An Experimental Study on Sequential Auctions with Privately Known Capacities[*]

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Abstract

We experimentally study sequential first-price procurement auctions where bidders' capacity constraints are private information. Our results show that: (i) bids in the second auction conforms with sequential rationality; (ii) although first auction's bids negatively depend on capacity, bidders seem unable to recognize this link when, after the first auction, they state their beliefs. We conjecture that bidding in the first auction is also affected by a hidden, behavioral type – related to the strategic sophistication of bidders – that obfuscates the link between capacity and bids. Building on this intuition, we show that a simple level-k model may help explain the inconsistency.

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1 Introduction

Several goods and services are procured or sold through auctions that are run in sequence. The most prominent example is represented by electricity markets: in these markets, the delivery of electricity is usually procured by means of auctions – the Day-Ahead Market and the IntraDay Market – where sellers commit to deliver a certain amount of power in a specific time interval next day or in the same day, followed by a real time auction – the Balance Market –, meant to secure real time balance between actual demand and supply. Other examples of sequential auctions include the sale of spectrum rights, oil and gas leases, greenhouse gas emission permits, treasury bonds.

The distinguishing feature of sequential auctions is that the outcome of one auction may alter the setting in which the following auction takes place and/or may convey additional, though imperfect, information on some relevant elements of the environment. This introduces a strategic linkage between the auctions, as rational bidders should anticipate that their behavior in one auction will, directly or indirectly, affect their payoffs in the next.

In procurement contexts, this strategic linkage is certainly relevant when sellers have capacity constraints. This has been documented empirically by Jofre-Bonet and Pesendorfer (2000, 2003) and De Silva (2005), who show that, in auctions for road construction contracts, firms that won previous auctions typically participate less and bid less aggressively in later auctions. The idea is that, given that completing a project requires several months whereas new contracts are auctioned off at high frequency, a firm that is awarded a contract, having more committed capacity, may not have the necessary resources to carry out future projects, or can obtain them only at relatively high cost. In other words, when firms have capacity constraints, winning an early auction entails a opportunity cost, as the firm will lose the opportunity to effectively compete in the next, where market conditions may possibly be more favorable. Inspired by these findings, Brosig and Reiß (2007) and Saini and Suter (2015) have then tested in the lab whether subjects do indeed properly account for this opportunity cost. However, these papers assume that bidders' capacities are common knowledge, thus potentially missing other concurring strategic effects. In particular, when bidders may have limited capacities but this information is privately held, the opportunity cost of winning an early auction for capacity constrained bidders interplays with a signaling cost: bidders that are far from their capacity limits may anticipate that their bids might signal their actual capacity, thereby affecting the intensity of competition in future auctions. This adds complexity to the bidders' tasks and makes the analysis of observed bids more challenging.

To investigate how bidders react to these strategic forces, we design an experiment with two sequential first-price auctions, each involving a single unit, and two sellers, who may have one or two units to sell. While the information on capacity is privately held, costs are common knowledge and, for simplicity, normalized to zero. At the end of the first auction, the outcome and the winning bid are revealed and bidders elicit their beliefs on the opponent's capacity. To match real-world situations where firms are unsure about the demand or even the occurrence of future auctions, we also introduce (exogenous) uncertainty about the realization of the second auction. We consider four treatments, which differ in the ex-ante probability distribution of bidders' capacities and in the degree of uncertainty about the second auction's implementation.

Our results can be summarized as follows. In the second auction, and given the beliefs expressed at the end of the first, observed bids match the main predictions associated with the Perfect Bayesian Equilibrium of the game: we take this as evidence of sequentially rational

behavior. On the other hand, by jointly analyzing the behavior in the first auction and the belief elicitation phase, we detect a clear-cut inconsistency between bids and beliefs: although bids in the first auction are significantly and negatively affected by capacity, bidders, upon stating their beliefs on the opponent's capacity, seem to be unable to recognize this link. In fact, beliefs, net of a partial reversion to the 50-50 odds, are very much aligned with prior probabilities.

In an attempt to reconcile this inconsistency with some form of behavioral rationality, we conjecture that bids in the first auction, beyond the type capacity, are also affected by a hidden, behavioral type, that we relate to the strategic sophistication of bidders. Loosely speaking, more sophisticated subjects have a superior ability to anticipate the opponent's behavior: as a consequence, as long as anticipated bids are not too low, they will underbid less sophisticated bidders. Therefore, a low observed bid can be rationalized in two ways: as the bid of a lowly sophisticated bidder with large capacity, or as the bid of a highly sophisticated bidder with small capacity. As a result, a particular observed bid does not provide enough information to distinguish the actual capacity of the opponent. We show that this intuition can be supported by a simple level-k model of strategic interaction.

We believe, that, beyond its theoretical interest, investigating the consequences on behavior of private information about capacity constraints has also practical relevance. Hortaçsu and Puller (2008) claim that, in electricity markets, it is realistic to assume that generation costs are common knowledge across firms. Instead, it is less likely that firms have accurate information regarding each other's available capacities at the time of bidding. This is because generating firms that participate in the auctions usually also trade electricity through bilateral forward contracts with electricity users. It is unrealistic to believe that firms perfectly know the exact contract positions of their rivals at the time of bidding. Similar considerations are likely to be applicable also in other procurement markets, especially those in which firms, beyond competing for contracts tendered by public authorities, also operate in the private sector.

The rest of the article is organized as follows. Section 2 reviews the related literature. Section 3 presents the experimental design and the theoretical predictions under standard bidders' preferences and equilibrium behavior. Section 4 analyzes the experimental results. Section 5 discusses the results and proposes some behavioral arguments to organize the empirical findings. Finally, Section 6 concludes.

2   Related Literature

In the benchmark model of sequential auctions (see Milgrom and Weber, 2000), all bidders are assumed to have unit demand and private valuations.[1] This model highlights that, when a bidder has limited demand, the following trade-off emerges: a bidder active in a certain round knows that, if she does not win the current auction, there will be one less opportunity to get the unit on sale; on the other hand, in the next round, she will face fewer and weaker (i.e. with lower valuations) competitors. The remarkable result, which follows from an arbitrage argument, is that these two forces perfectly offset, leading to the *law of one price*: expected

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[1]It is worth remarking here that, whereas in standard auctions the auctioneer is the seller and the bidders are the buyers, in procurement auctions it is the opposite. A procurement auction with capacity constrained bidders corresponds to a standard auction with limited demand bidders. A bidder's cost in a procurement auction plays the same role as a bidder's valuation in a standard auction.

prices are equal across different rounds.

Inspired by the observation that, in the real world, when homogenous goods are sold sequentially, awarding prices seem rather to decrease over selling rounds (see, e.g., Ashenfelter, 1989), the subsequent literature on sequential auctions has mainly concentrated on finding theoretical explanations to this *declining price anomaly*. These contributions provide conditions on bidders' preferences or on the market setting such that this price path can arise in equilibrium.[2]

Keser and Olson (1996) are among the first to document the decreasing price anomaly in the lab. They argue that bidders' risk aversion can only partially explain the observed pattern of bids. Neugebauer and Pezanis-Christou (2007) run an experiment involving a sequence of first-price auctions where, in each round, bidders know that the probability there will be another auction is smaller than one. Their results provide support for the declining price path (they find that the larger the uncertainty over supply, the higher the decline in average purchasing prices); however, they record quantitative deviations from the theoretical predictions, which they attribute to a distorted perception by bidders in the likelihood that another auction will take place later on.

That capacity constraints/limited demand may constitute a crucial determinant of bidders' behavior in sequential auctions became clear after the appearance of a few empirical papers on recurring procurement auctions. In particular, using data from repeated highway construction procurement auctions in California, Jofre-Bonet and Pesendorfer (2000, 2003) show that firms with higher backlogs (measured as the dollar value of the amount of work that is left to do from previously won projects) are less likely to submit a bid and, when they do, they make higher bids. De Silva (2005), using data from procurement auctions of road construction contracts in Oklahoma, shows that firms with more committed capacity have lower probability of winning. This evidence suggests that, for a firm that is capacity constrained, winning the current auction entails an opportunity cost, as this would prevent her from competing effectively in later auctions.[3]

A few subsequent papers have then tested experimentally whether this opportunity cost is properly accounted for by individuals. In particular, Brosig and Reiß (2007) consider a sequence of two procurement auctions where each bidder has a capacity of one unit, and this is common knowledge; bidders' production costs are private information, but they are in general different in the two auctioned project. In their model, a bidder, knowing her costs for the two projects in advance, may want to skip the current auction if her cost for the second project is (sufficiently) lower.[4] Upon bidding in the second auction, bidders are informed of the entry decision of the other bidder (i.e. they know whether they will face an opponent or not). The results of their experiment show that a majority of the subjects makes the correct entry decision in the first auction, thus broadly supporting the idea that they are aware of the

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[2]Deltas and Kosmopoulou (2004) provide a summary of these explanations. Interestingly, based on their empirical exercise involving rare book sequential auctions, they argue that none of the proposed equilibrium theory can fully account for the price trend they observe and that non-strategic (i.e. behavioral) factors are likely to matter. In particular, they attribute the observed bidding pattern (which displays increasing prices and decreasing probability of sale) also to a limited attention effect related to the order in which the items are presented in the catalogue.

[3]Based on evidence on sequential water auctions run in Spain, Donna and Espín-Sánchez (2018) argue that the limited demand of water by farmers, especially in rainy seasons, explains part of the observed pattern of bids.

[4]In fact, if a bidder participates in the first auction, she may unwillingly win it even if she bids the reserve price.

opportunity cost associated with first auction bids. On the other hand, bidding strategies in the second auction significantly depart from the theoretical prediction: instead, they closely track those associated with the equilibrium of a one-shot auction, as if they neglected the signaling effect conveyed by the decision of the opponent to skip the first auction. However, having no information on the subjects' actual beliefs (there is no belief elicitation in their experiment), the authors are neither able to confirm this hypothesis, nor to assess whether subjects anticipate this lack of updating in their entry decisions.

Saini and Suter (2015) run an experiment in which sellers are not literally capacity constrained. However, the winner of the current auction will experience a (probabilistic) cost increase in the next. In their setup, the identity of the winner is communicated at the end of each round, so that the loser (winner) of the previous auction knows she is going to face an opponent with stochastically higher (lower) cost in the current one. In other words, no issue related to the signaling effect of previous bids is present. While their main focus is on collusive behavior in sequential auctions with potentially infinite horizon, they also run one treatment with a sequence of two auctions. Results from this latter treatment show that, although bidders appear to be aware of the opportunity cost associated with winning the first auction, they seem to underestimate its real magnitude, given that their first auction bids are below what predicted by theory.[5]

All the experimental literature reviewed above has focused on situations where the private information involves the cost/value parameter specific to each bidder. Instead, each bidder's capacity/demand has always been assumed to be common knowledge. The novelty in our experiment is that we assume that capacities are private information. This introduces an additional strategic element in the game, in that a bidder that is not capacity constrained has to take into account a different opportunity cost: her bid in the first auction may reveal some information about her actual capacity, and this may dramatically affect the intensity of competition in the second. To investigate the impact of this signaling cost, we explicitly require subjects to elicit their beliefs at the end of the first auction. This allows us to deeply scrutinize the determinants of their behavior and to provide a belief-based explanation to the observed departure from the theory.

In this last respect, our work provides a deeper understanding of the belief updating process in sequential auctions. This element has been largely overlooked in the received experimental literature. In fact, in most of these papers, the signaling issues were either fully avoided by the choice of the experimental design (this is the case in Saini and Suter, 2015, and Otamendi et al., 2018, where the type of each bidder is re-drawn in every round, so a bid in one auction does not convey any relevant information. See also Leufkens et al., 2012), or, when present, were not deeply investigated (like in Brosig and Reiß, 2007). One relevant exception in this sense is the article by Cason et al. (2011): in their experiment involving two procurement auctions with unconstrained sellers and privately known costs, they envisage a belief elicitation phase at the end of the first auction. The descriptive analysis of these results seems to indicate that the belief updating process by bidders is qualitatively correct.

More generally, our article contributes to the literature on belief updating in dynamic games with incomplete information. In particular, the fact that the behavior we observe in the second auction is (essentially) sequentially rational is consistent with several other works

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[5]In the context of standard sequential auctions, Février et al. (2007) and Otamendi et al. (2018) reached a similar conclusion that, in general, bidders tend to underestimate the opportunity cost of winning an early auction.

that suggest that subjects use their stated beliefs as the basis of their choices (see, e.g., Schotter and Trevino, 2014). On the other hand, we observe a clear inconsistency between behavior and stated beliefs, which is not a new result in the experimental literature. Our contribution is to provide an original explanation of this missed belief updating without resorting to any cognitive bias, but showing how this may be reconciled with rational, though non-equilibrium, behavior.

3 Experimental Design and Theory

3.1 Baseline Game and Treatments

Our experimental setup consists of two sequential first-price auctions with incomplete information. Two sellers participate in two consecutive auctions, $A1$ and $A2$: in each auction, they compete to sell one unit of a homogeneous good to an hypothetical buyer, and the buyer buys the unit from the seller posting the lowest price, provided this price does not exceed a commonly known reserve price, R, that was set equal to 120. Sellers do not bear any cost for producing the units they sell. However, sellers may or may not be capacity constrained: specifically, before $A1$ starts, each seller is randomly assigned 1 or 2 units of the good. Let c denote a seller's capacity (i.e. c is the seller's *type*), and p be the ex-ante probability that $c = 2$.[6] Clearly, if her capacity is $c = 2$, the seller will participate in $A2$, whatever the outcome of $A1$ is. Instead, if her capacity is $c = 1$, the seller can participate in $A2$ only if she does not win $A1$. Each seller knows her own but not the opponent's capacity, whereas p is common knowledge. At the end of $A1$, and before $A2$ begins, sellers are informed on whether they won $A1$ and on the winning bid. Finally, bidders know from the outset that the outcome of $A2$ will be implemented (the winner will sell the good and will be paid her bid) only with probability q; instead, with the remaining probability, $(1 - q)$, $A2$ will be revoked, and no bidder will receive anything.[7] Whether the second auction is implemented or revoked is communicated to bidders at the end of it.

We consider four treatments: $T1$, $T2$, $T3$ and $T4$. Treatments differ in the parameters p and q. Specifically, p, the prior probability of having capacity $c = 2$, is equal to 0.5 in $T1$ and $T3$, to 0.75 in $T2$ and $T4$; q, the probability that $A2$ is implemented, is equal to 0.5 in $T1$ and $T4$, to 1 in $T2$, to 0.25 in $T3$. These values have been chosen to produce pairwise equal (pooling) equilibrium bids in the two auctions: specifically, predicted bids in $A1$ are the same in $T1$ and $T2$, as they are in $T3$ and $T4$; predicted bids in $A2$ are the same in $T1$ and $T3$, as they are in $T2$ and $T4$. Treatments' parameters and the theoretical pooling equilibrium bids are reported in Table 1. Notice that, in all treatments, predicted bids are much higher in $A2$ than in $A1$.

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[6]Throughout the article, when we talk about a seller's capacity, we always intend her *initial* capacity. When, instead, we want to refer to the remaining capacity of a seller after the end of the first auction, we will explicitly talk about *residual* capacity.

[7]This element is introduced to match real world situations: in recurring procurement auctions, it is reasonable to believe that firms may expect another auction to take place in the near future, but are not certain on whether and when; in electricity markets, the demand for electricity in the balancing market is not known a priori; this uncertainty has recently been exacerbated by the increased penetration of intermittent renewable sources. Moreover, the assumption of uncertainty regarding future auctions is also made in other related experiments (see, e.g., Neugebauer and Pezanis-Christou, 2007, Saini and Suter, 2015).

Table 1 – Treatments: parameters and pooling equilibrium predictions.

	p	q	N	a	$b_{\min}$	$\mathbb{E}[b_W]$	$\mathbb{E}[b_L]$
$T1$	0.5	0.5	48	30	60	83.2	101.6
$T2$	0.75	1	48	30	30	55.5	71.6
$T3$	0.5	0.25	48	15	60	83.2	101.6
$T4$	0.75	0.5	48	15	30	55.5	71.6

Notes: N is the number of subjects; a is the pooling equilibrium bid in $A1$; $b_{\min}$ is the lower bound of the equilibrium bidding (mixed) strategy in $A2$; $\mathbb{E}[b_W]$ and $\mathbb{E}[b_L]$ are the expected values of equilibrium bids in $A2$ placed by Winners and Losers of $A1$, respectively.

3.2 Equilibrium Predictions

Our benchmark model is based on the hypothesis of risk neutral bidders and equilibrium behavior. For each auction, given that production costs are null, a bidder's payoff is simply given by the winning price in case of win, whereas it is equal to zero if she does not win the auction or, for $A2$, if she wins but the auction is revoked. A bidder's total payoff is simply given by the (undiscounted) sum of the payoffs obtained in the two auctions. Being a dynamic game with incomplete information, our equilibrium concept will be that of Perfect Bayesian Equilibrium (PBE). We will use the letters a and b to denote bids in $A1$ and $A2$, respectively; and the greek letters α and β to denote mixed strategies.

Following a backward induction logic, we begin our analysis with the second auction. We will refer to the bidder who won (lost) the first auction as the Winner (the Loser). This distinction is due to the fact that the outcome of $A1$ introduces a fundamental heterogeneity between the two bidders: whereas the Winner knows that she will surely face an opponent in $A2$, the Loser should rationally anticipate that this will be the case only if the Winner's (initial) capacity was $c = 2$.

The concept of PBE requires that bidders' behavior in $A2$ must be sequentially rational: *given their beliefs*, bids in $A2$ maximize bidders' expected payoffs. The proposition below characterizes it.[8]

PROPOSITION 1. *Let μ_L (μ_W) denote the probability that the Loser (the Winner) assigns to the fact that the capacity of the Winner (the Loser) was two units. In a Perfect Bayesian Equilibrium, bids in $A2$ are the following:*

(i) *For $\mu_L \in (0, 1]$, the (mixed) strategy of the Winner (i.e. a bidder with capacity $c = 2$ who won the first auction) is given by the following distribution function:*

$$\beta_W(b) = \begin{cases} 0 & if \quad b < (1 - \mu_L)R \\ \frac{b - (1 - \mu_L)R}{\mu_L \cdot b} & if \quad (1 - \mu_L)R \leq b \leq R \end{cases} .$$

The Loser bids $b_L = R$ with probability $1 - \mu_L$; conditional on not bidding R, the Loser bids according to the mixed strategy $\beta_L(b|b \neq R) = \beta_W(b)$.

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[8]Notice that, to allow us to use calculus, the theoretical analysis in this section is carried out under the hypothesis that bids can be any real number between 0 and R. This is quite an accurate approximation, given that, in the experiment, any integer bid was allowed. The proofs of the Propositions are in the Appendix.

(ii) For $\mu_L = 0$, the Loser bids $b_L = R$ and the Winner bids $b_W = R - \varepsilon$.

Proposition 1 shows that sequentially rational bidding strategies depend on the first auction's outcome in two respects: first, in general, the strategies of the Winner and the Loser are different; second, they depend on the Loser's belief μ_L.

In particular, the expected bid of the Loser is strictly decreasing in μ_L. In fact, for $\mu_L \in [0, 1)$, the expected bid of the Loser is $\mathbb{E}[\beta_L(b)] = (1 - \mu_L)R[1 - \ln(1 - \mu_L)]$, with first derivative (with respect to μ_L) equal to $R \times \ln(1 - \mu_L) < 0$. This is easily understandable once one considers that μ_L – the Loser's belief about the probability that the (initial) capacity of the Winner was $c = 2$ – coincides with the likelihood that the Loser will indeed face an opponent in $A2$. Clearly, for strategic response, also the Winner's bid depends (negatively) on μ_L: for $\mu_L \in (0, 1)$, the expected bid of the Winner is $\mathbb{E}[\beta_W(b)] = -\frac{(1-\mu_L)R}{\mu_L} \ln(1 - \mu_L)$, with first derivative (with respect to μ_L) equal to $R \times \frac{\mu_L + \ln(1 - \mu_L)}{\mu_L^2} < 0$.

On the other hand, μ_W does not enter anywhere in the bidding functions. This is obvious as, conditional on participating, the Winner knows that she will face an opponent for sure, because the Loser will certainly have strictly positive residual capacity. Hence, whether the Loser's initial capacity was one or two units is totally irrelevant to the Winner.

Notice also that, except for the case in which $\mu_L = 1$, the Loser's bid is, on average, strictly larger than the Winner's bid, i.e. $\mathbb{E}[\beta_L(b)] > \mathbb{E}[\beta_W(b)]$, for all $\mu_L \in [0, 1)$. This is due to the fact that the Loser bids R with strictly positive probability, whereas the Winner never does so (R is outside the support of $\beta_W(b)$). By bidding R, the Loser wins only if she does not face any opponent (the capacity of the Winner was equal to 1), but, if this happens, her payoff is the largest.

Finally, neither p nor q enter anywhere in the bidding functions: this means that the treatment variables p and q may affect bidding behavior in $A2$ only through the beliefs.

We summarize the above considerations in the following testable predictions:

A2-(a) Neither p nor q have a direct impact on bids: hence, for given beliefs, *bids are the same in the four treatments.*

A2-(b) The Loser's bid negatively depends on her belief; the Winner's bid is unaffected by her belief.

A2-(c) The expected bid of the Loser is higher than the expected bid of the Winner. This difference is due to the fact that the Loser bids R with strictly positive probability.

After characterizing rational bidders' behavior in $A2$ *for given beliefs*, let's now consider bidding in the first auction and how bidders update their beliefs at the end of the auction. We will denote by a_c and α_c, a pure and a mixed strategy of a bidder with capacity c. The following proposition characterizes the PBE bidding strategies in $A1$ and the corresponding beliefs.

PROPOSITION 2.

(i) There is no separating equilibrium, either in pure or in mixed strategies.

(ii) *The following is the unique pooling equilibrium: $a_1 = a_2 = a^* = q(1-p)R$, with Loser's belief: $\mu_L(a^*) = p$, $\mu_L(a) \geq (1+p)/2 - (a^* - a)/(qR)$ for $a < a^*$, $\mu_L(a) \in [0,1]$ for $a > a^*$.*

(iii) *Suppose that (α_1, α_2) is a hybrid equilibrium (necessarily in mixed strategies). Let A_1 and A_2 be the supports of α_1 and α_2, respectively, and let $A = A_1 \cap A_2$. Then: (a) $A_2 = A$; (b) either $A_1 \setminus A_2 = \emptyset$, or, for all $a' \in A_1 \setminus A_2$ and all $a'' \in A_2$, it must be $a' > a''$; (c) for all $a', a'' \in A$, if $a' < a''$, then $\mu_L(a') > \mu_L(a'')$; (d) $\inf A \geq q(1-p)R$.*

The intuition behind the absence of a separating equilibrium (point *(i)* of Proposition 2) is straightforward: for a bidder with capacity $c = 2$, revealing her type (as it would happen in a separating equilibrium) is extremely costly because this would imply a null payoff in $A2$. Hence, a bidder with capacity $c = 2$ is willing to reveal her type only if, by doing so, she expects to get a sufficiently large payoff from the first auction. But if this is the case, a bidder with initial capacity $c = 1$, who is totally uninterested in $A2$ in case of winning, would rather mimic a bidder with capacity $c = 2$.

To see why the one described in point *(ii)* of Proposition 2 is an equilibrium, consider first a bidder with capacity $c = 1$. This bidder knows that, if she pools with the other type (i.e. a bidder with capacity $c = 2$) at some bid $\hat{a}$, she may win the auction and obtain payoff equal to $\hat{a}$, or may lose it: in the latter case, she will enjoy the expected payoff from $A2$, equal to $q(1-p)R$. Hence, if $\hat{a} < q(1-p)R$, she will strictly prefer to lose $A1$ (bidding anything above $\hat{a}$); if, on the other hand, $\hat{a} > q(1-p)R$, she will strictly prefer to win $A1$ (bidding slightly below $\hat{a}$); only if $\hat{a} = a^* = q(1-p)R$, this bidder does not want to deviate from the pooling bid.[9] Consider now a bidder with capacity $c = 2$. Clearly, relative to a bidder with capacity $c = 1$, this bidder has stronger incentive to win the first auction. She will abstain from undercutting a^* (winning $A1$ for sure) only if such a lower bid significantly reduces her expected payoff from $A2$, i.e. significantly increases the Loser's belief.

The third part of Proposition 2 characterizes a fundamental property that any hybrid equilibrium, if it exists, must satisfy in our context: the Loser's belief must be strictly decreasing in bids.[10] Here is an intuitive argument: consider two bids $a', a'' \in A = A_1 \cap A_2$, with $a' < a''$.[11] Clearly, both types of bidders must be indifferent between bidding a' and a'', i.e. $\pi_1(a') = \pi_1(a'')$ and $\pi_2(a') = \pi_2(a'')$. But then we have $\pi_2(a') - \pi_1(a') = \pi_2(a'') - \pi_1(a'')$: the difference between type-2 and type-1 expected payoffs must be constant as well. Now, for given bid, the difference between the expected payoff of a type-2 bidder and the expected payoff of a type-1 bidder is associated to the fact that, in case of winning $A1$, a type-2 bidder will also take part in $A2$, where she expects to get an additional payoff equal to $q(1-\mu_L(a))R$. Hence, for given bid, we have $\pi_2(a) - \pi_1(a) = q(1-\mu_L(a))R \times \text{PW}(a)$, where $\text{PW}(a)$ is the probability of winning $A1$ with a bid equal to a. Now, consider what happens when a type-2 bidder decreases her bid from a'' to a': the probability of winning $A1$ clearly increases. Thus, to keep the payoff difference constant, the expected payoff obtainable from $A2$ ($q(1-\mu_L)R$) has to decrease, i.e. μ_L must increase.

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[9]This argument implies that there can be no pooling equilibrium in mixed strategies as well. It also implies that no bid below $q(1-p)R$ can be part of an equilibrium strategy in $A1$.

[10]More precisely, the Loser's belief is strictly decreasing on A (the intersection of the supports of the mixed strategies). As stated in Proposition 2, the mixed strategy of a bidder with capacity $c = 1$ may also include bids greater than A; for these bids, the Loser's belief must be equal to 0.

[11]Notice that, in a hybrid equilibrium, the intersection of the supports must be non-empty.

An immediate corollary of part *(iii)* of Proposition 2 is that the expected bid of a bidder with capacity $c = 2$ must be strictly lower than the expected bid of a bidder with capacity $c = 1$.

The previous analysis sheds light on the relative importance of the two strategic forces at play in our setting: the opportunity cost and the signaling cost of early bids. Everything else equal, the opportunity cost induces bidders with capacity $c = 2$ to bid more aggressively than those with capacity $c = 1$, as the former have a stronger pressure to sell immediately (they have two units to sell). On the other hand, the signaling cost of early bids pushes bidders with capacity $c = 2$ to mimic those with capacity $c = 1$ in order to avoid revealing their true capacity. Equilibrium predictions imply that the signaling cost is stronger: a bidder with capacity $c = 2$ prefers to significantly reduce her chance of winning the first auction than to fully reveal her capacity.

To sum up, equilibrium behavior in $A1$ produces the following testable predictions:

A1-(a) [POOLING EQUILIBRIUM] *Either bidders make the same bid $a^* = q(1-p)R$, regardless of their capacities: in this case, the Loser's belief must be equal to the prior probability;*

A1-(b) [HYBRID EQUILIBRIUM] *or, the expected bid of a bidder with capacity $c = 2$ is strictly lower than the expected bid of a bidder with capacity $c = 1$, and both are strictly larger than $q(1-p)R$: in this case, the Loser's belief must be strictly decreasing in the winning bid.*

Notice that, in these predictions, the treatment parameters, p and q, only affect the bid levels. Therefore, the qualitative features of the equilibria hold for all treatments.[12]

3.3 Procedures

Upon their arrival, subjects were randomly assigned to a computer terminal. In all sessions, instructions were distributed at the beginning of the experiment and read aloud. Before the experiment started, subjects were asked to answer a number of control questions to make sure they understood the instructions as well as the effects of their choices. When necessary, answers to these questions were privately checked and explained. At the beginning of the experiment, the computer randomly formed four rematching groups of 6 subjects each. The composition of the rematching groups was kept constant throughout the session. At the beginning of every period, subjects were randomly and anonymously divided into pairs. Pairs were randomly formed in every period within rematching groups. Subjects were told that pairs were randomly formed in a way that they would never interact with the same opponent in two consecutive periods.[13]

In every period, subjects participated in three consecutive phases: (i) $A1$, (ii) a belief elicitation procedure, and (iii) $A2$. In $A1$, after being informed about their own capacity,

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[12]Notice, in particular, that a change in q (the probability that the second auction will be implemented) affects the opportunity cost and the signaling cost of early bids in the same direction: a lower q reduces the incentive for a bidder with capacity $c = 2$ to mimic a bidder with capacity $c = 1$, but also reduces the incentive of the latter to bid higher. As a result, the equilibrium (qualitatively) does not change.

[13]Our rematching protocol implies that, given the size of the sub-groups (6 subjects), on average subjects interacted with the same opponent every 5 periods. Although this does not represent perfect stranger protocol, it leaves very little room for developing punishment-reward strategies over multiple periods. The rematching protocol was intended to increase the number of independent observations and perform non-parametric tests to check the robustness of the main parametric results.

subjects choose simulatenously their bids. Bids were restricted to be integer numbers between 1 and the reserve price, that was set to 120. Then, once choices were posted, each subject received feedback about the winning bid and her corresponding payoff. In the second phase, subjects were asked to elicitate their beliefs on the capacity of the opponent. To this end, we relied on the Binary Lottery Procedure (McKelvey and Page, 1990; Schlag and van der Weele, 2013; Hossain and Okui, 2013; Harrison et al. 2014) as a proper incentive compatible belief elicitation mechanism. In particular, subjects were presented two boxes on the screen. In the first box, they had to indicate the probability that the opponent's capacity was $c = 1$, whereas, in the second, the probability that the opponent's capacity was $c = 2$. Both probabilities were restricted to be integers between 0 and 100, and to sum up to 100. Let η and μ denote the subjective probabilities attached by a subject to an opponent's capacity of one and two units, respectively. Each subject was informed that, at the end of the period, she would have participated in either of two possible lotteries: the first implemented if the opponent's capacity was indeed $c = 1$, and the second if it was indeed $c = 2$. The number of tickets assigned to the subject to participate in each of the two lotteries depended on the reported probabilities. In particular, conditional on the actual capacity of the opponent, the number of lottery tickets were computed by using the following two quadratic rules:

$$tickets(1u) = 10000[1 - (1 - \eta/100)^2]$$

$$tickets(2u) = 10000[1 - (1 - \mu/100)^2]$$

Thus, from the previous expressions and depending on her stated probabilities, each subject received a number of tickets between 0 and 10000 for each lottery. For simplicity, the tickets were numbered in ascending order, starting from 0 to the total number of tickets assigned to the subject. At the end of the period, after being informed about the actual capacity of the opponent, each subject entered to the corresponding lottery. In particular, the computer randomly selected one of the 10000 tickets, numbered from 1 to 10000. If the subject possessed the selected ticket, then she received 20 points to be added to her overall earnings in the period.

Finally, in the third phase, subjects whose residual capacity at the end of $A1$ was not null, competed in $A2$. Again, subjects simultaneouslty chose their bids and were then informed about the winning bid as well as their payoffs.[14]

Two features of the experimental design were specifically intended to facilitate learning and make personal history easily accessible. First, at the beginning of the experiment subjects were endowed with an hard copy record sheet that organizes bids and stated beliefs by periods. Second, upon making their decisions in the second and third phase of the experiment, their choices and information about the previous tasks were visualized on the screen. At the end of every period, the results of the two auctions were summarized on the screen, together with the decision on the annulment of the second auction, with the outcome of the belief lottery and with their overall earnings in the period.

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[14]It might be argued that our payment scheme might induce risk-averse subjects to hedge with their stated beliefs against adverse outcomes in the two auctions. However, there are at least three reasons to believe that the (potential) hedging problem plays a marginal role in our setting. First, there is experimental evidence suggesting that hedging is not a major problem in strategic interaction settings, unless hedging opportunities are very prominent (Blanco et al, 2010). Second, the maximum amount subjects could get from the belief elicitation phase was relatively small when compared to the money at stake in the two auctions. Third, in order to avoid confusion-driven pseudo-hedging, subjects were explicitly instructed that, by stating their beliefs truthfully, they could have minimized the penalization due to errors and maximized the corresponding gains.

For each of the 4 treatments, we run 2 sessions, each involving 24 subjects, thus generating 8 independent observations at the rematching group level. The experiment took place at the Bocconi Experimental Laboratory for Social Sciences (BELSS) of Bocconi University, Milan, between November and December 2016. Participants were mainly undergraduate students, recruited by using the SONA recruitment system (http://www.sona-systems.com/default.aspx). The experiment was computerized using the *z-Tree* software (Fischbacher, 2007). At the end of the experiment, the number of points obtained by a subject during the experiment was converted at an exchange rate of 0.02 euro per 1 point and monetary earnings were paid in cash privately. On average, subjects earned 16.67 euro for sessions lasting 75 minutes, including the time for instructions and payments. Before leaving the laboratory, subjects completed a short questionnaire containing questions on their socio-demographics and their perception of the experimental task.

4   Experimental Results

In line with the structure of the theoretical part, we present the results backward: we first analyze behavior in $A2$, looking at the differences in bid levels across treatments and at the relationships with the outcome – either winning or losing – of $A1$ and with the stated beliefs. Then, we will jointly study bids in $A1$ and stated beliefs, investigating differences across treatments and determinants of bidding behavior, in order to test whether subjects' choices were coherent with equilibrium beavior, characterized in Section 3.2. The non-parametric tests presented below are based on 8 independent observations (at the rematching group level) per treatment. Similarly, in the parametric analysis, we either cluster standard errors or introduce random effects at the rematching group level to control for dependency of observations over repetitions. All regressions pool data from the four treatments and use $T3$ as baseline.

4.1   Bids in $A2$

In order to analyze bids in $A2$, we only consider subjects with strictly positive residual capacity at the end of $A1$, namely those who did not win $A1$ (the Losers) and those who won $A1$ and had (initial) capacity $c = 2$ (the Winners). Figure 1 shows, for every treatment, the distribution of bids in $A2$ and their evolution over periods, whereas the first column of Table 2 reports descriptive statistics. Moreover, as highlighted by the theoretical section, bids in $A2$ should depend on the outcome of $A1$. For this reason, Figure 2 and the last two columns of Table 2 provide information about bids in $A2$ of Winners and Losers, separately.

The preliminary descriptive analysis highlights several interesting facts. First, bids in $T1$ and $T3$ are above those in $T2$ and $T4$, both when pooling observations and when splitting the sample according to the outcome of $A1$. Second, Losers tend to bid higher than Winners. Finally, bids in $A2$ do not exhibit any clear time pattern over periods.

In order to test the statistical validity of these preliminary empirical observations, Table 3 reports parametric results on the determinants of bids in $A2$.

Estimates reported in column (1) can be used to assess differences in bids across treatments. Bids are higher in $T1$ than in $T2$ ($\chi^2(1) = 16.27$, $p < 0.001$) and $T4$ ($\chi^2(1) = 8.63$, $p = 0.003$), whereas the difference between $T1$ and $T3$ (although positive) is not significant ($p = 0.247$). No significant differences are found between $T2$ and $T4$ ($\chi^2(1) = 1.21$, $p = 0.272$). We document higher bids in $T3$ than in $T2$ ($p = 0.004$) and (marginally) $T4$
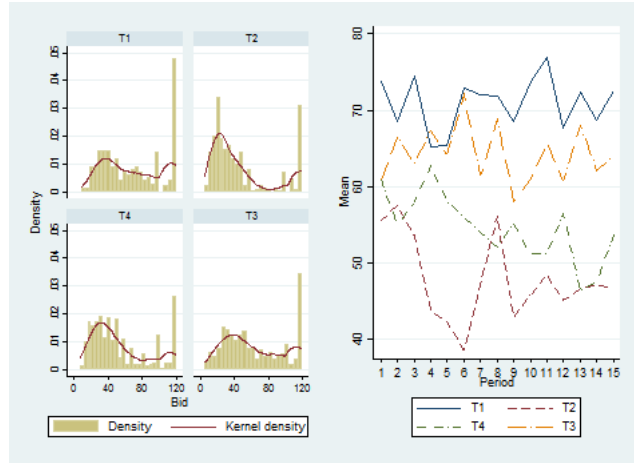
Figure 1 – Bids in $A2$: pooling.
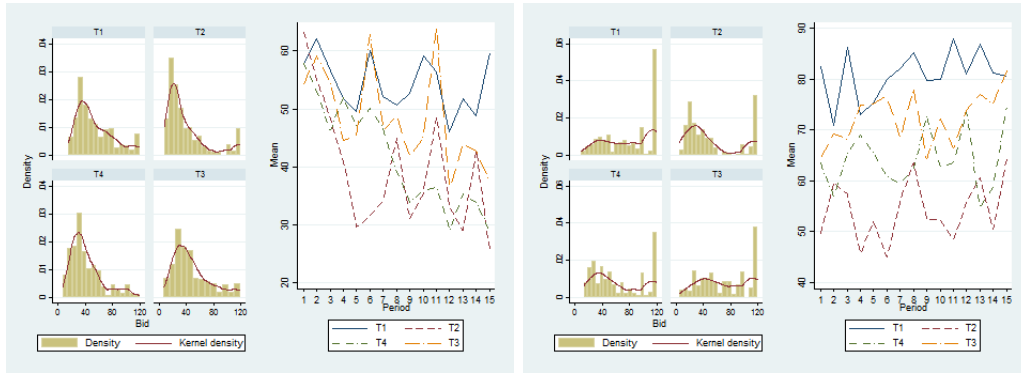
Figure 2 – Bids in $A2$: Winners (left panel) and Losers (right panel) of $A1$.

Table 2 – Bids in $A2$.

	b	$Obs.$	b_W	$Obs.$	b_L	$Obs.$
$T1$	70.960	569	53.937	209	80.842	360
	(35.251)		(26.595)		(35.911)	
$T2$	47.807	633	39.414	273	54.172	360
	(36.626)		(29.743)		(39.964)	
$T3$	64.147	544	48.033	184	72.383	360
	(35.528)		(27.435)		(36.392)	
$T4$	54.444	633	41.582	273	64.197	360
	(33.809)		(23.446)		(37.076)	

Notes: This table reports means and standard deviation (in parentheses) of bids in the second auction of each treatment and overall periods. Descriptives are presented both by pooling subjects (b), and by splitting the sample according to the outcome of the first auction (b_W for Winners and b_L for Losers).

Table 3 – Bids in $A2$: parametric analysis.

	b				b_W	b_L
	(1)	(2)	(3)	(4)	(5)	(6)
$T1$	6.603	1.615	9.630	8.003	1.096	11.799
	(5.704)	(6.702)	(6.783)	(7.108)	(8.182)	(8.218)
$T2$	-16.322***	-8.292	-2.823	-0.672	5.641	6.503
	(5.690)	(7.575)	(7.599)	(7.842)	(9.247)	(9.386)
$T4$	-10.095*	0.928	1.794	4.965	4.226	1.835
	(5.690)	(7.278)	(7.204)	(7.501)	(8.908)	(8.859)
μ		-0.240***	-0.247***	-0.247***	0.055	-0.387***
		(0.061)	(0.058)	(0.059)	(0.105)	(0.073)
$T1 \times \mu$		0.117	0.005	-0.001	-0.049	-0.048
		(0.079)	(0.075)	(0.076)	(0.132)	(0.093)
$T2 \times \mu$		-0.046	-0.121	-0.112	-0.238*	-0.218*
		(0.089)	(0.085)	(0.085)	(0.138)	(0.111)
$T4 \times \mu$		-0.104	-0.099	-0.093	-0.152	-0.017
		(0.085)	(0.080)	(0.080)	(0.132)	(0.101)
w_{A1}			-20.864***	-20.861***		
			(2.492)	(2.491)		
$T1 \times w_{A1}$			-5.222	-5.353		
			(3.538)	(3.538)		
$T2 \times w_{A1}$			3.801	3.892		
			(3.365)	(3.364)		
$T4 \times w_{A1}$			2.125	2.164		
			(3.343)	(3.341)		
$Period$				-0.007	-1.342***	0.715**
				(0.266)	(0.378)	(0.324)
$T1 \times Period$				0.273	0.856*	0.119
				(0.372)	(0.513)	(0.458)
$T2 \times Period$				-0.406	-0.325	-0.063
				(0.361)	(0.475)	(0.458)
$T4 \times Period$				-0.513	-0.387	-0.435
				(0.361)	(0.479)	(0.455)
$cons$	64.208***	75.044***	82.480***	82.519***	56.569***	83.629***
	(4.038)	(4.859)	(4.828)	(5.046)	(6.225)	(5.870)
lrl	-11536.58	-11506.98	-11355.97	-11353.57	-4283.03	-6906.19
$Wald - \chi^2$	19.42	95.66	411.16	420.40	102.14	196.21
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
$Obs.$	2379	2379	2379	2379	939	1440

Notes: this table reports estimates (with clustered standard errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the bid in $A2$. The table reports results by pooling observations (columns 1-4), and by splitting the sample between Winners and Losers of $A1$ (columns 5-6). $T1$, $T2$, and $T4$ are treatment dummies. μ is a subject's stated belief that the opponent's capacity was 2 units. w_{A1} is a dummy whose value is 1 if that subject won the first auction. $Period$ is a linear trend that takes value 0 in the first period. $T1 \times \mu$, $T2 \times \mu$, $T4 \times \mu$, $T1 \times w_{A1}$, $T2 \times w_{A1}$, $T4 \times w_{A1}$, $T1 \times Period$, $T2 \times Period$, $T4 \times Period$, are interaction terms. Significance levels are denoted as follows: * : $p < 0.1$; ** : $p < 0.05$; *** : $p < 0.01$.

($p = 0.076$).[15] Overall, this evidence is broadly consistent with the pooling equilibrium (see

[15]Parametric results are generally confirmed by non-parametric tests. According to a (two-sided) Mann-Whitney rank-sum test, we find significant differences in bids between $T1$ and $T2$ ($z = 2.731$, $p = 0.006$), $T1$

Table 1). However, to assess whether bidders' behavior is sequentially rational, we need to analyze it along with the beliefs, which may not be in line with the equilibrium ones.

Therefore, in column (2), we add the beliefs about the probability that the opponent's capacity was $c = 2$. According to the theoretical predictions developed in Section 3.2, differences in bids across treatments should solely be due to differences in beliefs. This prediction is empirically validated by the parametric analysis: after controlling for the stated beliefs, all the pairwise comparisons between average bids become nonsignificant (between $T1$ and $T2$, $\chi^2(1) = 1.78$, $p = 0.182$; between $T1$ and $T3$, $p = 0.810$; between $T1$ and $T4$, $\chi^2(1) = 0.01$, $p = 0.923$; between $T2$ and $T3$, $p = 0.274$; between $T2$ and $T4$, $\chi^2(1) = 1.35$, $p = 0.246$; between $T3$ and $T4$, $p = 0.899$). In all treatments, we also detect a negative and significant effect of beliefs on bids (for $T1$, $\chi^2(1) = 6.17$, $p = 0.013$; for $T2$, $\chi^2(1) = 19.61$, $p < 0.001$; for $T3$, $p < 0.001$; for $T4$, $\chi^2(1) = 34.77$, $p < 0.001$).

Still controlling for the beliefs, column (3) investigates the effects of the outcome of the first auction on second auction's bids. Consistent with the equilibrium predictions, we find that winning in $A1$ significantly reduces bids in $A2$ in all treatments (for $T1$, $\chi^2(1) = 107.88$, $p < 0.001$; for $T2$, $\chi^2(1) = 56.93$, $p < 0.001$; for $T3$, $p < 0.001$; for $T4$, $\chi^2(1) = 70.70$, $p < 0.001$).

Coherently with the previous result, we also detect a large difference between Winners and Losers in the frequency of bids that are equal to 120 (the reserve price). Indeed, whereas the proportion of Winners bidding the reserve price is 1.9% in $T1$, 3.7% in $T2$, 2.7% in $T3$, and 0.4% in $T4$, it is substantially larger when considering Losers: 19.4% in $T1$, 16.1% in $T2$, 20.6% in $T3$, and 15.0% in $T4$. These differences are highly significant in all treatments (according to a two-sided proportion test, $z = 5.99$, $p < 0.001$ in $T1$; $z = 5.01$, $p < 0.001$ in $T2$; $z = 5.59$, $p < 0.001$ in $T3$; $z = 6.47$, $p < 0.001$ in $T4$).

Column (4) adds a treatment specific time trend to the empirical model considered in column (3). Results confirm the initial impression that bids in $A2$ do not exhibit a clear time pattern: the time trend is not significant in $T1$ ($\chi^2(1) = 1.04$, $p = 0.308$) and in $T3$ ($p = 0.978$), it is negative and marginally significant in $T2$ ($\chi^2(1) = 2.87$, $p = 0.091$) and, finally, it is negative and significant in $T4$ ($\chi^2(1) = 4.54$, $p = 0.033$).

To assess how the effect exerted by beliefs on bids in $A2$ is related to the outcome of $A1$, columns (5) and (6) replicate the parametric specification presented in column (4) on the two subsamples of Winners and Losers, separately. As shown in the theoretical section, if bidders were sequentially rational, the effect of beliefs should be strong and negative for Losers, and negligible for Winners. The experimental results generally confirm this prediction: looking at column (5), the effect of Winners' beliefs on their own bids in $A2$ is not significant in $T1$ (the estimated effect is 0.006, $\chi^2(1) = 0.00$, $p = 0.945$), $T3$ (the estimated effect is 0.055, $p = 0.602$), and $T4$ (the estimated effect is -0.097, $\chi^2(1) = 1.43$, $p = 0.232$); only in $T2$, it is negative and significant (the estimated effect is -0.183, $\chi^2(1) = 4.18$, $p = 0.041$). On the contrary, as column (6) shows, this effect is always negative and significant for Losers (for $T1$, the estimated effect is -0.435, $\chi^2(1) = 55.79$, $p < 0.001$; for $T2$, the estimated effect is -0.605, $\chi^2(1) = 52.69$, $p < 0.001$; for $T3$, the estimated effect is -0.387, $p < 0.001$; for $T4$, the estimated effect is -0.404, $\chi^2(1) = 34.05$, $p < 0.001$).

We collect the main empirical findings regarding bidding behavior in $A2$ in the following statements.

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and $T4$ ($z = 2.100$, $p = 0.036$), $T2$ and $T3$ ($z = -2.310$, $p = 0.021$), and (marginally) $T3$ and $T4$ ($z = -1.680$, $p = 0.093$). In all other cases, the difference between treatments is not statistically significant.

R1. **Differences in bids in $A2$ across treatments.** *Differences in bids across treatments disappear once stated beliefs are controlled for.*

R2. **Bids in $A2$ and stated beliefs.** *In all treatments, stated beliefs reduce bids in $A2$. This effect is particularly strong for Losers and negligible for Winners.*

R3. **Bids in $A2$ and outcome of $A1$.** *In all treatments, Losers make substantially higher bids than Winners. Moreover, Losers are more likely to bid the reserve price.*

Together, *R1*, *R2*, and *R3* suggest that subjects' choices in $A2$ are coherent with the sequentially rational behavior summarized in predictions *A2-(a)*, *A2-(b)*, and *A2-(c)*.[16] First, treatment parameters have no direct effect on bids, as bidders rationally condition their behavior only on their beliefs, which summarize the whole relevant information to them. Second, bidders make correct use of their beliefs: Losers' bids depend negatively on their beliefs, whereas Winners' bids are unaffected by their beliefs, which are totally useless. Third, on average, Losers rationally bid more than Winners. This reflects the fact that, upon choosing their bids in $A2$, Winners know that they will compete against an opponent, whereas Losers are in general uncertain about that. In other words, Winners expect more competition than Losers: as a consequence, they make lower bids on average.

4.2 Bids in $A1$ and Beliefs

Figure 3 shows, for each treatment, the distribution of bids in $A1$ and their evolution over periods. Descriptive statistics are reported in the first column of Table 4. Given that, as suggested the by theory, bidders' capacity may affect the level of bids, we also present the same data splitted by capacity (see Figure 4 and the last two columns of Table 4). Notice that bids in $A1$ are lower than those observed in $A2$, though the difference is much less pronounced than what predicted by the (pooling) equilibrium.[17]

Four considerations emerge from a first look at the data. First, at least in early periods, we do not observe remarkable differences across treatments. Second, in all treatments and regardless of their initial capacities, subjects bid substantially more than what predicted by the theoretical pooling equilibrium. Third, in all treatments, bids made by subjects with capacity $c = 2$ are lower than those placed by subjects with capacity $c = 1$. Fourth, in all treatments, bids decline over repetitions.

In order to confirm these preliminary observations and provide further insights on the determinants of bids, Table 5 reports parametric results by pooling data from all treatments.

Column (1) shows that differences in bids across treatments are very small. Indeed, only the difference between $T1$ and $T3$ is marginally significant ($p = 0.086$). Any other difference is not statistically significant (between $T1$ and $T2$: $\chi^2(1) = 2.23$, $p = 0.135$; between $T1$ and $T4$: $\chi^2(1) = 1.42$, $p = 0.233$; between $T2$ and $T3$: $p = 0.824$; between $T2$ and $T4$: $\chi^2(1) = 0.09$, $p = 0.763$; between $T3$ and $T4$: $p = 0.599$).[18]

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[16]The absence of any significant time trend in bids reinforces our conclusion that the experimental evidence is consistent with equilibrium.

[17]Given that an incresing pattern of bids in a procurement auction corresponds to a decresing one in a standard auction, our model is consistent with the decreasing price anomaly, both theoretically and empirically.

[18]Non-parametric tests produce similar conclusions. A (two-sided) Mann-Whitney rank-sum test detects a weakly significant difference in bids only between $T1$ and $T3$ ($z = 1.680$, $p = 0.093$). In any other pairwise comparison, instead, the difference is not statistically significant (between $T1$ and $T2$: $z = 1.419$, $p = 0.156$;
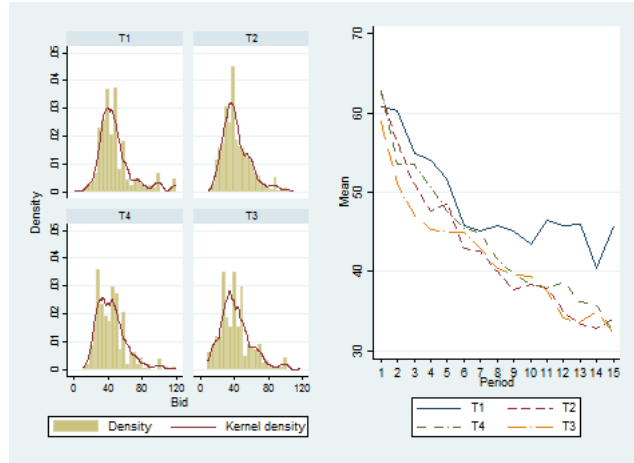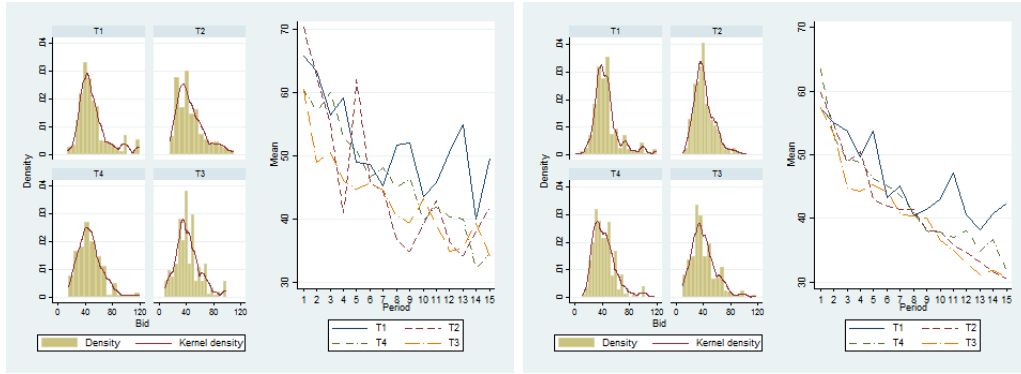
Figure 3 – Bids in $A1$: full sample.

Figure 4 – Bids in $A1$ by subjects' capacity: 1 unit (left panel) and 2 units (right panel).

Table 4 – Bids in $A1$.

	a	$Obs.$	a_1	$Obs.$	a_2	$Obs.$
$T1$	48.754	720	51.638	345	46.101	375
	(20.425)		(21.905)		(18.599)	
$T2$	42.731	720	46.189	190	41.491	530
	(16.308)		(19.480)		(14.835)	
$T3$	41.832	720	42.957	368	40.656	352
	(18.021)		(18.390)		(17.576)	
$T4$	43.949	720	47.173	179	42.882	541
	(17.344)		(18.279)		(16.906)	

Notes: this table reports means and standard deviations (in parentheses) of bids in the first auction for each treatment and overall periods. Descriptives are presented both by pooling subjects (a), and by splitting the sample according to the subjects capacities (a_1 for 1 unit, a_2 for two units).

between $T1$ and $T4$: $z = 0.945$, $p = 0.345$; between $T2$ and $T3$: $z = 0.105$, $p = 0.916$; between $T2$ and $T4$: $z = -0.210$, $p = 0.834$; between $T3$ and $T4$: $z = 0.420$, $p = 0.674$).

Table 5 – Bids in $A1$: parametric analysis.

	(1)	(2)	(3)
$T1$	6.922*	8.762**	6.040
	(4.031)	(4.107)	(4.235)
$T2$	0.899	4.110	5.950
	(4.031)	(4.171)	(4.278)
$T4$	2.117	4.367	5.421
	(4.031)	(4.181)	(4.283)
$2u$		-2.340**	-2.311**
		(1.135)	(0.994)
$T1 \times 2u$		-3.389**	-3.446**
		(1.601)	(1.401)
$T2 \times 2u$		-3.577**	-3.103**
		(1.706)	(1.494)
$T4 \times 2u$		-2.178	-1.440
		(1.729)	(1.514)
$Period$			-1.545***
			(0.111)
$T1 \times Period$			0.393**
			(0.157)
$T2 \times Period$			-0.314**
			(0.157)
$T4 \times Period$			-0.230
			(0.157)
$cons$	41.832***	42.976***	53.775***
	(2.850)	(2.901)	(2.992)
lrl	-12042.03	-12006.18	-11647.88
$Wald - \chi^2$	3.51	67.08	920.88
$p > \chi^2$	0.319	0.000	0.000
$Obs.$	2880	2880	2880

Notes: this table reports estimates (standard errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the bid placed by the subject in the first auction. $2u$ is a dummy whose value is equal to 1 if the subject has a capacity of 2 units. $T1 \times 2u$, $T2 \times 2u$, $T4 \times 2u$ are interaction terms. The other remarks of Table 3 apply.

Results in column (1) also support the observation that subjects bid above the pooling equilibrium levels reported in Table 1 (for $T1$: $\chi^2(1) = 43.30$, $p < 0.001$; for $T2$: $\chi^2(1) = 19.95$, $p < 0.001$; for $T3$: $\chi^2(1) = 88.63$, $p < 0.001$; for $T4$: $\chi^2(1) = 103.16$, $p < 0.001$).[19]

Column (2) shows that having capacity $c = 2$ strongly and persistently reduces bids (for $T1$: $\chi^2(1) = 25.74$, $p < 0.001$; for $T2$: $\chi^2(1) = 21.57$, $p < 0.001$; for $T3$: $p = 0.039$; for $T4$: $\chi^2(1) = 12.01$, $p < 0.001$). Despite the negative effect of capacity, in all treatments, bids are significantly higher than the predicted levels associated with the pooling equilibrium (for subjects with capacity $c = 1$: $\chi^2(1) = 55.91$, $p < 0.001$, in $T1$; $\chi^2(1) = 32.49$, $p < 0.001$, in $T2$; $\chi^2(1) = 93.03$, $p < 0.001$, in $T3$; $\chi^2(1) = 115.40$, $p < 0.001$, in $T4$; for subjects

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[19]Again, these results are confirmed by non-parametric tests: according to a (two-sided) Wilcoxon signed-rank test, the difference between average bids and predicted levels is significant in all treatments ($z = 2.521$, $p = 0.012$).

with capacity $c = 2$: $\chi^2(1) = 30.52$, $p < 0.001$, in $T1$; $\chi^2(1) = 15.18$, $p < 0.001$, in $T2$; $\chi^2(1) = 77.85$, $p < 0.001$, in $T3$; $\chi^2(1) = 94.31$, $p < 0.001$, in $T4$).

Finally, column (3) documents the decay of bids over repetitions: when considering all periods, we find a negative and highly significant linear time trend in $T1$ ($\chi^2(1) = 107.22$, $p < 0.001$), $T2$ ($\chi^2(1) = 278.93$, $p < 0.001$), $T3$ ($p < 0.001$) and $T4$ ($\chi^2(1) = 254.35$, $p < 0.001$).

The following three statements summarize the main findings about bids in $A1$.

$R4$. **Differences in bids in $A1$ across treatments.** *We detect no remarkable differences in bids across treatments.*

$R5$. **Bid levels in $A1$.** *In all treatments and regardless of the capacity, bids are larger than the one associated with the pooling equilibrium.*

$R6$. **Bids in $A1$ and capacity.** *In all treatments, subjects with capacity $c = 2$ bid less than those with capacity $c = 1$.*

Overall, results $R4$-$R6$ are clearly at odds with the characteristics of the pooling equilibrium (prediction $A1$-(a)). On the other hand, they provide *prima facie* evidence in favor of the hybrid equilibrium (prediction $A1$-(b)). However, to corroborate the validity of the hybrid equilibrium in explaining our evidence, bids must be looked at in connection with beliefs. In particular, as stated in prediction $A1$-(b), the negative relation between capacity and bids should reflect in a negative relation between bids and beliefs. Figure 5 shows, for each treatment, the kernel densities of the beliefs stated by subjects about the probability that the opponent's capacity was $c = 2$, both overall and after splitting subjects between Winners and Losers of $A1$. Table 6 reports the corresponding descriptive statistics. It is worth recalling that, upon stating their beliefs on the (initial) capacity of the opponent, Winners and Losers possess different information on the outcome of $A1$: whereas Losers are informed of the (winning) bid of the opponent, Winners only know that the opponent's bid was larger (or maybe equal) than their own.
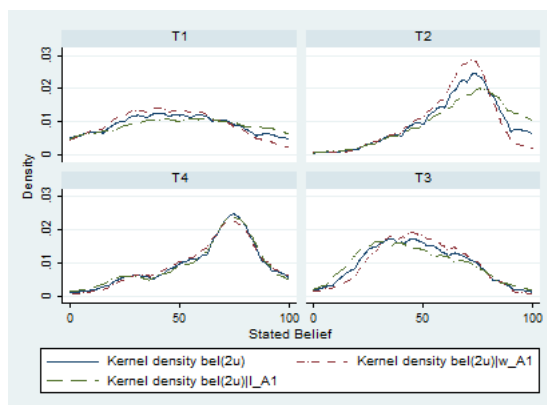

Figure 5 – Stated beliefs.

Looking at the data, three facts immediately emerge. First, in line with the ordering in the prior probabilities, average beliefs in $T1$ and $T3$ are quite close one another, as they are in $T2$ and $T4$; moreover, beliefs in the former treatments are lower than those in the latter.

Second, in $T2$, $T3$, and $T4$, both Winners' and Losers' stated beliefs are, on average, lower than the prior probabilities (50% in $T3$, 75% in $T2$ and $T4$); in $T1$, instead, whereas the average beliefs of Winners are lower than the prior probability (50%), the opposite occurs for Losers (overall, average beliefs are below the prior probability). Third, whereas in $T1$ and $T2$ Winners report, on average, lower beliefs than Losers, the opposite occurs in $T3$ and $T4$.

Table 6 – Stated beliefs.

	μ	$Obs.$	μ_W	$Obs.$	μ_L	$Obs.$
$T1$	48.825	720	45.544	360	52.105	360
	(28.010)		(25.164)		(30.272)	
$T2$	66.407	720	63.422	360	69.391	360
	(19.984)		(17.666)		(21.678)	
$T3$	46.765	720	48.203	360	45.328	360
	(21.127)		(19.457)		(22.610)	
$T4$	64.592	720	65.536	360	63.647	360
	(22.299)		(21.285)		(23.260)	

Notes: this table reports means and standard deviations (in parentheses) of the beliefs that the opponent's capacity was two units, in each treatment and overall periods. Descriptives are presented both by pooling subjects (μ), and by splitting the sample according to the outcome of the first auction (μ_W for Winners, μ_L for Losers).

We can assess the relevance of these preliminary observations in the first three columns of Table 7.

Column (1) confirms that the ordering in the stated beliefs across treatments follows the one in the priors. Beliefs in $T1$ and $T3$ are generally lower than those in $T2$ and $T4$: when pooling subjects, we find significant differences between $T1$ and $T2$ ($\chi^2(1) = 55.90$, $p < 0.001$), $T1$ and $T4$ ($\chi^2(1) = 44.96$, $p < 0.001$), $T2$ and $T3$ ($p < 0.001$), and $T3$ and $T4$ ($p < 0.001$). The remaining pairwise comparisons are not significant (between $T1$ and $T3$: $p = 0.381$; between $T2$ and $T4$: $\chi^2(1) = 0.60$, $p = 0.440$).[20]

Although they follow the same ordering, beliefs and prior probabilities do quantitatively differ, at least in most of the cases (see column (2)). In particular, Losers' beliefs are significantly lower than the prior probability in $T2$ ($\chi^2(1) = 11.24$, $p < 0.001$), $T3$ ($\chi^2(1) = 7.16$, $p = 0.007$), and $T4$ ($\chi^2(1) = 37.75$, $p < 0.001$), whereas the difference is not significant in $T1$ ($\chi^2(1) = 1.54$, $p = 0.215$). For Winners, we detect a negative and significant difference between beliefs and prior probability in $T2$ ($\chi^2(1) = 35.70$, $p < 0.001$) and $T4$ ($\chi^2(1) = 26.62$, $p < 0.001$). The difference is positive and significant in $T1$ ($\chi^2(1) = 6.32$, $p = 0.012$), whereas it is not significant in $T3$ ($\chi^2(1) = 0.71$, $p = 0.400$).[21]

Column (2) also shows that the effect on beliefs of winning in $A1$ is heterogeneous across treatments. Indeed, the difference between Winners' and Losers' beliefs is positive and sig-

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[20]These results are confirmed by non-parametric tests. A (two-sided) Mann-Whitney rank-sum test detects a significant difference in stated beliefs between $T2$ and $T1$, $T4$ and $T1$, $T2$ and $T3$, and $T4$ and $T3$ (in all cases, $z = 3.361$, $p < 0.001$). All other differences are not significant (between $T1$ and $T3$: $z = 1.260$, $p = 0.208$; between $T2$ and $T4$: $z = 0.735$, $p = 0.462$).

[21]Non-parametric tests generally confirm these results. According to a (two-sided) Wilcoxon signed-rank test, the difference between Winners' stated beliefs and prior probabilities is significant in $T2$ and $T4$ (in both cases, $z = -2.521$, $p = 0.012$). It is not significant in $T1$ ($z = -1.260$, $p = 0.208$) and $T3$ ($z = -0.840$, $p = 0.401$). For Losers, the difference is significant in $T2$, $T4$ (in both cases, $z = -2.521$, $p = 0.012$), and (marginally) $T3$ ($z = -1.680$, $p = 0.093$). It is not significant in $T1$ ($z = 0.700$, $p = 0.484$).

Table 7 – Stated beliefs: parametric analysis.

	μ			$Pr(correct\ belief)$		
	(1)	(2)	(3)	(4)	(5)	(6)
$T1$	2.060	7.207***	5.602*	0.067***	0.058*	0.029
	(2.352)	(2.604)	(3.141)	(0.023)	(0.031)	(0.047)
$T2$	19.642***	23.743***	23.001***	0.189***	0.186***	0.175***
	(2.352)	(2.603)	(3.141)	(0.020)	(0.030)	(0.040)
$T4$	17.826***	18.605***	18.025***	0.161***	0.139***	0.088*
	(2.352)	(2.602)	(3.140)	(0.026)	(0.037)	(0.051)
w_{A1}		3.372**	3.372**		−0.047	−0.047
		(1.589)	(1.587)		(0.035)	(0.034)
$T1 \times w_{A1}$		−10.295***	−10.296***		0.019	0.019
		(2.264)	(2.261)		(0.047)	(0.047)
$T2 \times w_{A1}$		−8.203***	−8.203***		0.006	0.007
		(2.260)	(2.257)		(0.049)	(0.049)
$T4 \times w_{A1}$		−1.558	−1.558		0.048	0.047
		(2.257)	(2.254)		(0.056)	(0.056)
$Period$			0.195			0.003
			(0.178)			(0.003)
$T1 \times Period$			0.229			0.004
			(0.251)			(0.005)
$T2 \times Period$			0.105			0.002
			(0.251)			(0.004)
$T4 \times Period$			0.083			0.008
			(0.251)			(0.005)
$cons$	46.765***	45.079***	43.710***			
	(1.663)	(1.838)	(2.219)			
lrl	−12957.31	−12935.21	−12932.31	−1732.67	−1731.18	−1726.05
$Wald - \chi^2$	114.73	148.64	161.01	89.36	92.79	109.62
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000
$Obs.$	2880	2880	2880	2574	2574	2574

Notes: columns (1)-(3) report estimates (std. errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dep. variable is the belief that the opponent's capacity was 2. Columns (4)-(6) report probit marginal effect estimates (robust std. errors clustered at the rematching group level in parentheses) for subjects whose beliefs were not equal to 50. The dep. variable is a dummy whose value is equal to 1 if the belief is correct. The other remarks of Table 3 apply.

nificant in $T3$ ($p = 0.034$), negative and significant in $T1$ ($\chi^2(1) = 18.42$, $p < 0.001$) and $T2$ ($\chi^2(1) = 9.04$, $p < 0.001$), whereas it is positive but not significant in $T4$ ($\chi^2(1) = 1.28$, $p = 0.258$).

To control for potential learning effects, column (3) adds treatment specific time trends to the previous parametric specification. We detect a positive and significant trend in $T1$ ($\chi^2(1) = 5.72$, $p = 0.017$) and (marginally) in $T2$ ($\chi^2(1) = 2.88$, $p = 0.090$), whereas it is not significant in the remaining two treatments (for $T3$: $p = 0.271$; for $T4$: $\chi^2(1) = 2.46$, $p = 0.117$).

We now analyze in more details how subjects update their beliefs using the information revealed at the end of $A1$. We focus on Losers because, upon stating their beliefs, they possess more precise information than Winners on the opponent's behavior in $A1$.

The effect of the winning bid on Losers' beliefs is parametrically investigated in the first

two columns of Table 8. We employ two different empirical strategies: in column (1), we assess the direct effect of the winning bid; in column (2), we use the difference between the Loser's bid and the (observed) winning bid as the main regressor.

Table 8 – Beliefs of Losers: parametric analysis.

	μ_L		$Pr(correct\ belief)$	
	(1)	(2)	(3)	(4)
$T1$	26.921***	6.260	-0.280***	0.080
	(7.848)	(4.678)	(0.086)	(0.061)
$T2$	35.249***	23.520***	0.041	0.152*
	(8.804)	(4.749)	(0.099)	(0.079)
$T4$	18.414**	26.422***	0.056	0.114
	(8.790)	(4.676)	(0.175)	(0.071)
a_W	0.165		-0.001	
	(0.117)		(0.001)	
$T1 \times a_W$	-0.463***		0.006***	
	(0.156)		(0.002)	
$T2 \times a_W$	-0.300*		0.002	
	(0.177)		(0.002)	
$T4 \times a_W$	0.032		-2.53×10^{-4}	
	(0.175)		(0.003)	
$(a - a_W)$		0.098		0.002
		(0.079)		(0.001)
$T1 \times (a - a_W)$		0.013		-0.003
		(0.105)		(0.002)
$T2 \times (a - a_W)$		-0.059		-1.966×10^{-4}
		(0.124)		(0.003)
$T4 \times (a - a_W)$		-0.303**		-0.004*
		(0.118)		(0.002)
$Period$	0.604**	0.542**	-3.063×10^{-4}	0.002
	(0.279)	(0.266)	(0.004)	(0.004)
$T1 \times Period$	-0.336	0.064	0.014**	0.005
	(0.386)	(0.369)	(0.007)	(0.007)
$T2 \times Period$	-0.234	0.059	0.010	0.006
	(0.422)	(0.374)	(0.007)	(0.008)
$T4 \times Period$	-0.267	-0.631*	0.014*	0.012
	(0.417)	(0.370)	(0.008)	(0.008)
$cons$	35.273***	40.056***		
	(5.668)	(3.337)		
lrl	-6500.17	-6503.72	-863.72	-864.94
$Wald - \chi^2$	89.40	84.02	82.87	75.91
$p > \chi^2$	0.000	0.000	0.000	0.000
$Obs.$	1440	1440	1299	1299

Notes: this table employs the same parametric strategy of Table 7 to study the determinants of stated beliefs and the probability of correct beliefs by focusing on Losers of the first auction. a_W is the (observed) winning bid in $A1$. $(a - a_W)$ is the difference between a subject's bid and the (observed) winning bid. $T1 \times a_W$, $T2 \times a_W$, $T4 \times a_W$, $T1 \times (a - a_W)$, $T2 \times (a - a_W)$, $T4 \times (a - a_W)$ are interaction terms. The other remarks of Table 3 and Table 7 apply.

In general, the information revealed by the winning bid in $A1$ exerts limited effects on the Losers' beliefs: the level of the winning bid reduces Loser's stated beliefs only in $T1$

($\chi^2(1) = 8.48$, $p = 0.003$), whereas this effect is not significant in the other three treatments (in $T2$: $\chi^2(1) = 1.05$, $p = 0.306$; in $T3$: $p = 0.160$; in $T4$: $\chi^2(1) = 2.32$, $p = 0.128$). Similarly, when we use the difference between the Loser's bid and the observed winning bid as a regressor, we find a negative and significant effect in $T4$ only ($\chi^2(1) = 5.40$, $p = 0.020$), and no significant effect in the other treatments (in $T1$: $\chi^2(1) = 2.57$, $p = 0.109$; in $T2$: $\chi^2(1) = 0.16$, $p = 0.685$; in $T3$: $p = 0.212$).

We summarize our findings on stated beliefs in the following statements.

R7. **Differences in beliefs across treatments.** *In line with the ordering in the prior probabilities, there is no statistical difference in beliefs between $T1$ and $T3$, and between $T2$ and $T4$. Moreover, beliefs in $T1$ and $T3$ are lower than those in $T2$ and $T4$.*

R8. **Beliefs and prior probabilities.** *In $T2$ and $T4$, beliefs of both Winners and Losers of $A1$ are below the prior probabilities. In $T1$, Losers' beliefs are aligned with the prior probability, whereas Winners report lower beliefs. Finally, in $T3$, Winners' beliefs are aligned with the prior probability, whereas Losers report lower beliefs.*

R9. **Beliefs and outcome of $A1$.** *Winning in $A1$ substantially reduces beliefs in $T1$ and $T2$. Instead, this effect is positive in $T3$ and $T4$, although limited in magnitude.*

R10. **Beliefs and winning bid of $A1$.** *In most treatments, Losers' beliefs are insensitive to the winning bid.*

Results $R7$-$R10$, when combined with results $R4$-$R6$, suggest that the belief updating process by bidders is imperfect, as their beliefs are inconsistent with actual behavior in $A1$. To see this, notice first that, in $T2$ and $T4$ (where the prior probability is equal to 75%), average beliefs are significantly biased downward (result $R8$); in particular, they lie around the midpoint between 50% and the prior probability (75%). On the other hand, beliefs in $T1$ and $T3$ (where the prior probability is equal to 50%), are overall in line with the prior.

However, it is the observed relation with the bidding behavior in $A1$ that clearly points toward an inconsistency of the beliefs. Although subjects with capacity $c = 2$ tend to make lower bids than those with capacity $c = 1$ (result $R6$), this is not reflected in the beliefs: Losers do not properly adjust their beliefs when they observe a lower winning bid (result $R10$); moreover, in two treatments out of four ($T3$ and $T4$), Winners' beliefs are larger than Losers' (result $R9$), whereas result $R6$ would imply the opposite. Taken together, these considerations greatly weaken the empirical validity of the hybrid equilibrium (prediction $A1$-(b)).[22]

This conclusion is further confirmed when we analyze the degree at which subjects are able to formulate correct beliefs. To this end, we construct a (very conservative) indicator of belief correctness: we classify a belief as correct if it is strictly larger (smaller) than 50 and the opponent has indeed a capacity $c = 2$ ($c = 1$).

Notice first that, in general, in spite of the conservativeness of our indicator, the proportion of incorrect beliefs is relatively large: 47.80% in $T1$, 35.23% in $T2$, 54.68% in $T3$, and 38.10% in $T4$, respectively. Second, even though Losers have more precise information than Winners on the actual bid placed by the opponent, this advantage does not reflect in more correct beliefs: as it is shown by columns (5) and (6) of Table 7, winning $A1$ does not exert significant

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[22]Moreover, the presence of a significant decreasing time trend in bids is suggestive that behavior is off equilibrium.

effects on the probability of stating correct beliefs in any treatment (in $T1$: $\chi^2(1) = 0.72$, $p = 0.395$; in $T2$: $\chi^2(1) = 1.38$, $p = 0.241$; in $T1$: $p = 0.174$; in $T4$: $\chi^2(1) = 0.00$, $p = 0.997$). Finally, as documented in the last two columns of Table 8, we detect a limited effect of the information provided by the (observed) winning bid on the probability that Losers' beliefs are correct. From column (3), the effect of the winning bid is not significant in $T2$ ($\chi^2(1) = 0.75$, $p = 0.387$), $T3$ ($p = 0.346$), and $T4$ ($\chi^2(1) = 0.08$, $p = 0.783$). In treatment $T1$, instead, the effect is significant ($\chi^2(1) = 7.91$, $p = 0.005$), but is unexpectedly positive: in fact, in light of result $R6$, we would expect a lower winning bid to be more informative of the actual capacity of the opponent; as such, if the Loser updated this information correctly, she should be more confident in formulating a larger belief. Similar conclusion can be drawn from column (4), which shows that, in all treatments, the effect of the difference between the winning bid and the Loser's bid does not exert any significant on the correctness of the Loser's belief (in $T1$: $\chi^2(1) = 0.65$, $p = 0.420$; in $T2$: $\chi^2(1) = 0.39$, $p = 0.531$; in $T3$: $p = 0.182$; in $T4$: $\chi^2(1) = 1.13$, $p = 0.288$).

We summarize the evidence on beliefs correctness in the following statement.

$R11$. **Correctness of beliefs.** *The probability that beliefs are correct is unaffected by the outcome of $A1$ and by the (observed) winning bid.*

5 Discussion

A direct comparison between our experimental evidence, summarized in results *R1-R11* in Section 4, and the predictions implied by the (Perfect Bayesian) equilibrium theory (presented in Section 3.2), leads us to draw mixed conclusions: whereas bidding behavior in $A2$ basically conforms with sequential rationality, behavior in $A1$ departs from equilibrium requirements in that beliefs seem to be inconsistent with actual bids. In particular, although bids in $A1$ are differentiated across types – bidders with capacity $c = 2$ make significantly lower bids than those with capacity $c = 1$ –, subjects do not seem able to appreciate the informative content revealed by the outcome of the auction (observed by all) and by the winning bid (observed by the Loser).

In this section, we discuss a potential explanation of our evidence. In so doing, we will take as given the fact that behavior in $A2$ is (sequentially) rational, and we will concentrate our attention on the relationship between behavior in $A1$ and corresponding beliefs.

In particular, our explanation satisfies two conditions, that conform with a broad notion of rationality:

(i) observed bids in $A1$ should be the result of some optimizing mental process, given rational expectations on the belief formation process. In this respect, the evidence that type-2 bidders make lower than type-1 does indeed seem to be a rational response to the expectation that the opponent fails to appreciate the informative content revealed by the outcome of $A1$. In fact, everything else being equal, a bidder with capacity $c = 2$ has an incentive to make a lower bid in $A1$, because she has to sell two units; instead, a bidder with capacity $c = 1$ knows that she will possibly have a second opportunity to sell her (unique) unit. The theoretical analysis, however, highlighted that, for a bidder with capacity $c = 2$, making a lower bid may be extremely costly if this bid reveals her capacity: indeed, if the other bidder understands that her capacity is $c = 2$, there will be tough competition in $A2$, leading prices to zero. The absence of a fully separating

equilibrium demonstrates that this signaling cost is so high that a bidder with capacity $c = 2$ prefers to pool with the other type, at least partially. Clearly, if the opponent fails to understand this, then the signaling cost will be absent, and making a lower bid is likely to be rational;

(ii) beliefs, that, as we have seen, are not consistent in the game-theoretic sense with actual behavior, should still respond to some *behavioral* consistency; by this, we mean that the fact that a subject purposefully makes different bids depending on her own capacity, but, at the same time, does not adjust her beliefs to the observed bidding behavior of the opponent, should be somehow rationalizable.

A possible way to reconcile the two conditions above is to hypothesize that bidding behavior in $A1$ is also driven by a latent factor that obfuscates the effect of the initial capacity. More precisely, there is a *hidden* type that affects bids in $A1$ and operates simultaneously with the type capacity; the interplay between these two types makes it more difficult for a bidder to infer the opponent's capacity from the observation of the outcome of the auction. Anticipating this, a bidder rationally decides to place a lower bid when she has larger capacity.

One characteristic that could work as a hidden type in the way described above is a bidder's strategic ability. There is a large experimental literature that relates the heterogeneous behavior usually observed in experimental games to the capacity of thinking strategically. The level-k model, as introduced by Stahl and Wilson (1994, 1995) and Nagel (1995), formalizes this idea. This model has been applied to a variety of experimental games, including auctions (see Crawford and Iriberri, 2007). The level-k model postulates that there are some players, called level-0, that are strategically naïve in that they follow some simple rule of thumb. The other players anchor their beliefs to level-0 players' behavior, but differ in the depth of their strategic reasoning: in particular, level-1 players believe that their opponents are level-0 and best respond to this belief; level-2 players believe that their opponents are level-1 and best respond to this belief; and so on. In other words, higher level players are more sophisticated than lower level ones, in that they are able to push their strategic reasoning process further on.

Our claim is that, applied to our context, a level-k model is able to rationalize our evidence on the presumed inconsistency between bids in $A1$ and beliefs, in a way that satisfy the two conditions above.

To have a sense on it, assume that bidders are potentially characterized by different degrees of strategic thinking ability, and that this is private information. This means that each bidder is actually identified by a two-dimensional type: her initial capacity c, and her level of strategic thinking k. Let a_c^k denote the bid placed by a level-k bidder with capacity c. Suppose, moreover, that bidders do not update their beliefs once the outcome of $A1$ is revealed, and this is common knowledge. The intuition suggests that: (i) $a_2^k < a_1^k$, i.e. a level-k bidder will typically make a lower bid when her capacity is $c = 2$. This because, ceteris paribus, a bidder with capacity $c = 2$ has a stronger incentive to win $A1$, and, moreover, we are assuming that a lower bid has no signaling cost (the opponent will stick to the prior probabilities in any case); (ii) as long as a_2^{k-1} and a_1^{k-1} are not too low, a level-k bidder will find it optimal to undercut them, i.e. $a_2^k < a_2^{k-1}$ and $a_1^k < a_1^{k-1}$.

To see that, under certain conditions, the above intuition is correct, suppose that a level-0 bidder, regardless of her capacity, bids randomly according to the strictly increasing cumulative distribution function F^0, with density f^0 and support A^0. The expected payoff of a

level-1 bidder with capacity $c = 2$ is

$$\pi_2^1(a) = (a + \omega)\left(1 - F^0(a)\right) + \omega\, F^0(a) = a\left(1 - F^0(a)\right) + \omega,$$

where $\omega = q(1 - p)R$ is the expected payoff from the second auction when the other bidder does not update her belief. Notice that, if the optimal bid for this bidder, a_2^1, is in the interior of A^0, then necessarily

$$\left.\frac{d\pi_2^1}{da}\right|_{a=a_2^1} = 1 - F^0(a_2^1) - a_2^1 \times f^0(a_2^1) = 0.$$

Similarly, the expected payoff of a level-1 bidder with capacity $c = 1$ is

$$\pi_1^1(a) = a\left(1 - F^0(a)\right) + \omega\, F^0(a).$$

It is immediate to see that

$$\left.\frac{d\pi_1^1}{da}\right|_{a=a_2^1} = 1 - F^0(a_2^1) + \left(\omega - a_2^1\right) \times f^0(a_2^1) > 0.$$

Hence, a_1^1, the optimal bid for a level-1 bidder with capacity $c = 1$, is larger than a_2^1.

Consider now higher level bidders. The following Proposition shows that, under certain conditions, their bids have a monotone pattern.[23]

PROPOSITION 3. *Suppose that bidders do not update their beliefs at the and of the first auction, and this is common knowledge. Let a_c^k denote the bid placed by a level-k bidder with capacity c. Then, for all $k \geq 2$, if $a_1^{k-1} \geq \omega + 2$, $a_2^{k-1} \geq 2/p$, and*

$$\max\left[p\left(a_1^{k-1} - \omega - 1\right); \frac{p\left(a_1^{k-1} - \omega - 1\right)}{2 - p} + 1\right] \leq a_1^{k-1} - a_2^{k-1} \leq p\left(a_1^{k-1} - 1\right),$$

then $a_1^k = a_1^{k-1} - 1$ and $a_2^k = a_2^{k-1} - 1$.

Proposition 3 states that, if a level-$(k - 1)$ bidder's bids are strictly decreasing in her capacity, these bids are not too low, and their difference is neither too high nor too low, then a level-k bidder will find it optimal to undercut a_1^{k-1} when her capacity is $c = 1$, to undercut a_2^{k-1} when her capacity is $c = 2$. As a consequence, also a level-k bidder's bids will be strictly decreasing in her capacity. Hence, the level-k model is able to explain our evidence on bidding behavior in $A1$ and it does so in a way that satisfies condition (i) above: bidders, expecting that beliefs will not be updated at the end of $A1$, find it optimal to place a lower bid when their capacity is $c = 2$.

More importantly, the level-k model may be used to rationalize why the lack of belief updating may still be consistent with the fact that bids in $A1$ are decreasing in capacity (condition (ii) above). Notice that Proposition 3 implies that, under certain conditions, and neglecting level-0 bidders, bids are strictly decreasing not only in the bidder's capacity, but also in her level. If we let $\Delta = a_1^k - a_2^k$,[24] then the model implies that $a_2^k = a_1^{k+\Delta}$ (again,

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[23]The proof is in the Appendix. Notice that, in Proposition 3, the continuous approximation is abandoned: only integer bids are considered, just like in the experiment.

[24]Notice that, as long as the conditions are satified, Δ is constant in k.

provided all the conditions are satisfied): a specific bid could originate from a bidder of type $(c = 2, k)$, as well as from a bidder of type $(c = 1, k + \Delta)$. In this sense, for a bidder that contemplates the possibility that the opponent's actual level of strategic reasoning can be any k, the observation of a particular bid does not provide enough information to infer the capacity of the opponent. In particular, if a bidder observes a bid $a = a_2^k = a_1^{k+\Delta}$, and the two levels k and $k + \Delta$ are deemed as equally likely to occur in the population, then the bidder could rationally conclude that the ex-post probability that the opponent's capacity is c coincides with the prior probability p.

Broadly speaking, the mental process that leads a bidder to stick to the prior belief when she observes the outcome of $A1$ could be described as follows. Consider a level-k bidder with capacity $c = 2$. This bidder initially believes that k', her opponent's level of strategic thinking, is $k - 1$; thus, she bids a_2^k, expecting to win $A1$ regardless of the capacity of the other bidder. If she happens to win, this will confirm that her expectations were indeed correct, and she will have no reason to update her belief. If instead she loses and observes a winning bid below her own, she will at first find this outcome unexpected, as it contrasts with her initial expectations. However, after a few seconds of thoughts, she will eventually realize that the other bidder has simply performed more steps of reasoning than she herself did. In other words, she will certainly realize that her initial belief on the opponent's level of strategic thinking ($k' = k - 1$) was incorrect, and that k' is actually higher. What about her belief on the capacity of the other bidder? Knowing that bids should decrease both in the capacity and in the level of strategic thinking, she will eventually conclude that either the opponent had capacity $c = 2$ and a level of strategic thinking only slightly higher than k, or her capacity was $c = 1$ and her level of strategic thinking was significantly higher than k. Hence, in any case this bidder may find a rational argument that justifies an opponent's capacity equal to $c = 2$, and an equally rational argument that is consistent with an opponent's capacity equal to $c = 1$. As a result, this bidder, having no sufficient elements to precisely assess the actual capacity of the opponent, will stick to the prior probabilities.

The implications of the level-k model set out before should not be taken too literally; rather, they should be interpreted in their qualitative contents. Consider the following example: suppose that a level-0 bidder bids according to a uniform distribution over the interval $[30, 90]$. Using the parameters of our treatment $T1$ ($p = q = 1/2$), it obtains: $[a_1^1 = 60, a_2^1 = 45]$, $[a_1^2 = 59, a_2^2 = 44]$, $[a_1^3 = 58, a_2^3 = 43]$, and so on (until we reach level-29, for which bids are $[a_1^{29} = 32, a_2^{29} = 17]$. A level-30 bidder with capacity $c = 1$ would not go on undercutting, as she prefers to lose $A1$ bidding anything above 32. Hence, we have that, for $1 \le k \le 14$, $a_2^k = a_1^{k+15}$, i.e. a level-k bidder with capacity $c = 2$ makes the same bid as a level-$(k + 15)$ with capacity $c = 1$. It is probably unrealistic to think that bidders will perform more than 3 or 4 levels of strategic thinking (the experimental literature has shown that subjects typically perform no more than 2-3 levels of iteration; see, e.g., Crawford et al. 2013). It is even less realistic to think that a bidder who observes a certain bid judges as equally likely that the opponent performed k or $k + 15$ steps of iterations. However, if we (realistically) modify the model allowing bidders to make payoff-sensitive errors (e.g. logistic), then our interpretation becomes more credible. In particular, if we suppose that every bid is played with positive probability but the bidding strategy of a level-k bidder with capacity c has density $f_c^k(a) = \exp(\pi_c^k(a)) / \int_{A^0} \exp(\pi_c^k(a)) da$, then we obtain the following expected bids (until $k = 3$): $[\mathbb{E}[a_1^1] = 60, \mathbb{E}[a_2^1] = 45.1]$, $[\mathbb{E}[a_1^2] = 48.9, \mathbb{E}[a_2^2] = 39.6]$, $[\mathbb{E}[a_1^3] = 41.1, \mathbb{E}[a_2^3] = 35.8]$. Notice that, in this case, the expected bid of level-k bidder with capacity $c = 2$ is quite close to the expected bid of level-$(k + 1)$ bidder with capacity $c = 1$. Hence, our interpretation

that, upon observing a certain bid, a bidder realizes that this bid could have been placed by an opponent of type $(c = 2, k)$ as well as by a type $(c = 1, k + 1)$, and that the levels k and $k + 1$ are essentially equally likely to occur, thus leading the bidder not to update her belief on the opponent's capacity, is certainly realistic.

The interpretation of our evidence in terms of a level-k model allows us to shed light on the dynamics of bidding over periods that we observe in the experiment. The analysis presented in Section 4.2 pointed out that bids in $A1$ are characterized by a significant negative time trend. The relationship between bids in $A1$ in two consecutive periods $((t - 1)$ and $t)$ is examined more in depth in Table 9.

The specifications in column (1) and (2) include a dummy variable that captures the outcome (winning or losing) of $A1$ in the previous period of the experiment. They show that, after controlling for a subject's capacity, winning in one period does not significantly affect bids in the subsequent period (in $T1$: $\chi^2(1) = 0.27$, $p = 0.603$; in $T2$: $\chi^2(1) = 0.78$, $p = 0.376$; in $T3$: $p = 0.131$; in $T4$: $\chi^2(1) = 0.84$, $p = 0.359$). On the other hand, the negative effect of the capacity on bids (result $R6$) remains highly significant (in $T1$, $\chi^2(1) = 32.99$, $p < 0.001$; in $T2$, $\chi^2(1) = 23.37$, $p < 0.001$; in $T3$, $p = 0.016$; in $T4$, $\chi^2(1) = 17.06$, $p < 0.001$).

Column (3) presents the results of a regression in which the dependent variable is the difference between the bids in $A1$ placed by the same bidder in two consecutive periods. Columns (4) and (5) replicate the same regression after splitting the sample between Winners and Losers of $A1$ in the previous period.[25] Notice that, in all treatments, Winners (column(4)) tend to increase their bids in the next period (in $T1$: $\chi^2(1) = 66.86$, $p < 0.001$; in $T2$: $\chi^2(1) = 25.18$, $p < 0.001$; in $T3$: $p < 0.001$; in $T4$: $\chi^2(1) = 18.91$, $p < 0.001$), whereas Losers (column (5)) adjust their bids in the opposite direction (in $T1$: $\chi^2(1) = 35.33$, $p < 0.001$; in $T2$: $\chi^2(1) = 26.11$, $p < 0.001$; in $T3$: $p < 0.001$; in $T4$: $\chi^2(1) = 30.64$, $p < 0.001$). This explains why, between subjects, and after controlling for bidders' capacities, there is no significant difference between Winners' and Losers' bids (columns (1) and (2)). Columns (4) and (5) show that the negative effect of capacity on bids is still present for the splitted samples (for Winners: $\chi^2(1) = 28.52$, $p < 0.001$ in $T1$; $\chi^2(1) = 17.57$, $p < 0.001$ in $T2$; $p = 0.294$ in $T3$; $\chi^2(1) = 3.37$, $p = 0.067$ in $T4$; for Losers: $\chi^2(1) = 22.36$, $p < 0.001$ in $T1$; $\chi^2(1) = 4.49$, $p = 0.034$ in $T2$; $p = 0.032$ in $T3$; $\chi^2(1) = 1.40$, $p = 0.237$ in $T4$). Notice, however, that the downward adjustment in Loser's bids is, on average, larger than the upward adjustment in Winner's bids (the constant is 8.401 in column (4) and -13.370 in column (5)), so that the net effect is negative.

These results can be easily reconciled with our level-k interpretation: when a bidder loses $A1$, she realizes that her opponent's level of strategic reasoning was higher than expected. A natural reaction to this conjecture is that, in the next period, this bidder should perform (at least) one additional step of strategic reasoning, eventually placing a lower bid. On the other hand, the Winner of $A1$ may think that, perhaps, she performed too many steps of strategic reasoning, placing too low a bid, and may consider increasing her bid in the next period. However, as the Winner does not observe the opponent's bid but only the outcome, she cannot be sure that her bid was indeed too low, so it is reasonable that her (upward) adjustment next period will be less pronounced. As a result, bids, on average, will tend to decrease over time.

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[25]It is worth remarking that, whereas in the analysis of the beliefs and of the bids in $A2$, the terms Winners and Losers referred to the outcome of $A1$ *in that period*, here, in the analysis of first auction bids, the terms Winners and Losers refer to the outcome of $A1$ *in the previous period*.

Table 9 – Adjustment of bids in $A1$: parametric analysis.

	a		Δa	$\Delta a\|w_{A1(t-1)}$	$\Delta a\|l_{A1(t-1)}$
	(1)	(2)	(3)	(4)	(5)
$T1$	9.661**	7.207	2.775	3.543*	2.138
	(4.253)	(4.391)	(1.848)	(2.071)	(2.632)
$T2$	3.974	5.916	1.608	−0.365	2.212
	(4.313)	(4.433)	(1.979)	(2.172)	(2.851)
$T4$	4.757	5.775	0.315	−1.268	1.605
	(4.314)	(4.430)	(1.979)	(2.201)	(2.806)
$w_{A1(t-1)}$	1.583	1.706*			
	(1.048)	(0.939)			
$T1 \times w_{A1(t-1)}$	−1.031	−0.987			
	(1.493)	(1.337)			
$T2 \times w_{A1(t-1)}$	−0.639	−0.606			
	(1.495)	(1.339)			
$T4 \times w_{A1(t-1)}$	−0.609	−0.623			
	(1.492)	(1.336)			
$2u$	−2.529**	−2.395**	−2.574**	−1.231	−3.401**
	(1.046)	(0.936)	(1.123)	(1.174)	(1.586)
$T1 \times 2u$	−3.457**	−3.482***	−4.471***	−5.058***	−4.105*
	(1.477)	(1.322)	(1.588)	(1.663)	(2.244)
$T2 \times 2u$	−3.156**	−2.827**	−2.898*	−4.115**	−0.542
	(1.574)	(1.409)	(1.699)	(1.734)	(2.445)
$T4 \times 2u$	−2.414	−1.622	$5.04 * 10^{-4}$	−1.276	1.254
	(1.589)	(1.424)	(1.715)	(1.802)	(2.412)
$Period$		−1.335***	0.251*	−0.334**	0.846***
		(0.112)	(0.139)	(0.143)	(0.197)
$T1 \times Period$		0.325**	0.060	0.085	0.048
		(0.158)	(0.197)	(0.202)	(0.278)
$T2 \times Period$		−0.298*	0.138	0.282	0.029
		(0.158)	(0.197)	(0.202)	(0.278)
$T4 \times Period$		−0.219	0.012	0.159	−0.123
		(0.158)	(0.197)	(0.202)	(0.278)
$cons$	41.051***	50.939***	−2.535*	8.401***	−13.370***
	(3.007)	(3.104)	(1.300)	(1.468)	(1.833)
lrl	−10912.19	−10636.73	−11003.88	−5112.40	−5491.91
$Wald - \chi^2$	87.59	738.81	87.26	71.01	107.92
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000
$Obs.$	2688	2688	2688	1344	1344

Notes: this table employs the same parametric strategy of Table 7 to study the adjustment of bids in $A1$. The dependent variable used in columns (1)-(2) is the bid placed by the subject in $A1$. The dependent variable used in columns (3)-(5) is the difference between the bids placed by the same subject in $A1$ in two consecutive periods. Regressions in columns (4) and (5) are restricted to the subsamples of Winners and Losers of $A1$ in the previous period, respectively. $w_{A1(t-1)}$ is a dummy whose value is equal to 1 if the subject won the first auction in the previous period. $T1 \times w_{A1(t-1)}$, $T2 \times w_{A1(t-1)}$, $T4 \times w_{A1(t-1)}$ are interaction terms. The other remarks of Table 5 apply.

6 Conclusion

This article reported the results of an experiment involving two sequential first-price procurement auctions, where sellers may be capacity constrained, and this information is privately

held.

That capacity constraints may crucially affect behavior in repeated procurement auctions is a well documented empirical fact. However, we are not aware of other experimental papers that study sequential auctions where bidders' capacities are private information.

We designed four treatments that differ in the ex-ante probability distribution of sellers' capacities and in the (exogenous) likelihood that the second auction will actually be carried out. This last assumption was made to match real world situations, such as electricity markets, where the quantity that will be demanded in later auctions is not known a priori, depending on exogenous, institutional and environmental factors. In all treatments, at the end of the first auction, bidders were informed of the outcome and of the winning bid, and were required to elicit their beliefs (through an incentive compatible procedure) on the initial capacity of the opponent.

The results of the experiment show that: (i) observed behavior in the second auction is overall consistent with sequential rationality; (ii) although bids in the first auction are decreasing in the capacity of the bidder, beliefs do not respond to the outcome of the first auction.

To provide an explanation of this last result, we conjectured that bidders have different strategic abilities. Being unobservable, strategic ability represents an additional type that concur with the type capacity in the determination of a bidder's bid. Under this hypothesis, a particular observed bid does not provide enough information to distinguish the actual capacity of the opponent, because the same bid can be rationalized in two ways: as the bid of a bidder with large capacity but low level of strategic ability, or as the bid of a bidder with small capacity but high level of strategic ability. We showed that a simple level-k model may produce exactly this outcome.

One may wonder how to reconcile level-k, i.e. non-equilibrium, behavior in the first auction with fully fledged equilibrium behavior (given beliefs) in the second. Now, there are several considerations that support our argument. First, observed bids display a significant trend in the first auction, which is absent in the second. Second, that bids in the second auction are "closer" to the theoretical prediction than those in the first is also found by Février et al. (2007), Saini and Suter (2015), Otamendi et al. (2018).[26] Third, the level-k behavior is not incompatible with equilibrium, in the sense that the former usually converges to the latter (and it certainly does so in our game) as a player's level of strategic reasoning gets higher and higher. In this respect, our evidence may suggest that, in our experiment, convergence to equilibrium was not reached yet in the first auction, while it was almost immediate in the second. In fact, bidding in the first auction is admittedly harder: while the second auction is a one-shot game, bidding in the first requires bidders not only to try and guess how the opponent is going to bid currently but also to look into future behavior.[27]

Of course, the observed inconsistency between first auction bids and beliefs could also be interpreted as a lack of rationality by subjects. However, such an explanation would (even more) hardly be reconciled with the sequential rationality recorded in the second auction. On the other hand, any explanation based on non-standard preferences but that retains the

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[26]Notice that in these experiments signaling issues are absent. Interestingly, Brosig and Reiß (2007), where, instead, first auction bids have signaling content, find a significant departure from equilibrium bids in the second auction. However, given that, in their experiment, beliefs are not elicited, they cannot assess whether these bids are consistent with sequential rationality.

[27]By the way, at the end of the experiment, more than one subject confided us that, while it was clear to her/him what strategy to follow in the second auction, it was much less clear what to do in the first.

equilibrium condition (risk aversion, loss aversion, quantal response equilibrium) would, per se, be unable to rationalize why subjects do not correctly update their beliefs.

From the theoretical point of view, our paper suggests that a lack of belief updating in dynamic games can coexist with rationality, once one admits that individuals' actual behavior is affected by unobservable cognitive characteristics. When this is the case, upon observing the behavior of others, a rational individual will simultaneously update her beliefs on these cognitive characteristics along with the non-cognitive charactericts (the capacity of the opponent in our case). These two processes mix together, possibly leading to outcomes that are inconsistent with equilibrium and appear, at first glance, irrational.

On more practical grounds, our contribution suggests that, when a procurer considers buying multiple goods or services sequentially, she must take into account that the information revealed from one round to the next may not be interpreted as the theory predicts, thus producing unexpected outcomes, and this may happen even when bidders act rationally. Hence, the amount and the quality of the information that is disclosed to bidders, not only during the course of the auction, but also before it, is a crucial element that has to be carefully considered by the procurer.

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A Proofs

A.1 Proof of Proposition 1

(i) Suppose $\mu_L \in (0, 1]$. Let the cumulative distribution functions $\beta_W(b)$ and $\beta_L(b)$ denote the equilibrium bidding strategies of the Winner and the Loser, respectively. Let S_W and S_L denote the corresponding supports.

Notice, first, that bidding $b < (1 - \mu_L)R$ is strictly dominated for the Loser, as the Loser can guarantee herself an expected payoff equal to (at least) $(1 - \mu_L)R$ by bidding exactly R. This immediately implies that, in equilibrium, the Winner will not bid below $(1 - \mu_L)R$ either. Moreover, for the Winner bidding R cannot be optimal because either it gives her a zero probability of winning, or, if R is a mass point of $\beta_L(b)$, bidding slightly less than R is a profitable deviation. Hence, we have that, necessarily, $S_L \subseteq [(1 - \mu_L)R, R]$, $S_W \subseteq [(1 - \mu_L)R, R)$.

Second, equilibrium bidding strategies are necessarily continuous in $[(1 - \mu_L)R, R)$, i.e. there are no mass points in that interval. To see this, suppose, to the contrary, that the strategy of, say, the Loser attaches strictly positive probability to some $b \in [(1 - \mu_L)R, R)$. Then, for sufficiently small ε, the Winner will never make a bid in $[b, b + \varepsilon]$, as these bids are certainly worse than bidding just below b. But if the Winner never makes a bid in $[b, b + \varepsilon]$, then bidding b for the Loser cannot be optimal.

Third, the Winner will not place a bid in an interval in which the Loser never places any bid. This is also true for the Loser, unless the Loser bids exactly R. This means that $S_W = S_L \setminus \{R\}$.

Fourth, $S_W = [(1-\mu_L)R, R)$, $S_L = S_W \cup \{R\}$ and the Loser bids R with strictly positive probability. To see this, suppose that R is not played with strictly positive probability by the Loser: then a bid close to $\sup(S_W)$ cannot be optimal for the Winner, as it would give her (essentially) a null probability of winning, and bidding $(1 - \mu_L)R$ would be a profitable deviation. Given that $R \in S_L$, the payoff of the Loser must be equal to $(1 - \mu_L)R$ for any bid in S_L; but this implies that $\inf(S_W) = \inf(S_L) = (1 - \mu_L)R$: in fact, if $\inf(S_W) > (1-\mu_L)R$, then bidding just below $\inf(S_W)$ would guarantee the Loser a payoff that is strictly larger than $(1 - \mu_L)R$. Moreover, $\sup(S_W) = \max(S_L) = R$: if $\sup(S_W) < R$, then bidding $\sup(S_W)$ would give the Loser the same probability of winning as bidding R, but with a strictly lower payoff in case of winning. Finally, there is no interval in $[(1 - \mu_L)R, R)$ to which equilibrium bidding strategies attach null probability: if there were such an interval (x, y), then any bid $b \in (x, y)$ would give strictly lower payoff than any bid larger than b, still within (x, y).

To sum up, the equilibrium bidding strategy of the Winner, $\beta_W(b)$, has no mass points and it is strictly increasing over $[(1 - \mu_L)R, R)$; the equilibrium bidding strategy of the Loser, $\beta_L(b)$, has a mass point at R and it is strictly increasing over $[(1 - \mu_L)R, R)$.

Now, the expected payoff in $A2$ of the Loser when she bids $b \in [(1 - \mu_L)R, R)$ and the Winner bids according to $\beta_W(b)$ is

$$\pi_L^2(b \in [(1 - \mu_L)R, R)) = [(1 - \mu_L) + \mu_L(1 - \beta_W(b))]\, b = [1 - \mu_L \beta_W(b)]\, b.$$

The expected payoff in $A2$ of the Winner when she bids $b \in [(1 - \mu_L)R, R)$ and the Loser bids according to $\beta_L(b)$ is

$$\pi_W^2(b \in [(1 - \mu_L)R, R)) = [\gamma + (1 - \gamma)(1 - \beta_L(b))]\, b = [1 - (1 - \gamma)\beta_L(b)]\, b,$$

where $\gamma > 0$ is the probability with which the Loser bids R. In a mixed strategy equilibrium, the bidder's expected payoff must be constant over its support. In particular, given that we know that $\pi_L^2(b = (1 - \mu_L)R) = \pi_W^2(b = (1 - \mu_L)R) = (1 - \mu_L)R)$, it must be that, for all $b \in ((1 - \mu_L)R, R)$:

$$[1 - \mu_L \beta_W(b)] \, b = (1 - \mu_L)R,$$

and

$$[1 - (1 - \gamma)\beta_L(b)] \, b = (1 - \mu_L)R.$$

The above system admits a unique solution, which is the one stated in the Proposition. In particular, $\gamma = (1 - \mu_L)$. This completes the proof for the case in which $\mu_L \in (0, 1]$.

(ii) When, instead, $\mu_L = 0$ (the Loser believes that the Winner is not going to bid in the second auction), the Loser will certainly bid R. The Winner will thus best respond by bidding just below R.

$\square$

A.2   Proof of Proposition 2

Let A_i $(i = 1, 2)$ denote the support of the equilibrium bidding strategy of a bidder with capacity i.

Notice, first, that, in equilibrium, the expected payoff of a bidder with capacity $c = 2$ must be strictly larger than the expected payoff of a bidder with capacity $c = 1$, i.e. $\pi_2(a_2 \in A_2) > \pi_1(a_1 \in A_1)$.[28] To see this, notice that the difference between the expected payoff of a bidder with capacity $c = 2$ and the expected payoff of a bidder with capacity $c = 1$ is due to the fact that, in case of winning the first auction, a bidder of type $c = 2$ will also take part in $A2$, where she can possibly sell a second good. In symbols, we have that, for all a,

$$\pi_2(a) - \pi_1(a) = \mathrm{PW}(a) \times q(1 - \mu_L(a))R \geq 0,$$

where $\mathrm{PW}(a)$ is the probability of winning the first auction with a bid equal to a, and $(1 - \mu_L(a))R$ is the payoff in $A2$ that a bidder with capacity $c = 2$ can expect to obtain if she wins the first with a bid equal to a (see Proposition 1). Now, for all $a' \in A_1$ and $a'' \in A_2$ (i.e. a' and a'' are equilibrium bids for type $c = 1$ and $c = 2$, respectively), it must be:

$$\pi_1(a') \geq \pi_1(a''), \qquad \pi_2(a') \leq \pi_2(a'').$$

Given that, for all a, $\pi_2(a) - \pi_1(a) \geq 0$, we have the following chain of inequalities:

$$\pi_1(a'') \leq \pi_1(a') \leq \pi_2(a') \leq \pi_2(a'').$$

Now, suppose, by contradiction that, in equilibrium, expected payoffs of types $c = 2$ and $c = 1$ are equal, i.e. $\pi_1(a') = \pi_2(a'')$. Then it must also be that $\pi_1(a') = \pi_2(a')$, or $\mathrm{PW}(a') \times q(1 - \mu_L(a'))R = 0$. But this inequality cannot be satisfied for all $a' \in A_1$: in fact, if the equilibrium bidding strategy of a type-1 bidder attaches strictly positive probability to some

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[28]For the sake of clarity, here we suppress the dependence of a bidder's expected payoff from the other bidder's strategy.

a', then $\text{PW}(a') > 0$ and $\mu_L(a') < 1$; if, instead, no element of A_1 is played with strictly positive probability, then, necessarily, $\text{PW}(a') > 0$ for all $a' < \sup(A_1)$ and $\mu_L(a')$ cannot be equal to 1 for all of them. Hence, it must necessarily be $\pi_1(a') < \pi_2(a'')$, and the above chain of inequalities reduces to

$$\pi_1(a'') \leq \pi_1(a') < \pi_2(a') \leq \pi_2(a''). \tag{1}$$

(i) **Separating Equilibrium.** Suppose that there is a separating equilibrium. As before, let $a' \in A_1$ and $a'' \in A_2$ be equilibrium bids for type $c = 1$ and $c = 2$, respectively (notice that, in a separating equilibrium, A_1 and A_2 are disjoint sets). Notice that, because $a'' \in A_2$ but $a'' \notin A_1$, it must be $\mu_L(a'') = 1$. Therefore, we have

$$\pi_2(a'') - \pi_1(a'') = \text{PW}(a'') \times q(1 - \mu_L(a''))R = 0,$$

i.e. $\pi_2(a'') = \pi_1(a'')$. But then (1) would be violated. We conclude that there is no separating equilibrium.[29]

(ii) **Pooling Equilibrium.** In a pooling equilibrium, $A_1 = A_2 = A$ and $\mu_L(a) = p$ for all $a \in A$. Consider a bidder with capacity $c = 1$. This bidder can always guarantee herself a payoff equal to $q(1-p)R$ (by bidding R in the first auction, *de facto* skipping it). This implies that $\inf(A) \geq q(1-p)R$. Moreover, $\max(A) = q(1-p)R$. To see this, notice first that $\sup(A) \in A$ (therefore $\sup(A) = \max(A)$): in fact, $\pi_2(a) - \pi_1(a)$ must be strictly positive and constant for all $a \in A$; if $\sup(A) \notin A$, you can always find $\hat{a}$ sufficiently close to $\sup(A)$ such that $\pi_2(\hat{a}) - \pi_1(\hat{a})$ is lower than any strictly positive constant. Second, A must attach strictly positive probability to $\max(A)$: if this were not true, the condition $\pi_1(a) < \pi_2(a)$, that must always hold in equilibrium, would be violated for $a = \max(A)$. Third, if $\max(A) > q(1-p)R$, then a bidder with capacity $c = 1$ would rather slightly reduce her bid. We conclude that $A = \{q(1-p)R\}$. Now, given that a bidder with capacity $c = 1$ can always guarantee herself a payoff equal to $q(1-p)R$ by bidding R, such a bidder is certainly willing to play $a^* = q(1-p)R$ in the first auction, if the other bidder does so. For a bidder with capacity $c = 2$, the expected payoff by bidding $a^* = q(1-p)R$ (when the other bidder does so) is equal to

$$\pi_2(a^*) = \frac{3}{2}a^*.$$

Bidding more than a^* is certainly not profitable. Bidding $a < a^*$ is not profitable either if

$$\frac{3}{2}a^* \geq a + q(1 - \mu_L(a))R,$$

or

$$\mu_L(a) \geq \frac{1+p}{2} - \frac{a^* - a}{qR}.$$

Hence, if the Loser's beliefs satisfy the above condition for all $a < a^*$, the pure strategy a^* constitutes a pooling equilibrium (and no other pooling equilibrium is possible).

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[29]This shows that, more generally, there is no equilibrium in which type $c = 2$ fully reveals her type with strictly positive probability.

(iii) **Hybrid Equilibrium.** (a) In a hybrid equilibrium, $A = A_1 \cap A_2$ is non-empty. It is immediate to observe that $A_2 \setminus A_1 = \emptyset$, i.e. $A_2 = A$ (see point *(i)* of this proof).

(b) Moreover, if $A_1 \setminus A_2 \neq \emptyset$, then, for all $a' \in A_1 \setminus A_2$ and all $a'' \in A_2$, it must be $a' > a''$. To see this, observe that

$$\pi_1(a') = \pi_1(a''), \qquad \pi_2(a') \leq \pi_2(a''),$$

which implies

$$\pi_2(a') - \pi_1(a') \leq \pi_2(a'') - \pi_1(a''),$$

or

$$\mathrm{PW}(a') \times qR \leq \mathrm{PW}(a'') \times q(1 - \mu_L(a''))R,$$

where, in the last expression, we used the fact that $\mu_L(a') = 0$. Notice that the RHS of the inequality above must be strictly positive; hence, to satisfy it, it must be that, for any $a' \in A_1 \setminus A_2$, either $\mathrm{PW}(a') = 0$, or $\mathrm{PW}(a') < \mathrm{PW}(a'')$. In both cases, the implication is the same: any $a' \in A_1 \setminus A_2$ must be larger than any $a'' \in A_2 = A$.

(c) Consider now any two bids $a', a'' \in A$, with $a' < a''$, Since they both belong to A, it must be

$$\pi_1(a') = \pi_1(a''), \qquad \pi_2(a') = \pi_2(a''),$$

which implies

$$\pi_2(a') - \pi_1(a') = \pi_2(a'') - \pi_1(a''),$$

or

$$\mathrm{PW}(a') \times q(1 - \mu_L(a'))R = \mathrm{PW}(a'') \times q(1 - \mu_L(a''))R.$$

Notice that it is not possible that, in equilibrium, $\mathrm{PW}(a') = \mathrm{PW}(a'')$ (which would imply $\mu_L(a') = \mu_L(a'')$): if this were the case, bidding a' would not be optimal, as a bid equal to a'' would guarantee the same probability of winning the first auction, with no impacts on beliefs, but a larger first auction payoff in case of winning. Hence, it must be $\mathrm{PW}(a') > \mathrm{PW}(a'')$; but then, to satisfy the equality above, we necessarily have $\mu_L(a') > \mu_L(a'')$, i.e. μ_L is strictly decreasing over A.

(d) See point *(ii)* of this proof.

$\square$

A.3   Proof of Proposition 3

From Proposition 1, we know that a bidder's expected payoff from the second auction is $q(1 - \mu_L)R$. Under the assumption that bidders' beliefs coincide with the prior probability p, a bidder's expected payoff from the second auction is $\omega = q(1 - p)R$. Given this, the total expected payoff of a bidder with capacity $c = 2$ that bids a in $A1$ is

$$\pi_2(a) = (a + \omega)\,\mathrm{PW}_1(a) + \omega\,(1 - \mathrm{PW}_1(a)) = a\,\mathrm{PW}_1(a) + \omega,$$

whereas the total expected payoff of a bidder with capacity $c = 1$ that bids a in $A1$ is

$$\pi_1(a) = a\,\mathrm{PW}_1(a) + \omega\,(1 - \mathrm{PW}_1(a)),$$

where $\mathrm{PW}_1(a)$ is the probability of winning $A1$ with a bid equal to a.

Consider a level-k bidder: this bidder believes that the opponent is a level-$(k-1)$ bidder who, depending on her capacity, bids a_2^{k-1} or a_1^{k-1}. The only relevant bids to be considered are the following:

- $a > a_1^{k-1}$: with such bid, the bidder will not win $A1$;

- $a = a_1^{k-1}$: with such bid, the bidder will win $A1$ only if the opponent has capacity $c = 1$ and the tie is resolved in her favor;

- $a = a_1^{k-1} - 1$: with such bid, the bidder will win $A1$ only if the opponent has capacity $c = 1$ (notice that, under the assumptions of the Proposition, $a_1^{k-1} - a_2^{k-1} > 1$, i.e. $a_1^{k-1} - 1 > a_2^{k-1}$);

- $a = a_2^{k-1}$: with such bid, the bidder will win $A1$ if the opponent has capacity $c = 1$ or the opponent has capacity $c = 1$ and the tie is resolved in her favor;

- $a = a_2^{k-1} - 1$: with such bid, the bidder will win $A1$ for sure.

All other bids are certainly suboptimal.

Now, consider a level-k bidder with capacity $c = 2$. Her expected payoff associated with each of the bids specified above is:

- $\pi_2^k(a > a_1^{k-1}) = \omega$;

- $\pi_2^k(a = a_1^{k-1}) = a_1^{k-1}(1-p)/2 + \omega$;

- $\pi_2^k(a = a_1^{k-1} - 1) = (a_1^{k-1} - 1)(1-p) + \omega$;

- $\pi_2^k(a = a_2^{k-1}) = a_2^{k-1}(1 - p/2) + \omega$;

- $\pi_2^k(a = a_2^{k-1} - 1) = a_2^{k-1} - 1 + \omega$.

It is immediate to see that a bid $a > a_1^{k-1}$ cannot be optimal. We then have that π_2^k is maximized at $a = a_2^{k-1} - 1$ if the following conditions simultaneously hold:

(a) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_1^{k-1}) \iff a_1^{k-1} - a_2^{k-1} \leq \frac{1+p}{2}a_1^{k-1} - 1$;

(b) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_1^{k-1} - 1) \iff a_1^{k-1} - a_2^{k-1} \leq p(a_1^{k-1} - 1)$;

(c) $\pi_2^k(a = a_2^{k-1} - 1) \geq \pi_2^k(a = a_2^{k-1}) \iff a_2^{k-1} \geq \frac{2}{p}$.

Now, consider a level-k bidder with capacity $c = 1$. Her expected payoff associated with each of the bids specified above is:

- $\pi_1^k(a > a_1^{k-1}) = \omega$;

- $\pi_1^k(a = a_1^{k-1}) = a_1^{k-1}(1-p)/2 + \omega(1+p)/2$;

- $\pi_1^k(a = a_1^{k-1} - 1) = (a_1^{k-1} - 1)(1-p) + \omega p$;

- $\pi_1^k(a = a_2^{k-1}) = a_2^{k-1}(1 - p/2) + \omega p/2$;

- $\pi_1^k(a = a_2^{k-1} - 1) = a_2^{k-1} - 1$.

π_1^k is maximized at $a = a_1^{k-1} - 1$ if the following conditions simultaneously hold:

(d) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a > a_1^{k-1}) \iff a_1^{k-1} \geq \omega + 1$;

(e) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_1^{k-1}) \iff a_1^{k-1} \geq \omega + 2$;

(f) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_2^{k-1}) \iff a_1^{k-1} - a_2^{k-1} \geq \frac{p(a_1^{k-1} - \omega - 1)}{2-p} + 1$;

(g) $\pi_1^k(a = a_1^{k-1} - 1) \geq \pi_1^k(a = a_2^{k-1} - 1) \iff a_1^{k-1} - a_2^{k-1} \geq p(a_1^{k-1} - \omega - 1)$.

Hence, if conditions (a)-(g) simultaneously hold, a level-k bidder will find it optimal to bid $a_2^{k-1} - 1$ when her capacity is $c = 2$, and $a_1^{k-1} - 1$ when her capacity is $c = 1$. After noticing that (d) is implied by (e) and (a) is implied by (b) and (e), we are left with conditions (b), (c), (e), (f) and (g), i.e. $a_1^{k-1} \geq \omega + 2$, $a_2^{k-1} \geq 2/p$, and

$$\max\left[p(a_1^{k-1} - \omega - 1); \frac{p(a_1^{k-1} - \omega - 1)}{2 - p} + 1\right] \leq a_1^{k-1} - a_2^{k-1} \leq p(a_1^{k-1} - 1).$$

$\square$

B   Experimental Instructions

[Instructions were originally written in Italian. The following instructions refer to treatment T1, where p (the probability of being assigned a capacity c = 2) and q (the probability of implementing the second auction) were both set to 50%.]

Instructions

Welcome! Thanks for participating in this experiment. By following these rules carefully, you can earn an amount of money that will be paid cash at the end of the experiment.

There are 24 subjects participating in this experiment. Both the identities and the final payments of the subjects will remain anonymous throughout the experiment.

During the experiment you are not allowed to communicate with the other participants. If you have questions, raise your hand and one of the assistants will come to your seat and assist you. The following instructions are the same for all the participants.

General rules

The experiment will consist of 15 periods, and in every period you will be presented with the same economic situation that is described in what follows.

At the beginning of each period, you will be randomly and anonymously assigned to a new group of two subjects. This means that the composition of the group will change in every period and you will never interact with the same participant in two consecutive periods.

During the experiment, your earnings will be expressed in points. At the end of the experiment, the total number of points earned in the 15 periods will be exchanged in euro at the exchange rate: 1 point = 2 euro cents.

How earnings are determined in each period

In each period of the experiment, you and the other subject in your group will compete as sellers for the sale of some (identical) objects to a buyer. In particular, you and the other subject in your group will participate in two sequential auctions, each involving a single object. The rules for determining the winner in each auction are very simple: the winner is the seller who offers the lowest selling price for the object.

The maximum price that the buyer is willing to pay for each object is 120 points. This means that you and the other subject cannot post a price for an object that is higher than 120 points.

At the beginning of the period, the computer will randomly assign an endowment of objects to each subject in the group. The endowment represents the maximum number of objects that the subject can sell to the buyer in that period. Specifically, each seller has a 50% probability of receiving an endowment of 2 objects, and a 50% probability of receiving an endowment of 1 object. The number of objects assigned to each seller is independent from both the number of objects assigned to the other seller and the number of objects assigned in previous periods. You will receive information about your endowment at the beginning of each period and before participating in the first auction. However, you will receive no information about the endowment assigned to the other subject in your group.

The first auction

You and the other seller in your group will choose the price for the first object simultaneously and anonymously. Then, the computer will compare the two prices: the seller who offers the lowest price wins the first auction. If sellers offer the same price, the computer randomly selects the winner (each seller has a 50% probabilility of being selected). The winner's earnings are equal to the winning price and his/her endowment is reduced by one object. The loser's earnings are equal to zero, and his/her endowment remains unchanged.

The second auction

Only sellers that, at the end of the first auction, have a non-zero endowment of objects make an offer in the second auction. If your endowment is empty, you will not make any offer in the second auction. To determine the winner of the second auction, the computer will use the same rule of the first auction: the seller who chose the lowest price wins the second auction and ties are broken by the computer randomly and with equal probability. If only one seller makes an offer in the second auction, he/she wins regardless of his/her choice.

There is the possibility that the second auction will be revoked. If the auction is not revoked, the winner's earnings are equal to the winning price and are added to those obtained in the first auction, whereas the loser does not receive any additional point. If, instead, the second auction is revoked, the earnings of both subjects in that auction are equal to 0. Whether the second auction is revoked or not will be determined by the computer randomly: in particular, the probability that the auction is revoked is equal to 50%. Moreover, whether the second auction in a period is revoked or not does not depend from what happened in the previous periods. You will be informed about whether the second auction has been revoked only at the end of the period, after you and the other subject have made your choices.

Conjectures on the endowment of the other subject in the group

After being informed about the outcome of the first auction and before making your offer in the second auction, you will be asked to formulate two conjectures: the first about the probability that the other seller in your group has an endowment of 1 object, and the second about the probability that he/she has an endowment of 2 objects.

Every conjecture must be a number between 0 and 100, where 0 means that you assign no chance to the fact that other subject has that specific endowment, and 100 means that you are sure that the other subject has that specific endowment. The two conjectures must sum up to 100.

Given your conjectures, at the end of the period you will take part in a lottery that, in case of success, will add 20 additional points to your earnings in that period. The lottery is designed in a way that it is in your interest to formulate your conjectures carefully: the higher the probability you expect the other seller to have a specific endowment, the larger should be your conjecture associated with that endowment.

The computer will assign a number of tickets included between 0 and 10000 to each of the two conjectures according to the following expressions:

$$tickets \ (endowment \ of \ 1 \ object) = 10000[1 - (1 - conjecture \ 1 \ object/100)^2],$$

$$tickets \ (endowment \ of \ 2 \ objects) = 10000[1 - (1 - conjecture \ 2 \ objects/100)^2].$$

Tickets assigned to each of the two conjectures will be numbered from 1 to the number determined according to the previous expressions.

At the end of the experiment, the computer will randomly draw an integer number between 1 and 10000 with equal probability. This random number will be compared with the tickets associated, according to the previous expressions, with the conjecture for the actual endowment of the other subject. If the random number is smaller than or equal to the number of tickets associated with that conjecture, you will earn 20 additional points; otherwise you will earn nothing.

Example. You expect the other subject to have an endowment of 1 object with a probability of 70 over 100 and, therefore, you expect the other subject to have an endowment of 2 objects with a probability of 30 over 100. In this case, according to expressions above, the computer assigns 9100 tickets to the former conjecture and 5100 tickets to the latter conjecture. Note that formulating a higher conjecture for the event you expect to be more likely to occur is always convenient for you as, in case your expectation turns out to be correct, you will have a higher probability to win the lottery. Suppose that the actual endowment of the other subject is 2 objects: this means that you will take part in the lottery with 5100 tickets. Suppose that the random number drawn by the computer is 4812. Since the random number is lower than the overall number of tickets assigned to your conjecture that the other subject's endowment is 2 objects, you earn the 20 additional points.

If you attach a probability of 100 over 100 to a certain endowment, then, regardless of the random number drawn by the computer, you will earn the additional 20 points if the other subject has actually been assigned that endowment, whereas you will earn nothing in the opposite case.

Before confirming your choices, you will have the opportunity to simulate the number of tickets assigned to each of the two conjectures using the "Calculate tickets" button. You can modify your conjectures how many times you wish. To confirm your conjectures, click on the "Confirm your conjectures" button.

At the end of each period, you will be informed about the actual endowment assigned to the other seller, the result of the lottery, the winning bids in the first and in the second auction, and your earnings in that period.