Demographic structure and the (un)employment volatility of young workers*

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Abstract

This paper offers a theoretical explanation of a novel fact: the unemployment volatility within the group of young workers increases with their share in the labor force. Using a search and matching model with a labor demand structure that differentiates between young and old workers, I show how the dynamics of labor market transitions depends on the demographic structure of the workforce. In addition to a composition effect, there is a spillover effect among the young: an increase in the share of the young depresses the price of output typically produced by them. The volatility of job finding and separation rates of this group increases accordingly, jointly raising unemployment volatility.

JEL: E24, E32, J11

Keywords: spillover effect, demographic change, age-specific unemployment volatility

1 Introduction

The volatility of unemployment rises with the share of young workers in the labor force, as young workers lose jobs more often and their unemployment (rate) is generally more cyclically sensitive. This age-specific difference in the cyclical volatility of the labor market has been

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well documented and explanations have been offered by several authors[1]. With the existence of this difference, changes in the relative share of young workers naturally have composition effects.

In this paper I focus on a new fact: the unemployment volatility within the group of young workers also increases with their share in the labor force, which is referred to as a spillover effect here. Empirically, this phenomenon does not exist in other age groups as I show in Han (2018). The reason, as argued in this paper, lies in a capital-experience complementarity as also observed in the data I use.

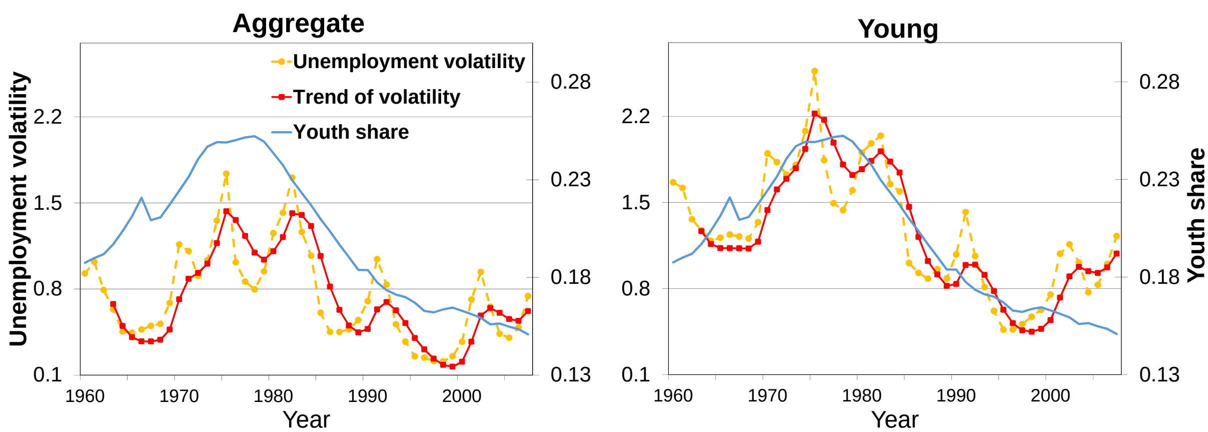

Figure 1: Unemployment volatility and the youth share (U.S.)

Notes: Dotted, circle-hatched line is the time-varying unemployment volatility (left axis), solid, square-hatched line is the trend of this measure (left axis), and the solid line is the youth share (right axis).

Figure 1 shows the time series for unemployment volatility (together with its trend, filtered using a one-sided HP filter with a smoothing parameter of 100)[2] and the share of young workers[3] in the labor force (hereafter referred to as the youth share) for the U.S. from 1960 to 2007. We see that aggregate unemployment volatility comoves with the youth share over

[1] The empirical evidence is available both in intensive (see, e.g., Gomme et al., 2004 and Jaimovich and Siu, 2009) and extensive margin (see Han, 2018). Theoretical explanations include the capital-experience complementarity by Jaimovich, Pruitt, and Siu (2013) and the higher initial training sunk cost for more educated workers by Cajner and Cairo (2013).

[2] Unemployment volatility is calculated by the author and measured by a stochastic volatility process which assumes that the unemployment rate follows an AR(1) process and and the volatility equation follows an AR(1) stochastic volatility process (see Fernández-Villaverde et al., 2011, Born and Pfeifer, 2014, and Han, 2018). Both unemployment rate and volatility are at an annual frequency. Through out this paper, I use a smoothing parameter of 100 with the one-sided HP filter for annual data, which is commonly used in the literature, see, e.g., Cooley and Ohanian (1991) and Rogerson and Shimer (2011). Besides, I have also repeated the analysis with a parameter of 6.25, as suggested by Ravn and Uhlig (2002), and got similar results.

[3] Young workers refer to those from age 15 to 24 and aggregate measure covers from age 15 to 64.

time (left panel). Moreover, an even closer comovement exists between this share and the unemployment volatility of the young (right panel), while the common underlying assumption used in the literature is that changes in the labor force composition do not have any effect on the labor market outcomes of a specific group[4]. Ignoring this spillover effect automatically leads to an overestimation of the composition effect. In addition, it could also lead one to overlook the necessity of age-specific labor market policies in maintaining macroeconomic stability. Therefore, the investigation of the existence of this spillover effect and its possible explanations is of importance for both empirical estimation and policy evaluation.

To explain this new finding, I incorporate the Mortensen-Pissarides (MP) job matching model with endogenous job separation into the real business cycle (RBC) model. Workers search for jobs in labor markets segmented by experience levels and produce distinct intermediate goods with labor as the only input, as inexperienced workers are more suitable for less sophisticated products and vice versa. Different intermediate goods are then combined with capital for the production of final goods. This generates age-specific differences in labor demand, when age is equated with labor market experience.

When the youth share increases, the supply of goods produced by young workers increases and their price decreases accordingly. This price change triggered by demographic change is the key to understand the spillover effect. Firstly, since the decline of price can shrink profits, firm raises its selection criteria of young workers, which then pushes the productivity cutoff of young workers to the mode of its distribution. This implies more young workers will be affected when aggregate productivity changes. Furthermore, similar as Hagedorn and Manovskii (2008), firm's profits and thereby firm's vacancy posting also become more sensitive to aggregate productivity since the level of profits shrinks. Therefore both aspects imply that the unemployment volatility of young workers increases with the youth share.

One contribution of this paper is to fill a theoretical gap between demographic change and the response of age-specific labor market to productivity shocks. Filling this gap is not only important for our understanding of the speciality of the labor market dynamics of the young, but also helpful in the explanation of the falling macroeconomic volatility in the U.S. since the mid-1980s, referred to as "The Great Moderation". I demonstrate that when the youth share increases, the unemployment rate of the young responds more strongly to productivity shocks. This is important because it establishes a new channel for the explanation of the Great Moderation. Instead of proposing composition effect as its main demographic explanation as Jaimovich and Siu (2009), I show theoretically that the changes in the unemployment volatility of the young due to demographic change can also be an important contributor.

As mentioned, Jaimovich and Siu (2009) have started the recent literature on the role of

[4]See, e.g., Jaimovich and Siu (2009) and Cajner and Cairo (2013).

demographics in labor market dynamics by focusing on the composition effect of the workforce's age structure. They link demographic changes to the variation in cyclical volatility in G7 countries after World War II, and find that change in the age composition of the labor force has a large and significant effect on cyclical volatility. They claim the composition effect among different age groups accounts for a sizable fraction of the Great Moderation. Lugauer and Redmond (2012) and Lugauer (2012) also find similar results. Although the replication of these results by Everaert and Vierke (2016) suggests that the relationship identified in these papers between variation in the age composition of the labor force and cyclical volatility may well be spurious[5], Han (2018) reconfirms the validity of this relationship with a broader dataset by measures unemployment volatility with a stochastic volatility process proposed by Fernández-Villaverde et al. (2011) and accounting for the effects of labor market institutions. By focusing on the difference in jobs done by young workers, I show that the unemployment volatility of young workers also increases with the youth share. In contrast to the finding of Jaimovich and Siu (2009), I find that the spillover effect is the main contributor for the Great Moderation.

Meanwhile, this paper is related to the growing studies on the state dependence of labor market fluctuations over the business cycle. Michaillat (2014) and Cacciatore et al. (2016) show that the labor market reforms have different effects on labor market fluctuations across phases of business cycles. Pizzinelli and Zanetti (2017) develop a search model with endogenous job separation and on-the-job search which replicates the asymmetries in the fluctuations of labor market variables across different states of aggregate productivity. As an extension, I show that the fluctuations of labor market variables also depend on the state of demographics.

Finally, this paper is also related to the study of Hagedorn and Manovskii (2008), who provide a different calibration method[6] to remedy the failure of basic search and matching model in generating sufficient volatility to match the empirical data, a puzzle raised by Shimer (2005). As an alternative solution, Hall (2005) proposes the introduction of wage stickiness[7]. This paper starts from a different angle and explores the capability of the Mortensen and Pissarides (1994) job matching model in generating unemployment volatility after consideration of the variation in the youth share.

The paper proceeds as follows. The next section gives further evidence for the existence of the spillover effect. Section 3 lays out the model. Section 4 shows the calibration strat-

[5]The series used in these papers for demographic change and cyclical volatility are found to be non-stationary, and even more, no co-integrating relation is detected between these series.

[6]Their solution relies on the parameter specification of unemployment benefit and worker's bargaining power, which gives rise to unrealistically high opportunity cost of unemployment and suggests that workers are motivated only by a small share of profit, as mentioned by Mortensen and Nagypal (2007).

[7]However Haefke, Sonntag, and Van Rens (2013) suggests that there is little evidence for the existence of wage rigidity in empirical data.

egy. Section 5 discusses the model's performance. Section 6 shows the robustness of the results when jobs are exogenously separated. Section 7 provides supporting evidence for the mechanism. The final section concludes the paper.

2 Empirical evidence

Back to the right panel of Figure 1, we can see a hump-shaped demographic trend for the youth share. It climbs up to its maximum in the mid-1970s as baby boomers entering into labor force from 1960s, then falls down persistently from 1980 with the decreasing inflow of young labor force. Meanwhile this hump-shaped trend is also closely accompanied by the unemployment volatility of the young over the whole period. This comovement serves as a preliminary evidence for the relationship between the youth share and the unemployment volatility of the young. To identify the causality, empirical evidence is provided in this part. First I show the cyclicality of labor force participation is not of primary concern for cyclical unemployment volatility of the young. With this potential endogeneity concern taken care of, I then show a statistically significant relationship exists between these two series using data from 20 OECD countries[8]. As the timing and extent of demographic changes are different among these countries, the validity of the regression results is ensured.

2.1 The role of labor force participation

It is generally acknowledged that labor market dynamics is affected disproportionately across sex and age groups by the labor force participation decision (see, e.g., Clark and Summers, 1982). An increase of participation rate, mostly during booms, will push up unemployment rate, while a decline of participation rate during recession will drag down unemployment rate. As the young have more options in case of job separation, their unemployment volatility might display greater cyclicality compared to other age groups. For instance, with worsening labor market conditions, unemployed young workers are more likely to reconsider the trade-off between job searching and further education. This cyclicality of participation might be more pronounced for females, given the fact that the participation rate of females is more sensitive to labor market conditions. Therefore, in the following, I examine the role of the cyclicality of participation on the unemployment dynamics of the young.

Note that the unemployment rate of a specific group can be expressed as a ratio between the unemployment-to-population ratio and the labor force participation rate of this group,

[8] Which includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the U.K., and U.S.

$$u_i \equiv \frac{U_i}{L_i} = \frac{U_i}{P_i} / \frac{L_i}{P_i} = \frac{up_i}{lp_i},$$

where U_i is the size of unemployment of group i, L_i is the size of its labor force, P_i is the size of its population, up_i is the unemployment-to-population ratio and lp_i stands for the labor force participation rate. Thus, unemployment volatility can be decomposed as

$$\text{Var}\left(\log u_i\right) = \text{Var}\left(\log up_i\right) + \text{Var}\left(\log lp_i\right) - 2\text{Cov}\left(\log up_i,\, \log lp_i\right).$$

If the unemployment volatility from the variation of labor force participation rate accounts for only a minor share, the cyclicality of participation rate is less of a concern.

Table 1: Decomposition of unemployment volatility: participation's share

	Female		Male		Both	
	15-24	15-64	15-24	15-64	**15-24**	15-64
Cov. excl.	2.67	0.39	1.11	0.05	**1.53**	0.10
Cov. incl.	7.60	1.68	4.01	1.30	**4.93**	1.19

Notes: "Cov. excl." means the covariance terms, $Cov\left(1 - e_i,\, lfpr_i\right)$, are excluded for the calculation of the contribution of participation to unemployment volatility. "Cov. incl." means total variance also includes covariance terms. Annual U.S. data from the OECD Labour Force Statistics database, 1960-2016. Cyclical volatility of each series is the percentage standard deviation of each series filtered by one-sided HP-filter with a smoothing parameter 100.

Table 1 reports the share of unemployment volatility that can be attributed to the participation rate of the young and the aggregate of both sex groups in the U.S.. I use one-sided HP filter to keep the time ordering of the data undisturbed. The decomposition results in the first row do not take account the effect of the covariance terms. As expected, the contribution of participation decision to unemployment volatility is higher for young and female. But even for young female, who have the highest value, the contribution is only 2.67 percent. For the young of both sex groups, the contribution of participation only accounts for 1.53 percent, leaving the rest unexplained to the unemployment-to-population ratio. When the effect of the covariance between the unemployment-to-population ratio and the labor force participation rate is also taken into account, still about 95 percent of the unemployment volatility of the young can be attributed to the unemployment-to-population ratio of the young.

Thus, in aggregate, and even for the young, the cyclicality of participation rate is not of primary importance for cyclical unemployment volatility.

2.2 The identification of the role of demographic change

To identify the relationship between the unemployment volatility of the young and the youth share, I consider the following regression,

$$\sigma_{it} = \alpha_i + \beta_t + \gamma s_{it} + \lambda X_{it} + \varepsilon_{it},$$

where σ_{it} stands for the unemployment volatility of the young for country i at time t measured by a stochastic volatility process as mentioned earlier, s_{it} is the youth share, X_{it} includes indicators for labor market institutions and an external shock. Of which, the indicators of labor market institutions include union density, union centralization of wage bargaining, the strictness of employment protection legislation, tax wedge and gross replacement rate. The external shock is a world demand shock, which is proxied by the log-difference of the sum of real GDP of other 19 countries. Besides, α_i is a country fixed effect and β_t is a full set of time dummies to control for time effects.

Table 2: Unemployment volatility of the young and demographic change

	Youth share	Young population share	Native young population share
Coefficient	8.97***	6.53*	9.41**
	(3.43)	(3.47)	(4.45)
Nobs	644	659	627
R^2	0.77	0.77	0.78

*Notes: Dependent variable is the unemployment volatility of the young. I use annual data from 1960 to 2007. Newey-West standard errors are in the parentheses. ***, **, * stand for a significance level of 1 percent, 5 percent and 10 percent respectively. Model used in this table is the CCEP estimator, which projects out fixed effects and eliminates the effects of the unobserved common factors.*

The data covers only up to 2007 because the unemployment dynamics afterwards contains too much noisy signal from the very recent financial crisis. It is technically difficult to disentangle its effect from that of demographic change.

The first column of Table 2 reports the pooled common correlated effects (CCEP) estimator[9], which projects out fixed effects and eliminates the effects of the unobserved common factors. The coefficient of the youth share is positive and significant at the 1 percent level, which suggests that the change in the youth share does have a statistically significantly positive impact on the unemployment volatility of the young. In the second column, I also report the results when the youth share is replaced by the share of young population to take care

[9] With fixed effects and time effects (FETE) and mean group (MG) estimators, cross-sectional dependence test suggests the existence of cross-sectional dependence, which can lead to biased estimation. Thus, CCEP estimator is used to take care of this. See Han (2018) for details.

the potential participation endogeneity. The coefficient is still significant at 10 percent level and at a similar level. Besides, in the last column, I also use the share of native young population[10] as a proxy to deal with the potential endogeneity from migration and get similar result.

3 The model

Here, I lay out the details of the model. There are three sectors in the economy: households, the firm producing intermediate goods, and the firm producing final goods. Households provide labor as the only input for the production of intermediate goods and lease capital to the firm producing final goods. Household members search for jobs in segmented labor markets. The setup of job searching behavior shares similarities with Krause and Lubik (2007) and Trigari (2009). Distinct intermediate goods and capital are then used as inputs for the production of final goods.

3.1 Households

Time is discrete and households have an infinite time horizon. Household members are characterized by age and experience levels. There are two types of age levels, young and old, and two types of experience levels, inexperienced and experienced. As the accumulation of experience takes time, for simplicity, I assume that young members are all inexperienced and old members are all experienced, which results in only two types of workers. Following Merz (1995), I use the pooling assumption for the consumption of each type of members. Suppose the utility function is of form: $u^i(c_t^i) = \frac{(c_t^i)^{1-\tau}-1}{1-\tau}$, where $i \in \{Y, O\}$ denotes either young or old, τ is the inverse of the intertemporal elasticity of substitution. Let S_t^Y denotes the youth share[11] and normalize the labor force to one.

Taking labor income and capital holdings at time t as given, a representative household decides the amount of consumption for each type of members and the amount of capital to

[10]The share of young population and the share of native young population are calculated using data from Maddison (2003) and Mitchell (2008). Details can be found in Han (2018).

[11]A special case is that the youth share remains constant, which can be realized if the birth rate of the young, ρ^B, the exit rate of the old, ρ^D, and the transition rate from young to old, ρ^{YO}, satisfy

$$\rho^{YO} = \frac{\rho^B}{S_t^Y} - S_t^Y \rho^D,$$

which is simply the solution of

$$\frac{L_{t+1}^Y}{L_{t+1}} = \frac{\rho^B L_t + (1-\rho^{YO})L_t^Y}{(1+\rho^B)L_t - \rho^D L_t^O} = \frac{L_t^Y}{L_t}.$$

hold at time $t+1$. Then the problem of a representative household can be written as

$$\max_{\{c_t^Y, c_t^O, K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t [S_t^Y u^Y(c_t^Y) + (1-S_t^Y) u^O(c_t^O)],$$

subject to[12]

$$S_t^Y c_t^Y + (1-S_t^Y) c_t^O + K_{t+1} = r_t K_t + (1-\delta) K_t + S_t^Y n_t^Y \omega_t^Y + (1-S_t^Y) n_t^O \omega_t^O,$$

where $\beta \in (0,1)$ is household's discount factor, K_t denotes capital holdings at time t, $\delta \in (0,1)$ is the depreciation rate of capital holdings, n_t^Y and n_t^O are, respectively, the employment rates of young and old workers, ω_t^Y and ω_t^O are wage rates, and z^Y and z^O are unemployment benefits. With these setups, optimality conditions imply that consumption is the same for both types of family members,

$$c_t^Y = c_t^O = c_t,$$

and intertemporal first-order condition is

$$c_t^{-\tau} = \beta E_t [c_{t+1}^{-\tau} (r_{t+1} + 1 - \delta)],$$

from which we can construct the implied stochastic discount factor, $E_t \beta_{t,t+1} = \beta E_t c_{t+1}^{-\tau} / c_t^{-\tau}$, to evaluate the activities of firms and workers.

3.2 Labor markets

The labor markets are segmented by experience levels. A worker-firm match can produce an output level of $A_t a_t$, where A_t and a_t are respectively aggregate and match-specific productivity shock. The aggregate productivity follows an AR(1) process: $ln\, A_t = \psi ln\, A_{t-1} + e_t$. The match-specific productivities are drawn independently from a time-invariant lognormal distribution with cumulative distribution function $F(a_t)$ and positive support. At the beginning of each period, both productivity shocks are realized, then workers face an exogenous separation shock ρ^x in both markets and an endogenous separation in each labor market if the realization of the match-specific productivity is lower than certain threshold for worker at age i, $\tilde{a}_t^i$, which leads to an overall separation rate $\rho_t^i = \rho^x + (1-\rho^x) F(\tilde{a}_t^i)$. If separation occurs, production does not take place; if not, production starts. Labor markets open afterwards. Unmatched firm posts exactly one vacancy at the cost of either ς^Y or ς^O depending on where

[12] Here I assume that unemployment benefit is financed by the tax income on wage. These two cancel out each other and thus do not appear in the budget constraint.

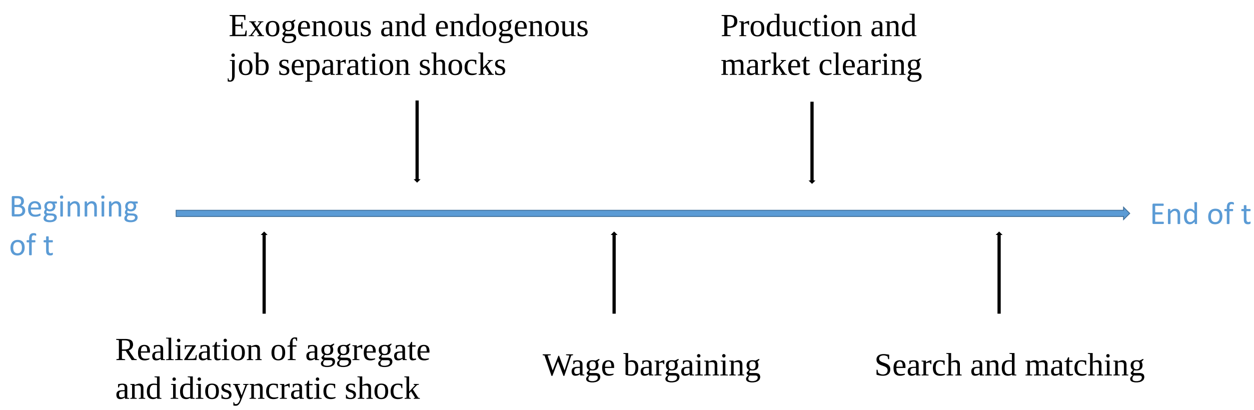

Figure 2: Timing of events

the vacancy is posted. New firms can enter into both labor markets without restrictions until the value of posting an additional vacancy equals to zero. The matching process is depicted by a constant returns to scale matching function:

$$m_t^i = \phi^i (v_t^i)^{1-\mu} (u_t^i)^{\mu},$$

where ϕ^i stands for matching efficiency of labor market i and μ is the elasticity of the matching function with respect to unemployment. The probability of a vacancy being filled equals to $q_t^i = m_t^i / v_t^i = \phi^i \theta_t^{i-\mu}$ and the probability for an unemployed worker to be matched equals $f_t^i = m_t^i / u_t^i = \phi^i \theta_t^{i 1-\mu}$, where $\theta_t^i \equiv v_t^i / u_t^i$ is the market tightness of labor market i at time t. As unemployment benefit is usually proportional to experience level, I assume that the value for old workers, z^O, is higher than that of young workers, z^Y.

With this sequence of events, employment rate at each segmented labor market evolves according to

$$n_t^i = (1 - \rho_t^i)(n_{t-1}^i + f_{t-1}^i u_{t-1}^i),$$

which shows that employment at t is given by the sum of the existing and re-employed workers at $t-1$, conditional on being unseparated at t. As the amount of unemployment workers when production starts is the same as that when job searching starts, this setup ensures that the ratio of workers searching for jobs at t, u_t^i, is equal to the unemployment rate at t, $1 - n_t^i$.

3.3 Firms

I assume that the goods produced by young and old worker are distinct intermediate goods. Both goods use labor as the only input and are subject to common aggregate productivity shock. As the labor force is normalized to one, the size of young labor force equals to S_t^Y

and the size of young matched workers are $n_t^Y S_t^Y$, where n_t^Y is the employment rate of the young. The total output of intermediate goods produced by young workers is

$$Y_t^Y = A_t n_t^Y S_t^Y \int_{\widetilde{a}_t^Y}^{\infty} a \frac{dF(a)}{1 - F(\widetilde{a}_t^Y)},$$

where the term with integral is simply the conditional expectation of match-specific productivity, $E(a/a \geqslant \widetilde{a}_t^Y)$, as only matches not lower than the threshold, $\widetilde{a}_t^Y$, can survive. Similarly, the output of intermediate goods produced by old workers is

$$Y_t^O = A_t n_t^O (1 - S_t^Y) \int_{\widetilde{a}_t^O}^{\infty} a \frac{dF(a)}{1 - F(\widetilde{a}_t^O)}.$$

Both intermediate goods together with capital are used as inputs for the production of final goods in perfectly competitive firms. In order to comply with the empirical observation that the unemployment volatility of young workers is higher than that of the old, I follow Jaimovich, Pruitt, and Siu (2013)'s strategy by introducing a nested constant-elasticity-of-substitution (CES) functional form for final output[13],

$$Y_t = \left[\lambda_1 (Y_t^Y)^\sigma + (1 - \lambda_1)\Big(\lambda_2 (K_t)^\rho + (1 - \lambda_2)(Y_t^O)^\rho\Big)^{\sigma/\rho}\right]^{1/\sigma},$$

where $\rho, \sigma \in (0,1)$, $(1-\rho)^{-1}$ is the elasticity of substitution between intermediate goods produced by old and capital, $(1-\sigma)^{-1}$ is the elasticity of substitution between intermediate goods produced by young and the composite of other two inputs; and $\lambda_1, \lambda_2 \in (0,1)$ are the factor share parameters. If ρ is far smaller than σ, final production features capital-experience complementarity. As capital is in inelastic supply in the short-run, the complementarity suggests that the intermediate goods produced by the old does not respond simultaneously to productivity shocks, so does the unemployment rate of the old.

Firms in final goods sector maximize profit by choosing the optimal levels of intermediate goods and capital,

$$\max_{Y_t^Y, Y_t^O, K_t} Y_t - P_t^Y Y_t^Y - P_t^O Y_t^O - r_t K_t,$$

and the FOCs are:

[13] Notice that final output does not feature a constant marginal product of labor. If labor enters directly into final production and replaces intermediate goods, the marginal product of labor becomes diminishing, implying the match surplus will depend on the number of workers within a firm. As a result, firms cannot bargain with each worker independently and its solution will be much more complicated than the Nash bargaining solution. See Elsby and Michaels (2013) for details. Therefore, I make the distinction of intermediate and final goods and assume that labor is the only input for intermediate goods. In this way, the simplicity of the bargaining solution is kept.

$$
\begin{aligned}
P_t^Y &= (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-1} \\
P_t^O &= (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t(1-\lambda_2)(Y_t^O)^{\rho-1} \\
r_t &= (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t\lambda_2(K_t)^{\rho-1},
\end{aligned}
$$

where $\Omega_t \equiv [\lambda_2(K_t)^\rho + (1-\lambda_2)(Y_t^O)^\rho]^{(\sigma-\rho)/\rho}$.

To close the model, it's easy to see that the aggregate resource constraint is

$$
C_t + K_{t+1} = Y_t + (1-\delta)K_t - C_t^h
$$

where $C_t = S_t^Y c_t^Y + (1-S_t^Y)c_t^O$ is the aggregate consumption and $C_t^h \equiv v_t^Y S_t^Y \varsigma^Y + v_t^O(1-S_t^Y)\varsigma^O$ is the aggregate hiring costs incurred by firms.

3.4 Characterization of agents' asset values

Bellman equations that characterize the problems of firms and workers are similar as that in the standard search and matching model with endogenous separation. The Bellman equation for the asset value of a job with productivity $(A_t,\, a_t)$ in term of final goods is

$$
J_t^i(A_t,\, a_t) = P_t^i A_t a_t - \omega_t^i(A_t,\, a_t) + E_t\beta_{t,\,t+1}\Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},\, a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)}\Big],
$$

where P_t^i and ω_t^i are respectively the relative price of the intermediate good and the wage rate at time t. The value of a job is composed of the current profits $P_t^i A_t a_t - \omega_t^i$ and the continuation value. In the next period, jobs survive with probability $1-\rho_{t+1}^i$, and with probability ρ_{t+1}^i they are destroyed and firms get a future value of zero.

The value of a vacancy is

$$
V_t^i = -\varsigma^i + E_t\beta_{t,\,t+1}\Big[q_t^i(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},\, a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} + (1-q_t^i)V_{t+1}^i\Big],
$$

where ς^i is the hiring cost in the current period. In the next period, a vacancy is filled and production happens with probability $q_t^i(1-\rho_{t+1}^i)$. Free entry ensures that $V_t^i = 0$ for both types of vacancies at any time t, which yields

$$
\frac{\varsigma^i}{q_t^i} = E_t\beta_{t,\,t+1}\Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} J_{t+1}^i(A_{t+1},\, a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)}\Big],
$$

suggesting the expected cost of a vacancy equals to the expected benefit.

For workers, the asset value of being matched is

$$W_t^i(A_t,\, a_t) = \omega_t^i(A_t,\, a_t) + E_t\beta_{t,\,t+1}\Big[(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} W_{t+1}^i(A_{t+1},\, a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} + \rho_{t+1}^i U_{t+1}^i\Big].$$

It includes the current inflow ω_t^i and the continuation value. Employed worker gets the expected value W_{t+1}^i if not separated from the labor market and U_{t+1}^i in case of separation in the next period.

The unemployed worker gets utility z^i in the current period, to be matched and produces with probability $f_t^i(1-\rho_{t+1}^i)$ or stays unemployed and get the expected value of being unemployed with probability $1-f_t^i(1-\rho_{t+1}^i)$ in the next period,

$$U_t^i = z^i + E_t\beta_{t,\,t+1}\Big[f_t^i(1-\rho_{t+1}^i)\int_{\widetilde{a}_{t+1}^i}^{\infty} W_{t+1}^i(A_{t+1},\, a)\frac{dF(a)}{1-F(\widetilde{a}_{t+1}^i)} + (1-f_t^i(1-\rho_{t+1}^i))U_{t+1}^i\Big].$$

With Nash bargaining, wage is chosen to satisfy the optimality condition

$$\eta J_t^i(A_t,\, a_t) = (1-\eta)\Big(W_t^i(A_t,\, a_t) - U_t^i\Big),$$

where $\eta \in (0,1)$ reflects the relative bargaining power of worker. Together with the free entry conditions, it is straightforward to get the wage equation, after substituting the equations for the asset values of workers and firms into the above optimality condition:

$$\omega_t^i(A_t,\, a_t) = \eta(P_t^i A_t a_t + \varsigma^i\theta_t^i) + (1-\eta)z^i.$$

The wage equation implies that the worker and the intermediate firm share the match output and the savings of hiring costs in accordance with their bargaining powers, besides, the worker is also compensated for the foregone unemployment benefits with a fraction of $1-\eta$.

Free entry implies $V_t^i = 0$. Summing up the Bellman equations, match surplus, $S_t^{Mi}(A_t,\, a_t) \equiv J_t^i(A_t,\, a_t) + W_t^i(A_t,\, a_t) - V_t^i - U_t^i$, can be written as

$$S_t^{Mi}(A_t,\, a_t) = P_t^i A_t a_t - z^i + E_t\beta_{t,\,t+1}\,(1-\rho^x)(1-f^i\eta)\int_{\widetilde{a}^i}^{\infty} S_{t+1}^{Mi}(A_{t+1},\, a)dF(a).$$

It is easy to see that $\frac{\partial S_t^{Mi}(A_t,a_t)}{\partial a_t} = P_t^i A_t > 0$, which ensures the existence and uniqueness of reservation productivity $\widetilde{a}^i$.

3.5 Comparative statics and the youth share

In this section, I first show how price and reservation productivity interact with the youth share. Based on these results, I then describe how the responses of market tightness and unemployment of the young with respect to aggregate productivity change with the youth share.

Proposition 1. *With the model setup mentioned, it can be shown that the price of intermediate goods produced by young workers decreases with the youth share, that is,* $\frac{\partial P_t^Y}{\partial S_t^Y} < 0$.

Proof: See the Appendix. ■

The intuition of Proposition 1 is straightforward: When the youth share increases, output by young workers increases accordingly if the average productivity and employment rate of the young remains unchanged; with the increase of its supply, the price drops.

But notice that the reservation productivity of the young could also change with the youth share, which then leads to the corresponding changes in the average productivity and employment rate of the young. Suppose the reservation productivity of the young increases with the youth share, then the average productivity of remaining young workers increases while employment rate decreases. These two happen to cancel each other, therefore output responds fully to the increase of the young. This gives rise to increase of output by the young and decrease of output by the old. Notice that final output is homogeneous of degree 1, together with the fact that capital is in inelastic supply in the short-run, it is easy to see that there is an oversupply of goods by the young. Its price decreases accordingly. This result is important because the response of price to demographic change also affects firm's profits, which then influences firm's vacancy posting behavior.

Corollary 1. *With the model setup mentioned, it can be shown that the reservation productivity of the young increases with the youth share, that is,* $\frac{\partial \widetilde{a}^Y}{\partial S_t^Y} > 0$.

Proof: See the Appendix. ■

Intuitively, as more young workers leads to the decline of the price of goods by the young, marginal young workers need to be more productive, which automatically pushes up the reservation productivity of the young.

If there is no aggregate uncertainty, it can be shown that the elasticity of market tightness of the young with respect to aggregate productivity is[14]

$$\frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{1 - \beta(1 - \widetilde{\rho}^Y) + \eta\beta(1 - \rho^x)\widetilde{f}^Y}{\mu[1 - \beta(1 - \widetilde{\rho}^Y)] + \eta\beta(1 - \rho^x)\widetilde{f}^Y} \frac{\frac{\partial \ln P^Y}{\partial \ln A} + 1}{1 - \tilde{z}^Y},$$

where $\tilde{z}^Y \equiv z^Y/[P^Y AE(a/a \geqslant \tilde{a}_t^Y)] \in (0, 1)$ is the effective unemployment benefit rate for

[14] Proofs of the equations for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ and $\frac{\partial \ln u^Y}{\partial \ln A}$ can be found in the Appendix.

young workers. The first term of the elasticity is a function of the overall separation rate, elasticity of the matching function with respect to unemployment, worker's bargaining power and the overall job-finding rate. Of which, only job-finding rate responds to the change in the youth share. The increase of young workers could lead to overcompetition for the young and decrease job-finding rate, this means a higher value of the first term when the youth share increases. But the limited variation of this term is well discussed in Shimer (2005).

The second term bears a resemblance to Hagedorn and Manovskii (2008) in the sence that its value depends on the gap between unity and the effective unemployment benefit rate of the young. The difference lies in that this effective rate is negatively related to the price of goods produced by the young. With a higher youth share, according to Proposition 1, this price decreases, which pushes the effective unemployment benefit rate closer to unity and gives rise to higher elasticity of market tightness. The increase of the youth share can shrink the profits generated by the match between firm and young worker, since the price of goods produced by the young decreases. In a boom, there is a large percentage increase in firm's profit by hiring young workers, and firm reacts with more vacancies to post. In a recession, firm's profit declines further by hiring young worker and firm rarely posts any vacancy. Therefore, firm's vacancy posting becomes more sensitive to aggregate productivity with a higher youth share, so does the labor market tightness of the young.

Besides, it can be shown that the elasticity of the unemployment rate of the young with respect to aggregate productivity is

$$\begin{aligned}\frac{\partial \ln u^Y}{\partial \ln A} &= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\Big[-(1-\tilde{\rho}^Y)(1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A} \\ &\quad + F'(\tilde{a}^Y)\tilde{a}^Y(1-\rho^x)(1+\frac{1}{\tilde{\rho}^Y})\frac{\partial \ln \tilde{a}^Y}{\partial \ln A}\Big],\end{aligned}$$

which is mainly composed of two terms. The first term is a function of the elasticity of market tightness with respect to aggregate productivity. It represents the response of job-finding rate to aggregate productivity. As mentioned before, the elasticity of market tightness increases with the youth share.

The second term contains the probability distribution function at reservation productivity and other functions of reservation productivity. It represents the response of job separation rate to aggregate productivity. Of which, probability distribution function is highly responsive to the youth share, as long as the distribution of the idiosyncratic productivity is concentrated and does not have any fat tail. According to Corollary 1, a higher youth share means a higher reservation productivity of the young. The increase of this threshold per se could be rather mild, but it can induce a large increase in the probability distribution function, suggesting a dramatic increase in the mass of marginal workers. As more marginal young workers are subjected to aggregate productivity shock, the unemployment rate of the young becomes more volatile. Therefore, the volatilities of job-finding rate and job separation rate of the young both increase with the youth share, and jointly contribute to the spillover effect.

4 Calibration

This section describes the parameter specification of the model for the U.S. First, I estimate the two parameters governing the elasticities of substitution in final goods production. Then, other parameter values are set to be consistent with the literature or empirical observations.

4.1 The two parameters governing the elasticities of substitution

As there is not any first-order moment in the data to be matched with parameters ρ and σ, which describe the elasticity of substitution among inputs, we need to estimate them, for which I take advantage of the FOCs in final goods production, in a similar way as Jaimovich, Pruitt, and Siu (2013).

Notice that the FOC with respect to Y_t^Y can be written in the following form

$$\triangle \log \left(P_t^Y Y_t^Y / Y_t\right) = \sigma \triangle \log \left(N_t^Y / Y_t\right) + \sigma \triangle \log \left(A_t\right),$$

where N_t^Y is the employment size of the young at time t. $P_t^Y Y_t^Y$ is the total value created by young workers in term of final goods, which can be measured by the labor income of the young. Note that the ratio between $P_t^Y Y_t^Y$ and Y_t is just the share of final goods created by the young. σ can be estimated as the response of the share created by the young with respect to the number of the young needed per final good, with the aggregate productivity shock as error term.

Similarly, from the FOCs with respect to Y_t^O and K_t, we can get

$$\triangle \log \left(P_t^O Y_t^O / (r_t K_t)\right) = \rho \triangle \log \left(N_t^O / K_t\right) + \rho \triangle \log \left(A_t\right).$$

It's easy to see that $P_t^O Y_t^O$ is the total value created by old workers in term of final goods and $r_t K_t$ is the total price paid for capital. The ratio between these two is also the ratio between the share of final goods created by old workers and the share created by capital. Then ρ can be estimated in the same way as σ. Following Jaimovich, Pruitt, and Siu (2013), I use lagged birth rates as instruments for the regressors in both equations, as both employment size of the young and final output are correlated with aggregate productivity. Considering that aggregate productivity enters into the error terms of both, I use GMM for simultaneous estimation. I use data from CPS up to 2007 for the estimation.

Table 3: Estimation results of the elasticities of substitution in final goods production

	σ	ρ
Point Estimate	0.701***	0.195***
	(0.056)	(0.025)
Hansen's J-test	1.474 [0.479]	
$\sigma = \rho$	47.19 [0.000]	

*Notes: Data from the March CPS, 1964-2007. Model used for estimation is GMM with IVs. HAC standard errors are in the parentheses. p-values are in the square brackets. *** stands for a significance level of 1 percent.*

Table 3 reports the estimation results. The point estimate of ρ is 0.195, indicating that the elasticity of substitution between intermediate goods by old and capital $(1-\rho)^{-1}$ is slightly more than one. The point estimate of σ is 0.701, suggesting the elasticity of substitution between intermediate goods produced by young and the composite of other two inputs $(1-\sigma)^{-1}$ is much larger than one. The p-value of Hansen's J-test is 0.479, suggesting no over-identification. Besides, the null hypothesis $\sigma = \rho$ is statistically rejected. Notice that $\sigma > \rho$ is not imposed as an assumption in the former analysis, it is a fact supported by data, which supports capital-experience complementarity.

4.2 Other parameters

The model is simulated at a quarterly frequency. In the benchmark calibration, the youth share, S^Y, is set at 0.20, which is the average from 1960 to 2007. In a standard manner, discount rate, β, is set at 0.99. Depreciation rate, δ, takes the value of 0.025 and is the same as the value used in Jaimovich, Pruitt, and Siu (2013), 0.025. The intertemporal elasticity of substitution is chosen to be 2, which is commonly used in literature. The exogenous job separation rate, ρ^x, is set to be 0.055 to target an average unemployment rate of 7 percent. The parameter of match efficiency, ϕ, is set to be the same for both markets and equal to 0.86 to match the empirical average job finding rate of 0.79 from Elsby, Hobijn, and Sahin (2010). Following Petrongolo and Pissarides (2001), the elasticity of market tightness with respect to vacancies, μ, is set at 0.5. To satisfy Hosios (1990) condition, I assume that worker's bargaining power, η, is equal to the elasticity of matching function.[15] The vacancy posting

[15]The Hosios condition makes sure the externalities inflicted by searchers and firms offset each other optimally. Although there is *ex ante* heterogeneity in the model, workers of different age and experience search in segmented labor market, thus no further externality is inflicted and the Hosios condition is guaranteed by equalizing the worker's bargaining power to the elasticity of the matching function with respect to unemployment.

cost, ς, is set to be the same for both markets and equals to 0.11 based on the evidence in the 1982 Employment Opportunity Pilot Project survey, see Cajner and Cairo (2013) for details.

Table 4: Parameter values in simulation (quarterly)

Parameter	Interpretation	Value	Rationale
σ	Define Elas. in final output	0.701	Estimated
ρ	Define Elas. in final output	0.195	Estimated
β	Discount factor	0.99	
δ	Depreciation rate	0.025	Jaimovich, Pruitt, and Siu (2013)
τ	Inverse of EIS	0.5	
ρ^x	Exogenous separation rate	0.055	Unemployment rate 7%
ϕ	Matching efficiency	0.86	Job-finding rate 79%
μ	Matching function elasticity	0.5	Petrongolo and Pissarides (2001)
η	Worker's bargaining power	0.5	Hosios condition
ς	Vacancy posting cost	0.11	Cajner and Cairo (2013)
z^O	Unemployment benefit -old	0.76	
z^Y	Unemployment benefit -young	0.55	Ratio of the young to the old (0.7)
λ_1	Factor share by young	0.335	National income share by young
λ_2	Factor share of capital	0.275	National income share of capital
ψ	Coef for productivity	0.81	Labour productivity (BLS)
σ^A	SD for productivity	0.011	Labour productivity (BLS)
μ_{LN}	Mean of log normal distribution	0	Standardization
σ_{LN}	SD of log normal distribution	0.12	Krause and Lubik (2007)

It's demonstrated by Hagedorn and Manovskii (2008) that a high value for non-market activity can generate more unemployment dynamics, with $z = 0.995$ for their case. But high value for non-market activity also implies unrealistically high opportunity cost of employment and unrealistic sensitivity of unemployment to changes in unemployment benefit. Here, I set the unemployment benefit of old workers, z^O, at 0.76, which is relatively higher than the value $z = 0.71$ used by Hall and Milgrom (2008). As unemployment benefit is usually proportional to experience level, I assume the unemployment benefit of the young is 70 percent of that of the old, which results the value for the young, z^Y, to be 0.55. For the specification of the factor share parameters λ_1, λ_2, I follow Krusell et al. (2000) and calibrate these parameters to match national income share from CPS, which gives that the share created by old workers amounts to 50 percent from 1964-2007, and the share by capital amounts to 36 percent. Parameters for the aggregate productivity shock are calibrated to match with the seasonally adjusted real output in the non-farm business sector. I take log transformation and then use one-sided HP filter to detrend the series with smoothing parameter[16] 1600. The standard deviation of the productivity is 0.013 and the quarterly autocorrelation is 0.81, which implies

[16] I follow the standard smoothing parameter used for quarterly data, see, e.g., Fujita and Ramey (2012).

the standard deviation of error term, σ^A, equals to 0.11[17]. Concerning the parameters of the lognormal distribution of the idiosyncratic productivity, a_t, I follow Krause and Lubik (2007), normalize its mean, μ_{LN}, at 1 and set the standard deviation, σ_{LN}, at 0.12 to ensure a concentrated distribution.

Table 5: Summary of the first-order moments in benchmark calibration (quarterly)

	Data	Model
Unemployment rate	0.06	0.07
Unemployment rate of the young	0.12	0.09
Unemployment rate of the old	0.05	0.06
Job-finding rate	0.81	0.79
Separation rate	0.06	0.06

Notes: Aggregate and age-specific unemployment rates are from St. Louis Fed, and transition rates are from CPS. All series are from 1960Q1 to 2007Q4.

Table 5 shows that the simulated first-order moments are well targeted, and close to their empirical counterparts.

5 Simulation results

This section presents the quantitative results. First, I report the simulation results for the aggregate economy. Second, I check the existence of the spillover effect in simulations, which is done by changing the value for the youth share and then comparing the age-specific unemployment volatility across demographic states. Finally, I present impulse response functions with respect to aggregate productivity shock to distinguish the role of job-finding and separation rates in the spillover effect. In order to solve the model numerically, I log-linearize the model in Section 3 around its steady state. Details can be found in the Appendix.

5.1 Aggregate volatility across demographic states

Table 6 reports the standard deviations (SD) of aggregate labor market variables in the data[18] from 1960 to 2007, together with the simulation results when the youth share varies around its empirical average (20 percent). As productivity shock is the main but not the only reason for labor market fluctuations, to make sure the comparability of the empirical and simulated

[17]As aggregate productivity shock follows an AR(1) process in the model: $ln\, A_t = \psi ln\, A_{t-1} + e_t$, it's easy to see that var $(ln\, A_t) = \frac{(\sigma^A)^2}{1-\psi^2}$.

[18]Unemployment rate is the seasonally adjusted unemployment rate, quarterly average of monthly data, and productivity is the real output per person of nonfarm business sector. Both data series are from St. Louis Fed. Vacancy rate is seasonally adjusted help-wanted advertising index from Conference Board.

moments, I identify the empirical business cycle component as the projection on current and lagged detrended output.

Table 6: Aggregate second-order moments in the data and the model

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	change	(4)	change
Unemployment rate	0.64	0.15	0.17	9%	0.21	24%
Job-finding rate	5.16	2.05	2.12	3%	2.28	7%
Separation rate	0.10	0.02	0.02	16%	0.04	83%
Market tightness	21.67	4.91	5.09	4%	5.50	8%
Productivity	1.30	1.30	1.30	-	1.30	-

Notes: Sample moments in the data are calculated from one-sided HP filtered data, 1960Q1-2007Q4. Those in the model are calculated from model-simulated data. For data in rate (first three rows), I filter on the rate itself; while for market tightness and productivity, I filter on the log form of the corresponding measures. Both empirical and simulated data series are in quarterly frequency, for which I use the standard smoothing parameter of 1600.

From the results we can see that all the simulated moments increase with the youth share, except that the SD of productivity is targeted at its empirical level. When the youth share increases from 18 to 20 percent, the simulated SD of unemployment rate increases 9 percent, from 0.15 to 0.17; when this share further increases from 20 to 22 percent, the simulated value for unemployment increases 24 percent, from 0.17 to 0.21. Similarly, the simulated SDs of job-find rate, separation rate and market tightness also increases with the youth share. The model predicts a positive relationship between aggregate labor market volatilities and the youth share. Meanwhile we see that the model underpredicts[19] labor market volatilities by comparing column (1) and (3), which have the same youth share.

5.2 Age-specific volatility across demographic states

This section assesses the capability of the model in generating the spillover effect.

Table 7 presents the SDs of age-specific labor market variables in the data and the simulation results when I vary the youth share from 18 to 22 percent. We can see that there is a clear pattern for the spillover effect. The simulated SD of the unemployment rate of the young increases from 0.22 to 0.31 as the youth share increases from 18 to 20 percent, and

[19]This underprediction is well known in the search and matching literature. For model with exogenous separation, see Shimer (2005); for model with endogenous separation, see Mortensen and Nagypal (2007) and Fujita and Ramey (2012).

further to 0.49 as the share increases to 22 percent. This means there is more than twenty percent increase on average in the unemployment volatility of the young with one percent increase of the youth share, clearly suggesting the existence of the spillover effect. As unemployment volatility can be attributed to variations in job-finding and separation rates, I also report the results for these two. The simulated SDs increase for both rates of the young as the youth share becomes higher. This is especially true for separation rate, whose volatility becomes about four times larger when the share increases from 18 to 20 percent and more than three times larger when it increases from 20 to 22 percent. Similar pattern is also found for the market tightness of the young, with relatively less increment.

Table 7: Age-specific second-order moments in the data and the model

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	change	(4)	change
Unemployment rate						
Young	1.03	0.22	0.31	41%	0.49	61%
Old	0.54	0.14	0.14	-4%	0.13	-3%
Job-finding rate						
Young	5.26	2.16	2.24	4%	2.26	1%
Old	4.06	2.02	2.02	0%	2.03	0%
Separation rate						
Young	0.15	0.01	0.03	391%	0.13	273%
Old	0.08	0.02	0.02	-14%	0.01	-13%
Market tightness						
Young	-	6.63	7.83	18%	8.96	14%
Old	-	4.62	4.55	-2%	4.51	-1%

Notes: Age-specific unemployment rate is from St. Louis Fed, 1960Q1-2007Q4, and age-specific transition rates are from CPS, 1976Q3 to 2007Q4. The young refers to workers from age 15 to 24 and the old covers from age 25 to 64. See Table 6 for further notes.

While for old workers, the volatilities of labor market variables decline slightly as the youth share increases, but overall remain stable. Since the increase of the youth share naturally means the decline of the share of the old, the volatilities of old workers should move into the opposite direction comparing to those of the young. As mentioned, the reason that they barely change lies in the complementarity between capital and experience as shown in the data. Besides, it is also worth noting that the simulated SD of the unemployment rate of the young is higher than that of the old over all demographic states.

To summarize, the model is capable of generating the spillover phenomenon as observed in the data. It predicts that there is approximately twenty percent increase in the unemployment volatility of the young with one percent more young workers in the labor force. If we use the first state (the youth share equals to 18%) to represent the period from 1985 to 2007 in the U.S., which has an average youth share of 17.1%, the second state (20%) to represent the period from 1960 to 1969, which has an average share of 20%, and the third state (22%) to represent the pre-great-moderation (1970-1984), which has a relatively higher youth share, the model can generate the hump-shaped trend of the unemployment volatility of the young over time as shown in Figure 1.

5.3 Dynamic response of labor market variables of the young

In order to highlight the mechanism for the spillover effect, Figure 3 plots the impulse response functions (IRFs) of key labor market variables of the young with respect to a positive aggregate productivity shock when the youth share varies, from 18 (dotted line) to 20 percent (solid line), and further to 22 percent (dotted, circle-hatched line). We can see that the responses of job-finding and separation rates of the young to shock both become stronger when there are more young workers in the labor force, which jointly contribute to the spillover effect.

The response of job-finding rate with respect to shock depends on the responses of market tightness and reservation productivity. For the former, a higher youth share is associated with a lower level of the price of goods by the young, in addition, we see that this price declines more steeply when there are more young workers (top-left panel). As the decline of price shrinks profits generated by the match between firm and young worker, the percentage increase in profits by hiring young workers is larger in case of a positive productivity shock. Therefore firm's vacancy posting become more sensitive to aggregate productivity when the youth share increases, so does the labor market tightness of the young. For the latter, we see that the response of the reservation productivity of the young remains almost the same when the youth share varies[20] (bottom-left panel). With the dominant role of the former, the volatility of job-finding rate of the young also increases with the youth share (top-right panel).

[20]This is true when we assume that the distribution of the match-specific productivities is concentrated.

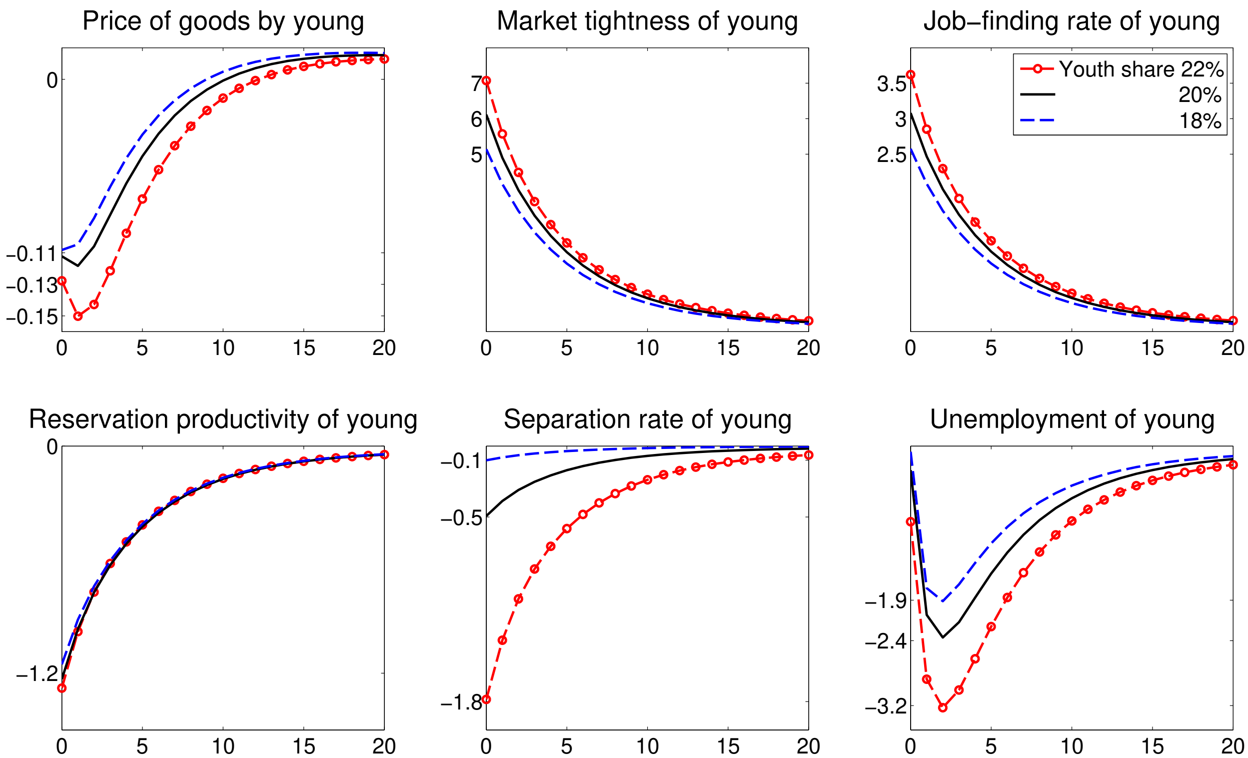

Figure 3: IRFs of variables of the young with respect to aggregate productivity shock across demographic states

To understand how the response of separation rate is affected by demographic change, it is necessary to look at the change of the size of marginal workers affected. As the response of the reservation productivity of the young barely changes when the youth share varies, its percentage change due to productivity shock is the same across all three demographic states. The difference lies in its level. A higher youth share means a higher reservation productivity of the young, thus the absolute change of this threshold is the most dramatic for the state with the highest youth share. Since a higher threshold is associated with a larger mass of marginal workers as pointed out by Pizzinelli and Zanetti (2017), there are more marginal young workers affected by aggregate shock when the youth share increases. Therefore, the volatility of separation rate of the young increases with the youth share (bottom-middle panel).

Therefore, both the volatilities of job-finding rate and job separation rate of the young increase with the youth share, and jointly contribute to the increase of the unemployment volatility of the young.

6 Introducing exogenous separation

In this section, I discuss the robustness of the former results in the case of exogenous job separation. The fact which necessitates this exercise is that the dynamics of unemployment rate in the U.S. comes mainly from the dynamics of job-finding rate, especially for the young. The purpose of this exercise is more about the generality of the spillover effect in different theoretical frameworks rather than the selection of suitable job matching model.

In Table 8, I carry out decompositions for each age group to quantify the contribution of job-finding and separation rates in accounting for age-specific unemployment volatility[21]. The results show that the fluctuation in the job-finding rate accounts for almost all the unemployment volatility for young workers. Even for other age groups, the role of job-finding rate is also dominant.

Table 8: Further decomposition of unemployment volatility

	15-24	25-54	55-64	15-64
Job-finding rate	**80.91**	68.80	84.97	73.36
Separation rate	**12.42**	26.21	15.86	19.38

Notes: Quarterly data from CPS, 1976Q3-2007Q4. All series are detrended by one-sided HP-filter with a smoothing parameter of 1600.

Bellman equations of the system with exogenous job separation are in the Appendix. If there is no aggregate uncertainty, it can be shown that the elasticity of market tightness of the young with respect to aggregate productivity is

$$e^Y_{\theta,A} = \frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{1-\beta(1-\rho^x)+\eta\beta(1-\rho^x)f^Y}{\mu[1-\beta(1-\rho^x)]+\eta\beta(1-\rho^x)f^Y}\frac{[\sigma+(1-\sigma)I^{Labor}]}{1-z/(P^Y A)},$$

where I^{Labor} is the income share of labor and is targeted at 0.725 in the calibration. Given the limited variation of the first term as discussed before, the change of this elasticity with the youth share is mainly through the price change triggered by demographic change. From Proposition 1, we see that the increase of the youth share decreases the price of goods produced by the young, thus leads to a higher effective unemployment benefit rate[22], $z/(P^Y A)$, which then implies more fluctuations in the labor market tightness of the young with respect to aggregate productivity shock.

Table 9 reports the second-order moments when job is exogenously separated. Similar as the former case, for the young, the simulated SD of the unemployment rate is higher when

[21] Following Shimer (2012) in the construction of worker flow rates with CPS data, I decompose the cyclical unemployment volatility of different age groups according to the contributions of job-finding and separation rates. Due to the effect of labor force participation, the numbers do not necessarily add up to 100.

[22] One necessary condition for the existence of match between firm and young worker is $P^Y A > z$, which ensures the non-negativeness of $1 - z/(P^Y A)$.

there are more young workers in the labor force; for the old, it is relatively stable, with a slight decline; in aggregate level, it also increases with the youth share. The dynamics of job-finding rate and market tightness also exhibit similar pattern. Notice that the values for unemployment volatilities (both aggregate and age-specific) are lower than those in the model with endogenous job separation[23], which is consistent with Fujita and Ramey (2012).

Table 9: The second-order moments in the data and the model (with exogenous separation)

	Data	Model				
Youth share	20%	18%	20%		22%	
	(1)	(2)	(3)	change	(4)	change
Unemployment rate						
Young	1.03	0.22	0.29	30%	0.38	33%
Old	0.54	0.13	0.13	-3%	0.12	-3%
Aggregate	0.64	0.15	0.16	8%	0.18	13%
Job-finding rate						
Young	5.26	2.30	2.40	4%	2.50	4%
Old	4.06	2.10	2.09	0%	2.09	0%
Market tightness						
Young	-	6.87	8.30	21%	10.22	23%
Old	-	4.70	4.61	-2%	4.52	-2%

Notes: Source of data is the same as that in Table 7. See Table 6 for the calculation method used for second-order moments.

Besides, I also plot the IRFs with respect to aggregate productivity shock when job is exogenously separated, which can be found in the appendix. The results are similar as the case with endogenous job separation. The price of goods produced by the young decreases more when the youth share increases, which shrinks the level of firm's profits, therefore firm's vacancy posting becomes more sensitive to aggregate productivity. The responses of the market tightness and job-finding rate of the young also become stronger when the youth share increases. As now the dynamics of unemployment is determined by the dynamics of job-finding rate, unemployment volatility of the young also increases.

To summarize, even without the dynamics of separation rate, the positive relationship between the volatility of job-finding rate of the young and the youth share still exists and leads to the spillover effect.

[23]The reason lies in the irresponsiveness of job separation rate to productivity shock in the model with exogenous job separation. With a positive shock, the value for a new match increases and firm posts more vacancies. This increases worker's job-finding rate and reduces unemployment. If separation rate is endogenous determined, firm lowers the threshold of individual productivity and therefore separation rate, which further decreases unemployment.

7 Empirical assessment of the mechanism

This section provides supporting evidence for the mechanism presented in this paper. The first one investigates the negative relationship between the price of goods by young and the youth share, and the second checks the positive relationship between the dynamics of transition rates and the youth share.

The first evidence is important for the aforementioned mechanism since the price change of intermediate goods connects demographics and labor market dynamics. In particular, the decline of the price of intermediate goods produced by the young when the youth share increases, as claimed in Proposition 1. Since this price level is not directly available in the data, I use the wage of the young as a proxy. From the wage equation of the young, we have

$$\omega_t^Y(A_t, a_t) = \eta(P_t^Y A_t a_t + \varsigma^Y \theta_t^Y) + (1-\eta) z^Y,$$

from which we see that the wage of the young is positively related to the price of goods by young. If the price decreases when there are more young workers, we should also see a decrease in the wage of the young. As the wage of the young also depends on aggregate and individual productivities and market tightness of the young, using annual data from the U.S., I regress the log form of wage of the young[24] on the youth share, the log form of real GDP per capita, and the job-finding rate of the young[25]. Although real GDP per capita is likely to be correlated with the error term, it is included for more efficient estimation of the coefficient of the youth share.

Table 10: The price of goods by the young and the share of the young

	Youth share	Young population share	Native young population share
Coefficient	-2.92***	-2.31***	-2.48***
	(0.92)	(0.75)	(0.78)
Nobs	32	32	32
R^2	0.98	0.98	0.98

*Notes: Dependent variable is the log form of the wage of young worker. Newey-West standard errors are in the parentheses. ***, **, * stand for a significance level of 1 percent, 5 percent and 10 percent respectively.*

The results are reported in Table 10. From the first column, we see that the youth share has a statistically significantly negative effect on the wage of the young. Besides, I use, respectively, the share of young population and the share of native young population as

[24] Here the wage of the young is wage per capita, which is not directly affected by the size of young workers.

[25] Since data of age-specific market tightness is not available, I use the job-finding rate of the young as proxy.

proxy for the youth share, in case of potential endogeneity due to labor force participation or migration. The results are similar, which support the mechanism in Proposition 1.

Table 11: The second-order moments of the transition rates of the young over time

Period	1976Q3-1986Q2	1986Q3-1996Q2	1996Q3-2006Q2
Youth share	23.3%	18.0%	15.9%
Job-finding rate (young)	7.59	3.98	3.86
Separation rate (young)	0.22	0.11	0.10

Table 11 shows the SDs of the transition rates of the young from 1976Q3 to 2006Q2. This period is characterized by a sharp and strict decrease in the youth share. It is then divided into three subperiods with a ten-year span for each. There is a 5.3 percent decrease of the young from the first subperiod to the second, and a 2.1% decrease from the second to the third. Similarly, the SDs of job-finding and separation rates of the young also have a decreasing trend over the whole period, with more decrease from the first subperiod than the second[26]. These results are consistent with the prediction of the model, which shows that the variations in the dynamics of job-finding and separation rates of the young, triggered by demographic change, jointly contribute to the spillover effect.

8 Conclusion

In this paper, I highlight the role of demographic structure of the workforce in shaping aggregate and age-specific business cycle, and provide a theoretical explanation to the positive relationship between the unemployment volatility of the young and the share of young workers in the labor force.

By incorporating a job matching model with endogenous separation into the RBC model featuring age-specific differences in labor demand, I show that the dynamics of transition rates of the young depends on the state of demographics, which are connected through the general equilibrium price of the intermediate goods of the young. Quantitative results show that the responses of job-finding and separation rates of the young to productivity shock become stronger as there are more young workers in the labor force, which jointly contribute to the spillover effect, i.e., the unemployment volatility within the group of young workers

[26]Comparing to the empirical counterparts, the variation of the simulated job-finding rate of the young is less obvious. The empirical results suggest a decrease of 13 percent in the SD of job-finding rate with one percent less young workers, while the model only predicts a corresponding value of one percent. The opposite is true for separation rate: the empirical results suggest a decrease of 16 percent, while the model predicts a value of 50 percent. This inconsistency in level is one direction for future research. Still it does not invalidate the mechanism of the spillover effect, since both the volatilities of job-finding rate and separation rate in the model share the same pattern as their empirical counterparts, that is they increase with the youth share.

increases with their share in the labor force, which has important implication for policy maker in the evaluation of age-specific labor market policy.

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Appendix

A1 - Participation's share in unemployment volatility (with different age groups categorization)

Table 12: Decomposition of Unemployment Volatility: Participation's Share

	Female			Male			Both		
	15-29	30-64	15-64	15-29	30-64	15-64	**15-29**	**30-64**	**15-64**
Cov. excl.	17.66	14.63	11.61	2.18	0.31	0.47	**5.96**	**2.02**	**1.89**
Cov. incl.	29.88	24.65	22.96	11.28	3.09	4.84	**18.28**	**7.77**	**9.70**

Notes: Robustness check for Table 1. See Table 1 for further notes.

A2 - Proof of Proposition 1.

Taking partial derivative of $P_t^Y = (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-1}$ with respect to S_t^Y gives

$$\begin{aligned}\frac{\partial P_t^Y}{\partial S_t^Y} &= (1-\sigma)\Big[(Y_t)^{-\sigma}\lambda_1(Y_t^Y)^{\sigma-1}\frac{\partial Y_t}{\partial S_t^Y} - (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-2}\frac{\partial Y_t^Y}{\partial S_t^Y}\Big] \\ &= (1-\sigma)P_t^Y(\frac{1}{Y_t}\frac{\partial Y_t}{\partial S_t^Y} - \frac{1}{Y_t^Y}\frac{\partial Y_t^Y}{\partial S_t^Y}),\end{aligned}$$

from which we see that the effect of demographic change on the price of output produced by young workers is negatively related to its effect on output produced by young workers and positively related to its effect on final goods.

As the output produced by young workers can be written as

$$Y_t^Y = A_t n_t^Y S_t^Y E(a/a \geqslant \tilde{a}_t^Y),$$

we have

$$\frac{\partial Y_t^Y}{\partial S_t^Y} = A_t\Big(n_t^Y E(a/a \geqslant \tilde{a}_t^Y) + S_t^Y E(a/a \geqslant \tilde{a}_t^Y)\frac{\partial n_t^Y}{\partial S_t^Y} + n_t^Y S_t^Y \frac{\partial E(a/a \geqslant \tilde{a}_t^Y)}{\partial S_t^Y}\Big).$$

Besides, we have

$$\frac{\partial n_t^Y}{\partial S_t^Y} = -\frac{n_t^Y F'(a)}{1-F(\tilde{a}^Y)}\frac{\partial \tilde{a}_t^Y}{\partial S_t^Y}$$

and[27]

$$\frac{\partial E(a/a \geqslant \widetilde{a}_t^Y)}{\partial S_t^Y} = E(a/a \geqslant \widetilde{a}_t^Y)\frac{F'(a)}{1-F(\widetilde{a}^Y)}\frac{\partial \widetilde{a}_t^Y}{\partial S_t^Y},$$

thus the equation for $\frac{\partial Y_t^Y}{\partial S_t^Y}$ can be simplified as

$$\frac{\partial Y_t^Y}{\partial S_t^Y} = A_t n_t^Y E(a/a \geqslant \widetilde{a}_t^Y).$$

For $\frac{\partial Y_t}{\partial S_t^Y}$, as capital is also in inelastic supply in the short-run, we have

$$\begin{aligned}\frac{\partial Y_t}{\partial S_t^Y} &= (Y_t)^{1-\sigma}[\lambda_1(Y_t^Y)^{\sigma-1}A_t n_t^Y E(a/a \geqslant \widetilde{a}_t^Y) - (1-\lambda_1)\Omega_t(1-\lambda_2)(Y_t^O)^{\rho-1}A_t n_t^O E(a/a \geqslant \widetilde{a}_t^O)] \\ &= P_t^Y A_t n_t^Y E(a/a \geqslant \widetilde{a}_t^Y) - P_t^O A_t n_t^O E(a/a \geqslant \widetilde{a}_t^O).\end{aligned}$$

Plugging the equations for $\frac{\partial Y_t^Y}{\partial S_t^Y}$ and $\frac{\partial Y_t}{\partial S_t^Y}$ back into the equation of $\frac{\partial P_t^Y}{\partial S_t^Y}$, one gets

$$\frac{\partial P_t^Y}{\partial S_t^Y} = (1-\sigma)P_t^Y A_t\Big[E(a/a \geqslant \widetilde{a}_t^Y)n_t^Y(\frac{1}{Y_t}P_t^Y - \frac{1}{Y_t^Y}) - \frac{1}{Y_t}P_t^O n_t^O E(a/a \geqslant \widetilde{a}_t^O)\Big].$$

From the optimization of final firm's problem, it's easy to see that $Y_t > P_t^Y Y_t^Y$, which leads to $\frac{1}{Y_t}P_t^Y A_t n_t^Y - \frac{1}{Y_t^Y}A_t n_t^Y = A_t n_t^Y(\frac{1}{Y_t}P_t^Y - \frac{1}{Y_t^Y}) < 0$, therefore we have $\frac{\partial P_t^Y}{\partial S_t^Y} < 0$ since $\sigma \in (0,1)$, . ■

A3 - Proof of Corollary 1.

In the following, I try to identify the sign of $\frac{\partial \widetilde{a}^Y}{\partial S_t^Y}$. With free entry condition and wage equation, the asset value of a job for the young can be written as

$$J_t^Y(A_t,\, a_t) = (1-\eta)(P_t^Y A_t a_t - z) - \eta\varsigma^Y\theta_t^Y + \frac{\varsigma^Y}{q_t^Y}.$$

Besides from $J_t^Y(A_t,\, \widetilde{a}_t{}^Y) = 0$, we can get the reservation productivity for the young explicitly as

[27] For the derivative of the conditional expectation, I used the fact that

$$\int_{\widetilde{a}^Y}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^Y)} = \widetilde{a}^Y + \frac{\int_{\widetilde{a}^Y}^{\infty}[1-F(\widetilde{a}^Y)]da}{1-F(\widetilde{a}^Y)}$$

is true for any function F such that $F(\infty) = 1$ and the Leibniz integral rule.

$$\tilde{a}_t^Y = \frac{1}{P_t^Y A_t}[z + \frac{1}{1-\eta}(\eta\varsigma^Y\theta_t^Y - \frac{\varsigma^Y}{q_t^Y})].$$

Taking derivative with respect to S_t^Y yields

$$\frac{\partial \tilde{a}_t^Y}{\partial S_t^Y} = -\frac{\tilde{a}_t^Y}{P_t^Y}\frac{\partial P_t^Y}{\partial S_t^Y} + \frac{\varsigma^Y}{P_t^Y A_t}\frac{1}{1-\eta}(\eta - \frac{\mu}{f_t^Y})\frac{\partial \theta_t^Y}{\partial S_t^Y}.$$

which suggests the effect of the youth share on the reservation productivity of the young depends on not only its effect on the price of intermediate goods produced by young workers, but also its effect on the market tightness of young labor market. It is necessary to express the second effect with known expressions. After abstracting from aggregate productivity shock, the asset equation for young can be written as

$$S^{MY}(A,\, a) = P^Y Aa - z + \beta\,(1-\rho^x)(1-f^Y\eta)\int_{\tilde{a}^Y}^{\infty} S^{MY}(A,\, a)dF(a),$$

and free entry condition as

$$\frac{\varsigma^Y}{q^Y} = \beta\,(1-\rho^x)(1-\eta)\int_{\tilde{a}^Y}^{\infty} S^{MY}(A,\, a)dF(a).$$

As $S^{MY}(A,\, \tilde{a}^Y) = 0$, subtracting $S^{MY}(A,\, \tilde{a}^Y)$ from $S^{MY}(A,\, a)$, we get

$$S^{MY}(A,\, a) = P^Y A(a - \tilde{a}^Y),$$

then free entry condition becomes

$$\frac{\varsigma^Y}{q^Y} = \beta\,(1-\rho^x)(1-\eta)AP^Y\int_{\tilde{a}^Y}^{\infty}(a - \tilde{a}^Y)dF(a).$$

Taking derivative with respect to S_t^Y yields

$$\frac{\partial \theta^Y}{\partial S_t^Y} = \frac{f^Y\beta(1-\rho^x)(1-\eta)A}{\varsigma^Y\mu}[(E(a/a>\tilde{a}^Y) - (1-F(\tilde{a}^Y))\tilde{a}^Y)\frac{\partial P^Y}{\partial S_t^Y} - (1-F(\tilde{a}^Y))P^Y\frac{\partial \tilde{a}^Y}{\partial S_t^Y}].$$

Combining the above two equations with partial derivatives gives

$$\frac{\partial \tilde{a}^Y}{\partial S_t^Y} = \frac{\beta(1-\rho^x)\Big[E(a/a>\tilde{a}^Y) - \Big(1-F(\tilde{a}^Y)\Big)\tilde{a}^Y\Big] + \tilde{a}^Y/(1-f^Y)}{P^Y M}\frac{\partial P^Y}{\partial S_Y},$$

where $M = -1/(1-f^Y) + \beta(1-\rho^x)\Big(1-F(\tilde{a}^Y)\Big)$.

As the probability of a young worker being matched is lower than 1, we can get that $1/(1-f^Y) > 1$ and the second part of M is smaller than 1, thus it is easy to see that $M < 0$. Besides, the conditional expectation of idiosyncratic productivity is higher than the reservation productivity, thus, $E(a/a > \widetilde{a}^Y) - (1 - F(\widetilde{a}^Y))\widetilde{a}^Y > 0$. Together with the result from Proposition 1, which suggests that higher supply of young labor will drive down the price of intermediate goods produced by young, $\frac{\partial P^Y}{\partial S_Y} < 0$, we can conclude that the youth share has a positive effect on the reservation productivity of the young.■

A4 - Derivation of the equations for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ and $\frac{\partial \ln u^Y}{\partial \ln A}$ in case of endogenous job separation

Part 1: $\frac{\partial \ln \theta^Y}{\partial \ln A}$

In steady state, the free entry condition becomes[28]

$$\frac{\varsigma^i}{q^i} = \beta\,(1-\rho^x)(1-\eta)\int_{\widetilde{a}^i}^{\infty} S^{Mi}(A,\,a)dF(a) = \beta\,(1-\rho^x)(1-\eta)P^iA\int_{\widetilde{a}^i}^{\infty}[1-F(a)]da. \tag{1}$$

Evaluating the equation for match surplus at $\widetilde{a}^i$ gives the condition for reservation productivity

$$P^iA(\widetilde{a}^i + \beta\,(1-\rho^x)\int_{\widetilde{a}^i}^{\infty}[1-F(a)]da) = z^i + \frac{\eta\theta^i\varsigma^i}{1-\eta}. \tag{2}$$

Equations (1) and (2) uniquely determine the steady values of market tightness, θ^i, and reservation productivity, $\widetilde{a}^i$. With these two equations, I can derive the comparative statics for the responsiveness of market tightness with respect to aggregate productivity.

From equation (1), we have

$$\frac{\varsigma^Y}{q^Y} = \beta\,(1-\rho^x)(1-\eta)P^YA\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da. \tag{3}$$

Taking log form and totally differentiating with respect to $\ln A$ gives

$$\frac{\partial \ln \theta^Y}{\partial \ln A}\mu = 1 + \frac{\partial \ln P^Y}{\partial \ln A} - \frac{[1-F(\widetilde{a}^Y)]\widetilde{a}^Y}{\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}\frac{\partial \ln \widetilde{a}^Y}{\partial \ln A}. \tag{4}$$

From equation (2), we have

$$P^YA\{\widetilde{a}^Y + \beta\,(1-\rho^x)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da\} = z + \frac{\eta\theta^Y\varsigma^Y}{1-\eta}. \tag{5}$$

Taking log form and totally differentiating with respect to $\ln A$ gives

[28] The second equality follows from integration by parts for any function such that $F(\infty) = 0$ and the condition $\frac{\partial S^i_t(A_t,\,a_t)}{\partial a_t} = P^i_tA_t$.

$$
\begin{aligned}
&(1+\tfrac{\partial \ln P^Y}{\partial \ln A})\{\widetilde{a}^Y+\beta\,(1-\rho^x)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da\}+\\
&\tfrac{\partial \ln \widetilde{a}^Y}{\partial \ln A}\widetilde{a}^Y\{1-\beta(1-\rho^x)[1-F(\widetilde{a}^Y)]\}=\tfrac{\partial \ln \theta^Y}{\partial \ln A}\tfrac{\eta\theta^Y\varsigma^Y}{P^Y A(1-\eta)}.
\end{aligned}
\tag{6}
$$

Combining equation (3) and (4) to substitute out $\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da$, and the same for equation (5) and (6), then solving for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ and get

$$
\frac{\partial \ln \theta^Y}{\partial \ln A}=(\frac{\partial \ln P^Y}{\partial \ln A}+1)\frac{\frac{z(1-\eta)\beta(1-\rho^x)\widetilde{f}^Y}{\varsigma^Y\theta^Y}+1-\beta(1-\widetilde{\rho}^Y)+\eta\beta(1-\rho^x)\widetilde{f}^Y}{\mu[1-\beta(1-\widetilde{\rho}^Y)]+\eta\beta(1-\rho^x)\widetilde{f}^Y}.
$$

Denote $\tilde{z}^Y\equiv z/(P^Y A\int_{\widetilde{a}^Y}^{\infty}a\frac{dF(a)}{1-F(\widetilde{a}^Y)})$ and $\widetilde{f}^Y\equiv f^Y[1-F(\widetilde{a}^Y)]$, and use the fact that[29] $\int_{\widetilde{a}^Y}^{\infty}a\frac{dF(a)}{1-F(\widetilde{a}^Y)}=\widetilde{a}^Y+\frac{\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}{1-F(\widetilde{a}^Y)}$, then $\tilde{z}^Y$ becomes

$$
\tilde{z}^Y=\frac{z}{\widetilde{a}^Y+\frac{F(\widetilde{a}^Y)\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}{1-F(\widetilde{a}^Y)}+\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da}.
$$

Using equation (3) to substitute out the first integral in the denominator and equation (5) to substitute out the second integral, then get

$$
\tilde{z}^Y=\frac{z\beta(1-\rho^x)\widetilde{f}^Y(1-\eta)}{P^Y A\widetilde{a}^Y\widetilde{f}^Y(1-\eta)[\beta(1-\rho^x)-1]+\theta^Y\varsigma^Y-(\widetilde{f}^Y\eta-[1-F(\widetilde{a}^Y)])\theta^Y\varsigma^Y+z\widetilde{f}^Y(1-\eta)}.
$$

Combining equation (3) and (5) to get

$$
(\widetilde{f}^Y\eta-[1-F(\widetilde{a}^Y)])\theta^Y\varsigma^Y=(P^Y A\widetilde{a}^Y-z)(1-\eta)\widetilde{f}^Y.
$$

Substituting this term into the expression of $\tilde{z}^Y$ to get

$$
\frac{z(1-\eta)\beta(1-\rho^x)\widetilde{f}^Y}{\varsigma^Y\theta^Y}=\frac{\tilde{z}^Y\beta(1-\rho^x)(\widetilde{f}^Y\eta-[1-F(\widetilde{a}^Y)])+\tilde{z}^Y}{1-\tilde{z}^Y}.
$$

After the substitution of this term into the expression of $\frac{\partial \ln \theta^Y}{\partial \ln A}$, it can be simplified as

$$
\frac{\partial \ln \theta^Y}{\partial \ln A}=\frac{1-\beta(1-\widetilde{\rho}^Y)+\eta\beta(1-\rho^x)\widetilde{f}^Y}{\mu[1-\beta(1-\widetilde{\rho}^Y)]+\eta\beta(1-\rho^x)\widetilde{f}^Y}\frac{\frac{\partial \ln P^Y}{\partial \ln A}+1}{1-\tilde{z}^Y}.
$$

[29]Which can be obtained from the integral of $\int_{\widetilde{a}^Y}^{\infty}[1-F(a)]da$ by parts, actually the upper bound is limited here as it follows a lognormal distribution.

Part 2: $\frac{\partial \ln u^Y}{\partial \ln A}$

Notice that the elasticity of the overall job-finding rate with respect to aggregate productivity can be written as

$$\begin{aligned}\frac{\partial \ln \tilde{f}^Y}{\partial \ln A} &= \frac{\partial \ln ([1-F(\tilde{a}^Y)]f^Y)}{\partial \ln A} \\ &= (1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A} - \frac{f(\tilde{a}^Y)\tilde{a}^Y}{1-F(\tilde{a}^Y)}\frac{\partial \ln \tilde{a}^Y}{\partial \ln A},\end{aligned}$$

and the elasticity of the overall separation rate with respect to aggregate productivity can be written as

$$\begin{aligned}\frac{\partial \ln \tilde{\rho}^Y}{\partial \ln A} &= \frac{\partial \ln [\rho^x + (1-\rho^x)F(\tilde{a}^Y)]}{\partial \ln A} \\ &= \frac{(1-\rho^x)f(\tilde{a}^Y)\tilde{a}^Y}{\rho^x + (1-\rho^x)F(\tilde{a}^Y)}\frac{\partial \ln \tilde{a}^Y}{\partial \ln A}.\end{aligned}$$

With endogenous separation, employment rate at each segmented labor market evolves according to

$$n_t^Y = (1-\tilde{\rho}^Y)(n_{t-1}^Y + \tilde{f}_{t-1}^Y u_{t-1}^Y).$$

Therefore the steady state value for unemployment rate is

$$u^Y = \frac{\tilde{\rho}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}.$$

Thus the elasticity of unemployment with respect to aggregate productivity is

$$\begin{aligned}\frac{\partial \ln u^Y}{\partial \ln A} &= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\frac{\partial \ln \tilde{\rho}^Y}{\partial \ln A} - \frac{(1-\tilde{\rho}^Y)\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\frac{\partial \ln \tilde{f}^Y}{\partial \ln A} \\ &= \frac{\tilde{f}^Y}{\tilde{\rho}^Y + (1-\tilde{\rho}^Y)\tilde{f}^Y}\{f(\tilde{a}^Y)\tilde{a}^Y[(1-\rho^x)(1+\frac{1}{\tilde{\rho}^Y})]\frac{\partial \ln \tilde{a}^Y}{\partial \ln A} \\ &\quad -(1-\tilde{\rho}^Y)(1-\mu)\frac{\partial \ln \theta^Y}{\partial \ln A}\}.\blacksquare\end{aligned}$$

A5 - Bellman equations for model with exogenous job separation

The Bellman equation for the asset value of a job in term of final good is

$$J_t^i = P_t^i A_t - \omega_t^i + (1-\rho^x)E_t\beta_{t,t+1}J_{t+1}^i.$$

The value of a vacancy is

$$V_t^i = -\varsigma^i + E_t \beta_{t,\,t+1}[q_t^i(1-\rho^x)J_{t+1}^i + (1-q_t^i)V_{t+1}^i].$$

For workers, the asset value of being matched is

$$W_t^i = \omega_t^i + E_t \beta_{t,\,t+1}[(1-\rho^x)W_{t+1}^i + \rho U_{t+1}^i].$$

The expected value of being unemployed is

$$U_t^i = z + E_t \beta_{t,\,t+1}[f_t^i(1-\rho^x)W_{t+1}^i + (1-f_t^i(1-\rho^x))U_{t+1}^i].$$

A6 - Derivation of the equation for $\frac{\partial \ln \theta^Y}{\partial \ln A}$ in case of exogenous job separation

In case of no aggregate uncertainty ($A_t = A$), we can solve for the asset value of a job,

$$J^i = \frac{(1-\eta)(P^i A - z) - \eta \varsigma^i \theta^i}{1 - \beta(1-\rho^x)}.$$

Plugging this into the free entry condition, we can get a implicit function for market tightness θ^i

$$\frac{\varsigma^i}{\beta(1-\rho^x)q^i} = \frac{(1-\eta)(P^i A - z) - \eta \varsigma^i \theta^i}{1 - \beta(1-\rho^x)}.$$

Denote aggregate productivity $\alpha \equiv [1-\beta(1-\rho^x)]/[\beta(1-\rho^x)]$ and rearrange the equation,

$$\alpha \frac{\varsigma^i}{q^i} + \eta \varsigma^i \theta^i = (1-\eta)(P^i A - z).$$

Totally differentiate the above equation,

$$-\alpha \varsigma^i \frac{[q^i(\theta^i)]'}{[q^i(\theta^i)]^2} d\theta^i + \eta \varsigma^i d\theta^i = (1-\eta) d(P^i A),$$

then, we can get

$$\begin{aligned} e^Y_{\theta,A} &= \frac{\partial \theta^Y}{\partial A}\frac{A}{\theta^Y} \\ &= \frac{\partial \theta^Y}{\partial (P^Y A - z)}\frac{P^Y A - z}{\theta^Y}\frac{\partial (P^Y A - z)}{\partial A}\frac{A}{P^Y A - z} \\ &= \frac{1-\eta}{\eta\varsigma^Y - \alpha\varsigma^Y\frac{[q^Y(\theta^Y)]'}{[q^Y(\theta^Y)]^2}}\frac{\alpha\frac{\varsigma^Y}{q^Y\theta^Y} + \eta\varsigma^Y}{1-\eta}\frac{\partial \ln (P^Y A - z)}{\partial \ln A} \\ &= \frac{\alpha + \eta f^Y(\theta^Y)}{\alpha\mu + \eta f^Y(\theta^Y)}\frac{\partial \ln (P^Y A - z)}{\partial \ln A}. \end{aligned}$$

For the second term, we can get that $\frac{\partial \ln (P^Y A - z)}{\partial \ln A} = \frac{\partial (P^Y A - z)}{\partial A}\frac{A}{P^Y A - z} = (A\frac{\partial P^Y}{\partial A} + P^Y)\frac{A}{P^Y A - z}$. Using similar steps as the derivation of $\frac{\partial P^Y}{\partial S^Y}$ in Proposition 1, one can get

$$\begin{aligned} A\frac{\partial P^Y}{\partial A} + P^Y &= (1-\sigma)P^Y(\frac{1}{Y}P^Y A n^Y S^Y + \frac{1}{Y}P^O A n^O(1-S^Y) - \frac{1}{Y^Y}A n^Y S^Y) + P^Y \\ &= P^Y[\sigma + \frac{(1-\sigma)}{Y}(P^Y A n^Y S^Y + P^O A n^O(1-S^Y))]. \end{aligned}$$

Denote $I^{Labor} \equiv \frac{1}{Y}(P^Y A n^Y S^Y + P^O A n^O(1-S^Y))$, which is simply the share of labor in final good and is targeted at the national income share of labor, we have

$$\frac{\partial \ln (P^Y A - z)}{\partial \ln A} = \frac{[\sigma + (1-\sigma)I^{Labor}]}{1 - z/(P^Y A)},$$

which gives

$$e^Y_{\theta,A} = \frac{\partial \ln \theta^Y}{\partial \ln A} = \frac{\alpha + \eta f^Y(\theta^Y)}{\alpha\mu + \eta f^Y(\theta^Y)}\frac{[\sigma + (1-\sigma)I^{Labor}]}{1 - z/(P^Y A)}. \blacksquare$$

A7 - (Linearized) Equation System for model with endogenous separation

1. Euler equation:

$$c_t^{-\tau} = \beta E_t[c_{t+1}^{-\tau}(r_{t+1} + 1 - \delta)]$$

$$0 = E_t[\tau(\hat{c}_t - \hat{c}_{t+1}) + \frac{r}{r + 1 - \delta}\hat{r}_{t+1}]$$

2. Output of intermediate goods:

$$Y_t^Y = A_t n_t^Y S^Y \int_{\tilde{a}_t^Y}^{\infty} a\frac{dF(a)}{1 - F(\tilde{a}_t^Y)}$$

$$Y_t^O = A_t n_t^O (1-S^Y)\int_{\widetilde{a}_t^O}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}_t^O)}$$

$$\hat{Y}_t^i = \hat{A}_t + \hat{n}_t^i + \frac{H'(\widetilde{a}^i)\widetilde{a}^i}{H(\widetilde{a}^i)}\hat{\widetilde{a}}_t^i$$

where $H(\widetilde{a}^i) \equiv \int_{\widetilde{a}^i}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^i)}$ and $H'(\widetilde{a}^i) = \frac{dH(\widetilde{a}^i)}{d\widetilde{a}^i} = \frac{f(\widetilde{a}^i)[H(\widetilde{a}^i)-\widetilde{a}^i]}{1-F(\widetilde{a}^i)}$[30]

3. Output of final good:

$$Y_t = [\lambda_1(Y_t^Y)^\sigma + (1-\lambda_1)(\lambda_2(K_t)^\rho + (1-\lambda_2)(Y_t^O)^\rho)^{\sigma/\rho}]^{1/\sigma}$$

$$\hat{Y}_t = \lambda_1(\frac{Y^Y}{Y})^\sigma \hat{Y}_t^Y + \frac{1-\lambda_1(Y^Y/Y)^\sigma}{\lambda_2(K)^\rho + (1-\lambda_2)(Y^O)^\rho}(\lambda_2 K^\rho \hat{K}_t + (1-\lambda_2)(Y^O)^\rho \hat{Y}_t^O)$$

4. Price of intermediate goods produced by young:

$$P_t^Y = (Y_t)^{1-\sigma}\lambda_1(Y_t^Y)^{\sigma-1}$$

$$\hat{P}_t^Y = (1-\sigma)\hat{Y}_t + (\sigma-1)\hat{Y}_t^Y$$

5. Price of intermediate goods produced by old:

$$P_t^O = (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t(1-\lambda_2)(Y_t^O)^{\rho-1}$$

$$\hat{P}_t^O = (1-\sigma)\hat{Y}_t + \hat{\Omega}_t + (\rho-1)\hat{Y}_t^O$$

where $\hat{\Omega}_t = \frac{\sigma-\rho}{\lambda_2(K)^\rho+(1-\lambda_2)(Y^O)^\rho}(\lambda_2 K^\rho \hat{K}_t + (1-\lambda_2)(Y^O)^\rho \hat{Y}_t^O)$.

6. Price of capital:

$$r_t = (Y_t)^{1-\sigma}(1-\lambda_1)\Omega_t\lambda_2(K_t)^{\rho-1}$$

$$r_t = (1-\sigma)\hat{Y}_t + \hat{\Omega}_t + (\rho-1)\hat{K}_t$$

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[30] As $H(\widetilde{a}^i) \equiv \int_{\widetilde{a}^i}^{\infty} a\frac{dF(a)}{1-F(\widetilde{a}^i)}$, $H'(\widetilde{a}^i) = \frac{d\int_{\widetilde{a}^i}^{\infty} a\,dF(a)/d\,\widetilde{a}^i}{1-F(\widetilde{a}^i)} - \frac{\int_{\widetilde{a}^i}^{\infty} a\,dF(a)}{(1-F(\widetilde{a}^i))^2}(-f(\widetilde{a}^i))$. With Leibniz integral rule, it's easy to see that:

$$d\int_{\widetilde{a}^i}^{\infty} a\,dF(a)/d\,\widetilde{a}^i = -f(\widetilde{a}^i)\widetilde{a}^i$$

therefore,

$$\begin{aligned} H'(\widetilde{a}^i) &= \frac{-f(\widetilde{a}^i)\widetilde{a}^i}{1-F(\widetilde{a}^i)} + \frac{f(\widetilde{a}^i)H(\widetilde{a}^i)}{1-F(\widetilde{a}^i)} \\ &= \frac{f(\widetilde{a}^i)[H(\widetilde{a}^i)-\widetilde{a}^i]}{1-F(\widetilde{a}^i)} \end{aligned}$$

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7. Aggregate resource constraint:

$$c_t + K_{t+1} = Y_t + (1-\delta)K_t - v_t^Y S^Y \varsigma^Y - v_t^O(1-S^Y)\varsigma^O$$

$$c\hat{c}_t + K\hat{K}_{t+1} = Y\hat{Y}_t + (1-\delta)K\hat{K}_t - S^Y\varsigma^Y v^Y \hat{v}_t^Y - (1-S^Y)\varsigma^O v^O \hat{v}_t^O$$

8. Evolution of employment rate:

$$n_t^i = (1-\rho_t^i)(n_{t-1}^i + \phi^i (v_{t-1}^i)^{1-\mu}(u_{t-1}^i)^{\mu})$$

$$\hat{n}_t^i + \frac{\rho^i}{1-\rho^i}\hat{\rho}_t^i = (1-\rho^i)\hat{n}_{t-1}^i + \rho^i(1-\mu)\hat{v}_{t-1}^i + \rho^i\mu\hat{u}_{t-1}^i)$$

9. Evolution of unemployment rate:

$$u_t^i = 1 - n_t^i$$

$$u^i\hat{u}_t^i = -n^i\hat{n}_t^i$$

10. Labor market tightness:

$$\theta_t^i = v_t^i / u_t^i$$

$$\hat{\theta}_t^i = \hat{v}_t^i - \hat{u}_t^i$$

11. Overall separation rate:

$$\rho_t^i = \rho^x + (1-\rho^x)F(\widetilde{a}_t^i)$$

$$\hat{\rho}_t^i = \frac{(1-\rho^x)\widetilde{a}^i f(\widetilde{a}^i)}{\rho^i}\hat{\widetilde{a}}_t^i$$

12. Overall job-finding rate:

$$fr_t^i = (1-\rho^x)[1-F(\widetilde{a}_{t+1}^i)]\phi^i\theta_t^{i\,1-\mu}$$

$$\hat{fr}_t^i = -\frac{\widetilde{a}^i f(\widetilde{a}^i)}{1-F(\widetilde{a}^i)}\hat{\widetilde{a}}_{t+1}^i + (1-\mu)\hat{\theta}_t^i$$

13. Reservation productivity:

$$\widetilde{a}_t^i = \frac{1}{P_t^i A_t}[z + \frac{\varsigma^i}{1-\eta}(\eta\theta_t^i - (\theta_t^i)^{\mu}/\phi^i)]$$

$$\hat{\widetilde{a}}_t^i = -\hat{P}_t^i - \hat{A}_t + \frac{\frac{\varsigma^i}{1-\eta}(\eta\theta^i - \mu(\theta_t^i)^{\mu}/\phi^i)}{z + \frac{\varsigma^i}{1-\eta}(\eta\theta^i - (\theta^i)^{\mu}/\phi^i)}\hat{\theta}_t^i$$

14. Free entry condition in differential equation:

$$\frac{\varsigma^i(\theta_t^i)^\mu}{\phi^i} = \beta E_t\{\frac{c_{t+1}^{-\tau}}{c_t^{-\tau}}(1-\rho_{t+1}^i)[(1-\eta)P_{t+1}^i A_{t+1} H(\tilde{a}_{t+1}^i) - \varsigma^i \eta \theta_{t+1}^i - (1-\eta)z + \frac{\varsigma^i(\theta_{t+1}^i)^\mu}{\phi^i}]\}$$

$$\begin{aligned} \mu\hat{\theta}_t^i - \tau\hat{c}_t &= -\tau E_t\hat{c}_{t+1} - \frac{\rho^i}{1-\rho^i}E_t\hat{\rho}_{t+1}^i + \frac{\varsigma^i(\mu(\theta^i)^\mu/\phi^i - \eta\theta^i)E_t\hat{\theta}_{t+1}^i}{B} \\ &+ \frac{(1-\eta)P^i A[H(\tilde{a}^i)(E_t\hat{P}_{t+1}^i + E_t\hat{A}_{t+1}) + H'(\tilde{a}^i)\tilde{a}^i E_t\hat{\tilde{a}}_{t+1}^i]}{B} \end{aligned}$$

where $B = (1-\eta)P^i A H(\tilde{a}^i) - \varsigma^i\eta\theta^i - (1-\eta)z + \varsigma^i(\theta^i)^\mu/\phi^i$.

15. Aggregate productivity shock:

$$ln\, A_t = \psi^A ln\, A_{t-1} + e_t^A$$

$$\hat{A}_t = \psi^A \hat{A}_{t-1} + e_t^A$$

16. Aggregate unemployment rate:

$$u_t = u_t^O(1-S^Y) + u_t^Y S^Y$$

$$u\hat{u}_t = u^O(1-S^Y)\hat{u}_t^O + u_t^Y S^Y \hat{u}_t^Y$$

17. Aggregate vacancies:

$$v_t = v_t^O(1-S^Y) + v_t^Y S^Y$$

$$v\hat{v}_t = v^O(1-S^Y)\hat{v}_t^O + v_t^Y S^Y \hat{v}_t^Y$$

18. Aggregate market tightness:

$$\theta_t = v_t/u_t$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t$$

19. Aggregate job-finding rate:

$$fr_t = [u_t^O(1-S^Y)fr_t^O + u_t^Y S^Y fr_t^Y]/u_t$$

$$\hat{fr}_t = \frac{u_t^O(1-S^Y)fr_t^O(\hat{fr}_t^O + \hat{u}_t^O) + u_t^Y S^Y fr_t^Y(\hat{fr}_t^Y + \hat{u}_t^Y)}{u_t^O(1-S^Y)fr_t^O + u_t^Y S^Y fr_t^Y} - \hat{u}_t$$

20. Aggregate separation rate[31]:

$$\rho_t = 1 - (1 - u_t)/[(1 - S^Y)(n^O_{t-1} + f^O_{t-1}u^O_{t-1}) + S^Y(n^Y_{t-1} + f^Y_{t-1}u^Y_{t-1})]$$

$$-\frac{\tilde{\rho}}{1-\tilde{\rho}}\hat{\rho}^i_t = -\frac{u}{1-u}\hat{u}_t - \frac{(1-S^Y)[n^O\hat{n}^O_{t-1} + \tilde{f}^O u^O((1-\mu)\hat{\theta}^O_{t-1} + \hat{u}^O_{t-1})]}{(1-S^Y)(n^O + f^O u^O) + S^Y(n^Y + f^Y u^Y)} - \frac{S^Y[n^Y\hat{n}^Y_{t-1} + f^Y u^Y((1-\mu)\hat{\theta}^Y_{t-1} + \hat{u}^Y_{t-1})]}{(1-S^Y)(n^O + f^O u^O) + S^Y(n^Y + f^Y u^Y)}$$

A8 - IRFs with respect to an aggregate productivity shock (with exogenous job separation)

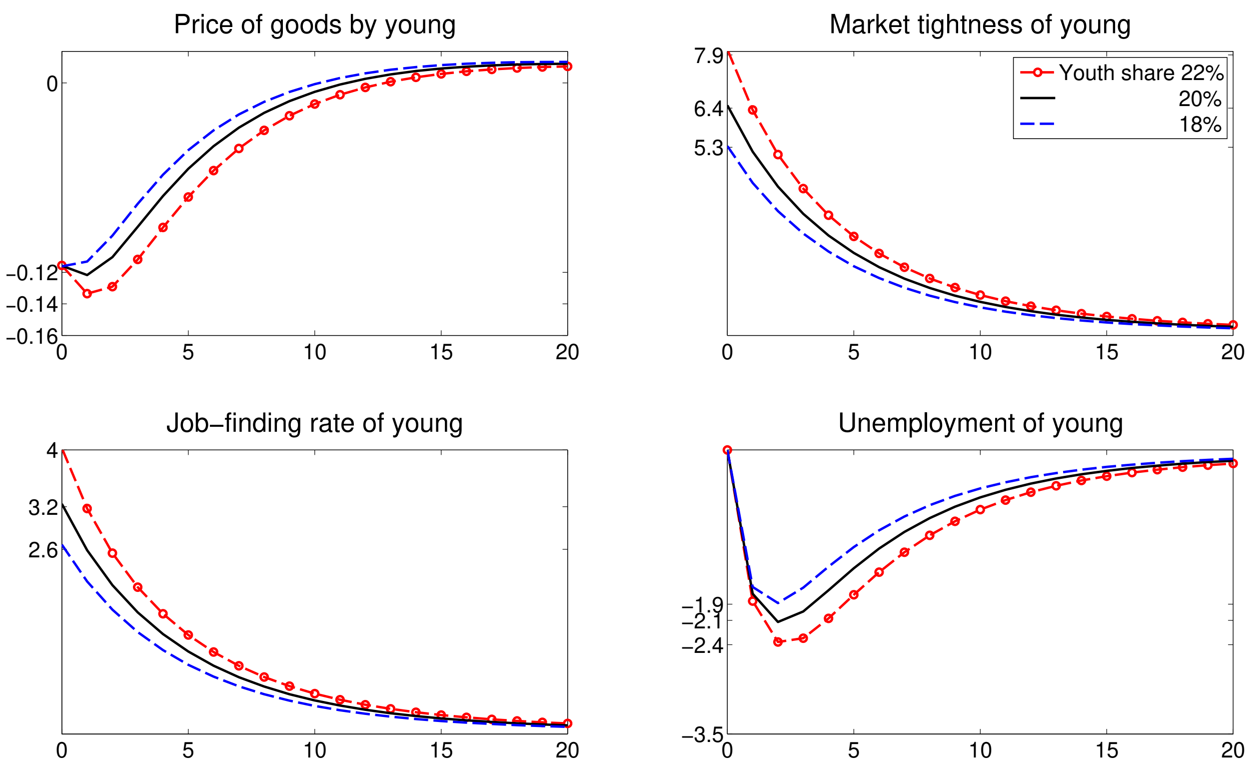

Figure 4: IRFs of variables of the young with respect to an aggregate productivity shock across demographic states (with exogenous separation)

[31]It comes from the evolution of age-specific employment rate, which gives $1 - \tilde{\rho}_t = \frac{n_t}{n_{t-1} + f_{t-1}u_{t-1}}$.