Favoritism in Auctions: A Mechanism Design Approach

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Abstract

The auction designer has one favorite among bidders and maximizes his utility by choosing an auction format. To prevent favoritism, several restrictions are imposed on the designer. I show that even if the designer is restricted to using anonymous and dominant strategy incentive compatible auctions, for any allocation rule she can transfer all potential revenue to her favorite and guarantee him the interim utility at least equal to his value. The equivalence of anonymity with respect to bids and anonymity with respect to true values is also established in this case.

When the non-positive transfers restriction is added, the auction choice still depends on the favorite's value. The designer chooses a second-price auction with pooling, where she commits to not distinguishing values in pooling regions and using lotteries to determine a winner. To fully prevent favoritism, the deterministic auctions restriction is added. Altogether, these restrictions allow implementing only a specific class of second-price auctions with a generalized reserve price. For each bidder, this reserve price depends on other bids. The designer chooses the standard second-price auction from this class and no favoritism is possible.

Keywords: Collusion, Favoritism, Lotteries, Mechanism design, Second-price auction

JEL Classification Codes: D45, D82, H42

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1 Introduction

In this paper, I approach the problem of favoritism from the mechanism design perspective. Thus, I study how the designer chooses the auction format to maximize her favorite's utility under different sets of restrictions on the implemented auction rules. In my model, favoritism does not arise due to some hidden actions, unfair manipulation with bids, fictitious bidders, cheating or other unfair actions; rather, favoritism is solely due to the design of the mechanism. There are two main reasons that make the problem interesting: first, there are some restrictions on the auctions formats, which the designer needs to meet while choosing an auction; and second, in addition to the knowledge of her favorite's identity, the designer has information about how much her favorite values the good. This information can be used in the auction design, whereby the designer can choose different auctions for different values of her favorite. The main questions are what auction the designer chooses under different sets of restrictions to make her favorite better depending on his value and what is a good set of restrictions to prevent different forms of favoritism. The first question can be paraphrased in terms of the situation where the bidder chooses an auction. Namely, what auction would be chosen, if one particular bidder could choose an auction he would like to participate in.

There are many real-life auctions where the problem of favoritism is relevant. For example, consider a situation where the principal intends to sell some good using an auction. If she is not sufficiently informed about the market and potential buyers, she could hire an expert to design the auction format to achieve goals such as revenue maximization, efficiency maximization, etc. However, the designer's incentives can differ from those of the principal. As a result, the auction format chosen by the designer can substantially differ from that preferred by the principal. In this paper, this conflict of interest arises in a situation where the designer has a favorite among potential buyers. One possible reason for this would be a bribe from this particular buyer or any other form of collusion. Subsequently, the designer's objective could be maximization of this particular buyer's utility and the principal could not achieve her goal in the auction outcome. If the principal's objective is revenue maximization and the designer chooses an auction format where the good directly goes to her favorite, this outcome is a disaster for the principal in terms of collected revenue, which is equal zero.

Thus, the principal would like to limit the freedom of an auction format choice given to the designer to prevent favoritism. Another situation is a government auction; for example, a government procurement auction, where one of the participating companies may be partially or fully owned by a government. In this case, the government could prefer to choose the auction format that favors this company.

However, one essential requirement for the rules of a procurement is that they guarantee fair competition. Institutions like the European Commission and the WTO set procurement guidelines that should ensure the absence of positive and negative discrimination. In particular, "equal treatment, non-discrimination, mutual recognition, proportionality and transparency" (European Commission, 2014) are required. "Each Party shall seek to avoid introducing or continuing discriminatory measures that distort open procurement" (World Trade Organization, 2011). Nevertheless, statistics show that discrimination in procurement is present. According to an estimate (PwC and Ecorys, 2013), the costs of corruption in public procurement in eight EU countries ranged from €1.4billion to €2.2 billion in 2010. More than half of foreign bribery cases occurring involved obtaining a public procurement contract (OECD, 2014). 10-30% of the investment in a publicly-funded construction project may be lost through mismanagement and corruption (COST, 2012). The question is why the implemented legal restrictions cannot prevent discrimination and favoritism and how legal restrictions should be changed.

It is obvious that if there are no restrictions imposed on the designer, then the designer can simply allocate the good to her favorite and not charge him anything. This is an example of a situation, which I call perfect favoritism. Namely, perfect favoritism is possible if the designer can guarantee her favorite the ex-post utility higher than his value in any equilibrium of the auction. Hence, some restrictions are needed to prevent this. Probably the most natural attempt to avoid such obvious favoritism is to impose an *anonymity* restriction to eliminate direct discrimination by identity of the bidder. Anonymity means that the allocation and transfer rules should only depend on the submitted bids, rather than the identities of bidders. However, it emerges that anonymity alone is not a particularly useful restriction for several reasons. First, as shown by Deb and Pai (2015), given some asymmetric auction the designer is often able to construct an anonymous auction, which has an equilibrium such that it provides the same expected outcome as the original auction. Thus, if we assume that the designer can choose an equilibrium, then anonymity restriction alone is not a binding constraint at all. One further reason is the first main result of my paper (Theorem 2), showing that if there is some anonymous and *dominant strategy incentive compatible* (DIC) auction that generates revenue R, then there exists another anonymous and DIC auction that has the same allocation rule and where the whole revenue R is transferred to the favorite. For example, the designer can implement the allocation rule of a second-price auction and transfer all collected revenue to his favorite. Hence, the favorite either obtains the good for free or obtains the revenue weakly higher than his value. Therefore, the designer can implement perfect favoritism in an anonymous and dominant strategy incentive compatible auction. This result is stronger than the result of Deb and Pai (2015) in the sense that it does not use the fact that the designer chooses a particular equilibrium. It should be also emphasized that if the auction is DIC, then standard anonymity restriction with respect to bids implies "true" anonymity with respect to values in the corresponding direct auction (Theorem 1).

I call intra-auction favoritism a situation where the designer can discriminate bidders within the auction (a bidder with a higher value obtains lower utility than a bidder with a lower value). To avoid intra-auction discrimination via transfers that results to perfect favoritism, I additionally impose the *non-positive transfers* restriction, which does not allow the designer to transfer collected revenue to her favorite. I analyze the case with two bidders and show that under these three restrictions the intra-auction favoritism is not possible and the favorite's preferred auction is a *second-price auction with pooling* (Proposition 1). This is the second important result of the paper. Pooling means that the designer commits to not distinguishing among the bids in certain regions of the values domain and using a lottery to determine a winner. Pooling is always optimal when the favorite's and his opponent's values are sufficiently close. In this case, the winner is determined by a lottery and the payment is lower than in a second-price auction. Additionally, pooling may be used to reduce payments when the favorite wins. I also provide comparative statics results concerning how the choice of mechanism depends on the favorite's value (Proposition 3). Only the pooling region at the top changes its size, with all other things being equal. If the favorite's value is too low, then the top pooling region covers the whole set of possible values and the optimal mechanism emerges as a simple lottery.

Although intra-auction favoritism is not possible under anonymity, DIC and non-positive transfers[1], the designer makes the choice of the auction dependent on her favorite's value. Even if the chosen auction is fair (non-discriminatory), this is still a form of favoritism. I call this situation inter-auction favoritism. To illustrate the last point, consider a situation in which the designer can only choose among two auction formats: 1) a second-price auction and 2) a symmetric lottery. Both of these formats can be called fair. Indeed, in a second-price auction the bidder with the highest value wins the auction and has to pay the second highest bid. In a lottery, all bidders do not need even to make bids and thus they have the same probabilities of winning the good. However, bidders with different values could still prefer one of these formats to another. For example, if one of n bidders has a low value, he would certainly prefer a lottery rather than a second-price auction, since it gives him a chance to obtain the good for free with probability $1/n$. Meanwhile, a bidder with a high value could prefer a second-price auction rather than a lottery, since his chances of winning the good in the competition are high. Thus, although both described auction formats are fair, they are not equally valued by different bidders.

I show that by imposing one more restriction on the designer, it is possible to prevent any form of favoritism. Thus, I impose a *deterministic auctions* restriction, which does not allow the designer to use randomization to determine a winner if there is a unique highest bid. The third main result characterizes a class of auctions feasible under these four restrictions as *second-price auctions with a generalized reserve price* (Theorem 3). A generalized reserve price is different from the standard reserve price in the sense that it is unique for each bidder and depends on all bids of his opponents. However, it is constructed in a symmetric way to preserve anonymity restriction. Independent of the favorite's value, the auction maximizing the utility of the favorite in this class of auctions is a standard second-price auction without any reserve price (Proposition 4). Thus, this combination of four restrictions allows preventing any form of favoritism.

[1]This is true in the model with two bidders. If there are more than two bidders, then the intra-auction favoritism can still be possible.

I also analyze what kind of favoritism is possible under different subsets of restrictions. I show that the above restrictions form a hierarchy with *non-positive transfers* at the top, *deterministic auctions* at the bottom and *anonymity+DIC* in the middle (Proposition 5). In other words, *non-positive transfers* always reduce the scope of favoritism. *Anonymity* helps if and only if *DIC* is imposed and vice versa. *Deterministic auctions* only matter in combination with *anonymity+DIC*.

My paper is related to those by Deb and Pai (2015) and Azrieli and Jain (2016). They show that for many mechanisms that are not anonymous, one can find a symmetric auction such that it has a Bayes-Nash equilibrium with the same expected revenue and bidder's utilities. Manelli and Vincent (2010) and Gershkov et al. (2013) show that in the independent private values model, there is equivalence of Bayesian and dominant strategy implementation in expected terms. This equivalence does not hold here due to the additional restrictions and in particular anonymity.

Collusion among buyers is studied in Graham and Marshall (1987) and Mailath and Zemsky (1991) for second-price auctions, as well as McAfee and McMillan (1992) for first-price auctions. Robinson (1985), Caillaud and Jehiel (1998), Che and Kim (2006), Marshall and Marx (2007) and Che and Kim (2009) compare possibilities of collusion among buyers or between a buyer and seller in different auction formats. In a setting with non-transferable payments, Condorelli (2012) and Chakravarty, Kaplan (2013) find the social welfare maximizing mechanism with a benevolent designer. They show that the optimal mechanism comprises contest and lottery regions depending on a distribution of values. In my paper, the favorite's preferred auction under the restriction of non-positive transfers exhibits similar properties.

Extensive literature exists on the informed principal problem (see Myerson (1983), Maskin and Tirole (1990, 1992), Severinov (2008), Mylovanov and Tröger (2012, 2014) and Yilankaya (1999)). In such models, the design of a mechanism can reflect the information that the designer possesses. Thus, the choice of the mechanism can partially or fully reveal information to the agents. In my paper, all main results are formulated for dominant strategy incentive compatible auctions. Since each bidder has a dominant strategy, he does not pay attention to the information revealed by the designer.

The remainder of this paper is structured as follows. In the next section, I present the auction model used in the paper. Then, I introduce the concept of favoritism. Subsiquently, I introduce the restrictions sequentially and discuss how they help (or ortherwise) to prevent different forms of favoritism. I conclude with a discussion of open issues. All major proofs are delegated to Appendix A.

2 Auction model

The designer has to conduct an auction to sell one indivisible good (object) to a set $N = \{1, ..., n\}$ of potential bidders. The bidders are characterized by independent private values v_i coming from continuously differentiable distributions F_i on $V_i = [\underline{v}_i, \overline{v}_i]$ with a positive density[2]. The designer has a favorite among the bidders and without loss of generality, I assume that it is the first bidder[3]. The designer knows the value of the favorite $v_1 = v^*$ and maximizes his interim utility[4].

The auction proceeds in the following steps:

1. The designer announces the rules of the auction.
2. Agents simultaneously decide whether they want to participate in the auction and if yes they make their bids.
3. The winner is determined according to the auction rules defined on step 1.

Each bidder i chooses a bid from a given set of admissible bids, $b_i \in \{\oslash\} \cup B_i$, where $B_i \subset \mathbb{R}_+$ and $b_i = \oslash$ mean that the bidder i does not participate in the bidding. By $\mathbf{B} = \times_{i=1}^{n} (\{\oslash\} \cup B_i)$ we denote the product set of admissible bid sets. $M \subset N$ is a set of bidders who participate in the bidding, namely $\forall i \in M : b_i \neq \oslash$. The number of participating bidders is $m = |M|$. I denote a vector of values $\mathbf{v} = (v_1, ..., v_n) \in \times_{i=1}^{n} V_i$ and vector of bids $\mathbf{b} = (b_1, ..., b_n) \in \mathbf{B}$. $N_{-i} = N \backslash \{i\}$, $\mathbf{v}_{-i}$, $\mathbf{b}_{-i}$ are used for the set of bidders without a

[2] $\overline{v_i}$ could be equal to $+\infty$

[3] Otherwise, we can renumerate the bidders such that the favorite obtains a number 1.

[4] The assumption that the designer knows the favorite's value is quite natural. Since the designer wants to maximize the utility of the favorite, their incentives are completely aligned and the favorite would like to disclose the information about his value to the designer regardless.

bidder i. When the bids are submitted, an outcome of the auction has to be determined. Denote by a_j an allocation of the object where an agent j obtains the object. By a_0, I denote the allocation when the object remains unassigned. The set of possible allocations is $A = \{a_j\}_{j=0}^{n}$. An allocation is chosen according to an allocation rule $\mathbf{y} : \mathbf{B} \to [0,1]^n$, $\mathbf{y}(\mathbf{b}) = (y_1(\mathbf{b}), ..., y_n(\mathbf{b}))$, where $y_i(\mathbf{b}) := \Pr(a_i|\mathbf{b})$[5]. The allocation rule determines how often each allocation is chosen. Transfer rule $\mathbf{p} : A \times \mathbf{B} \to \mathbb{R}^n$, $\mathbf{p}(a, \mathbf{b}) = (p_1(a, \mathbf{b}), ..., p_n(a, \mathbf{b}))$, where $p_i(a, \mathbf{b})$ specifies how much agent i receives in the allocation a, given that a vector of bids $\mathbf{b}$ is submitted. Transfers $\mathbf{t} : \mathbf{B} \to \mathbb{R}^n$, $\mathbf{t}(\mathbf{b}) = (t_1(\mathbf{b}), ..., t_n(\mathbf{b}))$, where $t_i(\mathbf{b}) := \sum_{a\in A} p_i(a, \mathbf{b}) \Pr(a|\mathbf{b}) = \sum_{j=0}^{n} p_i(a_j, \mathbf{b}) \Pr(a_j|\mathbf{b}) = \sum_{j=0}^{n} p_i(a_j, \mathbf{b}) y_j(\mathbf{b})$ can be computed after the bids have been submitted, but before an allocation has been chosen.

Example 1 *The auction format is a simple lottery, where the winner and only the winner pays a fixed price γ independent of bids. Subsequently, the allocation rule is $\mathbf{y}(\mathbf{b}) = (1/n, ..., 1/n)$, bidder i pays $-\gamma$ if he obtains the object and 0 otherwise, namely $p_i(a_j, \mathbf{b}) = -\gamma$ if $i = j$ and $p_i(a_j, \mathbf{b}) = 0$ if $i \neq j$, and the transfers are $\mathbf{t}(\mathbf{b}) = (-\gamma/n, ..., -\gamma/n)$.*

The utility of an agent i who participates in the auction is

$$U_i(v_i|a) = v_i I\{a = a_i\} + p_i,$$

where $I : A \to \{0, 1\}$ is an indicator function equal to 1 if $a = a_i$ and 0 otherwise. The ex-post utility of a bidder i given vector of bids $\mathbf{b}$ is as follows:

$$U_i(v_i|\mathbf{b}) = \sum_{a\in A} U_i(v_i, a) \Pr(a|\mathbf{b}) = v_i y_i(\mathbf{b}) + t_i(\mathbf{b}).$$

For any vector of bidding strategies $\boldsymbol{\beta}(\mathbf{v}) = (\beta_1(v_1), ..., \beta_n(v_n))$ where $\beta_i : V_i \to \{\varnothing\} \cup B_i$, we can define the interim utility of a bidder i as an expectation of his ex-post utility taken with respect to a vector of other bidders' values $\mathbf{v}_{-i}$, given that $\beta(\mathbf{v})$ is played. Thus,

$$U_i(v_i|\boldsymbol{\beta}) = v_i \mathbb{E}_{\mathbf{v}_{-i}} y_i(\boldsymbol{\beta}(\mathbf{v})) + \mathbb{E}_{\mathbf{v}_{-i}} t_i(\boldsymbol{\beta}(\mathbf{v})).$$

[5] By $\Pr(a_i|\mathbf{b})$, I mean the probability that an allocation $a_i \in A$ is chosen conditional on a vector $\mathbf{b} \in \mathbf{B}$ is submitted.

When it is clear which bidding strategy we consider, I simply use $U_i(v_i)$ rather than $U_i(v_i|\boldsymbol{\beta})$. Each bidder i participates in the auction, making a bid $b_i \neq \oslash$ if and only if the individual rationality constraint holds:

$$U_i(v_i|\boldsymbol{\beta}) \geq 0. \tag{1}$$

Definition 1 (feasible auction)

A feasible auction $FA = (\mathbf{B}, \mathbf{y}, \mathbf{p})$ *is a collection of bid sets* $\mathbf{B}$*, an allocation rule* $\mathbf{y}$ *and a transfer rule* $\mathbf{p}$*, such that*

$$\begin{aligned} \forall i, \mathbf{b} \quad & 0 \leq y_i(\mathbf{b}) \leq 1, \\ \forall \mathbf{b} \quad & \sum_i y_i(\mathbf{b}) \leq 1, \\ \forall i, a, \mathbf{b}_{-i} \quad & y_i(\mathbf{b}) = p_i(a, \mathbf{b}) = 0 \;\; if \;\; b_i = \oslash. \end{aligned}$$

Any feasible auction should completely ignore bidders who do not participate in the bidding. These bidders never receive the good or transfers. The solution concept is Bayes-Nash equilibrium (BNE). The profile of bidding strategies $\psi = \{\beta_i^*(v_i)\}_{i=1}^n$ constitutes a Bayes-Nash equilibrium of an auction if the interim utility from playing the equilibrium strategy is greater than any other strategy, i.e. for any v_i and for any $\beta_i(v_i)$:

$$\begin{aligned} & v_i \mathbb{E}_{\mathbf{v}_{-i}} y_i(\boldsymbol{\beta}^*(\mathbf{v})) + \mathbb{E}_{\mathbf{v}_{-i}} t_i(\boldsymbol{\beta}^*(\mathbf{v})) \geq \qquad (2) \\ \geq \;\; & v_i \mathbb{E}_{\mathbf{v}_{-i}} y_i(\beta_1^*(v_1), ..., \beta_i(v_i), ..., \beta_n^*(v_n)) + \mathbb{E}_{\mathbf{v}_{-i}} t_i(\beta_1^*(v_1), ..., \beta_i(v_i), ..., \beta_n^*(v_n)). \end{aligned}$$

Definition 2 (no deficit)

An equilibrium ψ *of a feasible auction* FA *is feasible if it does not run ex-post deficit:*

$$\psi : \sum_{i=1}^n t_i(\boldsymbol{\beta}^*(\mathbf{v})) \leq 0. \tag{3}$$

In any feasible equilibrium, the sum of transfers to bidders is non-positive. However, without any further restrictions, a transfer to some particular bidder could be positive. It is important to emphasize here that non-positive transfers are only a restriction only on equilibrium outcome. Thus, it may not hold for any vector $\mathbf{v}$, but should hold for those vectors that appear in equilibrium ψ. Since the designer knows the value of the favorite, the auction can be such that it runs the deficit if the favorite makes a bid b_1 different from $\beta_1^*(v^*)$. However, this never happens in equilibrium ψ and hence it is sufficient that $\sum_i t_i(\boldsymbol{\beta}^*(\mathbf{v})) \leq 0$ only for $v_1 = v^*$ and for any $\mathbf{v}_{-1}$. This concludes the description of a model and now we continue with a concept of favoritism.

3 Favoritism

Denote by $\Psi(A)$ the set of all undominated feasible BNE of some auction A. I will now use notation $U_i(v_i, \psi)$ to denote the interim utility of a bidder i in a particular equilibrium $\psi \in \Psi(A)$.

Definition 3 (favorite's preferred equilibrium)

A favorite's preferred equilibrium (FPE) $\psi^*(A) : A \to \Psi(A)$ is the equilibrium that generates the highest interim utility for the favorite given his value v^* among all feasible undominated equilibria, namely for any $\psi \in \Psi(A)$:

$$U_1(v^*, \psi^*(A)) \geq U_1(v^*, \psi)$$

Definition 4 (favorite's preferred auction)

A favorite's preferred auction (FPA) is a feasible auction that maximizes the favorite's interim utility in FPE, namely,

$$FPA = \arg\max_{FA} U_1(v^*, \psi^*(FA)) \tag{4}$$

Since the choice of an auction may generally depend on the actual value of the favorite, it means that the favorite and all other bidders can be in different information sets when the auction starts. Hence, when an auction format is announced, other bidders can make an inference about a favorite's value. By manipulation with the auction format, the designer can exclude the participation of some potential bidders.

Taking into account the possibility of favoritism, some restrictions can be imposed on auctions proposed by the designer. I denote by $C = \{c_i\}_{i=1}^K$ the set of restrictions on $(\mathbf{y}, \mathbf{p}, \mathbf{t})$. Thus, the designer is not completely free in the choice of an auction. I introduce the following two definitions to take this into account.

Definition 5 (auction feasible under restrictions)

A feasible auction under set of restrictions C (later $FA(C)$) is a feasible auction $FA = (\mathbf{B}, \mathbf{y}, \mathbf{p})$ such that $(\mathbf{B}, \mathbf{y}, \mathbf{p})$ satisfy C.

Definition 6 (favorite's preferred auction under restrictions)

A favorite's preferred auction under set of restrictions C (later $FPA(C)$) is a feasible under C auction, which maximizes favorite's interim utility in FPE, namely,

$$FPA(C) = \arg\max_{FA(C)} U_1(v^*, \psi^*(FA(C)))$$

The concept of favoritism is formulated in the next definitions.

Definition 7 (intra-auction favoritism)

The auction allows intra-auction favoritism if there exist an equilibrium $\psi \in \Psi(A)$, two bidders i, j and a vector of values $\mathbf{v}$, such that $v_i \geq v_j$ and $U_i(v_i|\boldsymbol{\beta}^*(\mathbf{v})) < U_j(v_j|\boldsymbol{\beta}^*(\mathbf{v}))$.

This definition means that intra-auction favoritism exists if there exist an equilibrium and two bidders such that one of them has a greater value and at the same time a lower level of ex-post utility in this equilibrium compared to the other. It also implies that all bidders with the same values should obtain the same utilities. If intra-auction favoritism is possible, it means that the designer can discriminate bidders by their identities within the same auction.

Definition 8 (inter-auction favoritism)

The auction allows inter-auction favoritism if the favorite's preferred auction depends on the favorite's value v^*.

In other words, for two different values of the favorite the choice of the auction format will differ. Thus, even if intra-auction favoritism is not possible, the designer could favor one bidder by a particular choice of a mechanism.

Definition 9 (perfect favoritism)

Perfect favoritism is possible under set of restrictions C if there exists a feasible auction $FA(C)$ such that in any equilibrium in undominated strategies $\psi \in \Psi(FA(C))$ and for any $v^* \in V_1$ the following holds

$$U_1(v^*, \psi) \geq v^*$$

Thus, perfect favoritism is possible when the designer is always able to guarantee her favorite the interim utility greater than or equal to his value of the good. One trivial example is an allocation of the good to the favorite independent of bids. Another example is when rather than allocating a good she sends him a transfer $p_i > v_i$. Of course, these examples may not be feasible under an appropriate set of constraints. Next, we discuss what the designer can do under different sets of restrictions C.

4 Unrestricted favoritism

First, suppose that $C = \oslash$. Thus, no restrictions are imposed on the designer's choice of an auction. In this case, the designer can simply give the object to her favorite for free. However, it is not the favorite's preferred auction and it is possible to construct an even better mechanism. The next proposition provides a characterization of FPA.

Claim 1 (favorite's preferred auction)

If no restrictions are imposed on the designer, the favorite's preferred auction has a favorite's preferred equilibrium in dominant strategies ψ^* and treats the favorite and other bidders differently. The favorite obtains the object if nobody else obtains it and receives all collected revenue. All other bids are treated as in the optimal auction proposed by Myerson (1981), where a seller's reservation value is equal to v^*.

Proof. Since the designer is always able to transfer all collected revenue to his favorite, it is always possible to have an equality in (3) and hence $t_1(\mathbf{b}) = -\sum_{i\neq 1} t_i(\mathbf{b})$. Subsequently, (4) can be rewritten as

$$v^* \mathbb{E}_{\mathbf{v}_{-1}} y_1(\boldsymbol{\beta}^*(\mathbf{v})) - \mathbb{E}_{\mathbf{v}_{-1}} \sum\nolimits_{i\neq 1} t_i(\boldsymbol{\beta}^*(\mathbf{v})) \to \max_{FA}$$

This problem is essentially similar to a problem of profit maximization when the seller has a reservation value equal to v^* and all bidders aside from the favorite participate in the bidding. The result follows directly. ■

The Myerson's optimal auction allocates the good to a bidder with the highest "ironed virtual value" $\phi_i(v_i)$[6], provided that this value is greater than the reservation value r of a seller. The winner should pay the amount that is equal to the lowest $\widehat{v}$, such that it lets him win, i.e. $\widehat{v}$ is the solution to $\phi_i(\widehat{v}) = \max(\{\phi_j(v_j)\}_{j\neq i}, r\}$ In the model of favoritism, we can think about a favorite's value as a reserve value of a designer and thus $r = v^*$. Hence, in FPA the favorite obtains the object if all other bidders have virtual values smaller than v^*, i.e. $\forall j \neq 1$, $\phi_j(v_j) < v^*$. Suppose that the bidder k wins in FPA. The smallest value $\widehat{v_k}$ that lets the him win the auction is always greater than or equal to v^*. Indeed, otherwise, since $\phi_k(v_k) < v_k$, we would have $\phi_k(\widehat{v_k}) < \widehat{v}_k < v^*$, which contradicts the fact that $\widehat{v}$ lets win the auction. Thus, when the favorite does not win the FPA, he always receives a monetary transfer $\widehat{v} > v^*$ and hence even his ex-post utility is greater than his value.

Observation 1 *If the designer is unrestricted, then the perfect favoritism is possible.*

Since all of the collected money goes to the favorite, the actual revenue is always zero. In order to prevent the perfect favoritism and the zero revenue, restrictions on feasible auction should be imposed. To understand what would be the reasonable set of restrictions, I discuss what the designer uses to implement perfect favoritism if she is unrestricted. First, we observe from Claim 1 that the designer always wants to differentiate her favorite and all other bidders. This possibility should be excluded and the natural way to achieve this is to impose a restriction that requires the designer to treat all bidders equally, namely anonymity.

5 Anonymity

Let $\pi : \{1, ..., n\} \to \{1, ..., n\}$ be a permutation. Denote Θ as the set of all permutations of n elements. Later, for simplicity, I will also use expressions like $\pi(i) = j$, where I mean that

[6] $\varphi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ if it is increasing, otherwise $\varphi_i(v_i)$ is equal to a special "ironed" transformation of $v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$, such that it makes it monotone.

the element in i-th position moves to j-th position when permutation π is applied. Denote by $\mathbf{b}_\pi = (b_{\pi(1)}, ..., b_{\pi(i)}, ..., b_{\pi(n)})$

Definition 10 (anonymity)

A feasible auction FA is anonymous (feasible under c_A) if the names of the bidders do not matter, namely if any permutation of bids among bidders alters $(\mathbf{y}, \mathbf{t})$ symmetrically. Precisely, for any bidders $i, j \in N$, for any allocation $a_k \in A$ for any permutation $\pi \in \Theta$ and for any vector of bids $\mathbf{b} \in \mathbf{B}$:

$$\begin{aligned} B_i &= B_j, \\ y_i(b_{\pi(1)}, ..., b_{\pi(i)}, ..., b_{\pi(n)}) &= y_{\pi(i)}(b_1, ..., b_i, ..., b_n), \\ p_i(a_k, b_{\pi(1)}, ..., b_{\pi(i)}, ..., b_{\pi(n)}) &= p_{\pi(i)}(a_{\pi(k)}, b_1, ..., b_i, ..., b_n). \end{aligned}$$

This definition means that if after a permutation π a bidder i makes a bid that an agent $\pi(i)$ has made before the permutation, he should have the same probability of winning the auction and the same transfer at any allocation a_k as the agent $\pi(i)$ before the permutation at the allocation $a_{\pi(k)}$. Note that this also implies that

$$\begin{aligned} & t_i(b_{\pi(1)}, ..., b_{\pi(i)}, ..., b_{(n)}) = \sum_{j=0}^{n} p_i(a_j, \mathbf{b}_\pi) y_j(\mathbf{b}_\pi) = \\ = & \sum_{j=0}^{n} p_{\pi(i)}(a_{\pi(j)}, \mathbf{b}) y_{\pi(j)}(\mathbf{b}) = \sum_{j=0}^{n} p_{\pi(i)}(a_j, \mathbf{b}) y_j(\mathbf{b}) = \\ = & t_{\pi(i)}(b_1, ..., b_i, ..., b_n). \end{aligned}$$

Hence, expected transfers are also symmetric with respect to a permutation. To understand how it works, consider an example with three bidders, and a vector of bids (b_1, b_2, b_3). Consider now the permuted vector of bids (b_2, b_3, b_1). By the anonymity restriction, the probability that bidder 1 wins bidding b_2, when bidder 2 bids b_3 and bidder 3 bids b_1, should be equal to the probability that bidder 2 wins bidding b_2 and his opponents bidding b_1 and b_3. Consider also the allocation a_3, i.e. the third bidder wins the good bidding b_1. Accordingly, the transfer to bidder 1 in this allocation given that he bids b_2, and bidder 2 bids b_3 must be equal to the transfer to bidder 2 when he bids b_2 in the allocation, where bidder 1 wins and bids b_1, with bidder 3 making a bid b_3.

This restriction holds strong importance. Without anonymity, the designer can simply give the object to her favorite for free. By contrast, when anonymity is imposed, the designer is no longer able to discriminate bidders directly by making different rules for different bidders. However, as shown in Deb and Pai (2015), the anonymity restriction often does not truly restrict the designer in the ability to implement the auction that she wants. Suppose that the designer wants to implement the nonanonymous allocation rule, such that it allocates the object to a bidder with the highest index I_i, where $I_i(v_i)$ is some increasing function of a bidder's value.

They show that there exists an anonymous auction and an equilibrium of this auction such that it implements the same allocation and the same expected payments as the original auction. One of their main results is also that the designer is able to implement in a symmetric way, particularly the optimal auction, which is not anonymous if the distributions of agents' values differ. Indeed, since the optimal auction allocates the object to a bidder with the highest ironed virtual value, we can define $I_i(v_i) = \phi_i(v_i)$. In terms of allocation rule, FPA only differs from the optimal auction in the index function for the first agent, namely $I_1(v_1) = v_1$. Hence, this implementability result also holds in our model and the designer can implement FPA as an anonymous mechanism.

In Appendix B, we show ex-post implementability for the case of symmetric bidders. This theorem states that for symmetric bidders it is possible to construct an auction that has an equilibrium such that the outcome of this equilibrium is *ex-post identical* in terms of allocation and payments to the equilibrium of FPA. This equilibrium has the property that among all bidders with values smaller or equal than $\widehat{v}$ only the favorite participates and bids his true value v^*.If there is no other bidder with a value greater than $\widehat{v}$, the favorite wins the object and pays zero; otherwise, the highest bid wins the auction and this bidder pays a maximum of the second highest bid and $\widehat{v}$. This payment goes to the favorite. The intuition behind this result is that when such an auction is announced, all standard bidders with values lower than $\widehat{v}$ know that if they participate they cannot end up with a profit in the case when there is somebody else with a value below $\widehat{v}$, who participates. In this case, the revenue for the designer is also equal to zero.

This construction above — as well as one by Deb and Pai (2015) — has the weakness that there could be many equilibria of the symmetric auction and we emphasize one particular equilibrium where the favorite is preferred. In fact, it is assumed that the designer can choose among different equilibria. There are also $n-1$ similar equilibria where one of bidders participates and the others, including the favorite, do not participate. Since our notion of perfect favoritism requires that the favorite obtains sufficiently high utility in any equilibrium, the construction above does not allow preventing the perfect favoritism. Hence, at this point one could think that perfect favoritism is not possible if anonymity restriction is imposed. However, it is not true and, as we show below, the perfect favoritism is still possible; namely, there exists an auction such that it has only one equilibrium in undominated (in our case, it would even be dominant!) strategies that provides the favorite with the level of utility higher than his value.

Thus, anonymity restriction itself is not sufficient for the absence of favoritism. It is clear that the opportunity to exclude the participation of other bidders has to be disabled. Thus, we consider dominant strategy incentive compatibility restriction.

6 Dominant strategy incentive compatibility

Definition 11 (dominant strategy incentive compatibility)

A feasible auction FA is dominant strategies incentive compatible (DIC, feasible under c_{DIC}) if for any bidder there exists a strategy $\beta_i^*(v_i)$ that provides higher utility than any other strategy independent of how the other bidders play, namely for all $\{\beta_j(v_j)\}, j=1,...n$:

$$v_i \mathbb{E}_{\mathbf{v}_{-i}} y_i(\beta_1(v_1),...,\beta_i^*(v_i),...,\beta_n(v_n)) + \mathbb{E}_{\mathbf{v}_{-i}} t_i(\beta_1(v_1),...,\beta_i^*(v_i),...,\beta_n(v_n)) \geq$$
$$\geq \quad v_i \mathbb{E}_{\mathbf{v}_{-i}} y_i(\beta_1(v_1),...,\beta_i(v_i),...,\beta_n(v_n)) + \mathbb{E}_{\mathbf{v}_{-i}} t_i(\beta_1(v_1),...,\beta_i(v_i),...,\beta_n(v_n))$$

Since the inequality should hold for all $\beta_j(v_j)$, it should also hold for any constant strategies, $\forall j \neq i, \forall v_j : \beta_j(v_j) = b_j$. In turn, if the equality holds for any bids b_j plugged instead of $\beta_j(v_j)$, this means that it would hold in expectation. Thus, the $\beta_i^*(v_i)$ is a dominant strategy for a bidder i if and only if for any $b_j \in \{\varnothing\} \cup B_i$ and for any $\beta_i(v_i)$

$$\begin{aligned} & v_i y_i(b_1, ..., \beta_i^*(v_i), ..., b_n) + t_i(b_1, ..., \beta_i^*(v_i), ..., b_n) \geq \\ \geq \quad & v_i y_i(b_1, ..., \beta_i(v_i), ..., b_n) + t_i(b_1, ..., \beta_i(v_i), ..., b_n) \end{aligned}$$

Although dominant strategy implementation is robust in the sense that the behavior of each player does not depend on what others do, it can have more than one dominant strategy[7]. However, in the auction setting with bidders who have private values and linear utilities, the dominant strategy is unique if it exists. The next result shows this:

Lemma 1 (uniqueness of dominant strategy)

For any FA, there could be at most one dominant strategy in the sense that if there are other dominant strategies they also provide the same allocation and transfers, namely for any two dominant strategies of each player $\beta_i^*(v)$, $\beta_i^{**}(v)$ and for any bids of other bidders $\mathbf{b}_{-i}$ the following holds:

$$\begin{aligned} y_i(b_1, ..., \beta_i^*, ..., b_n) &= y_i(b_1, ..., \beta_i^{**}, ..., b_n), \\ t_i(b_1, ..., \beta_i^*, ...b_n) &= t_i(b_1, ..., \beta_i^{**}, ...b_n), \end{aligned}$$

The next simple lemma is also crucial for our further results and it only holds for anonymous auctions.

Lemma 2 (universality of dominant strategy)

If $b^*(v)$, $v \in V_i$, is a dominant strategy for a bidder i in some anonymous auction $FA(C)$, it is also a dominant strategy for any other bidder j with any value $v_j \in V_i \cap V_j$.

Later on, when we talk about "equilibrium" we mean the unique equilibrium in dominant strategies where all bidders use the same strategy. It is also convenient to consider direct auctions. An auction is called direct if for any bidder $i \in N$ the allowed bidding set is equal to a union of sets of possible values, namely $B_i = \underset{j \in N}{\cup} V_j$ for any i. Subsequently, describing direct auctions, instead of $(\mathbf{B}, \mathbf{y}, \mathbf{p})$, I use simplified notation $(\mathbf{y}, \mathbf{t})$, keeping in

[7] Here, I mean a weakly dominant strategy. If there exists a strictly dominant strategy, it is unique.

mind that $\mathbf{B} = \underset{i \in N}{\times} (\underset{j \in N}{\cup} V_j\}$ and $t_i(\mathbf{v}) = \sum_{j=0}^{n} p_i(a_j, \mathbf{b}) y_j(\mathbf{b})$. The classical revelation principle claims that without loss of generality it is possible to restrict attention to direct mechanisms in which truth-telling is a Bayes-Nash equilibrium. However, under anonymity restriction, it is not possible to directly apply the revelation principle and preserve this restriction for a direct auction. Note that anonymity restriction imposes constraints on allocation and transfers based on bids $\mathbf{b}$, not the values. If anonymity is the only restriction, namely $C = \{c_A\}$, then the anonymity with respect to bids does not imply the anonymity with respect to values of the direct mechanism. To illustrate this idea, consider the auction from Proposition 1. This auction is anonymous with respect to bids, although the bidding behavior is different for different bidders. Thus, bidders with the same values can make different bids in the auction. Hence, the class of anonymous direct auctions is smaller than the class of all anonymous auctions. Hence, while considering anonymous auctions, we cannot simply restrict our attention to direct anonymous auctions. However, under additional DIC restriction, I can show the equivalence between anonymity with respect to bids of the original auction and anonymity with respect to values of the corresponding direct auction.

Theorem 1 (anonymity with respect to valuations)

Anonymity with respect to bids of any DIC auction implies anonymity with respect to values of the corresponding direct auction.

Proof. Suppose that each agent has a dominant strategy $\beta_i^*(v)$ in the original anonymous auction. In the corresponding direct auction, then:

$$\begin{aligned} & y_i(v_{\pi(1)}, ..., v_{\pi(n)}) = y_i(\beta_1^*(v_{\pi(1)}), ..., \beta_n^*(v_{\pi(n)})) = \\ = \;& y_i(\beta_{\pi(1)}^*(v_{\pi(1)}), ..., \beta_{\pi(n)}^*(v_{\pi(n)})) = \\ = \;& y_{\pi(i)}(\beta_1^*(v_1), ..., b_n^*(v_n)) = y_{\pi(i)}(v_1, ..., v_n), \end{aligned}$$

where the first equality follows from Lemma 1, the second equality follows from Lemma 2, the third equality is due to anonymity and the final one is again due to Lemma 1. The similar logic holds for transfers. ■

In other words, for any feasible auction that is DIC and anonymous, the corresponding direct auction is also anonymous. Thus, we do not exclude any feasible auctions when instead of using original anonymous DIC auctions we consider corresponding anonymous direct auctions. Maskin and Laffont (1979) characterizate all DIC direct mechanisms and show that the necessary and sufficient conditions for bidders reporting their true values are as follows:

$$1)\ y_i(\mathbf{v}) \text{ is nondecreasing in } v_i \text{ for all } \mathbf{v}_{-i}, \tag{5}$$

$$2)\ v_i y_i(\mathbf{v}) + t_i(\mathbf{v}) = h_i(\underline{v}_i, \mathbf{v}_{-i}) + \int_{\underline{v}_i}^{v_i} y_i(v_1, ..., \underset{i}{q}, ...v_n)dq \tag{6}$$

where $h_i(\underline{v}_i, \mathbf{v}_{-i})$ are some arbitrary functions that do not depend on the bidder i's value. Using this characterization, we can consider auctions where all bidders report their true values.

We should note that if DIC is the only restriction, i.e. $C = \{c_{DIC}\}$, then it is never binding for the construction of the favorite's optimal auction, namely $FPA(\{c_{DIC}\}) = FPA$. Indeed, the favorite's optimal auction is dominant strategy incentive compatible, since the favorite does not participate in the bidding and his opponents have a dominant strategy to bid their true values in the optimal auction. As Theorem 1 shows, imposing DIC and anonymity is indeed a binding restriction that allows implementing only "true" anonymous auctions. However, I show below that despite Theorem 1, *anonymity* + DIC do not prevent even perfect favoritism. It is almost always possible to send revenue to the favorite.

Theorem 2 (transferring revenue to the favorite)

For any direct feasible anonymous and dominant strategy incentive compatible auction $(\mathbf{y}', \mathbf{p}')$ *that generates the equilibrium revenue* $R(v^*, \mathbf{v}_{-1}) = -\sum_{i=1}^{n} t'_i(v^*, \mathbf{v}_{-1})$ *there exists another direct feasible anonymous and dominant strategy incentive compatible auction* $(\mathbf{y}'', \mathbf{p}'')$ *that has the same allocation rule* $\mathbf{y}''(.) = \mathbf{y}'(.)$ *and such a transfer rule* $\mathbf{p}''(.)$ *that implements the same equilibrium transfers for all bidders except the favorite, namely* $t''_j(v^*, \mathbf{v}_{-1}) = t'_j(v^*, \mathbf{v}_{-1})$ *for any* $j \neq 1$, *and the favorite's equilibrium transfer is such that* $t''_1(v^*, \mathbf{v}_{-1}) = t'_1(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1})$ *almost always.*

In other words, it is almost always possible to transfer all collected revenue to the favorite even in an anonymous and DIC auction. Almost always means that the statement is true for all realizations of values, except those where one or more bidders' values coincide with the favorite's value v^*. However, since the distributions of values are atomless and the number of bidders is finite, the probability of such event is equal to zero. The intuition behind this surprising result is that the designer manipulates the transfer rule taking into account that the favorite bids v^*. Then, the auction is constructed in such a way that for any value it is a dominant strategy to bid the true value and the auction transfers all collected revenue to the bidder who submitted a bid v^*, namely to the favorite. Since the probability that there is more than one bidder having a value v^* (in this case, it is not possible to transfer money to the favorite) is zero, such events do not affect the favorite's utility. In order to maintain dominant strategy incentive compatibility, the designer should promise substantially high payments to bidders in the case when nobody bids v^*. In general, if there is no such bidder that reports v^*, the auction would not be budget balanced. However, since the bidder reports v^*, this is not an equilibrium path and hence the auction would be budget balanced in equilibrium for every realization of opponents' values.

Now, consider a standard second-price auction. It is anonymous and DIC. Hence, Theorem 2 implies the following:

Corollary 1 (transferring revenue in the second-price auction)

If the designer is restricted to using only anonymous and DIC auctions, then there exists a feasible auction that implements the same allocation rule for all bidders and the same transfers in equilibrium for all bidders except the favorite as in the second-price auction. Instead, the favorite receives all collected revenue and has the ex-post utility greater than his value v^* in equilibrium almost always.

It is possible to send the revenue collected in a second-price auction to the favorite for almost all opponents' bids. In the constructed auction, the favorite wins if and only if he has the highest value and almost always pays nothing in this case. If the value of the favorite is not the highest, then the bidder with the highest value obtains the good and the favorite receives monetary transfer equal to the second highest value. Thus, in all cases when the favorite's value is not the highest or the second highest one, the utility obtained by the favorite is equal to the second highest value and strictly exceeds v^*. Since the equilibrium strategy is unique due to Lemma 1, the following is true:

Corollary 2 *Anonymity and DIC together do not prevent perfect favoritism.*

Thus, even a strong combination of anonymity and dominant strategy incentive compatibility that allows to implement only those rules that are symmetric with respect to real values does not prevent even perfect favoritism. It is important to notice here that unlike the mechanisms discussed in the previous section this construction has the unique equilibrium in dominant strategies. Although my notion of favoritism assumes that the designer can choose among equilibria in undominated strategies, Corollary 1 implies that the designer can construct an auction that has the unique equilibrium in undominated strategies and in this equilibrium the favorite obtains utility weakly greater than his value for (almost) any realization of opponent values.

Furthermore, notice that I do not claim that the proposed auction is the constrained favorite's preferred auction. In fact, the designer can do even more for his favorite by imposing reserve prices, similar to the unconstrained case. However, the problem of finding the favorite's preferred auction is complicated in this case, because there are many anonymity constraints that have to be satisfied.

The auction proposed in Corollary 1 is efficient, namely the good is always allocated to the bidder with the highest value. Thus, if the designer is restricted to using only efficient auctions, she can achieve perfect favoritism while implementing efficient auctions. Thus, we can formulate the following corollary.

Corollary 3 *It is possible to achieve efficiency and perfect favoritism simultaneously.*

In order to reduce favoritism, it is important to prevent the designer from sending all revenue to her favorite. Since anonymity and dominant strategy incentive compatibility do not restrict the designer's ability to transfer money to her favorite, an additional restriction should be imposed.

7 Non-positive transfers

Definition 12 (non-positive transfers)

A feasible auction FA satisfies non-positive transfers (NT, feasible under c_{NT}) if for any vector of bids $\mathbf{b} \in \mathbf{B}$ and any allocation $a \in A$

$$\mathbf{p}(a, \mathbf{b}) \leq \mathbf{0}$$

This restriction is crucial for preventing favoritism. We see from Theorem 2 that the designer always wants to transfer all collected revenue to her favorite. Even anonymity and DIC are insufficient to prevent the designer from doing this. It is clear that to prevent favoritism this possibility should be excluded. The natural way to do this is to impose a restriction that allows the designer to only collect money from the agents but not to give it. In other words, the principal may want to prohibit positive transfers.

If NT is the only restriction, namely $C = \{c_{NT}\}$, then the best that the designer can do is to allocate the good to her favorite for sure, independent of all bids. Imposing anonymity restriction jointly with NT, that is $C = \{c_A, c_{NT}\}$, does not particularly help. Again, with the result of Deb and Pai (2015) the designer is still able to allocate the good to her favorite (in some equilibrium) without extracting money from him. $C = \{c_{DIC}, c_{NT}\}$ works in the same way as $C = \{c_{NT}\}$, since allocating the good to the favorite independent of the bids is trivially incentive compatible. However, the combination of all three constraints, $C = \{c_A, c_{DIC}, c_{NT}\}$ substantially limits the scope of favoritism. In this case, as I show below, the designer has to use stochastic mechanisms and pool bidders having values in some regions to one specific value. I provide a complete solution to the problem in the case with two bidders. In the case with many bidders, it seems impossible to obtain an analytical solution due to the increased number of anonymity constraints that have to be satisfied. In general, there are $n!$ constraints only due to anonymity. Since the problem of maximizing the favorite's utility is asymmetric, it is incredibly difficult to take all of them into account. However, even the case with two bidders is sufficiently rich to shed some light on what is happening here.

There are two bidders, with bidder 1 being a favorite and bidder 2 being his opponent. For this case, we are able to characterize the $FPA(\{c_A, c_{DIC}, c_{NT}\})$ for any continuously differentiable distribution of the opponent's value $F(v)$, $v \in [0, \overline{v}]$. Note that I allow the case

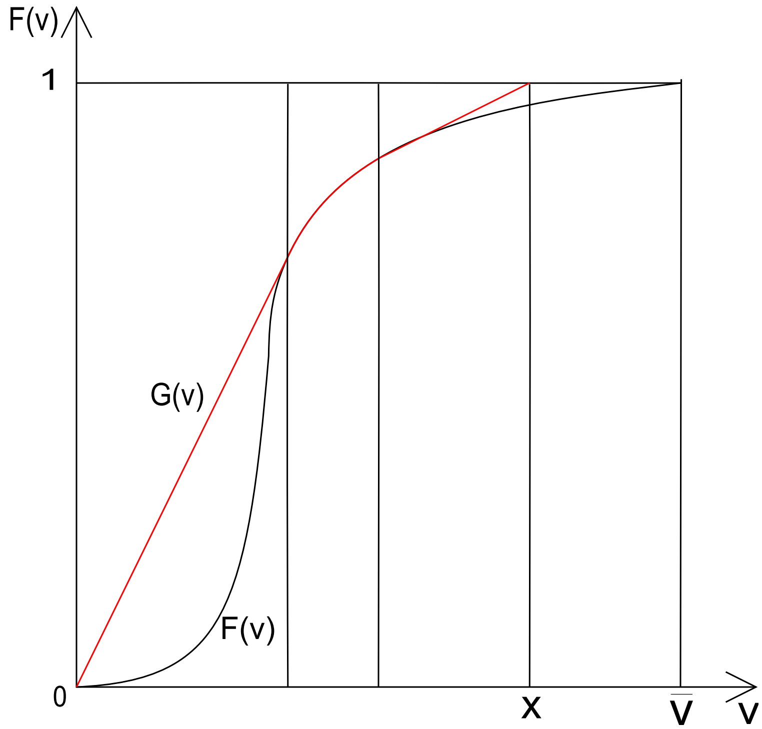

Figure 1: Concave envelope

when the favorite's value is greater than any possible value of his opponent and thus $v^* > \overline{v}$ is possible. In order to formulate the main result, I need some additional notations. Denote $G : \mathbb{R}_+ \to \mathbb{R}_+$,

$$G_x(z) = \left\{ \begin{array}{c} conc_x \langle F(z) \rangle \,, \text{ if } z \leq x \leq \overline{v} \\ conc_{\overline{v}} \langle F(z) \rangle \,, \text{ if } z \leq \overline{v} < x \\ 1 + \left(\lim\limits_{q \to x^-} \frac{dG_x(q)}{dz} \right) * (z - x), \text{ if } z, \overline{v} > x \\ 1, \text{ if } z, x > \overline{v} \end{array} \right\}^8$$

where $conc_x \langle F(z) \rangle$ is the lowest function that is concave, weakly greater than $F(v)$ and takes a value equal to 1 at the point $z = x$. It is illustrated in Figure 1 for the case $x < \overline{v}$. Denote $g_x(z) := dG_x(z)/dz$[9]

[8] $\lim\limits_{q \to x^-}$ is the limit from the left at the point x. We use it to define $G_x(z)$ for values beyond the domain of F.

[9] If $z = q \in \{\overline{v}, x\}$ then $G_x(z)$ is not differentiable. In this case, let the derivative $g_x(q)$ equal the limit from the left of $g_x(z)$ at the point $z = q$, i.e. $g_x(q) = \lim\limits_{z \to q^-} \frac{dG_x(z)}{dz}$.

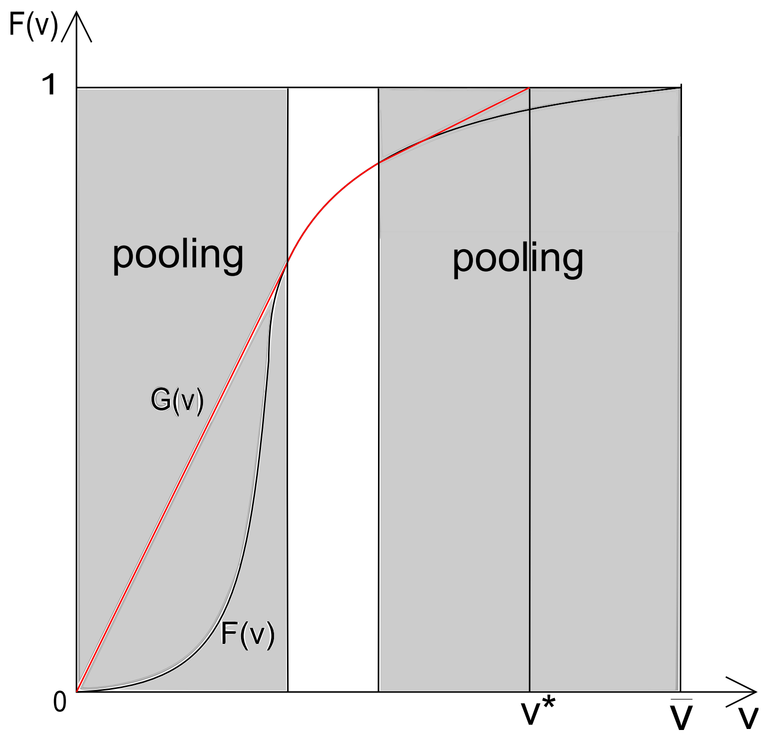

Figure 2: Second-price auction with pooling

Proposition 1 (FPA under anonymity, DIC and NT)

Assume that there are only two bidders. The favorite's preferred auction under anonymity, DIC and NT allocates the object to a bidder with the lowest $g_{v^*}(v_i)$. In the case of equality, a simple lottery is used to determine a winner. Transfers are computed by (6) with $h_i(\underline{v}_i, \mathbf{v}_{-i})$.

The auction described in Proposition 1 has a very clear economic description and is easy to implement. I call it a second-price auction with pooling. It is possible to think about a standard second-price auction with a slight modification; namely, there are intervals on the value domain such that if a bidder reports a value in one of these regions, he is treated as a bidder having value in the middle of this interval. For example, assume that there is only one such "pooling" or "lottery" interval (a, b). Suppose the second bidder has a value $v_2 \in (a, b)$. If a value of the favorite v^* is greater than b, he obtains the object and pays $(a + b)/2$. If $v^* \in (a, b)$ both bidders have an equal chance $1/2$ of winning the object. In the case of a win, the winner pays a, i.e. the left bound of the interval. If $v^* < a$, the second bidder obtains the object and pays v^*. The structure of pooling and contest regions is illustrated in Figure 2.

There are two reasons why pooling arises in the solution. The first one is that it is a way for the designer to give the object to her favorite when the opponent's value is higher. In order to better understand this, I can formulate the following proposition:

Proposition 2 (pooling at the top)

For any $v^* < \overline{v}$, there exists a cutoff $\widehat{v} < v^*$, such that the $FPA(\{c_A, c_{DIC}, c_{NT}\})$ pools all bidders with values above $\widehat{v}$. This cutoff $\widehat{v}(v^*)$ is a monotone increasing function of the favorite's value.

Thus, the designer prefers to use lottery if a value of a second bidder is higher than a value of her favorite or lower, but sufficiently close. In the first case, it gives a chance to allocate the object to her favorite and in the second case it reduces payments in the case of win.

Example 2 $F(v) = \sqrt{v}$, $v \in [0, 1]$, $v^* = 0.75$

By Propositions 1 and 2, we know that since the distribution function is concave, the restricted favorite's preferred auction is a second-price auction with one pooling region at the top. The pooling cutoff $\widehat{v}$ can be computed from $\sqrt{\widehat{v}} + \frac{1}{2\sqrt{\widehat{v}}}(v^* - \widehat{v}) = 1$, which gives $\widehat{v} = 0.25$. Thus, the result of the favorite's optimal auction is as follows: if the opponent has a value lower than 0.25, *the favorite obtains the object and pays an amount equal to his opponent's value. If the opponent has a value higher than* 0.25, *there will be a lottery among two bidders and the winner pays* 0.25. *The expected utility of the favorite in the favorite's preferred auction is* 0.458, *which is larger than the utility* 0.433 *of a standard second-price auction and* 0.375 *of a standard lottery.*

The second reason why pooling may be optimal is that it reduces expected payments made by the favorite when his value is substantially higher than a value of his competitor. Indeed, in the regions where pooling is used the graph of the cumulative distribution function lies below the straight line and hence the average value in each such region is smaller than the middle value. Suppose that a value of the second bidder belongs to that region. In the case of no pooling, the first bidder would have to pay in expectation the amount that is equal to the average value. When pooling is used he pays only the amount equal to the middle value and this reduces payments. We can also observe how the $FPA(\{c_A, c_{DIC}, c_{NT}\})$ depends on the favorite's value.

Proposition 3 (comparative statics)

If $v^* < \overline{v}$, then the only difference between $FOA(\{c_A, c_{DIC}, c_{NT}\})$ for different values is the size of a pooling region above the cutoff function $\widehat{v}(v^*)$. For any $v^* > \overline{v}$, $FPA(\{c_A, c_{DIC}, c_{NT}\})$ is the same as the one for $v^* = \overline{v}$.

Thus, if the favorite's value becomes smaller, the designer wants to increase pooling in the region of high values and keep the same allocation rule for low realizations of values. Thus, the change of the favorite's value has only a local effect on the auction design. If the favorite has a value higher than any possible value of his opponent, then the optimal auction does not depend on the specific value.

One can easily see that intra-auction favoritism and perfect favoritism are not possible under restrictions of anonymity, DIC and NT. However, inter-auction favoritism can be successively used by the designer to make the auction better for her favorite. In the next section, I show how any form of favoritism can be prevented by adding one additional constraint.

8 Deterministic auctions

Definition 13 (deterministic auctions)

A feasible auction FA is deterministic (DA, feasible under c_{DA}) if for any two bidders $i \neq j$ and for any two bids b_i, b_j submitted by these bidders, such that $b_i \neq b_j$, the allocation is such that $y_i \in \{0, 1\}$ and $y_j \in \{0, 1\}$.

This restriction does not allow the designer to use any randomization in the case when submitted bids are different. For example, the second price with probability 1. However, a second-price auction with pooling is not DA, because it uses randomization to determine the winner.

Definition 14 (second-price auction with a generalized reserve price)

An auction is called a second-price auction with a generalized reserve price if the following is satisfied:

$$\left\{\begin{array}{c} y_i(\mathbf{b}) = 1 \\ t_i(\mathbf{b}) = -\max_{j\neq i}(b_j, r(\mathbf{b}_{-i})) \end{array}\right\}, \qquad \text{if } b_i > \max_{j\neq i}(b_j, r(\mathbf{b}_{-i}))$$
$$y_i(\mathbf{b}) = t_i(\mathbf{b}) = 0, \qquad \text{if } b_i < \max_{j\neq i}(b_j, r(\mathbf{b}_{-i}))$$

where $r : \mathbb{R}^{n-1} \to R$ *is a componentwise symmetric function* [10].

The difference between a second-price auction with a generalized reserve price and a standard second-price auction is only that the reserve price is not the same for different bidders but rather for each player it may depend on bids made by his opponents. If a generalized reserve price is a constant function, i.e. $r(\mathbf{x}) = const\ \forall \mathbf{x} \in \mathbb{R}^{n-1}$, we obtain a second-price auction with a standard reserve price. In a special case of a zero reserve price, we get a standard second-price auction. The next important result characterizes the set of all feasible auctions.

Theorem 3 (auctions feasible under full set of constraints)

Any feasible anonymous, DIC, deterministic auction with nonpositive transfers is a second-price auction with a generalized reserve price.

This result shows that the set of all auctions feasible under $C = \{c_A, c_{DIC}, c_{NT}, c_{DA}\}$ is only a very specific class of auctions, described above. Hence, the ability to favor some participant is substantially limited. The next result shows that there is actually no scope for favoritism in this case.

Proposition 4 (no favoritism)

For any favorite's value v^*, *the favorite's preferred auction feasible under restrictions of anonymity, strategyproofness, nonpositive transfers and determinism is a standard second-price auction.*

[10] In the zero probability case, when there are two or more bids that are equal and the highest among all bids, only a symmetric lottery can be used to determine the winner who obtains the object and pays his bid if it is greater than a generalized reserve price. Note also that due to symmetry the reserve price is the same for all agents with the highest bids.

From Theorem 3, we know that all the designer can do is to choose some auction from a class of second-price auctions with a generalized reserve price. Proposition 4 shows that the reserve price that makes the favorite better off is zero. Hence, $FPA(\{c_A, c_{DIC}, c_{NT}, c_{DA}\})$ is a standard second-price auction.

Thus, I have shown that if the designer is allowed to use only anonymous, dominant strategy incentive compatible, deterministic auctions such that bidders never obtain money from it, then any kind of favoritism is impossible. The best the designer can do is always choose a second-price auction independent of her favorite's value and value distributions. Note also that although this set of restrictions substantially limits the freedom to choose the auction format, the revenue maximizing auction is still available for the designer if the agents are symmetric. Indeed, in this case the revenue maximizing auction will be the second-price auction with a reserve price that can be implemented under $\{c_A, c_{DIC}, c_{NT}, c_{DA}\}$. Such a set of restrictions guarantees collected revenue as the revenue in a second-price auction even if the designer only cares about the favorite.

9 Hierarchy of restrictions

Based on the previous results, we can understand how the restrictions interact with each other, namely how each restriction helps to prevent favoritism depending on the other restrictions in place.

First, DA is a binding restriction if and only in the situation without this restriction the favorite's optimal auction uses lotteries. As we have seen above, it can be only the case when anonymity and DIC are imposed jointly. Next, without anonymity, the requirement of dominant strategies does not restrict the designer, because in this case the situation is equivalent to a situation where the designer has her own value of v^* and there is no favorite. In this case, the optimal mechanism is DIC even without imposing DIC. The opposite direction is also true, namely without the DIC restriction, the requirement of anonymity does not restrict the designer. This follows from Deb and Pai (2015) argument. Hence, we conclude that the anonymity restriction is binding if and only if the DIC restriction is imposed and vice versa. From the previous discussion, we have seen that NT restriction binds if DA is not imposed. Since DA is binding if and only if a combination of *anononymity*$+DIC$ is present, we are left to consider only the case with *anonymity* $+ DIC + DA$ as restrictions to fully understand the role of NT. Note that in this case the allocation rule is uniquely determined, since due to *anonymity* and DA the allocation rule has to be such that the highest bid wins the auction for sure, which jointly with DIC implies that the bidder with the highest value wins the auction. Hence, the favorite's preferred auction is the one described in Corollary 1, where the designer transfers all collected revenue to her favorite in equilibrium. Thus, NT always reduces the scope of favoritism independent of other restrictions imposed. Thus, we obtain the following result:

Proposition 5 (hierarchy of restrictions)

The set of restrictions comprising anonymity, DIC, NT, DA forms a hierarchy with NT at the top, $DIC + NT$ in the middle and DA at the bottom. NT restricts the scope of favoritism independent of whether other constraints are imposed. DIC reduces the scope of favoritism if and only if anonymity is imposed and vice versa. DA reduces the scope of favoritism if and only if a combination anonymity $+ DIC$ is imposed.

10 Discussion

The fact that the designer knows not only the identity of the favorite but also his value is the main driving force of favoritism in choosing the particular auction format. If the designer did not know the value of the favorite, a combination of anonymity and dominant strategy incentive compatibility would turn the problem of favoritism to a problem of buyers' welfare maximization irrespective of their identities. Hence, anonymity and dominant strategy incentive compatibility would be a sufficient condition to prevent any kind of favoritism.

My results are robust to the imperfect knowledge of the designer. In particular, if the designer's belief v_d^* about the favorite's value is sufficiently precise, namely there exist such ε and δ such that $\Pr(|v_d^* - v^*| > \varepsilon) < \delta$, she can provide him the interim utility $U_d(v^*)$ such that $U_d(v^*) > U(v^*) - O(\max\{\varepsilon, \delta\})$, where $U(v^*)$ is the utility of the favorite in the perfect knowledge case. When ε and δ are sufficiently small, the favorite's expected utility approaches the perfect information case. Thus, our results are not simply an artifact of precise information about the favorite's value.

For a set of restrictions which that comprises anonymity, DIC and non-positive transfers, I have analyzed the case with only two bidders. The favorite's preferred auction for many agents and one favorite would have the similar properties and would be a second-price auction with pooling. For many agents, pooling in general is partial, namely not all bidders above some cutoffs are pooled, although due to increased number of anonymity restrictions it is much more difficult to compute. In order to observe what happens when we increase the number of bidders, we can compare a standard lottery and a second-price auction. The expected utility of the favorite from participation in a lottery is $U_1(v^*) = v^*/n$ and from participation in a second-price auction is $U_1(v^*) = F^n(v^*) * (v^* - E[v_{(1)}|v_{(1)} \leq v^*])$, where $v_{(1)}$ is the first-order statistic out of $(n-1)$ variables. Obviously, both expressions go to zero when n increases, although the speed of convergence is $1/n$ in the case of a lottery and $F^n(v^*)$ in the second-price auction. Since for any $v^* < \overline{v}$ we have $F^n(v^*) < v^*/n$ we can conclude that when the number of bidders increases, all of them would prefer a lottery to an auction[11].

11 Conclusion

In this paper, I have analyzed the problem of favoritism in auctions from a mechanism design perspective. In my model, the designer has one favorite among the bidders, whose value is known to the designer. I have characterized feasible auctions that the designer can implement to maximize the utility of her favorite under different sets of restrictions on these auctions. Deb and Pai (2015) have shown that assuming that the designer can choose between different undominated equilibria, anonymity is not a binding restriction for the designer. I have shown that even if the designer is restricted not only by anonymity but also by dominant strategy incentive compatibility, it is insufficient to prevent perfect favoritism. Namely, the designer is almost always able to transfer all collected revenue to her favorite in any auction. Hence, it is

[11]This does not imply that a lottery is socially preferable to an auction. See Condorelli (2012) for a description of a socially optimal mechanism.

possible to guarantee him the interim utility greater than or equal to his value in the unique equilibrium of the constructed auction. To prevent this possibility, I additionally impose the non-positive transfers restriction. Subsequently, the designer cannot discriminate bidders within any auction. However, although intra-auction favoritism is not possible, the inter-auction favoritism could still be possible, whereby the designer chooses different auction formats for different favorite's values. I have shown that the favorite's preferred auction is a second-price auction with pooling where the designer commits to not distinguishing some value reports. The size of the pooling region for the highest values depends on the favorite's value. Thus, the designer uses inter-auction favoritism. Finally, I have shown that it is possible to completely prevent any form of favoritism if the designer is restricted to using only deterministic auctions in addition to anonymity, dominant strategy incentive compatibility and non-positive transfers restrictions. In this case, any feasible mechanism is a second-price auction with a generalized reserve price, whereby the reserve price for each bidder depends on bids submitted by other bidders. The favorite's preferred auction in this class is a standard second-price auction without any reserve price.

My results imply that while delegating the decision about the auction format choice to the designer, the principal should care about how much freedom should be given to the designer and in what way this freedom can be limited. If the final goal of the principal is revenue maximization, then along with anonymity and dominant strategy incentive compatibility, restrictions of non-positive transfers and deterministic auctions should be imposed. Non-positive transfers would help to prevent discrimination of bidders via transfers. Determinism is used to sustain competition, since without it the designer would like to make it less intensive by using lotteries.

Traditional problems of mechanism design (revenue maximization, efficiency maximization, social welfare maximization) are symmetric and hence they have symmetric solutions. I have considered essentially asymmetric problems and have found symmetric (anonymous) solutions for them. Thus, my results can also serve as a mathematical approach to solving such kind of problems.

12 References

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13 Appendix A

Proof of Lemma 1

Similar to Maskin, Laffont (1979), if $\beta_i^*(v_i)$ is a dominant stratefy for bidder i, then for $b_i = \beta_i^*(v_i)$ and for any $b_j \in \{\oslash\} \cup B_j$, $j \neq i$:

$$U_i(v_i|\mathbf{b}) = \int_{\underline{v}_i}^{v_i} y_i(b_1, ..., \beta_i^*(q), ..., b_n))dq + h_i(\underline{v}_i, \mathbf{b}_{-i}).$$

Since $\beta_i^*(v_i)$ and $\beta_i^{**}(v_i)$ are both dominant strategies:

$$\begin{aligned} U_i(v_i|b_1, ..., \beta_i^*(v_i), ..., b_n) &\geq U_i(v_i|b_1, ..., \beta_i^{**}(v_i), ..., b_n), \\ U_i(v_i|b_1, ..., \beta_i^{**}(v_i), ..., b_n) &\geq U_i(v_i|b_1, ..., \beta_i^*(v_i), ..., b_n). \end{aligned}$$

Hence,

$$U_i(v_i|b_1, ..., \beta_i^{**}(v_i), ..., b_n) = U_i(v_i|b_1, ..., \beta_i^*(v_i), ..., b_n).$$

Taking a derivative of both sides with respect to v_i we obtain for any v_i:

$$y_i(b_1, ..., \beta_i^*(v_i), ..., b_N) = y_i(b_1, ..., \beta_i^{**}(v_i), ..., b_N).$$

Then,

$$t_i(b_1, ...\beta_i^*(v_i), ...b_N) = U_i(v_i|b_1, ..., \beta_i^*(v_i), ..., b_N) - v_i y_i(b_1, ..., \beta_i^*(v_i), ..., b_N) =$$
$$= U_i(v_i, |b_1, ..., \beta_i^{**}(v_i), ..., b_N) - v_i y_i(b_1, ..., \beta_i^{**}(v_i), ..., b_N) = t_i(b_1, ..., \beta_i^{**}(v_i), ...b_N).$$

Proof of Lemma 2

Suppose that $\beta^*(v)$ is a dominant strategy for an agent i and consider some value v from the intersection of possible values sets for bidders i and j :

$$U_i(v|b_1, ..., \underset{i}{\beta^*(v)}, ..., b_N) \geq U_i(v|b_1, ..., \underset{i}{\beta(v)}, ..., b_N) \text{ for any } \beta(v) \text{ and } b_k \in \{\oslash\} \cup B_k,\ k \neq i.$$

This means that for any $\widetilde{b}$:

$$\begin{aligned} & v y_i(b_1, ..., \underset{i}{\beta^*(v)}, ..., \underset{j}{\widetilde{b}}, ..., b_N) + t_i(b_1, ..., \underset{i}{\beta^*(v)}, ..., \underset{j}{\widetilde{b}}, ..., b_N) \geq \\ \geq \quad & v y_i(b_1, ..., \underset{i}{\beta(v)}, ..., \underset{j}{\widetilde{b}}, ..., b_N(v_N)) + t_i(b_1, ..., \underset{i}{\beta(v)}, ..., \underset{j}{\widetilde{b}}, ..., b_N). \end{aligned}$$

If we switch bids of agents i and j, by anonymity agent j should have the same allocation as an agent i had before. Hence, the previous inequality can be rewritten as:

$$\begin{aligned} & v y_j(b_1, ..., \underset{i}{\widetilde{b}}, ..., \underset{j}{\beta^*(v)}, ..., b_N) + t_i(b_1, ..., \underset{i}{\widetilde{b}}, ..., \underset{j}{\beta^*(v)}, ..., b_N) \geq \\ \geq \quad & v y_j(b_1, ..., \underset{i}{\widetilde{b}}, ..., \underset{j}{\beta(v)}, ..., b_N) + t_i(b_1, ..., \underset{i}{\widetilde{b}}, ..., \underset{j}{\beta(v)}, ..., b_N). \end{aligned}$$

Hence,

$$U_j(v|b_1, ..., \underset{j}{\beta^*(v)}, ..., b_N) \geq U_j(v|b_1, ..., \underset{j}{\beta(v)}, ..., b_N).$$

Proof of Theorem 2

Step 1 (Construction of $h_i''(\underline{v}_i, \mathbf{v}_{-i})$).

Consider some anonymous and DIC auction that has the allocation rule $\mathbf{y}'(\mathbf{v})$ and the transfer rule $\mathbf{t}'(\mathbf{v})$. The new constructed auction also has to be DIC. By (6) functions

$\{h'_i(\underline{v}_i, \mathbf{v}_{-i})\}_{i=1}^n$, $\{h''_i(\underline{v}_i, \mathbf{v}_{-i})\}_{i=1}^n$ have to satisfy:

$$t'_i(\mathbf{v}) = -v_i y'_i(\mathbf{v}) + h'_i(\underline{v}_i, \mathbf{v}_{-i}) + \int_{\underline{v}_i}^{v_i} y'_i(v_1, ..., \underset{i}{q}, ...v_n) dq \tag{7}$$

$$t''_i(\mathbf{v}) = -v_i y''_i(\mathbf{v}) + h''_i(\underline{v}_i, \mathbf{v}_{-i}) + \int_{\underline{v}_i}^{v_i} y''_i(v_1, ..., \underset{i}{q}, ...v_n) dq \tag{8}$$

It is required that the new allocation rule is the same as before. Accordingly, for any vector of reported values $\mathbf{v}$, we must have $y'_i(\mathbf{v}) = y''_i(\mathbf{v})$. However, transfers should be (almost always) the same only in equilibrium. In equilibrium, the favorite always reports v^*. Hence, only vectors $\mathbf{v} = (v^*, \mathbf{v}_{-1})$ can be on equilibrium path. For any i and for any $\mathbf{v}_{-i}$, define

$$h''_i(\underline{v}_i, \mathbf{v}_{-i}) := h'_i(\underline{v}_i, \mathbf{v}_{-i}) \tag{9}$$

if at least one component of $\mathbf{v}_{-i}$ is equal to v^* and

$$h''_i(\underline{v}_i, \mathbf{v}_{-i}) := v^* y''_i(v_1, ..., \underset{i}{v^*}, ...v_n) - \int_{\underline{v}_i}^{v^*} y''_i(v_1, ..., \underset{i}{q}, ..., v_n) dq + \tag{10}$$

$$+ \sum_{j \neq i} v_j y''_j(v_1, ..., \underset{i}{v^*}, ...v_n) - \sum_{j \neq i} \int_{\underline{v}_j}^{v_j} y''_j(v_1, ..., \underset{i}{v^*}, ..., \underset{j}{q}, ..., v_n) dq - \sum_{j \neq i} h''_j(\underline{v}_j, \mathbf{v}_{-j} | v_i = v^*)$$

if none of $\mathbf{v}_{-i}$ components is equal to v^*, where $h'_j(\underline{v}_j, \mathbf{v}_{-j} | v_i = v^*)$ means that the value of component v_i in $\mathbf{v}_{-j}$ is replaced by v^*.

Step 2. (Computing transfers).

Equation (8) then uniquely defines $t''_i(\mathbf{v})$ given $y''_i(\mathbf{v})$ and $h''_i(\underline{v}_i, \mathbf{v}_{-i})$. Thus, if $\mathbf{v}_{-i}$ has a component equal to v^*, then $h''_i(\underline{v}_i, \mathbf{v}_{-i}) = h'_i(\underline{v}_i, , \mathbf{v}_{-i})$ and, hence,

$$t''_i(\mathbf{v}) = t'_i(\mathbf{v}). \tag{11}$$

If all components of $\mathbf{v}_{-i}$ are different from v^*, plugging the expression (10) to (8), using

$\mathbf{y}'' = \mathbf{y}'$ and $h''_j(\underline{v}_j, \mathbf{v}_{-j}|v_i = v^*) = h'_j(\underline{v}_j, \mathbf{v}_{-j}|v_i = v^*)$, $j \neq i$ we obtain

$$\begin{aligned} t''_i(\mathbf{v}) =& -v_i y'_i(\mathbf{v}) + \int_{v^*}^{v_i} y'_i(v_1, ..., \underset{i}{q}, ...v_n)dq + v^* y'_i(v_1, ..., \underset{i}{v^*}, ...v_n) + \\ & + \sum_{j\neq i} v_j y'_j(v_1, ..., \underset{i}{v^*}, ...v_n) - \sum_{j\neq i} \int_{\underline{v}_j}^{v_j} y'_j(v_1, ..., \underset{i}{v^*}, ..., \underset{j}{q}, ..., v_n)dq - \sum_{j\neq i} h'_j(\underline{v}_j, \mathbf{v}_{-j}|v_i = v^*) = \\ =& -v_i y'_i(\mathbf{v}) + \int_{v^*}^{v_i} y'_i(v_1, ..., \underset{i}{q}, ...v_n)dq + v^* y'_i(v_1, ..., \underset{i}{v^*}, ...v_n) + \\ & + \sum_{j\neq i} v_j y'_j(v_1, ..., \underset{i}{v^*}, ...v_n) - \sum_{j\neq i} t'_j(v_1, ..., \underset{i}{v^*}, ..., v_n) - \sum_{j\neq i} v_j y'_j(v_1, ..., \underset{i}{v^*}, ..., v_n) = \\ =& -v_i y'_i(\mathbf{v}) + \int_{v^*}^{v_i} y'_i(v_1, ..., \underset{i}{q}, ...v_n)dq + v^* y'_i(v_1, ..., \underset{i}{v^*}, ...v_n) - \sum_{j\neq i} t'_j(v_1, ..., \underset{i}{v^*}, ..., v_n), \end{aligned} \tag{12}$$

where we also used that (7) implies $\sum_{j\neq i} \int_{\underline{v}_j}^{v_j} y'_j(v_1, ..., \underset{i}{v^*}, ..., \underset{j}{q}, ..., v_n)dq + \sum_{j\neq i} h'_j(\underline{v}_j, \mathbf{v}_{-j}|v_i = v^*) = \sum_{j\neq i} t'_j(v_1, ..., \underset{i}{v^*}, ..., v_n) + \sum_{j\neq i} v_j y'_j(v_1, ..., \underset{i}{v^*}, ..., v_n)$. Now, we need to verify that the constructed auction satisfies anonymity and in equilibrium it almost always implements the described transfers.

Step 3. (Check anonymity of $(\mathbf{y}'', \mathbf{t}'')$).

Since $(\mathbf{y}', \mathbf{t}')$ is an anonymous auction and $\mathbf{y}'' = \mathbf{y}'$, the allocation rule is trivially symmetric. Now, consider $\mathbf{t}''(\mathbf{v})$. If $\mathbf{v}_{-i}$ has a component equal to v^*, then $t''_i(\mathbf{v}) = t'_i(\mathbf{v})$. Since $t'_i(\mathbf{v})$ is symmetric, then $t''_i(\mathbf{v})$ is also symmetric. If all components of $\mathbf{v}_{-i}$ are different from v^*, then $t''_i(\mathbf{v})$ is described by expression (12), which does not depend on $\{\underline{v}_i\}_{i=1}^n$ and has only symmetric functions inside. Thus, anonymity is satisfied.

Step 4. (Equilibrium transfers).

In equilibrium the favorite reports v^*. Hence, (11) implies that $t''_i(v^*, \mathbf{v}_{-1}) = t'_i(v^*, \mathbf{v}_{-1})$ for all bidders, except the favorite. Since the number of bidders is finite and the distributions are strictly increasing the probability that some other bidder is going to report v^* is zero. Thus, the favorite's transfer in equilibrium is almost always described by (12) and plugging $v_1 = v^*$, we obtain $t''_1(v^*, \mathbf{v}_{-1}) = -\sum_{j\neq 1} t'_j(v^*, \mathbf{v}_{-1}) = t'_1(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1})$. The no-deficit requirement is trivially satisfied in equilibrium, because the constructed auction transfers all revenue to the favorite making the budget balanced. This completes the proof.

Lemma 3 *If $\{c_A, c_{DIC}, c_{NT}\} \subset C$, then in any direct $FA(C)$: $h_i(0, \mathbf{v}_{-i}) = 0$ for any i, $\mathbf{v}_{-i}$*

Proof of Lemma 3

Suppose that bidder i has a value $v_i = 0$. Then, by (6)

$$t_i(\mathbf{v}) = U_i(\mathbf{v}) = h_i(0, \mathbf{v}_{-i})$$

Since $c_{NT} \in C$, we should have $h_i(0, \mathbf{v}_{-i}) = t_i(\mathbf{v}) \leq 0$ for any $\mathbf{v}_{-i}$. Simultaneously, $U_i(\mathbf{v})$ has to be positive otherwise it would not be the dominant strategy to report the true value and bidder i could exclude himself from participation. Hence, $h_i(0, \mathbf{v}_{-i}) \geq 0$ should also hold for any $\mathbf{v}_{-i}$. Combining last two inequalities we have $h_i(0, \mathbf{v}_{-i}) = 0$.

Proof of Proposition 1

Using characterization (6) and Lemma 3, transfers $t_i(v_1, v_2)$ are fully determined by the allocation rule $y_i(v_1, v_2)$. Since the function G_x has a different form depending on a relationship between v^* and $\overline{v}$, we consider possible cases separately.

Case 1 $v^* \leq \overline{v}$.

By the anonymity restriction we need to specify an allocation rule only on the cone $\Gamma = \{\mathbf{v} =(v_1, v_2) \in [0, \overline{v}]^2 : v_1 \geq v_2\}$. Indeed, suppose we have specified some allocation rule on Γ. Then, for all reported values $(v_1, v_2) \notin \Gamma$ we have $v_2 > v_1$. Since for $v_2 > v_1$ the bid vector $(v_2, v_1) \in \Gamma$, we know the allocation probabilities $y_1(v_2, v_1)$ and $y_2(v_2, v_1)$. Then, by anonymity we have the allocation for $(v_1, v_2) \notin \Gamma$ as $y_1(v_1, v_2) = y_2(v_2, v_1)$ and $y_2(v_1, v_2) = y_1(v_2, v_1)$.

To illustrate our proof we plot for convenience simultaneously two things on the same figure. The first one is a graph of a distribution function $F(v)$ of the opponent's value. The second one is the value space (v_1, v_2). The auction described in the statement implies that the whole value space is cut into a certain number of triangles and rectangles (see Figure 3 as an example). I use R_i to talk about region i on the figure 3. The rectangles can be only of two types: 1) interior rectangles, like R_2, in general there could be many of them; and 2) at most one boundary rectangle with values $v_1 \geq v^*$ inside, like R_4. For all pairs (v_1, v_2) inside each such rectangle $y_1(v_1, v_2) = 1$ and $y_2(v_1, v_2) = 0$. Triangles can be of three types:

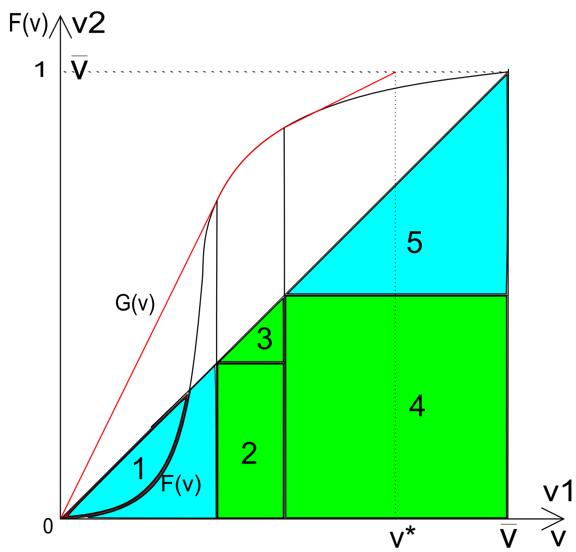

Figure 3: Illustration of a proof

1) interior triangles like R_3, 2) the unique boundary triangle containing $v_1 = v_2 = 0$, like R_1, 3) the unique boundary triangle with values $v_1 \geq v^*$ inside, like R_5. If a triangle is the region, where $g_{v^*}(v_1)$ is constant (R_1, R_5 on Figure 3), then $y_1(v_1, v_2) = y_2(v_1, v_2) = 1/2$, for all pairs (v_1, v_2) inside this triangle. If a triangle lies in the region, where $g_{v^*}(v_1)$ is strictly decreasing (R_3 on Figure 3), then $y_1(v_1, v_2) = 1$ and $y_2(v_1, v_2) = 0$. Our task is to prove that the described allocation is indeed optimal for the favorite having a value v^*.

Using a notation $k(v_1, v_2) := y_1(v_1, v_2) + y_2(v_1, v_2)$, where $0 \leq k(v_1, v_2) \leq 1$, we can rewrite the interim utility of the first agent:

$$
\begin{aligned}
U_1(v^*) &= \int_0^{\overline{v}} (v^* y_1(v^*, v_2) + t_1(v^*, v_2)) f(v_2) dv_2 = \int_0^{\overline{v}} (\int_0^{v^*} y_1(v_1, v_2) dv_1) f(v_2) dv_2 = \\
&= \int_0^{v^*} \int_{v_2}^{v^*} y_1(v_1, v_2) f(v_2) dv_1 dv_2 + \int_0^{v^*} \int_{v_1}^{\overline{v}} y_1(v_1, v_2) f(v_2) dv_2 dv_1 = \\
&= \int_0^{v^*} \int_{v_2}^{v^*} y_1(v_1, v_2) f(v_2) dv_1 dv_2 + \int_0^{v^*} \int_{v_1}^{\overline{v}} y_2(v_2, v_1) f(v_2) dv_2 dv_1 = \\
&= \int_0^{v^*} \int_{v_2}^{v^*} y_1(v_1, v_2) f(v_2) dv_1 dv_2 + \int_0^{v^*} \int_{v_2}^{\overline{v}} y_2(v_1, v_2) f(v_1) dv_1 dv_2 = \\
&= \int_0^{v^*} \int_{v_2}^{v^*} y_1(v_1, v_2) f(v_2) dv_1 dv_2 + \int_0^{v^*} \int_{v_2}^{\overline{v}} [k(v_1, v_2) - y_1(v_1, v_2)] f(v_1) dv_1 dv_2, \qquad (13)
\end{aligned}
$$

where the equality in the first line follows from (6) and Lemma 3, the next one is changing the order of integration, then we apply anonymity, and finally we switch notations of v_1 and v_2 in the second summand. Here, we can notice that it is always optimal to put $k(v_1, v_2) = 1$ for any v_1, v_2. This means that it is never optimal to throw the object away. From now onwards, I will skip the term $\int_0^{v^*} \int_{v_2}^{\overline{v}} k(v_1, v_2) f(v_1) dv_1 dv_2$, which is constant in the $FPA(C)$. Now, we need to maximize (13) subject to monotonicity constraints (5).

I prove that even separately in each of the described regions, i.e. neglecting global monotonicity constraints, it is not possible to change an allocation rule to increase utility of the favorite. Denote by $z_1, z_2, ...$ such points where $g_{v^*}(v_1)$ changes its type from linear to strictly concave and vice versa. Suppose that there exists any interior or boundary triangle with $(0, 0)$ inside, called R_1, such that $g_{v^*}(v_1)$ is linear for any $v_1 \in R_1$ and $y_1(v_1, v_2) \neq 1/2$ for some $(v_1, v_2) \in R_1$. Due to anonymity on the diagonal $y_1(v, v) = 1/2$ for any v and due to monotonicity $y_1(v_1, v_2) \geq 1/2$ in each of the regions. Thus, $y_1(v_1, v_2) > 1/2$ is only possible in the low-right corner of the triangle R_1, which I denote by $A_1 \subset R_1$. But if it is the case, we can reduce $y_1(v_1, v_2)$ by a small $\varepsilon > 0$[12]. In R_1 the following holds: $\int\int_{A_1} \varepsilon f(v_2) dv_1 dv_2 < \int\int_{A_1} \varepsilon f(z_1) dv_1 dv_2$ and $\int\int_{A_1} \varepsilon f(v_1) dv_1 dv_2 > \int\int_{A_1} \varepsilon f(z_1) dv_1 dv_2$. The

[12]Strictly speaking, we cannot always reduce allocation probability by ε everywhere, since it could prove to be lower than 1/2 and violate monotonicity constraint. Thus, in the points where it occurs, we only reduce by $y_1(v_1, v_2) - 1/2$. Hence, the decrease is $\min\{\varepsilon, y_1(v_1, v_2) - 1/2\}$. But it matters only in the region with at least one dimension of order ε and hence it would be a second-order effect, which we can neglect.

change in utility is:

$$\begin{aligned}\Delta U_1 &= -\int\int_{A_1} \varepsilon f(v_2)dv_1dv_2 + \int\int_{A_1} \varepsilon f(v_1)dv_1dv_2 > \\ &> -\int\int_{A_1} \varepsilon f(z_1)dv_1dv_2 + \int\int_{A_1} \varepsilon f(z_1)dv_1dv_2 > 0\end{aligned}$$

Hence, it is not possible to improve upon $y_1(v_1, v_2) = 1/2$ in the region R_1.

Now consider any interior rectangle R_2. I claim that it is always optimal to give the object to the first agent. I use the similar logic as above. Assume that it is not true and there exists a subset $A_2 \subset R_2$: for any $(v_1, v_2) \in A_2$ we have $y_1(v_1, v_2) < 1$. Due to monotonicity, it could only be the upper-left corner. Now we increase probability of allocation to the first agent by ε in A_2[13]. In R_2 the following inequalities hold $\int\int_{A_2} \varepsilon f(v_2)dv_1dv_2 > \int\int_{A_2} \varepsilon f(z_1)dv_1dv_2$ and $\int\int_{A_1} \varepsilon f(v_1)dv_1dv_2 < \int\int_{A_1} \varepsilon f(z_1)dv_1dv_2$. Hence, the utility change is:

$$\begin{aligned}\Delta U_1 &= \int\int_{A_2} \varepsilon f(v_2)dv_1dv_2 - \int\int_{A_2} \varepsilon f(v_1)dv_1dv_2 > \\ &> \int\int_{A_2} \varepsilon f(z_1)dv_1dv_2 - \int\int_{A_2} \varepsilon f(z_1)dv_1dv_2 > 0\end{aligned}$$

So it is never optimal to put $y_1(v_1, v_2) < 1$ anywhere in R_2 , i.e. in the $FPA(C)$ the first agent always get the object in R_2.

While considering any interior or boundary triangle R_3 such that $g_{v^*}(v_1)$ is strictly increasing for any $v_1 \in R_3$, we notice that for any point $(v_1, v_2) \in R_3$ the following relation holds: $f(v_1) < f(v_2)$. Hence, from (13) it is optimal even pointwise in R_3 to make $y(v_1, v_2)$ as high as possible, i.e. $y(v_1, v_2) = 1$.

Boundary rectangle R_4 and boundary triangle R_5 such that $(v^*, v_2) \in R_4 \cap R_5$ for any $v_2 \leq v^*$ are specific regions. The logic of a proof is a modified logic of the proof for regions R_1 and R_2. We start from R_4 and assume that for some $A_4 \subset R_4$ it is optimal to allocate the good to the favorite with a probability $y_1(v_1, v_2) < 1$. Again, it could only be the upper-low corner of the rectangle. We again increase probability of allocation in A_4 by ε[14]. The change

[13] $\min\{\varepsilon, 1 - y_1(v_1, v_2)\}$
[14] $\min\{\varepsilon, 1 - y_1(v_1, v_2)\}$

in utility is:

$$\Delta U_1 = \int\int_{A_4\cap\{v_1\le v^*\}} \varepsilon f(v_2)dv_1dv_2 - \int\int_{A_4} \varepsilon f(v_1)dv_1dv_2$$

In this region $\int\int_{A_4\cap\{v_1\le v^*\}} \varepsilon f(v_2)dv_1dv_2 > \int\int_{A_4\cap\{v_1\le v^*\}} \varepsilon f(z_2)dv_1dv_2$ and $\int\int_{A_4} \varepsilon f(v_1)dv_1dv_2 < \int\int_{A_4\cap\{v_1\le v^*\}} \varepsilon f(z_2)dv_1dv_2$. Hence, $\Delta U > 0$ and $y_1(v_1, v_2) = 1$ must be optimal.

In R_5 we need to show that $y_1(v_1, v_2) = 1/2$ is optimal. By contrast, assume that there is $A_5 \subset R_5$ in the low-right corner where $y_1(v_1, v_2) > 1/2$. As before, reduce allocation probability by ε[15]. Since $\int\int_{A_5\cap\{v_1\le v^*\}} \varepsilon f(v_2)dv_1dv_2 < \int\int_{A_5\cap\{v_1\le v^*\}} \varepsilon f(z_2)dv_1dv_2$ and $\int\int_{A_5} \varepsilon f(v_1)dv_1dv_2 > \int\int_{A_5\cap\{v_1\le v^*\}} \varepsilon f(z_2)dv_1dv_2$, the utility change is:

$$\Delta U_1 = -\int\int_{A_5\cap\{v_1\le v^*\}} \varepsilon f(v_2)dv_1dv_2 + \int\int_{A_5} \varepsilon f(v_1)dv_1dv_2 > 0$$

Thus, $y_1(v_1, v_2) = 1/2$ is optimal in $R_{5.}$

Since we worked with each region independently, this proof holds for any number and any combination of these regions. Since for any distribution function we can divide the subset of values below the diagonal $v_1 = v_2$ into regions of described values, we can apply the above logic to any distribution function and corresponding partition.

To complete the proof, we must show that the global monotonicity conditions are satisfied. Indeed, $y_1(v_1, v_2) \in \{1, 1/2\}$ for any $v_1 > v_2$. The regions where $y_1(v_1, v_2) = 1/2$ are only the triangles close to the diagonal. Thus, the proposed auction is indeed monotone. Transfers are chosen according to (6) taking into account that by Lemma 3 we have $h_i(0, \mathbf{v}_{-i}) = 0$.

Case 2 $v^* > \overline{v}$.

The idea here is to consider the characterization for the case $v^* = \overline{v}$ which follows from the previous case, and then to show for $v^* > \overline{v}$ that for all (v_1, v_2) such that $v_1 \in (\overline{v}, v^*]$ and $v_2 < v_1$ the optimal allocation is $y(v_1, v_2) = 1$, and for all (v_1, v_2) such that $(v_1, v_2) \in [0, \overline{v}] \times [0, \overline{v}]$ the allocation remains unchanged.

Indeed, suppose we consider $v^* > \overline{v}$. Then, similarly to the previous case we obtain the

[15] $\min\{\varepsilon, y_1(v_1, v_2) - 1/2\}$.

following.

$$\begin{aligned}
U_1(v^*) &= \int_0^{\overline{v}}(v^* y_1(v^*,v_2)+t_1(v^*,v_2))f(v_2)dv_2 = \int_0^{\overline{v}}(\int_0^{v^*} y_1(v_1,v_2)dv_1)f(v_2)dv_2 = \\
&= \int_0^{\overline{v}}\int_{v_2}^{v^*} y_1(v_1,v_2)f(v_2)dv_1dv_2 + \int_0^{\overline{v}}\int_{v_1}^{\overline{v}} y_1(v_1,v_2)f(v_2)dv_2dv_1 = \\
&= \int_0^{\overline{v}}\int_{v_2}^{v^*} y_1(v_1,v_2)f(v_2)dv_1dv_2 + \int_0^{\overline{v}}\int_{v_1}^{\overline{v}} y_2(v_2,v_1)f(v_2)dv_2dv_1 = \\
&= \int_0^{\overline{v}}\int_{v_2}^{v^*} y_1(v_1,v_2)f(v_2)dv_1dv_2 + \int_0^{\overline{v}}\int_{v_2}^{\overline{v}} y_2(v_1,v_2)f(v_1)dv_1dv_2 = \\
&= \int_0^{\overline{v}}\int_{v_2}^{v^*} y_1(v_1,v_2)f(v_2)dv_1dv_2 + \int_0^{\overline{v}}\int_{v_2}^{\overline{v}} [k(v_1,v_2)-y(v_1,v_2)]f(v_1)dv_1dv = \\
&= U_1(\overline{v}) + \int_0^{\overline{v}}\int_{\overline{v}}^{v^*} y_1(v_1,v_2)f(v_2)dv_1dv_2
\end{aligned}$$

Hence, it is optimal to have $y_1(v_1,v_2)=1$ if $v_1>\overline{v}$. At the same time, it does not violate monotonicity constraint. Thus, the optimal allocation for $v_1\le\overline{v}$ when $v^*>\overline{v}$ should coincide with the allocation for $v_1\le\overline{v}$, when $v^*=\overline{v}$. The auction described in the statement implements exactly this allocation[16]. Once again, transfers can be computed according to (6) taking into account that by Lemma 3 we have $h_i(0,\mathbf{v}_{-i})=0$.

Proof of Proposition 2

Since distribution $F(v)$ is atomless, we have $F(v)<1$ for any $v<\overline{v}$. Thus, $1=G_{v^*}(v^*)>F(v^*)$ for any $v^*<\overline{v}$. Since $G_{v^*}(v)$ and $F(v)$ are different at $v=v^*$, it means that $v=v^*$ belongs to a subset where $G_{v^*}(v)$ is linear, i.e. there is a pooling interval $(\widehat{v},\widehat{\widehat{v}})$ such that $v^*\in(\widehat{v},\widehat{\widehat{v}})$. $\widehat{v}$ is a point, tangent line from which goes directly to $(v^*,1)$. By construction $g_{v^*}(v)=const$ for $v>v^*$ and hence $v>v^*$ is a pooling region. This implies that all values $v>\widehat{v}$ must be pooled.

To show monotonicity of $\widehat{v}$ as a function of v^*, suppose that it is not true, i.e. there exist v_1^* and v_2^* such that $v_1^*<v_2^*$ and $\widehat{v}(v_1^*)>\widehat{v}(v_2^*)$. By definition of G_{v^*} the following holds: $G_{v_2^*}(\widehat{v}(v_1^*))\ge F(\widehat{v}(v_1^*))$. Since $G_{v_1^*}(v_1^*)-G_{v_1^*}(\widehat{v}(v_1^*))=1-F(\widehat{v}(v_1^*))>G_{v_2^*}(v_1^*)-G_{v_2^*}(\widehat{v}(v_1^*))$

[16] The allocation for $(v_1,v_2):\overline{v}<v_1\le v_2$ does not affect the utility of the favorite. For definiteness sake, in the statement we have specified $y(v_1,v_2)=1$ for $\overline{v}<v_1\le v_2$.

we must have $g_{v_1^*}(\widehat{v}(v_1^*)) > g_{v_2^*}(\widehat{v}(v_1^*))$. Then

$$\begin{aligned}
G_{v_1^*}(\widehat{v}(v_2^*)) &= G_{v_1^*}(\widehat{v}(v_1^*)) - (\widehat{v}(v_1^*) - \widehat{v}(v_2^*))g_{v_1^*}(\widehat{v}(v_1^*)) \\
&= F(\widehat{v}(v_1^*)) - (\widehat{v}(v_1^*) - \widehat{v}(v_2^*))g_{v_1^*}(\widehat{v}(v_1^*)) \\
&< F(\widehat{v}(v_1^*)) - (\widehat{v}(v_1^*) - \widehat{v}(v_2^*))g_{v_2^*}(\widehat{v}(v_1^*)) = F(\widehat{v}(v_2^*))
\end{aligned}$$

However, $G_{v_1^*}(\widehat{v}(v_2^*)) < F(\widehat{v}(v_2^*))$ is impossible by construction of $G_{v_1^*}$. Thus, $\widehat{v}(v^*)$ has to be monotone.

Proof of Proposition 3

The result follows from the proof of Proposition 1. Suppose $v^* < \overline{v}$. If the favorite's value changes, the corresponding change of the $FPA(\{c_A, c_{DIC}, c_{NT}\})$ is related to the change of the function $G_{v^*}(v)$. The only change of this function happens on the subset $[\widehat{v}(v^*), \overline{v}]$, which is a pooling region. For all favorite's values above the maximal possible value of his opponent, the function $G_{v^*}(v)$ is the same function for all v^*, which brings the same $FPA(\{c_A, c_{DIC}, c_{NT}\})$ for all $v^* > \overline{v}$.

Proof of Theorem 3

Step 1:

First, consider a value v_i of a bidder i such that $v_i < \max_{j \neq i}\{v_j\}$. Then, due to DA, anonymity and monotonicity condition (5), the bidder i should receive the object with zero probability and $y_i(v_1, ..., \underset{i}{q}, ...v_n) = 0$ for all $q \leq v_i$. Indeed, to show this, suppose that $y_i(v_1, ..., v_i, ..., v_n) = 1$ for some $v_i < \max_{j \neq i}\{v_j\}$. Then, monotonicity implies that $y_i(v_1, ..., \max_{j \neq i}\{v_j\}, ...v_n) = 1$. However, due to anonymity the bidder k who has the value $v_k = \max_{j \neq i}\{v_j\}$ should also have probability of assigning the good equal to one. Thus, we obtain that $y_i(v_1, ..., \max_{j \neq i}\{v_j\}, ...v_n) = y_k(v_1, ..., \max_{j \neq i}\{v_j\}, ...v_n) = 1$, which contradicts feasibility. Thus, all bidders whose value is not the highest one should receive the good with zero probability, namely if $v_i < \max_{j \neq i}\{v_j\}$, then $y_i(\mathbf{v}) = 0$.

Step 2:

From Step 1, it follows that for any realization of values there could be only two possible cases: 1) the bidder with the highest value obtains the object for sure, 2) nobody gets the object. Monotonicity constraint (5) implies that if for some vector of values bidder i receives the good, he should also receive the good when he has a higher value keeping values of his opponents fixed. Thus, for any $FA(C)$ there is a cutoff r_i for each bidder, which can depend on other bidders' values, such that the bidder obtains the object with a probability of 1 if and only if his value is 1) the greatest among values of other bidders and 2) greater or equal than the cutoff r_i. Thus, $y_i(\mathbf{v}) = 1$ if and only if $v_i > \max_{j\neq i}(v_j, r_i)$, otherwise $y_i(\mathbf{v}) = 0$.

Step 3:

Now we need to understand how these values $\{r_i\}_{i=1}^n$, or essentially reserve values, are constructed. First, notice that for each bidder i his reserve value r_i can depend on his opponents' bids. Hence, r_i can depend on $v_1, ..., v_{i-1}, v_{i+1}, ..., v_n$. By anonymity, the allocation probability for a bidder i should not be affected by any permutation of other players' bids. Hence, r_i has to be a symmetric function of $n-1$ variables $r_i : \mathbb{R}^{n-1} \to \mathbb{R}$. Then, again due to anonymity, since the allocation rule must be symmetric among bidders, then for any bidders i and j and for any $\mathbf{x} \in \mathbb{R}^{n-1}$ the function $r_i(\mathbf{x})$ and $r_j(\mathbf{x})$ have to be equal, $r_i(\mathbf{x}) = r_j(\mathbf{x})$. Hence, the reserve value function should be common for all bidders: $r_1(\mathbf{x}) = ... = r_n(\mathbf{x}) = r(\mathbf{x})$.

Step 4:

Take any bidder $i : v_i < \max_{j\neq i}\{v_j\}$. Then, from Step 1 we have $y_i(v_1, ..., \underset{i}{q}, ...v_n) = 0$ for all $q \leq v_i$. Hence, from (6) we have

$$t_i(\mathbf{v}) = U_i(\mathbf{v}) = h_i(\underline{v}_i, \mathbf{v}_{-i})$$

Since transfers have to be non-positive, it follows that $\forall \mathbf{v}_{-i}$: $h_i(\underline{v}_i, \mathbf{v}_{-i}) \leq 0$. However, simultaneously to satisfy DIC, utility of bidder i has to be at least non-negative, otherwise he could refrain himself from participation. Thus, it must be the case that $h_i(\underline{v}_i, \mathbf{v}_{-i}) \geq 0$. Combining the two inequalities we obtain $h_i(\underline{v}_i, \mathbf{v}_{-i}) = 0$. It means that transfers are uniquely defined when the allocation is chosen. Thus, plugging the obtained allocation rule and $h_i(\underline{v}_i, \mathbf{v}_{-i}) = 0$ to (6) we get $t_i(\mathbf{v}) = -\max_{j\neq i}(v_j, r(\mathbf{v}_{-i}))$ if and only if $v_i > \max_{j\neq i}(v_j, r(\mathbf{v}_{-i}))$, and $t_i(\mathbf{v}) = 0$ otherwise. Since we have $y_i(\mathbf{v}) = 1$ if and only if $v_i >$

$\max_{j \neq i}(v_j, r(\mathbf{v}_{-i}))$, the statement follows.

Proof of Proposition 4

From the proof of Theorem 3, the utility of any bidder under the full set of restrictions $C = \{c_A, c_{DIC}, c_{NT}, c_{DA}\}$ must be $U_i(\mathbf{v}) = \int_{\underline{v}_i}^{v_i} y_i(v_1, ..., \underset{i}{q}, ...v_n)dq$, where $y_i(\mathbf{v}) = 1$ if and only if $v_i > \max_{j \neq i}(v_j, r_i(\mathbf{v}_{-i}))$, otherwise $y_i(\mathbf{v}) = 0$. The choice of a reserve value function completely determines the auction format. Hence, the utility of each bidder including the favorite can be written as follows:

$$
\begin{aligned}
U_i(\mathbf{v}) &= \int_{\max_{j \neq i}(v_j, r_i(\mathbf{v}_{-i}))}^{v_i} y_i(v_1, ..., \underset{i}{q}, ...v_n)dq = \\
&= \max\{0, v_i - \max_{j \neq i}(v_j, r_i(\mathbf{v}_{-i}))\}
\end{aligned}
$$

Hence, making positive reserve prices can only reduce the utility of each bidder including the favorite. Thus, it is optimal to put zero reserve price, so $r_i(\mathbf{x}) = 0 \ \forall \mathbf{x} \in \mathbb{R}^{n-1}$.

14 Appendix B

Assumption 1 (symmetric bidders) $\forall i, j \in N : V_i = V_j = V$ *and* $F_i(v) = F_j(v) = F(v)$

Proposition 6 (ex-post equivalence of unrestricted and anonymity-restricted favoritism)

If bidders are symmetric and $C = \{c_A\}$ *then there exists* $FA(C) = (\widehat{B}, \widehat{\mathbf{y}}, \widehat{\mathbf{p}})$ *such that it has an equilibrium* $\widehat{\psi}$, *in which* $\widehat{B} = [\underline{v}, \overline{v}]$, $\widehat{M} = M^{FPA}$, $\widehat{\mathbf{p}}(a, \widehat{\mathbf{b}}(\mathbf{v})) = \mathbf{p}^{FPA}(a, \mathbf{b}^{FPA}(\mathbf{v}))$, $\widehat{\mathbf{y}}(\widehat{\mathbf{b}}(\mathbf{v})) = \mathbf{y}^{FPA}(\mathbf{b}^{FPA}(\mathbf{v}))$, *where* $\widehat{\mathbf{b}}(\mathbf{v})$ *and* $\mathbf{b}^{FPA}(\mathbf{v})$ *stand for the biddings in the equilibrium* $\widehat{\psi}$ *of* $FA(C)$ *and* ψ^* *of* FPA *respectively.*

Proof of Proposition 6

I prove the theorem by directly constructing the equivalent anonymous auction $FA(C)$. Consider the set of admissible bids equal to the set of possible values, $B = [\underline{v}, \overline{v}]$. Denote $\widehat{v}$ as the smallest value such that $v^* = \widehat{v} - \frac{1-F(\widehat{v})}{f(\widehat{v})}$. If no solution to this equation exists, assume $\widehat{v} = \overline{v}$. The allocation rule is such that the bidder with the highest bid wins, i.e. $\widehat{\mathbf{y}}(b_1, ..., b_i, ..., b_m) = (0, ..., \underset{i}{1}, ..., 0)$ if $b_i > b_j$ for any $j \neq i$. If $k \geq 2$ bidders make exactly the same bids, there is a symmetric lottery between them with $1/k$ being a probability of securing the good for each of them. Transfers $\widehat{\mathbf{p}}(\mathbf{b})$ are such that if there is only one bid on the interval $[\underline{v}, \widehat{v}]$, then this bidder pays nothing, although if there are two or more bidders who make bids from this interval, all of them should pay $\widehat{v}$. If the winning bid is greater than $\widehat{v}$, the payment is the maximum between the second highest bid and $\widehat{v}$. Subsequently, there is an equilibrium $\widehat{\psi}$, in which the favorite bids v^*, all bidders with values smaller than $\widehat{v}$ do not participate in the bidding and all bidders with values greater than $\widehat{v}$ participate and bid their true values. This equilibrium outcome is always the same as in the FPA.