

Government Debt, Dividend Growth and Stock Returns*

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Abstract

This paper documents the debt-to-GDP ratio can predict both dividend growth and stock returns. These findings are consistent with Lettau and Ludvigson (2005)'s argument that there exists a common component among stock returns and dividend growth. To rationalize these findings, we propose a production-based asset pricing model incorporating firm's trade-off behavior. The model can produce testable predictions that the debt-to-GDP can capture the common component. We also relate debt-to-GDP factor to the government inter-temporal budget constraint and propose a VAR system to verify the non-explosive public debt behavior of U.S. government to ensure the debt-to-GDP can act as a rational forward-looking factor. We find the tax and spending uncertainty accounts for 31.36% and 68.56% of variations of debt-to-GDP ratio respectively.

JEL classification: G10, G12, G30, G32, C50.

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"The variance of dividend yields or price-dividend ratios corresponds entirely to discount-rate variation, but as much as half of the variance of ... returns $r_{t+1} \approx \rho dp_{t+1} + dp_t + \Delta d_{t+1}$ corresponds to current dividends Δd_{t+1} . This fact seems trivial but has caused a lot of confusion." - Cochrane, (2011's Presidential Address)

1. Introduction

The mechanism through which fiscal variables impact stock returns remains an open question. We conjecture that the presidency affects the stock market through its fiscal and debt policy. There is an extensive literature that analyzes the impact of monetary policy on financial markets. However, the literature on the effect of fiscal policies on the stock market has been growing. Tracing the unambiguous effect of economic policies on stock returns would necessitate data on government taxation, spending and deficit, which are what we concern. We start this research to explore whether certain fiscal variables can predict stock returns. We document that the debt-to-GDP ratio can predict the dividend growth and therefore can better predict the stock returns.

The finding is directly related to the confusing "facts" introduced by Cochrane (2011). The debt-to-GDP ratio can predict the dividend growth and therefore can better predict the stock returns. One way to say is that $ret_{t+1} = f(dp_t, \Delta d_{t+1})$. Another way to say is what Lettau and Ludvigson (2005) argued that there exists a common component between ret_{t+1} and Δd_{t+1} that dp_t fails to capture. The similar factor loading of debt-to-GDP on dividend growth and stock returns suggest the debt-to-GDP captures this common component.

There is a large literature on dividend growth predictability. Many papers provide evidence of strong predictability on dividend growth rates. These studies verify that dividend growth rates are not *i.i.d.* Examples are Lacerda and Santa-Clara (2010), Binsbergen, Jules, and Kojien (2010), Bansal and Yaron (2004), and Lettau and Ludvigson (2005). Lacerda and Santa-Clara (2010) assume that investors forecast dividend growth from a past average and adjust the dividend price factor to obtain better forecast power. Binsbergen et al. (2010) model expected dividend growth as a persistent AR(1) process. Bansal and Yaron (2004) model dividend growth as containing a persistent observable component that is common to consumption growth. Lettau and Ludvigson (2005) are able to forecast dividend growth from a stationary linear combination of consumption, dividends, and labor income. These studies make evident the need to depart from the assumption that expected dividend growth is known and constant. Chen (2009) documents the reversal of return and dividend growth predictability around 1950s. The prewar data suggests a constant return and time-varying dividend growth while the postwar data suggests the time-varying return and a constant

dividend growth.

The reasons we choose the debt-to-GDP ratio $(\frac{B}{Y})_t$ are two-fold: One is that the debt-to-GDP ratio can be derived from the government budget constraint and can act as a rational forward-looking factor under the non-explosive public debt policy. We can relate this factor to stock returns like what Lettau and Ludvigson (2001) develop the consumption-wealth ratio from the consumer's budget constraint. The second reason is that we identify the changes in the steady states of debt-to-GDP ratios and then relate the debt-to-GDP ratios to U.S. public debt policy. The empirical results support that the identified steady-states of debt-to-GDP can be interpreted as the time-varying debt target during each partisan regime.

We start from a simplified version of government budget constraint and show debt-to-GDP ratio can be represented as steady state debt level and short-run fiscal adjustment. $(\frac{B}{Y})_t \simeq \overline{(\frac{B}{Y})}_t + \widetilde{(\frac{B}{Y})}_t$. We use Bai and Perron (2003) and Lettau and Van Nieuwerburgh (2007)'s method to test the structural breaks in debt-to-GDP ratio and find (i) the shifts in steady states of debt-to-GDP coincide with the U.S. presidential election cycles. (ii) we adopt a method to show that our way of decomposition can well fit the debt dynamics in data and be consistent with the structural breaks interpretation. (iii) the steady states of the debt-to-GDP level $\overline{(\frac{B}{Y})}_t$ rather than the deviation term $\widetilde{(\frac{B}{Y})}_t$ explain most of the variations in expected dividend growth.

Here our research is related to the public debt policy literature at two aspects. One is that we modelling the debt-to-GDP as mean-reverting process and identify the state-dependent time-varying constant which is driven by political factors and we interpret the constant as debt target. Bohn (1998) documented that the debt-GDP ratio displays mean-reversion if one controls for war-time spending and for cyclical fluctuations. Sarno (2001) fits a smooth transition autoregressive (STAR) model for the debt-GDP ratio to capture the non-linearity in this process. In this paper, we first apply the structural break method to identify the changes in steady states of debt-to-GDP and then fit the mean-reverting process for each break. Our way can be regarded as a way to deal with the non-linearity in debt-to-GDP process. Second is that we propose an empirical framework to verify the sustainability of public debt. There is a large literature on indicators of public debt sustainability and empirical tests of fiscal solvency (Buiter (1985); Blanchard (1990); Blanchard, Chouraqui, Hagemann, and Sartor (1991); Afonso (2005); Bohn (2008); Neck and Sturm (2008); Escolano (2010); DErasmo, Mendoza, and Zhang (2016)). These surveys generally start by formulating standard concepts of government accounting, and then build around them the arguments to construct indicators of debt sustainability or tests of fiscal solvency. In this paper we propose a new approach to verify the sustainability of U.S. public debt which ensures that the debt-to-GDP can act as a rational forward looking factor. Our result is consistent with previous

research by Bohn (2008). Bohn (2008) examined the sustainability of U.S. fiscal policy and found evidence in favor of sustainability. The sustainability ensures that debt-to-GDP can be a rational forward looking factor.

To rationalize the finding we propose a production based asset pricing framework incorporating the firm's dynamic trade-off model. Methodologically, our theoretical work builds on recent papers by Livdan, Saprizza, and Zhang (2009), Croce, Kung, Nguyen, and Schmid (2012), Palazzo (2012), Croce (2014), and Croce, Nguyen, Raymond, and Schmid (2018). Following Croce et al. (2012), they study the role of fiscal policy and taxation on investment, growth and returns. We extend the framework to incorporate firm's cash holding behavior and the setting allows the model to produce testable predictions consistent with the common component argument.

Besides the trade-off between the corporate debt benefit and expected tax shield, we consider the conflict among shareholders and debt holders: the asset substitution and dividend payout. When economy uncertainty increases, firms may choose under-investment behavior, increase cash holding and payout more dividend as welfare transfer from the shareholders to the debt holders. This welfare transfer theory of conflict among shareholders and debt-holders has been reflected in papers by Jensen (1986), Leland (1998), Gilje (2016), Chen and Manso (2016), Chu (2017), DeAngelo, Gonçalves, Stulz, et al. (2017) and Ham, Kaplan, and Leary (2018). Therefore, the way we extend the model by adding cash holding (and setting dividend payout) should produce more realistic predictions.

We consider our contribution as following. First, we document that the debt-to-GDP factor captures the common component between dividend growth and stock returns. The finding is consistent with the argument made by Lettau and Ludvigson (2005). Second, we propose a production based asset pricing framework to rationalize the finding and the model generates testable predictions regarding the common component argument. Third, we combine the $(\frac{B}{Y})_t$ decomposition and structural breaks method. We propose a VAR system to verify the U.S. public debt policy is sustainable which justifies that the debt-to-GDP can be acting as a rational forward-looking factor. The variance decomposition of debt-to-GDP ratio shows that tax ratio and government spending variations account for 31.4% and 68.6% of total variations.

The rest of the paper is organized as follows. In the next section we present evidence on the predictability of both stock returns and dividend growth. We also interpolate the data back to 1950s and add the predictive comparison in Section 3. We then move to Section 4 to discuss the debt-to-GDP ratio. We first decompose the factor into two parts: time-varying mean and the deviation parts. By applying the structural break methods and assuming the mean-reverting property of debt-to-GDP process, we can interpret the state dependent

time-varying mean as debt target for each partisan regime. Then we relate the debt-to-GDP ratio to government budget constraint and propose a classic VAR system to verify the non-explosive debt behavior in U.S. to ensure the rationality of using debt-to-GDP as a forward-looking factor. For the story part in Section 5, we propose a general equilibrium framework to incorporate public debt and study how the effect is transmitted in a firm’s dynamic trade-off model. Section 6 concludes.

2. Empirical Evidence

We start from the long horizon regressions and then interpolate the data back to 1950s to test the relationship again. The predictability of dividend growth and stock returns hold for different samples and different specification. High debt-to-GDP ratio can predict high future dividend growth and high future stock returns.

2.1. Long-Horizon Forecasting Regressions

We report in Tables 1-2 the evidence from the long-horizon forecasting regression. In this paper, we consider predictive regressions for quarterly data with horizons ranging from 1 to 5 years. We consider the quarterly data for the Standard&Poor’s (*S&P*) 500 index from 1966Q1 to 2017Q4 taken from Robert Shiller’s Web site, and dividends are 12-month moving sums of dividends paid on the *S&P* 500 index. These series coincide with those used in Welch and Goyal (2008), and made available at Amit Goyal’s Web site. A full description of all data used in our empirical analysis is provided in Appendix.

Tables 1-2 report the evidence for forecasting returns and dividend growth based on the following two benchmark models:

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 \widetilde{dp}_t + \epsilon_{t,t+j}^r \quad (1)$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 \widetilde{dp}_t + \epsilon_{t,t+j}^d \quad (2)$$

$$H = 1, \dots, 5$$

Here we adjust the dividend price ratios with structural break methods applied by Lettau and Van Nieuwerburgh (2007). LVN use a century of U.S. data to show evidence on the breaks in the constant mean dp_t . As a matter of fact, the evidence from univariate tests for nonstationarity and bivariate cointegration tests does not lead to the rejection of the null of

the presence of a unit root in dp_t . LVN (2007) identify 2 statistically significant breaks in the mean of dp_t , in 1954 and 1994. They then provide evidence that deviations of dp_t , from its time-varying mean have a much stronger forecasting power for stock market returns than deviations of dp_t , from a constant mean. The evidence toward a slowly evolving mean in dp_t , has been reported as a pure statistical fact. LVN (2007) give some hints on possible causes for the breaks arising from economic fundamentals due to technological innovation, changes in expected return, etc., but do not explore the possible effects of fundamentals any further. Favero, Gozluklu, and Tamoni (2011) illustrate how the theoretical model by Geanakoplos, Magill, and Quinzii (GMQ) (2004) implies that a specific demographic variable, MY, the proportion of middle-aged to young population, explains fluctuations in the dividend yield. In figure 1 we show the adjusted dividend-price ratio and we document the consistent struck breaks at 1954Q2 and 1995Q2 based on quarterly data .

[Insert Figure 1 near here]

[Insert Table 1 near here]

Using US poster-war data, we document that the dividend price ratio can predict the stock returns but fail to predict the dividend growth which is consistent with previous research.

We introduce the debt-to-GDP factor into the regressions and the new specification can be represented as:

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r \quad (3)$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \epsilon_{t,t+j}^d \quad (4)$$

$$H = 1, \dots, 5$$

Using US data, we document the facts that high government debt-to-GDP ratio is related to high equity returns and high dividend growth. We provide the results at different specifications. All regressions are run at quarterly frequency but all dependent variables in regressions are expected annual returns and expected annual dividend growth. The evidence is summarized as follows:

i) DGDP is always significant along with dp_t in all the forecasting regressions for real stock market returns (Panel A, Table 2). The R^2 of the predictive regression at quarterly frequency increases with the horizon from 0.182 at the 1-year horizon to 0.307 at the 5-year horizon.

ii) DGDP is always significant along with dp_t in all the forecasting regressions for real dividend growth (Panel B, Table 2). The R^2 of the predictive regression at quarterly frequency increases with the horizon from 0.193 at the 1-year horizon to 0.416 at the 5-year horizon.

[Insert Table 2 near here]

2.2. Revisit dp_t Ratio

The Debt-to-GDP ratio has prediction power on both returns and dividend growth. Therefore one way to argue is that we represent the returns as function of dividend growth.

$$ret_{t+j} = f(dp_t, \Delta d_{t+j}) \quad (5)$$

Therefore we can better predict stock returns through better predicting the dividend growth.

Another potential argument is that the raw debt-to-GDP ratio captures the offsetted common component between expected returns and dividend growth. The argument that dividend yield can not capture the common component has been made by previous papers such as Lettau and Ludvigson (2005).

We start from the most initial argument made by Campbell and Shiller (1988). They decompose the dp_t ratio as

$$dp_t \approx E_t \left[\sum_{j=0}^{\infty} \rho_p^j (r_{t+1+j} - \Delta d_{t+1+j}) \right] \quad (6)$$

where $\rho_p = \frac{1}{1+\exp(d-p)}$ and r_{t+1+j} is the log return to stock market wealth. This equation says that if the log dividend-price ratio is high, agents must be expecting either high future returns on stock market wealth or low dividend growth rates. However, this expression does not predict which variables on the right-hand side should be forecastable. Many studies have documented that dp_t forecasts returns over long horizons but explains little of the variability in future dividend growth. Other studies find that the forecasting power of single dp_t for future excess returns over shorter horizons is statistically weak.

Lettau and Ludvigson (2005) argue that the positively correlated fluctuations in expected dividend growth and expected returns have offsetting effects on the log dividend-price ratio. They argue that the dp_t ratio fails to capture the common variations among the expected

returns and the dividend growth while both *cay* and *cdy* can capture this common component.

Following the argument above, we can say that fluctuations in expected returns and expected dividend growth have common component which offset the effects on dp_t ratio. Therefore a single dp_t factor cannot predict the stock returns. Based on the empirical evidence of predictability on dividend growth and stock returns, we can argue that debt-to-GDP ratio can predict the embedded common component.

We argue that the losing explanation power of dp_t is due to the common component of r_{t+j} and Δd_{t+j} . Take a simple example:

$$r_{t+j} = x_{r,t+j} + \eta_{t+j} \quad (7)$$

$$\Delta d_{t+j} = x_{d,t+j} + \eta_{t+j} \quad (8)$$

where η_{t+j} is the common component. Then

$$r_{t+j} - \Delta d_{t+j} = x_{r,t+j} - x_{d,t+j} \quad (9)$$

Theoretically we can recover the missing component η_{t+j} by controlling a factor which can predict both r_{t+j} and Δd_{t+j} . Based on the empirical evidence, we can argue that debt-to-GDP ratio can predict the embedded common component and therefore we can control it in return regressions.

Joint Hypothesis Test: To test the prediction power on returns is transmitted through the prediction power on dividend growth and the debt-to-GDP ratio captures the common component, we run the following GMM estimation and run the hypothesis test.

$$\begin{aligned} \sum_{j=1}^H \Delta d_{t+j} &= \gamma_{0,H} + \gamma_{1,H} DGDP_t + \epsilon_{t,t+j}^d \\ \sum_{j=1}^H r_{t+j} &= \beta_{0,H} + \beta_{1,H} \left(\sum_{j=1}^H \Delta d_{t+j} \right) + \beta_{2,H} \widetilde{dp}_t + \epsilon_{t,t+j}^r \end{aligned} \quad (10)$$

$$H = 1, \dots, 5$$

The results show the fitted dividend growth is significant in explaining the variations of stock returns. The factor loading $\beta_{1,H}$ is close to 1 and in longer horizon regressions it approaches 1. The dividend growth is instrumented by the debt-to-GDP ratio and the explanation power increases with horizons. We run the joint test whether the factor loadings of dividend growth on corresponding stock returns are equal to one which is $\beta_{1,1} = \beta_{1,2} =$

$\beta_{1,3} = \beta_{1,4} = \beta_{1,5} = 1$. The value is 0.3614 and we cannot reject the null hypothesis that the debt-to-GDP ratio can capture the common component among stock returns and dividend growth.

[Insert Table 3 near here]

3. Debt-to-GDP Ratio and CAY

3.1. Debt-to-GDP Ratio and CAY

In this section, we include further evidence controlling the CAY factor. Lettau and Ludvigson (2001), (2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long-run equilibrium value is defined by Lettau and Ludvigson (2001) alternatively as a linear combination of labor income and financial wealth, CAY. CAY is much less persistent time series than dp_t , and it is predictor of both stock returns and dividend growth, and when included in a predictive regression relating stock market returns to dp_t , CAY swamp the significance of this variable.

The new specification with CAY is as following

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \beta_3 CAY_t + \epsilon_{t,t+j}^r \quad (11)$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \gamma_3 CAY_t + \epsilon_{t,t+j}^d \quad (12)$$

$$H = 1, \dots, 5$$

[Insert Table 4 near here]

Evaluating the effect of the inclusion of CAY in the long-run forecasting regressions that also include DGDP, is important for a number of reasons. First, it is a parsimonious way of evaluating the model with DGDP, against all financial ratios traditionally adopted to predict returns. In fact, Lettau and Ludvigson (2001), (2005) show superior performance in predicting long-run returns of CAY, with respect to all the traditionally adopted financial ratios, such as the detrended short-term interest rate (Campbell (1991), Hodrick (1992)),

the log dividend-earnings ratio and the log price-earnings ratio (Lamont (1998)), the spread of long-term bond yield (10Y) over 3M T-bill, and the spread between the BAA and the AAA corporate bond rates. Second, it would allow further investigation on the presence of a common component in dividend and stock market returns suggested by Lettau and Ludvigson (2005) but not consistent with our findings in Table 4, that witness the significance of DGDP, for predicting long-run returns and long-run returns adjusted for dividend growth. Third, it could shed further light on the relative importance of CAY to DGDP, for predicting returns and dividend growth in the dynamic dividend growth model.

We found that CAY and DGDP are both significant in long-run stock return regressions but CAY lose significance in the long-run dividend growth. The evidence supports that DGDP have stronger predictive power and we relate this to the presence of a common component in dividend growth and stock market returns.

3.2. Empirical Evidence back to 1950s

Since the quarterly data on total public debt to GDP ratio has been published since 1966Q1, to obtain the post-war results and to be consistent with observations of CAY(available after 1950s) we interpolate the debt data from the annual observations by cubic spline interpolation. Both the raw and interpolated data has been plotted in figure 6 We document the interpolated data can match the actual data and we find the predictability on returns and dividend growth.

The evidence is similar to what we obtained from the actual quarterly data and is summarized as follows:

i) DGDP is always significant along with dp_t in all the forecasting regressions for real stock market returns (Panel A, Table 5). The R^2 of the predictive regression at quarterly frequency increases with the horizon from 0.186 at the 1-year horizon to 0.380 at the 5-year horizon.

ii) DGDP is always significant along with dp_t in all the forecasting regressions for real dividend growth (Panel B, Table 5). The R^2 of the predictive regression at quarterly frequency increases with the horizon from 0.146 at the 1-year horizon to 0.295 at the 5-year horizon.

[Insert Table 5 near here]

3.3. Prediction Comparison

Following Welch and Goyal (2007)'s way to construct the OOS statistics, we first evaluate the predictive performance at quarterly frequency and annual frequency. In this section we analyze the performance of several specifications from the perspective of a real-time investor. We therefore consider out-of-sample evidence for the 1-, 3- and 5-year horizons.

Two indicators we adopt here are OOS and MSE_F . Both factors help us to compare the predictive performance.

For OOS R^2 , we construct as following:

$$R_{OOS}^2 = 1 - \frac{MSE_A}{MSE_N}$$

For MSE_F statistics, we adopt it to evaluate the equal MSE (see McCracken's F statistics):

$$MSE_F = (T - h + 1) * \frac{MSE_N - MSE_A}{MSE_A}$$

where T is observation and h is the overlapping period.

We run recursive forecasting regressions for the 1-, 3- and 5-year horizons. Table 6 report the OOS statistics as quarterly frequency and annual frequency. For quarterly sample, the whole sample is from 1950Q1 to 2017Q4.

The evidence is as follows: the two factor model(DGDP+ dp_t) beats the *CAY* model for most cases and the three factor model beats the rest for all cases at quarterly frequency. At annual frequency, we only present the results of two factor model compared to single dp_t model because the consumption to wealth ratio, *CAY* is unavailable before the year 1952.

[Insert Table 6 near here]

[Insert Figure 2 near here]

4. Debt-to-GDP Ratio and Public Debt Rule

In this section, we derive the debt-to-GDP ratio from the government budget constraint and relating it to the public debt policy. Then we verify the U.S. debt policy is sustainable which ensures that the debt-to-GDP can act as a forward-looking factor. By variance decomposition, we show the tax uncertainty accounts for 31.36% of the variations and government spending accounts for 68.56% of variations of the debt-to-GDP.

4.1. Public Debt Policy

We make a proposition that the debt-to-GDP follows a mean-reverting process during each partisan cycle. We show the debt-to-GDP can be represented as two parts, the steady states mean values (level factor) and the deviation part (residual factor) from the steady state means. To identify the two terms, we adopt the structural break methods and found the break points coincide with U.S. partisan cycle. Then the evidence supports our proposition that during each partisan cycle, the debt-to-GDP follows mean-reverting process. Related to public debt policy, it is natural to assume the debt-to-GDP process is mean-reverting and the non-linearity has also been documented. To deal with the non-linearity property in the debt process, we take one step further to assume that there exist structural breaks in the whole process and the mean value of debt-to-GDP is time varying and state dependent while the mean-reverting property holds only in each regime. Empirical evidence shows that the U.S. debt policy is sustainable.

The interesting things we found are that the predictability of dividend growth mainly comes from the changes in the steady states means of debt-to-GDP. It reveals that the level factor matters and firms adjust their dividend payments based on their expected debt-to-GDP in certain periods.

Suppose the government follows a mean-reverting public debt policy and the mean of debt-to-GDP ratio is state dependent.

$$\left(\frac{B}{Y}\right)_{t+1} = (1 - \rho)\left(\frac{B}{Y}\right)_{t,s}^{Mean} + \rho\left(\frac{B}{Y}\right)_t + \epsilon_{t+1} \quad (13)$$

where the mean of debt-to-GDP, $\left(\frac{B}{Y}\right)_{t,s}^{Mean}$, is time varying and depends on the state variable s (e.g. presidential cycles). Therefore we have the steady states mean should be equal to the debt target in each regime.

$$E_t\left[\left(\frac{B}{Y}\right)_T\right] = \left(\frac{B}{Y}\right)_{t,s}^{Mean} = \left(\frac{B}{Y}\right)_{ss} \quad (14)$$

Lettau and Van Nieuwerburgh (2007) first found shifts in the steady states of financial ratios and proposed a method to adjust those financial ratios. To disentangle the time-varying steady states debt-to-GDP, we adopt a similar method to disentangle the steady states of debt-to-GDP ratio by using Bai and Perron (2003) method.

We use Bai and Perron(2003)'s method to test the structural breaks in debt-to-GDP ratio and take the mean value of each break period as the steady state values as Lettau and Van Nieuwerburgh(2007) interpreted. The debt-to-GDP can be represented as the following

form:

$$\left(\frac{B}{Y}\right)_t \simeq \left(\frac{B}{Y}\right)_{ss} + \widetilde{\left(\frac{B}{Y}\right)}_t; \quad (15)$$

In Lettau and Van Nieuwerburgh(2007)'s paper, they adjust the dividend yield and find that the deviation from the steady state dividend yield can explain both the adjusted stock returns and expected dividend growth. However, the same pattern does not hold here. We find the deviation from the steady state of debt-to-GDP ratio(the residual factor) can not predict the expected returns but the steady states of debt-to-GDP(the level factor) can explain most of the variations in dividend growth.

We use the quarterly data (1966:Q1 to 2017Q4) of total federal debt and test the multiple structural breaks in $\frac{B}{Y}$ by Bai and Perron(2003)'s method. We find the breaks at 1981Q4, 1994Q1, 2001Q1 and 2008Q1. The breaks periods are consistent with U.S. presidential election cycles. We find the shifts in steady states of debt-to-GDP is more corresponding to the political cycles in U.S.

[Insert Figure 3 near here]

We use the documented steady states level to explain the expected stock returns and expected dividend growth and find the steady states rather than the deviation part can explain most of the variations. Results in Table 7 are actually similar to the empirical evidence in first section. The only difference is that we replace the raw debt-to-GDP level with our new steady states debt-to-GDP level, $\left(\frac{B}{Y}\right)_{ss}$.

We present the new specifications here

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_{ss,t} + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r \quad (16)$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_{ss,t} + \gamma_2 \widetilde{dp}_t + \epsilon_{t,t+j}^d \quad (17)$$

$$H = 1, \dots, 5$$

The evidence is summarized as follows:

i) DGDP is always significant along with dp_t in all the forecasting regressions for real stock market returns (Panel A, Table 6). The R^2 of the predictive regression at quarterly frequency increases with the horizon from 0.199 at the 1-year horizon to 0.526 at the 5-year horizon.

ii) DGDP is always significant along with dp_t in all the forecasting regressions for real dividend growth (Panel B, Table 6). The R^2 of the predictive regression at quarterly fre-

quency increases with the horizon from 0.207 at the 1-year horizon to 0.568 at the 5-year horizon.

The interesting thing we found is that the changes in debt-to-GDP steady states $(\frac{B}{Y})_{ss}$ can explain most of the variations in both stock returns and dividend growth.

[Insert Table 7 near here]

4.2. Variance Decomposition of debt-to-GDP

To adopt debt-to-GDP as a forward-looking factor, we need to ensure the public debt policy is sustainable. That is, whether the outstanding stock of public debt matches the projected present discounted value of the primary fiscal balance, measuring both at the general government level and including all forms of fiscal revenue as well as all current expenditures, transfers and entitlement payments. There is a large literature on indicators of public debt sustainability and empirical tests of fiscal solvency (Buiter (1985); Blanchard (1990); Blanchard et al. (1991); Afonso (2005); Bohn (2008); Neck and Sturm (2008); Escolano (2010); DErasmo et al. (2016)). These surveys generally start by formulating standard concepts of government accounting, and then build around them the arguments to construct indicators of debt sustainability or tests of fiscal solvency. In this section we propose a new approach to verify the sustainability of U.S. public debt which can ensure that the debt-to-GDP can act as a rational forward looking factor. Meantime the framework provides us the variance decomposition of the debt-to-GDP factor.

We first derive the debt-to-GDP from the government inter-temporal budget constraint and iterating it forward as following. See proof in appendix.

$$\left(\frac{B}{Y}\right)_t \simeq E_t\left[\left(\frac{B}{Y}\right)_T\right] + \sum_{j=1}^{T-t} E_t[tax_{t+j} - g_{t+j}]; \quad (18)$$

With $\left(\frac{B}{Y}\right)_t = \left(\frac{B}{Y}\right)_{ss} + \widetilde{\left(\frac{B}{Y}\right)}_t$ and $E_t(\widetilde{\left(\frac{B}{Y}\right)}_t) = 0$, we have

$$\widetilde{\left(\frac{B}{Y}\right)}_t = \left(\frac{B}{Y}\right)_t - E_t\left(\left(\frac{B}{Y}\right)_t\right) \simeq \sum_{j=1}^{T-t} E_t[\widetilde{tax}_{t+j} - \widetilde{g}_{t+j}]; \quad (19)$$

There for we propose the following VAR system to study the variance decomposition of debt-to-GDP ratio. We found that the tax uncertainty accounts for 31.36% of the variations and government spending uncertainty accounts for 68.56% of variations of the debt-to-GDP. Meanwhile in long run these two uncertainty adds up around 1 which proves the sustainability

of U.S. public debt. Those three equations are as

$$\sum_{j=1}^H \widetilde{tax}_{t+j} = \beta_{tax}^H \left(\frac{\widetilde{B}}{\widetilde{Y}} \right)_t + \epsilon_{t,t+H}^{tax} \quad (20)$$

$$\sum_{j=1}^H \widetilde{g}_{t+j} = \beta_g^H \left(\frac{\widetilde{B}}{\widetilde{Y}} \right)_t + \epsilon_{t,t+H}^g \quad (21)$$

$$\left(\frac{\widetilde{B}}{\widetilde{Y}} \right)_{t+1} = \beta_{by} \left(\frac{\widetilde{B}}{\widetilde{Y}} \right)_t + \epsilon_{t,t+1}^{by} \quad (22)$$

For this method holds, in short run we should have the relationship as

$$\beta_{tax} - \beta_g + \beta_{by} \approx 1, \quad \epsilon_{t+1}^{tax} - \epsilon_{t+1}^g + \epsilon_{t+1}^{by} \approx 0 \quad (23)$$

in long run we should have the relationship as

$$\beta_{tax}^\infty - \beta_g^\infty \approx 1, \quad (24)$$

Therefore to verify the sustainability, we need to ensure both the short-run and long-run conditions hold.

i) In short run, we compare the coefficients of β_{tax}^H and β_g^H from three methods: OLS, OLS Implied and GMM. The results show those coefficients are matched in short-run (within eight quarters).

[Insert Figure 4 near here]

ii) In long run, the relationship $\beta_{tax}^\infty - \beta_g^\infty \approx 1$ holds because it's analogy to variance decomposition in long run. We have the result as

$$\beta_{tax}^\infty = 0.3136, \quad \beta_g^\infty = -0.6856, \quad \beta_{tax}^\infty - \beta_g^\infty = 0.9992 \approx 1$$

If we replace $\frac{\widetilde{B}}{\widetilde{Y}}$ with the debt-to-GDP held by public, the results are quite similar

$$\beta_{tax}^\infty = 0.4627, \quad \beta_g^\infty = -0.6263, \quad \beta_{tax}^\infty - \beta_g^\infty = 1.0890 \approx 1$$

The satisfied short and long run conditions ensure that the government budget constraint holds which implies the sustainability of public debt policy.

Therefore we start from a proposition that debt policy should be mean-reverting during each presidential cycle and the empirical evidence supports the U.S. public debt policy is

non-explosive. By verifying this framework, we manage to show the debt-to-GDP can act as forward looking factor and find the tax and spending uncertainty accounts for 31.36% and 68.56% respectively.

5. A General Equilibrium Model

To rationalize the documented empirical evidence, we build a general equilibrium model to incorporate both public debt and dividend payout policy to study how the effect is transmitted to dividend growth.

For general equilibrium model, Croce (2014) provide the framework for production-based asset pricing and Croce et al. (2012) introduce the public debt into the framework. By introducing the tax channel, the fiscal policy's effect on stock returns can be analyzed and reasonable risk premia can be reproduced from the model. Another paper to emphasize is the work did by Croce et al. (2018), they highlight a novel and distinct mechanism shaping the link between public debt and future growth, namely a risk channel. They identify innovations to government indebtedness as a risk factor priced in both the cross section and the time series of stock returns. By affecting their cost of capital, movements in government debt impact firms investment and innovation decisions. Empirically, they test these links on the entire cross-section of US stock returns, and interpret and quantify them through the lens of a production-based asset pricing model with endogenous innovation and growth.

For corporate financing and dividend payout policy framework, we refer the following several papers Lintner (1956), Easterbrook (1984), Andres, Doumet, Fernau, and Theissen (2015), Chen and Manso (2016) and Chen, Karabarbounis, and Neiman (2017). As Easterbrook (1984) augured, any dividend policy (or any other corporate policy) should be designed to minimize the sum of capital, agency, and taxation costs. Besides the trade-off between the corporate debt benefit and expected tax shield, we consider the conflict among shareholders and debt holders: the asset substitution and dividend payout. When economy uncertainty increases, firms may choose under-investment, increase cash holding and payout more dividend as welfare transfer from the shareholders to the debt holders. Fan and Sundaresan(2000) model the endogenous cash holding and optimal dividend policy where cash flow-based bond covenants lead to more conservative payout policies. We cover all these three effect in our model. (i) introducing the corporate debt shielding for taxation; (ii) endogenizing the equity returns; (iii) introducing the dividend payout function(or the cash holding channel).

5.1. *The Story Behind*

We adopt the firm's dynamic trade-off setting and explain the transmission channel through the lens of capital structure and firm investment. The financing behavior (corporate bond issuance) and cash holding behavior are considered and related to firm's investment which bring us more reasonable setting.

As Leland (1998) argue, the joint determination of capital structure and investment risk should be examined. Optimal capital structure reflects both the tax advantages of debt less default costs (Modigliani-Miller), and the agency costs resulting from asset substitution (Jensen-Meckling). Modigliani and Miller argue that the optimal amount of debt balances the tax deductions provided by interest payments against the external costs of potential default. This feature is captured in Croce et al. (2012) where the tax-based risk channel is first investigated. What we introduce here is the second feature, the shareholder-creditor conflict feature.

The shareholder-creditor conflict feature can be represented in several ways. One is the "asset substitution". Jensen and Meckling(1976) argue that equity-holders of a levered firm can potentially extract value from debt-holders by increasing investment risk after debt is in place: the "asset substitution" problem. However Gilje (2016) finds that exacerbated shareholder-creditor conflict in financial distress causes firms to take less risk, a finding inconsistent with the theory. The second way is through the dividend payout. The conflict of interest between shareholders and creditors can induce agency costs in the form of excessive dividend payments, claim dilution, asset substitution, and underinvestment (Jensen and Meckling 1976; Smith and Warner 1979). Excessive dividend payments, in particular, may lead to significant wealth transfers from creditors to shareholders. The same may apply to excessive dividend payment as well. Chu (2017) provide direct evidence that the shareholder-creditor conflict affects payout policy. Chu (2017) documents that firms pay out less when there is less conflict between shareholders and creditors, suggesting that the shareholder-creditor conflict induces firms to pay out more at the expense of creditors.

For the asset substitution, Jensen and Meckling(1976) argue equity holders benefit from successful outcomes of high-risk projects, while losses from unsuccessful outcomes are borne by debt holders. This asymmetry between who receives the gains and losses from a project could make it optimal for equity holders to maximize the amount of risk a firm undertakes when leverage is high. Gilje (2016) documents the contradicting fact that firms reduce risk-taking as they approach distress. He argues that firms could have risk-mitigating incentives that outweigh risk-shifting incentives. For example, firms could have incentives to ensure that they have a good reputation to facilitate access to debt markets (Diamond 1989), which can affect their ability to pursue future positive NPV projects (Almeida, Campello, and

Weisbach 2009) and firms with shorter maturity debt and bank debt tend to have more risk-reducing incentives.

For the dividend payout, a growing number of studies have found that the level of firm leverage (the usual indicator of agency conflict) negatively affects dividend policy (Jensen (1986); Jensen, Solberg, and Zorn (1992); Agrawal and Jayaraman (1994); Crutchley and Hansen (1989); DeAngelo and DeAngelo (2007); Byoun (2008); Frank and Goyal (2009); Byoun (2011)). Their studies inferred that highly levered firms look forward to maintaining their internal cash flow to fulfill duties, instead of distributing available cash to shareholders and protect their creditors while mature firms reserve moderate leverage and limit agency costs on free cash flows through large dividend payout. An increase in dividends could be caused either by an increase in the firm's profits (implying higher stock prices) or by the commencement of disinvestment as the firm has fewer profitable opportunities (implying lower stock prices). Following Easterbrook (1984)'s argument there are two potential channel that affect the firms' dividend payment. (i) Shareholders can change the risk of the firm by altering its mix of projects. Creditors recognize this and try to control it by adjusting the rate of interest they demand. Debt-holders assume that given the limits set by their contracts, shareholders prefer to take the maximum advantage. (ii) Shareholders can also change the risk of the firm by altering its debt-equity ratio. The lower the ratio of debt to equity, the lower the chance of bankruptcy of the firm. Once again, debt holders consider this in deciding what rate of interest to demand. Once again, given the existence of debt, shareholders can control the amount of risk.

We model the shareholders risk-taking behavior by picking the dividend and cash holding policy. If managers first issue debt and then finance new projects out of retained earnings, the debt-equity ratio will fall. The lower it falls, the lower the shareholders' risk and the greater the boon bestowed on the debt holders, who receive their contracted-for interest but escape the contracted-for risk. Financing projects out of retained earnings if unanticipated by bondholders transfers wealth from shareholders to debt-holders. Just as bondholders want to limit dividends, to prevent advantage-taking by shareholders once a rate of interest has been set, so shareholders want to increase dividends to the extent possible in order to avoid being taken advantage of by bondholders.

Here we model the cash holding behavior of firms because it is the precautionary saving behavior of firms and there should be a risk channel. Intuition suggests that they may have an incentive not to pay out all the available cash flows to reduce the chance that the cash flow constraint becomes binding. Ham et al. (2018) argue that theories based on the agency costs of free cash flow receive the majority of support in recent review articles (Allen and Michaely (2003); DeAngelo, DeAngelo, Skinner, et al. (2009); Kalay and Lemmon (2011)).

Asymmetric information models provide a potential reconciliation between their findings and governance-based theories (Fudenberg and Tirole (1995); DeMarzo and Sannikov (2016)). In these models, managers map earnings information into dividends to maximize the probability of retaining control. If directors and outside investors "manage by exception", in that their degree of involvement is a decreasing function of payout, with decreases leading to asymmetrically more supervision (Hilton and Platt (2011)), managers have an incentive to increase dividends when they foresee a sustainable increase in earnings. However, they will do so conservatively to minimize the probability of having to decrease dividends in case of an unexpected earnings shortfall. A related research by DeAngelo et al. (2017) studies the firms' deleveraging process from the retaining financing flexibility point-view. While debt repayment is the most important direct contributor to deleveraging, its deleveraging impact is not fully independent of the new equity capital that firms obtain through earnings retention and share-issuance proceeds. Moreover, they also documented that many sample firms effectively trade off rebuilding flexibility through leverage reductions and cash-balance build ups in order to deliver increasing payouts to shareholders. The fact they documented indicating that traditional trade-off models of capital structure need to move beyond financial distress costs as a motive to limit leverage and also include a non-distress-related motive for cash-balance accumulation. The most obvious such motive is to acquire flexibility, with firms accumulating cash to meet possible future funding needs. However, agency problems could also lead to the choice of financial policies with ample cash holdings coupled with low leverage. All managers had to do was hold dividends constant and use the incremental retention to pay down debt and/or build up cash balances. Their decisions reveal that they willingly accept muted deleveraging in order to deliver larger payouts to shareholders. In other words, firms often treat payout considerations as important in their own right, and not as dominated by the benefits of rebuilding flexibility by deleveraging and accumulating larger cash balances. This emphasis on payouts may still reflect financial-flexibility considerations since, as DeAngelo and DeAngelo (2007) note, a demonstrated commitment to making equity payouts should help provide a firm with reliable future access to equity capital. The financing flexibility can be described as a precautionary saving behavior.

To relate the government debt to firm dynamic behavior, we focus on how firms' investment and cash holding behavior would react to future uncertainty. Increase in government leverage raise the uncertainty concerns in future and firms would react to the increased uncertainty by adjusting firms' investment and financing behavior. An increasing body of literature investigates the impact of uncertainty on corporate financing decisions. For example, Chen, Wang, and Zhou (2014) find that stock return volatility significantly predicts active leverage adjustment, and falling earning growth appears to be the channel through

which increasing volatility predicts leverage reduction and investment contraction. Chen and Manso (2016) show that investment and capital structure decisions, as well as debt overhang costs depend on the cyclical nature of cash flows from assets-in-place and growth opportunities. More cyclical cash flows from assets-in-place make underinvestment more likely in bad times, exacerbating the costs of debt overhang when macroeconomic risk is taken into account. Ha, Im, and Shon (2017) find that uncertainty is likely to increase potential financial distress costs and exacerbate shareholder-debtholder conflicts, thereby leading to a lower optimal leverage ratio. High uncertainty makes both assets in place and investment projects riskier, followed by more severe under-investment and asset substitution problems. To resolve the shareholder-debt-holder agency conflicts, a firm is likely to choose a lower optimal/target leverage ratio which implies that uncertainty lowers leverage targets more significantly when agency conflicts between shareholders and debt-holders are higher. They show that firms with higher uncertainty tend to invest more cautiously given investment opportunities, and they use less debt than the firms with lower uncertainty. Chen and Manso (2016) study the debt-overhang problem and provide the intuition for how macroeconomic risks and agency problems interact. One way is that in recessions marginal utilities are higher and the distortions caused by agency problems will affect investors more than in booms. The other way is that credit spreads tend to rise significantly in aggregate bad times. Thus, for a given investment opportunity, the transfers from equity holders to debt holders in a typical pro-cyclical firm will tend to concentrate in bad times. These two effects both raise the ex ante costs of debt overhang and cause larger distortions to investment.

Another way to read the government debt is through the investors' portfolio allocation viewpoint. Taggart Jr (1981) points out that aggregate firm leverage is determined by the interaction of the supply of securities by firms and demands for those securities by investors. Graham, Leary, and Roberts (2014) show that U.S. federal government debt issuance significantly affects corporate financial policies and balance sheets. Government debt is strongly negatively correlated with corporate debt and investment, but strongly positively correlated with corporate liquidity. Their results suggest that large, financially healthy corporations act as liquidity providers by supplying relatively safe securities to investors when alternatives are in short supply, and that this financial strategy influences firms' capital structures and investment policies. The results are consistent with a recent research by Demirci, Huang, and Sialm (2018) and they document a negative relationship between government debt and corporate leverage. They document that the government debt level in the U.S. has increased from around 50% to more than 100% of GDP between 1990 and 2014 while the book leverage of U.S. firms has declined from 33% to 28%. Graham, Leary, and Roberts (2015) also document that when the government reduces debt issuance, corporations increase their use

of debt relative to equity, resulting in an increase in corporate leverage.

High debt-to-GDP implies higher financing costs and high uncertainty for firms which implies higher probability of distress where firms face higher agency conflicts and pay higher dividends. To sum, we will solve the agency-conflict problem in a dynamic trade-off model (the benefits trade-off between debt-holders and shareholders) through the investment channel and cash holding channel. We proceed the paper in following (i) First we relates the dividend payout to the corporate financing and investment behavior in a dynamic trade-off model. (ii) Second we propose a general equilibrium framework incorporating both public debt and firm behavior to study how the effect is transmitted.

5.2. Model

The economy is composed of three sectors: the production sector, the household sector and the government sector. We propose the model in this section and provide model-based evidence in the following section.

Production Sector The final consumption good is produced in a competitive sector. There is a representative firm that uses capital K_t , labor L_t to produce the final goods according to production technology

$$Y_t = (K_{t-1})^\alpha (\Omega_t N_t)^{1-\alpha} \quad (25)$$

where α is the physical capital share, let Ω_t denotes the level of productivity at time t and lowercase letters denote log-units. The decomposition of the productivity growth rate is specified as that $\Omega_t = e^{a_t}$, $\Delta a_t = \mu + z_{t-1} + \epsilon_{a,t}$, with $\epsilon_{a,t} \sim N(0, \sigma_a^2)$, and $z_t = \rho z_{t-1} + \epsilon_{z,t}$, with $\epsilon_{z,t} \sim N(0, \sigma_z^2)$, where z_t refers to the long-run risk (LRR) component in productivity growth, and $\epsilon_{a,t}$ is short-run growth risk (SRR).

The firm's objective is to maximize the shareholder's value. This can be formally stated as

$$\max_{\{D_t, I_t, N_t, K_t, CH_t, B_t^C\}} E_0 \left[\sum_{t=1}^{\infty} M_t D_t \right] \quad (26)$$

where

$$\begin{aligned} CF_t &\leq Y_t - W_t N_t - T_t - I_t + B_t^C - R_{b,t-1} B_{t-1}^C - C_t^D - C_t^E + R_{l,t-1} CH_{t-1} \\ D_t &= \beta_d CF_t \\ CH_t &= (1 - \beta_d) CF_t \\ K_t &\leq (1 - \delta) K_{t-1} + \Lambda \left(\frac{I_t}{K_{t-1}} \right) K_{t-1} \end{aligned} \quad (27)$$

where D_t are the firm's dividends, M_t is the stochastic discount factor, I_t is investment in physical capital, W_t is the wage rate, $\Lambda(I_t)$ is the convex adjustment cost function, B^C is corporate bond, CH_t is cash holding and C_t^D is the financing distress costs associated with corporate bonds. δ is the depreciation rate of physical capital and $\Lambda(\cdot)$ is the capital adjustment cost function. We specify $\Lambda(\cdot)$ as in Jermann(1998), $\Lambda(\frac{I_t}{K_{t-1}}) = \frac{b}{1-a_1}(\frac{I_t}{K_{t-1}})^\zeta + c$, where $\frac{1}{1-\zeta}$ represents the elasticity of the investment rate with respect to Tobin's Q. The parameters b and c are set so that there are no adjustment costs in the deterministic steady states. Following Hennessy, Levy, and Whited (2007) and Livdan, Saprizza, and Zhang (2009), we assume the saving rate is smaller than the borrowing rate so that firms are not indifferent between savings and cash distributions. $R_{l,t}$ is return of cash holding where $R_{l,t} = R_{b,t} - b$ where b captures the dis-utility of cash holding. We model the cash retention ratio(or the dividend payout ratio) is proportional to cash flow which is in support of precautionary savings motives at the firm level being positively driven by expected equity returns(Palazzo (2012)).

The total distress costs associated with debt can be represented as

$$\frac{C_t^D}{\Omega_{t-1}} = \phi_0 e^{-\phi_1(\frac{\theta K_t}{B_t^C} - 1)} \quad (28)$$

where θ, ϕ_0 and ϕ_1 are positive constants and the maximum corporate bonds B_t^C can be issued should be up-bounded by θK_t .

We introduce the costly counter-cyclical agency costs as equation (29). The setting allows the costly debt issuance in bad times than good times.

$$\frac{C_t^E}{\Omega_{t-1}} = \lambda_1 \left(\frac{B^C}{Y} - \frac{B^C}{Y_{ss}} \right)^2 \quad (29)$$

The cash holding costs for each period is

$$C_t^{CH} = bCH_t = b(1 - \beta_d)CF_t \quad (30)$$

Here we do not model the firm leverage dynamics directly as Croce et al. (2012). Based on the empirical evidence that the debt-to-GDP ratio is negatively correlated to the corporate leverage, we adopt the way to model dividend process and check the corporate leverage process after that. There are numerous dividend signaling models predict that dividend changes convey information about cash flows; i.e., a dividend increase (decrease) conveys favorable (unfavorable) information about the current and/or future cash flows of the firm (Bhattacharya (1979), John and Williams (1985), Miller and Rock (1985)). Empirical evidence

on earnings behavior following dividend changes provides support for this hypothesis (Denis, Denis, and Sarin (1994)). Here we take the dividend adjustment model as the signaling model.

Turning to capital structure, we should have the following

$$\frac{\partial(C_t^D + C_t^E + C_t^{CH})}{\partial B_t} = E_t[M_{t+1}\tau_{t+1}]r_{f,t} \quad (31)$$

The left-hand side of the equation is related to marginal costs from adding one additional unit of corporate debt and the right-hand refers to the marginal benefit from debt (from tax advantage of debt) by adding one additional unit of debt.

Household Sector The representative agent has the Epstein and Zin (1989) preferences defined over the consumption bundle \tilde{C}_t :

$$U_t = [(1 - \delta)\tilde{C}_t^{1-\frac{1}{\Psi}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\Psi}}{1-\gamma}}]^{\frac{1}{1-\frac{1}{\Psi}}} \quad (32)$$

The parameters Ψ and γ measure the IES and the RRA of the agent, respectively. The consumption bundle aggregates consumption, C_t , and leisure, L_t , as follows:

$$\tilde{C}_t = C_t^o \cdot (\Omega_{t-1}L_t)^{1-o} \quad (33)$$

where Ω_{t-1} denotes aggregate productivity and o is a weight determining the average share of hours worked. Multiplying leisure by productivity guarantees balanced growth, and it is interpreted as an adjustment for the standards of living.

The consumer's budget constraint is

$$C_t + Z_t Q_t + B_t + B_t^C \leq R_{b,t-1}(B_{t-1} + B_{t-1}^C) + Z_{t-1}(D_t + Q_t) + W_t N_t + G_t \quad (34)$$

where Z_t is number of equity shares, Q_t is the ex-dividend price of stocks, B_t is number of government bonds and B_t^C is number of corporate bonds.

The stochastic discount factor takes the following usual form:

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{1-\frac{1}{\Psi}} \left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\Psi}-\gamma} \quad (35)$$

The risk free rate is $R_{b,t} = E_t[M_{t+1}]^{-1}$.

The optimality condition for the labor implies that the marginal rate of substitution

between consumption and leisure should be equal the marginal product of labor:

$$1 \geq N_t + L_t$$

$$\frac{\partial \tilde{C}_t}{\partial L} / \frac{\partial \tilde{C}_t}{\partial C_t} = (1 - \alpha) \frac{Y_t}{N_t}$$

Government Sector In the model government debt enhances the liquidity of households by providing an additional means of smoothing consumption (in addition to claims to capital) and by effectively loosening borrowing constraints. With distorting taxes there is a role for government debt as a means of smoothing tax distortions over time. Optimal debt policy in such a model (see Barro, 1979; Chamley, 1985; Chamley, 1986) generally implies that the steady-state level of debt depends on the initial level of debt.

Here the budget constraint follows:

$$B_t = R_{b,t-1}B_{t-1} + G_t - T_t \quad (36)$$

Where Government spending and Taxation are represented as G_t and T_t . $R_{b,t-1}$ is the one period bond return and B_t is total public debt at time t .

Here we assume the government expenditure is exogenous stochastic process.

$$\begin{aligned} \frac{G_t}{Y_t} &= \frac{1}{1 + e^{-g_t}} \\ g_t &= (1 - \rho_g)\bar{g} + \rho_g g_t + \epsilon_{g,t} \end{aligned} \quad (37)$$

Here we assume the average tax rate is endogenously determined by the government inter-temporal budget constraint.

$$\begin{aligned} \frac{T_t}{Y_t} &= \frac{\tau_t * \text{taxbase}_t}{Y_t} \\ \text{taxbase}_t &= (Y_t - W_t N_t - r_{b,t-1} B_{t-1}^C) \end{aligned} \quad (38)$$

Suppose the public debt policy follows mean-reverting process.

$$\left(\frac{B}{Y}\right)_{t+1} = (1 - \rho_{\frac{B}{Y}})\left(\frac{\bar{B}}{Y}\right) + \rho_{\frac{B}{Y}}\left(\frac{B}{Y}\right)_t + \epsilon_{\frac{B}{Y},t} \quad (39)$$

For example, the rule can be aimed at stabilizing long-horizon consumption dynamics measured by $E_t(\Delta c_{t+1})$, which is $\epsilon_{\frac{B}{Y}} = \psi(\mu_c - E_t(\Delta c_{t+1}))$ proposed by Croce et al. (2012)

5.2.1. *Optimal Investment and Financing Decisions*

We have the market clearing condition as

$$Y_t = C_t + I_t + C_t^D + C_t^E + C_t^{CH} \quad (40)$$

The optimal investment and financing decision satisfies the following Euler equation:

$$q_t = E_t[M_{t+1}(\beta_d((1 - \tau_{t+1})\frac{\partial Y_{t+1}}{\partial K_t} - \frac{\partial C_{t+1}^E}{\partial K_t}) + q_{t+1}(1 - \delta - \frac{\phi'_{t+1}I_{t+1}}{K_{t+1}} + \phi_{t+1}))] - \beta_d \frac{\partial C_{t+1}^D}{\partial K_t} \quad (41)$$

5.3. *Statistics and Predictive Regressions*

We report our baseline calibration in table 8.

[Place Table 8 about here]

After calibration, we have our model produce reasonable moments matching what we obtained from data and we report the moments in table 9. The upper panel presents the basic moments and the lower panel reports the moments of dividend growth, tax rates and debt-to-GDP ratio. We find the model without firm's cash holding behavior fails to produce several key moments. First the mean of excess returns is 1.08% which is much lower than the 5.70% documented in data. Second the dividend growth 5.24% is double the number 2.40% in data. After introducing the retained earnings, the model can produce reasonable moments matching the moments from the data.

[Place Table 9 about here]

Our model predicts that debt-to-GDP ratio has predictive power for dividend growth and stock returns. Based on the simulated results, we find the factor loadings on both dividend growth and stock returns are in the same scale which is consistent with the common component argument. In table 10, we also include results from the model where firms have no cash holding and we find that the model can predict the positive stock returns but fail to capture the positive predictability of dividend growth.

[Place Table 10 about here]

6. Conclusion

This paper documents the debt-to-GDP ratio can predict both dividend growth and stock returns. These findings are consistent with Lettau and Ludvigson (2005)'s argument that there exists a common component among stock returns and dividend growth. The significance of debt-to-GDP ratio remains after controlling the consumption-wealth ratio. To rationalize these findings, we propose a production-based asset pricing model incorporating firms' trade-off behaviors. The simulated results from our model also suggest the debt-to-GDP captures the common component.

In this paper, we also propose a VAR system to show that the U.S. public debt policy is sustainable and find the tax and spending uncertainty accounts for 31.36% and 68.56% of variations respectively. The sustainability of debt policy ensures that the debt-to-GDP can act as a forward looking factor.

For policy implication, if steady-states of debt-to-GDP matters in determining the expected return and expected dividend growth, it is reasonable to make the public debt rule(or debt projection) explicit like targeting the inflation in monetary policy. For further research, (i) we can study how the current and past debt projection by Congressional Budget Office may affect people's expectation. Since firms adjust their leverage and payout based on their expectation on government leverage, we can expect the government leverage may affect other corporate behaviors. (ii) the empirical method to test public debt sustainability can also be applied to other countries and potential public debt sustainability indicator can be constructed from this method. (iii) the common variations documented can partially explain the confusing fact in returns' variations decomposition and can be applied to prediction and asset allocation.

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Table 1: In this table, we use the sample to show the results.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 \widetilde{dp}_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 \widetilde{dp}_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A					
1966Q1:2017Q4 (Quarterly)					
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$
\widetilde{dp}_t	0.3072 [3.77]**	0.2590 [3.99]**	0.1930 [3.54]**	0.1591 [3.39]**	0.1509 [3.61]**
Valkanov t-test	0.26**	0.28**	0.25**	0.24**	0.25**
Observations:	208	208	208	208	208
R-squared:	0.155	0.226	0.199	0.180	0.196
F-statistic:	37.735	60.266	51.244	45.304	50.062
Panel B					
1966Q1:2017Q4 (Quarterly)					
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$
\widetilde{dp}_t	-0.0150 [-0.31]	0.0038 [0.11]	0.0173 [0.71]	0.0172 [0.90]	0.0088 [0.53]
Valkanov t-test	-0.02	0.01	0.05	0.06	0.04
Observations:	208	208	208	208	208
R-squared:	0.003	0.000	0.006	0.008	0.003
F-statistic:	0.591	0.050	1.332	1.755	0.612

Table 2: In this table, we use the sample to show the results.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A		1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$	
$DGDP_t$	0.1463 [2.03]*	0.1457 [2.58]*	0.1512 [3.20]**	0.1558 [3.55]**	0.1547 [3.52]**	
\widetilde{dp}_t	0.3019 [3.78]**	0.2670 [4.26]**	0.2083 [3.80]**	0.1753 [3.60]**	0.1673 [3.81]**	
Valkanov t-test	0.14*	0.18**	0.23**	0.26**	0.26**	
Observations:	204	200	196	192	188	
R-squared:	0.182	0.285	0.295	0.297	0.307	
F-statistic:	22.395	39.196	40.288	39.829	40.926	
Panel B		1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$	
$DGDP_t$	0.1257 [3.46]**	0.1366 [4.33]**	0.1425 [5.33]**	0.1416 [5.74]**	0.1372 [5.61]**	
\widetilde{dp}_t	-0.0127 [-0.24]	0.0125 [0.34]	0.0309 [1.25]	0.0314 [1.88]	0.0266 [1.91]	
Valkanov t-test	0.24**	0.31**	0.38**	0.41**	0.41**	
Observations:	204	200	196	192	188	
R-squared:	0.193	0.262	0.343	0.394	0.416	
F-statistic:	24.073	34.999	50.430	61.484	65.886	

Table 3: Revisit dp_t : i) $ret_{t+j} = f(dp_t, \Delta d_{t+j})$ and ii) Common component argument by Lettau and Ludvigson (2005). In first part we estimate all β_1 by GMM. In second part we run the null hypothesis test that $\beta_{1,H} = 1$.

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \epsilon_{t,t+j}^d$$

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 \left(\sum_{j=1}^H \Delta d_{t+j} \right) + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r$$

$$H = 1, \dots, 5$$

Part 1 1966Q1:2017Q4 (Quarterly)					
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$
$DGDP_t$	0.1293 [9.66]**	0.1377 [10.40]**	0.1417 [11.14]**	0.1343 [10.79]**	0.1247 [10.23]**
Observations:	204	200	196	192	188
R-squared:	0.189	0.256	0.319	0.361	0.383

Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$
$\Delta d_{t,t+j}$	0.9285 [4.42]*	0.7904 [5.93]*	0.9091 [8.54]**	0.9555 [9.05]**	0.9866 [8.71]**
\widetilde{dp}_t	0.2405 [3.14]**	0.1476 [3.28]**	0.1184 [3.16]**	0.1143 [3.42]**	0.0899 [2.87]**
Observations:	204	200	196	192	188
R-squared:	0.103	0.232	0.273	0.280	0.243

Part 2 Wald Test	
Null Hypothesis: $\beta_{1,1} = \beta_{1,2} = \beta_{1,3} = \beta_{1,4} = \beta_{1,5} = 1$	
Chi-square:	5.4686 p-value: 0.3614

Table 4: In this table, we use the sample to show the results.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \beta_3 CAY_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \gamma_3 CAY_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A	1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$
$DGDP_t$	0.1460 [2.26]*	0.1280 [2.99]**	0.1123 [3.42]**	0.1023 [3.28]**	0.0941 [3.15]**
\widetilde{dp}_t	0.2960 [3.64]**	0.2557 [4.48]**	0.1923 [3.83]**	0.1587 [3.39]**	0.1500 [3.51]**
CAY_t	2.5395 [2.87]**	2.7724 [4.66]**	2.5043 [5.82]**	2.1069 [5.57]**	1.7673 [4.94]**
Observations:	204	200	196	192	188
R-squared:	0.272	0.493	0.542	0.516	0.486
F-statistic:	24.940	63.431	75.840	66.711	57.956
Panel B	1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$
$DGDP_t$	0.1257 [3.48]**	0.1345 [4.28]**	0.1367 [4.89]**	0.1337 [4.87]**	0.1300 [4.67]**
\widetilde{dp}_t	-0.0135 [-0.25]	0.0111 [0.30]	0.0285 [1.14]	0.0289 [1.71]	0.0245 [1.74]
CAY_t	0.3502 [1.68]	0.3420 [1.86]	0.3685 [1.89]	0.3129 [1.44]	0.2118 [0.96]
Observations:	204	200	196	192	188
R-squared:	0.206	0.278	0.365	0.414	0.428
F-statistic:	17.335	25.123	36.827	44.273	45.836

Table 5: In this table, we use the interpolated quarterly data to show the results.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A		1950Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$	
$DGDP_t$	0.1751 [2.30]*	0.1742 [2.86]**	0.1836 [3.55]**	0.2041 [4.24]**	0.2134 [4.63]**	
\widetilde{dp}_t	0.3223 [4.34]**	0.2525 [4.37]**	0.1910 [3.81]**	0.1728 [3.96]**	0.1760 [4.51]**	
Valkanov t-test	0.14*	0.18**	0.22**	0.27**	0.29**	
Observations:	268	264	260	256	252	
R-squared:	0.186	0.249	0.270	0.331	0.380	
F-statistic:	30.337	43.256	47.485	62.620	76.460	
Panel B		1950Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$	
$DGDP_t$	0.1194 [3.44]**	0.1152 [3.56]**	0.1166 [3.89]**	0.1157 [4.16]**	0.1118 [4.26]**	
\widetilde{dp}_t	-0.0285 [-0.61]	-0.0047 [-0.15]	0.0100 [0.46]	0.0129 [0.84]	0.0127 [0.97]	
Valkanov t-test	0.21**	0.22**	0.24**	0.26**	0.27**	
Observations:	268	264	260	256	252	
R-squared:	0.146	0.172	0.219	0.266	0.295	
F-statistic:	22.728	27.143	36.034	45.749	52.103	

(Robust)Table 5.1: In this table, we use the interpolated quarterly data to show the results.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_t + \beta_2 \widetilde{dp}_t + \beta_3 CAY_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_t + \gamma_2 \widetilde{dp}_t + \gamma_3 CAY_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A	1950Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$
$DGDP_t$	0.1581 [2.37]*	0.1468 [3.27]**	0.1353 [3.94]**	0.1218 [3.82]**	0.1133 [3.81]**
\widetilde{dp}_t	0.3060 [4.00]**	0.2329 [4.26]**	0.1688 [3.51]**	0.1448 [3.35]**	0.1454 [3.79]**
CAY_t	2.8566 [3.51]**	2.8862 [5.12]**	2.5226 [6.10]**	2.1437 [6.45]**	1.7825 [5.55]**
Observations:	260	256	252	248	244
R-squared:	0.278	0.443	0.490	0.508	0.495
F-statistic:	32.924	66.893	79.421	84.033	78.294
Panel B	1950Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$
$DGDP_t$	0.1257 [3.76]**	0.1322 [4.43]**	0.1329 [4.79]**	0.1289 [4.54]**	0.1222 [4.03]**
\widetilde{dp}_t	-0.0252 [-0.52]	0.0002 [0.01]	0.0144 [0.64]	0.0163 [1.02]	0.0153 [1.13]
CAY_t	0.3451 [1.68]	0.3089 [1.71]	0.3014 [1.53]	0.2356 [1.09]	0.1540 [0.71]
Observations:	260	256	252	248	244
R-squared:	0.185	0.238	0.302	0.338	0.340
F-statistic:	19.336	26.270	35.778	41.584	41.268

Table 6: Predictive Performance (Recursive Window)

Predictive Performance	1950Q1:2017Q4		Quarterly	
	In-Sample		Out-of-Sample	
Panel A (1y)	R^2	RMSE	R^2	RMSE
dp_t	0.197	0.143	0.093**	0.168
\widetilde{dp}_t	0.306	0.133	0.146**	0.163
CAY	0.174	0.145	0.149**	0.163
$DGDP + \widetilde{dp}_t$	0.308	0.133	0.216**	0.156
$DGDP + \widetilde{dp}_t + CAY$	0.363	0.127	0.368**	0.140
Panel B (3y)	R^2	RMSE	R^2	RMSE
dp_t	0.570	0.052	0.237**	0.087
\widetilde{dp}_t	0.013	0.079	0.255**	0.086
CAY	0.186	0.072	0.374**	0.079
$DGDP + \widetilde{dp}_t$	0.237	0.069	0.413**	0.076
$DGDP + \widetilde{dp}_t + CAY$	0.351	0.064	0.599**	0.063
Panel C (5y)	R^2	RMSE	R^2	RMSE
dp_t	0.696	0.023	0.343**	0.062
\widetilde{dp}_t	-0.516	0.051	0.246**	0.066
CAY	0.086	0.040	0.331**	0.062
$DGDP + \widetilde{dp}_t$	0.351	0.064	0.599**	0.063
$DGDP + \widetilde{dp}_t + CAY$	0.243	0.036	0.536**	0.052

Table 7: In this table, we use the quarterly sample to show the results after including the steady states of debt-to-GDP ratio.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_{ss,t} + \beta_2 \widetilde{dp}_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_{ss,t} + \gamma_2 \widetilde{dp}_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A		1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$	
$DGDP_{ss,t}$	0.1435 [2.04]*	0.1264 [2.41]*	0.1341 [3.28]**	0.1348 [3.91]**	0.1214 [3.64]**	
\widetilde{dp}_t	0.2851 [3.46]**	0.2510 [3.78]**	0.1905 [3.27]**	0.1567 [3.00]**	0.1484 [3.11]**	
Valkanov t-test	0.14*	0.17*	0.23**	0.28**	0.27**	
Observations:	204	200	196	192	188	
R-squared:	0.179	0.268	0.274	0.270	0.268	
F-statistic:	21.843	36.057	36.429	34.949	33.788	
Panel B		1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$	
$DGDP_{ss,t}$	0.1191 [2.51]*	0.1396 [3.85]**	0.1509 [5.35]**	0.1540 [6.63]**	0.1513 [7.39]**	
\widetilde{dp}_t	-0.0266 [-0.51]	-0.0046 [-0.12]	0.0123 [0.46]	0.0125 [0.70]	0.0080 [0.61]	
Valkanov t-test	0.18**	0.27**	0.38**	0.48**	0.54**	
Observations:	204	200	196	192	188	
R-squared:	0.161	0.263	0.379	0.473	0.537	
F-statistic:	19.269	35.141	58.782	84.902	107.350	

(Robust)Table 7.1: In this table, we use the quarterly sample to show the results after including the steady states of debt-to-GDP ratio.

$$\sum_{j=1}^H r_{t+j} = \beta_0 + \beta_1 DGDP_{ss,t} + \beta_2 \widetilde{dp}_t + \beta_3 CAY_t + \epsilon_{t,t+j}^r$$

$$\sum_{j=1}^H \Delta d_{t+j} = \gamma_0 + \gamma_1 DGDP_{ss,t} + \gamma_2 \widetilde{dp}_t + \gamma_3 CAY_t + \epsilon_{t,t+j}^d$$

$$H = 1, \dots, 5$$

Panel A	1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$ret_{t,t+1}$	$ret_{t,t+2}$	$ret_{t,t+3}$	$ret_{t,t+4}$	$ret_{t,t+5}$
$DGDP_{ss,t}$	0.1379 [2.08]*	0.1053 [2.24]*	0.0944 [2.66]**	0.0835 [2.64]**	0.0650 [2.19]*
\widetilde{dp}_t	0.2799 [3.31]**	0.2421 [3.93]**	0.1793 [3.35]**	0.1464 [2.97]**	0.1383 [3.06]**
CAY_t	2.5097 [2.84]**	2.7776 [4.64]**	2.5325 [5.69]**	2.1574 [5.34]**	1.8518 [4.72]**
Observations:	204	200	196	192	188
R-squared:	0.266	0.476	0.527	0.501	0.467
F-statistic:	24.211	59.440	71.438	62.961	53.791
Panel B	1966Q1:2017Q4 (Quarterly)				
Dep. Var:	$\Delta d_{t,t+1}$	$\Delta d_{t,t+2}$	$\Delta d_{t,t+3}$	$\Delta d_{t,t+4}$	$\Delta d_{t,t+5}$
$DGDP_{ss,t}$	0.1183 [2.49]*	0.1371 [3.77]**	0.1453 [4.97]**	0.1469 [5.87]**	0.1455 [6.62]**
\widetilde{dp}_t	-0.0273 [-0.51]	-0.0057 [-0.15]	0.0107 [0.40]	0.0111 [0.62]	0.0070 [0.53]
CAY_t	0.3246 [1.59]	0.3272 [1.90]	0.3568 [2.03]*	0.2974 [1.58]	0.1913 [1.04]
Observations:	204	200	196	192	188
R-squared:	0.172	0.277	0.399	0.491	0.547
F-statistic:	13.863	25.054	42.524	60.519	74.016

Table 8: Calibrated Parameter Values

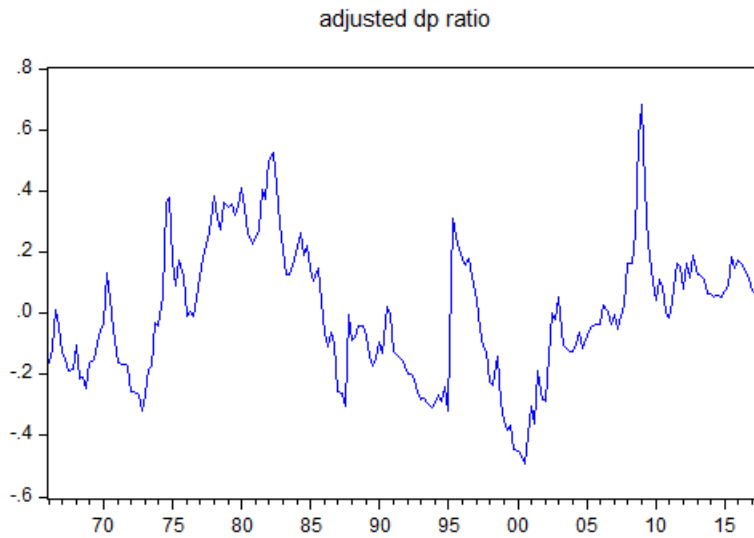
δ	γ	ψ	α	μ	ρ	a_1	ζ	ϕ_1	θ	λ_1
0.996	10.000	1.500	0.345	0.002	0.982	0.143	0.83	2000.000	0.330	0.400
b	φ	ρ_q	ρ_{by}	σ_c	σ_x					
0.003	0.205	0.98	0.98	0.010	0.001					

Table 9: Simulated Moments and Statistics: The bootstrapped statistics were calculated from 1000 simulations, each with 2000 observations. All values are annualized. The upper panel reports required variables(moments) in the framework and lower panel reports variables(moments) we investigated. We compare moments from three cases: Case 1-Data; Case 2-Model; Case 3-Model without cash holding.

Variable(Macro)	Data	Model	(w/o cash holding)
$\sigma(\Delta y)$ (%)	3.56	3.50	3.34
$\sigma(\Delta c)/\sigma(\Delta y)$	0.71	0.68	0.64
$\sigma(\Delta i)/\sigma(\Delta y)$	4.49	5.93	3.51
$E[I/Y]$ (%)	17.49	11.79	24.81
$\rho(\Delta c, \Delta i)$	0.39	0.23	0.66
$\rho(\Delta c, r_{exr})$	0.25	0.12	0.28
$\sigma(q)$	0.29	0.27	0.22
$ACF_1[q]$ (%)	0.86	0.99	1.00
$ACF_1[\Delta c]$ (%)	0.50	0.50	0.57
Variable(Return)	Data	Model	(w/o cash holding)
$E[r_{exr,t+1}]$ (%)	5.70	5.96	1.08
$\sigma(r_{exr,t+1})$ (%)	20.89	8.83	7.24
$E[r_t^f]$ (%)	0.19	0.04	-0.51
$\sigma(r_t^f)$ (%)	1.86	1.22	0.89
$ACF_1[r_{exr,t+1}]$ (%)	0.09	-0.00	-0.00
$ACF_1[r_t^f]$ (%)	0.64	0.70	0.57
Variable(Key)	Data	Model	(w/o cash holding)
$E[\Delta div]$ (%)	2.40	2.78	5.24
$\sigma(\Delta div)/\sigma(\Delta y)$	1.85	2.38	2.95
$\sigma(\frac{B}{Y})$ (%)	44.00	32.25	31.78
$E[\frac{B}{Y}]$ (%)	45.00	44.97	45.05
$\sigma(\tau)$ (%)	12.80	14.60	14.29
$E[\tau]$ (%)	36.50	34.82	33.72

Table 10: In this table, we report key statistics obtained from both data and model, where γ and β are the factor loadings of debt-to-GDP. We compare results from three cases: Case 1-Data; Case 2-Model; Case 3-Model without cash holding.

Panel A	Data		Model		<i>(w/o cash holding)</i>	
Dep. Var:	γ	R^2	γ	R^2	γ	R^2
$\Delta d_{t,t+1}$	0.1257	0.191	0.1438	0.065	-0.0912	0.144
$\Delta d_{t,t+3}$	0.1404	0.323	0.1377	0.102	-0.0928	0.185
$\Delta d_{t,t+5}$	0.1332	0.390	0.1270	0.116	-0.0956	0.182
Panel B	Data		Model		<i>(w/o cash holding)</i>	
Dep. Var:	β	R^2	β	R^2	β	R^2
$ret_{t,t+1}$	0.1470	0.034	0.1575	0.133	0.1460	0.298
$ret_{t,t+3}$	0.1371	0.075	0.1251	0.134	0.1242	0.284
$ret_{t,t+5}$	0.1293	0.080	0.0979	0.112	0.1056	0.240



Test statistics employ HAC covariances (Bartlett kernel, Newey
-West fixed bandwidth) assuming common data distribution

Sequential F-statistic determined breaks: 2

Break Test	Break	F-statistic	Scaled F-statistic
0 vs. 1 *	1921Q1	15.94486	31.88972
1 vs. 2	1870Q4	2.613803	5.227607
1 vs. 2 *	1966Q2	6.309086	12.61817
2 vs. 3	1870Q4	0.725486	1.450973
2 vs. 3	---	---	---

* Significant at the 0.05 level, Bai-Perron (Econometric Journal,
2003) critical value 11.47.

Break dates:

	Sequential	Repartition
1	1950Q1	1954Q2
2	1995Q2	1995Q2

Fig. 1. \widetilde{dp}_t : We follow Lettau and Van Nieuwerburgh (2007) and apply the structural break methods to the quarterly data of dp_t . Here we show the adjusted dp_t ratio which is the dp_t demeaned by the steady states values.

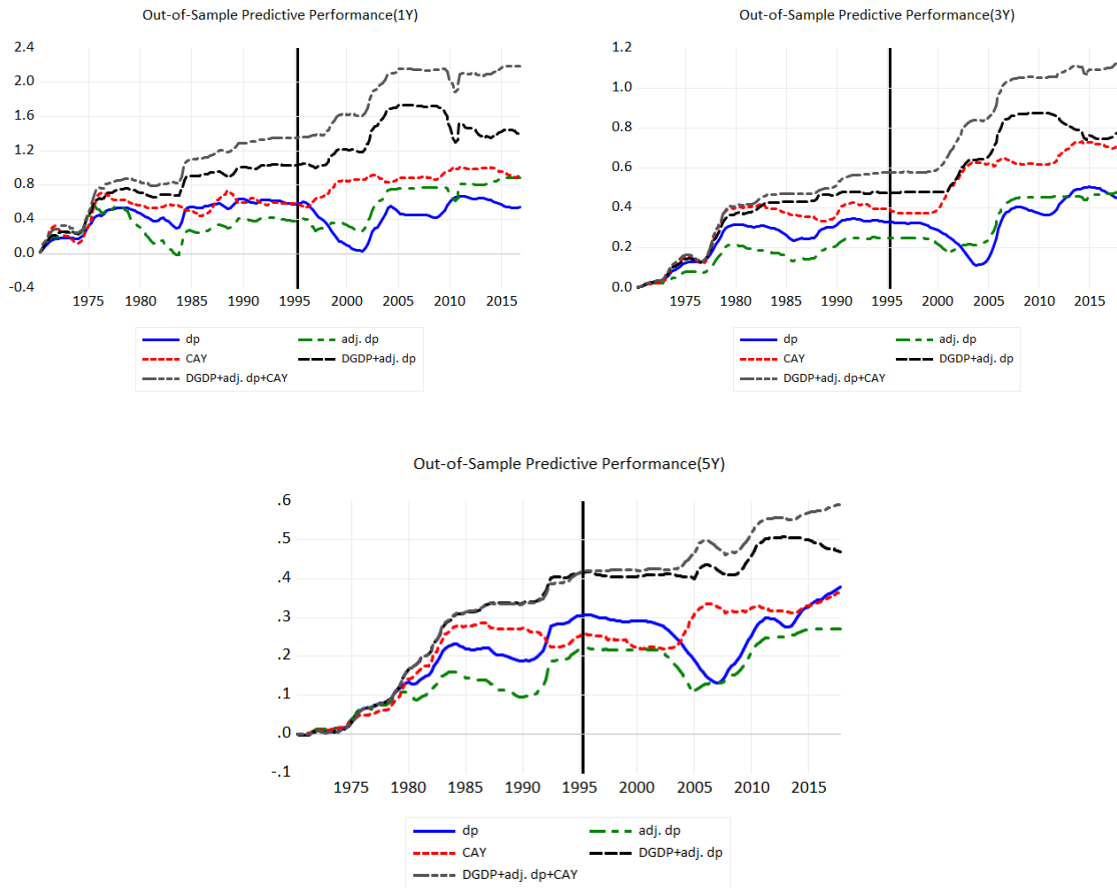


Fig. 2. Out-of-sample performance: a)1Y Return, b)3Y Return, c)5Y Return. The above figure plots the difference between the cumulative RMSE of forecasts based on four different specifications. Quarterly Data 1950Q1:2017Q4.

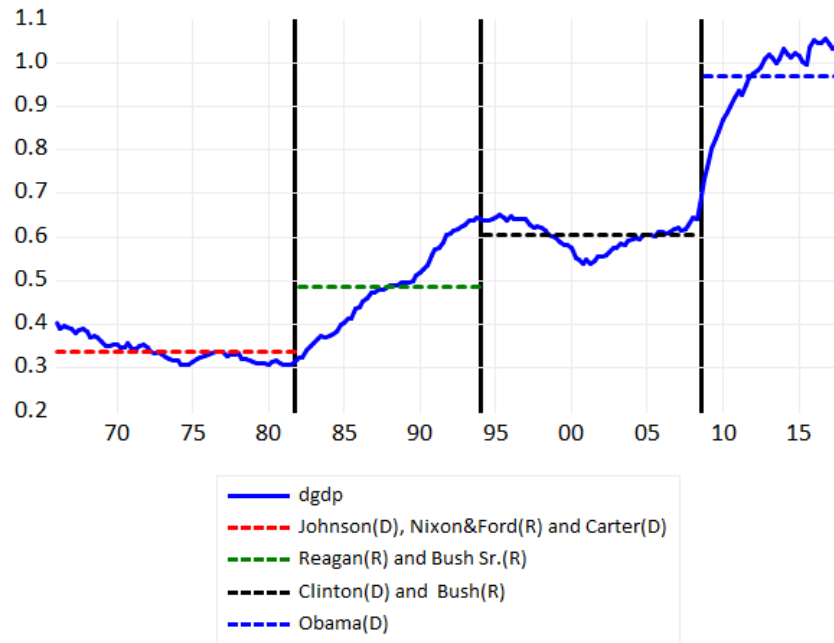


Fig. 3. Change in the Mean of DGDP. The above figure plots the debt-to-GDP ratio after 1966Q1(solid line). Three breaks are identified in this time series.

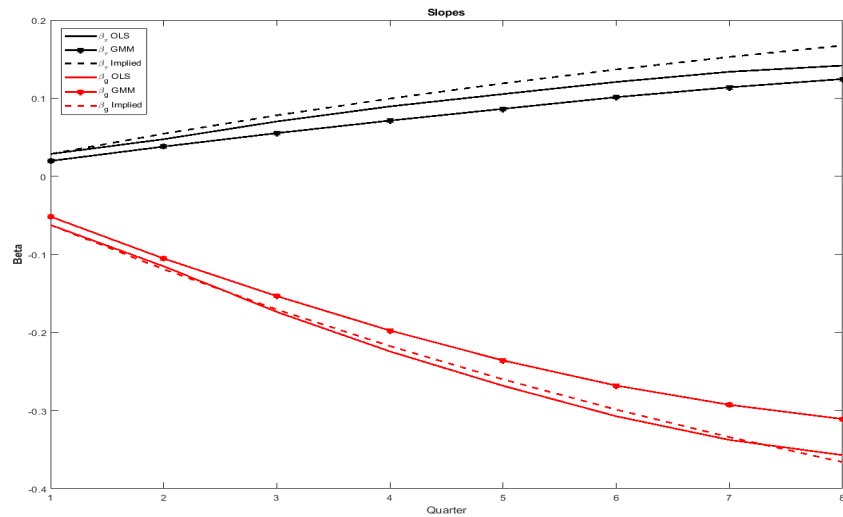


Fig. 4. β : OLS estimation, GMM estimation and Model implied. This figure compares the univariate short-horizon regression coefficients β_{tax}^H (black) and β_g^H (red): OLS, OLS-Implied and the GMM estimates. The system contains 16 equations, 8 tax ratio and 8 spending ratio equations. The horizons (in quarters) are H from 1 to 8.

Appendix A.

This Online Appendix reports data description and data sources.

The Debt-to-GDP Ratio. We download Total Public Debt as Percent of Gross Domestic Product (GFDEGDQ188S) from 1966Q1 to 2017Q4 and Federal Debt Held by the Public as Percent of Gross Domestic Product (FYGFQDQ188S) from 1970Q1 to 2017Q4 from St. Louis (FRED).

Stock Market Prices. *S&P* 500 index yearly prices from 1950 to 2017 are from Robert Shiller's Web site (see Data Sources at the end of this Appendix); we take December observations.

Stock Market Dividends. Dividends are 12-month moving sums of dividends paid on the *S&P* 500 index. They are from the Robert Shiller Web site (see Data Sources) for the period 1870-2017.

Stock Market Earnings. Earnings are 12-month moving sums of dividends paid on the *S&P* 500 index. They are from the Robert Shiller Web site (see Data Sources) for the period 1870-2017.

Stock Market Payout Ratio. Payout ratio is constructed by total dividends over total earnings at each time node.

Stock Market Returns. For the *S&P* 500 index, to construct the continuously compounded return ret_t , we take the ex-dividend price P_t , add dividend D_t over P_{t-1} .

Risk-Free Rate. We download secondary market 3-month T-bill rates from St. Louis (FRED) from 1966 to 2017.

Log Dividend-Price Ratio (dp_t). The difference between the log of dividends and the log of prices.

Consumption, Wealth, Income Ratio (CAY). The series is taken from Lettau and Ludvigson (2001). Data are available from Martin Lettau's Web site (see Data Sources) at quarterly frequency from 1952 to 2017.

Taxation. We download Federal Government Current Receipts (FGRECPT) from 1966Q1 to 2017Q4 from St. Louis (FRED).

Government Spending. We download Federal government total expenditures (W019RCQ027SBEA) from 1966Q1 to 2017Q4 from St. Louis (FRED).

Corporate Leverage. The corporate leverage is constructed as book leverage of firms in *S&P* 500 index. Take the sum of long-term debt and short term debt over firms' book value of total assets as aggregate leverage. The firms' data are downloaded from the WRDS(Compustat Daily Updates-Fundamentals Annual) and matched with the *S&P*

500 index list(Compustat - North America-Index Constituents).

Data Sources

Martin Lettau's Web site: <http://faculty.haas.berkeley.edu/lettau/>

Sydney Ludvigson's Web site: <https://www.sydneyludvigson.com/>

Robert Shiller's Web site: <http://www.econ.yale.edu/shiller/>

Congressional Budget Office: <https://www.cbo.gov/about/products/>

FRED: <https://fred.stlouisfed.org/>

WRDS: <https://wrds-web.wharton.upenn.edu/wrds/>

Appendix B.

This Online Appendix reports the variance decomposition of debt-to-GDP.

We start from the government budget constraint and allow the government to finance expenditure G_t through a mix of corporate taxes Tax_t , and public debt B_t , according to the following budget constraint:

$$B_{t+1} = R_{b,t}B_t + G_{t+1} - Tax_{t+1} \quad (42)$$

Taxation and government spending are represented as $Tax_t = \tau_t * Y_t$ and $G_t = g_t * Y_t$ where τ_t, g_t are total tax ratio and spending ratio. Divide both sides with the GDP at that period and we have

$$\frac{B_{t+1}}{Y_{t+1}} = R_{b,t} \frac{B_t}{Y_t} \frac{Y_t}{Y_{t+1}} + \frac{G_{t+1}}{Y_{t+1}} - \frac{Tax_{t+1}}{Y_{t+1}}$$

Define $s_t = \tau_t - g_t$ as surplus ratio, $k_{t+1} = \frac{Y_{t+1}}{Y_t \cdot R_{b,t}}$ as discount ratio and $H_{t+1}^N = \prod_{i=1}^N k_{t+i}$ as N-period cumulative discount ratio at time $t + 1$. Let $R_{y,t} = \frac{Y_{t+1}}{Y_t}$, we have $k_{t+1} = \frac{R_{y,t}}{R_{b,t}}$. Iterating the equation forward and then the $\frac{B}{Y}$ can be decomposed as following:

$$\left(\frac{B}{Y}\right)_t = E_t[H_{t+1}^{T-t} \cdot \left(\frac{B}{Y}\right)_T] + E_t\left[\sum_{j=1}^{T-t} H_{t+1}^j \cdot s_{t+j}\right]; \quad (43)$$

where $\left(\frac{B}{Y}\right)_T$ is the debt-to-GDP level at time T .

Therefore in a rational forward looking world, the current $\left(\frac{B}{Y}\right)_t$ can be decomposed into the discounted long-term debt level $E_t[H_{t+1}^{T-t} \left(\frac{B}{Y}\right)_T]$, and the discounted surplus ratio $E_t[\sum_{j=1}^{T-t} H_{t+1}^j s_{t+j}]$. Since $\left(\frac{B}{Y}\right)_t$ factor summarizes expected long-term debt level and expected series of government budget surplus(deficit) which are closely related to stock returns, we can justify the economic prediction power in asset returns.

Before we move to next step, we first discuss the k_{t+1} . It follows a ratio distribution and after Geary-Hinkley transformation, we can transform it into a normal distribution with small variance. Suppose $R_{y,t} \sim N(\mu_y, \sigma_y^2)$ and $R_{b,t} \sim N(\mu_b, \sigma_b^2)$. Follow the Geary-Hinkley transformation, we have $k_t \sim N(\mu_k, \sigma_k^2)$. Therefore, by $H_{t+1}^N = \prod_{i=1}^N k_{t+i}$ as N-period cumulative discount ratio at time $t + 1$, we have

$$E_t(H_{t+1}^j) \simeq (\mu_k)^j$$

where j is bounded by a finite number T .

Summary Statistics

	Mean	Median	Std. Dev.
k_t	1.006	1.006	0.0096

Correlation Matrix

Correlation	tax_t	g_t	k_t
tax_t	1	-0.555	-0.005
g_t	-0.555	1	-0.080
k_t	-0.005	-0.080	1

Here we choose the annualized 3-Month Treasury Bill rates as safe returns and we can find the μ_k is around 1. We document that the k_{t+1} is less correlated with both fiscal variables: tax rates and spending ratios. We can assume k_t is independent of tax_t and g_t . Here we approximate the expected value as $E_t(H_{t+1}^j) \simeq (\mu_k)^j$.

Till now, we have:

$$\left(\frac{B}{Y}\right)_t \simeq \mu_k^{T-t} E_t\left[\left(\frac{B}{Y}\right)_T\right] + \sum_{j=1}^{T-t} \mu_k^j E_t[s_{t+j}]; \quad (44)$$

In this subsection, we relate the debt-to-GDP to the public debt policy. Here we simply refer the public debt policy as which process the debt-to-GDP follows (e.g. mean-reverting process). Define $\left(\frac{B}{Y}\right)_{ss} \simeq \mu_k^{T-t} E_t\left[\left(\frac{B}{Y}\right)_T\right]$, $\widetilde{\left(\frac{B}{Y}\right)}_t \simeq \sum_{j=1}^{T-t} \mu_k^j E_t[s_{t+j}]$ and $\mu_k = 1$. Here we take an approximation and let μ_k be a very highly persistent constant factor approaching 1 from left hand side. This approximation is consistent with the non-explosive debt condition and is consistent with our later empirical decomposition. Equation 44 will be reduced to the following form:

$$\begin{aligned} \left(\frac{B}{Y}\right)_t &\simeq E_t\left[\left(\frac{B}{Y}\right)_T\right] + \sum_{j=1}^{T-t} E_t[s_{t+j}]; \\ &\simeq \left(\frac{B}{Y}\right)_{ss} + \widetilde{\left(\frac{B}{Y}\right)}_t \end{aligned} \quad (45)$$

The way we interpret the $E_t\left[\left(\frac{B}{Y}\right)_T\right] = \left(\frac{B}{Y}\right)_{ss}$ has been justified in the previous section. We interpret the $\widetilde{\left(\frac{B}{Y}\right)}_t$ as the short-run fiscal adjustment besides the steady state debt-to-GDP target.

To verify that the debt-dynamics is non-explosive, we adopt a way which is firstly proposed by Cochrane (2007) and applied by many papers like Lettau and Van Nieuwerburgh(2007), Maio and Santa-Clara (2015), and Golez and Koudijs (2017). The initial

framework is based on the dividend yield decomposition

$$\widetilde{dp}_t \approx E_t[\sum_{j=1}^{\infty} \rho_p^j (\widetilde{r}_{t+j} - \widetilde{\Delta d}_{t+j})] \quad (46)$$

and the three equations to derive the term structure of 'equity yield' are following:

$$\sum_{j=1}^H \widetilde{r}_{t+j} = \beta_r^H \widetilde{dp}_t + \epsilon_{t,t+H}^r \quad (47)$$

$$\sum_{j=1}^H \widetilde{\Delta d}_{t+j} = \beta_{\Delta d}^H \widetilde{dp}_t + \epsilon_{t,t+H}^{\Delta d} \quad (48)$$

$$\widetilde{dp}_{t+1} = \beta_{dp} \widetilde{dp}_t + \epsilon_{t,t+1}^{dp} \quad (49)$$

The above method has been widely applied and appeared in many finance papers. The main focus in those papers is testing the driver of variations in dividend yield in their return predictability regressions. In our paper, we apply this method firstly to the variance decomposition of debt-to-GDP ratio.

Appendix C.

This Online Appendix reports the additional figures.

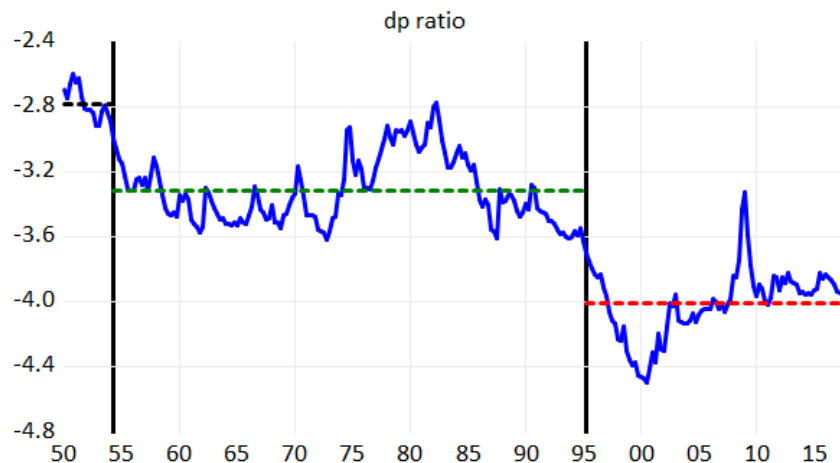


Fig. 5. dp_t : We follow Lettau and Van Nieuwerburgh (2007) and apply the structural break methods to the quarterly data of dp_t . Here we show the dp_t ratio and the structural breaks.

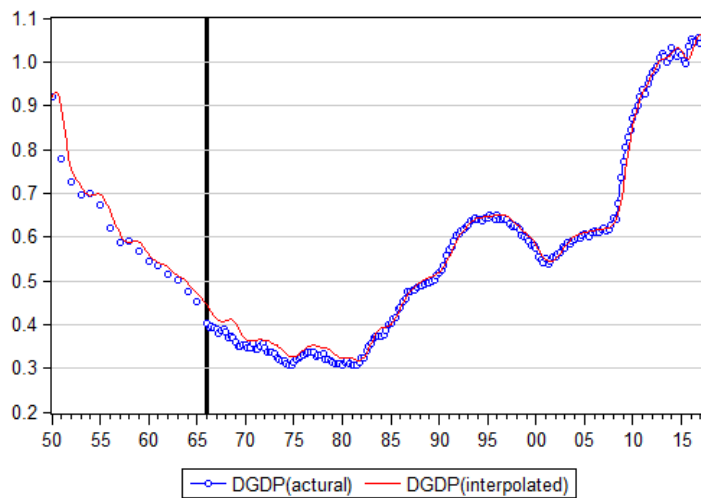


Fig. 6. DGDP: raw quarterly data and data interpolated from the annual data. Actual data $DGDP(\text{actual})$ are observed at quarterly frequency from 1966 onwards and at annual frequency from 1870. The $DGDP(\text{interpolated})$ are cubic spline interpolated using annual data from the 1950.