Optimal Bank Capitalization in Crowded Markets*

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Abstract

We study banks' optimal equity buffer in general equilibrium and their response to under-capitalization. Making progress towards a "pecking order theory" for private recapitalizations, our benchmark model identifies equity issuance as individually and socially optimal, compared to deleveraging, as well as conditions that invert the individually optimal ranking. Imperfectly elastic supply of capital, incomplete insurance markets and costly bankruptcies give rise to inefficiently high leverage ex-ante, and to excessive capital shortfalls and insolvencies ex-post. Abstracting from moral hazard and informational asymmetries, we therefore provide a novel rationale for macroprudential capital regulation and new testable implications about banks' capital structure management.

Keywords: Bank capital, recapitalization, macroprudential regulation, incomplete markets, financial market segmentation, constrained inefficiency

JEL classifications: D5, D6, G21, G28

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1 Introduction

While the scarcity of specialized investment capital has received considerable attention in conjunction with asset fire sales (Shleifer and Vishny, 1992), its broader implications for banks' capital structure management are less well understood. At the same time, existing evidence suggests that equity offerings are important for the recapitalization of under-capitalized banks (De Jonghe and Öztekin, 2015; Dinger and Vallascas, 2016), and that large system-wide issuance volumes increase banks' cost of capital (Lambertini and Mukherjee, 2016).[1] We offer a theoretical underpinning for this evidence by developing a general equilibrium model of bank capital that encompasses private recapitalizations by equity issuance and deleveraging, as well as optimal ex-ante leverage.

We analyze situations in which banks need to recapitalize simultaneously and over a relatively short period, so that markets are crowded and the supply of specialized investment capital is imperfectly elastic.[2] We study implications for banks' optimal equity buffer and derive a *pecking order theory* for ex-post recapitalization strategies. Our benchmark model identifies equity issuance as individually and socially optimal, compared to deleveraging, provided the latter reduces portfolio returns in capital-constrained states. Deleveraging then destabilizes the individual bank, as well as the sector as a whole, by exacerbating the (aggregate) capital shortage. Banks' individually optimal ranking of recapitalization strategies can be inverted in the presence of additional frictions, related, for example, to corporate governance or debt renegotiation.

Ex-ante, and due to a *pecuniary externality*, banks do not fully incorporate the general equilibrium effect of initial equity buffers on their and others' ability to recapitalize. Under plausible distributional assumptions about the riskiness of banks' assets, this implies inefficient over-leveraging and excessive insolvencies.

Our framework resonates with the definition by Brownlees and Engle (2017), according to which *systemically risky banks* are prone to under-capitalization when the system-wide capital shortfall is particularly severe. We show that excessive exposure to such systemic capital shortfalls, as the result of inefficient

[1] Additional evidence consistent with the observation that banks recapitalize through share issuance is available from studies focusing on German (Memmel and Raupach 2010), Swiss (Rime 2001), British (Ediz et al. 1998), European (Kok and Schepens 2013) and Middle Eastern banks (Alkadamani 2015). The evidence on the link between share issuance volumes and banks' cost of capital is empirically less well-developed. However, some evidence from related security markets exists and is discussed below.

[2] Evidence of limits to arbitrage in equity markets goes back at least to Asquith and Mullins (1986) and Pontiff (1996). More recently, Mitchell et al. (2007), Mitchell and Pulvino (2012), and Duffie (2010) in his presidential address to the American Finance Association, argue that capital, for example, in the market for convertible debt is slow-moving. In addition, there is some evidence on bank equity markets in Cornett and Tehranian (1994), who show that banks' share price tends to drop in response to *involuntary* share issuance, i.e. for reasons likely to be unrelated to adverse selection à la Myers and Majluf (1984).

over-leveraging, can be individually optimal, even in the absence of deposit insurance or *Too-Big-To-Fail* guarantees. We therefore provide a novel and complementary rationale for macroprudential capital regulation that does not require moral hazard or informational asymmetries.

Our theory allows for a positive analysis of banks' capital structure management and yields the normative insight that inefficiencies may be amplified if banks select deleveraging over equity issuance. Moreover, it delivers a set of novel and testable predictions about banks' recapitalization strategies, their respective stability implications, and the link between ex-ante equity buffers and future costs of capital. Finally, our analysis is also relevant for the design and communication of supervisory stress tests, since they can create the kind of aggregate capital shortages that we have in mind. Recent evidence on crowded markets for bank equity, for instance, is available from the *Supervisory Capital Assessment Program* (SCAP). In May 2009 the Federal Reserve assessed the 19 largest bank holding companies and identified 10 with a significant capital shortfall. These 10 "excessively risky" banks were mandated to raise equity privately within 6 month, or to face permanent recapitalization from the government.[3] Within a few weeks U.S. firms raised a record $60bn (about $45bn by banks) in new common equity and over $125bn by the end of 2009.[4] Consistent with our model's predictions, Lambertini and Mukherjee (2016) show that these issuance volumes were associated with an increasing cost of capital for banks failing SCAP.[5,6]

We build a two period general equilibrium model of financial intermediation, in which banks manage the maturity mismatch between long-term assets and short-term deposits. They have exclusive access to a risky investment technology and choose their optimal capital structure ex-ante. To this end, they issue uninsured demandable debt and equity to households.[7] Since bank-run induced bankruptcies are costly and ex-ante insurance is not feasible (incomplete insurance markets), the role of bank equity is to protect depositors'

[3] The results of the Supervisory Capital Assessment Program, as well as the details on its design and implementation are available online: http://www.federalreserve.gov/newsevents/press/bcreg/20090507a.htm.

[4] See Hanson et al. (2011) and an US equity market issuance summary by Reuters: http://www.lse.co.uk/ukIpoNews.asp?ArticleCode=4a39ycmc7drz9zm.

[5] As mentioned earlier, elevated issuing costs may also arise due to an adverse selection problem (Myers and Majluf, 1984). Hanson et al. (2011), however, argue that the strong regulatory involvement in the SCAP likely muted the adverse selection problem associated with equity issuance in this case.

[6] Systemic capital shortfalls have also occured in response to the turning of the subprime mortgage market in the US, or when Italian banks were forced collectively to write down large proportions of the non-performing loans on their balance sheets.

[7] We acknowledge the relevance of deposit insurance and guarantee schemes in practice. However, we want to stress that our mechanism does not hinge on the assumption that deposits are uninsured or only partially insured. If mandatory recapitalization is triggered by a regulator (e.g. when a regulatory constraint is hit as the result of a stress test) and not in response to the risk of bank-runs, our qualitative results also hold with full deposit insurance.

claims. Financial market segmentation separates households into *investors* and *depositors*, and captures administrative charges or informational costs, for example, due to financial literacy (Guiso and Sodini, 2013).[8] It implies an endogenous equity premium and an imperfectly elastic supply of specialized investment capital (Holmstrom and Tirole, 1997), which we consider to be a short-term property of the relevant financial markets.[9,10] We assume that households have to pay an idiosyncratic utility cost to become investors, which allows them to purchase equity claims or bank assets. Banks' investments are correlated and the actual risk of their portfolios is revealed at an intermediate date. This makes it necessary for under-capitalized banks to recapitalize, in order to prevent depositor runs and bankruptcy. Whether they can recapitalize, however, depends on individual portfolio risk *and* on market depth, which is−in turn−a function of the aggregate capital shortfall. Banks trade off the cost of ex-ante capital with recapitalization costs and the likelihood of bankruptcy, but do not fully incorporate the role of the aggregate capital shortage.[11] This is due to an endogenous equity premium, which arises because the required compensation of the marginal investor increases with system-wide recapitalization needs, and which implies a pecuniary externality.

Central to our mechanism is that some banks' portfolios are too risky for recapitalization. The upside they can offer to new shareholders is insufficient to compensate households for their participation cost; even if initial shareholders' claims are fully diluted. Ex-ante equity then affects the capital shortfall at the bank level *(intensive margin)*, as well as the threshold level of risk for which recapitalizations remain feasible *(extensive margin).* Through the extensive margin, which is a function of endogenous market conditions, initial leverage thus affects the frequency of bankruptcies. Inefficient under-capitalization occurs when this extensive margin exists and when the intensive margin is sufficiently important.

As a result, our paper is closely related to the literature on fire sales (Shleifer and Vishny, 1992, 2010). Reminiscent of the precautionary and speculative motives, which are a characteristic of this literature (Allen

[8]This micro-foundation for financial market segmentation is motivated by empirical evidence on the low direct and indirect participation in stock markets. See Vissing-Jorgensen (2003) on fixed costs of participation, Barberis et al. (2006) on loss aversion and Guiso et al. (2008) on heterogeneous beliefs or trust.

[9] Similar setups have been used by De Nicolò and Lucchetta (2013), Allen et al. (2014), and Carletti et al. (2017).

[10]Due to positive financial market participation costs, equity is *always* more costly than debt in our model. The magnitude of the cost differential, however, depends on endogenous market conditions. Our assumption is w.l.o.g., as long as *some* of the equity that banks issue is more costly than debt. Notice also, that the reason for the elevated cost of equity is different from traditional reasons related to adverse selection problems à la Myers and Majluf (1984).

[11]The optimal leverage ratio in our model is thus determinate. That bankruptcy costs generate a determinate capital structure is known from Bradley et al. (1984) and Myers (1984), as well as from the literature on firms' optimal capital structure following Modigliani and Miller (1958) and Modigliani and Miller (1963).

and Gale, 1994, 2004b, 2007), we identify an insufficient precautionary motive for ex-ante capitalization. Because a pecuniary externality, in combination with incomplete markets for ex-ante risk-sharing and costly bankruptcies, gives rise to a *constrained inefficiency*, our paper is furthermore related to Lorenzoni (2008) and Dávila and Korinek (2017). Akin to the *borrowing constraint* in Lorenzoni (2008), which depends on asset prices and affects leverage, our *recapitalization threshold* depends on the endogenous crowdedness of the market for specialized investment capital. Korinek (2012) characterizes financial amplification, building on a pecuniary externality that links asset fire sales to falling prices and tightening borrowing constraints. In his model, financially constrained firms inefficiently under-insure (due to the under-valuation of liquidity during crises).[12] Conversely, a tighter recapitalization constraint reduces market crowdedness in our model (due to the extensive margin effect). Using the language of Dávila and Korinek (2017), the externality in our model can thus be characterized as a *collateral externality*.[13]

Next, our paper also relates to the extensive literature on the optimal capital structure of firms. In contrast to the classical model of Modigliani and Miller (1958), markets in our model are incomplete, implying that excessively risky banks are subject to depositor runs and insolvency. Because bankruptcies are costly, the optimal capital structure is determinate. While we analyze banks' capital structure and implications for macroprudential regulation, Gale and Gottardi (2015) study firms that choose leverage and investment, to balance the tax advantages of debt with default risk.[14] Related papers, specifically on banks' capital structure and regulation, include Gorton and Pennacchi (1990), Admati et al. (2011), DeAngelo and Stulz (2015), Allen et al. (2014), Gale and Gottardi (2017) and Carletti et al. (2017). Prominently, Admati et al. (2013) build on an agency conflict between shareholders and creditors and find banks to be under-capitalized due to a leverage ratchet effect. DeAngelo and Stulz (2015), instead, argue that liquidity creation by banks can justify high leverage.[15]

[12]In a recent paper, Walther (2016) develops a model with a price externality in asset markets where banks under-invest in liquidity and build up excessive leverage. While in our model the endogenous cost of equity issuance relates to the frequency of insolvency, Walther's inefficiency is not related to solvency and arises because banks do not internalize the possibility of a socially costly transfer of assets to investors.

[13]The shadow value of the financial constraint and the sensitivity of the financial constraint to asset prices are negative, while the sensitivity of the equilibrium capital price to changes in aggregate state variables may be positive or negative. The sign of the inefficiency depends on the intensity of the recapitalization margin. Bankers inefficiently under-insure if the extensive margin exists and the intensive recapitalization margin is sufficiently strong.

[14]In their dynamic general equilibrium model with asset fire sales by insolvent firms, inefficient under-investment occurs because firms do not internalize how aggregate debt reduces the tax burden. Conversely, we find in our model that banks tend to be inefficiently over-leveraged when they trade off the ex-ante cost of equity with the likelihood of costly bankruptcy.

[15]For different types of agency problems see Kashyap et al. (2008) and Philippon and Schnabl (2013), who study efficient

While moral hazard problems play a prominent role in the literature on macroprudential capital regulation (Farhi and Tirole, 2012; Begenau, 2015), our paper is more closely related to papers motivating the need for regulation based on externalities (see, e.g., Gersbach and Rochet, 2012; De Nicolò et al., 2012; Malherbe, 2015; Klimenko et al., 2016 and De Nicolò et al., 2012 for a survey) and emphasizing the buffer function of equity (Repullo and Suarez, 2013). Similar to the dynamic model of Klimenko et al. (2016), banks in our model also fail to internalize the effect of their individual decisions on the loss-absorbing capacity of the banking sector. However, our work is complementary in that we do not study implications for lending but focus on the efficiency implications of different forms of private bank recapitalizations. This focus also separates us from papers studying the joint regulation of capital and liquidity (Calomiris et al., 2015; Eichenberger and Summer, 2005; Boissay and Collard, 2016; Hugonnier and Morellec, 2017). However, we recognize the role of reserves as a buffer against losses (as emphasized, e.g., in Calomiris et al., 2015) and introduce cash in an extension. In our model the issuance of additional equity claims is equivalent to the bail-in of deposits. While we acknowledge the growing literature on conditionally convertible (CoCo) bonds (Flannery, 2002; Kashyap et al., 2008; Martynova and Perotti, 2017), we focus on common equity, which is without loss of generality as long as convertible debt has to be held by specialized investors.

The remainder of the paper is organized as follows. Section 2 presents the baseline economy, analyses the recapitalization choice and solves for the laissez-faire equilibrium. Section 3 analyses efficiency with the help of an appropriate second-best benchmark and draws policy conclusions. Thereafter, Section 4 considers various extensions. Testable implications are discussed in Section 5. Finally, Section 6 concludes.

2 Model

Time, agents and endowments. There are three dates $t \in \{0,1,2\}$ and one perishable good that can be consumed or invested. The economy is populated by unit continuums of households and bank managers, indexed by i and j, respectively. Households, as well as managers are ex-ante identical, risk-neutral and live for two periods. Each household is endowed with one unit of the good at $t = 0$, and can either consume immediately or invest. Households' preferences are given by $U_i = \sum_{t\in\{0,1,2\}} C_{it}$, where C_{it} is the level of consumption of household i at date t. Bank managers have no endowment, but exclusive access to a risky

government interventions in the presence of debt overhang.

investment technology. They collect funds from households by issuing bank deposits and shares, and invest them at a (potentially small) effort cost $\gamma > 0$, which is proportional to the size of the bank. We consider the case of a competitive banking sector, so that managers' participation constraint holds with equality.[16]

Risky investment technology. At $t = 0$, each bank invests the collected funds in a portfolio of risky long-term assets with stochastic returns (e.g. in risky loans to an unmodelled corporate sector).[17] Portfolio returns depend on the aggregate state of the economy, $\theta \in \{G, B\}$, and bank-specific risk, Δ_j. Uncertainty about both is resolved at the beginning of $t = 1$. The aggregate state is *good* ($\theta = G$) with probability $0 < p < 1$, or *bad* ($\theta = B$) with probability $(1-p)$. If $\theta = G$, the portfolios of all banks generate a safe return of $R > 1$; if $\theta = B$ portfolio returns are stochastic and heterogeneous across banks:

$$\tilde{R}^B(\Delta_j) = \begin{cases} R + \varepsilon\Delta_j, & \text{with probability } \frac{1}{2} \\ R - \Delta_j, & \text{with probability } \frac{1}{2}, \end{cases}$$

where $\Delta_j > 0$ captures the bank-specific portfolio risk and $0 \leq \varepsilon \leq 1$. A fraction $0 < s \leq 1$ of banks is characterized by $\Delta_j = \underline{\Delta}$ with $\underline{\Delta} \in (0, R]$, while $\Delta_j \sim U[\underline{\Delta}, R]$ for the remaining fraction $(1-s)$.[18] To capture the systemic events that we are interested in, banks' portfolio returns are perfectly correlated. Specifically, the realization of returns in state B is either $R + \varepsilon\Delta_j$ or $R - \Delta_j, \forall j$ with $\mathbb{E}[\tilde{R}^B(\Delta_j)] = R + \frac{\varepsilon-1}{2}\Delta_j \ \forall j$.[19] Figure 3 in Appendix A.1 illustrates the payoff structure.

Financial intermediation. Banks are protected by limited liability and collect uninsured deposits on a perfectly competitive market at $t = 0$. We denote the endogenous deposit rate of bank j by r_j and the corresponding expected return by $\overline{r}_j$. Banks also raise capital on a spot market at $t = 0$, by offering shares of their franchise to households. As explained below, the latter endogenously segment into depositors and specialized investors. The premium for specialized investment capital is $\overline{\tau}_0 > 0$ at $t = 0$ and $\overline{\tau}_1^{\theta} \geq 0$ at

[16]The focus on a competitive banking sector is justified if the systemic capital shortfall state is a rare event. The key insights prevail when banks have a monopoly in the deposit market; a case we analyze in an earlier working paper.

[17]For reasons discussed earlier, our baseline model has no storage technology and therefore no portfolio choice problem. If equity issuance (or retained earnings) are cost-effective ways of building a loss buffer, our results generalize (see Section 4).

[18]We assume a uniform distribution of Δ_j for convenience, although it is *not* required for our results.

[19]For $\varepsilon = 1$ we have a mean-preserving spread and the expected return per unit invested is unaffected by the realization of idiosyncratic and aggregate risk. Otherwise, banks with a riskier portfolio receive a lower expected return. In an earlier version of this paper we considered conditionally independent portfolio returns across banks. The key results on efficiency are unaffected.

$t = 1$. Deposits are demandable at $t = 1$ and we impose a sequential service constraint as in Diamond and Dybvig (1983). Both features are motivated by real world deposit contracts and could be endogenized as tools against renegotiation (Diamond and Rajan, 2001).[20] Bank j's deposit and equity funding at $t = 0$ is given by $d_j \geq 0$ and $e_{0j} \geq 0$. The corresponding *capital shortfall* in the bad aggregate state, i.e. if $\theta = B$, is:

$$r_j d_j \geq (R - \Delta_j)(d_j + e_{0j}). \tag{1}$$

As a result of sequential service and public knowledge about Δ_j at $t = 1$, banks are at risk of a run $-$and thus bankruptcy$-$ if they cannot guarantee full repayment of $r_j d_j$ at $t = 2$. Formally this is the case if condition (1) holds with strict inequality. Throughout this paper we limit our attention to runs against undercapitalized banks and abstract from panic runs (Pareto criterion). Insolvent banks face a bankruptcy cost that leaves a small but positive liquidation value (e.g. because they are placed under receivership). Only an arbitrarily small mass of depositors who withdraw can therefore expect to receive the promised r_j. Depositors who withdraw too late and equity investors receive nothing.

Recapitalizations. Banks with a capital shortfall can avoid a run by recapitalizing during the *recapitalization stage* at the beginning of $t = 1$. Our focus is on private recapitalizations in crowded markets. That is, we abstract from public interventions and the accumulation of capital from internal sources over time. Instead, we allow for recapitalizations via the issuance of new equity claims ("liability side operations"), or deleveraging ("asset side operations"). Banks make a take-it-or-leave-it offer to either bail-in depositors, or to exchange deposit claims for a fraction of the bank's assets. In both cases, depositors decide simultaneously whether to accept the banks' offer.[21] Our full information assumption implies that it is observable to the remaining depositors whether a bank can close its capital shortfall or not.

Financial market segmentation. A key feature of our model is that the supply of specialized investment capital is imperfectly elastic in the short-term. It results from the assumption of segmented financial markets, by which the household sector is endogenously partitioned into depositors and investors. Becoming an

[20]Alternative rationales for demandable debt contracts, based on liquidity risk or a disciplining role for deposits, have been proposed by Calomiris and Kahn (1991) or Grossman and Hart (1982).

[21]For a paper with a similar debt-renegotiation problem see Gale and Gottardi (2015).

investor requires paying a utility cost that is akin to a financial market participation cost, which can be interpreted as the cost of acquiring the necessary financial literacy to understand banks' risky cash-flows.

Segmentation at $t = 0$. Each household can either consume her endowment right away, or give it to a bank in the form of deposits or equity. Equity investments, however, i.e. the provision of specialized investment capital, necessitate a utility cost ($\underline{\alpha} > 0$), which enables financial market participation. For simplicity (w.o.l.g.) we assume identical participation costs at $t = 0$ for all households.

Segmentation at $t = 1$. Households who decided at $t = 0$ to become equity investors are passive at $t = 1$, while depositors may engage in bank-runs or participate in the recapitalization of banks, via debt bail-ins or the exchange of their deposit claims against bank assets. As in period $t = 0$, investing in bank equity or assets, requires paying a specialization cost. All households draw an individual financial market participation cost $\alpha_{1i} \sim U[\underline{\alpha}, \overline{\alpha}]$, where $\overline{\alpha} > \underline{\alpha}$. Households who are depositors at the beginning of $t = 1$ can, upon observing the level of portfolio risk of each bank, decide to pay their idiosyncratic utility cost $\alpha_{1i} > 0$ and either participate in debt bail-ins or exchange their deposit claims for bank assets. As discussed in the introduction, the heterogeneity of depositors at $t = 1$ gives rise to an imperfectly elastic supply of specialized investment capital, which we think of as a short-term property of global equity markets.

Timeline. Figure 1 depicts the timeline. The intermediate date is split into a *recapitalization stage* (stage 1) and a *bank run stage* (stage 2). During stage 1 there are no restrictions on depositors of different banks to exchange contracts with each other and with other banks as long as all parties are willing to participate.

Since banks' portfolio risk becomes known at the beginning of $t = 1$ there is no asymmetric information. The crucial frictions are that financial markets and contracts are incomplete, that the supply of investment capital is imperfectly elastic due to segmented financial markets, and that bankruptcies are costly. This completes the description of our model setup.

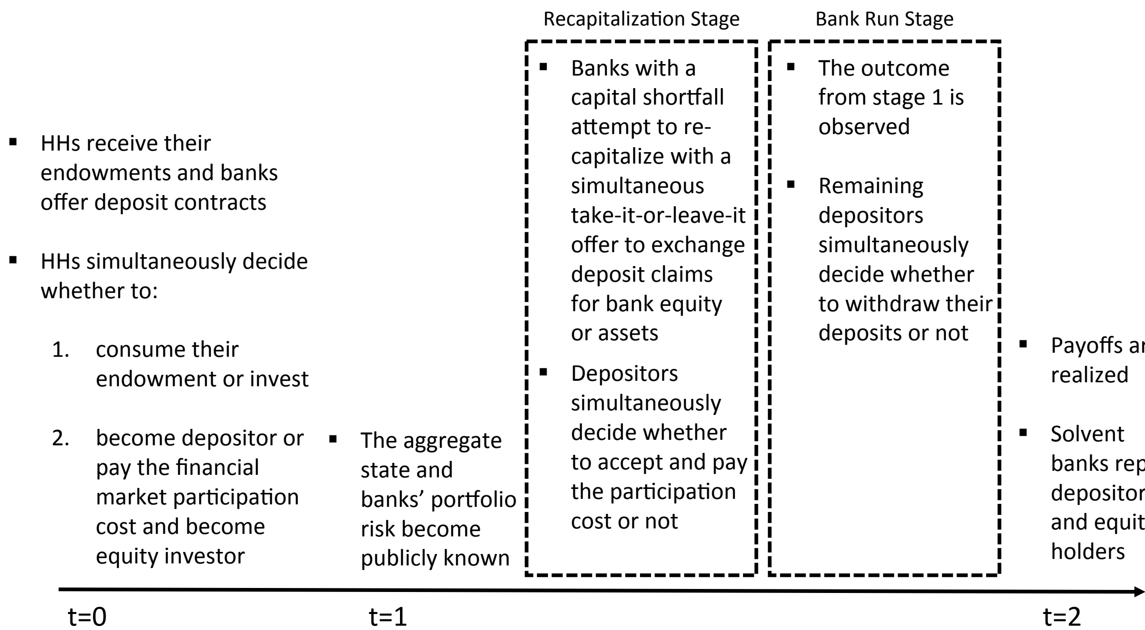

Figure 1: Timeline

2.1 Optimal bank recapitalization

We start our analysis by focusing on the continuation equilibrium at $t = 1$ and study banks' recapitalization choice. To this end, we consider symmetric equilibria and take the ex-ante capital structure $e_{0j} = e_0, \forall j$ and the deposit rate $r_j = r, \forall j$ as given.[22] It follows that all banks have a debt obligation of rd_0, where we normalize bank size to one, so that $d_0 = 1 - e_0$. In the good aggregate state, $\theta = G$, bank returns are deterministic and deposits are safe. There is no risk of runs and, hence, no need for recapitalization.

Instead, in the bad aggregate state, $\theta = B$, bank portfolios are risky and default on deposits is possible. To avoid triggering depositor withdrawal during the *bank run stage*, banks need to eliminate the capital shortfall during the *recapitalization stage*. This requires encouraging initial depositors to incur the financial market participation cost, and to exchange their deposit claims for bank equity or a fraction of the bank's assets. Whether a bank is able to recapitalize depends on the cost of specialized investment capital, which increases in the aggregate capital demand, and on the bank-specific portfolio risk Δ_j, which determines the capital shortfall $x_j(e_0, r; \Delta_j) = \max\{0, r(1 - e_0) - (R - \Delta_j)\}$. Observe that $x_j > 0$ whenever the debt obligation exceeds the lower bound of the return on the risky technology. In our baseline analysis, we restrict

[22] In Section 2.2 we provide conditions under which such outcomes arise in general equilibrium.

attention to scenarios where even the safest banks, i.e. those of type $\underline{\Delta}$, encounter a positive capital shortfall. Formally, this implies $x(e_0, r; \underline{\Delta}) > 0$. We discuss the case $x(e_0, r; \underline{\Delta}) < 0$ as an extension in Section 4.

As mentioned before, we focus on the comparison between equity issuance and deleveraging via asset side operations and abstract, in particular, from recapitalization through retained earnings. This modeling choice is motivated by the empirical literature on the capital structure management of undercapitalized banks (Dinger and Vallascas, 2016; De Jonghe and Öztekin, 2015), and consistent with the short recapitalization horizon that also motivates our assumption of imperfectly elastic supply of investment capital. To avoid a run at $t = 1$, banks can thus either extend take-it-or-leave-it offers for bail-ins of deposit, or offer to exchange a fraction of their assets against deposit claims. If successful, both strategies allow an undercapitalized bank j to eliminate the capital shortfall x_j; if not, depositors run and the bank enters bankruptcy. We first consider the two options for recapitalization in isolation and then jointly, to identify banks' optimal choice.

2.1.1 Bank profits under equity issuance

We begin with the discussion of equity issuance (or bail-ins), which affect the liability side of the balance sheet while keeping bank size unaltered. Recall that recapitalization is only necessary in the bad aggregate state, $\theta = B$. Using the subscript E to indicate that we are considering equity financing, let $e_{1j,E} > e_0$ denote the fraction of equity financing after recapitalization. Equity is endogenously costly due to financial market participation costs. It follows that banks will never raise more capital than necessary. That is, distressed banks optimally recapitalize only up to the point where $r(1 - e_{1j,E}) = R - \Delta_j$.

Raising enough equity at $t = 1$ requires to promise a sufficiently high expected return of $r + \overline{\tau}_1^B$ to new equity investors, where $\overline{\tau}_1^B > 0$ is the expected premium demanded by the marginal depositor, who agrees to the banks' bail-in offer at $t = 1$. Due to limited liability banks' profits in state $\theta = B$ equal:

$$\mathbb{E}[\pi_{j,E}(e_0, \Delta_j)] = \left\{ \begin{array}{l} \frac{1}{2}\max\left\{R + \varepsilon\Delta_j - r(1 - e_{1j,E}) - \left(r + \tau_1^{B,H}\right)(e_{1j,E} - e_0), 0\right\} \\ +\frac{1}{2}\max\left\{R - \Delta_j - r(1 - e_{1j,E}) - \left(r + \tau_1^{B,L}\right)(e_{1j,E} - e_0), 0\right\} \end{array} \right\} \tag{2}$$
$$= \max\left\{R + \frac{\varepsilon - 1}{2}\Delta_j - r(1 - e_0) - \overline{\tau}_1^B\left(e_{1j,E}(\Delta_j, r) - e_0\right), 0\right\},$$

where $e_{1,E}(\Delta_j, r) = 1 - \frac{R - \Delta_j}{r}$ follows from bank j's solvency constraint. Since the equity premium is equal to $\tau_1^{B,H}$ if the payoff is high and $\tau^{B,L}$ if it is low, the expected premium is given by $\overline{\tau}_1^B = \frac{1}{2}(\tau_1^{B,H} + \tau^{B,L})$. A bank's ability to recapitalize in state $\theta = B$ is maximized for $\tau_1^{B,L} = -r$, so that the ability to recapitalize depends entirely on the potential upside that a bank can promise, i.e. on whether the bank can offer $\tau_1^{B,H} = 2\overline{\tau}_1^B + r$. In conclusion, we have $e_{1j,E} > e_0, \forall \Delta_j \in \left[\underline{\Delta}, \hat{\Delta}_E\right] \wedge e_0 < \overline{e}_0 \equiv 1 - \frac{R-\underline{\Delta}}{r}$ if $x\left(e_0, r; \underline{\Delta}\right) > 0$ and $\mathbb{E}\left[\pi_{j,E}\left(e_0, \underline{\Delta}\right)\right] > 0$.

2.1.2 Bank profits under deleveraging ("asset side operations")

We proceed to the analysis of bank recapitalizations through deleveraging via asset side operations, i.e. through an exchange of depositors' claims against a fraction of the assets on banks' balance sheets.[23] Different to equity issuance, this results in a shortening of the balance sheet. We denote by ℓ_j the fraction of assets that are divested by bank j, and by $e_{1j,A} - e_0$ the fraction of depositors that can be retired with the proceeds thereof. As before, participation costs imply that solvent banks will never divest more assets than necessary, so that $r\left(1 - e_{1j,A}\right) \leq \left(1 - \ell_j\right)\left(R - \Delta_j\right)$ will always hold with equality. Expected bank profits in state $\theta = B$ are therefore equal to:

$$\begin{aligned}
\mathbb{E}\left[\pi_{j,A}\left(e_0, \Delta_j\right)\right] &= \left\{ \begin{array}{l} \frac{1}{2}\max\left\{1 - \ell\left(e_0, \Delta_j, r\right)\left(R + \varepsilon\Delta_j\right) - r\left(1 - e_{1,A}\left(e_0, \Delta_j, r\right)\right), 0\right\} \\ +\frac{1}{2}\max\left\{1 - \ell\left(e_0, \Delta_j, r\right)\left(R - \Delta_j\right) - r\left(1 - e_{1,A}\left(e_0, \Delta_j, r\right)\right), 0\right\} \end{array} \right\} \\
&= \left(1 - \ell\left(e_0, \Delta_j, r\right)\right)\frac{\varepsilon + 1}{2}\Delta_j,
\end{aligned} \tag{3}$$

where $e_{1,A}\left(e_0, \Delta_j, r\right) = 1 - \frac{\left(1 - \ell\left(e_0, \Delta_j, r\right)\right)\left(R - \Delta_j\right)}{r}$ and for all $\Delta_j \in [\underline{\Delta}, R]$:

$$\ell\left(e_0, \Delta_j, r\right) = \begin{cases} 1 & \text{, if } \left(1 - e_0\right)\left(r + \overline{\tau}_1^B\right) \geq R + \frac{\varepsilon - 1}{2}\Delta_j \geq \frac{R - \Delta_j}{r}\left(r + \overline{\tau}_1^B\right) \\ \frac{\left(1 - \frac{R - \Delta_j}{r} - e_0\right)\left(r + \overline{\tau}_1^B\right)}{R + \frac{\varepsilon - 1}{2}\Delta_j - \frac{R - \Delta_j}{r}\left(r + \overline{\tau}_1^B\right)} \in (0, 1) & \text{, if } R + \frac{\varepsilon - 1}{2}\Delta_j > \max\left\{\left(1 - e_0\right), \frac{R - \Delta_j}{r}\right\}\left(r + \overline{\tau}_1^B\right) \\ z \in [0, 1] & \text{, if } R + \frac{\varepsilon - 1}{2}\Delta_j < \frac{R - \Delta_j}{r}\left(r + \overline{\tau}_1^B\right), \end{cases} \tag{4}$$

with $\frac{d\ell\left(e_0, \Delta_j, r\right)}{d\Delta_j} > 0$ and $\frac{d\ell\left(e_0, \Delta_j, r\right)}{d\varepsilon} < 0$ for all $\ell \in (0, 1)$. Notice that the bank is bankrupt in the last case of equation (4). In our baseline we assume that insolvent banks instantaneously offload all their assets, i.e. $z = 1$. We have $\mathbb{E}\left[\pi_{j,A}\right] = 0$ when recapitalization is impossible or requires divesting the entire portfolio.

[23] Notice that we consider swaps of individual assets on the banks' balance sheet and abstract from securitization with trenching that would allow to replicate the liability side operation discussed in Section 2.1.1.

2.1.3 No-arbitrage condition

Households deciding to participate in financial markets at $t = 1$ must be *indifferent* between investing in equity or assets of recapitalizing banks. Formally, the *no-arbitrage* condition requires that investors expect the same premium under equity issuance and deleveraging:

$$\overbrace{\overline{\tau}_1^B \left(e_{1j,A} - e_0\right)}^{\text{premium on equity investments}} = \overbrace{\ell_j \left(e_0, \Delta_j\right) \left[R + \frac{\varepsilon - 1}{2} \Delta_j\right]}^{\text{fundemental value of divested assets}} - \overbrace{r\left(e_{1j,A} - e_0\right)}^{\text{forgone return of retired deposits}} .$$

2.1.4 "Pecking order" for bank recapitalizations

Recall that we restrict attention to strictly positive capital shortfalls, $x\left(e_0, r; \underline{\Delta}\right) > 0$. Conditional on a bank-specific shortfall, banks can recapitalize either through operations on the liability (Section 2.1.1) or asset side (Section 2.1.2). Which option they prefer depends on whether $\mathbb{E}\left[\pi_{j,E}\right] \gtrless \mathbb{E}\left[\pi_{j,A}\right]$. Proposition 1 summarizes.

Proposition 1. *For a given $e_0 = [0, \overline{e}_0)$, r and $\overline{\tau}_1^B$. If either $\Delta_j < R$ or $\varepsilon < 1$, banks strictly prefer equity issuance to asset side operations whenever recapitalization is possible and profitable. Banks are indifferent between recapitalization choices if $\Delta_j = R$ and $\varepsilon = 1$.*

Proof. See Appendix Section A.3.1. □

The results hinge on the immediate divestment of assets being less effective in eliminating depositors' downside risk (as assets have a strictly positive return in the low return state). Allowing for securitization with trenching would enable banks to exactly replicate the liability side operation discussed previously. Thus, our model suggests that equity issuance is strictly preferable whenever the latter is impossible. While this finding may be inconsistent with the traditional "pecking order theory" in corporate finance, evidence for the banking sector by-and-large suggests that in particular undercapitalized banks *do* see equity issuance as an important way to raise capital (e.g., De Jonghe and Öztekin, 2015; Dinger and Vallascas, 2016).

2.1.5 Recapitalization constraints

Key determinant of banks' recapitalization needs, as well as of their ability to raise capital on the market, is their exposure to portfolio risk. For a given Δ_j the *recapitalization* or *solvency constraint* is given by

$\mathbb{E}[\pi_{j,E}] = 0$. We solve for the *critical threshold level of portfolio risk* $\hat{\Delta}_E$ below which banks can recapitalize:

$$\hat{\Delta}_E\left(e_0, r; \overline{\tau}_1^B\right) \equiv \begin{cases} R & if\ R\frac{1+\varepsilon}{2} \geq \left(r+\overline{\tau}_1^B\right)(1-e_0) \\ \frac{R-(1-e_0)\left(r+\overline{\tau}_1^B\right)+R\frac{\overline{\tau}_1^B}{r}}{\frac{\overline{\tau}_1^B}{r}+\frac{1-\varepsilon}{2}} & if\ R\frac{1+\varepsilon}{2} < \left(r+\overline{\tau}_1^B\right)(1-e_0) \wedge \varepsilon > \widehat{\varepsilon} \\ \underline{\Delta} & if\ R\frac{1+\varepsilon}{2} < \left(r+\overline{\tau}_1^B\right)(1-e_0) \wedge \varepsilon \leq \widehat{\varepsilon}, \end{cases} \tag{5}$$

where $\widehat{\varepsilon} \equiv 2(\frac{\overline{\tau}_1^B}{r}(1-\frac{R}{\underline{\Delta}}) - \frac{R-(r+\overline{\tau}_1^B)(1-e_0)}{\underline{\Delta}}) - 1$. Notice that $\hat{\Delta}_E$ is decreasing in r and $\overline{\tau}_1^B$, while it is increasing in e_0 and R. Moreover, provided $\frac{1+\varepsilon}{2} < \left(r+\overline{\tau}_1^B\right)(1-e_0)$ interiority, i.e. $\hat{\Delta}_E \in (\underline{\Delta}, R)$, is guaranteed for $\overline{\tau}_1^B$ sufficiently small. This is because $\widehat{\varepsilon} < 0$ if $\overline{\tau}_1^B \to 0$.

Instead, if recapitalization is achieved through deleveraging via asset side operations, the marginal bank is characterized by $\mathbb{E}[\pi_{j,A}(e_0, \Delta_j)] = 0$. Here the threshold level of portfolio risk below which banks are unable to recapitalize can be computed as:

$$\hat{\Delta}_A\left(e_0, r; \overline{\tau}_1^B\right) \equiv \begin{cases} R & if\ \varepsilon \geq \frac{2(1-e_0)\left(r+\overline{\tau}_1^B\right)-R}{R} \\ \frac{R-(1-e_0)\left(r+\overline{\tau}_1^B\right)}{\frac{1-\varepsilon}{2}} & if\ \frac{2(1-e_0)\left(r+\overline{\tau}_1^B\right)-R}{\underline{\Delta}} < \varepsilon < \frac{2(1-e_0)\left(r+\overline{\tau}_1^B\right)-R}{\underline{\Delta}} \\ \underline{\Delta} & otherwise. \end{cases} \tag{6}$$

Notably $\hat{\Delta}_A$ is never interior if $\varepsilon = 1$. This is because investors are risk neutral and, hence, do not take risk into account when it merely constitutes a mean-preserving spread, i.e. if $\varepsilon = 1$. As a result, the level of Δ_j only affects the expected profits of banks. Moreover, $\hat{\Delta}_A < \underline{\Delta}$ implies that asset side recapitalizations are not feasible. Whenever $\varepsilon \leq \frac{2(1-e_0)\left(r+\overline{\tau}_1^B\right)-R}{\underline{\Delta}}$, there exist values of $\Delta_j \in \left[\underline{\Delta}, \hat{\Delta}_E\right]$ and $\varepsilon \leq 1$ where it is impossible for bank's to recapitalize via asset side operations, while it is possible to recapitalize via equity issuance.

Corollary 1. *Deleveraging via asset side operations is destabilizing on the bank-specific level. Formally, if either $\Delta_j < R$ or $\varepsilon < 1$ equity issuance tends to be associated with fewer bankruptcies than recapitalizations through asset side operations for a given $e_0 = [0, \overline{e}_0)$, r and $\overline{\tau}_1^B$. That is $\hat{\Delta}_E \geq \hat{\Delta}_A$, which holds with strict inequality whenever $\hat{\Delta}_E\left(e_0, r; \overline{\tau}_1^B\right) \in (\underline{\Delta}, R)$.*

Proof. The proof follows from the previous argument in conjunction with Proposition 1. □

The result of Corollary 1 is illustrated graphically in Figure 2. Both thresholds are weakly increasing in the level of ex-ante equity buffers e_0. For $\varepsilon = 1$ an interior solution to $\hat{\Delta}_A$ does not exist, i.e. $\hat{\Delta}_A \in \{\underline{\Delta}, R\}$. For a large e_0, all banks can recapitalize irrespective of using liability or asset side operations, i.e. $\hat{\Delta}_E = \hat{\Delta}_A = R$. Instead, liability side operations enable more banks to recapitalize (i.e. banks with a higher Δ_j) for lower levels of e_0. Notice that an interior solution with $\hat{\Delta}_E > \{\hat{\Delta}_A, \underline{\Delta}\}$ arises naturally in scenarios when there are few incentives to provision ex-ante equity buffers (e.g. if p is high), while the premium for specialized investment capital in the bad aggregate state at $t = 1$, $\overline{\tau}_1^B$, is high. In our general equilibrium analysis in Section 2.2 we provide sufficient conditions for such scenarios to arise.

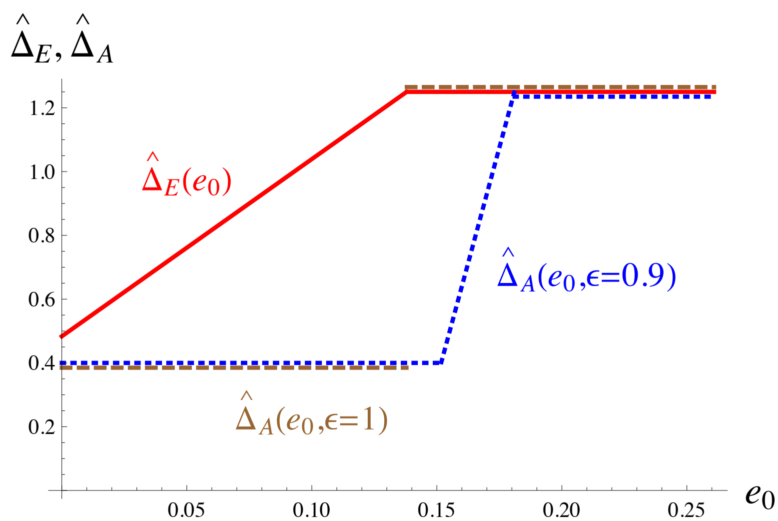

Figure 2: Comparison of threshold levels of banks' portfolio risk for liability and asset side recapitalizations using the parameter values of Table 1. Specifically, $\underline{\Delta} = 0.4$ and $R = 1.25$.

Destabilizing role: the bank level. An important insight is that recapitalization strategies requiring asset side operations can be regarded as *destabilizing* relative to liability side operations since, *ceteris paribus*, $\hat{\Delta}_E > \hat{\Delta}_A$ on a bank-specific level. Importantly, the reason for the destabilizing effect is distinct from, and complementary to, other destabilizing features of fire sales, such as mark-to-market, contagion, or margin effects (e.g., Allen and Carletti, 2008; Brunnermeier and Pedersen, 2009; Acharya and Thakor, 2016).

Destabilizing role: the systemic level. Besides the destabilizing role of assets side operations on the individual bank level, they are also associated with a destabilizing role on a systemic level. This is because of a higher aggregate demand for specialized investment capital under asset side operations relative to equity issuance, which, *ceteris paribus*, results in a higher cost of specialized investment capital and, hence, a higher incidence of insolvencies under asset side operations. To see this, recall that both critical threshold levels of portfolio risk, $\hat{\Delta}_E$ and $\hat{\Delta}_A$, are decreasing in $\overline{\tau}_1^B$. We next analyze how $\overline{\tau}_1^B$ is determined.

Given e_0, the aggregate supply of investment capital at $t = 1$ in the bad state, $\theta = B$, is:

$$FS_1^B \equiv \int_{\underline{\alpha}}^{\overline{\tau}_1^B} \frac{1-e_0}{\overline{\alpha}-\underline{\alpha}} d\alpha_{1i}. \tag{7}$$

Notice that households who invested in risky bank equity at $t = 0$ are passive, which leaves the fraction $1-e_0$ of depositors as potential suppliers of specialized investment capital at $t = 1$. Given the idiosyncratic financial market participation cost, only depositors with cost below the equilibrium premium for investment capital, $\alpha_{1i} \in \left[\underline{\alpha}, \overline{\tau}_1^B\right]$, will decide to participate in the banks' debt bail-ins or asset side operations.

Under equity issuance the aggregate demand for specialized investment capital at $t = 1$ in state $\theta = B$ is:

$$FD_{1,E}^B \equiv E_1^B + X_1^B. \tag{8}$$

The first summand of equation (8) is the total equity issuance (or bail-in volume) and the second summand is the demand stemming from insolvent banks laying off all their assets. Recall that with probability $(1-s)$ the bank-specific realization of Δ_j is drawn from a uniform distribution with pdf $h(\Delta_j) \equiv \frac{1}{R-\underline{\Delta}}$ and with probability s we have $\Delta_j = \underline{\Delta}$. Hence, for a given e_0 and r :

$$\begin{aligned} E_1^B \equiv & \quad s\left(1-\frac{R-\underline{\Delta}}{r}-e_0\right)+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E\left(e_0,r,\overline{\tau}_1^B\right)}\left(1-\frac{R-\Delta_j}{r}-e_0\right)h(\Delta_j)\,d\Delta_j \\ X_1^B \equiv & \quad (1-s)\int_{\hat{\Delta}_E\left(e_0,r,\overline{\tau}_1^B\right)}^{R} P_j^B\, h(\Delta_j)\,d\Delta_j. \end{aligned}$$

where $P^B\left(\Delta_j, r, \overline{\tau}_1^B\right) \equiv \frac{R+\frac{\varepsilon-1}{2}\Delta_j}{r+\overline{\tau}_1^B}$ is the price of assets from bank j by no-arbitrage. Under asset side operations the aggregate demand for specialized investment capital equals:

$$FD_{1,A}^B \equiv s\ell(e_0,\underline{\Delta},r)P^B\left(\underline{\Delta},r,\overline{\tau}_1^B\right)+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E\left(e_0,r,\overline{\tau}_1^B\right)}\ell(e_0,\Delta_j,r)P^B\left(\Delta_j,r,\overline{\tau}_1^B\right)\,h(\Delta_j)\,d\Delta_j+X_1^B. \tag{9}$$

Observe that $FD_{1,E}^B < FD_{1,A}^B$ for a given e_0 and r. As a result, the $\overline{\tau}_1^B$ implied by $FS_1^B = FD_{1,A}^B$ is higher than the one implied by $FS_1^B = FD_{1,E}^B$, which further reduces $\hat{\Delta}_A$ relative to $\hat{\Delta}_E$.

Corollary 2. *For a given* $e_0 = [0, \bar{e}_0)$ *and r, deleveraging via asset side operations is also destabilizing on a systemic level relative to recapitalizations through equity issuance.*

Proof. The proof follows from the previous argument in conjunction with Corollary 1. □

In sum, the destabilizing role of asset side recapitalizations extends from the bank-specific to the systemic level. Next, we consider variations of our baseline model where the pecking order may be overturned.

2.1.6 Factors that influence the recapitalization choice

In practice, banks use both liability and asset side operations to recapitalize (e.g., De Jonghe and Öztekin, 2015). We acknowledge that there are a number of important factors that may affect the recapitalization choice and overturn the result in Proposition 1. These factors include regulatory considerations, informational frictions, renegotiation costs, agency problems or corporate governance frictions. Our focus on equity issuance and deleveraging stems from the urgent recapitalization need, which renders other instruments, like retained earnings, impracticable. In the context of our model the preference for equity issuance is a general feature, driven by the lower need for specialized investment capital. However, factors such as debt renegotiation costs or corporate governance frictions can entice bank managers to prefer deleveraging. Conversely, an asymmetric information on the asset side is likely to strengthen the result in Proposition 1. Similarly it can be argued that bankers may derive control-right-related benefits that increase with the duration of assets under management. In this case, they would be inclined to veto a balance sheet shortening via deleveraging.

Corporate governance frictions. We first discuss a variation of our model where the dilution of initial equity investors' stakes is costly. This is motivated by the corporate finance literature and could reflect disincentives by controlling shareholders that result from diluted cash flow rights, or the reduction of private rents due to a loss of control, depending on whether shareholders at $t = 1$ acquire voting or non-voting shares.[24] To fix ideas, assume that any attempt to conduct a liability side recapitalization via equity issuance involves a utility cost $\phi > 0$ for the owners of the bank (the bank manager and equity investors at $t = 0$) due to a loss in their economic influence. Assume that, at the same time, society gains in influence resulting in a utility benefit ϕ that is allocated lump-sum to all households at the end of $t = 2$ such that only the

[24] La Porta et al. (2002) find that firm value is higher when controlling shareholders own a larger fraction of the cash flow; Dyck and Zingales (2004) document the value of private benefits of control across 39 countries.

distribution of benefits and costs is affected. Conversely, there is no such redistribution of benefits from economic influence when banks deleverage via asset side operation. It shows that for a sufficiently high ϕ banks optimally recapitalize exclusively the via asset side. Proposition 2 summarizes the result formally.

Proposition 2. *If the loss of control rights is sufficiently costly, $\phi \geq \underline{\phi}$, banks strictly prefer deleveraging to recapitalizations through equity issuance despite the higher frequency of bank failures, $\hat{\Delta}_A \geq \hat{\Delta}_E$.*

Proof. It suffices to compare $\mathbb{E}\left[\pi_{E,i}\right]$ and $\mathbb{E}\left[\pi_{A,i}\right]$ as in Section 2.1, where $\mathbb{E}\left[\pi_{E,i}\right]$ is appended by $-\phi$ to account for the cost. By continuity and monotonicity, there exists a $\underline{\phi} > 0$ such that the result in Proposition 2 holds for all $\phi \geq \underline{\phi}$. □

Intuitively, the banks' owners are trading off the higher cost of asset side operations with the benefit of retaining control rights (i.e. avoiding the utility cost ϕ).

Costly Debt Re-negotiations. Next consider a variation where debt re-negotiation involves some cost $\varphi > 0$ that is proportional to $e_{1j,E}\left(\Delta_j, r\right) - e_0$. Now $\mathbb{E}\left[\pi_{E,i}\right]$ needs to be appended by $-\varphi\left(e_{1j,E} - e_0\right)$. Similar to Proposition 2 we can find a $\underline{\varphi} > 0$ such that asset side operations are preferred for all $\varphi \geq \underline{\varphi}$ and $\Delta \geq \underline{\Delta}$.

Asymmetric information friction. Finally, consider a variant of the model where banks' portfolio risk Δ_j is not fully revealed so that the exact level of risk is only observed privately. In this case, the market has only partial information about each bank's portfolio. Specifically, we assume that individual bank assets vary in the expected return they deliver. Moreover, each bank has a certain proportion $0 \leq \kappa \leq 1$ of its portfolio consisting of "standard" or "easy to value" assets for which the market is able to identify individual asset values. Conversely, the proportion $1 - \kappa$ of the portfolio consists of "non-standard" or "opaque" assets.

In such a world, asset side operations can become relatively more costly, as soon as banks have to shed opaque assets at a discount due to an Akerlof-type adverse selection problem. This is because banks have an incentive to trade on their private information and offload assets with a lower expected return first (Malherbe, 2014). Thus, equity issuance may be preferable even if $\phi > \underline{\phi}$ since investors can participate in the potential upside of banks' average portfolios by investing in equity claims and thereby circumvent this type of lemons problem. In other words, asymmetric information frictions between banks and the market–which are arguably bigger on the asset-by-asset level than on the bank level–suggest to favor recapitalizations via equity issuance due to the *destabilizing* role of divesting opaque assets at potentially steep discounts.[25] This is particularly true for bank stress test scenarios where adverse selection problems on the liability side à la Myers and Majluf (1984) are likely to be muted (Hanson et al., 2011).

2.2 Equilibrium

So far, we discussed the continuation equilibrium at $t = 1$ and performed a partial equilibrium analysis by taking ex-ante equity buffers, e_0, and the endogenousdeposit rate, r, as given. Next we solve for the general equilibrium of the full model with $\phi = 0$. Given the result in Proposition 1 we focus on recapitalizations via equity issuance in the bad aggregate state at $t = 1$. Later we also consider the case $\phi \geq \underline{\phi}$.

We start by defining the decentralized equilibrium. Let $\overline{r}_j$ denote the expected return on deposits offered by bank j at $t = 0$ and d_j its demand for deposit funding. The supply of deposits by household i to bank j is given by d^s_{ij}, so that $d_i \equiv \int_i d^s_{ij} di$ is the aggregate supply of deposits from one household to all banks. Moreover, we denote by $w_i = \left(w^G_i, w^B_i\right)$ the vector of depositors' withdrawal decisions at $t = 1$, where $w^\theta_{ij} \in \{0,1\}, \forall\theta \in \{G,B\}$. The action $w^\theta_{ij} = 1$ is to withdraw from bank j in state θ.

For the market for specialized investment capital we adopt the following notation. The vector $\overline{\tau} = \left(\overline{\tau}_0, \overline{\tau}^G_1, \overline{\tau}^B_1\right)$ collects the expected premium demanded by sophisticated investors at $t = 0,1$. Next, the vector $f_i = (f_{0i}, f^G_{1i}, f^B_{1i})$ collects the supply of specialized investment capital by household i at $t = 0$ and for the two aggregate states at $t = 1$. Finally, the equity demand vector, $e_j = \left(e_{0j}, e^G_{1j,E} - e_0, e^B_{1j,E} - e_0\right)$, collects bank j's demand for equity funding at $t = 0,1$. We abuse notation and skip the subscript E so that $e^\theta_{1j} \geq 0$ denotes bank $j's$ desired new level of equity funding. If bank j wants to recapitalize, then $e^\theta_{1j} > e_{0j}$.

Definition. *The allocation (d^s_{ij}, f_i, d_j, e_j) and depositors' withdrawal decisions w_{ij}, $\forall i, j$, as well as the price vector ($\overline{r}$, $\overline{\tau}$) constitute a competitive equilibrium if:*

(i) given ($\overline{r}$, $\overline{\tau}$), the vector (r_j, d_j, e_j) solves the optimization problem for each bank j;

(ii) given ($\overline{r}$, $\overline{\tau}$), d^s_{ij}, f_i and w_{ij} solve the optimization problems for all households i at $t = 0,1$;

(iii) given $\overline{r}$, deposit markets clear at $t = 0$:

(1) on the bank level: $d_j = \int_i \left(d^s_{ij}\right) di \; \forall j$

(2) in the aggregate: $\int_j d_j dj = \int_i \left[\int_j \left(d^s_{ij}\right) dj\right] di$;

[25] The notion of destabilization differs from Corollaries 1 and 2 in that it hinges on asymmetric information.

(v) given ($\bar{r}$, $\bar{\tau}$), markets for specialized investment capital clear $\forall \theta, t$:

(1) at $t = 0$: $FS_0 = E_0$, *where* $F_0 \equiv \int_i (f_{0i})\, di$ *and* $E_0 \equiv \int_j (e_{0j})\, dj$

(2) at $t = 1$: $FS_1^\theta = \int_j \left(e_{1j}^\theta\right) dj - E_0 + X_1^\theta$,$\forall \theta \in \{G, B\}$, *where* $FS_1^\theta \equiv \int_i (f_{1i}^\theta) di$.

The model has to be solved backwards. The problem of banks at $t = 1$ is analyzed in Section 2.1. We continue by discussing the household problem in Section 2.2.1. Thereafter, Section 2.2.2 studies the problem of banks at $t = 0$. For convenience, and w.l.o.g., we henceforth normalize the size of a bank to $e_{0j} + d_j = 1$.

2.2.1 Household problem

At the beginning of $t = 0$ households simultaneously decide whether to consume their endowment right away, or to become debt or equity holders, facing a homogeneous financial market participation cost of $\hat{\alpha}_0 > \alpha_0$, which we derive below. The banking sector is competitive and individual households may invest in multiple banks, i.e. $d_j^s = \int_i d_{ji}^s di$. Households form correct expectations about banks' insolvency risks and care about the expected deposit repayment $\bar{r}$, as well as the return on becoming an equity investor at $t = 0$, or from participating in a bail-in at $t = 1$. Formally, the problem of household i is to select d_{ij}^s and f_{0i}, with $d_{ij}^s, f_{0i} \in [0, 1]$ and $\int_j \left(d_{ij}^s\right) dj + f_{0i} \leq 1$, to maximize expected utility:

$$\mathbb{E}[U_i] = \mathbb{E}\left[\textstyle\sum_{t=0}^2 C_{ti}\right] = \overbrace{\left(1 - \int_j \left(d_{ij}^s\right) dj - f_{0i}\right)}^{consumption\ at\ t=0} + \overbrace{\int_j \left(d_{ij}^s\right) dj \left[\bar{r} + \frac{(1-p)\left(\bar{\tau}_1^B - \frac{\alpha}{2}\right)^2}{\overline{\alpha} - \underline{\alpha}}\right]}^{expected\ return\ from\ t=0\ deposits} \tag{10}$$
$$+ \underbrace{f_{0i}\left[\bar{r} + \bar{\tau}_0\right]}_{expected\ return\ for\ t=0\ equity\ investors} - \underbrace{\underline{\alpha}\, \mathbb{1}_{f_{0i}>0}}_{participation\ cost\ at\ t=0},$$

where $\bar{r}$ is adjusted for the expected surplus from participating in a bail-in with financial market participation costs below those of the marginal investor at $t = 1$. To see this, consider the situation of households at $t = 1$.

In state $\theta = G$ all bank portfolios are risk-less. As a result, initial equity investors and depositors are passive, i.e. $w_{ij}^G = 0\ \forall i, j$. Instead, in state $\theta = B$ bank portfolios are risky, implying that deposits are potentially at risk as well. At the beginning of $t = 1$ the idiosyncratic portfolio risk of each bank, Δ_j, becomes publicly known. Household decisions are made in *two stages* which we consider in reverse order:

In the *stage 2 of* $t = 1$ the mass $\int_j e_{1j}^B dj$ of households who invested in bank equity at $t = 0$ or $t = 1$ are passive. Instead, depositors find it optimal to withdraw their funds from bank j whenever its portfolio turns out to be risky and the bank is known to have an insufficient equity buffer. Formally, deposit repayment cannot be assured in the bank's low return state if $r_j\left(1-e^B_{1j}\right) > R-\Delta_j$. As a result of the sequential service constraint, banks are therefore at risk of a run, and thus bankruptcy, if they cannot guarantee full repayment. This is because bankruptcies are costly, leaving a small but positive liquidation value. Thus the first atomistic depositors who withdraws can expect to receive the promised return r_j, while the remaining stakeholders (other depositors and equity investors) receive nothing.

In *stage 1 of* $t=1$ households who invested in risky bank equity at $t=0$ are passive. Risky banks offer depositors to convert their debt claim into equity. The equity demand is given by $e^B_{1j}-e_0$. The debt-renegotiation is modeled as a take-it-or-leave-it bail-in offer by bank j. Given the idiosyncratic financial market participation cost of depositors, only depositors with cost, α_{1i}, below the equilibrium premium for specialized investment capital, $\overline{\tau}^B_1$, will decide to become investors and participate in debt bail-ins. Formally, the aggregate supply of specialized investment capital available for bail-ins is given by equation (7).

Depositors at $t=0$ can therefore expect the return $\overline{r}$ on deposits, i.e. the return r_j in state G or state B when the bank j is solvent, and the additional return from potentially agreeing to a bail-in at $t=1$,[26] net of their idiosyncratic financial market participation costs, i.e.:

$$\overline{r}+(1-p)\int_{\alpha_0}^{\overline{\tau}^B_1}\frac{\overline{\tau}^B_1-\alpha_{1i}}{\overline{\alpha}-\underline{\alpha}}\,d\alpha_{1i}=\overline{r}+\overline{\tau}_0-\underline{\alpha}\;\Leftrightarrow\;\overline{\tau}_0=\hat{\alpha}\left(\overline{\tau}^B_1\right)\equiv\underline{\alpha}+\frac{(1-p)\left(\overline{\tau}^B_1-\frac{\underline{\alpha}}{2}\right)^2}{\overline{\alpha}-\underline{\alpha}}. \tag{11}$$

The left-hand side of equation (11) simplifies to the expected return on deposits in equation (10). In equilibrium, households are indifferent between becoming depositors or equity investors. The right-hand side of equation (11) follows. Due to fixed participation costs households optimally select either $d^s_i=1$ or $f_{0i}=1$. Provided this indifference holds, market clearing, as described in the equilibrium definition, demands that $\int_j(e_{0j})\,dj=\int_i(f_{0i})\,di$ and $\int_j(d_j)\,dj=\int_i\left[\int_j\left(d^s_{ij}\right)dj\right]di$.

2.2.2 Bank problem at $t=0$

In Section 2.1 we discussed banks' recapitalization choices for a given e_{0j} and r_j. Next, we solve the bank problem at $t=0$ to pin down the optimal capital structure and deposit rate. At $t=0$ bank i chooses the optimal capital structure, i.e. the amount of deposits and initial equity, as well as the deposit rate. Recall

[26]Notably, the bail-in must not be offered by their own bank, but also by a different bank.

that we normalize (w.l.o.g.) the size of each bank to one, i.e. $e_{0j}+d_j=1$. The problem reads:

$$\max_{r_j,e_{0j}}\mathbb{E}\left[\Pi\right]=\left\{\begin{array}{c} p\overbrace{\Gamma}^{\text{return in aggregate state G w/o recapitalization need}}+ \\ \\ (1-p)\overbrace{\left(s\left[\pi_{j,E}\left(e_{0j},\underline{\Delta}\right)\right]+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left[\pi_{j,E}\left(e_{0j},\Delta_j\right)\right]h\left(\Delta_j\right)d\Delta_j\right)}^{\text{return in aggregate state B w/ recapitalization need}}\mathbb{1}_{\hat{\Delta}_E\geq\underline{\Delta}} \\ \\ +(1-p)(1-s)\;0\;\mathbb{1}_{\hat{\Delta}_E<\underline{\Delta}} \qquad -\underbrace{e_{0j}\left(\overline{r}+\overline{\tau}_0\right)}_{\text{expected return for t=0 equity investors}} \end{array}\right\} \tag{12}$$

subject to:

$$\begin{aligned}
&0\leq e_{0j}\leq 1\\
&\Gamma\left(e_{0j},r_j\right)\equiv R-r_j\left(1-e_{0j}\right)\\
&\overline{r}\leq\left(p+(1-p)\left[s+(1-s)\textstyle\int_{\underline{\Delta}}^{\hat{\Delta}_E}h\left(\Delta_j\right)d\Delta_j\right]\;\mathbb{1}_{\hat{\Delta}_E\geq\underline{\Delta}}\right)\;r_j\equiv\overline{p}\left(\hat{\Delta}_E\right)\;r_j\\
&\pi_{j,E}\left(e_{0j},\Delta_j\right)=R+\tfrac{\varepsilon-1}{2}\Delta_j-r_j\left(1-e_0\right)-\overline{\tau}_1^B\left(1-\tfrac{R-\Delta_j}{r_j}-e_0\right)\\
&\mathbb{E}\left[\Pi\right]\geq\gamma\\
&\hat{\Delta}_E\left(e_{0j},r_j;\overline{\tau}_1^B\right)=\frac{R-\left(1-e_{0j}\right)\left(r_j+\overline{\tau}_1^B\right)+R\frac{\overline{\tau}_1^B}{r_j}}{\frac{\overline{\tau}_1^B}{r_j}+\frac{1-\varepsilon}{2}}.
\end{aligned}$$

The third condition ensures depositor participation, where $\overline{r}$ is the equilibrium expected return on deposits, and the fifth condition ensures the participation of bank managers. The fourth condition follows from the fact that it is optimal to just meet the bank-specific capital short-fall. Finally, $\hat{\Delta}_E$ follows from the recapitalization constraint in equation (6).

At $t=0$ banks take into account that their portfolio contains risky assets in state $\theta=B$, with the bank-specific realization of the Δ_j drawn from a uniform distribution with pdf $h\left(\Delta_j\right)=\frac{1}{R-\underline{\Delta}}$. If $\varepsilon=1$ bank portfolios are subject to a mean-preserving spread. $\hat{\Delta}_E$ is the threshold for which a recapitalization is just feasible. All banks with a portfolio that exceeds the threshold level of risk, i.e. $\Delta_j>\hat{\Delta}_E$, cannot recapitalize and are subject to a depositor run that renders them insolvent (zero return). We consider the limit case in which depositors of insolvent banks expect a zero payoff for simplicity and empirical plausibility.[27] The ex-ante probability of bankruptcy is $1-\overline{p}\left(\hat{\Delta}_E\right)=(1-p)\left[(1-s)\int_{\hat{\Delta}_E}^{R} h(\Delta_j)\,d\Delta_j\,\mathbb{1}_{\hat{\Delta}_E\geq\underline{\Delta}}+s\mathbb{1}_{\hat{\Delta}_E<\underline{\Delta}}\right]$.

The problem can be simplified substantially. We consider a competitive deposit markets where the prices $\overline{\tau}_0$, $\overline{\tau}_1^B$ and $\overline{r}$ adjust such that $\mathbb{E}[\Pi]=\gamma$.[28] Optimality demands that r_j is chosen such that the third condition holds with equality. The remaining choice variable is e_{0j}. Since equity is costly, there exists an upper bound to ex-ante equity buffers, i.e. $e_{0j}\in(0,\overline{e}_{0j})$. Thus, it cannot be optimal to select a higher e_{0j} than needed, as to avoid recapitalization needs in all states of the world. For $s=1$ it follows that $\overline{e}_{0j}\equiv 1-\frac{R-\underline{\Delta}}{r_j}$. Assuming an interior solution for $\hat{\Delta}_E$ exists, the first-order necessary condition with respect to e_{0j} reads:

$$\begin{aligned} e_{0j}:\ K_1\left(e_{0j};\overline{\tau}_0,\overline{\tau}_1^B\right)\equiv\ & (1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left[\overline{\tau}_1^B-\left(1+\frac{R-\Delta_j}{r_j^2}\overline{\tau}_1^B\right)\frac{dr_j}{de_{0j}}\right]h(\Delta_j)\,d\Delta_j \\ & -\overline{\tau}_0+(1-p)\,s\left[\overline{\tau}_1^B-\left(1+\frac{R-\underline{\Delta}}{r_j^2}\overline{\tau}_1^B\right)\frac{dr_j}{de_{0j}}\right]=0,\ \forall e_{0j}\in(0,\overline{e}_{0j}). \end{aligned} \tag{13}$$

At $t=0$ banks trade off the cost of the ex-ante equity buffer, $\overline{\tau}_0$, with the benefits of higher precautionary equity buffers coming from the reduction of recapitalization costs in state $\theta=B$ at $t=1$, as well as from the reduction in the incidence of insolvencies, $\frac{d\hat{\Delta}_E}{de_{0j}}>0$, which also allows to lower the deposit rate since $\frac{dr_j}{de_{0j}}$ tends to be negative for plausible parameter values. To see this, notice that:

$$\frac{dr_j}{de_{0j}}=-\frac{(1-p)(1-s)\,h\left(\hat{\Delta}_E\right)r_j\frac{d\hat{\Delta}_E}{de_{0j}}}{\overline{p}\left(\hat{\Delta}_E\right)+(1-p)(1-s)\,h\left(\hat{\Delta}_E\right)r_j\frac{d\hat{\Delta}_E}{dr_j}}$$

is negative, whenever the second-order effect via the critical threshold of risk, $\frac{d\hat{\Delta}_E}{dr_j}<0$, is not too important. By continuity, the latter is assured if s is sufficiently high.

Our focus is on an interior equilibrium with $\hat{\Delta}_E\in(\underline{\Delta},R)$. In Appendix Section A.3.2 we derive sufficient conditions for the existence of a unique equilibrium and show that it is characterized by a symmetric choice, i.e. $e_{0j}=E_0^*\in[0,\overline{e}_0]$. For interiority E_0^* we need parameters assuring that $\overline{\tau}_0$ is not too high relative to

[27]Notice that bankers and investors share the return of the productive technology net of the obligatory debt repayment. In this sense, both parties can be viewed as bank shareholders (absent any agency conflict). Since we allow for new equity investors to dilute the claims of initial shareholders and bankers, securities issued at $t=1$ might be interpreted as preferred stock with differential voting rights (which would allow new shareholders to expropriate resources from early investors and bankers).

[28]If $\varepsilon=0$ it is always the case that bank bid up the deposit rate so that managers are at their participation constraint, i.e. $\mathbb{E}[\Pi]=\gamma$. For high values of ε there can be extreme parameters where the set of constraints gives rise to an equilibrium compensation for managers that is strictly larger than their outside option, i.e. $\mathbb{E}[\Pi]>\gamma$. This is because the high payoff in state $\theta=B$ can be large and managers are the residual claimants. Under plausible parameters when systemic capital shortfalls are rare events, it must, however, be true in equilibrium that $\mathbb{E}[\Pi]=\gamma$. A simple sufficient condition is given by $\gamma>(1-p)\left(R-\frac{1}{2}\right)$.

$\overline{\tau}_1^B$, so that there is an incentive for ex-ante equity buffers. Therefore, the slope of the imperfectly elastic supply curve for specialized investment capital at $t = 1$ cannot be too steep. Then a sufficient condition for existence, uniqueness and symmetry is that s is sufficiently high, which results in an important role played by the *intensive recapitalization margin* that assures that bankruptcies are rare events. In Section 3 we will see that a sufficiently high value of s also implies that higher ex-ante equity buffers (a higher E_0) are associated with a reduction of the recapitalization need in the bad aggregate state of $t = 1$. We consider this to be the most plausible relationship between ex-ante buffers and recapitalization needs. As illustrated in a numerical example in Appendix Section A.2, the parameter restrictions are not restrictive.

Before turning to the efficiency analysis in Section 3, we conclude the description of the laissez-faire equilibrium by stating the market clearing conditions. A key feature of our model is that future recapitalization needs are more costly to meet, the more crowed the market, i.e. the higher $\overline{\tau}_1^B$ relative to $\overline{\tau}_0$.

Market clearing. Using the previous results and assumptions we have, clearing of the deposit market and of the market for specialized investment capital at $t = 0, 1$. The general equilibrium cost of capital is:

$$\begin{aligned}
&\text{at } t = 1: && \overline{\tau}_0(E_0) = \hat{\alpha}\left(\overline{\tau}_1^B\right) \text{ from equation (11),which implies } E_0 = FS_0 \\
&\text{at } t = 1 \text{ in state } \theta = G: && \overline{\tau}_1{}^G(E_0) \geq \underline{\alpha} \\
&\text{at } t = 1 \text{ in state } \theta = B: && \overline{\tau}_1{}^B(E_0) \text{ solves } FD_{1,E}^B = FS_1^B \text{ from equations (8) and (7),}
\end{aligned}$$

with $\overline{\tau}_1^G$ indeterminate, since $FD_{1,E}^G = FS_1^G = 0$.

3 Efficiency analysis

Households' first-best level of welfare in a frictionless economy is R. Here risk-neutral households can directly and costlessly invest in the technology with expected return R, without incurring the intermediation cost γ. Our efficiency analysis considers, however, an appropriate second-best benchmark with financial intermediation and the same frictions as in the laissez-faire equilibrium from Section 2.2.

Specifically, we first focus on the choice at $t = 0$ in Section 3.1 and consider the baseline model with $\phi = 0$, where equity issuance is the optimal recapitalization choice at $t = 1$. The constrained planner maximizes total welfare by selecting e_{0j} for each bank. Different from individual banks, the planner takes into account how ex-ante equity buffers affect the cost of investment capital at both dates, including–in particular–the implications for recapitalizations at $t = 1$. The laissez-faire equilibrium exhibits inefficiently under-capitalized banks, which has implications for optimal macroprudentional regulation, such as leverage requirements targeting ex-ante equity buffers. Next, Section 3.2 analyzes the model with $\phi \geq \phi$. Specifically, we consider a planner who can manipulate the recapitalization choice by compensating banks' managers and initial investors for their loss of control rights. Notably, the inefficient choice at $t = 1$ to conduct asset side operations in the laissez-faire equilibrium magnifies the inefficient under-capitalization at $t = 0$.

3.1 Regulation of ex-ante equity buffers

To analyze efficiency, we use an envelope argument. In particular, we compare the optimality condition of the banks' problem to the one of a constrained planner and evaluate the planner's optimality condition at the decentralized equilibrium allocation for the model with $\phi = 0$. This requires focusing on an interior equilibrium. Since the planner maximizes total welfare and all agents are risk-neutral, we can reformulate the planner problem of maximizing household payoffs to an equivalent, but for our purpose simpler, problem. Specifically, we study the maximization of bank profits, keeping the expected equilibrium deposit rate, $\bar{r}$, for households fixed. Thereby, we account for extra rents, denoted by $X\left(\overline{\tau}_1^B, E_0\right)$, which are generated by investors who's individual participation costs at $t = 1$ in the bad aggregate state is smaller than that of the marginal investor, i.e. $\alpha_{1i} \leq \overline{\tau}_1^B$. The constrained planner problem can be written as:

$$\max_{e_0 = E_0 \geq 0} \left\{ \mathbb{E}\left[\Pi\right] + (1-p)\, X\left(\overline{\tau}_1^B, E_0\right) \right\} \tag{14}$$

subject to the same constraints as the problem in (12), as well as three additional constraints. Namely, $\overline{\tau}_0\left(E_0\right)$ and $\overline{\tau}_0\left(E_0\right)$ as implicitly defined in market clearing and:

$$X\left(\overline{\tau}_1^B, E_0\right) \equiv \int_{\underline{\alpha}}^{\overline{\tau}_1^B} \frac{\overline{\tau}_1^B \left(1-E_0\right)}{\overline{\alpha}-\underline{\alpha}}\, d\alpha_{1i} - \int_{\underline{\alpha}}^{\overline{\tau}_1^B} \frac{\alpha_{1i}\left(1-E_0\right)}{\overline{\alpha}-\underline{\alpha}}\, d\alpha_{1i}.$$

The additional conditions result from the equilibrium cost of specialized investment capital and the financial market participation costs. This is because the constrained planner takes into account how $\overline{\tau}_0$, $\overline{\tau}_1^B$ and $X\left(\overline{\tau}_1^B\right)$ depend on the ex-ante equity issuance volume for a given $\bar{r}$. Critically, she internalizes how the threshold level of portfolio risk, $\hat{\Delta}_E\left(e_0, r, \overline{\tau}_1^B\right)$, is affected by the general equilibrium effect through $\overline{\tau}_1^B$.

Assuming that an interior solution exists, the constrained efficient solution is characterized by a similar system of equations as the laissez-faire equilibrium. The only difference is that condition (13) has to be replaced by the optimality condition derived from the problem in (14). As said, we use an envelope argument to analyze efficiency. Let e_0^* be the individually optimal equity buffer so that $K_1\big|_{E_0=e_0^*}=0$ by definition, and denote the first derivative of the planner problem with respect to e_0 by K_2. Evaluating K_2 at $E_0=e_0^*$ gives:

$$
\begin{aligned}
K_2\big|_{E_0=e_0^*} \quad & = K_1\big|_{E_0=e_0^*} + W_E \, \frac{d\overline{\tau}_1^B}{dE_0}\Big|_{E_0=e_0^*} \\
W_E \quad & \equiv -p\left(1-E_0\right)\, \frac{dr}{d\overline{\tau}_1^B} - E_0 \frac{d\overline{\tau}_0}{d\overline{\tau}_1^B} + (1-p)\left(\frac{\overline{\tau}_1^B-\underline{\alpha}}{\overline{\alpha}-\underline{\alpha}}\left(\left(1-E_0\right)\frac{d\overline{\tau}_1^B}{dE_0}-\overline{\tau}_1^B\right)+\int_{\alpha_1}^{\overline{\tau}_1^B}\frac{\alpha_{1i}}{\overline{\alpha}-\underline{\alpha}}\,d\alpha_{1i}\Big/\frac{d\overline{\tau}_1^B}{dE_0}\right) \\
& \quad -(1-p)\left(\left(1-E_0\right)\frac{dr}{d\overline{\tau}_1^B}+s\left(e_{1,E}\left(\underline{\Delta}\right)-E_0\right)-(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left(e_{1,E}\left(\Delta_j\right)-E_0\right)\,h\left(\Delta_j\right)d\Delta_j\right).
\end{aligned}
$$

The result in Proposition 3 follows.

Proposition 3. *If an interior equilibrium exists and s is sufficiently high, then banks are inefficiently under-capitalized in equilibrium, i.e.* $K_2\big|_{E_0=e_0^*} = W_E\,\frac{d\overline{\tau}_1^B}{dE_0}\big|_{E_0=e_0^*} > 0.$

Proof. See Appendix A.3.3. □

We show in the proof that a lower bound on s exists, which is smaller than one and sufficient, but not necessary for the result to hold. A numerical example is given in Appendix Section A.2. Proposition 3 holds for many plausible supply schedules for specialized investment capital, including the linear schedule generated by uniformly distributed participation costs. Importantly, the direction of the inefficiency hinges on the direction of the endogenous cost effect. If the future cost of equity capital is decreasing in ex-ante equity buffers, $\frac{d\overline{\tau}_1^B}{dE_0}<0$, as is the case for uniformly distributed participation costs, this gives rise to inefficient under-capitalization. As mentioned before, we view $\frac{d\overline{\tau}_1^B}{dE_0}<0$ as the empirically most relevant scenario.

The inefficiency arises due to a pecuniary externality, which materializes in the recapitalization constraint and is governed by $\hat{\Delta}_E\left(e_0, r_i, \overline{\tau}_1^B\right)$. This feature is in the spirit of the extensive literature on collateral constraints that depend on market prices (Lorenzoni, 2008; Dávila and Korinek, 2017). Distinctively, the direction of the inefficiency depends in our model on the relationship between ex-ante choices and the direction of the general equilibrium cost effect at $t = 1$. Moreover, the inefficiency results differ for recapitalizations by deleveraging and equity issuance, as we highlight in the welfare and policy discussion below.

3.2 Recapitalization policy and welfare comparison

In this section we analyze the model with $\phi \geq \underline{\phi}$, considering a constrained planner who can manipulate the recapitalization choice at $t = 1$ by compensating bank managers and initial investors for their loss of control rights. We assume that such a policy intervention can be financed by a non-distortionary lump-sum tax to households ex-post. While it may, in practice, be more realistic to observe mandatory recapitalizations where the regulator also specifies how the recapitalization shall be conducted, a plausible interpretation for the compensation of banks managers and initial investors is to think of a 'golden parachute' clause that allows corporations and regulators to adequately deal with corporate governance frictions.

We find that the magnitude of the inefficient under-capitalization ex-ante is higher when banks expect to recapitalize by deleveraging in the bad aggregate state of $t = 1$. Intuitively, this result arises because asset side operations are less resourceful than equity issuance (Proposition 1). As a result, the pecuniary externality via the extensive recapitalization margin is stronger, leading to a relatively higher incidence of bank failures. The key message is that interventions at $t = 0$ and $t = 1$ should be coordinated. If $\phi \geq \underline{\phi}$, then the constrained efficient allocation can be obtained by regulating ex-ante equity buffers and simultaneously introducing a *tax-subsidy program* at $t = 1$, which is designed to incentivize managers and initial equity investors to agree on equity issuance. Comparing welfare levels, we find that the laissez-faire equilibrium characterized by asset side recapitalizations is inferior to the decentralized equilibrium under the subsidy-tax program, which in turn is inferior to the constrained efficient allocation. Proposition 4 summarizes.

Proposition 4. *Suppose $\phi \geq \underline{\phi}$. If an interior equilibrium exists and s is sufficiently high, the laissez-faire equilibrium with deleveraging in the bad aggregate state at $t = 1$ is characterized by lower ex-ante equity buffers than the decentralized equilibrium under the subsidy-tax program, i.e. $E^*_{0,E} > E^*_{0,A}$. This causes an additional increase in bank failures with asset side operations, i.e. $\hat{\Delta}_A\left(E^*_{0,A}\right) < \hat{\Delta}_A\left(E^*_{0,E}\right) < \hat{\Delta}_E\left(E^*_{0,E}\right)$. The constrained efficient allocation can be implemented using two instruments simultaneously:*

(1) a subsidy-tax program at $t = 1$, which induces equity issuance,

(2) a leverage requirement at $t = 0$, which induces $E^*_{0,E}$.

Proof. See Appendix A.3.4. □

Together with Proposition 3 it follows that the inefficient choice at $t = 1$, to deleverage if $\phi \geq \underline{\phi}$, magnifies the extend of inefficient under-capitalization at $t = 0$ since $E^*_{0,A} < E^*_{0,E} < E^{SP}_0$ (where the superscript SP indicates the solution to the planner problem). In other words, we show how policy interventions in the context of mandatory recapitalizations and macroprudential leverage regulation are intertwined.

4 Discussion

In this section we sketch three further extensions to our baseline model. The first extension is to consider a portfolio choice problem with the introduction of a cash good, which means that banks now have the additional choice on how much of the available resources to invest in a storage technology at $t = 0$. The formal derivations are relegated to the Appendix and Proposition 5 summarizes the result.

Proposition 5. *Suppose $s = 1$. If the effectiveness of storage as a loss buffer is insufficient relative to the long-term asset, i.e. if $R - \underline{\Delta} \geq 1$, then banks favor the exclusive use of equity as a buffer against potential withdrawals. Instead, if $R - \underline{\Delta} < 1$ and the cost of storage is sufficiently high relative to the premium on specialized investment capital in the bad aggregate state, i.e. if $R - 1$ is sufficiently high relative to $\overline{\tau}^B_1$, then banks exclusively use equity as a loss buffer. Formally, a sufficient condition for this to be true is given by:*

$$\overline{\alpha} < \frac{R - \gamma}{1 - (R - \underline{\Delta})} \frac{R - 1}{1 - p}.$$

For the general case $s \in [0, 1]$, a sufficient condition for banks to exclusively use equity is $\overline{\alpha} < R - 1$.

Proof. See Appendix Section A.3.5. □

In short, all the main results of our paper go through in the modified model whenever equity is a cost-effective tool to install a loss-buffer, that is if $\overline{\alpha}$ is not too high relative to opportunity cost of storage.

The second extension we discuss is to consider a bankruptcy scenario where insolvent banks do not instantaneously offload all their assets in the market at $t = 1$. This assumption appears plausible from an institutional perspective, i.e. in light of time consuming bankruptcy procedures, and could also be the result of a moral hazard problem or absconding. Specifically, we assume that insolvent banks divest a fraction $0 \leq \tilde{\ell} \leq 1$ of their assets at $t = 1$, so that X_1^B is given by:

$$X_1^B \equiv (1-s) \int_{\hat{\Delta}_E}^{R} \frac{R + (\varepsilon_j - \Delta_j)}{r + \overline{\tau}_1^B} \, \tilde{\ell} \, h(\Delta_j) \, d\Delta_j.$$

The formulation entails our baseline scenario where all assets are instantaneously offloaded to the market (i.e. $\tilde{\ell} = 1$ as in Allen and Gale (2004a)). If $\tilde{\ell} < 1$, insolvent banks exert less pressure on the cost of specialized investment capital. However, the proposed modification of the baseline model has little effect on the equilibrium analysis and does not materially change the qualitative results of our paper.

The third extension is to introduce an interbank market. So far, we have restricted attention to the role of specialized investment capital supplied from outside the banking system, and ignored the possibility for well-capitalized banks to provide capital to under-capitalized banks. One could argue that an interbank market is not essential for a model designed to capture dynamics that occur under unfavorable *aggregate* economic conditions. Yet, for completeness, we sketch below that our key insights should extend even to the case with interbank trade. More specifically, one could consider the following modification (which nests our baseline model): Suppose a fraction $0 \leq v < s$ of banks is of type $\Delta_j = 0$ with a risk-less portfolio, while a fraction $s(1-v)$ is of type $\Delta_j = \underline{\Delta} > 0$ with a low-risk portfolio and a fraction $(1-s)$ is characterized by a portfolio risk drawn from $\Delta_j \sim U[\underline{\Delta}, R]$. We can partition the banking sector as follows:

$$\Delta_j = \begin{cases} > \hat{\Delta}_E & \textit{insolvent banks} \\ \in \left(\tilde{\Delta}, \hat{\Delta}_E\right] & \textit{recapitalizing banks} \\ 0 \leq \tilde{\Delta} & \textit{banks without recapitalization need}, \end{cases}$$

where $\tilde{\Delta} \in (0, \underline{\Delta})$ solves $r(1-e_0) = R - \Delta_j$ and $x_j \leq 0$ for all $\Delta_j \leq \tilde{\Delta}$.

If $v > 0$, then there is a positive mass of banks with loss-bearing capacity in the bad state $\theta = B$. These banks can offer asset swaps to under-capitalized banks and thereby reduce the aggregate need for specialized investment capital from outside the banking system. If the *inside supply* of investment capital is sufficiently high, then recapitalizing banks do not face crowded markets.[29] Conversely, the imperfectly elastic supply

[29] See, e.g., Walther (2016) for a banking model with asset fire sales that uses a similar assumption.

from outside the banking system matters, as long as the inside supply small, which is precisely the case that we want to emphasize. This suggests that the explicit modeling of an interbank market is likely to leave insights regarding the recapitalization strategy and efficiency fundamentally unaltered. Notably information asymmetries on the asset side, as discussed in Section 3, are arguably less pronounced between banks than between banks and investors from outside the banking system. This would imply that a higher loss-bearing capacity inside the banking system is associated with lower premia on specialized investment capital.

5 Testable implications

Our theory gives rise to several novel testable implications. This section summarizes testable implications and puts them in the context of the existing empirical literature.

Liability side vs. asset side bank recapitalizations. There is an empirical literature analyzing banks' recapitalization strategies during normal times (De Jonghe and Öztekin, 2015; Dinger and Vallascas, 2016) and in response to regulatory interventions (Lambertini and Mukherjee, 2016; Eber and Minoiu, 2016; Gropp et al., 2016). Our model can give guidance on the interpretation of some of the results and point in directions for future research. First, our theory suggests that there is a tendency for banks to prefer liability-side recapitalizations (Proposition 1). This tendency may be overturned or, instead, strengthened depending on different measurable bank properties. If bank managers and/or the controlling shareholders are more averse to the loss of control, e.g. because it allows them to extract private rents, then banks tend to prefer asset-side recapitalizations (Proposition 2). Thus, we would expect a higher aversion, i.e. a higher ϕ, to be associated with environments in which managers or majority shareholders are more likely to extract private rents, or in which a dilution of control leads to less efficient operations. On the other hand, banks with higher holdings of opaque assets are likely to prefer liability side recapitalizations (Section 2.1). Empirically, opaque assets typically include asset-backed securities or own-named covered bonds. In addition, asymmetric information frictions related to asset quality are likely to be more pronounced when the interbank market capacity is exhausted and assets are bought by investors outside the banking system (Section 4).

Different to the Fed's SCAP in May 2009 where equity issuance played an important role, the European Central Bank's (ECB's) Comprehensive Assessment in 2013/14 was associated with bank recapitalizations relying mostly on the asset side (Eber and Minoiu, 2016; Gropp et al., 2016). Our theoretical framework suggests that factors like government frictions or asset opacity may have played a role in explaining this difference. However, we also acknowledge the importance of stress test design (i.e. toughness and transparency). In fact, liability side operations are more attractive after tough bank stress tests when effects related to informational asymmetries (Myers and Majluf, 1984) are likely to be minimized (see, e.g., Hanson et al., 2011). Conversely, a recapitalization strategy of reduced asset growth over an extended period may be attractive depending on the treatment of non-performing loans in the central collateral framework.[30]

Asset prices and recapitalization costs. Asset prices are inversely related to banks' recapitalization costs, which implies a positive relationship between asset prices and banks' ex-ante equity buffers under the most plausible parameter conditions in our model (Proposition 3). If this is indeed the case empirically, then our theory has implications for the relationship between ex-ante equity buffers in the banking system and asset price volatility. More specifically, it predicts that higher equity buffers in the banking system should be associated with lower volatility in asset prices and in the cost of specialized investment capital, i.e. $\frac{d\left(\bar{\tau}_1^B-\bar{\tau}_1^G\right)}{dE_0} < 0$. This is because in the bad state, $\theta = B$, asset prices are more sensitive to the level of equity buffers with $\frac{d\bar{\tau}_1^B}{dE_0} < 0$, while asset prices in the good state, $\theta = G$, are inelastic in the level of ex-ante equity buffers.

Destabilizing role of asset side recapitalizations. Our theory suggests that the frequency of bankruptcies is higher for asset side recapitalizations due to their destabilizing role relative to equity issuance (Corollary 1 and 2). When taking this prediction at face value, we would expect asset side recapitalizations to have a more negative effect on banks' share prices than liability side recapitalizations, everything else equal. In line with our model this prediction is best tested in the context of bank stress tests during episodes of financial distress, where potential opposing effects stemming from information asymmetries between investors and banks à la Myers and Majluf (1984) suggest to be of limited importance.

6 Conclusion

We develop a general equilibrium model of bank capitalization. After system-wide capital short-falls many banks face recapitalization needs to protect against potential withdrawals of uninsured deposits. We find

[30]Non-performing loans in the context of the central bank collateral framework have received renewed attention (ECB 2017).

a tendency for inefficient under-capitalization of banks under plausible parameter assumptions. The constrained inefficiency arises due to a pecuniary externality in banks' recapitalization constraints and, hence, provides a new rational for macroprudential regulation. Moreover, we study the recapitalization choice and how it is affected by different factors. We identify a destabilizing role of asset side recapitalizations on the bank-specific and systemic level. Finally, we present a set of novel testable implications that may be of particular interest for an empiricist studying bank stress tests during episodes of financial distress.

Our analysis highlights that policy interventions in the context of mandatory recapitalizations and macroprudential leverage regulation should be coordinated. Since the magnitude of the inefficiency increases with asset side recapitalizations, but banks nonetheless find them privately optimal, leverage requirements may have to be accompanied with a tax-subsidy scheme that induces banks to issue new equity. Key to our efficiency analysis is an imperfectly elastic supply of investment capital in the short-term, which emerges in a general equilibrium model with financial market segmentation. Hence, banks' ability to recapitalize depends on endogenous future market conditions. Notably, the inefficiency does not rely on common model features, such as deposit insurance or moral hazard. Our results hold for competitive and monopolistic deposit markets, and–because of the systemic implications of the pecuniary externality–point to the benefits of cooperation of national financial regulators (or the creation of supranational regulators such as the European Systemic Risk Board). Moreover, our paper offers insights for public bank stress tests design, as it may be interpreted as an argument favoring staggered stress tests over extensive simultaneous testing exercises.

While we acknowledge that a fully dynamic macro-prudential model can give additional insights, e.g. on payout policies, we develop a tractable two-period general equilibrium model with financial market segmentation that allows to focus on system-wide capital shortfalls. This contributes to a better understanding of banks' recapitalization choices, as well as their implications on stability and efficiency. Importantly, our main qualitative results do not hinge on the assumption that deposits are uninsured. Conditional on correlated portfolios (Acharya and Yorulmazer, 2007; Farhi and Tirole, 2012), for example, private recapitalization efforts may also be triggered when asset risk translates into a required level of capital, and therefore in the presence of risk-sensitive capital regulation.

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A Appendix

A.1 Figures

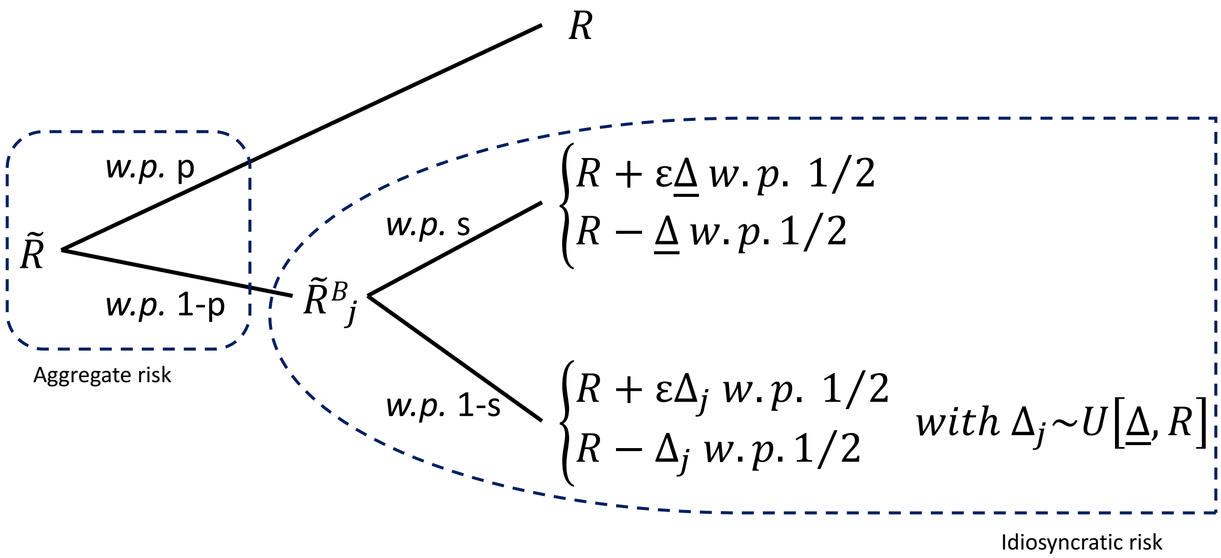

Figure 3: Bank portfolio payoffs per unit invested

A.2 Numerical example

We support the equilibrium and efficiency analysis of our baseline model ($\phi = 0$) with a numerical example that illustrates some of the key insights. Using the parameters in Table 1, the model generates a need for recapitalization with a probability of 15%. Notably, the parameters assure a strong intensive recapitalization margin ($s \geq 0.8$) and rare bankruptcies.

Variable	R	$\underline{\Delta}$	ε	p	γ	$\underline{\alpha}$	$\overline{\alpha}$	s
Value	1.25	0.4	1	0.85	0.05	0.2	1.5	$\in \{0.8, 1\}$

Table 1: Model parameters

Table 2 summarizes the results. For the model with only the *intensive recapitalization margin* ($s = 1$), the equilibrium is constrained efficient with $E_0^* = E_0^{SP} = 0$. This is because all conditions of Remark 1 in Appendix Section A.3.2 hold. Instead, banks are inefficiently under-capitalized in the presence of the *intensive* and *extensive recapitalization margin* ($s = 0.8$): $E_0^* \approx 0.03 < E_0^{SP} \approx 0.17$. The supply of specialized investment capital is imperfectly elastic with a fairly steep slope, $\frac{\overline{\alpha}-\alpha}{1-E_0} > 1$. The premium $\overline{\tau}_1^B$ is relatively high compared to $\overline{\tau}_0$ and, as is plausible, decreasing in the level of ex-ante equity buffers.

Variable	E_0	$\hat{\Delta}_E$	$E_1 - E_0$	$\overline{\tau}_0$	$\overline{\tau}_1^B$	r	$\overline{r}$
Laissez-faire if $s = 1$	0		0.293	0.2	0.527	1.202	$= r$
Planner solution if $s = 1$	0		0.293	0.2	0.527	1.202	$= r$
Laissez-faire if $s = 0.8$	0.030	0.433	0.190	0.203	0.425	1.158	1.055
Planner solution if $s = 0.8$	0.168	1.083	0.136	0.212	0.419	1.149	1.083

Table 2: Results

The increase in social welfare due to higher ex-ante equity buffers is reflected in a higher expected return for depositors. The key welfare effect comes through the impact on the incidence of bankruptcies. While the critical portfolio threshold of risk is $\hat{\Delta}_E \approx 1.08$ for the planner solution, it is considerably lower in the decentralized equilibrium ($\hat{\Delta}_E \approx 0.43 > \underline{\Delta}$) where banks are under-capitalized. The implied ex-ante probability of a bankruptcy is about 0.6% under the planner solution and about 2.9% under laissez-faire.

A.3 Proofs

A.3.1 Proof of Proposition 1

First notice that for any $\Delta_j \in [\underline{\Delta}, R)$ and $\varepsilon \leq 1$ there exist some $e_0 = [0, \overline{e}_0)$ and $\overline{\tau}_1^B > 0$ such that recapitalization via equity issuance is possible, i.e. $\mathbb{E}[\pi_{j,E}] > 0$. From equations (2) and (3):

$$\mathbb{E}[\pi_{j,E}(e_0, \Delta_j) - \pi_{j,A}(e_0, \Delta_j)] = \begin{cases} = 0 & if\ \Delta_j = R \wedge \varepsilon = 1 \\ > 0 & if\ \Delta_j < R \vee \varepsilon < 1, \end{cases}$$

where for any $\varepsilon < 1$ we have that $\mathbb{E}[\pi_{j,A}] = 0$, $\forall e_0 = [0, \overline{e}_0)$. Hence, banks strictly prefer recapitalizations through equity issuance to asset side operations for all combinations of $e_0 = [0, \overline{e}_0)$ and $\overline{\tau}_1^B > 0$ such that $\mathbb{E}[\pi_{j,E}] > 0$. Both results of Proposition 1 follow.

A.3.2 Existence and uniqueness

To prove existence and uniqueness, we first derive conditions such that an interior equilibrium exists. It is convenient to begin with the special case $s = 1$, i.e. the case in which all banks are equally risky in the bad state $\theta = B$. We first express conditions in terms of the $t = 1$ premium for specialized investment capital, which is ultimately endogenous. This intermediate step helps to build intuition, and facilitates–later on–the derivation of sufficient conditions that depend only on exogenous model parameters.

Assuming $s = 1$ it follows trivially, from the bank problem in (12), that $e_0^* = 0$, if all bank portfolios are risk-less (i.e. if $\underline{\Delta} = 0$). Similarly, $e_0^* = 0$ continues to be true for positive–but not too large–risk levels, i.e. $\underline{\Delta} \in (0, \hat{\gamma}_1]$ where $\hat{\gamma}_1 \equiv \frac{2\gamma}{1+\varepsilon+p(1-\varepsilon)}$. To see this, notice first that competition induces banks to bid up deposit rates until managers' participation constraint is satisfied with equality:

$$p\overbrace{[R-\overline{r}]}^{\equiv\gamma^G} + (1-p)\left\{\frac{1}{2}\overbrace{[R+\varepsilon\underline{\Delta}-\overline{r}]}^{\equiv\gamma^{B,H}} + \frac{1}{2}\overbrace{[R-\underline{\Delta}-\overline{r}]}^{\equiv\gamma^{B,L}}\right\} = \gamma \Leftrightarrow R - \frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta} - \overline{r} = \gamma.$$

Together with $\underline{\Delta} \leq \hat{\gamma}_1$, $R - \underline{\Delta} \geq \overline{r}$ follows immediately. It must then be that $e_0^* = 0$ (because equity is costly and has no upside under these circumstances), and that $r_j = \overline{r}, \forall j$ (since no bank ever defaults).[31] Conversely, a capital shortfall necessarily occurs if portfolio risk is "too high", i.e. if $\underline{\Delta} > \hat{\gamma}_1$. Yet, even in this scenario, a bank without any initial equity financing ($e_0 = 0$) would still be able to recapitalize, if it is not too risky, i.e. $\underline{\Delta} < \hat{\gamma}_2 \equiv \frac{2\gamma}{p(1-\varepsilon)}$, and if recapitalization costs are not too high, i.e. if $\overline{\tau}_1^B \leq \overline{\Phi}(R, \varepsilon, \underline{\Delta}, p, \gamma)$, where:

$$\overline{\Phi}(R,\varepsilon,\underline{\Delta},p,\gamma) \equiv p(1-\varepsilon)(\hat{\gamma}_2 - \underline{\Delta})\left[\frac{1}{2} + \frac{R-\underline{\Delta}}{(1+\varepsilon+p(1-\varepsilon))(\underline{\Delta}-\hat{\gamma}_1)}\right]$$

and $\overline{\Phi} > 0$ for $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_2)$. However, that recapitalization at $t = 1$ is feasible for banks that chose $e_{0j} = 0$, does not necessarily imply that a zero equity buffer is also optimal. Whether it is, depends on the cost of capital at $t = 0$ relative to the cost at $t = 1$, the trade-off between which is reflected in the derivative with respect to e_{0j} of the simplified bank problem in equation (12):

$$\frac{d\mathbb{E}[\Pi]}{de_{0j}} = pr_j + (1-p)\left(r_j + \overline{\tau}_1^B\right) - (\overline{r} + \overline{\tau}_0) = (1-p)\,\overline{\tau}_1^B - \overline{\tau}_0, \tag{15}$$

[31]Notice that bank managers are residual claimants, so that $R - \underline{\Delta} \geq \overline{r}$ is sufficient to ensure repayment; $\gamma^{B,L} = 0$ is perfectly feasible, as long as the participation constraint is satisfied and bank managers are compensated in other states of the world.

where we continue to assume $s=1$, $\overline{\tau}_1^B \leq \overline{\Phi}$, and $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_2)$. The derivative is negative–so that $e_0^* = 0$–if ex-ante equity is expensive relative to expected recapitalization costs, i.e. if $\overline{\tau}_0 > (1-p)\,\overline{\tau}_1^B$. Remark 1 summarizes conditions under which–for a given $\overline{\tau}_1^B$ and provided that $s=1$–banks optimally choose $e_0^* = 0$.

Remark 1. *Assume portfolio risk is identical for all banks and equal to $\underline{\Delta}$ (i.e. $s=1$). Since perfect competition on the deposit market implies binding managerial participation constraints ($R - \frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta} - \overline{r} = \gamma$), it follows that $\underline{\Delta} \leq \hat{\gamma}_1 \Leftrightarrow R - \underline{\Delta} \geq \overline{r}$. If $\underline{\Delta} \in (0, \hat{\gamma}_1]$ banks never need to recapitalize, i.e. $e_{1j}^* = 0, \forall j$, and optimally select $e_{0j}^* = 0, \forall j$. If $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_2)$, banks with no initial equity necessarily face a capital shortfall. They can recapitalize as long as $\overline{\tau}_1^B \leq \overline{\Phi}(R, \varepsilon, \underline{\Delta}, p, \gamma)$ and continue to optimally select $e_0^* = 0, \forall j$ if $\overline{\tau}_0 > (1-p)\,\overline{\tau}_1^B$.*

Under the conditions of Remark 1 banks are always solvent. We say that only an *intensive recapitalization margin* exists, because the trade off is between the cost of recapitalization in state B and the cost of a higher ex-ante capital buffer. Importantly, individual equity buffers do not affect the own or other banks' *ability to recapitalize*, i.e. to stay solvent. We say that there is no *extensive recapitalization margin*.

Building on the case with homogeneous bank risk and no bankruptcies, we can derive implications for more general circumstances with heterogeneous portfolio risk and bankruptcies. If we, e.g., consider an s strictly smaller but close to 1 and $\overline{\tau}_1^B > \overline{\Phi}(\Delta_j)$ for some bank j, then the first derivative of the bank problem in (12) can become larger than in (15). Intuitively, the reason is that default risk introduces survival as an additional benefit of higher initial equity buffers. Now banks weigh the cost of recapitalization *and* that of a higher survival probability (which–in turn–reduces depositors' risk premium) against the cost of a higher initial equity buffer. As long as the cost of equity at $t=0$ is not too high relative to recapitalization costs, $e_{0j} > 0$ will therefore be optimal if $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_2)$ and $\overline{\tau}_0 > (1-p)\,\overline{\tau}_1^B$.[32] The same first-order condition also implies that it will never be optimal for the ex-ante capital buffer to reach its maximum, i.e. $e_{0j} < \overline{e}_{0j}$ (since $\overline{\tau}_0 > (1-p)\,\overline{\tau}_1^B$ implies $K_1\big|_{e_{0j} \geq \overline{e}_{0j}} < 0$), so that interiority of e_{0j} is guaranteed ($e_{0j}^* \in (0, \overline{e}_{0j}), \forall j$).[33]

Interiority. We focus on symmetric equilibria and are interested in an interior solution to the problem in (12), where $e_{0j} \in (0, \overline{e}_{0j})$ and $\hat{\Delta}_E \in (\underline{\Delta}, R)$. Lemma 1 provides sufficient conditions that build on Remark 1 and ensure interiority of the threshold level of risk using only deep model parameters. Afterwards,

[32] To see this more formally, consider the first-order necessary condition of the bank problem in equation (13). It assumes $0 < s \leq 1$ and the existence of an interior solution for $\hat{\Delta}_E$, and showsthe cost of future recapitalizations to increase with market crowdedness (as we explain in Section 2.2.2).

[33] $\frac{dr_j}{de_{0j}} = \frac{d\hat{\Delta}_E(\overline{\tau}_1^B)}{de_{0j}} = 0$ for $e_{0j} \geq \overline{e}_{0j}$ and $(1-p)\,\overline{\tau}_1^B \left(s + (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h(\Delta_j)\, d\Delta_j\right) < (1-p)\,\overline{\tau}_1^B < \overline{\tau}_0$.

Lemma 2 shows that all equilibria must be characterized by a symmetric choice at $t = 0$, while Proposition 6 establishes existence and uniqueness.

Lemma 1. *Suppose that* $\underline{\alpha} + (1-p)\frac{\left(\frac{\underline{\alpha}}{2}\right)^2}{\overline{\alpha}-\underline{\alpha}} > (1-p)\left(\overline{\alpha} - \frac{R-\underline{\Delta}}{R-\frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta}-\gamma}\left(\overline{\alpha}-\underline{\alpha}\right)\right)$ *and* $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_3)$*, where:*

$$\hat{\gamma}_3 \equiv \min\left\{\hat{\gamma}_2, \frac{2+p(1-\varepsilon)}{1+\varepsilon+p(1-\varepsilon)}\right\} > \hat{\gamma}_1.$$

Interiority of the critical level of portfolio risk, $\hat{\Delta}_E \in (\underline{\Delta}, R)$*, and* $e_{0j} < \overline{e}_{0j}$ *are ensured for sufficiently large s, if* $\overline{\alpha} - \frac{R-\underline{\Delta}}{R-\frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta}-\gamma}\left(\overline{\alpha}-\underline{\alpha}\right) < \overline{\Phi}(R,\varepsilon,\underline{\Delta},p,\gamma)$ *and* $\underline{\alpha} > \underline{\Phi}(R,\varepsilon,\underline{\Delta},p,\gamma)$*, with:*

$$\underline{\Phi}(R,\varepsilon,\underline{\Delta},p,\gamma) \equiv \left[R - \frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta} - \gamma\right]\frac{\underline{\Delta} - R\frac{1-\varepsilon}{2}}{R-\underline{\Delta}}.$$

Proof. The proof is based on a generalization of Remark 1, and relies on the same arguments that we laid out it the previous section. The first inequality implies $\overline{\tau}_0 > (1-p)\overline{\tau}_1^B$, i.e. that the derivative of the bank problem with respect to e_{0j} is negative in the absence of an extensive recapitalization margin and that $K_1\big|_{e_{0j}\geq\overline{e}_{0j}} < 0$. Capital market clearing at $t = 1$, for the case where all banks are initially without equity ($e_{0j} = 0$) and raise the maximum amount at $t = 1$ ($e_{1j} = \overline{e}_{0j}$), implies for $s \to 1$ an upper bound for the premium that is given by $\overline{\tau}_1^B < \overline{\alpha} - \frac{R-\underline{\Delta}}{\overline{r}}\left(\overline{\alpha}-\underline{\alpha}\right)$. At the same time, the condition that households need to be indifferent between becoming a depositor and an investor at $t = 0$ implies $\overline{\tau}_0 = \underline{\alpha} + (1-p)\frac{\left(\overline{\tau}_1^B - \frac{\underline{\alpha}}{2}\right)^2}{\overline{\alpha}-\underline{\alpha}}$, which yields a lower bound for $\overline{\tau}_0$ if we replace $\overline{\tau}_1^B$ with its lower bound $\underline{\alpha}$, i.e. $\overline{\tau}_0 \leq \underline{\alpha} + (1-p)\frac{\left(\frac{\underline{\alpha}}{2}\right)^2}{\overline{\alpha}-\underline{\alpha}}$. The condition that $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_3)$ combines the condition $\underline{\Delta} \in (\hat{\gamma}_1, \hat{\gamma}_2)$ from before, i.e. the condition that banks of type $\underline{\Delta}$ experience a capital shortfall, but are not too risky to recapitalize, with an additional requirement ($\underline{\Delta} < \hat{\gamma}_3$) that ensures $\underline{\Phi} < \overline{\Phi}$. Finally, $\overline{\alpha} - \frac{R-\underline{\Delta}}{\overline{r}}\left(\overline{\alpha}-\underline{\alpha}\right) < \overline{\Phi}$ and $\underline{\alpha} > \underline{\Phi}$ imply $\overline{\tau}_1^B \leq \overline{\Phi}$, where an upper bound for $\overline{r}$ is given by $\overline{r} = R - \frac{(1-p)(1-\varepsilon)}{2}\underline{\Delta} - \gamma$, so that equity premia costs are not too high to prevent recapitalization, and $\overline{\tau}_1^B \geq \underline{\Phi}$, which ensures interiority of $\hat{\Delta}_E$ in equation (5). By continuity, there exists some $s^I(0,1)$ such that the result in Lemma 1 holds for all $s < s^I$. This concludes the proof. □

Lemma 1 demonstrates that the results from Remark 1 extend to the general case under the sufficient condition that s is larger than some threshold $\underline{s}^I$ (which we can only define implicitly). As discussed earlier, the extensive recapitalization margin creates additional incentives for higher ex-ante equity buffers. Such incentives can be balanced with a higher participation cost at $t = 0$ ($\underline{\alpha}$), or a more elastic supply schedule of specialized investment capital at $t = 1$ (i.e. by reducing $\overline{\alpha} - \underline{\alpha}$). The additional inequalities assure that all banks need to recapitalize, but only some banks are able to, leading to an interior portfolio risk threshold.

Symmetry. Next, we establish sufficient conditions for symmetric equilibria, i.e. for $e_{0j} = e_0, \forall j$. To do this, we take the equilibrium expected return on deposits $\overline{r}$ and the vector of equity premia $\overline{\tau}$ as given. This leaves a system of two equations in two unknowns (e_{0j} and $\hat{\Delta}_E$), consisting of the optimality condition and the recapitalization constraint from equation (5). Symmetry requires the existence of a unique pair $(e_{0j}, \hat{\Delta}_E)$ solving these two equations.

Lemma 2. *If an interior equilibrium exists and s is sufficiently high, it must be characterized by a symmetric choice at $t = 0$, i.e. by $e_{0j} = E_0^* \in [0, \overline{e}_0), \forall i$.*

Proof. We analyze the following system of two equations in two unknowns, resulting from the first order necessary condition of the bank problem and the condition that determines the threshold level of risk $\hat{\Delta}_E$ at which banks are just able to recapitalize after learning about their potential shortfall. Since we are interested in symmetry of banks' choices, we take prices, $\overline{r}$ and $\overline{\tau}$, as given:

$$
\begin{aligned}
K_1\left(e_{0j}, \hat{\Delta}_E; \overline{r}, \overline{\tau}\right) = & \quad 0 \\
L_1\left(e_{0j}, \hat{\Delta}_E; \overline{r}, \overline{\tau}\right) \equiv & \quad \hat{\Delta}_E - \frac{R - \left(1 - e_{0j}\right)\left(r\left(e_{0j}, \hat{\Delta}_E; \overline{r}\right) + \overline{\tau}_1^B\right) + R \frac{\overline{\tau}_1^B}{r\left(e_{0j}, \hat{\Delta}_E; \overline{r}\right)}}{\frac{\overline{\tau}_1^B}{r\left(e_{0j}, \hat{\Delta}_E; \overline{r}\right)} + \frac{1-\varepsilon}{2}} = 0.
\end{aligned}
$$

We assume interiority and present a proof in three steps. Step 1 provides preliminary results. Step 2 establishes $\frac{d(K_1/(1-s))}{de_{0j}} \approx 0$ and $\frac{d(K_1/(1-s))}{d\hat{\Delta}_E} > 0$ for $s > \underline{s}^{II}$, where $\underline{s}^{II} < 1$. Our argument assumes $\Delta_j \sim U$, but could be extended to more general distributions. It follows that $K_1\left(e_{0j}, \hat{\Delta}_E\right) = 0$ implies $\frac{d\hat{\Delta}_E}{de_{0j}} \to 0$ for $s > \underline{s}^{II}$. Step 3 then establishes $\frac{dL_1}{de_{0j}} < 0$ and $\frac{dL_1}{d\hat{\Delta}_E} > 0$ for $s > \underline{s}^{II}$, so that $L_1\left(e_{0j}, \hat{\Delta}_E\right) = 0$ implies $\frac{d\hat{\Delta}_E}{de_{0j}} > 0$.

Together, the three steps imply that there can exist at most one crossing in the $(e_{0j}, \hat{\Delta}_E)$-space. If an equilibrium exists, it must therefore be characterized by a symmetric choice at $t = 0$: either by an interior solution with $e_{0j}^* = E_0^* > 0$, or by a corner solution with $e_{0j}^* = E_0^* = 0$.

Step 1: We first derive the following derivatives, as they will be useful for the subsequent analysis:

$$\begin{aligned}
\frac{dr_j}{de_{0j}} &= -\frac{(1-p)(1-s)\overline{r}\,h\left(\hat{\Delta}_E\right)}{\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h\left(\Delta_j\right)d\Delta_j\right]\right)^2}\frac{d\hat{\Delta}_E}{de_{0j}} < 0 \\
\frac{dr_j}{d\hat{\Delta}_E} &= -\frac{(1-p)(1-s)\overline{r}\,h\left(\hat{\Delta}_E\right)}{\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h\left(\Delta_j\right)d\Delta_j\right]\right)^2} < 0 \\
\frac{d^2r_j}{de_{0j}^2} &= \frac{2(1-p)^2(1-s)^2\overline{r}\,h\left(\hat{\Delta}_E\right)^2}{\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h\left(\Delta_j\right)d\Delta_j\right]\right)^4}\left(\frac{d\hat{\Delta}_E}{de_{0j}}\right)^2 > 0 \\
\frac{d^2r_j}{de_{0j}d\hat{\Delta}_E} &= -\frac{(1-p)(1-s)\overline{r}\,h'\left(\hat{\Delta}_E\right)\frac{d\hat{\Delta}_E}{de_{0j}}}{\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h\left(\Delta_j\right)d\Delta_j\right]\right)^2} + \frac{(1-p)^2(1-s)^2\overline{r}\,h\left(\hat{\Delta}_E\right)h'\left(\hat{\Delta}_E\right)\frac{d\hat{\Delta}_E}{de_{0j}}}{\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h\left(\Delta_j\right)d\Delta_j\right]\right)^3} = 0.
\end{aligned}$$

Step 2: Next, we analyze $K_1\left(e_{0j},\hat{\Delta}_E\right)$:

$$\begin{aligned}
\frac{dK_1}{de_{0j}} \equiv\ & (1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left(\left(1+\frac{R-\Delta_j}{r_j^2}\overline{\tau}_1^B\right)\frac{d^2r_j}{de_{0j}^2}+\frac{R-\Delta_j}{r_j^3}\overline{\tau}_1^B\left(\frac{dr_j}{de_{0j}}\right)^2\right)h(\Delta_j)\,d\Delta_j \\
& -(1-p)\,s\left(\left(1+\frac{R-\underline{\Delta}}{r_j^2}\overline{\tau}_1^B\right)\frac{d^2r_j}{de_{0j}^2}+\frac{R-\Delta_j}{r_j^3}\overline{\tau}_1^B\left(\frac{dr_j}{de_{0j}}\right)^2\right) < 0,\text{ if }\frac{d^2r_j}{de_{0j}^2} > 0 \\
\frac{dK_1}{d\hat{\Delta}_E} \equiv\ & (1-p)(1-s)\left[\overline{\tau}_1^B-\left(1+\frac{R-\Delta_j}{r_j^2}\overline{\tau}_1^B\right)\frac{dr_j}{de_{0j}}\right]h\left(\hat{\Delta}_E\right) \\
& -(1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left(\left(1+\frac{R-\Delta_j}{r_j^2}\overline{\tau}_1^B\right)\frac{d^2r_j}{de_{0j}d\hat{\Delta}_E}+\frac{R-\Delta_j}{r_j^3}\overline{\tau}_1^B\frac{dr_j}{d\hat{\Delta}_E}\frac{dr_j}{de_{0j}}\right)h(\Delta_j)\,d\Delta_j \\
& -(1-p)\,s\left(1+\frac{R-\underline{\Delta}}{r_j^2}\overline{\tau}_1^B\right)\frac{d^2r_j}{de_{0j}d\hat{\Delta}_E}+\frac{R-\underline{\Delta}}{r_j^3}\overline{\tau}_1^B\frac{dr_j}{d\hat{\Delta}_E}\frac{dr_j}{de_{0j}} \\
& > 0,\text{ if }\frac{dr_j}{de_{0j}} < 0,\ \frac{dr_j}{d\hat{\Delta}_E} < 0,\ \frac{d^2r_j}{de_{0j}d\hat{\Delta}_E} \leq 0.
\end{aligned}$$

It follows that $lim_{s\to1}\frac{d(K_1/(1-s))}{de_{0j}} = 0$ and $lim_{s\to1}\frac{d(K_1/(1-s))}{d\hat{\Delta}_E} = (1-p)\,\overline{\tau}_1^B\,h\left(\hat{\Delta}_E\right) > 0$.

Step 3: Finally, we analyze $L_1\left(e_{0j},\hat{\Delta}_E\right)$:

$$\begin{aligned}
\frac{dL_1}{de_{0j}} <\ & 0\ if\ \frac{dr_j\left(e_{0j},\hat{\Delta}_E\right)}{de_{0j}} < 0 \\
\frac{dL_1}{d\hat{\Delta}_E} =\ & 1-\frac{\frac{\overline{\tau}_1^B}{r^2}\left(R\frac{1+\varepsilon}{2}-\left(1-e_{0j}\right)\overline{\tau}_1^B\right)-\left(1-e_{0j}\right)\left(2\frac{\overline{\tau}_1^B}{r}+\frac{1-\varepsilon}{2}\right)}{\left(\frac{\overline{\tau}_1^B}{r}+\frac{1-\varepsilon}{2}\right)^2/\frac{dr_j\left(e_{0j},\hat{\Delta}_E\right)}{d\hat{\Delta}_E}}.
\end{aligned}$$

By continuity, there exists a $s^{II} < 1$ such that $\frac{dL_1}{d\hat{\Delta}_E} > 0$ and there exists a unique pair $(e_{0j},\hat{\Delta}_E)$ solving the system of equations. The result in Lemma 2 follows. □

Intuitively, $s > \underline{s}^{II}$ ensures that the probability mass around the critical threshold level of risk $\hat{\Delta}_E$ is "not too high", which in turn implies that the extensive recapitalization margin is not "too strong". In the contribution to expected recapitalization needs of bank j, the *direct effect* of higher ex-ante equity buffers ($\frac{de_{1j}}{de_{0j}} < 0$), i.e. the effect relying on the intensive recapitalization margin, therefore dominates the *indirect effect* of lower default risk ($\frac{dr_j}{de_{0j}} < 0$), i.e. the effect relying on the extensive margin.

Using the results of Lemmas 1 and 2, we can then conclude our proof of existence and uniqueness for $s > \max\left\{\underline{s}^I, \underline{s}^{II}, \underline{s}^{III}\right\}$, where $\underline{s}^{III}$ is implicitly defined in the proof of Proposition 6.

Proposition 6. *Suppose the sufficient conditions of Lemma 1 hold and that s is sufficiently large. Then a symmetric equilibrium choice exists, if the slope of the imperfectly elastic supply for specialized investment capital is sufficiently small.*

Proof. Under the conditions of Lemmas 2 and 1, we can focus on symmetric and interior choices at $t = 0$ ($e_{0j} = e_0 \in (0, \overline{e}_0), \forall j$). The proof then consists of two steps. We first examine how the cost of specialized investment capital depends on ex-ante equity buffers and thereafter how this changes the derivatives of $K_1\left(e_0, \hat{\Delta}_E; \overline{r}, \overline{\tau}\right)$ and $L_1\left(e_0, \hat{\Delta}_E; \overline{r}, \overline{\tau}\right)$ from the proof of Lemma 2.

Step 1: The following derivatives will be useful for the subsequent analysis:

$$\frac{d\overline{r}}{de_0} = -\frac{d\mathbb{E}[\Pi]/de_0}{d\mathbb{E}[\Pi]/d\overline{r}} = -\frac{K_1}{d\mathbb{E}[\Pi]/d\overline{r}} \qquad \frac{d\overline{r}}{d\hat{\Delta}_E} = -\frac{d\mathbb{E}[\Pi]/d\hat{\Delta}_E}{d\mathbb{E}[\Pi]/d\overline{r}} = 0,$$

where:

$$\begin{aligned} \frac{d\mathbb{E}[\Pi]}{d\overline{r}} = & -\left(p + (1-p)\left[s + (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E} h(\Delta_j)\, d\Delta_j\right]\right)\frac{dr}{d\overline{r}}(1-e_0) \\ & -(1-p)\,\overline{\tau}_1\left[s\frac{R-\underline{\Delta}}{r^2} + (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\frac{R-\Delta_j}{r^2}h(\Delta_j)\, d\Delta_j\right]\frac{dr}{d\overline{r}} - e_0 < 0 \\ \frac{d\mathbb{E}[\Pi]}{d\hat{\Delta}_E} = & \ 0. \end{aligned}$$

Moreover:

$$\frac{d\overline{\tau}_1^B}{de_0} = \frac{\begin{pmatrix} -s\left(1 - \frac{R-\underline{\Delta}}{r^2}\frac{dr}{de_0}\right) - (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left(1 - \frac{R-\Delta_j}{r^2}\frac{dr}{de_0}\right)h(\Delta_j)\, d\Delta_j \\ +(1-s)\frac{d\hat{\Delta}_E}{de_0}\left(1 - \frac{R-\hat{\Delta}_E}{r} - e_0 - \frac{R+\frac{1-\varepsilon}{2}\hat{\Delta}_E}{r+\overline{\tau}_1^B}\right)h\left(\hat{\Delta}_E\right) \end{pmatrix}}{\frac{1-e_0}{\overline{\alpha}-\underline{\alpha}}} + \frac{\overline{\tau}_1^B - \underline{\alpha}}{1-e_0}.$$

By continuity, there must then exist a sufficiently high s such that $\frac{d\overline{\tau}_1^B}{de_0} < 0$. We denote the corresponding threshold with $s^{III} < 1$.

Step 2: Next, we analyze the derivatives:

$$\begin{aligned}
\frac{dK_1}{de_0} &= \frac{\partial K_1}{\partial e_0} + \frac{\partial K_1}{\partial \bar{r}}\frac{\partial \bar{r}}{\partial e_0} + \frac{\partial K_1}{\partial \overline{\tau}_0}\left(\frac{\partial \overline{\tau}_0}{\partial e_0} + \frac{\partial \overline{\tau}_0}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial e_0}\right) \\
&\quad + \frac{\partial K_1}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial e_0} < 0, \text{ if } s \text{ is large and } \frac{\partial \overline{\tau}_1^B}{\partial e_0} \text{ is small} \\
\frac{dK_1}{d\hat{\Delta}_E} &= \frac{\partial K_1}{\partial \hat{\Delta}_E} + \frac{\partial K_1}{\partial \bar{r}}\frac{\partial \bar{r}}{\partial \hat{\Delta}_E} + \frac{\partial K_1}{\partial \overline{\tau}_0}\left(\frac{\partial \overline{\tau}_0}{\partial \hat{\Delta}_E} + \frac{\partial \overline{\tau}_0}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial \hat{\Delta}_E}\right) \\
&\quad + \frac{\partial K_1}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial \hat{\Delta}_E} > 0, \text{ if } s \text{ is large and } \frac{\partial \overline{\tau}_1^B}{\partial e_0} \text{ is small} \\
\frac{dL_1}{de_0} &= \frac{\partial L_1}{\partial e_0} + \frac{\partial L_1}{\partial \bar{r}}\frac{\partial \bar{r}}{\partial e_0} + \frac{\partial L_1}{\partial \overline{\tau}_0}\left(\frac{\partial \overline{\tau}_0}{\partial e_0} + \frac{\partial \overline{\tau}_0}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial e_0}\right) + \frac{\partial L_1}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial e_0} < 0, \text{ if } s \text{ is large and } \frac{\partial \overline{\tau}_1^B}{\partial e_0} \text{ is small} \\
\frac{dL_1}{d\hat{\Delta}_E} &= \frac{\partial L_1}{\partial \hat{\Delta}_E} + \frac{\partial L_1}{\partial \bar{r}}\frac{\partial \bar{r}}{\partial \hat{\Delta}_E} + \frac{\partial L_1}{\partial \overline{\tau}_0}\left(\frac{\partial \overline{\tau}_0}{\partial \hat{\Delta}_E} + \frac{\partial \overline{\tau}_0}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial \hat{\Delta}_E}\right) + \frac{\partial L_1}{\partial \overline{\tau}_1^B}\frac{\partial \overline{\tau}_1^B}{\partial \hat{\Delta}_E} > 0, \text{ if } s \text{ is large,}
\end{aligned}$$

where the partial derivatives $\frac{\partial K_1}{\partial e_0}$, $\frac{\partial K_1}{\partial \hat{\Delta}_E}$, $\frac{\partial L_1}{\partial e_0}$ and $\frac{\partial L_1}{\partial \hat{\Delta}_E}$ are the same as in the Proof of Lemma 2, where we did not (have to) take into account general equilibrium price effects. Observe that $\frac{dL_1}{de_0} < 0$ and $\frac{dL_1}{d\hat{\Delta}_E} > 0$ implies that $\frac{d\hat{\Delta}_E}{de_0} > 0$. As in Lemma 2, a sufficiently high s assures that there can exist at most one crossing, provided that the magnitude of the general equilibrium price effect, $\frac{\partial \overline{\tau}_1^B}{\partial e_0}$, is small. To see this, observe that we have $\lim_{\overline{\alpha} \to \underline{\alpha}} \frac{dK_1}{de_0} < 0$, for large s, since $\lim_{\overline{\alpha} \to \underline{\alpha}} \frac{\partial \overline{\tau}_1^B}{\partial e_0} = 0$. By continuity, we can define a $\underline{\alpha} < \alpha'$ such that result extends to all $\overline{\alpha} < \alpha'$, which result in $K_1\left(e_0, \hat{\Delta}_E\right) = 0$ and implies $\frac{d\hat{\Delta}_E}{de_0} \to 0$ (for sufficiently large s and sufficiently small $\overline{\alpha}$). Given the existence of an interior solution to $\hat{\Delta}_E$, there must either exist an equilibrium characterized by a symmetric choice at $t = 0$ and an interior solution $e_0^* = E_0^* > 0$ or a corner solution with $e_0^* = E_0^* = 0$. This concludes the proof. □

A.3.3 Proof of Proposition 3

The proof consists of showing that $W_E\big|_{e_0 = e_0^*} < 0$ provided s is sufficiently high and $\frac{d\overline{\tau}_1^B}{dE_0} < 0$. From equation (11) we know that $\frac{d\overline{\tau}_0^B}{d\overline{\tau}_1^B} > 0$. Moreover:

$$\frac{dr}{d\overline{\tau}_1^B} = -\frac{(1-p)(1-s)\,h\left(\hat{\Delta}_E\right) r_j \frac{d\hat{\Delta}_E}{d\overline{\tau}_1^B}}{\overline{p}\left(\hat{\Delta}_E\right) + (1-p)(1-s)\,h\left(\hat{\Delta}_E\right) r_j \frac{d\hat{\Delta}_E}{dr_j}\frac{dr_j}{d\overline{\tau}_1^B}},$$

with $\frac{d\hat{\Delta}_E}{d\overline{\tau}_1^B} < 0$ and $\frac{d\hat{\Delta}_E}{dr_j} < 0$. By continuity, there exists a sufficiently high $\underline{s}^{IV} < 1$ such that $\frac{dr}{d\overline{\tau}_1^B} > 0$ holds for all $s \in \left[\underline{s}^{IV}, 1\right)$. In that case $W_E\big|_{E_0 = e_0^*} < 0$, provided $\frac{d\overline{\tau}_1^B}{dE_0} < 0$. From the Proof of Proposition 6 we have $\frac{d\overline{\tau}_1^B}{de_0} < 0$ provided $s < \underline{s}^{III}$. As a result, a sufficient condition for the result in Proposition 3 to hold is that

$s \in \left[\max\left\{\underline{s}^{III}, \underline{s}^{IV}\right\}, 1\right)$. This concludes the proof.

A.3.4 Proof of Proposition 4

The proof consists of three steps. We first solve the bank's problem for the case $\phi \geq \underline{\phi}$. Thereafter, we compare the optimality conditions of the bank's problem with and without the tax-subsidy program. That is, with recapitalizations via the liability and asset side, respectively. Finally, we complete the analysis by accounting for general equilibrium effects, which allow to compare the incidence of bank failures. For the subsequent analysis it is useful to distinguish the deposit rates in the different equilibria with liability and asset side recapitalizations with $r_{j,E}$ and $r_{j,A}$, respectively.

Step 1: From Proposition 2 we know that if $\phi > \underline{\phi}$, banks prefer to deleverage via asset side operations when facing a recapitalization need at $t = 1$. The bank's problem reads:

$$\max_{r_j, e_{0j}} \mathbb{E}[\Pi] = p\Gamma - e_{0j}(\bar{r} + \bar{\tau}_0) + (1-p)\left(s\pi_{j,A}(e_{0j}, \underline{\Delta}) + (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A} \pi_{j,A}(e_{0j}, \Delta_j) h(\Delta_j) d\Delta_j\right) \mathbb{1}_{\hat{\Delta}_A \geq \underline{\Delta}}$$

subject to the same constraints as the problem in (12), but with the following modifications:

$$\begin{aligned}
&\bar{r} \leq \left(p + (1-p)\left[s + (1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A} h(\Delta_j) d\Delta_j\right] \mathbb{1}_{\hat{\Delta}_A \geq \underline{\Delta}}\right) r_{j,A} \equiv \bar{p}\left(\hat{\Delta}_A\right) r_{j,A} \\
&\pi_{j,A}(e_{0j}, \Delta_j) = \left(1 - \frac{\left(1 - \frac{R-\Delta_j}{r_{j,A}} - e_{0j}\right)\left(r_{j,A} + \bar{\tau}_1^B\right)}{R + \frac{\varepsilon-1}{2}\Delta_j - \frac{R-\Delta_j}{r_{j,A}}\left(r_{j,A} + \bar{\tau}_1^B\right)}\right) \frac{\varepsilon+1}{2}\Delta_j \\
&\hat{\Delta}_A\left(e_{0j}, r_{j,A}; \bar{\tau}_1^B\right) = \left(R - (1-e_{0j})\left(r_{j,A} + \bar{\tau}_1^B\right)\right) \frac{2}{1-\varepsilon}.
\end{aligned}$$

Assuming an interior solution for $\hat{\Delta}_A$ exists, the first-order necessary condition with respect to e_{0j} reads:

$$\begin{aligned}
e_{0j}:\ K_3\left(e_{0j}; \bar{\tau}_0, \bar{\tau}_1^B\right) \equiv \quad & -(1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left(\frac{d\ell\left(e_{0j}, \Delta_j\right)}{de_{0j}} \frac{(\varepsilon+1)\Delta_j}{2} + e_{0j}\frac{dr_{j,A}}{de_{0j}} + r_{j,A}\right) h(\Delta_j) d\Delta_j \\
& -\bar{\tau}_0 - (1-p)s\left(\frac{d\ell\left(e_{0j}, \underline{\Delta}\right)}{de_{0j}} \frac{(\varepsilon+1)\underline{\Delta}}{2} + e_{0j}\frac{dr_{j,A}}{de_{0j}} + r_{j,A}\right) = 0,\ \forall e_{0j} \in (0, \bar{e}_{0j}),
\end{aligned} \tag{16}$$

where:

$$\begin{aligned}
\frac{d\ell\left(e_{0j},\Delta_j\right)}{de_{0j}} &= \frac{-\left(r_{j,A}+\overline{\tau}_1^B\right)\left(R+\frac{\varepsilon-1}{2}\Delta_j\right)+\left[R+\frac{\varepsilon-1}{2}\Delta_j-\left(1-e_{0j}\right)\left(r_{j,A}+\overline{\tau}_1^B\right)\right]\frac{R-\Delta_j}{r_{j,A}^2}\overline{\tau}_1^B\frac{dr_{j,A}}{de_{0j}}}{\left(R+\frac{\varepsilon-1}{2}\Delta_j-\frac{R-\Delta_j}{r_j}\left(r_j+\overline{\tau}_1^B\right)\right)^2} \\
\frac{dr_{j,A}}{de_{0j}} &= -\frac{(1-p)(1-s)h\left(\hat{\Delta}_A\right)r_{j,A}\frac{d\hat{\Delta}_A}{de_{0j}}}{\overline{p}\left(\hat{\Delta}_A\right)+(1-p)(1-s)h\left(\hat{\Delta}_A\right)r_{j,A}\frac{d\hat{\Delta}_A}{dr_{j,A}}} \\
\frac{d\hat{\Delta}_A}{de_{0j}} &= \left(r_{j,A}+\overline{\tau}_1^B\right)\frac{2}{1-\varepsilon} > 0.
\end{aligned}$$

By continuity, there exists some $\underline{s}^V < 1$ such that $\frac{dr_{j,A}}{de_{0j}} < 0$ provided $s \in \left[\underline{s}^V, 1\right)$. For $\frac{dr_{j,A}}{de_{0j}} < 0$ we have that $\frac{d\ell\left(e_{0j},\Delta_j\right)}{de_{0j}} < 0\ \forall \Delta_j \in \left[\underline{\Delta}, \hat{\Delta}_A\right]$, given $\hat{\Delta}_A < \hat{\Delta}_E$ and $R+\frac{\varepsilon-1}{2}\Delta_j-(1-e_{0j})\left(r_{j,A}+\overline{\tau}_1^B\right) > 0\ \forall \Delta_j \in \left[\underline{\Delta}, \hat{\Delta}_E\right]$. Note that from equation (6) the last inequality must hold if $\hat{\Delta}_A \in \left[\underline{\Delta}, \hat{\Delta}_E\right]$ as required.

Step 2: With the tax-subsidy program the bank manager and initial equity investors are compensated for the loss of control rights in the bad state $\theta = B$ so that they are willing to issue new equity to recapitalize even if $\phi \geq \underline{\phi}$. Since the policy intervention is financed with a non-distortionary tax at $t = 2$ to all households, the bank's problem is identical to 2.2.2 and the optimality condition is given by equation (13).

We, henceforth, compare the optimality conditions in equations (13) and (16). Suppose an interior solution exists and let $e_{0,E}^*$ be the choice of ex-ante equity buffers such that $K_1\left(e_{0,E}^*; \overline{\tau}_0, \overline{\tau}_1^B\right) = 0$ for a given $\overline{\tau}_0$ and $\overline{\tau}_1^B$. We can show that $K_3\left(e_{0,E}^*; \overline{\tau}_0, \overline{\tau}_1^B\right) > 0$. First notice that $r_{j,E} < r_{j,A}$ since $\hat{\Delta}_A\left(e_{0,E}^*\right) < \hat{\Delta}_E\left(e_{0,E}^*\right)$. Moreover:

$$\frac{d\hat{\Delta}_E}{de_{0j}} = \frac{r_{j,E}+\overline{\tau}_1^B}{\frac{\overline{\tau}_1^B}{r_{j,E}}+\frac{1-\varepsilon}{2}} < \frac{d\hat{\Delta}_A}{de_{0j}} = \frac{r_{j,A}+\overline{\tau}_1^B}{\frac{1-\varepsilon}{2}}$$

and:

$$\frac{dr_{j,A}}{de_{0j}} < \frac{dr_{j,E}}{de_{0j}} < 0,$$

where the last inequality holds under the sufficient condition that $s \geq \underline{s}^V$.

Thus, we arrive at:

$$\begin{aligned}
K_3\left(e_{0,E}^*; \overline{\tau}_0, \overline{\tau}_1^B\right) - K_1\left(e_{0,E}^*; \overline{\tau}_0, \overline{\tau}_1^B\right) = \quad & -(1-p)\,s\left(\frac{d\ell\left(e_{0j},\underline{\Delta}\right)}{de_{0j}}\frac{(\varepsilon+1)\underline{\Delta}}{2}+e_{0j}\frac{dr_{j,A}}{de_{0j}}+r_{j,A}\right) \\
& -(1-p)\,s\left[\overline{\tau}_1^B-\left(1+\frac{R-\underline{\Delta}}{r_{j,E}^2}\overline{\tau}_1^B\right)\frac{dr_{j,E}}{de_{0j}}\right] \\
& -(1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left(\frac{d\ell\left(e_{0j},\Delta_j\right)}{de_{0j}}\frac{(\varepsilon+1)\Delta_j}{2}+e_{0j}\frac{dr_{j,A}}{de_{0j}}+r_{j,A}\right)h\left(\Delta_j\right)d\Delta_j \\
& -(1-p)(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E}\left[\overline{\tau}_1^B-\left(1+\frac{R-\Delta_j}{r_{j,E}^2}\overline{\tau}_1^B\right)\frac{dr_{j,E}}{de_{0j}}\right]h\left(\Delta_j\right)d\Delta_j.
\end{aligned}$$

By continuity, we have that $K_3\left(e^*_{0,E};\overline{\tau}_0,\overline{\tau}^B_1\right)-K_1\left(e^*_{0,E};\overline{\tau}_0,\overline{\tau}^B_1\right)<0$ for s sufficiently high, since $\lim_{s\to 1}\frac{dr_{j,E}}{de_{0j}}=\lim_{s\to 1}\frac{dr_{j,A}}{de_{0j}}=0$ and:

$$\lim_{s\to 1}\left(\frac{d\ell}{de_{0j}}\frac{(\varepsilon+1)\underline{\Delta}}{2}+e_{0j}\frac{dr_{j,A}}{de_{0j}}+\left(r_{j,A}+\overline{\tau}^B_1\right)-\left(1+\frac{R-\underline{\Delta}}{r^2_{j,E}}\overline{\tau}^B_1\right)\frac{dr_{j,E}}{de_{0j}}\right)>0.$$

Thus, by continuity, there exists some $\underline{s}^{VI}<1$ such that for a given $\overline{\tau}_0$ and $\overline{\tau}^B_1$, banks' optimally chosen ex-ante equity buffer is higher if they expect to conduct liability side recapitalizations. Formally, $e^*_{0,E}>e^*_{0,A}$ for a given $\overline{\tau}_0$ and $\overline{\tau}^B_1$.

Step 3: Next, we account for the endogenous response of market prices. That is, we take into account that $\overline{\tau}_0$ and $\overline{\tau}^B_1$ systematically differ under asset and liability side operations. When evaluated at $e^*_{0,E}$ we have that $\hat{\Delta}_A\left(e^*_{0,E}\right)<\hat{\Delta}_E\left(e^*_{0,E}\right)$. As a result, relative to liability side operations there is more demand for specialized investment capital (extensive margin) stemming from the higher incidence of failed banks that are winded down. On the other hand, asset side recapitalizations of solvent banks also result in more demand for specialized investment capital (intensive margin) since $\mathbb{E}\left[\pi_{j,E}\left(e^*_{0,E},\Delta_j\right)\right]>\mathbb{E}\left[\pi_{j,A}\left(e^*_{0,E},\Delta_j\right)\right]\ \forall\Delta_j$.

The two equilibrium prices are determined as follows:

$$\overline{\tau}_0=\hat{\alpha}\left(\overline{\tau}^B_1\right)\equiv\underline{\alpha}+(1-p)\frac{\left(\overline{\tau}^B_1-\frac{\underline{\alpha}}{2}\right)^2}{\overline{\alpha}-\underline{\alpha}}.$$

For liability side operations $\overline{\tau}^B_1$ solves:

$$\begin{aligned}\frac{\overline{\tau}^B_1-\underline{\alpha}}{1-e^*_{0,E}}= \quad & s\left(1-\frac{R-\underline{\Delta}}{r_{j,E}}-e^*_{0,E}\right)+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_E\left(e^*_{0,E}\right)}\left(1-\frac{R-\Delta_j}{r_{j,E}}-e^*_{0,E}\right)h(\Delta_j)\,d\Delta_j\\ &+(1-s)\int_{\hat{\Delta}_E\left(e^*_{0,E}\right)}^{R}\frac{r_{j,E}+\overline{\tau}^B_1}{R+(\varepsilon-1)\Delta_j}h(\Delta_j)\,d\Delta_j.\end{aligned}$$

whereas, for asset side operations $\overline{\tau}^B_1$ solves:

$$\begin{aligned}\frac{\overline{\tau}^B_1-\underline{\alpha}}{1-e^*_{0,E}}= \quad & s\frac{1-\frac{R-\underline{\Delta}}{r_{j,A}}-e^*_{0,E}}{R+\frac{\varepsilon-1}{2}\underline{\Delta}-\frac{R-\underline{\Delta}}{r_{j,A}}\left(r_{j,A}+\overline{\tau}^B_1\right)}\frac{\left(r_{j,A}+\overline{\tau}^B_1\right)^2}{R+(\varepsilon-1)\underline{\Delta}}\\ &+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A\left(e^*_{0,E}\right)}\left(\frac{1-\frac{R-\Delta_j}{r_{j,A}}-e^*_{0,E}}{R+\frac{\varepsilon-1}{2}\Delta_j-\frac{R-\Delta_j}{r_{j,A}}\left(r_{j,A}+\overline{\tau}^B_1\right)}\frac{\left(r_{j,A}+\overline{\tau}^B_1\right)^2}{R+(\varepsilon-1)\Delta_j}\right)h(\Delta_j)\,d\Delta_j\\ &+(1-s)\int_{\hat{\Delta}_A\left(e^*_{0,E}\right)}^{R}\frac{r_{j,A}+\overline{\tau}^B_1}{R+(\varepsilon-1)\Delta_j}h(\Delta_j)\,d\Delta_j.\end{aligned}$$

First, it can be seen that $\frac{d\overline{\tau}_0}{d\overline{\tau}_1^B} > 0$, and, second that both $\overline{\tau}_0$ and $\overline{\tau}_1^B$ are higher under asset side recapitalizations when evaluated at $E_{0,E}^* = e_{0,E}^*$. Moreover, $\frac{dK_3(e_{0,E}^*;\overline{\tau}_0,\overline{\tau}_1^B)}{d\overline{\tau}_1^B} < 0$ provided $s \in \left[\max\left\{\underline{s}^V, \underline{s}^{VI}\right\}, 1\right)$. As a result, the desire for lower ex-ante equity buffers under asset, relative to liability side operations, is further strengthened by the general equilibrium price effect, i.e. $E_{0,E}^* > E_{0,A}^*$. The results in Proposition 4 follow. This completes the proof.

A.3.5 Proof of Proposition 5

We prove the result of Proposition 5 for the case $s = 1$. The argument can be readily extended to the general case, yielding the sufficient condition for $s \in [0,1]$.

Formally, bank i chooses at $t = 0$ the optimal capital structure, i.e. the amount of deposits and initial equity, the optimal portfolio composition, the amount of investments in the risky long-term technology and in storage, as well as the deposit rate. Recall that we normalize (w.l.o.g.) the size of each bank to one, i.e. $e_{0j} + d_j = 1$ and let $0 \leq x_j \leq 1$ indicate the fraction of resources invested in the the risky long-term technology. Focusing on liability side operations and supposing that $\underline{\Delta}$ is sufficiently high such that a capital short-fall arises, but at the same time not too high such that there are no insolvencies, the problem reads:

$$\max_{r_j,e_{0j},x_j} \mathbb{E}[\Pi] = \left\{p\Gamma(r_j,x_j,e_{0j}) + (1-p)\left(s\left[\Gamma(r_j,x_j,e_{0j}) - \left(e_{1j}^B(r_j,x_j;\underline{\Delta}) - e_{0j}\right)\overline{\tau}_1^B\right]\right)e_{0j}(\overline{r}+\overline{\tau}_0)\right\}, \quad (17)$$

subject to:

$$\begin{aligned}
&0 \leq e_{0j} \leq \overline{e}_0(x_j) \\
&0 \leq 1 - x_j \leq \overline{1-x_j}(e_{0j}) \\
&\Gamma(r_j,x_j,e_{0j}) \equiv x_j R + (1-x_j) - r_j(1-e_{0j}) \\
&\overline{r} = r_j \\
&e_{1j}^B(r_j,x_j;\Delta_j) = 1 - \tfrac{x_j(R-\Delta_j)+(1-x_j)}{r_j} \\
&\mathbb{E}[\Pi] \geq \gamma.
\end{aligned}$$

The interpretation of the last three conditions is as before. Notice that the upper bound on ex-ante equity buffers is $\overline{e}_0(x_j) = 1 - \frac{x_j(R-\underline{\Delta})+(1-x_j)}{r_j}$ since nobody would be willing to hold higher buffers than necessary.

Similarly we compute the upper bound on storage as $\overline{1-x_j}\left(e_{0j}\right)=\frac{r_j\left(1-e_{0j}\right)-(R-\underline{\Delta})}{1-(R-\underline{\Delta})}$, which decreases in e_{0j}.

Given that $\bar{r}$ is taken as given, the bank problem can simplified. The Kuhn-Tucker conditions are:

$$
\begin{aligned}
\frac{d\mathbb{E}[\Pi]}{de_{0j}} &= (1-p)\,\overline{\tau}_1^B-\overline{\tau}_0+\lambda_1-\lambda_2-\lambda_4\left[1-x_j-\frac{r_j\left(1-e_{0j}\right)-(R-\underline{\Delta})}{1-(R-\underline{\Delta})}\right]=0\\
\frac{d\mathbb{E}[\Pi]}{dx_j} &= (R-1)+(1-p)\,\frac{(R-\underline{\Delta})-1}{\bar{r}}\overline{\tau}_1^B-\lambda_2\left[e_{0j}-\left(1-\frac{x_j(R-\underline{\Delta})+\left(1-x_j\right)}{r_j}\right)\right]+\lambda_3-\lambda_4=0\\
0 &= \lambda_1\left[e_{0j}-0\right]\equiv\lambda_1 g_1\\
0 &= \lambda_2\left[e_{0j}-\overline{e}_0\left(x_j\right)\right]\equiv\lambda_2 g_2\\
0 &= \lambda_3\left[1-x_j-0\right]\equiv\lambda_3 g_3\\
0 &= \lambda_4\left[1-x_j-\overline{1-x_j}\left(e_{0j}\right)\right]\equiv\lambda_4 g_4\\
&\lambda_m\geq 0\;\forall m=1,2,3,4\\
&g_m\geq 0\;\forall m=1,2,3,4.
\end{aligned}
$$

Notice that $x_j=1$ provided $(R-1)+(1-p)\,\frac{(R-\underline{\Delta})-1}{\bar{r}}\overline{\tau}_1^B>0$ independent of e_{0j}. Moreover, $x_j=1$ if $e_{0j}=\overline{e}_0\left(1\right)=1-\frac{R-\underline{\Delta}}{r_j}$. The sufficient conditions in Proposition 5 follow. This concludes the proof.

A.4 Asset side recapitalizations

In this section we focus on asset side recapitalizations in isolation and sketch the bank's problem at $t=0$ and the constrained planner problem. Recall from Section 2.1.5 that $\hat{\underline{\Delta}}_A$ cannot be interior if $\varepsilon=1$. Since our envelop argument for the efficiency analysis relies on interiority of $\hat{\underline{\Delta}}_A$, we focus on the case $\varepsilon<1$.

Bank's problem. The simplified bank's problem writes:

$$
\max_{e_{0j}}\mathbb{E}\left[\Pi\right]=\left\{\begin{array}{c}
p\left[R-r_j\left(1-e_{0j}\right)\right]+0\;\mathbb{1}_{\hat{\Delta}_E<\underline{\Delta}}\;-\;e_{0j}\left(\bar{r}+\overline{\tau}_0\right)+\\
(1-p)\left(\begin{array}{c}
\frac{s}{2}\left[\left(1-\ell_j\right)\left(R+\varepsilon\underline{\Delta}\right)-r_j\left(1-e_{1j,A}^B\right)\right]+\\
\frac{1-s}{2}\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left[\left(1-\ell_j\right)\left(R+\varepsilon\Delta_j\right)-r_j\left(1-e_{1j,A}^B\right)\right]h\left(\Delta_j\right)d\Delta_j
\end{array}\right)\mathbb{1}_{\hat{\Delta}_A\geq\underline{\Delta}}
\end{array}\right\}
$$

subject to the same constraints as the problem in (12), but with the following modifications:

$$
\begin{aligned}
&\ell_j\left(e_{0j},\Delta_j\right)=\frac{\left(1-\frac{R-\Delta_j}{r_j}-e_{0j}\right)\left(r_j+\overline{\tau}^B_{1,A}\right)}{R+\frac{\varepsilon-1}{2}\Delta_j-\frac{R-\Delta_j}{r_j}\left(r_j+\overline{\tau}^B_{1,A}\right)}\\
&\overline{r}=\left(p+(1-p)\left[s+(1-s)\int_{\underline{\Delta}}^{\hat{\Delta}_A}h\left(\Delta_j\right)d\Delta_j\right]\mathbb{1}_{\hat{\Delta}_A\geq\underline{\Delta}}\right)\ r_j\equiv\overline{p}\left(\hat{\Delta}_A\right)\ r_j\\
&e^B_{1j,A}\left(r_j,\ell_j;\Delta_j\right)=1-\frac{\left(1-\ell_j\right)\left(R-\Delta_j\right)}{r_j}\\
&\hat{\Delta}_A\left(e_{0j},r_j;\overline{\tau}^B_1\right)=2\frac{R-\left(1-e_{0j}\right)\left(r_j+\overline{\tau}^B_1\right)}{1-\varepsilon}.
\end{aligned}
$$

Assuming that an interior solution exists, the first-order necessary condition writes:

$$
\begin{aligned}
e_{0j}:\ K_3\equiv\quad &-\overline{\tau}_0+p\left[-\frac{dr_j}{de_{0j}}\right]+\frac{(1-p)s}{2}\left[-\frac{d\ell_j}{de_{0j}}\left(R+\varepsilon\underline{\Delta}\right)-\left(\frac{d\ell_j}{de_{0j}}\left(R-\underline{\Delta}\right)+r_j\right)-\frac{dr_j}{de_{0j}}e_{0j}\right]\\
&+\frac{(1-p)(1-s)}{2}\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left[-\frac{d\ell_j}{de_{0j}}\left(R+\varepsilon\Delta_j\right)-\left(\frac{d\ell_j}{de_{0j}}\left(R-\Delta_j\right)+r_j\right)-\frac{dr_j}{de_{0j}}e_{0j}\right]h\left(\Delta_j\right)d\Delta_j\\
&-\frac{(1-p)(1-s)}{2}\frac{d\hat{\Delta}_A}{de_{0j}}\left[e_{0j}r_j\right]h\left(\hat{\Delta}_A\right)=0,\ for\ e_{0j}\in(0,1),
\end{aligned}
\tag{18}
$$

where:

$$
\begin{aligned}
\frac{d\hat{\Delta}_A\left(\overline{\tau}^B_1\right)}{de_{0j}}\quad&=2\frac{r_j+\overline{\tau}^B_1}{1-\varepsilon}>0\\
\frac{d\hat{\Delta}_A\left(\overline{\tau}^B_1\right)}{dr_j}\quad&=-2\frac{\left(1-e_{0j}\right)\overline{\tau}^B_1}{1-\varepsilon}<0<0\ \forall e_{0j}\in(0,1)\\
\frac{dr_j\left(e_{0j},\hat{\Delta}_A;\overline{\tau}^B_1\right)}{de_{0j}}\quad&=\frac{-(1-p)(1-s)\frac{d\hat{\Delta}_A\left(\overline{\tau}^B_1\right)}{de_{0j}}h\left(\hat{\Delta}_A\right)}{\overline{p}\left(e_{0j},\hat{\Delta}_A,r_j\right)-r_j(1-p)(1-s)\frac{d\hat{\Delta}_A\left(\overline{\tau}^B_1\right)}{dr_j}h\left(\hat{\Delta}_A\right)}<0\ \forall e_{0j}\in(0,1)\\
\frac{d\ell_j\left(e_{0j},\Delta_j\right)}{de_{0j}}\quad&=\frac{\begin{pmatrix}\left(-r_j+\left(1-e_{0j}\right)\frac{dr_j}{de_{0j}}+\left(\frac{R-\Delta_j}{r_j^2}-1\right)\overline{\tau}^B_{1,A}\right)[\cdot]\\-\left(1-\frac{R-\Delta_j}{r_j}-e_{0j}\right)\left(r_j+\overline{\tau}^B_{1,A}\right)\frac{R-\Delta_j}{r_j^2}\overline{\tau}^B_{1,A}\end{pmatrix}}{\left[R+\frac{\varepsilon-1}{2}\Delta_j-\frac{R-\Delta_j}{r_j}\left(r_j+\overline{\tau}^B_{1,A}\right)\right]^2}<0\ \forall e_{0j}\in\left(0,1-\frac{R-\Delta_j}{r_j}\right)\\
\frac{de^B_{1j,A}\left(r_j,\ell_j;\Delta_j\right)}{de_{0j}}\quad&=-\frac{-\frac{d\ell_j}{de_{0j}}\left(R-\Delta_j\right)-\frac{dr_j}{de_{0j}}\left(1-\ell_j\right)\left(R-\Delta_j\right)}{r_j^2}<0\ \forall e_{0j}\in\left(0,1-\frac{R-\Delta_j}{r_j}\right).
\end{aligned}
$$

The intuition is analog to the model with liability side recapitalizations. Importantly, the volume of asset sales in the aggregate state $\theta=B$ at $t=1$ decreases when ex-ante equity buffers are higher.

Market clearing. While the market clearing conditions for the deposit market and the market for specialized investment capital at $t=1$ are unaltered, market clearing in state $\theta=B$ needs to be altered. The aggregate demand for specialized investment capital is given by equation (9), with $\overline{\tau}^B_1>\underline{\alpha}\ \forall\hat{\Delta}_A\in(\underline{\Delta},R)$.

Constrained planner problem. We construct the planner problem as in Section A.3.3:

$$\max_{e_0 \geq 0} \left\{ \mathbb{E}[\Pi] + (1-p) X\left(\overline{\tau}_1^B\right) \right\}$$
$$= \left\{ \begin{array}{c} p\left[R - r(1-e_0)\right] - e_0\left(\overline{r} + \overline{\tau}_0\right) + (1-p) X\left(\overline{\tau}_1^B\right) + \\ (1-p) \left(\begin{array}{c} \frac{s}{2}\left[(1-\ell_j)(R + \varepsilon\underline{\Delta}) - r_j\left(1 - e^B_{1j,A}\right)\right] + \\ \frac{1-s}{2}\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left[(1-\ell_j)(R + \varepsilon\Delta_j) - r_j\left(1 - e^B_{1j,A}\right)\right] h(\Delta_j)\, d\Delta_j \end{array} \right) \end{array} \right\} \tag{19}$$

subject to the appropriately modified condition from before.

Assuming an interior solution exists, the constrained efficient solution is characterized by a similar system of equations as the laissez-faire equilibrium. However, condition (18) has to be replaced by the optimality condition from the problem in (19). Again, let e_0^* be the individually optimal equity buffer and denote the first derivative of the planner's problem with respect to e_0 by K_4. Evaluating K_4 at e_0^* leads to:

$$K_4\big|_{e_0=e_0^*} = K_3\big|_{e_0=e_0^*} + W_A\, \frac{d\overline{\tau}_1^B}{de_0}\big|_{e_0=e_0^*}$$

where $K_3\big|_{e_0=e_0^*} = 0$ by definition, and:

$$\begin{aligned} W_A \equiv\ & -p(1-e_0)\, \frac{dr}{d\overline{\tau}_1^B} - e_0 \frac{d\overline{\tau}_0}{d\overline{\tau}_1^B} + (1-p)\left(\int_{\alpha_1}^{\overline{\tau}_1^B} \overline{\tau}_1^B \frac{dg(\alpha_i, e_0)}{de_0}\, d\alpha_i - \int_{\alpha_1}^{\overline{\tau}_1^B} \alpha_i \frac{dg(\alpha_i, e_0)}{de_0}\, d\alpha_i\right) / \frac{d\overline{\tau}_1^B}{de_0} \\ & + (1-p)\frac{s}{2}\left[-\frac{d\ell_j}{dr}\frac{dr}{d\overline{\tau}_1^B}(R + \varepsilon\underline{\Delta}) - \frac{d\ell_j}{dr}\frac{dr}{d\overline{\tau}_1^B}(R - \underline{\Delta})\right] \\ & + (1-p)\frac{1-s}{2}\int_{\underline{\Delta}}^{\hat{\Delta}_A}\left[-\frac{d\ell_j}{dr}\frac{dr}{d\overline{\tau}_1^B}(R + \varepsilon\Delta_j) - \frac{d\ell_j}{dr}\frac{dr}{d\overline{\tau}_1^B}(R - \Delta_j)\right] h(\Delta_j)\, d\Delta_j. \end{aligned}$$

Recall that $\frac{d\overline{\tau}_1^B}{de_0} < 0$ and $\frac{dr}{d\overline{\tau}_1^B} > 0$ provided a sufficiently high s. Moreover $\frac{d\ell_j}{dr} > 0\, \forall \hat{\Delta}_A \in (\underline{\Delta}, R)$.