "black" information (credit remarks, past arrears, defaults and bankruptcies); others also collect positive, or "white" information that describes the borrower's overall position (patterns of repayments, open and closed credit accounts, new loans, debt maturity, guarantees and assets).[3]

As for the memory, negative records are erased after a certain number of years, typically in accordance with privacy protection laws designed to enable people to make a fresh start. Positive records, where collected, are typically not subject to this regulation, but they are also erased after a number of years. While up to 90 percent of the countries where a credit register is operating adopt privacy provisions limiting its memory for negative records, the limit varies wildly, ranging from one or two years in Bangladesh, Malaysia and Pakistan, to three years in Germany, Italy and Sweden, up to ten in the US and fourteen and fifteen in Russia, Canada and Greece. Positive records are again erased after a limited number of years, ranging from two in New Zealand, South Africa and Greece, to ten in the US, to twenty in Canada and twenty five in Colombia.

Let us now describe with a little more details our main results. It is known that in environments like the one we study, under certain conditions there can be stationary equilibria with trade (Kovbasyuk and Spagnolo (2016)), and we start characterizing the

[3]In about half of the countries on which we have information both positive and negative credit records are available, and in the remaining half only negative records are (Bottero and Spagnolo (2012)).

conditions under which this is the case in the present model as a benchmark for what will follow (Theorem 1). We also show that also in this environment the length of the memory of positive and negative records play opposite roles with respect to the existence of stationary equilibria, and at a different intensity (Corollary 1).

We then proceed to study non-stationary equilibria, the focus of this paper. Our first main result (Theorem 2) shows that whenever the average quality of borrowers is neither too low (so that a stationary equilibrium with no lending does not exist) nor too high (so that a stationary equilibrium with lending does not exist), our economy will be in a regime switching equilibrium, i.e. will display endogenous, deterministic, and persistent credit cycles in the absence of any exogenous shock. The logic behind the theorem is the one we described earlier and will be described in further details in the coming sections. It is important to note though, that the result characterizes sufficient conditions, not necessary ones. This means that our model may have equilibria displaying endogenous credit cycles also in other parameter spaces.

We then analyze infinitely repeated credit cycles where each period of lending (no lending) is identical in terms of distributions of borrowers with different records over time. In other words, the credit market goes through exactly the same cycle over and over again indefinitely. Our second main result regards markets where the memory of negative records is at least as long as the memory of positive records, a condition satisfied for most, although not all countries and credit bureaus.[4] Under this assumption, we characterize sufficient conditions under which ergodic limit cycles, repeating themselves identically forever, exist (Theorem 3). The conditions ensure that periods of lending and of no lending are finite but also long enough for the fractions of good and bad borrowers in the pool of unknown ones, the driving forces behind cyclicality, to stabilize. We also show that there is a non-negligible set of the parameter space in which both stationary and ergodic cycle equilibria co-exist (Proposition 1) so that initial conditions like the diistribution of records will determinein which equilibrium the economy will end up. We then characterize how the frequency of these cycles is affected by the parameters of the economy by studying comparative statics (Corollary 2), showing that the memory of

[4]All countries where the credit bureau do not collect positive records satisfy this condition trivially, as no positive records can be thought of zero memory. Of the countries we are aware of where the credit bureaus collect both positive and negative information, positive records are stored longer than negatives only in very few cases (for records of bankruptcies, we are aware of two cases, Canada and Colombia).

negative records plays a crucial role: both lending and no lending regimes last longer than the time negative records are retained, while lending regimes lasts no longer than twice that time.

Finally, we characterize welfare for both stationary and ergodic cyclical equilibria and compare it for the parameter space in which both exist, showing that welfare is always higher in stationary equilibria than in cyclical ones (Theorem 4), so that a public intervention that would move the economy from an ergodic cyclical equilibrium to a stationary one at low cost would be justified.

Our paper contributes to a recent strand of literature showing how financial market frictions, limited commitment/moral hazard (Gu et al. (2013), Azariadis et al. (2015), Myerson (2012)) or adverse selection (Martin (2004), Benhabib et al. (2014)) may indeed be the very origin of cyclical or chaotic economic dynamics. Within this strand, we are perhaps closest to Martin (2004), where regime switches in lender policy analogous to those emerging in our model are caused by endogenous changes in borrowers' wealth, value of collateral, and relative cost of screening vs pooling. The mechanism we identify is very different, however, and we focus on unsecured debt rather than on collateralized loans. This is an important difference in the light of Azariadis et al. (2015) who have recently shown, using U.S. data from the period 1981–2012, that it is mostly unsecured firm credit that moves procyclically and tends to lead GDP, while secured firm credit is instead almost acyclical.

Although the mechanisms at work in our model are very different, the connection between asymmetric information on borrowers' types, changes in lending standards and economic fluctuations they produce relate closely our model to of the work of Dell'Ariccia and Marquez (2006) and Gorton and He (2008), where banks reduce screening/loosen lending standards offering credit to the pooled set of borrowers when the externality in competitive screening identified in Broeker (1990) is softened respectively by the entry of new (non-screened) borrowers or collusion among banks, respectively.

Our paper is probably closest, at least in spirit, to a recent paper by Liu and Skrzypacz (2014) where in a very different environment - a classic reputation model - and for rather different reasons - incentive issues, absent from our environment - limited records generate “reputation bubbles”.

# 1 Environment

Consider an economy populated by numerous competitive lenders and a unit mass of borrowers indexed by $i \in [0,1]$. All agents are risk neutral. Time is continuous $t \in [0,\infty)$. Lenders have enough resources to finance all potential borrowers at any period $t$. Each period $t$ a borrower $i$ may face an investment opportunity that requires an investment of 1 in the beginning of the period that delivers $R > 1$ at the end of the period, if successful, and 0 otherwise. The arrival of investment opportunities to each borrower is described by a Poisson process with arrival intensity $\lambda$. The investment opportunities are short-lived: if not implemented immediately they disappear. When a borrower faces an investment opportunity he seeks a loan on the market, and is matched with a lender. If the loan is granted the lender receives fraction $\varepsilon \in (0,1)$ of the net expected surplus generated by the investment opportunity. To simplify calculations we focus on the case of competitive lenders and consider $\varepsilon \to 0$.

Borrowers have no wealth to begin with and when they successfully exercise the investment opportunity they consume all the proceeds, so that they start with no resources in the next period.

Each borrower can either be of good type ($G$) or of bad type ($B$). The fraction of good borrowers in the population is $\mu$ and that of bad borrowers $\eta = 1 - \mu$. Good borrowers have good projects, i.e. investiment opportunities that always succeed, and always repay their loans. Bad borrowers' projects always fail and lead to default. For tractability we assume that projects returns realize instantaneously: as soon as the borrower gets financing the final returns realize. For simplicity we also assume that the borrower is never matched with the same lender twice, and that the lenders' only source of information about the borrower's past is the credit bureau (credit register).

Whenever a borrower defaults, the lender truthfully reports it and the credit register records a "negative" $D$ record. Conversely, whenever the borrower repays a loan, the lender reports it the credit register records a "positive" $S$ record. As in reality the credit register makes past records accessible for lenders but deletes past records after a specified period of time.

**Assumption 1.** *Negative records are stored for time interval $T^- \in [0,\bar{T}]$, while positive records are stored for time interval $T^+ \in [0,\bar{T}]$.*

Given that in reality the memory of credit registers is never infinite, we require $\bar{T} < \infty$. Moreover, Kovbasyuk and Spagnolo (2016) showed that an information intermediary with limited memory may be optimal. A credit history $h_i^t$ of any borrower $i \in [0,1]$ at any $t \in [0,\infty)$ contains records of the borrower's past defaults that happened between $t - T^-$ and $t$, and the borrower's successful repayment records that happened between $t - T^+$ and $t$. Note, that if a borrower has not borrowed for more than $\max[T^-, T^+]$ his credit history is empty, and we denote his record by $N$ (empty record).

As will become clear from the subsequent analysis the dynamics of the lending market will be determined by the model parameters and by the initial histories $h_i^0$, $i \in [0,1]$. For brevity we refer to the space of all possible model parameters by $\Omega = \{(\mu, \lambda, R, T^-, T^+)\} \subset (0,1) \times R_+^2 \times [0, \bar{T}]^2$, and by $\omega \in \Omega$ to a particular combination of these parameters.

**Credit Market Equilibrium.** For each combination of model parameters $\omega \in \Omega$ and initial histories the equilibrium in the credit market is characterized as follows. Lenders observe credit histories $h_i^t$ for all borrowers $i \in [0,1]$ at any $t \in [0,\infty)$ and form beliefs about the borrowers type, which determined by the posterior probability $\pi(h_i^t) \in [0,1]$ that the borrower is of good type. If a borrower $i \in [0,1]$ faces an investment opportunity at $t$ he asks for a loan. Competitive lenders get a small fraction $\varepsilon \to 0$ of the investment's net expected value, and provide the loan if this value is non-negative, i.e. if $\pi(h_i^t)R \geq 1$. In this case they charge the rate of interest $R_i^t = 1/\pi(h_i^t)$. If the net expected value is exactly zero $\pi(h_i^t)R = 1$ the lenders are indifferent about granting the loan or not and do so with probability $x \in [0,1]$. Once the loan is granted to a borrower $i$, he finances his project and the project's returns realize. If borrower $i$ is bad, he defaults, lenders get nothing and a $D$ record is recorded in the credit history of borrower $i$. If borrower $i$ is good, he succeeds, lenders get the interest $R_i^t$ and an $S$ record is recorded in the credit history of borrower $i$. Lenders update their beliefs about borrowers using the Bayes' rule.

Within this credit market framework, many results that were established in Kovbasyuk and Spagnolo (2016) hold. First, it is easy to see that the only information relevant for the lenders is the latest record in the borrower's history. Indeed, if the latest record of a borrower is a default record and default happened $\tau^- \in [0, T^-]$ periods ago his history is summarized by a record $D(\tau^-)$, the lenders' posterior $\pi(D(\tau^-)) = 0$ and the borrower can't get a loan. On the other hand, if the latest record is a success record and success happened $\tau \in [0, T^+]$ periods ago his history is summarized by a record $S(\tau)$, the lenders' posterior is $\pi(S(\tau)) = 1$ and the borrower can get a loan. Finally, if the credit history of a borrower has no default or success records, the lenders' posterior $\pi(N)$ depends on the composition of the pool of borrowers that have no default record in the last $T^-$ periods and no success records in the last $T^+$ periods. For brevity we say that these borrowers have an empty record denoted by $N$. The composition of the pool of borrowers with an $N$ is key for the credit market dynamics, if the average quality of this pool is high enough $\pi(N)R \geq 1$ a borrower from that pool can get a loan, if the average quality of this pool is low $\pi(N)R < 1$ no borrower from that pool can get a loan. The composition of that pool in turn depends on past credit market dynamics, on record keeping limits $T^-$ and $T^+$, and on the joint distribution of borrowers' types and records $\mathbf{\Delta}_t = \{\mu_t^N, \mu_t^S(.), \eta_t^N, \eta_t^D(.)\}$. Here, $\mu_t^N$ and $\eta_t^N$ are masses of good and bad borrowers with an $N$ record at time $t$, the density function $\mu_t^S(\tau)$, $\tau \in [0, T^+]$ determines the mass of good borrowers with record $S_t(\tau)$ at time $t$, and the density function $\eta_t^D(\tau^-)$, $\tau^- \in [0, T^-]$ determines the mass of bad borrowers with record $D(\tau^-)$ at time $t$.

Given the model parameters $\omega \in \Omega$, the initial distribution of types and records $\mathbf{\Delta}_0 = \{\mu_0^N, \mu_0^S(.), \eta_0^N, \eta_0^D(.)\}$ pins down the equilibrium evolution of the distribution $\mathbf{\Delta}_t$. Note, that $\mu_t^N + \int_0^{T+} \mu_t^S(\tau)d\tau = \mu$ and $\eta_t^N + \int_0^{T-} \eta_t^D(\tau^-)d\tau^- = 1 - \mu$ for any $t \in [0, \infty)$.

# 2 Stationary Equilibrium

As a starting point of the analysis we establish several results about the stationary credit market equilibrium that or model admits under certain conditions. Naturally, a **stationary equilibrium** in our model is the situation when the joint distribution of types and records is time invariant $\mathbf{\Delta}_t = \mathbf{\Delta}_{ST}$.

An example of a stationary equilibrium is one in which there is no lending and all borrowers have an $N$ record. It is easy to check that such an equilibrium exists if the average quality of borrowers is low $\mu R < 1$. Stationary equilibria have been studied in Kovbasyuk and Spagnolo (2016) in a more general environment, where it has been shown that a stationary equilibrium with trade exists if and only if negative records are kept for a long time while positive records are deleted relatively fast. Analogous results hold in the credit market model studied here. The following theorem describes all possible stationary equilibria. Besides the one with no trade there are 2 more cases.

**Theorem 1.** *A stationary equilibrium with no lending exists with interest rate*

$$R_{NL}(N) = 1/\mu$$

*and distribution* $\mathbf{\Delta}_{NL}$

$$\begin{aligned} \mu^N_{NL} = \mu,\ \mu^S_{St}(\tau) = 0,\ \tau \in [0, T^+], \\ \eta^N_{NL} = 1-\mu,\ \eta^D_{NL}(\tau) = 0,\ \tau \in [0, T^-], \end{aligned} \tag{1}$$

*Two stationary equilibria with lending exist:*

*A pure strategy equilibrium with the interest rate*

$$R_L(N) = 1 + \frac{1-\mu}{\mu}\frac{e^{\lambda T^+}}{1+\lambda T^-},$$

*and the stationary distribution* $\mathbf{\Delta}_L$

$$\begin{aligned} \mu^N_L = \mu e^{\lambda T^+},\ \mu^S_L(\tau) = \lambda\mu e^{\lambda\tau},\ \tau \in [0, T^+], \\ \eta^N_L = \tfrac{1-\mu}{1+\lambda T^-},\ \eta^D_L(\tau) = \tfrac{\lambda(1-\mu)}{1+\lambda T^-},\ \tau \in [0, T^-], \end{aligned} \tag{2}$$

*and a mixed strategy equilibrium with the interest rate*

$$R_M(N) = 1 + \frac{1-\mu}{\mu}\frac{1-x+xe^{\lambda T^+}}{1+x\lambda T^-} = R$$

.

*and the stationary distribution* $\mathbf{\Delta}_M$ *is*

$$\begin{aligned} \mu^N_M = \tfrac{\mu}{1-x+xe^{\lambda T^+}},\ \mu^S_{St}(\tau) = \lambda(\mu-(1-x)\mu^N_M)e^{-\lambda\tau},\ \tau \in [0, T^+], \\ \eta^N_0 = \tfrac{1-\mu}{1+x\lambda T^-},\ \eta^D_0(\tau) = \tfrac{\lambda(1-\mu)}{1+x\lambda T^-},\ \tau \in [0, T^-]. \end{aligned} \tag{3}$$

A stationary equilibrium with lending exists if and only if a borrower with an $N$ record can get a loan, i.e. his interest rate $R_L(N)$ does not exceed $R$:

$$R_L(N) = 1 + \frac{1-\mu}{\mu}\frac{e^{\lambda T^+}}{1+\lambda T^-} \leq R, \tag{4}$$

Intuitively, in a stationary equilibrium with lending a borrower with an $N$ record must be able to borrow, otherwise any borrower whose last record gets deleted (that is, his record becomes $N$) will never borrow again. As a result the mass of borrowers that can borrow would be decreasing towards zero with time, which can't happen in a stationary equilibrium.

When $x = 0$ mixed strategy equilibrium coincides with the no-lending pure strategy equilibrium, when $x = 1$ with the lending pure strategy equilibrium. In terms of interest rates it can be explained as following: $R_M(N, x) = R$ is monotone in $x$, $R_M(N, 1) = R_L(N)$ and $R_M(N, 0) = R_{NL}(N)$. So conditions on all stationary equilibria to exist depends on $R_L(N), R_{NL}(N) \leq\geq R$. There are 2 distinct cases, depending on correlation of $R_L(N)$ and $R_{NL}(N)$.

**Theorem 2.** *When $R_{NL}(N) < R_L(N)$ stationary equilibria mutually exclude each other: stationary equilibrium with no lending exists for $R < R_{NL}(N)$, stationary equilibrium with lending exists for $R \geq R_L(N)$ and mixed stationary equilibrium exists for $R_{NL}(N) \leq R < R_L(N)$.*

*When $R_{NL}(N) \geq R_L(N)$ all three stationary equilibria can co-exist. For $R < R_L(N)$ only stationary equilibrium with no lending exists, for $R \geq R_{NL}(N)$ only stationary equilibrium with lending exists and for $R_L(N) \leq R < R_{NL}(N)$ all three stationary equilibria co-exist.*

If the initial distribution of types to records coincides with the stationary one, then at any $t \in [0, \infty)$ the market is in the stationary equilibrium.[5]

Condition (4) guarantees that the pool of borrowers with an $N$ records is good enough, so that these borrowers can get a loan whenever they have an investment opportunity.

**Corollary 1.** *The effects of changes in $T^-$ and in $T^+$ on the interest rate $R(N)$ are the opposite, moreover the adverse effect of an increase $T^+$ is stronger than the beneficial effect of an increase in $T^-$:*

$$\frac{\partial R(N)}{\partial T^+} < 0, \ \frac{\partial R(N)}{\partial T^-} > 0, \ \left|\frac{\partial R(N)}{\partial T^+}\right| > \left|\frac{\partial R(N)}{\partial T^+}\right|. \tag{5}$$

The proof can be obtained immediately by differentiating $R(N)$ with respect to $T^+$ and $T^-$. The corollary implies that condition (4) can be satisfied when the memory for positive records ($T^+$) is short, while memory for negative records ($T^-$) is long. This is very intuitive. Long memory for defaults keeps bad borrowers out of the pool of borrowers with no records for a long time, improving the average quality of this pool, thereby making it easier for borrowers with no records to borrow and for a stationary equilibrium with lending to exist. On the contrary, long memory for positive records allows good borrowers to keep out of the pool for a long time, making it more difficult for borrowers with no records to borrow and sustain the stationary equilibrium with lending. The effect of the positive memory on the interest rate $R(N)$ is stronger than the effect of the negative memory, because a positive records allows good borrowers to borrow again an renew or “refresh” their positive records. As a result in expectation good records are stored for longer than $T^+$ and create a significant adverse effect on the

[5]In principle, the market can start with a different initial distribution and still converge to a stationary equilibrium. But this is not always the case, Proposition **??** shows that for the same $w$ both stationary and cyclical equilibrium can take place depending on the initial distribution $\Delta_0$.

average quality of borrowers with an $N$ record. The effect of negative memory is less strong, a negative record prevent the borrower from accessing the credit market, as a result such a record is never "refreshed" and simply disappears exactly after $T^{-}$.

# 3 Regime switching equilibrium

Departing from Kovbasyuk and Spagnolo (2016), in this paper we study non-stationary equilibria. More precisely, we are focusing on the case of cyclical dynamics of the economy when periods of boom (in which borrowers with no records can get a loan) alternate with periods of bust (in which borrowers with no records can't get a loan). In equilibrium, good borrowers with a $S(\tau)$, $\tau \in [0, T^{+}]$ record can always get a loan any moment they need it, while bad borrowers with a $D(\tau^{-})$, $\tau^{-} \in [0, T^{-}]$ record do not get any loans.

The key for the regime switch from boom to bust is the composition of the pool of borrowers with an $N$ record. Lenders' decision whether to give a loan to a borrower with an $N$ record depends on the probability that this borrower is of a good type. Lenders calculate this probability using Bayes' rule and all available information about the borrowers. When the mass of bad borrowers in the pool of borrowers with an $N$ record is low, this probability is high enough, and lenders are willing to give loans to borrowers with an $N$ record. We call this part of the credit cycles the *regime of lending.* On the other hand, when the mass of bad borrowers in the pool of borrowers with an $N$ record is high, it is not profitable for banks give loans to borrowers with an $N$ record. We call this past fo the credit cycle the *regime of no lending.* As our first result we would like to show that model of credit market with limited memory, under some reasonable restrictions on the parameters, has an equilibrium, in which regimes of lending and no lending are finite and alternate each other infinitely.

**Definition 1.** *A regime switching equilibrium* is an equilibrium in which periods of lending and no lending are finite and alternate each other infinitely.

Before moving on to the theorem on existence we would like to introduce some more formal definitions, that are going to be useful in the following chapters.

Define by $\mu_t^N$ and $\eta_t^N$ the masses of good and bad borrowers with $N$ record at time $t$. As it was explained above the economy is in lending regime at $t$ if and only if the expected return to the bank exceed the loan provided. Good borrowers' investment always delivers $R$ , while bad borrowers' return is zero. The lenders are willing to give a loan to a borrower with an $N$ record when

$$\Pr(G|N)\cdot R+\Pr(B|N)\cdot 0=\frac{\mu_t^N}{\mu_t^N+\eta_t^N}\cdot R>1,$$

or, equivalently

$$\frac{\eta_t^N}{\mu_t^N}<R-1.$$

Let us then define the *relative quality of the pool* as

$$\sigma_t^N=\frac{R-1}{\eta_t^N/\mu_t^N}.$$

With this definition, the condition for lending to borrowers with an $N$ record is the condition on relative quality of the pool of borrowers with an $N$ record

$$\sigma_t^N>1. \tag{6}$$

Also let us define relative quality of the whole population of borrowers as

$$\sigma=\frac{R-1}{\eta/\mu}.$$

It is easy to see, that condition $\sigma>1$ ensures that without any history recorded (when every borrower is in the pool with an $N$ record) there will be lending to borrowers with an $N$ record. Now we explain why a finite memory of the credit bureau naturally generates cyclcal, regime switching equilibria in the credit market.

First, let us explain the mechanism that leads the credit market regime to switch between lending and no lending to the pool. During a lending period any borrower, except those who have defaulted less than $T^-$ before, are getting financing as soon as they face an investment opportunity. The quality of the pool is respectively high. Borrowers with no available history are also receiving loans if they have an investment opportunity.

Good and bad borrowers in the pool face an investment opportunity with the same intensity, reveal their type, update their history record and leave the pool. As the intensity is the same for both types, the fraction of subjects leaving the mass in the pool is equal for both types, so borrowers in the pool who receive a new loan do not influence on the quality of the pool.

The masses of borrowers whose records are being erased because of the limited memory and who therefore enter the pool are instead changing the ratio of types in the pool (its quality). The logic is the following. When a bad borrower gets a loan, he defaults and gets a $D$ record, after which he is excluded from the market for exact $T^{-}$ period, i.e. as long as his negative record remains publicly available. He has no possibility to change his $D$ record except waiting until it is erased. When a good borrower receives a loan, he repays it and gets an $S$ record, hence he remains able to obtain a new loan if a new possibility to invest arises. So it is "harder" for a good borrower to reenter the pool once he leaves it. This difference in types results in a different rate at which the masses in the pool change. And if at the beginning of the period of lending a substantial mass of bad and good borrowers receives a loan, the mass of bad borrowers will return to the pool simultaneously, when the $D$ records are erased after $T^{-}$ time, while good borrowers re-enter the pool slowler and in a smoother way, as they go on producing and getting new positive feedbacks with positive probability, which keep them out of the pool as long as they do not get an investment opportunity for $T^{+}$ time. So it is reasonable to expect a large drop in the quality of the pool that can result in switching to a regime of no lending to unknown borrowers in the pool.

What happens with the pool after a switch to no lending? The pool becomes a "black hole" for some time — once a borrower gets in, he is not able to get a loan again and update his record. During the no lending regime only good borrowers with an $S$ record are able to trade. So after $T^{-}$ time all the bad borrowers will end up in the pool. Any good borrower will aslo face a positive probability to re-enter the pool by facing a long enough period with no investment opportunity, so that his $S$ record is erased. For any of such individual this will happen at a different moment, so the quality of the pool will improve slowly. But in the limit, after a long enough time, every good borrower will face such a time span with no investment opportunities and will end up in the pool. Under the assumption that the average quality of the population is reasonably high ($\sigma > 1$) we know that there is lending to the pool when this contains every borrower in the economy. So there will exist a moment when the quality of the pool rises high enough for lenders to switch back to the lending regime. Theorem 3 below provides sufficient conditions that guarantee that the credit market is in a regime switching cyclical equilibrium of the type just described. This is important to notice, that those conditions do not include initial distributions of records.

**Theorem 3.** *The credit market is in a regime switching equilibrium if:*

*1) the average quality of borrowers is high enough, so that a stationary equilibrium with no lending does not exist $\mu R > 1$, and*

*2) the average quality of borrowers is not too high, so that a stationary equilibrium with lending does not exist, that is (4) is violated.*

Note that condition (4) is violated when

$$\sigma = \frac{R-1}{\eta/\mu} < \frac{e^{\lambda T^+}}{1+\lambda T^-}. \tag{7}$$

*Proof.* The proof of the theorem is very simple. Suppose the market is in a no-lending regime. Let's prove that this regime will end in a finite time and the market switches to the lending regime. If the lending regime were infinite, the economy would end up in a no-lending stationary equilibrium. This can't happen because $\mu R > 1$, and the no-lending stationary equilibrium is not possible, therefore the no lending regime is finite. Suppose the credit market is in the lending regime, then this regime ends in finite time and the credit market switches to the no-lending regime. Indeed, if the lending regime were infinite, the economy would end in the stationary equilibrium with lending, which can't happen because (4) is violated. Hence, the lending regime is finite. Our model always has an equilibrium, therefore we conclude that this equilibrium is the regime switching equilibrium. □

Having shown that the credit market with a finite memory can be in a regime switching equilibrium and exhibit cyclical boom-bust dynamics for a broad range of model parameters, we proceed to study the properties of the credit cycle. For that we want to explicitly solve for the credit cycles, characterize lending dynamics for each category of borrowers, and show the evolution of the aggregate information in the credit market pinned down by the distribution of borrowers types to credit records $\Delta_\mathbf{t}$. To do so, we introduce some additional assumptions, and focus on ergodic limit cycles, in which exactly the same credit cycle repeats infinitely.

# 4 Ergodic limit cycles

In this section we study ergodic limit cycles, formally *ergodic cyclical equilibria* in the credit market. An ergodic cyclical equilibrium is a regime switching equilibrium, where each period of lending (no lending) is identical, in terms of dynamic changes in the composition of borrowers with different records through the cycle.

**Definition 2.** Consider a regime switching equilibrium and take any two distinct moments of time $T_1 > 0$ and $T_2 > 0$ when the credit market switches from the lending to the no-lending regime (or vice versa). This equilibrium is an *ergodic cyclical equilibrium* if at any $T_1 + t$ and $T_2 + t$, $t \geq 0$ distributions over seller types and records are identical: $\Delta_{T_1+t} = \Delta_{T_2+t}$.

Focusing on ergodic cyclical equilibria is of course very convenient for the analysis: the same boom-bust cycle repeats indefinitely, and it is therefore enough to characterize a single boom-bust cycle to determine the dynamic behavior of the credit market at any moment of time. Below we derive sufficient conditions that guarantee that the credit market is in an ergodic cyclical equilibrium. Clearly, the equilibrium path critically depends on the initial conditions: different equilibrium paths can realize depending on what information about borrowers is available at date $t = 0$. It is natural to assume that no past information about borrowers is available at $t = 0$. So for parameters $\omega = \{\mu, \lambda, R, T^-, T^+\}$ we introduce the initial distribution of records with "no information" $\mathbf{\Delta}_0^{NI}(\omega)$, where $\mu_0^N = \mu$, $\mu_0^S(\tau) = 0$, $\eta_0^N = \eta$, $\eta_0^D(\tau) = 0$.

We can then state the following theorem, that states that erogic cyclical equilibrium exists.

**Theorem 4.** *The credit market with parameters $\omega = \{\mu, \lambda, R, T^-, T^+\}$ is in an ergodic cyclical equilibrium if*

*1) the market starts with no information ($\mathbf{\Delta}_0 = \mathbf{\Delta}_0^{NI}(\omega)$),*

*2)default records are stored for at least as long as records of successful repayment $T^+ \leq T^-$,*

*3)the average quality of borrowers is high enough, so that a stationary equilibrium with no lending does not exist $\mu R > 1$, and*

*4) the average quality of borrowers is not too high:*

$$\eta e^{-1+e^{-\lambda T^-}} \geq (R-1)\mu e^{-\lambda T^+}. \tag{8}$$

The formal proof of the Theorem is in the Appendix. Here we provide the intuition behind the theorem. The key feature of the model, which lays in the basis for the ergodicity of the economy, is the stabilization of the masses of different borrower's types with different records after certain time within each cycle. If a lending period is long enough (continues for at least $T^+$), then the mass of good borrowers with an $N$ record stabilizes at a certain "stationary" level $\bar{\mu}^N = \mu e^{-\lambda T^+}$ until the regime switches. In other words, if lending continues for longer than $T^+$, then at any moment before the regime switches to no lending the mass of good borrowers with an $N$ record receiving a loan and "loosing" an $N$ record, is equal to the mass of good borrowers "loosing" an $S(T^+)$ record and getting an $N$ record instead. The total mass of good borrowers with an $N$ record after the first $T^+$ periods of lending remains constant until regime switches to no lending. Analogous reasoning holds for bad borrowers during the no lending regime. In a no lending regime borrowers with an $N$ record or with a $D(\tau')$, $\tau \in [0, T^-]$ record can't borrow. After $T^-$ of no lending all bad borrowers have their bad records erased and all of them end up with an $N$ record, so that $\eta_t^N = \eta$ is constant until the lending regime starts over. In other words, if the lending regime is longer than $T^+$ and the no lending regime is longer than $T^-$ the distributions of borrowers types with different records within a boom-bust cycle become independent of the distributions in the previous cycles, and information from previous cycles becomes irrelevant for the market dynamics.

The mechanism described above works under two conditions: periods of lending and no lending are to be **finite** (to guarantee regime switching) and **"long"** (to guarantee stabilization and ergodicity, as a result). The conditions in the Theorem 4 guarantee that this is indeed the case.

The third condition of the Theorem requires the average quality of borrowers to be high enough for lending with no information to exist (so that the no lending regime is finite). The last condition guarantees that there are enough bad borrowers in the economy, so the fluctuation of the mass of bad borrowers with an $N$ record during lending periods has high enough amplitude. At some point, when mass of good borrowers with an $N$ record has already stabilized, the mass of bad borrowers with an $N$ record rises because records of the defaults that happened in the beginning of the lending regime are erased. As a result, the quality of the pool of borrowers with an $N$ record goes down and eventually hits a critical level when the lenders stop lending to these borrowers and the market switches to the no lending regime.

Condition (8) requires the average quality of borrowers not to be too high, so that the mass of good borrowers in the economy is respectively low, ensuring that the no lending period is "long" (which helps to establish the equilibrium's ergodicty). The idea is as follows: when the market switches to the no lending regime, many good borrowers have

$S(\tau)$, $\tau \in [0, T^+]$ records and are out of the pool of borrowers with an $N$ record. This pool contains many bad borrowers, and that is why the switch to no lending happens in the first place. In order for the no lending regime to last longer than $T^-$, the rate with which positive records of good borrowers are erased and good borrowers are entering the pool of borrowers with an $N$ records should be not too high. Otherwise the average quality of the pool may rise too quickly, hit a critical threshold and provoke the switch to lending regime too soon. The rate with which good borrowers fill up the pool directly depends on their total mass in the economy. If there are not too many good borrowers in the economy then the time, needed for their mass in the pool to reach the level necessary for the lending regime to start, is longer than $T^-$.

Condition $T^+ \leq T^-$ ensures that periods of lending are longer than $T^+$. Indeed, in the beginning of the lending regime all bad borrowers are in the pool of borrowers with an $N$ record, hence during next $T^-$ periods of the lending regime bad borrowers only exit the pool and the average quality of the pool increases (good borrowers exit the pool at the same rate but some also enter the pool as in the beginning of the lending regime some good borrowers have $S$ records that are later erased). As a result, the lending regime lasts for at least $T^- \geq T^+$.

Note that the existence conditions for a *regime switching equilibium* in Theorem 3 are different from the conditions in Theorem 4 that guarantee the existence of an *ergodic cyclical equilibrium*. Theorem 3 states that if $\mu R > 1$ and the stationary equilibrium does not exists, then regardless of the initial conditions the credit market is in a regime switching equilibrium. In Theorem 4 we show that if the economy starts with no information and the model parameters are withing a certain set then the market is in an *ergodic cyclical equilibrium*. It turns out this set of parameters does not contradict the existence of a stationary equilibrium, in other words starting with some different initial conditions the credit market can converge to a stationary equilibrium with lending without any cycles. Essentially, for a certain set of parameters the dynamics of the market critically depends on the initial conditions. More on co-existence of different equilibria we are going to study in the following chapters.

## 4.1 Overlapping of ergodic limit cycles and stationary equilibria

Having established the existence of ergodic limit cycles we are able to study the relationship between their set of parameters and those of stationary equilibria. As it was stated in Theorem 2 existence of any of stationary equilibria depends on correlations of the interest rates of unknown borrowers $R^N_{NL}, R^N_L, R^N_M$ and $R$. In theorem 4 it is included that ergodic cycles do not coexist with stationary no lending equilibrium. Condition (8) can be rewritten as condition on the interest rate of unknown borrowers, which has to rise high enough to switch from lending to no-lending regime for cycles to exist. Let denote such a threshold as $R^N_C = 1 + \frac{\eta e^{-1+e^{-\lambda T^-}}}{\mu e^{-\lambda T^+}}$. Thus we ends up with one more threshold to compare with $R$. Since theorem 3 guarantees that negation of (4) implies existence of cycles, one can argue that $R^N_C$ is always higher than $R^N_L$. Combining all the above and assuming as before that $T^+ \geq T^-$ we have the following overlapping picture:

**Proposition 1.** *An overlapping of equilibria's parameter sets is different with a different order in which the thresholds $R^N_{NL}, R^N_L, R^N_M$ and $R^N_C$ are grouped. This order depends on the correlation between memories $T^+$ and $T^-$:*

*1) If positive memory significantly shorter then negative $\left(T^+ < \frac{1-e^{-\lambda T^-}}{\lambda}\right)$ there is no ergodic cycles. With sufficiently high rate of return $R > R^N_{NL}$ an expected profit of any loan is high enough to be granted and only stationary equilibrium with lending exists. On the other hand if $R$ is too small $(R < R^N_L)$ then no borrower without good record can get a loan and only stationary no-lending equilibrium exists. Finally for $R$ in the middle of these two extremes $\left(R^N_L \leq R \leq R^N_{NL} = \frac{1}{\mu}\right)$ all three stationary equilibria coexist together.*

*2) If positive memory is commensurable to logarithm of negative $\left(\frac{1-e^{-\lambda T^-}}{\lambda} \leq T^+ \leq \frac{ln(1-\lambda T^-)}{\lambda}\right)$, the situation is similar to the described above, but with the only difference that limited ergodic cyclical equilibrium appears. Again if rate of return is too small $\left(R < R^N_L\right)$ there can be no lending to unknown borrowers and only stationary no-lending equilibrium exists. With $R$ in the range between $R^N_L$ and $R^N_{NL}$ the rate of return is high enough for all three stationary equilibria coexist, but not for ergodic cycles since any period of no lending is going to last for infinite time. Starting from $R^N_{NL}$ no-lending periods become to be finite. Hence for $R^N_{NL} \leq R \leq R^N_C$ limited ergodic cycles and pure stationary lending equilibria coexist. And with rate of return greater then $R^N_C$ it becomes too high for switching to no-lending to occur and only possible equilibrium is stationary with lending.*

*3) Finally if positive memory shorter, but commensurable to negative $\left(\frac{ln(1-\lambda T^-)}{\lambda} < T^+ \leq T^-\right)$ then stationary no-lending equilibrium doesn't coexist with no other equilibrium. Mixed strategy lending equilibrium overlap only with ergodic cycles, when $R^N_{NL} \leq R < R^N_L$. Pure stationary lending coexist with ergodic cycles for $R^N_L \leq R \leq R^N_C$. And again when $R$ exceeds $R^N_C$ it becomes too high and only stationary lending equilibrium remain.*

*Proof.* to be added into the Appendix □

## 4.2 Frequency of the cycles

Having established the existence of ergodic limit cycles we are able to study how the duration of booms (lending regimes) and busts (no lending regime) depends on the parameters of the model. Let $T_l$ be the duration of the lending regime, and let $T_n$ be the duration of the no lending regime of the cycle.

**Corollary 2.** *Under the conditions in Theorem 4 both lending and no lending regimes of the ergodic credit cycle last longer than the time the negative records are stored by the credit bureau $T_l, T_n > T^-$. Moreover, lending regime lasts no longer than twice the time the negative records are stored $T_l < 2T^-$.*

Lower bounds stem directly from the fact that we study ergodic limit cycles and ergodicity requires cycles to be "long" enough. To see the intuition suppose both lending and no lending regimes are "short", that is last less than $T^-$. Given that negative records are recorded for $T^-$ this implies that in the beginning of lending regime which follows after some no lending regime some bad borrowers will have old default records generated during the previous lending regime. Thus, the information produced in the previous lending regime reaches the next lending regime and can affect dynamics and information production in this regime. In this case the information can perpetuate infinitely, that is the credit market may not "forget" bits of past information and the cycles may not be ergodic. If on the other hand, both lending and no lending regime last for at least $T^- \geq T^+$ then credit records from previous cycles do not reach subsequent cycles and the cycles are ergodic.

The intuition behind the upper bound on the duration of the lending regime is subtle. Intuitively, during the lending regime masses of good borrowers with different records stabilize after $T^+ < T^-$. At the same time, the mass of bad borrowers with an $N$ record fluctuates, and the amplitude of the fluctuation is the highest during the first $2T^-$. Essentially, if the mass of bad borrowers with an $N$ records does not reach the critical level to trigger the switch to no lending regime during the first $2T^-$, it will never reach this level afterwards and the market will stay in the lending regime permanently. Therefore, in and ergodic regime switching equilibrium the duration of the lending regime can't exceed $2T^-$.

Also note that per se, there is no upper bound on the duration of the no lending regime. If the average quality of the borrowers is sufficiently low, and all bad borrowers are in the pool of borrowers with an $N$ record (as is the case in the no lending regime after time $T^-$), then lending to this pool of borrowers may be profitable only when almost all good borrowers are inside this pool. As good borrowers with positive records are able to receive new loans even during no lending regimes, their positive history can be updated over and over again with each new loan. For the good borrower to loose his positive history, he must not face an investment opportunity for long enough time (longer than $T^+$). In order for almost all good borrowers to loose their positive records the market may stay in the no lending regime for a very long time.

We now would like illustrate how the duration of different phases of the cycle depends on the model's parameters. We have shown that the duration of the lending regime is limited $T_l \in [T^-, 2T^-]$, but the no lending regime can last very long. For tractability we restrict the analysis to ergodic limit cycles with a duration of the no lending regime in the interval $T_n \in [T^+, 2T^+]$. This allows us to explicitly express durations $T_l$ and $T_n$ and to perform comparative statics analysis.

**Proposition 2.** *Under the conditions in Theorem 4 there exist an ergodic cyclical equilibrium with the duration of the no lending regime $T_n \in [T^+, 2T^+]$. In this equilibrium the duration of the lending regime is*

$$T_l = T^- - \frac{1}{\lambda}(W(-\sigma exp(-(\lambda T^+ + e^{-\lambda T^-}))) + e^{-\lambda T^-}), \tag{9}$$

*where $W(x)$ — is a Lambert W function (the inverse of $f(z) = ze^z$).*

*The duration of the lending regime increases with $\sigma$ and $T^-$ but decreases with $T^+$.*

*Whenever*

$$\sigma \geq \frac{e^{2\lambda T^+}}{(1 + 2\lambda T^+)e^{\lambda T^+} - \lambda T^+ - \frac{(\lambda T^+)^2}{2}}, \tag{10}$$

*the duration of the no-lending period is bounded above by $2T^+$ and equals to*

$$T_n = T^+ + \frac{1}{\lambda}(e^{\lambda T^+} - 1 - \sqrt{e^{2\lambda T^+}(1 - 2/\sigma) + 2\lambda T^+ e^{\lambda T^+} + 1}). \tag{11}$$

*The duration of the no-lending period decreases with $\sigma$, increases with $T^+$, and does not depend on $T^-$.*

The duration of the lending regime is positively related with the average quality of the borrowers. When the average quality of borrowers increases, it takes longer for bad borrowers to gain enough mass in the pool with borrowers with $N$ records and make lending to them unprofitable. On the contrary, the no lending periods is negatively related with the average quality of the borrowers, as good borrowers are saturating the pool and reaching the threshold mass faster if there are more of them in the population.

If the memory on defaults $T^-$ increases then the duration of lending regimes $T_l$ increases as well. When default records are kept for a longer period it takes bad borrowers longer time to have their default records deleted, and they naturally stay longer out of the pool of borrowers with an $N$ records. This naturally prolongs lending to the pool of borrowers with an $N$ record and the lending regime.

# 5 Welfare comparison

Now, when the overlapping of equilibria are described it is possible to compare those equilibria. As a metric of comparison between co-existing equilibria we are going to study welfare, generated by the model.

At each instance loans issued to good borrowers produce $R-1$ of surplus. On the other hand loans that will not be repayed are count as a loss to the economy. The surplus generated over a small interval of time $[t, t+dt]$ with the parameters $\omega$ and the initial records $\Delta_0$ can be expressed as follows:

$$w(\omega, \Delta_0, t)dt = ((R-1)\mu_t^S(0) - \eta_t^D(0))dt. \tag{12}$$

The metric of comparison we are interested in is a limit of an average welfare over the infinite time.

$$W(\omega, \Delta_0) = \underset{T\to\infty}{limsup}\frac{1}{T}\int_0^T \left((R-1)\mu_t^S(0) - \eta_t^D(0)\right) dt. \tag{13}$$

**Proposition 3.** *Average welfare generated by the economy in the stationary equilibrium with lending (**SL**) is*

$$W(\omega, \Delta_0^{St}) = \lambda\left((R-1)\mu - \frac{\eta}{1+\lambda T^-}\right), \tag{14}$$

*where $\omega \in \Omega^{St}$.*

*Proof.* The economy under consideration is in the stationary equilibrium with lending. Hence at any moment of time $\lambda$ fraction of all good borrowers and $\lambda$ fraction of bad borrowers with no record are receiving a new loan. So we have $\bar{\mu}^S(0) = \lambda\mu$, $\bar{\eta}^D(0) = \lambda\bar{\eta}^N = \frac{\eta}{1+\lambda T^-}$. □

**Proposition 4.** *Average welfare generated by the economy in stationary mixed strategy lending equilibrium (**SMSL**) is*

$$W(\omega, \Delta_0^M) = \lambda\left((R-1)\mu - \eta_M^N\right), \tag{15}$$

*where $\omega \in \Omega^M$, $\eta_M^N$ is a mass bad borrowers with $N$ records in the **SMSL** equilibrium.*

*Proof.* Let denote $W_M$ as welfare generated by **SMSL**. $W_M = \lambda(R-1)(\mu - \mu_M^N) + \lambda x\left((R-1)\mu_M^N - \eta_M^N\right)$, where $x$ is a fraction of borrowers with no history applying for a loan actually receiving one and $\mu_M^N$, $\eta_M^N$ is a mass borrowers with $N$ records in the **SMSL** equilibrium. By definition of mixed strategy banks are expected return of unknown loans is 0, or in other terms $(R-1)\mu_M^N = \eta_M^N$. Hence the formula for the mixed statey welfare can be rewritten as $W_M = \lambda(R-1)\mu - \lambda\eta_M^N$. □

**Proposition 5.** *Average welfare generated by the economy in the ergodical cyclical equilibrium is equal to the mean of the surplus generated over consequential period of lending and no-lending and can be calculated by the formula.*

$$W(\omega, \Delta_0^{NI}) = \lambda(R-1)\mu - \frac{1}{T_l + T_n}\lambda\left(\int_0^{T_l}\eta_t^N dt + \int_{T_l}^{T_l+T_n}(R-1)\mu_t^N dt\right), \tag{16}$$

*where $\omega \in \Omega^{ErC}$.*

The formal proof of the Proposition 5 is in the Appendix.

Now, when all possible welfare are described, it is not difficult to find the most effective equilibrium. First, it is obvious, that stationary no-lending equilibrium generates nothing, while any other outcome is going to result in positive welfare gain. The following theorem states, that the most efficient equilibrium among pure stationary lending, mixed stationary lending and ergodic cyclical equilibria is the first one. Though it is worth stressing that there no overlapping of all three of them it is possible to compere pure stationary lending with both other in different parameter spaces.

**Theorem 5.** *Stationary lending generates more welfare then any other co-existing equilibrium.*

*Proof.* to be add in the Appendix. □

# 6 Discussion

In this paper we have build a simple model of unsecured credit and have illustrated that substantial and persistent cycles in the aggregate credit can happen in an economy without any aggregate shocks if credit histories of borrowers are limited. The credit cycle is likely to emerge when the average borrower in the population is credit worthy, records of past successful loan repayments are stored in the credit registry for a sufficiently long time, while records of credit defaults are deleted relatively fast.

The model is stylized and makes a number of simplifying assumptions. For instance, we have assumed that borrowers do not have relationships with lenders (banks), and each time they ask for a loan the lender's decision about granting the loan or not is based only on the information available at the credit registry. In reality, borrowers may have stable relationships with their banks, so that even when their record at the credit registry is empty their regular lender may still remember their past repayments and take this into account. Clearly, if the regular lender remembers that the borrower has defaulted in the past, the borrower with a clear history should ask for a loan from a new lender rather than asking the regular one. In other words, the possibility of borrower-lender relationships does not affect the effective length of the memory for negative records in the model. However, if the regular lender remembers that the borrower has repaid a loan in the past, he may grant a loan to the borrower even if the borrower has no positive record in the credit registry. This makes the effective memory for positive records longer. For instance, if the regular lender remembers pas repayments for $X > T^+$ periods, the borrower who had successfully repaid a loan between $T^+$ and $X$ periods ago should always ask for a loan from the regular lender. Note, that the model dynamics and the formulas obtained in the analysis are still valid, one simply needs to replace $T^+$ with $X$ in the expressions.

# Appendix

*Proof of theorem 1.* In a stationary equilibrium with lending a borrower with an $N$ records must be able to get a loan whenever he faces an investment opportunity. Suppose otherwise, a borrower with $N$ record can't get a loan, then the market is not in a stationary equilibrium with lending, as with time $S(\tau)$ records get deleted and the mass of borrowers than can't borrow is growing. This can't happen in a stationary equilibrium because $\Delta_{\mathbf{t}} = \mathbf{\Delta}$ must be time invariant. One can compute $\mathbf{\Delta}$ in an equilibrium in which borrower with an $N$ records can get a loan. For this $\mathbf{\Delta}$ condition (4) is equivalent to $\mu^N R/(\mu^N + \eta^N) \geq 1$, which guarantees that borrowers with an $N$ record can indeed get a loan. In other words, condition (4) is necessary. Clearly, if condition (4) holds, then there is a stationary distribution $\mathbf{\Delta}$ such that borrowers with an $N$ record can borrow, that is (4) is also sufficient. □

*Proof of Theorem 4.* We prove the theorem in two steps. First, we prove that in a "sufficiently" long cycle part of the distribution of borrowers to records $\delta_t$ stabilize. Then we use this fact to show how "lending" and "no-lending" regimes appear one after the other, that is regimes switch. First let us prove

**Lemma 1.** *If lending regime lasts longer than $T^+$, then the mass of good borrowers with an $N$ record $\mu_t^N$ stabilizes $\bar{\mu}^N = \mu e^{-\lambda T^+}$.*

*Proof.* At any moment of trading with the pool the $\lambda$ fraction of all good borrowers is financed. Fraction $e^{-\lambda\tau}$ of them will get financed in the following $\tau$ period. Let $X$ be the moment when trading with $N$ borrowers starts. Then at any moment $t > X + T^+$

$$\mu_t^N = \mu - \int_0^{T^+} \lambda \mu e^{-\lambda\tau} d\tau = \mu - \mu(1 - e^{-\lambda T^+}) = \mu e^{-\lambda T^+} = \bar{\mu}^N. \tag{17}$$

□

**Lemma 2.** *If there is no lending for more then $T^-$, then the mass of bad borrowers with $N$ record stabilizes at the level $\eta$ (all the bad borrowers end up in the pool).*

*Proof.* During no lending bad borrowers have no possibility to update there record. After $T^-$ there will be no bad borrower with remained history of his default. So all bad borrowers are in the pool with N records. □

The following steps of the proof consist of showing that under the conditions of the theorem regimes of lending and no lending are "long" and finite.

First we introduce two lemmas which insure switching of the regimes of lending and no lending:

**Lemma 3.** *If $\sigma > 1$, then any period of no lending is finite.*

*Proof.* For any of the good borrower the probability of being forgotten is equal to probability of not having an investment for $T^+$. This probability equals to $e^{-\lambda T^+}$. For any arbitrary small $\epsilon > 0$ there going to be a moment $t_\epsilon$, when less then $\epsilon$ of good borrowers are remain out of the pool, so $\mu^N_{t_\epsilon} > \mu - \epsilon$. But condition $\sigma > 1$ guaranties that there is lending to the pool in case of all good borrowers having N records. So at some point there will be enough good borrowers in the pool to make its average quality high enough to provide loans to borrowers without any available history. □

On the contrary to the mass of good borrowers during no lending, which is increase monotonically, the mass of bad borrowers during lending period fluctuating in a more complex way. To study the conditions on the regime switching from lending to no lending we have to determine the laws of dynamic of $\eta^N_t$ during lending period.

We are focusing on the case of "long" cycles. Hense it is convinient to devide the whole period of lending into parts of length $T^-$. We are going to call these segments "waves" of lending, due to the fluctuation of $\eta^N_t$. Let us define $\eta_k(t), t \in [0, T^-]$ as a function of changing of the mass of bad borrowers in the pool during the k-th wave of lending. Hense if $T_0$ is a moment of switching to lending regime, then

$$\eta_k(t) = \eta^N_{T_0 + kT^- + t}, \quad t \in [0, T^-]. \tag{18}$$

Without loosing of generality we are going to assume that each period of lending starts with all bad borrowers in the pool. Later in the proof we are going to show why this assumption holds.

So we have

$$\eta_0(0) = \eta. \tag{19}$$

After fixing initial conditions we are going to describe the rate of change of $\eta^N_t$ during a lending regime regime. First let notice, that at any moment of lending $\lambda$ fraction of all borrowers in the pool receives an investment opportunity and leaves the pool.

During the first wave of lending bad borrowers are only leaving the pool, as there is no one with $D$ record for long enough to be erased. During all the other waves ($k > 1$) the rate of change of $\eta_t^N$ consist of two parts: the mass of those, who receives a loan and leaves the pool and those, who has defaulted exactly $T^-$ before and being erased and join the pool. Hence we are able to sum up above reasoning into the system of differential equations

$$[right = \quad \text{where} \quad k > 1, t \in [0, T^-]]\{ \begin{aligned} \dot{\eta}_1(t) &= -\lambda\eta_1(t), \\ \dot{\eta}_k(t) &= -\lambda\eta_k(t) + \lambda\eta_{k-1}(t), \\ \eta_0(0) &= \eta, \\ \eta_k(0) &= \eta_{k-1}(T^-). \end{aligned} \tag{20}$$

We immediately are able to describe the first wave. $\eta_1(t) = e^{-\lambda t}$. So during the first $T^-$ the mass of bad borrowers in the pool exponentially decries.

To find formule for the higher number of waves lets look for the solution in a form

$$\eta_k(t) = c_k(t)e^{-\lambda t}, \quad t \in [0, T^-], k \in \{1, 2, 3\ldots\} \tag{21}$$

Differentiating left and right parts of this equation we get

$$\dot{\eta}_k(t) = -\lambda c_k(t)e^{-\lambda t} + \dot{c}_k(t)e^{-\lambda t}, \quad t \in [0, T^-], k \in \{1, 2, 3\ldots\} \tag{22}$$

Note, that $c_1(t) \equiv 1$.

Using the first equation of (20) we get

$$\dot{c}_k(t)e^{-\lambda t} = \lambda\eta_{k-1}(t) = \lambda c_{k-1}(t)e^{-\lambda t}, \quad \text{or} \tag{23}$$

$$\dot{c}_k(t) = \lambda c_{k-1}(t). \tag{24}$$

Recurrent formula (24) implies, that

$$\{ \begin{aligned} & c_k^{(i)}(t) = \lambda^i c_{k-i}(t), \quad 1 < k, \quad 0 \leq i \leq k \\ & c_1(t) = 1. \end{aligned} \tag{25}$$

Using Taylor expansion we have

$$c_k(t) = \sum_{i=0}^{\infty} c_k^{(i)}(0)\frac{t^i}{i!} = \sum_{i=0}^{k-1} c_{k-i}(0)\frac{(\lambda t)^i}{i!} \tag{26}$$

Note, that we can not include tale, as $c_1(t) \equiv 1$ is already constant.

Substituting this into (21) we get

$$\eta_k(t) = e^{-\lambda t} \sum_{i=0}^{k} \eta_{k-i}(0) \frac{(\lambda t)^i}{i!} \tag{27}$$

Now, after we have specified the dynamic of $\eta_t^N$ during lending period, we can prove the conditions which imply a finite lending regime.

**Lemma 4.** *If $\sigma < e^{\lambda T^+ + e^{-\lambda T^-} - 1}$, then any period of lending, that starts with all the bad borrowers being in the pool, is finite.*

*Proof.* The formula (27) tells us how $\eta_t^N$ behaves during landing periods. First $T^-$ it is exponentially declining till the level of $\eta e^{-\lambda T^-}$ and for the next $T^-$ period changes due to the law

$$\eta_{T^-+t}^N = \eta e^{-\lambda t}(e^{-\lambda T^-} + \lambda t), \quad t \in [0, T^-]. \tag{28}$$

Since in the case under consideration $T^+ < T^-$, we can be sure that during the second wave of $[T^-, 2T^-]$, the mass of good borrowers is already steady. So it is enough to show, that the maximum possible level of $\eta_t^N$ during the second wave is higher than level of threshold, that can be calculated as $\mu e^{-\lambda T+}(R-1)$.

Let consider $t_{max} > 0$, such that at the moment $(T^- + t_{max})$ the mass of the bad borrowers in the pool reaches its maximum.

We are going to obtain condition on the finite period of lending in form of

$$\eta_{T^-+t_{max}}^N > \mu e^{-\lambda T^+}(R-1). \tag{29}$$

After the substitution of (28)

$$\eta e^{-\lambda t_{max}}(e^{-\lambda T^-} + \lambda t_{max}) > \mu e^{-\lambda T^+}(R-1), \tag{30}$$

or equivalently

$$e^{-\lambda t_{max}}(e^{-\lambda T^-} + \lambda t_{max}) > \sigma e^{-\lambda T^+}, \tag{31}$$

where $\sigma = \frac{(R-1)}{\eta/\mu}$ — the indicator of quality of population.

Solving the maximization for $e^{-x}(e^{-y} + x)$ with respect to $x$:

FOC:

$$-e^{-x}(y + x) + e^{-x} = e^{-x}(1 - x - y) = 0. \tag{32}$$

hence

$$x_{max} = 1 - y. \tag{33}$$

we obtain

$$\lambda t_{max} = 1 - e^{-\lambda T^-}. \tag{34}$$

Substitution this into (31) we derive condition on the finite period of lending

$$e^{e^{-\lambda T^-}-1}(e^{-\lambda T^-} + 1 - e^{-\lambda T^-}) = e^{e^{-\lambda T^-}-1} > \sigma e^{-\lambda T^+}, \tag{35}$$

or

$$e^{\lambda T^+ + e^{-\lambda T^-}-1} > \sigma \tag{36}$$

□

We have proven so far, that periods of lending and no lending are finite, and during those periods masses are stabilizing. The last things left to be checked — are those periods long?

**Lemma 5.** *Under the condition on memories ($T^+ \leq T^-$) any period of lending, starting with all bad borrowers inside the pool, is "long" ($T^+ \leq T_l$).*

*Proof.* At the begging of lending period all bad borrowers are in the pool. When lending starts, borrowers in the pool start to receive loans, reveal their type and leave the pool. For the first $T^-$ bad borrowers are only leaving the pool, as there no borrowers with a $D(\tau)$ record, where $\tau < T^-$, that can be erased during this period. So rate of change of mass of bad borrowers during this period is $\dot{\eta}_t^N = -\lambda \eta_t^N$. On the other hand good borrower are leaving the pool with the same pace, but the are some good borrowers who is going to enter the pool during this period. So the total change in mass of good borrowers in the pool is less, then total change of bad ones. So the quality of the pool is only increasing for the first $T^-$ period. This proves, that the length of lending is not less then $T^-$. The last step is to remember the condition $T^+ \leq T^-$. So we have $T^+ \leq T^- < T_l$. □

**Lemma 6.** *Consider a period of lending starting with all the bad borrowers being in the pool. If this lending period lasts for $T_l$, such that $T^- \leq T_l < \infty$ then the following period of no-lending lasts for $T_n \geq T^-$.*

*Proof.* The condition we are going to present should insure $T^- \leq T_n$ so no-lending period is "long" enough for all bad borrowers to be forgotten and join the pool.

Without loss of generality, we can us assume that period of lending starts at 0 and lasts for $T_l$. So $T_l$ is the moment of switching to no lending. Hence $\eta^N_{T_l}/\mu^N_{T_l} = R-1$. We have to prove, that under the assumptions of the Lemma

$$\eta^N_{T_l+t}/\mu^N_{T_l+t} > R-1, \quad t \in (0, T^-], \tag{37}$$

which insures that there is no switching for the next $T^-$.

First let us study the changing of $\mu^N_t$ during no-lending.

Since during no-lending there is no trading with the pool, the mass of the good borrowers is changing only due to those borrowers, whose history is stored for $T+$ and going to be erased. To enter the pool at moment $T_l + t$ borrower have to get a loan at the moment $T_l + t - T^+$ and have no loans for the next $T+$. At any moment no more then $\lambda\mu$ of good borrowers receives a loan and only $e^{-\lambda T^+}$ fraction of them are not facing investment opportunities during the following $T^+$ period. So we obtain the upper bound on the rate of change of the mass of the good borrowers in the pool in form of

$$\dot{\mu}^N_{T_l+t} \leq \lambda\mu e^{-\lambda T^+}. \tag{38}$$

By integrating this inequality we obtain upper bound on the mass of good borrowers in the pool during no lending period

$$\mu^N_{T_l+t} \leq \mu^N_{T_l} + \lambda t \mu e^{-\lambda T^+} = \mu e^{-\lambda T^+}(1 + \lambda t). \tag{39}$$

So the rate of change of $\eta^N$ during $[T_l, T_l + T^-]$ directly depends on $\eta^N$ during $[0, T_l]$. Since preceding period of lending starts with all bad borrowers in the pool, we know that during $[0, T_l]$ the mass of bad borrowers in the pool decreasing for the first $T^-$ and increasing on $[T^-, T_l]$ until gets to the threshold level of $\mu e^{-\lambda T^+}(R-1)$.

One important observation is that at moment of switching to no-lending $T_l$ the mass of bad borrowers in the pool increase, since the mass of good borrowers is already stabilized on the stationary level and mass of bad borrowers have to rise to the threshold level to perform a switch to no-lending. So $\dot{\eta}^N_{T_l} = -\lambda\eta^N_{T_l} + \lambda\eta^N_{T_l-T^-} > 0$. Hence

$$\eta^N_{T_l-T^-} > \eta^N_{T_l} = \mu e^{-\lambda T^+}(R-1). \tag{40}$$

Now let consider function

$$f(t) = \eta^N_{T_l+t} - \mu e^{-\lambda T^+}(R-1)(1 + \lambda t), \quad t \in [0, T^-]. \tag{41}$$

As for any $t \in [0, T^-]$ there is no lending to the pool at the moment $T_l + t$, then bad borrowers are only entering the pool with the rate $\dot{\eta}^N_{T_l+t} = \lambda \eta^N_{T_l+t-T^-}$

For $t = 0$ we have

$$f(0) = \eta^N_{T_l} - \mu e^{-\lambda T^+}(R-1) = 0. \tag{42}$$

Using (40) we have

$$\dot{f}(0) = \dot{\eta}^N_{T_l} - \lambda \mu e^{-\lambda T^+}(R-1) = \lambda(\eta^N_{T_l - T^-} - \mu e^{-\lambda T^+}(R-1)) > 0. \tag{43}$$

Since after $T^-$ of no-lending all bad borrower are in the pool, for $t = T^-$ we have

$$f(T^-) = \eta^N_{T_l+T^-} - \mu e^{-\lambda T^+}(R-1)(1+\lambda T^-) = \eta^N - \mu e^{-\lambda T^+}(R-1)(1+\lambda T^-), \tag{44}$$

$$\dot{f}(T^-) = \dot{\eta}^N_{T_l+T^-} - \lambda \mu e^{-\lambda T^+}(R-1) = \lambda(\eta^N_{T_l} - \mu e^{-\lambda T^+}(R-1)), \tag{45}$$

using (7) and the fact, that $T_L$ is a moment of regime switching we get $f(T^-) > 0$ and $\dot{f}(T^-) = 0$.

If there exist point $t$, such that $f(t) < 0$, then there have to exist points $t_1$ and $t_2$ in $(0, T^-)$, such that $\dot{f}(t_1) = \dot{f}(t_2) = \dot{f}(T^-) = 0$. This would imply that $\eta^N_t$ reaches level of $\mu e^{-\lambda T^+}$ in 3 points on the $[0, T_l]$. But as we have discussed above the the mass $\eta^N_t$ during lending period has a monotonic decrease during $[0, T^-]$ and monotonic increase on $(T^-, T_l]$.

So we have proven that $f(t) \geq 0$ for all $t \in [0, T^-]$.

So we have $\eta^N_{T_l+t} \geq \mu e^{-\lambda T^+}(1+\lambda t)(R-1)$ for any $t \in [0, T^-]$. Using (39) we have

$$\eta^N_{T_l+t} \geq \mu^N_{T_l+t}(R-1), \quad \text{for any} \quad t \in [0, T^-]. \tag{46}$$

Hence no-lending period lasts for at least $T^-$. □

Combining all Lemmas together we have Theorem 4 proved. □

*Proof of Corollary 2.* Upper bound on $T_l$ also follows from the proof of Lemma 4, as it is shown, that the highest pick of $\eta^N_t$ reached during $[T^-, 2T^-]$ of lending. But at this period mass of good is already constant and if there is a regime switching it has to be happen at this time range. □

*Proof of Proposition 2.* In the Lemma 4 it was shown , that regime switching from lending to no-lending happens in the range between $T^-$ and $2T^-$. Let $\tau_l$, as $T_l - T^-$. The condition on regime switching at the moment $T_l$ is

$$\frac{\eta^N_{T_l}}{\mu^N_{T_l}} = R - 1. \tag{47}$$

Using stabilization of $\mu^N$ and formula (27) for $\eta^N$ during $[T^-, 2T^-]$ we have condition on the $\tau_l$

$$e^{-\lambda\tau}(e^{-\lambda T^-} + \lambda\tau) = \sigma e^{-\lambda T^+}, \tag{48}$$

that is equivalent to

$$e^{-(e^{-\lambda T^-}+\lambda\tau)}(e^{-\lambda T^-} + \lambda\tau) = \sigma e^{-(e^{-\lambda T^-}+\lambda T^-)} \tag{49}$$

$$-(e^{-\lambda T^-} + \lambda\tau) = W(-\sigma e^{-(e^{-\lambda T^-}+\lambda T^-)}) \tag{50}$$

$$\tau = -\frac{1}{\lambda}(W(-\sigma e^{-(e^{-\lambda T^-}+\lambda T^-)}) + e^{-\lambda T^-}) \tag{51}$$

where $W(x)$ — is a Lambert W function that defined as inverse of $f(z) = ze^z$.

Hence

$$T_l = T^- - \frac{1}{\lambda}(W(-\sigma e^{-(e^{-\lambda T^-}+\lambda T^-)}) + e^{-\lambda T^-}), \tag{52}$$

Comparative statics on $T_l$:

Define $y = -(e^{-\lambda T^-} + \lambda\tau)$, then (48) equivalent to

$$ye^y = -\sigma e^{-\lambda T^-} e^{-e^{-\lambda T^-}}. \tag{53}$$

As switching to no lending happen before $t_{\max} = (1 - e^{-\lambda T^-})/\lambda$ moment, in which mass of bad borrowers reaches its maximum, we know that $0 < \lambda\tau < 1 - e^{-\lambda T^-}$. This implies that $-1 \le y \le 0$. Within this range function $ye^y$ monotonically increasing. From (53) we know, that $ye^y$ is negatively correlated with $\sigma$ and positively with $T^+$. From definition of $y$ we also know, that $y$ and $\tau$ are negatively correlated, that immediately implies negative correlation of $y$ and $T_l$.

Combining all together implies that $T_l$ increases with $\sigma$ and $T_l$ decreases with $T^+$.

To understand dependance between length of period of lending $T_l$ and $T^-$ consider $y = e^{-\lambda T^-}$, $x = \lambda\tau$, $c = \sigma e^{-\lambda T^+}$. Substituting all this into (48) we get

$$e^{-x}(x + y) = c \tag{54}$$

$$e^{-(x+y)}(x + y) = ce^{-y} \tag{55}$$

$$(x + y) = -W(-ce^{-y}) \tag{56}$$

$$x = -W(-ce^{-y}) - y \tag{57}$$

where $W(x)$ — is a Lambert W function that defined as inverse of $f(z) = ze^z$. Lambert function has a derivative $W\prime(z) = 1/(z + e^{W(z)})$. By differentiation of (57) we have

$$\frac{dx}{dy} = -ce^{-y}W\prime(-ce^{-y}) - 1 \tag{58}$$

$$\frac{dx}{dy} = \frac{-ce^{-y}}{-ce^{-y} + e^{W(-ce^{-y})}} - 1 \tag{59}$$

$$\frac{dx}{dy} = \frac{e^{W(-ce^{-y})}}{ce^{-y} - e^{W(-ce^{-y})}} \tag{60}$$

From the last equation it is seen, that $\frac{dx}{dy} > 0$ iff $ce^{-y} > e^{W(-ce^{-y})}$

From (55) and (56) we can derive that $e^{W(-ce^{-y})} = e^{-(x+y)} = \frac{ce^{-y}}{x+y}$. So $\frac{dx}{dy} > 0$ iff $ce^{-y} > ce^{-y}/(x + y)$. As $c = \sigma e^{-\lambda T^+} > 0$ and $x + y = \lambda\tau + e^{-\lambda T^-} < 1$ we derive, that $\frac{dx}{dy} < 0$. So $\lambda\tau$ is negatively correlated with $e^{-\lambda T^-}$, which implies $\lambda\tau$ positively correlated with $\lambda T^-$. $T_l = T^- + \tau$ implies, that $T_l$ increases with $T^-$.

Now we are going to focus on the no-lending regime. On the contrary with lending case we have no primary bound on $T_n$ — the length of no-lending period.

Before studying the condition on the moment of switching, we are going to present recursive formula for for the mass of good borrowers during no-lending regime (similar to (27) for $\eta^N$ during lending).

Without lose of generality, let assume that $T_l$ is the moment of switching to the regime of no lending.

Instead of focusing on the $\mu_t^N$ mass of good borrowers in the pool, we are going to present recursive formula for $\mu_t^S$ mass of good borrowers with $S$ record. Obviously knowing $\mu_t^S$ it is easy to find $\mu_t^N$, as

$$\mu_t^N = \mu - \mu_t^S. \tag{61}$$

$\mu_t^S$ is the mass of borrowers who can actually borrow during no-lending period.

As we are focusing on a no-lending regime, the borrowers are only entering the pool and no one gets out. The mass of good borrowers with $S$ record is constantly declining. Similarly to the case of lending let divide period of no lending $[T_l, T_l + T_n]$ into segments of length $T^+$. As $T_l > T^+$ we can state, that during $[T_l - T^+, T_l]$ there was lending with the pool, so all good borrowers were able to get a loan and at each moment $\lambda\mu$ of them received an $S$ record. $e^{-\lambda T^+}$ of them haven't faced an investment opportunity for the next $T^+$ and get into the pool. So for any moment during the first wave of no-lending $[T_l, 2T_l]$ the rate of change of mass of borrowers with $S$ is

$$\dot{\mu}^S_{T_l+t} = -\lambda e^{-\lambda T^+}\mu, \quad t \in [0, T^+]. \tag{62}$$

At the moment $T_l$ the mass of good borrowers in the pool equal to the $\mu e^{-\lambda T^+}$ (due to Lemma 1). So $\mu^S_{T_l} = \mu(1 - e^{-\lambda T^+})$. Using this as an initial condition and integrating (62) we have

$$\mu^S_{T_l+t} = \mu(1 - e^{-\lambda T^+} - \lambda t e^{-\lambda T^+}), \quad t \in [0, T^+]. \tag{63}$$

For the following waves, as the only borrowers with $S$ can trade, the following formula holds

$$\dot{\mu}^S_{T_l+t} = -\lambda e^{-\lambda T^+}\mu^S_{T_l+t-T^+}, \quad t > T^+, \tag{64}$$

Define

$$\mu_k(t) = \mu^N_{T_l+kT^++t}, \quad t \in [0, T^+], k \in 1, 2, 3, \ldots. \tag{65}$$

Rewriting (64) with $\mu_k(t)$ we obtain system of differential equations

$$\left\{ \begin{array}{ll} \dot{\mu}_k(t) = -\lambda e^{-\lambda T^+}\mu_{k-1}(t), & t \in [0, T^+], k \in 1, 2, 3, \ldots \\ \mu_k(0) = \mu_{k-1}(T^+) & \end{array} \right. \tag{66}$$

Using a Taylor expansion we obtain recursive formule for $\mu_k(t)$

$$\left\{ \begin{array}{ll} \mu_k(t) = \sum_{i=0}^{k} \mu_{k-i-1}(T^+)\frac{(-\lambda e^{-\lambda T^+} t)^i}{i!}, & t \in [0, T^+], \\ \mu_k(0) = \mu_{k-1}(T^+) & \\ \mu_1(t) = \mu(1 - e^{-\lambda T^+} - \lambda t e^{-\lambda T^+}), & t \in [0, T^+] \\ \mu_1(0) = \mu & \text{(initial condition)} \end{array} \right. \tag{67}$$

And the mass of good borrowers in the pool can be calculated using $\mu^N_t = \mu - \mu^S_t$.

Now we are ready to show, that condition

$$\sigma \geq \frac{e^{2\lambda T^+}}{(1 + 2\lambda T^+)e^{\lambda T^+} - \lambda T^+ - \frac{(\lambda T^+)^2}{2}}, \tag{68}$$

switching from no-lending to lending happens within $[T^+, 2T^+]$.

Lemma 6 guaranties that no-lending period last at least for $T^-$, at the moment of switching $\eta^N_{T_l+T_n} = \eta$. As we are looking for a switching to happen during the second wave we do know the formula of $\mu^N_t$ as well. Let $T_n = T^+ + \tau_n$, then for $\tau_n \in [0, T^+]$ we have

$$\mu^N_{T_l+T_n} = \mu - \mu_2(\tau_n) =$$
$$\mu e^{-\lambda T^+} + \lambda T^+ \mu e^{-\lambda T^+} + \lambda \tau_n e^{-\lambda T^+}(\mu - \mu e^{-\lambda T^+}) + \frac{(\lambda\tau_n)^2}{2}\mu e^{-2\lambda T^+} = \tag{69}$$
$$\mu e^{-\lambda T^+}(1 + \lambda T^+ + \lambda\tau_n - e^{-\lambda T^+}(\lambda\tau_n + \frac{(\lambda\tau_n)^2}{2}))$$

We know that at the moment of switching to lending all bad borrowers are in the pool. The mass of good borrowers in the pool is permanently growing. Hence for checking, that no-lending lasts no longer then $2T^+$ all we need is compare $\frac{\eta}{\mu^N_{T_l+2T^+}}$ and $\sigma$. Substituting (69) with $\tau_n = T^+$ into switching condition we have exactly condition (68)

Comparative statics on $T_n$:

$T_n = T^+ + \tau_n$, where $\tau_n$ is a solution of

$$e^{-\lambda T^+}(1 + \lambda T^+ + \lambda\tau_n - e^{-\lambda T^+}(\lambda\tau_n + \frac{(\lambda\tau_n)^2}{2})) = \frac{1}{\sigma} \tag{70}$$

or

$$\tau_n = \frac{1}{\lambda}\sqrt{e^{2\lambda T^+}(1 - 2/\sigma) + 2\lambda T^+ e^{\lambda T^+} + 1} \tag{71}$$

From (71) it is seen, that $T_n$ does not depend from $T^-$ and decreases with $\sigma$

To understand dependance between length of period of no lending $T_n$ and $T^+$ consider $x = \lambda\tau_n$, $y = \lambda T^+$ and

$$f(x,y) = e^{-y}(1 + x + y - e^{-y}(x + \frac{x^2}{2})) = \frac{1}{\sigma} \tag{72}$$

Compute total differential

$$df(x,y) = e^{-y}(1 - e^{-y}(1+x))dx +$$
$$+ (-e^{-y}(1 + x + y - e^{-y}(x + \frac{x^2}{2})) + e^{-y}(1 + e^{-y}(x + \frac{x^2}{2})))dy = 0 \tag{73}$$

$$(1 - e^{-y}(1+x))dx = (x + y - 2e^{-y}(x + \frac{x^2}{2}))dy \tag{74}$$

Left hand side:

$$y > x \Rightarrow e^y > e^x > 1 + x \Rightarrow 1 - e^{-y}(1+x) > 0 \tag{75}$$

Right hand side:

$$e^y > 1 + y > 1 + \frac{x}{2} \tag{76}$$

$$\frac{1}{1 + x/2} > e^{-y} \tag{77}$$

$$\frac{2x}{x + x^2/2} > 2e^{-y} \Rightarrow x + y - 2e^{-y}(x + \frac{x^2}{2}) > 0 \tag{78}$$

This implies that $\tau_n$ increase with $T^+$, so also $T_n$ increase with $T^+$. □

*Proof of Proposition 5.* $\omega \in \Omega^{ErC}$

Let define $\bar{W}$ as a mean of the welfare over the first periods on lending and no-lending. $\bar{W} = \frac{1}{T_l+T_n}\int_0^{T_l+T_n} w(\omega, \Delta_0^{NI}, t)dt.$

Now let define mean of the welfare over a period $T$ as $W(T) = \frac{1}{T}\int_0^T w(\omega, \Delta_0^{NI}, t)dt$. So $W(\omega, \Delta_0^{NI}) = \underset{T\to\infty}{limsup} W(T)$.

Let $n$ be such a number that $n(T_l + T_n) \leq T < (n+1)(T_l + T_n)$. Then

$$\frac{1}{(n+1)(T_l+T_n)} \int\limits_0^{n(T_l+T_n)} w(t)dt < W(T) < \frac{1}{n(T_l+T_n)} \int\limits_0^{(n+1)(T_l+T_n)} w(t)dt. \tag{79}$$

Since economy is in a ergodic cyclical equilibrium we have

$$\frac{n}{n+1}\bar{W} < W(t) < \frac{n+1}{n}\bar{W}. \tag{80}$$

With $T$ going to infinity we have $W(\omega, \Delta_0^{NI}) = \bar{W}$.

Now let look at $\bar{W}$ more carefully. For any lending moment t $\mu_t^S(0) = \lambda\mu$, $\eta_t^D(0) = \lambda\eta_t^N$. For any moment t of no lending $\mu_t^S(0) = \lambda(\mu - \mu_t^N)$ and $\eta_t^D = 0$, as only good borrowers with positive record are financed. So the surplus generated over a small interval of time $[t, t + dt]$ is equal to $\lambda((R-1)\mu - \eta_t^N)dt$ for lending period and $\lambda(R - 1)(\mu - \mu_t^N)dt$. Hence

$$\begin{aligned}\bar{W} = \tfrac{1}{T_l+T_n}(\textstyle\int_0^{T_l} \lambda((R-1)\mu - \eta_t^N)dt + \int_{T_l}^{T_l+T_n} \lambda((R-1)\mu - \mu_t^N)dt) = \\ = \lambda(R-1)\mu - \tfrac{1}{T_l+T_n}\lambda(\textstyle\int_0^{T_l} \eta_t^N dt + \int_{T_l}^{T_l+T_n}(R-1)\mu_t^N dt).\end{aligned} \tag{81}$$

□

*Proof of Theorem 5.* 1) First let show, that if $\omega \in \Omega^{St}$ such that **SL** and **SMSL** co-exist, then

$$W(\omega, \Delta_0^{St}) > W(\omega, \Delta_0^M)$$

. Propositions 3 and 4 imply that $W(\omega,\Delta_0^{St}) - W(\omega,\Delta_0^{M}) = \lambda\left(\eta_M^N - \frac{eta}{1+\lambda T^-}\right)$. But $\eta_M^N = \frac{eta}{1+\lambda x T^-}$, with $x<1$. Hence $W(\omega,\Delta_0^{St}) > W(\omega,\Delta_0^{M})$.

2) Now we are going to compare stationary lending welfare with cyclical on:

Since $\omega \in \Omega^{St}$ the condition (4) hold. So

$$\bar{\eta} \le \bar{\mu}(R-1), \tag{82}$$

where $\bar{\eta} = \frac{\eta}{1+\lambda T^-}$ and $\bar{\mu} = \mu e^{-\lambda T^+}$.

The inequality (**??**) can be written as

$$\lambda(R-1)\mu - \frac{1}{T_l+T_n}\lambda(\int_0^{T_l}\eta_t^N dt + \int_{T_l}^{T_l+T_n}(R-1)\mu_t^N dt) \le \lambda((R-1)\mu - \frac{\eta}{1+\lambda T^-}), \tag{83}$$

which is equivalent to

$$\int_0^{T_l}\eta_t^N dt + \int_{T_l}^{T_l+T_n}(R-1)\mu_t^N dt \ge (T_l+T_n)\frac{\eta}{1+\lambda T^-}, \tag{84}$$

Lets notice that during no-lending period $\mu_t^N \ge \bar{\mu} = \mu e^{-\lambda T^+}$. Combining with (82) we have that $\int_{T_l}^{T_l+T_n}(R-1)\mu_t^N dt \ge T_n\frac{\eta}{1+\lambda T^-}$. Hence the last part left to be proven, is

$$\int_0^{T_l}\eta_t^N dt \ge T_l\frac{\eta}{1+\lambda T^-}. \tag{85}$$

We know the dynamic of the mass of bad borrowers with no record during lending period. For the first $T^-$ the mass of bad borrowers in the pool declines as $\eta_t^N = \eta e^{-\lambda t}$, and on the $[T^-, T_l]$ it is rise to the threshold level. At the moment $T_l$ of switching to no-lending the mass of bad borrowers is high enough to stop lending to the pool. More precisely $\eta_{T_l}^N = \bar{\mu}(R-1)$, which is greater then level $\bar{\eta}$ due to (82) . Hence there exist a moment $\tau \in [T^-, T_l]$ such that $\eta_\tau^N = \bar{\eta}$. Now let divide $[0, T_l]$ into $[0, \tau - T^-]$, $[\tau - T^-, \tau]$ and $[\tau, T_l]$.

First notice, that since $\tau \in [T^-, T_l]$ the mass of bad borrowers in the pool is increasing at that moment. Hence $\dot{\eta}_\tau^N = \lambda\eta_{\tau-T^-}^N - \lambda\eta_\tau^N > 0$, which implies, that $\eta_{\tau-T^-}^N > \eta_\tau^N = \bar{\eta}$. So we have $\eta_t^N$ being higher then $\bar{\eta}$ for any $t \in [0, \tau - T^-]$.

On the third segment$[\tau, T_l]$ $\eta_t^N$ is also higher then $\bar{\eta}$, as the mass of bad borrowers increases.

Finally we can use the fact, that $\eta_\tau^N + \int_{\tau-T^-}^{\tau}\lambda\eta_t^N dt = \eta$. Rearranging we have

$$\int_{\tau-T^-}^{\tau}\eta_t^N dt = \frac{\eta - \eta_\tau^N}{\lambda}, \tag{86}$$

But since $\eta_\tau^N = \bar{\eta} = \frac{\eta}{1+\lambda T^-}$, we have

$$\int\limits_{\tau-T^-}^{\tau} \eta_t^N dt = T^- \frac{\eta}{1+\lambda T^-}, \tag{87}$$

Combining all three parts $[0, \tau - T^-]$, $[\tau - T^-, \tau]$ and $[\tau, T_l]$ together we finally have $\int_0^{T_l} \eta_t^N dt \geq T_l \frac{\eta}{1+\lambda T^-}$. □

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