Label the Divorced: A Repeated-Game Analysis on (Re)Marriage Market with Community Enforcement

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Abstract

We study the marriage and remarriage market in a repeated-game framework with potentially alternating partners. Based on our preliminary findings, introducing marital labels by a credible institution can improve social efficiency by limiting inefficient punishment on divorced singles. We conjecture that with the presence of marital labels, the most efficient centralized matching protocol is to arrange as many bachelor-divorced marriage as possible. We also present our preliminary assessment of the equilibrium behavior for a decentralized society where marital status are observable. Preliminary welfare comparison across social institutions with different degrees of pre-marriage labels transparency is conducted.

1 Introduction

The very nature of marriage has been widely discussed on the grounds of sociology, philosophy and economics. Apart from love and passion, economists, in favor of self-interested market system, typically interpret marriage as long-term contract to facilitate two individuals generating surplus from forming a union rather than staying single, in either a formal or informal way.(See Matouschek and Rasul, 2008, for an overview on formal contract.)

In spite of a large body of literature on divorce and remarriage market in development economics literature, little has been contributed on the institutional effect of the marital status as civil labels. In this article, we conceptualize the marriage activities as a repeated partnership between two individuals with an implicit self-enforcing contract. Consider two single people meet and play the partnership game, in which they can either cooperate or defect. Frequently, players realizes their partner's behavior and decide whether to continue the marriage. For those who suffer from marriage, the possibility of opting out can

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serve as a punishment of the deviator. However, in a society where the information about players' behavior in their previous marriages is limited, the player in current marriage may have incentive to break their vow, cheat on their partner and start a relationship fresh new with others, thereby causing efficiency loss in the whole economy. Our paper is dedicated to analyzing such scenarios.

Our formal analysis begins with a repeated game model with anonymous random matching feature. The seminal work of Kandori(1992) and Ellison(199) initiates the literature of repeated Prisoner's Dilemma in which players are anonymous and know little about others' past history. They show that while the standard argument of long-term incentive for a fixed group (For instance, Fudenberg and Maskin(1986)) is not replicable if players are allowed to change their partner and start new relationship anonymously, collusive equilibrium is still sustainable because the punishment against individual's defection can be enforced by the social norm. The so-called "contagion" behavior, executed by the whole community turns out to be a substitute of the threat of personal retaliation in the classical repeated game models. Following the line of research, Takahashi(2010) examines the case where players have the information on their partner's past action history and shows that cooperation is sustainable. Deb(2018) investigates the role of unverifiable names in this type of games and also find out the efficiency cooperation can be achieved by asking the community to hold responsibility for any deviation. Although this stream of literature is germane, our model distinguish in two aspects. First we assume imperfect monitoring for all the stage games. Whilst most of papers in the community enforcement literature assume players have poor access to information about what happened within other partnerships, cheating in player's own stage game is always detected ex post. However in reality, apart from mutual efforts, happy marriage requires many other elements that are relatively random, such as match of idiosyncratic characteristics, which renders imperfect monitoring environment more reasonable. Second, our model allows players to continue their relationship once both parties agree to do so. The effect of such expansion on action space is ambiguous: on the one hand, the possibility of maintaining a stable partnership can encourage players to cooperate within the relationship; on the other hand, under imperfect monitoring, the potential of long-term relationship can make players' profitable deviation strategies more complex to identify, causing more troubles in sustaining cooperative behavior.

The stylized model we present here is build upon Datta (1993), Ghosh and Ray (1996), Lindsey et al (2001), Eeckhout (2006) and Fujiwara-Greve and Okuno-Fujiwara (2009), all of which adopts a long-term relationship model yet allowing unilateral separation from the partnership. For example, Lindsey et al (2001) derive the optimal symmetric social conventions always falls into one of two paths: one harsher rule that end up the relationship whenever one of the party defects and one more forgiving rule that requires mutual defection to terminate the partnership. They show that along the equilibrium path, the chance of breakup decreases over time. Eeckhout(2006) applies the tool to study discrimination and segregation. He shows that race and colors sometimes are employed simply as coordination devices to substitute public randomization device to achieve the second-best efficiency.

2 The Model

Environment

We consider a society of infinitely lived players. The whole economy begins with a population of measure one. In each of the following period, the population grows at rate λ, i.e., at the beginning of any period $T \geq 1$, the measure of total population is $(1+\lambda)^T$. Thus, a group of bachelor(ette) of size $\lambda(1+\lambda)^{T-1}$ join the society. We define the set of all players at period T as $\mathcal{I}^T$ .All players share common discount factor $\delta \in (0,1)$.

At the beginning of each period, every player is either single (bachelor(ette), or divorced), or matched in pairs (married). all the singles will be matched into pairs in two. We consider both directed matching and random matching in later section.

Once matched, all the couples play a partnership game with each other where both sides **privately** decide whether to exert effort (E) or shirk (S) to produce a public goods, of which the outcome is **publicly observable** as a binary $y \in \{\text{success}, \text{failure}\}$[1]. Conditional on two players' effort level $a \in A \times A$, the production outcome follows a stochastic process:

$$\rho(\text{success}|a) = \begin{cases} 1-\theta & \text{if } a = EE, \\ 1-\phi & \text{if } a = SE \text{ or } ES, \\ 0 & \text{if } a = SS. \end{cases}$$

The probability of successful production is $1-\theta$ if both players keep their promise to spend efforts, and $1-\phi$ if only one player shirks. When both players refuse to input, the production is going to fail for sure.

Here we slightly abuse the notation y to denote the value of successful public goods to each player. We normalize the value of failure production to be 0 and the effort cost for each player is c. The implied ex ante stage-game payoff matrix is therefore:

	E	S
E	$(1-\theta)y-c, (1-\theta)y-c$	$(1-\phi)y-c, (1-\phi)y$
S	$(1-\phi)y, (1-\phi)y-c$	$0, 0$

We assume the strictly positive correlation between the amount of efforts and likelihood of generating public goods, i.e. $\theta < \phi$. To capture the inefficient effortless prediction in static game, it is also assumed that $(1-\phi)y < c < (1-\theta)y$. To follow the notational tradition of the related literature, we normalize the ex-ante payoff matrix as follows:

[1]Mailath,Samuelson(2006), Chapter 1 and 7 provide an excellent discussion on modeling prisoner dilemma as partnership game under both perfect and imperfect monitoring.

	E	S
E	$1, 1$	$-l, 1+g$
S	$1+g, -l$	$0, 0$

In the model, we allow public randomization device for players to choose any lottery over action space $a_t \in \Delta(A \times A)$, where $A = \{E, S\}$.[2] We also allow **upfront** transfer payments is available within a partnership. Specifically, we denote transfer from i to j by $m_{ij}^t \in \mathbb{R}$. A negative transfer simply means that player i is the recipient of a monetary transfer made by j. Player i's expected lifetime payoff in this repeated game is therefore given by the quasi-linear representation:

$$V_i^T = (1-\delta)\mathbb{E}\{\sum_{t=T}^{\infty} \delta^{t-T}[u_i(a_t) - m_{ij}^t]\},$$

where $u_i(a_t) : A \times A \to \mathbb{R}$ is the payoff from the public goods less the cost of individual effort.

At the end of each period, upon the realization of public outcomes, the players choose whether to terminate the relationship. The marriage dissolves if at least one player decides to divorce. Later on we will show that on the equilibrium path, the relationship come to end when the bad outcome is realized, despite that both players did contribute efforts in the relationship. In other words, the overall divorce rate, simply triggered by the exogenous production failure, is equal to θ. If no divorce takes place, two players will continue their marriage in the next period as married couples. Otherwise, the relation terminates and two players will re-enter the single pool to start a new relationship in next period.

Hence, in the equilibrium, at the *beginning* of each period, the distribution of each type of players among all the population is stationary as follows:

	bachelor(ette)	divorced	married
Period 0	1	0	0
Period 1	λ	θ	$(1-\theta)$
Period 2	$\lambda(1+\lambda)$	$\theta(1+\lambda)$	$(1-\theta)(1+\lambda)$
...			
Period T	$\lambda(1+\lambda)^{T-1}$	$\theta(1+\lambda)^{T-1}$	$(1-\theta)(1+\lambda)^{T-1}$

It is easy to check that apart from the initial period, the ratio of divorced to bachelor(ette) is θ to λ, which is stationary across periods.

Marital Label

At time t, the public history of a relationship that survives k periods includes all the outcome of every single period within this marriage, namely, $H_{ij}^t = \{(m^{t-k+1} \times a^{t-k+1}), ..., (m^t \times a^t)\}$.. In the paper, we

[2]Ellison(1996) discuss how to employ a public random variable that follows a uniform distribution to implement the randomization.

assume *anonymous* matching. When a marriage comes to an end, players become oblivious of the current history and reset their new public history.

The new ingredient in our paper is label. Suppose now a social institution introduces a civil label to indicate players' marital status. The label records whether a particular player has been married or not and is publicly observable to the entire community. At period t, every member in the community is adhered with a specific label $L_i^t \in \{b, d, m\}$. A bachelor(ette) who enters into the economy at current period is endowed with label b, while players who arrive in previous periods =are either with label d or m, which stand for divorced or married, respectively.

Efficiency

To investigate how labels facilitates or deteriorate cooperative behavior in marriage, we first introduce the measurement of efficiency in the dynamic environment. For any candidate equilibrium, we define the social efficiency at any steady state τ as follows:

$$S = \frac{\int_{\mathcal{I}^\tau} V_i^\tau di + \sum_{t=\tau+1}^{\infty} \delta^{t-\tau} \int_{\mathcal{I}_t \setminus \mathcal{I}_{t-1}} V_i^t di}{|\mathcal{I}^\tau|} \tag{1}$$

The efficiency concept describes the aggregation of all players' life-time payoff, normalized by the population of current period such that it is constant across every steady state. We impose the regularity condition $\delta\lambda < 1$ to ensure the boundedness. As the best possible efficiency is achieved by full cooperation of the whole society, the first-best social efficiency S_{FB} is recursively defined by

$$\begin{aligned} S_{\text{FB}} &= 1 + \delta\lambda \sum_{t=0}^{\infty} [\delta(1+\lambda)]^t \\ &= \frac{1-\delta}{1-\delta(1+\lambda)} \end{aligned} \tag{2}$$

3 Benchmark: Label-less Society

In this section, we study the benchmark case where no label of marital status is at place or label is not credible.

3.1 The Second Best Solution

It is also well documented in the literature that every relationship needs to go through the trust building phase before proceeding to the cooperation phase(Eeckhout(2006)). For any strategy profile s, let $v(s)$ be the corresponding payoff to a newly-formulated couple and v_c be the payoff to a couple already in the cooperation phase:

$$v_c = (1-\delta)1 + \delta[\theta v(s) + (1-\theta)\, v_c]$$
$$= \frac{1-\delta+\delta\theta v(s)}{1-\delta\,(1-\theta)} \tag{3}$$

If the player chooses to deviate in the cooperation phase, her expected payoff is

$$v_{dev} = (1-\delta)\,(1+g) + \delta v(s)$$

The candidate equilibrium needs to satisfy the non-deviation constraint, that is,

$$v_c \geq v_{dev}$$
$$v(s) \leq 1 - \frac{1-\delta(1-\theta)}{\delta(1-\theta)} g \equiv \overline{V} \tag{4}$$

We rule out the uninteresting case where the constraint efficiency level is negative[3], i.e., $\overline{V} \geq 0$. Equivalently,

$$\delta \geq \frac{g}{(1+g)(1-\theta)} \tag{A1}$$

Therefore, if there is no marital status label, the highest attainable payoff is bounded above by $\overline{V}$ to guarantee no deviation in the cooperation phase. The upper bound is referred to as constraint efficiency in Eeckhout(2006).

Now we argue that it is without loss of generality to restrict our focus on symmetric equilibrium in label-less economy, for that with whatever asymmetric strategies s_i, all the players still have to satisfy the constraint (4). Hence, the constraint efficiency can be no higher than that obtained from the symmetric strategies. By the symmetry argument, there should be no positive upfront monetary transfer between the players .

3.2 The Attainability of Constraint Efficiency

Now we construct an equilibrium where second-best efficiency is achieved. Consider the well-known **K-incubation** strategy where a trust building phase takes newly matched couples K periods of (S,S) before they start cooperation phase[4]. Notice that here K is not necessarily an integer. When $K \notin \mathbb{N}$, a natural implementation is that the punishment phase lasts for $[K] \equiv k$ periods. In the upcoming period the couple, with the help of randomization device, maintain the mutual defection with probability $\varepsilon \in [0,1)$ [5] but kick

[3]If the constraint efficiency level is negative, the players would defect indefinitely.

[4]Throughout the paper, we restrict our attention on the symmetric Green-Porter equilibrium where punishment is enforced through immediate reversion to static Nash equilibrium for certain amount of periods, upon observing the bad outcome. K-incubation equilibrium is essentially equivalent implementation of Green-Porter equilibrium under the repeated random matching environment.

[5]Generically, $\varepsilon \neq K-[k]$ since by definition the punishment at K period is discounted by δ^{K-1} yet in (k,ε) implementation, this part is discounted by $\delta^{[k]}$ where $[k] > K-1$. Therefore we need some $\varepsilon \leq K-[k]$ to compensate this additional discount.

off the cooperation with probability $1-\varepsilon$ to implement the equivalent payoff. Afterwards, full cooperation is enforced and two players both spend efforts in producing the public goods. The cooperation phase will continue until the marriage dissolves exogenously. Throughout the paper, we use K-incubation strategy and (k,ε) strategy interchangeably.

v_s can be explicitly characterized by K as:

$$\begin{aligned}
& v_s(K) = \delta\theta[1+\delta(1-\theta)+[\delta(1-\theta)]^2+...+[\delta(1-\theta)]^k]v(s)+\delta^{k+1}(1-\theta)^{k+1}v_c \\
& + \delta^k(1-\theta)^k[(1-\varepsilon)(1-\delta)] \\
\Rightarrow\ & v_s(K) = \delta\theta\frac{1-[\delta(1-\theta)]^{k+1}}{1-\delta(1-\theta)}v(s)+\delta^{k+1}(1-\theta)^{k+1}v_c+\delta^k(1-\theta)^k[(1-\varepsilon)(1-\delta)] \\
\Rightarrow\ & \Big(1-\delta\theta\frac{1-[\delta(1-\theta)]^{k+1}}{1-\delta(1-\theta)}\Big)v(s) = \delta^{k+1}(1-\theta)^{k+1}v_c+\delta^k(1-\theta)^k[(1-\varepsilon)(1-\delta)] \qquad (5)
\end{aligned}$$

We can solve for $v(s)$ from Equations (3) and (5):

$$v(s) = [\delta(1-\theta)]^k\Big(1-[1-\delta(1-\theta)]\varepsilon\Big) \qquad (6)$$

It is easy to show that $v(s)$ decreases in K:

First, for $K\notin\mathbb{N}$, local change of K would not affect k, therefore we can directly check its derivative by treating k as a constant:

$$\frac{\partial v(K)}{\partial K} = \frac{\partial v(K)}{\partial\varepsilon} = -[\delta(1-\theta)]^k[1-\delta(1-\theta)] < 0 \qquad (7)$$

For any positive integer K, locally we check the definition of first-order derivative. As the right-hand derivative does not involve changing k therefore is similar to previous case. We only need to focus on the left-hand derivative, where $[\kappa]=k-1$ and $\varepsilon_\kappa=\varepsilon(\kappa)$:[6]

$$\begin{aligned}
\lim_{\kappa\to K^-}\frac{v(\kappa)-v(K)}{\kappa-K} &= [\delta(1-\theta)]^{k-1}\frac{(1-\varepsilon_k)[1-\delta(1-\theta)]}{\kappa-k} \\
&\approx [\delta(1-\theta)]^{k-1}\frac{(k-\kappa)[1-\delta(1-\theta)]}{\kappa-k} = -[\delta(1-\theta)]^{k-1}[1-\delta(1-\theta)] < 0
\end{aligned}$$

Therefore, the expected value for divorced marriage is decreasing in K.

When $K\to 0$, $v(s)=1>\overline{V}$. When $K\to\infty$, $v(s)\to 0$. By IVT, we can always find a unique strictly positive K_{sb} such that $v(s)=\overline{V}$ just holds.

We can easily calculate the social efficiency for the label-less society with K-incubation strategy.

Proposition 1. *For any K-incubation equilibrium in label-less society with $\delta<1$, the first-best result is not attainable and the highest possible social efficiency S_{sb} is achieved by K_{sb}-incubation strategy, where K_{sb} is pinned down by*

$$S_{sb} = \frac{1-\delta}{1-\delta(1+\lambda)}(\frac{1-\theta}{1+\lambda})^{k_{sb}}(1-\frac{\theta+\lambda}{1+\lambda}\varepsilon_{sb}) \qquad (8)$$

[6] At limit case, ε_κ and $\kappa+1-k$ converge to 0 at same speed as $\kappa\to k^-$.

4 Label the Divorced

Suppose now a social institution, say government, introduces an identity label that specifies a marital status to each member of the economy. Then at the beginning of each relationship, upon observing the opponent's label, every pair is to sequentially choose a strategy profile $s : L_i \times L_j \to \Big(\mathbb{R} \times \Delta(A \times A) \times \{divorce, continue\}^2\Big)^\tau$ for all τ periods until the relationship resolves.

4.1 Optimal Matching Protocol

First, we investigate the social planner's problem where she prescribes a matching protocol that all the single players have to follow. Upon the formulation of marriages, the social planner does not intervene anymore and each player chooses their action purely out of their own interest.

Again, we focus on the modified K-incubation equilibrium, where cooperation phase starts immediately for bachelor-bachelorette marriage; when a bachelor is assigned to a divorced, full cooperation follows an upfront transfer m_{db} from divorced player to bachelor(ette)s; the possible punishment phase is imposed on divorced-divorced marriage before they shift to the full cooperation.

All the possible matching protocols can be characterized by a dichotomy according to the relative proportion of bachelor(ette) and divorced in the single pool.

Excess Bachelor(ette): $\lambda \geq \theta$

Under the assumption $\lambda \geq \theta$, every period there will be more bachelor(ette)s in the single pool, therefore the matching protocol is fully captured by the proportion of divorced assigned to bachelors μ_{db}.

According to the equilibrium strategy profile, the expected payoffs of each type of players are:

$$\begin{aligned} v_c &= (1-\delta)1 + \delta[\theta v_d + (1-\theta)v_c] && (9) \\ v_b &= (1-\delta)[\frac{\mu_{db}\theta}{\lambda}(1+m_{db}) + \frac{\lambda - \mu_{db}\theta}{\lambda}] + \delta[\theta v_d + (1-\theta)v_c] && (10) \\ v_d &= \mu_{db} v_{db} + (1-\mu_{db})v_{dd} && (11) \end{aligned}$$

where v_{db} and v_{dd} refers to the expected payoff of a divorced when assigned to a bachelor and a divorced.

$$\begin{aligned} v_{db} &= (1-\delta)(1-m_{db}) + \delta[\theta v_d + (1-\theta)v_c] && (12) \\ v_{dd} &= \delta\theta \sum_{t=0}^{k} [\delta(1-\theta)]^t v_d + [\delta(1-\theta)]^{k+1} v_c + [\delta(1-\theta)]^k (1-\varepsilon)(1-\delta) \\ &= \delta\theta \frac{1-[\delta(1-\theta)]^{k+1}}{1-\delta(1-\theta)} v_d + [\delta(1-\theta)]^{k+1} v_c + [\delta(1-\theta)]^k (1-\varepsilon)(1-\delta) && (13) \end{aligned}$$

Substitute (12) and (13) into (11), we have

$$v_d = \mu_{db}\Big(1 - [1-\delta(1-\theta)m_{db}]\Big) + (1-\mu_{db})[\delta(1-\theta)]^k \Big(1 - [1-\delta(1-\theta)\varepsilon]\Big) \quad (14)$$

To ensure the prescribed equilibrium stable, we have to impose two constraints that eventually set an upper bound and a lower bound on the upfront transfer m_{db}.

First constraint is to guarantee that a Divorced is willing to exercise the upfront transfer when matched with an Bachelor(ette), that is,

$$v_{bd} \geq \delta v_d \tag{15}$$

.

It implies that the monetary transfer cannot be too high for the divorced couples such that initiating cooperation phase right now by paying the transfer is weakly preferable to rejecting the transfer duty and seeking for another relationship in next period.

The condition can be rewritten as an upper bound $\overline{m_{db}}$ imposed on the monetary transfer. On the other hand, the payment should be sufficiently high to ensure that the players in cooperation phase have no incentive to grab the myopic profit and become a divorced in the rest of their lives:

$$v_c \geq (1+g)(1-\delta) + \delta v_d \tag{16}$$

which will eventually sets a lower bound $\underline{m_{db}}$ on m. What remains to check is the existence of certain levels of transfer, namely whether $\overline{m_{db}} \geq \underline{m_{db}}$. We would conjecture that the final constraint can be rewritten in the following form:

$$f(\mu_{db}, K) < 0$$

where an educated guess is that $f(\mu_{db}, K)$ is decreasing in K and μ_{db}. As inefficient noncooperation can always serve a substitute of punishment for monetary transfer, while reducing the possibility of divorced-divorced marriage can force those divorced singles to voluntarily engage in transfer and starting a new relationship as soon as possible whenever they are matched to a bachelor. If that conjecture is true, then the most relaxed constraint will be satisfied by taking $\mu_{bd} = 1$ and $K = 0$. Hence, an equilibrium that avoids inefficient punishment phase is always implementable and we can define the minimum rate of divorced-bachelor marriage to support efficient equilibrium as

$$\underline{\mu_{db}} = \inf\{\mu_{db} \in [0,1] : f(\mu_{db}, 0) \leq 0\} \tag{17}$$

It is very likely that the in a society where $\lambda \geq \theta$, first-best efficiency can be attained by exhausting all the divorced to match with bachelor first. Since being a divorced will always face the transfer duty whenever she is engaged in a new relationship, any deviation will simply postpone the cooperation and therefore is non-profitable. It would be better for her to comply with the social convention and start cooperation phase.

Excess Divorced: $\lambda < \theta$

Symmetrically, if there are more divorced in the single pool, then the matching rule is captured by the proportion of bachelor(ette) pool who are assigned to the divorced μ_{bd}. However, under this setting the divorced-divorced marriage is inevitable as there will always be excess divorced players who have to be matched with each other.

Similarly, we can write down the expected payoffs for each type of players:

$$v_c = (1-\delta)1 + \delta[\theta v_d + (1-\theta)v_c] \tag{18}$$

$$v_b = (1-\delta)[(1-\mu_{bd}) + \mu_{bd}(1+m_{db})] + \delta[\theta v_d + (1-\theta)v_c] \tag{19}$$

$$v_d = \frac{\mu_{bd}\lambda}{\theta} v_{db} + (1 - \frac{\mu_{bd}\lambda}{\theta})v_{dd} \tag{20}$$

where

$$v_{db} = (1-\delta)(1-m_{db}) + \delta[\theta v_d + (1-\theta)v_c] \tag{21}$$

$$\begin{aligned} v_{dd} &= \delta\theta\sum_{t=0}^{k}[\delta(1-\theta)]^t v_d + [\delta(1-\theta)]^{k+1}v_c + [\delta(1-\theta)]^k(1-\varepsilon)(1-\delta) \\ &= \delta\theta\frac{1-[\delta(1-\theta)]^{k+1}}{1-\delta(1-\theta)}v_d + [\delta(1-\theta)]^{k+1}v_c + [\delta(1-\theta)]^k(1-\varepsilon)(1-\delta) \end{aligned} \tag{22}$$

Plug (21) and (22) into (20), we get

$$v_d = \frac{\mu_{bd}\lambda}{\theta}\Big(1 - [1-\delta(1-\theta)]m_{db}\Big) + (1-\frac{\mu_{bd}\lambda}{\theta})[\delta(1-\theta)]^k\Big(1-[1-\delta(1-\theta)]\varepsilon\Big) \tag{23}$$

Again, we can imagine that two non-deviation constraints lead to an upper bound and lower bound on the plausible transfer and the existence condition of equilibrium will be an inequality $f(\mu_{bd}, K) < 0$. Notice that now even we let $\mu_{bd} = 0$, the inefficient punishment K now is not always eliminable, since now divorced-divorced union is inevitable, it is possible that divorced singles, when matched with singles, would like to reject the monetary payment and wait until they can assigned with another divorced. Such concern limits the upper bound of m_{db} and extracts the plausible transfer space.

We can then characterize the optimal matching protocol by solving the constrained maximization problem.

Conjecture 1. *Suppose (A1) holds. In a society with excess bachelor, the optimal matching protocol is characterized by any* $\mu^*_{db} \in [\underline{\mu_{db}}, 1]$ *and the first-best efficiency* S_{FB} *is always attainable.*

In a society with excess divorced,

$$\mu^*_{bd} = \begin{cases} [\underline{\mu_{bd}}, 1] & \text{if } \delta \text{ is sufficiently large;} \\ 1 & \text{otherwise.} \end{cases}$$

with the associated optimal social efficiency level

$$S^*_{sp} = \begin{cases} S_{FB} & \text{if } \delta \text{ is sufficiently large;} \\ S^*_{sp}(K^*_{sp}) & \text{otherwise.} \end{cases}$$

The proposition specifies an algorithm to derive the optimal distribution of each type of marriage. When the society has more bachelors than divorced, we can always recover the first-best result by letting as many divorced as possible to marry bachelor. In this way, the punishment is only enforced by the upfront transfer, which is always feasible as the inequality is always satisfied due to assumption (A1), therefore there is no efficiency loss from the perspective of the whole society.

4.2 Unobservable Labels: Random Matching

Now we assume that at the beginning of each period, all the singles voluntarily match with each other without observing other's label before the relationship starts out. In other words, the matching is assumed to be purely random and uniform. We focus on the following modified K-incubation strategy:

when two bachelor(ette) match, no trust building phase is required; when a divorced is matched with an Bachelor(ette), they also coordinate by playing (C, C) while the divorced offers an upfront transfer m_{db} to the bachelor(ette). when two divorced match, they first go through a (k, ε) trust building phase, then maintain full cooperation indefinitely.

The strategy is supported by the off-equilibrium path where a deviation from the specified action in the prisoner's dilemma game results in immediate divorce and a deviation by the divorced of not offering transfer m_{db} or offering any amount different from m_{db} when matched with an bachelor(ette) results in (D, D) and then divorce in the current period.

The payoff for a married is

$$\begin{aligned} v_c &= (1-\delta)1 + \delta[\theta v_d + (1-\theta)\, v_c] \\ &= \frac{\delta\theta}{1-\delta\,(1-\theta)} v_d + \frac{1-\delta}{1-\delta\,(1-\theta)} \end{aligned} \tag{24}$$

The payoff of a divorced when matched with another Divorced is

$$\begin{aligned} v_{dd} &= \delta\theta[1 + \delta(1-\theta) + \left(\delta(1-\theta)\right)^2 + ... + \left(\delta(1-\theta)\right)^k] v_d + [\delta(1-\theta)]^{k+1} v_c \\ &\quad + [\delta(1-\theta)]^k [(1-\varepsilon)(1-\delta)] \\ &= \delta\theta \frac{1-[\delta(1-\theta)]^{k+1}}{1-\delta(1-\theta)} v_d + [\delta(1-\theta)]^{k+1} v_c + [\delta(1-\theta)]^k [(1-\varepsilon)(1-\delta)] \end{aligned}$$

Plug in (24), we have

$$v_{dd} = \frac{\delta\theta}{1-\delta\,(1-\theta)} v_d + [\delta(1-\theta)]^k (1-\delta) [\frac{1}{1-\delta(1-\theta)} - \varepsilon] \tag{25}$$

The payoff of a divorced when matched with a bachelor(ette) is

$$v_{db} = (1-\delta)(-m_{db} + 1) + \delta[\theta v_d + (1-\theta)\, v_c]$$

Plug in (24), we have

$$v_{db} = \frac{\delta\theta}{1-\delta\,(1-\theta)} v_d + \frac{1-\delta}{1-\delta\,(1-\theta)} - m_{db}(1-\delta) \tag{26}$$

The payoff of a divorced is

$$v_d = \frac{\theta}{\theta+\lambda} v_{dd} + \frac{\lambda}{\theta+\lambda} v_{db} \tag{27}$$

Plug in (25) and (26), we have

$$v_d = \frac{\theta}{\theta+\lambda}[\delta(1-\theta)]^k\Big(1-[1-\delta(1-\theta)]\varepsilon\Big) + \frac{\lambda}{\theta+\lambda}\Big(1-[1-\delta(1-\theta)m_{db}]\Big) \tag{28}$$

The payoff of an Bachelor(ette) when matched with a Divorced is

$$v_{bd} = (1-\delta)(m_{db}+1) + \delta[\theta v_d + (1-\theta)\, v_c]$$

Plug in (24), we have

$$v_{bd} = \frac{\delta\theta}{1-\delta\,(1-\theta)}v_d + \frac{1-\delta}{1-\delta\,(1-\theta)} + m_{db}(1-\delta) \tag{29}$$

The payoff of an Bachelor(ette) when matched with another Bachelor(ette) is

$$v_{bb} = (1-\delta) + \delta[\theta v_d + (1-\theta)\, v_c]$$

Plug in (24), we have

$$v_{bb} = \frac{\delta\theta}{1-\delta\,(1-\theta)}v_d + \frac{1-\delta}{1-\delta\,(1-\theta)} \tag{30}$$

The payoff of a bachelor(ette) is

$$v_b = \frac{\theta}{\theta+\lambda}v_{bd} + \frac{\lambda}{\theta+\lambda}v_{bb}$$

Plug in (29) and (30), we have

$$v_b = \frac{\delta\theta}{1-\delta\,(1-\theta)}v_d + \frac{1-\delta}{1-\delta\,(1-\theta)} + \frac{\theta}{\theta+\lambda}m_{db}(1-\delta) \tag{31}$$

The non-deviation constraint in the cooperation phase is

$$\begin{aligned} v_c &\geq (1-\delta)(1+g) + \delta v_d \\ v_d &\leq 1 - \frac{1-\delta(1-\theta)}{\delta(1-\theta)}g \end{aligned}$$

which translates into a lower bound on upfront transfer :

$$m_{db} \geq -\frac{\theta}{\lambda}\Big\{\frac{1}{(1-\delta(1-\theta))} - [\delta(1-\theta)]^k[\frac{1}{1-\delta(1-\theta)} - \varepsilon]\Big\} + \frac{\theta+\lambda}{\lambda}\frac{1}{\delta(1-\theta)}g = \underline{m_{db}} \tag{NDc rm}$$

The non-deviation constraint for a divorced to offer transfer m_{db} when matched with an bachelor(ette) is

$$\begin{aligned} v_{db} &\geq (1-\delta)0 + \delta v_d = \delta v_d \\ v_d &\leq \frac{1-m_{db}(1-\delta(1-\theta))}{\delta(1-\theta)} \end{aligned}$$

which boils down to an upper bound on the upfront transfer

$$m_{db} \leq \frac{1}{1-\delta(1-\theta)} - \frac{\theta[\delta(1-\theta)]^T}{\lambda[1-\delta(1-\theta)]+\theta}\big[\frac{1}{1-\delta(1-\theta)} - \varepsilon\big] = \overline{m_{db}} \tag{NDT rm}$$

The non-deviation conditions (NDc rm) and (NDT rm) are simultaneously satisfied if and only if the feasible set of transfer is non-empty, i.e., $\overline{m_{db}} \geq \underline{m_{db}}$, or

$$\frac{\theta+\lambda}{\lambda}\{[\delta(1-\theta)]^k(\frac{1}{1-\delta(1-\theta)}-\varepsilon)\frac{\theta}{\lambda[1-\delta(1-\theta)]+\theta}+\frac{1}{\delta(1-\theta)}g-\frac{1}{1-\delta(1-\theta)}\}\equiv f_{rm}(K)\leq 0 \quad (32)$$

It is easy to see that $\underline{m_{db}}$ decreases in T and ε and $\overline{m_{db}}$ increases in T and ε. Therefore, we can easily check whether there exists a trust-building period to support K-incubation equilibrium by checking the limit case where K goes to infinity.

$$\begin{aligned}\lim_{T\to\infty,\varepsilon\to 1}\underline{m_{db}} &= -\frac{\theta}{\lambda}\frac{1}{(1-\delta(1-\theta))}+\frac{\theta+\lambda}{\lambda}\frac{1}{\delta(1-\theta)}g\\ \lim_{T\to\infty,\varepsilon\to 1}\overline{m_{db}} &= \frac{1}{1-\delta(1-\theta)}\end{aligned}$$

and the non-emptiness of feasible transfer set yields:

$$\begin{aligned}\lim_{T\to\infty,\varepsilon\to 1}\overline{m_{db}} &\geq \lim_{T\to\infty,\varepsilon\to 1}\underline{m_{db}}\\ \frac{\theta+\lambda}{\lambda}\frac{1}{(1-\delta(1-\theta))} &\geq \frac{\theta+\lambda}{\lambda}\frac{1}{\delta(1-\theta)}g\\ \delta &\geq \frac{g}{(1-\theta)(1+g)} \quad (33)\end{aligned}$$

which always holds under Assumption (A1). Hence, a K-incubation equilibrium always exists. What remains to do is find the most efficient equilibrium.

The social efficiency at any steady state is captured by the following recursive structure:

$$S=(1-\delta)\{1-\frac{\theta}{1+\lambda}\frac{\theta}{\lambda+\theta}-\frac{1-\theta}{1+\lambda}[(1-f(1)-f^c)+f(1)\varepsilon]\}+\delta(1+\lambda)S$$

where $f(t)$ represents the fraction of players among those married who are t periods away from full cooperation:

$$f(t)=\frac{\frac{\theta}{1+\lambda}\frac{\theta}{\lambda+\theta}(1-\theta)^{T-t}}{(1+\lambda)^{T-t}\frac{1-\theta}{1+\lambda}}=\frac{(1-\theta)^{T-t-1}\theta^2}{(1+\lambda)^{T-t}(\lambda+\theta)} \quad (34)$$

and f^c represents the fraction of players among those married who are in cooperation phase:

$$\begin{aligned}f^c &= 1-[f(1)+f(2)+...+f(k)]=1-\frac{(1-\theta)^k\theta^2}{(1+\lambda)^k(\lambda+\theta)^2}[(\frac{1+\lambda}{1-\theta})^k-1]\\ &= 1-\frac{\theta^2}{(\lambda+\theta)^2}+\frac{(1-\theta)^k\theta^2}{(1+\lambda)^k(\lambda+\theta)^2} \quad (35)\end{aligned}$$

Plug in (34) and (35), we have

$$\begin{aligned}S^*_{rm} &= \frac{1-\delta}{1-\delta(1+\lambda)}\{1-\frac{\theta^2}{(1+\lambda)(\lambda+\theta)}-\frac{\theta^2}{(\lambda+\theta)^2}[\frac{1-\theta}{1+\lambda}-\frac{(1-\theta)^k}{(1+\lambda)^k}]-\frac{\theta^2(1-\theta)^k}{(\lambda+\theta)(1+\lambda)^T}\varepsilon\}\\ &= \frac{1-\delta}{1-\delta(1+\lambda)}\{1-\frac{\theta^2}{(\lambda+\theta)^2}+\frac{\theta^2}{(\lambda+\theta)^2}(\frac{1-\theta}{1+\lambda})^k(1-\frac{\lambda+\theta}{1+\lambda}\varepsilon)\} \quad (36)\end{aligned}$$

which is decreasing in T as well. Hence if the society is to achieve the first-best efficiency, the only way is to set $T=1$ and $\varepsilon=0$. Then $\underline{m_{db}}$ and $\overline{m_{db}}$ are respectively

$$\begin{aligned}\underline{m_{db}} &= \frac{\theta+\lambda}{\lambda}\frac{1}{\delta(1-\theta)}g\\ \overline{m_{db}} &= \frac{\theta+\lambda}{[\lambda(1-\delta(1-\theta))+\theta]}\end{aligned}$$

In other words, the first-best efficiency is implementable if and only if $\overline{m_{db}} \geq \underline{m_{db}}$, or equivalently:

$$\delta \geq \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)} \tag{A2}$$

Observe that (A2) is more restrictive than (A1). If it is satisfied, the divorced couple could directly enter into the cooperation phase. The punishment is enforced on a divorced player only when he meets an bachelorette, where an upfront transfer $m_{db} \in [\frac{\theta+\lambda}{\lambda}\frac{1}{\delta(1-\theta)}g, \frac{\theta+\lambda}{[\lambda(1-\delta(1-\theta))+\theta]}]$ is made to her partner before they formally start full cooperation.

When (A2) fails, or $\delta \in [\frac{g}{(1-\theta)(1+g)}, \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)})$, it is inevitable for a divorced couple to go through a trust building T^*_{rm} and ε^*_{rm} such that $\underline{m_{db}}$ and $\overline{m_{db}}$ are exactly equal or equation (32) binds. If a divorced meets a bachelorette, he needs to transfer $m_{db} = \underline{m_{db}} = \overline{m_{db}}$ to the bachelorette beforehand.

4.3 Observable Labels: Assortative Matching

In this subsection, we look for an equilibrium that is in accordance with the assortative matching. To be more specific, we look at a society where the marital labels publicly observable such that all the singles are able to search for partners of any particular type that they wish to marry. We call a distribution of various types of marriage **stable** if ex post no players wish and is able to marry another partner of different type who also has the incentive to accept. Since the bachelor receives additional upfront transfer from the divorced and follows the same subsequent equilibrium path as when matched with another bachelorette. Bachelors always weakly prefer the divorced to other bachelors, therefore they look for divorced partners until all of divorced are matched. On the other hand, whether the bachelor(ette) is able to marry a divorced depends on the divorced players' preference.

In particular, the divorced is indifferent between marrying an bachelor(ette) and another divorced if and only if $v_{dd} = v_{db}$. We conjecture that this indifference condition will simply yield a cutoff value $\overline{\overline{m_{db}}}$.

The divorced weakly prefers to marry an bachelor(ette) if

$$m_{db} \leq \overline{\overline{m_{db}}} \tag{DpB}$$

The Divorced weakly prefers to marry another Divorced if

$$m_{db} \geq \overline{\overline{m_{db}}} \tag{DpD}$$

Again, we discuss two scenarios according to the relative population of the bachelor(ette)s to to the divorced group.

Excess Bachelor(ette): $\lambda \geq \theta$

Then we can construct an equilibrium where both divorced and bachelor groups prefer each other and desire the inter-label marriage. In other words, all divorced are matched with the bachelor(ette), and the remaining bachelor(ette)s are matched with each other. All pairs initiate cooperation from the first period onwards. The conditions in previous random matching sections are also necessary here, which again imposes bounds on m_{db}. However, we conjecture that these two conditions, along with Condition (DpB), can be easily satisfied when we impose the harshest off-equilibrium punishment phase by taking $K \to \infty$.

Excess Divorced: $\lambda < \theta$

The optimal matching protocol derived above indicates that more mixed-label marriages yield weakly more efficient outcome.We thusly look for the most efficient equilibrium if such equilibria exist.

Write down the expected payoff for players of each label:

$$v_d = \frac{\theta-\lambda}{\theta}v_{dd} + \frac{\lambda}{\theta}v_{db} = \frac{\theta-\lambda}{\theta}[\delta(1-\theta)]^k[1-\varepsilon(1-\delta(1-\theta))] + \frac{\lambda}{\theta}[1-m_{db}(1-\delta(1-\theta))]$$
$$v_b = v_{bd} = \frac{\delta\theta}{1-\delta(1-\theta)}v_d + \frac{1-\delta}{1-\delta(1-\theta)} + m_{db}(1-\delta)$$
$$v_c = \frac{\delta\theta}{1-\delta(1-\theta)}v_d + \frac{1-\delta}{1-\delta(1-\theta)}$$

Still we have three conditions to be satisfied and the existence condition of equilibrium will be tighter as now we need one more constraint: $\overline{\overline{m_{db}}} \geq \underline{m_{db}}$. Intuitively, this condition should coincide with the non-deviation condition in the benchmark label-less society. As the indifference condition always holds, the punishment, either in the form of upfront transfer, or experiencing K periods of mutual defection should be equal. Then the incentive to sustain a cooperative relationship will be exactly the same as the label-less society, except now the punishment has an alternative form. It also implies that in this case, first-best efficiency can never be achieved just as in the label-less equilibria,.

Therefore, we can denote $K^{am} = K^{SB} > 0$ that obtains the constraint efficiency.

To conclude, we conjecture that the inter-type marriage K-incubation equilibrium always exists. k^{am}, ε^{am} and m_{db} are chosen such the the below equation holds

$$\begin{aligned} m_{db} &= \frac{1}{1-\delta(1-\theta)} - [\delta(1-\theta)]^{k^A-1}\Big[\frac{1}{1-\delta(1-\theta)} - \varepsilon^A\Big] \\ &= -\frac{\theta-\lambda}{\lambda}\Big\{\frac{1}{(1-\delta(1-\theta))} - [\delta(1-\theta)]^{k^A-1}\Big[\frac{1}{(1-\delta(1-\theta))} - \varepsilon^A\Big]\Big\} + \frac{\theta}{\lambda}\frac{1}{\delta(1-\theta)}g. \end{aligned} \quad (37)$$

On the equilibrium path, the Divorced is just indifferent between matching a Bachelor(ette) and another Divorced. The Bachelor(ette) all match with Divorced and after the transfer payment they play (C, C) from the first period onwards. The Divorced who are not matched with Bachelor(ette) match with each other and play the (T^A, ε^A) strategy according to Equation (37).

4.4 Comparison of Random Matching and Assortative Matching

Excess Bachelor(ette): $\lambda \geq \theta$

When $\delta \geq \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)}$, both Random Matching and Assortative Matching dictate (C, C) from first period onwards for all pairs after possible transfer payment and thus the two institutions achieve the same level of efficiency.

When $\delta < \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)}$ and Assumption (A1) is satisfied, Assortative Matching dictates (C, C) from first period onwards for Bachelor-Divorced pairs after the transfer payment. Bachelor-bachelorette pairs also obtain (C, C) from first period onwards. But for Random Matching, despite (C, C) for bachelor-bachelorette pairs from first period onwards and (C, C) from first period onwards for bachelor-divorced pairs after the transfer payment, divorced-divorced pairs have to start the game with (T^R, ε^R) punishment. Therefore, Assortative Matching dominates Random Matching in terms of efficiency.

Excess Divorced: $\lambda < \theta$

When $\delta \geq \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)}$, Random Matching dictates (C, C) from first period onwards for all pairs after possible transfer payments. But Assortative Matching requires (T^A, ε^A) punishment for Divorced-Divorced pairs. Therefore, Random Matching dominates Assortative Matching in terms of efficiency.

When $\delta < \frac{(\lambda+\theta)g}{\lambda(1-\theta)(1+g)}$ and Assumption (A1) is satisfied, Assortative Matching dictates (C, C) from first period onwards for bachelor-divorced pairs after the transfer payment. divorced-divorced pairs need to go through (k^A, ε^A) punishment. Under Random Matching, bachelor-divorced pairs obtain (C, C) from first period onwards after the transfer payment; bachelor-bachelorette pairs obtain (C, C) from first period onwards; and divorced-divorced pairs go through (T^R, ε^R) punishment.

(k^R, ε^R) punishment is shorter than (k^A, ε^A) punishment and is thus more efficient. This is because (k^R, ε^R) needs to satisfy $\overline{m_{db}} \geq \underline{m_{db}}$ while (k^A, ε^A) needs to satisfy both $\overline{m_{db}} \geq \underline{m_{db}}$ and one more voluntary matching constraint. This additional constraint is a tighter constraint so the trust-building phase is supposed to be longer.

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