Identifying the Best Agent in a Network

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- The mechanism is a function that defines a probability of being selected for every agent for any set of messages the principal might receive.
- The mechanism yields partial implementation for all networks where every agent has at least one neighbor,
  and full implementation for some networks.

2 Applications for this Environment

Competition for presentation slot at certain conferences (e.g. EC) or for a journal publication:

- we submit papers and write referee reports about other's submissions

"360 degree" framework used in firms:

- reports from self, peers, subordinates, bosses, managers are used to evaluate the performance of individuals;
  reports are usually given as ratings on a scale

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- Hagenbach and Koessler (2009), Galeotti, Ghiglino and Squintani (2009), Currarini and Feri (2013), Acemoglu, Bimpikis and Ozdaglar (2014), Bloch, Demange, Kranton (2014), Garcia (2014), Patty and Penn (2014), Calvó-Armengol, de Martí and Prat (2015), Foerster (2015):
  Communication and information transmission takes place between agents in a network.
- This paper: strategic communication and information transmission from a network of agents to a principal who wants to extract information.

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- Renou and Tomala (2012): implementation for different communication networks among agents and mechanism designer, agents' ex-ante information independent of network.

  This paper: communication network is a star with principal as center, agents' ex-ante information determined by the knowledge network.

- Dziubiński, Sankowski, Zhang (2016): optimal network protection when the network is unknown to the defender and imperfectly known to the nodes through a truthful mechanism.
  This paper: knowledge network is public knowledge, agents' "abilities" are unknown and independent of the network.

The Model

One Principal and Many Heterogeneous Agents

- A principal P has to assign one unit/prize to one agent.
- Set of agents: $N = \{1, ..., n\}$ with $n \geq 3$
- Every agent wants the prize.
- Agents are differently suited for the prize/are heterogeneous in their value for P:
    - agent i's distance to the ideal point: d_i
    - each d_i is independently drawn from a distribution with strictly positive density over $[0, 1]$
    - $d_i = 0$ would be the ideal, $d_i = 1$ the worst possible fit
    - P's utility is decreasing in the agent's distance to whom she assigns the prize
    - restrict attention to the case that no two agents have the same distance: $d_i \neq d_j$ for $i \neq j$

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    Agents know the distribution of values.
- Agent i's "type" $t_i := (d_i, (d_j)_{j \in N_i})$

Messages

- Agents simultaneously send private costless messages to P.
- Message m_i of agent i contains
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  If $d_i < d_j$, agent i "proves" better than $j \in N_i$ if $m_{ij} - m_{ii} > 2b$.
- $m = (m_1, m_2, ..., m_n)$ is the message profile sent by the agents to P, M is the set of all feasible message profiles.

Principal's Action

- Before agents choose their message strategies,
  P chooses a function $\pi : M \rightarrow [0,1]^n$ which specifies a probability $\pi_i(m) \in [0,1]$ for every agent i for any m.
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- P publicly announces π and commits to it.

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- Given π and $\hat{m}_{-i}$, each agent i chooses $\hat{m}_i$ such that
  - $\hat{m}_i(t_i)$ maximizes her expected probability of being selected, π_i^e, for each t_i,
  - and $\hat{m}_i(t_i)$ is the truth ($\hat{m}_{ik}(t_i) = d_k$ for all k), if the truth maximizes π_i^e for t_i.

Recap' on the Model and Goal of the Paper

The Model:

1. P chooses, announces, and commits to π.
2. Agents engage in a static Bayesian game. They choose $\hat{m}$, types realize and m is sent to P.

   $\Rightarrow$ The solution concept for this stage is Bayesian Nash equilibrium.
3. Given the messages sent by the agents, P assigns the prize according to $\pi(m)$.

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The "best" agent is agent i with $d_i = \min_{k \in N} d_k$ and will be called "the global minimum" g.

A Mechanism to Identify the Global Minimum

The Mechanism π

Claim.

Given the following mechanism π, there exists an equilibrium for every network in which each agent has at least one neighbor such that P assigns the prize with probability 1 to the global minimum.

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Structure of π:

For any message profile m,
the principal determines $N \supseteq B_1(m) \supseteq B_2(m) \supseteq B_3(m)$.

$\pi_i(m)$ depends on the properties of $B_2(m)$ and $B_3(m)$ and in which subsets agent i is.

Determining $B_1(m)$

All agents who send the best application are in $B_1(m)$.

Agent $i \in B_1(m)$, iff

$$m_{ii} = \min_{k \in N} m_{kk}.$$

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- the worst reference which i receives: $\bar{r}_i = \max_{j \in N_i} m_{ji}$
- Agent $i \in B_2(m)$, iff
  - $i \in B_1(m)$, and
  - her worst reference is the least bad among the worst references agents in $B_1(m)$ receive, i.e. the min-max reference about agents in $B_1(m)$

$$\bar{r}_i = \min_{k \in B_1(m)} \bar{r}_k$$

Determining $B_3(m)$

An agent $i \in B_3(m)$, iff $i \in B_2(m)$ and

- agent i's application conflicts with a reference from a neighbor or agent i's reference about a neighbor conflicts with this neighbor's application, i.e.

$$m_{ii} \neq m_{ji} \text{ or } m_{ij} \neq m_{jj} \text{ for at least one } j \in N_i,$$

or

- the message of agent i proves that agent i is better than each of her neighbors, i.e.

$$m_{ij} - m_{ii} > 2b \text{ for all } j \in N_i.$$

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    - there is agent $j \neq i$ to whom i is not linked,
      then $\pi_j(m) = \frac{1}{|N|-|N_i|-1}$ for every $j \notin N_i$.

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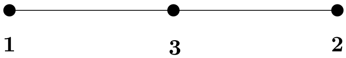
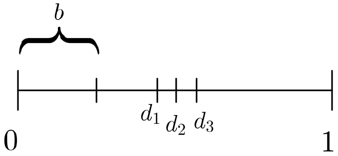
  - i is linked to every other agent $j \neq i$ and there is no $k \in N_i$ for whom $m_{ik} - m_{ii} > 2b$, then $\pi_j(m) = \frac{1}{|N|-1}$ for every $j \in N_i$.

Agents' Incentives given π. Example I

Suppose $n = 3$
in a line



and distances are



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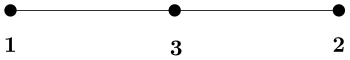

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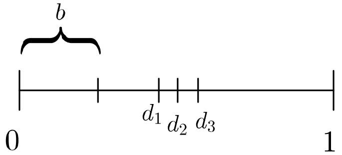

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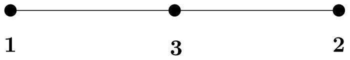

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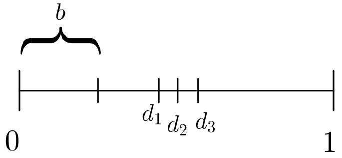

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Then **3** knows that for any m_3 she is selected with prob. 0 and tells the truth.

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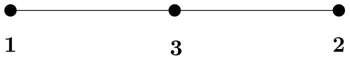

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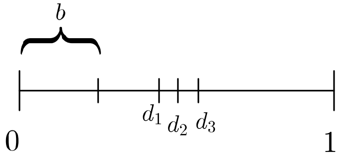


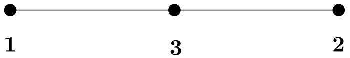
1 and **2** send messages with $m_{11} = d_1 - b$ and $m_{22} = d_2 - b$.

Then **3** knows that for any m_3 she is selected with prob. 0 and tells the truth.

1 wins.

Agents' Incentives given π. Example II

Suppose $n = 3$
in a line



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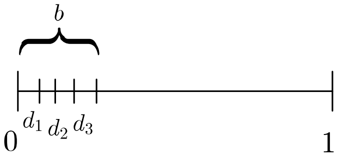

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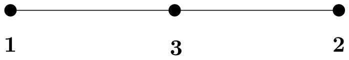

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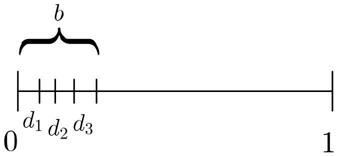

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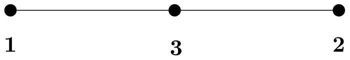

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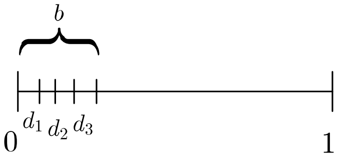

1 and **2** send $m_{11} = m_{22} = 0$ and $m_{13} = m_{23} = d_3 + b$.

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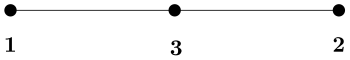

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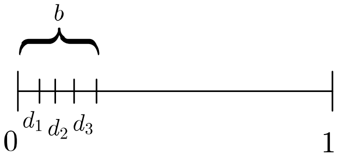

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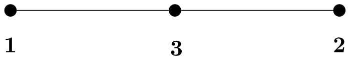

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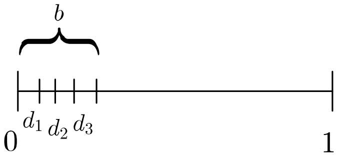

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Since **3** has a strict preference for truth telling, the principal correctly distinguishes between **1** and **2**, and **1** wins.

An Equilibrium

– Partial Implementation –

Equilibrium Messages given π

Claim.

The following strategy profile $\hat{m}$ is an equilibrium of the static Bayesian game among agents given π as defined before.

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A *local minimum* is an agent i who is better than all her neighbors, i.e. $d_i < d_j$ for all $j \in N_i$.

An agent i has *partial information* if she is not linked to every agent $j \neq i$.

An agent i has *full information* if she is linked to every agent $j \neq i$.

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Intuitively, $\hat{m}$ is such that...

- every agent who is not a local minimum says the truth, and
- every local minimum either proves that she is better than each of her neighbors or at least lies to the full extent about herself and her neighbors.

Strategy Profile $\hat{m}$ is such that...

- agent i who is not a local minimum says the truth:
  - $\hat{m}_{ii} = d_i$ and $\hat{m}_{ij} = d_j$ for all $j \in N_i$

Strategy Profile $\hat{m}$ is such that...

- agent i who is not a local minimum says the truth:
  - $\hat{m}_{ii} = d_i$ and $\hat{m}_{ij} = d_j$ for all $j \in N_i$
- every local minimum agent i with partial information chooses
  - the best application: $\hat{m}_{ii} = \max\{0, d_i - b\}$
  - worst references if some neighbor's distance is close:
    $\hat{m}_{ij} = \min\{d_j + b, 1\}$ for every $j \in N_i$, if $d_j - d_i \leq 2b$ for some $j \in N_i$
  - true references if all neighbors are far:
    $\hat{m}_{ij} = d_j$ for every $j \in N_i$, if $d_j - d_i > 2b$ for all $j \in N_i$

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- every local minimum agent i with full information chooses
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If P commits to π and agents choose $\hat{m}$, then in each case g

- sends the best application,
- receives the min-max reference if the best application is zero,
- and proves better than all of her neighbors or conflicts with some neighbor.

Strategy Profile $\hat{m}$ is a Bayesian Nash Equilibrium given π.

I show that strategy profile $\hat{m}$ is such that, given π and $\hat{m}_{-i}$, for each agent $i \in N$

1. $\hat{m}_i(t_i)$ maximizes π_i^e, and
2. the truth does not maximize π_i^e, if $\hat{m}_i(t_i)$ is not the truth

for $t_i =$ "local minimum with full information",
"local minimum with partial information",
"not a local minimum".

Full Implementation

– Always Identifying the Best Agent for Every Equilibrium –

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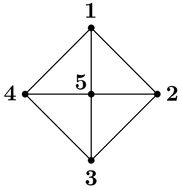
Full implementation!

E.g. the complete network, star, and "Dutch windmill" satisfy 1) and 2).

A Network Without Full Implementation

For the network on the RHS, there exists an equilibrium strategy profile $\hat{m}'$ such that g is not selected with probability 1.



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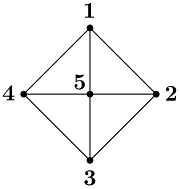

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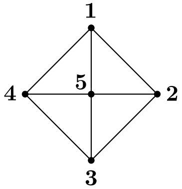

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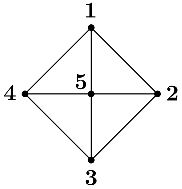

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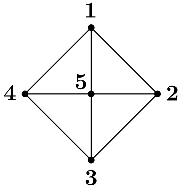

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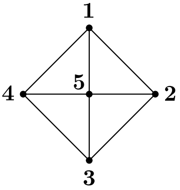

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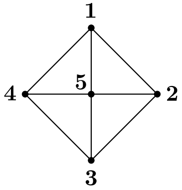

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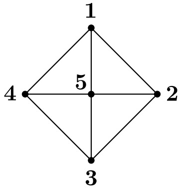

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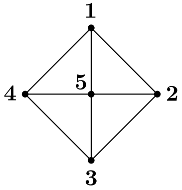

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Full implementation for star and complete network, but not for the wheel! The property of full implementation not monotonic in links added. More information and communication possibilities not always beneficial.

Conclusion

- This paper presents a model in which a principal wants to extract private information from agents who have a knowledge network and who communicate strategically with the principal.
- A mechanism is proposed to identify the best agent in the network with probability 1.
- Given this mechanism,
  - for any network in which every agent has at least one neighbor, there exists an equilibrium in which the best agent is always identified. (Partial Implementation)
  - for some networks (e.g. star, complete), every equilibrium is such that the best agent is always identified. (Full Implementation)
  - full implementation is not monotonic in links added to a network.
- Open Questions: Repeated game? Favoritism? Imperfect knowledge about neighbor? Private information about network? ...