Collateral Runs*

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May 30, 2018

Abstract: This paper models an unexplored source of liquidity risk faced by large broker-dealers: *collateral runs*. By setting different contracting terms on repurchase agreements with cash borrowers and lenders, dealers can source funds for their own activities. Cash borrowers internalize the risk of losing their collateral in case their dealer defaults, prompting them to withdraw it. This incentive creates strategic complementarities for counterparties to withdraw their collateral, reducing a dealer's liquidity position and compromising her solvency. Collateral runs are markedly different than traditional wholesale funding runs because they are triggered by a contraction in dealers' assets, rather than their liabilities.

JEL classification: G23, G33, G01, C72

Keywords: runs, repo, rehypothecation, dealer, liquidity, default, collateral

*We are grateful to Elizabeth Klee, David Rappoport, seminar participants at 14th Cowles Conference on General Equilibrium and its Applications at Yale, and brown bag participants at the Federal Reserve Board and Duke Fuqua School of Business for fruitful comments and suggestions. All remaining errors are ours. The views of this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. Please send comments to *sebastian.infantebilbao@frb.gov* or *alexandros.vardoulakis@frb.gov*.

1 Introduction

This paper presents a theoretical model to characterize a relatively unexplored risk that can affect large broker-dealers: a run from their collateral providers (cash borrowers). Broker-dealers are unique because they transform liquidity on both sides of their balance sheet. On the liabilities side, they borrow funds, typically short-term, from cash lenders, using financial securities as collateral. On the asset side, they extend credit, also typically short-term, against similar collateral provided by cash borrowers. In the context of the 2007–09 financial crisis, a large literature has noted that financial institutions faced run risk from their wholesale funding lenders (see, for example, Gorton and Metrick, 2012; Krihnamurthy, Nagel, and Orlov, 2014; and Copeland, Martin, and Walker, 2014). In this paper, we highlight an alternative channel, which operates on the asset side of the balance sheet. Unlike traditional wholesale funding runs, where dealers risk an abrupt withdrawal of funds from cash lenders, in collateral runs, dealers risk an abrupt withdrawal of collateral from collateral providers.

The main set-up of the model considers a dealer providing short-term secured financing, interpreted as repurchase agreements (repos), to a large number of counterparties, called hedge funds. Hedge funds borrow from the dealer because they want to take leveraged positions in the asset that they pledge as collateral. The dealer is able to extend said financing by re-using the collateral she receives to issue secured debt to cash lenders, called money market funds, a process known as rehypothecation.[1] The dealer's market power allows her to set favorable contracting terms on both secured financing transactions. In particular, she has incentives to distribute only a fraction of the cash she raises from money funds and use the difference to finance higher yielding risky assets, which are illiquid. This funding difference can be thought of as the amount of over-collateralization of the hedge funds' repo relative to the money funds' repo. In case the dealer defaults, money funds have immediate access to the collateral and can sell it to make their claims whole, essentially insulating them from the dealer.[2] In contrast, hedge funds risk losing their collateral altogether which is

[1] We assume that hedge funds cannot contact money market funds directly—that is, their only source of secured financing for the hedge fund is through the dealer. This assumption captures the idea that many hedge funds are small and relatively opaque firms, which wholesale cash lenders will not (or cannot) interact with directly. This is consistent with Gottardi et al. (2017) who show that an optimal rehypothecation chain can arise whenever a dealer is more trustworthy than a hedge fund counterparty. Note that we will use the terms re-use and rehypothecation interchangeably.

[2] Although an abrupt withdraw of cash lenders is an important consideration for repo market stability, we

more valuable than the initial loan they received. Specifically, hedge funds risk losing the amount of over-collateralization on their repo. This amount is an unsecured claim on the dealer's illiquid asset holdings, which is pooled with the unsecured claims of other hedge funds. This creates a first mover advantage for hedge funds that decide to withdraw their collateral.

The incentive to withdraw the collateral creates strategic complementarities amongst hedge funds' actions because each hedge fund's optimal action and payoff can depend on what other hedge funds do. For example, if all other hedge funds roll over their repo positions, then the dealer does not need to liquidate any of her illiquid assets, making it optimal for an individual hedge fund to roll over as well. On the contrary, if all other hedge funds withdraw their collateral, then the dealer may need to sell off all of her illiquid assets, making it optimal for an individual hedge fund to withdraw their collateral. Hence, an individual hedge fund's payoff not only depends on the dealer's solvency, but also on *its beliefs about the actions/beliefs of other hedge funds*. There are also extreme cases in which a hedge fund's actions are independent of other hedge funds' actions. For example, if the value of the dealer's illiquid assets are low enough, she will be insolvent, making it individually optimal for a hedge fund to withdraw, independent of others' actions. Conversely, if the value of the dealer's illiquid assets are high enough, she will have ample liquidity to repay all counterparties, making it individually optimal for a hedge fund to roll over, independent of others' actions. We establish when such extreme cases arise and show the existence of intermediate situations where the dealer is solvent but illiquid.

The situation of a solvent but illiquid dealer introduces a coordination problem among hedge funds akin to coordination problems in currency attacks (Morris and Shin, 1998), risky debt rollover and bank runs (Diamond and Dybvig, 1983; Morris and Shin, 2004; Rochet and Vives, 2004; Goldstein and Pauzner, 2005; He and Xiong, 2012; Vives, 2014), credit market freezes (Bebchuk and Goldstein, 2011), and investment funds (Chen, Goldstein, and Jiang, 2010; Liu and Mello, 2011). As is generally the case in coordination games, multiple equilibria may exist. In order to find a unique equilibrium and characterize the optimal contracting terms that give rise to it, we model an incomplete information game (global

purposefully shut down that channel to focus on fragility stemming from an abrupt withdrawal of collateral. In the model this comes naturally because the underlying collateral completely insures the cash lender from any loss.

game), similar to Goldstein and Pauzner (2005), where hedge funds receive noisy signals about the (fundamental) expected value of the dealer's risky investment. Moreover, we extend their framework by introducing a stochastic liquidation value for the risky investment, which is proportional to its fundamental value (see also Kashyap et al., 2017, for such an extension under a different source of stochastic uncertainty). On the one hand, a stochastic liquidation value enables the endogenous derivation of the regions for fundamentals where individual actions are independent of other funds' actions.[3] On the other hand, it adds additional complexity in the proof of the uniqueness of a threshold equilibrium. Thus, we extend the proof in Goldstein-Pauzner to account for stochastic liquidation values.

Establishing a unique threshold equilibrium shows the existence of a panic-based run. That is, even though fundamentals may not be bad enough to make the dealer insolvent, hedge funds incentives to withdraw early can render the dealer illiquid. This mechanism highlights a novel fragility in the short-term funding intermediation process: a coordination failure amongst collateral providers. The main contribution of this paper is to formalize counterparties' strategic complementarities and to characterize how these complementarities can lead to a dealer's endogenous default due to fundamental- or panic-based *collateral runs*. Moreover, we underscore the relationship between the amount of overcollateralization (i.e., repo haircuts), the repo rate, and the dealer's stability.

Note that the underlying collateral pledged by hedge funds and re-used by the dealer can differ significantly from the risky asset purchased by the dealer. Specifically, the underlying collateral can be extremely safe, yet there can still can be a collateral run. The risk that collateral providers face does not come from their own assets, but rather from the dealer's use of the excess funds she raises with them. Duffie (2013) recognizes that an important source of liquidity for dealers stems from their levered counterparties' assets pledged as collateral, while Infante (2017) characterizes the optimal contracting terms that lead to a liquidity windfall whenever a dealer intermediates repos from one cash lender to one cash borrower.

[3]These are known as upper and lower dominance regions and are essential for the existence of equilibrium. The upper dominance region is defined as the area where fundamentals are so good that an individual hedge fund rolls over its repo even if all other funds choose to withdraw. Allowing the liquidation value of risky investment to move with the realization of fundamentals facilitates the endogenous derivation of the upper dominance region. The lower dominance region is defined as the area where fundamentals are so bad that an individual hedge fund withdraws its collateral even if all other funds choose to roll over. The fact that the dealer defaults in some state of the world is sufficient to guarantee the existence of the lower dominance region.

However, neither of these two papers examines how such liquidity windfalls can introduce dealer illiquidity, coordination failures, and run risk.[4]

An important motivating example of our paper is the demise of Bear Stearns in March 2008. Anecdotally, in the days leading up to its collapse, the firm suffered a large outflow of counterparties that not only pulled their cash but also their collateral from the firm. Using the Federal Reserve Bank of New York's (FRBNY) weekly survey of primary dealers (FR 2004), we can estimate a lower bound on the total amount of cash that Bear Stearns accessed through rehypothecation. Specifically, the FR 2004 asks primary dealers to report the total amount of secured financing extended (Securities In), the total amount of secured financing received (Securities Out), and their outright positions for different asset classes. Importantly, the survey asks dealers to report the total amount of funds received and distributed though secured financing transactions, not the value of the collateral posted. Therefore, these data can be used to estimate the amount of liquidity obtained through contracting differences. Figure 1 shows Bear Stearns' repo activity in the months leading up to its default. The difference between Securities Out (green line) and Securities In plus the firm's net position (red line) is an estimate of a lower bound on the total amount of funds raised through differences in haircuts.[5] From the figure, it can be appreciated that 1) the lion's share of securities the firm could post in secured financing transactions came from collateral sourced from their counterparties (blue line), and 2) before the sharp drop in activity, the estimated cash stemming from different contracting terms reached $50 billion, approximately a third of the firm's entire repo book. A withdrawal of collateral effectively eliminated this additional liquidity windfall. To put this magnitude into perspective, Figure 2 plots the estimated cash windfall as a fraction of the total repo book for both Bear Stearns and average of the remaining primary dealers. These estimates suggest that, relative to its peers, Bear Stearns relied heavily on differences in contracting terms as a source of liquidity.

Our paper is also related to the theoretical literature that characterizes optimal contract-

[4]Infante (2017) provides a brief discussion of the relevant institutional details surrounding the re-use of collateral in the United States. In particular, in the context of repo, there are no limits to rehypothecation.

[5]The lower bound depends on an important restriction that securities dealers face: the box constraint. Broadly speaking, the box constraint is a physical restriction that forces dealers to have access to securities, either by owning them outright or by borrowing them, in order to deliver to a counterparty. Huh and Infante (2017) characterize how this constraint is important for bond market intermediation and how to interpret the data in the FR 2004. Details on the lower-bound calculation and some potential caveats are in subsection C of the Appendix.

ing terms and instability in collateralized short-term funding markets. Fostel and Geanakoplos (2015) derive the optimal haircuts on secured debt. Geanakoplos (2003), Fostel and Geanakoplos (2008)—including a series of subsequent papers—and Simsek (2013) study the interlinkages between asset prices, haircuts and leverage over the cycle as well as the implications for investment and financial stability. Martin et al. (2014) detail the contracting terms that lead to traditional cash-driven repo runs.Ahnert et al. (2018) study how the over-collateralization of long-term secured debt can affect the incentives of short-term unsecured debt holders to run.[6] We differ from these papers because we examine a distinct source of instability in repo intermediation. In the aforementioned papers, the instability stems from the liability side of the balance sheet; cash lenders may be less willing to provide funding and either require higher margins, leading to borrower deleveraging, or withdraw their funding altogether in a coordinated run episode. In contrast, the instability we study in this paper is borne from the asset side of an intermediaries balance sheet; borrowers may collectively withdraw their collateral even if cash-lenders' claims are safe with stable haircuts (and not procyclical) and have no incentive to run.[7]

The rest of the paper is structured as follows. Section 2 presents the model setup, detailing the economic environment, the main actors, and their incentives. Section 3 characterizes the coordination problem hedge funds face, their threshold strategies, and the regions where fundamental- or panic-based runs can materialize. Section 4 presents the problem the dealer faces given hedge funds' threshold strategies, characterizes the optimal contracting terms, and numerically shows how the equilibria can change with fundamentals. Finally, section 5 gives some concluding remarks. All proofs are relegated to the Appendix.

2 Model Setup

The model consists of three periods $t \in \{0, 1, 2\}$ and is populated by three types of agents; a broker-dealer (D), a continuum of hedge funds (H), and a continuum of money market funds

[6]Many other theoretical papers have studied spirals and freezes in short-term funding markets. Some examples are Brunnermeier and Pedersen (2009), Acharya et al. (2011), Diamond and Rajan (2011), and Ahnert (2016). As mentioned, we differ from this literature because we mute the rollover risk of cash lenders positions.

[7]In the context of micro finance, Bond and Rai (2009) also study instabilities arising from coordinated defaults impacting the asset side of a lender's balance sheet.

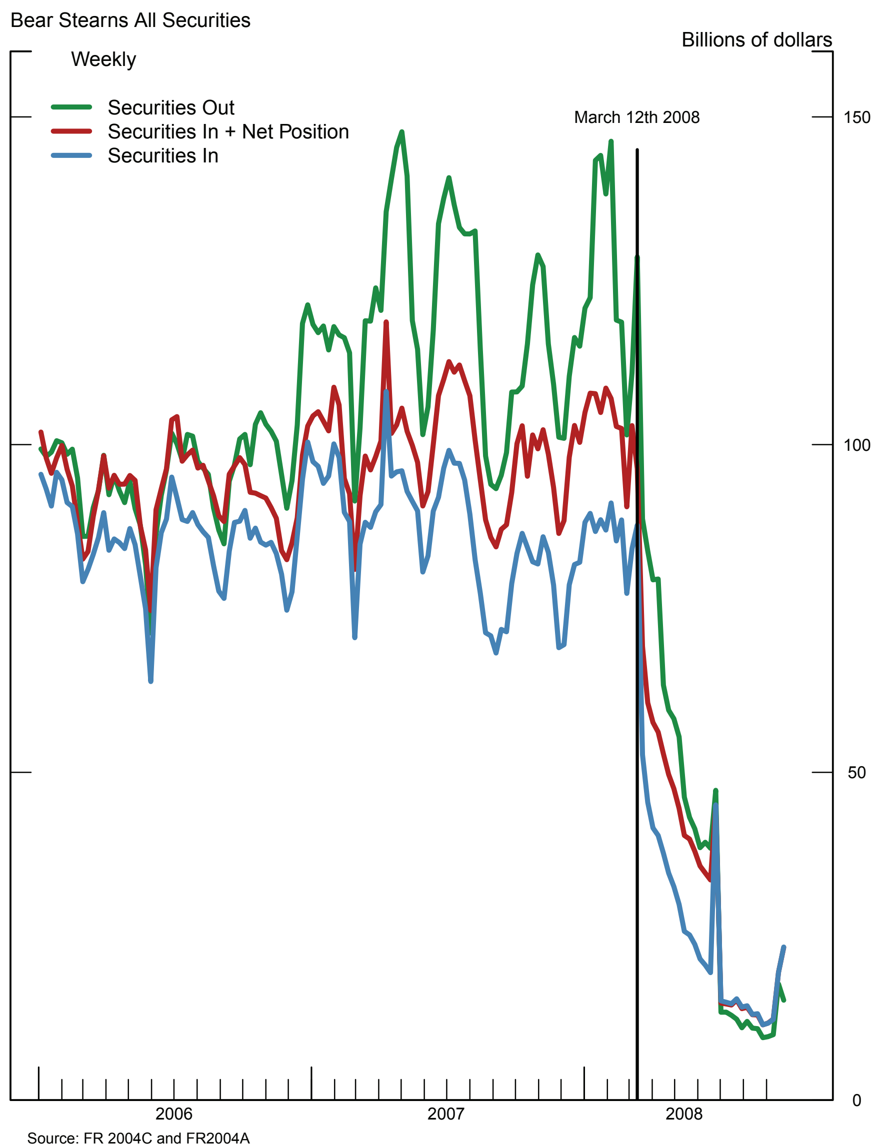


Figure 1: Securities Position of a Bear Stearns
Figure shows the total amount of secured financing extended (securities in—blue line), the total amount of secured financing received (securities out—green line), and the sum between securities in and the dealers' net-securities position (red line). The difference between the green line and the red line proxies for the additional liquidity the dealer reaps from re-using collateral. Source: FR 2004.

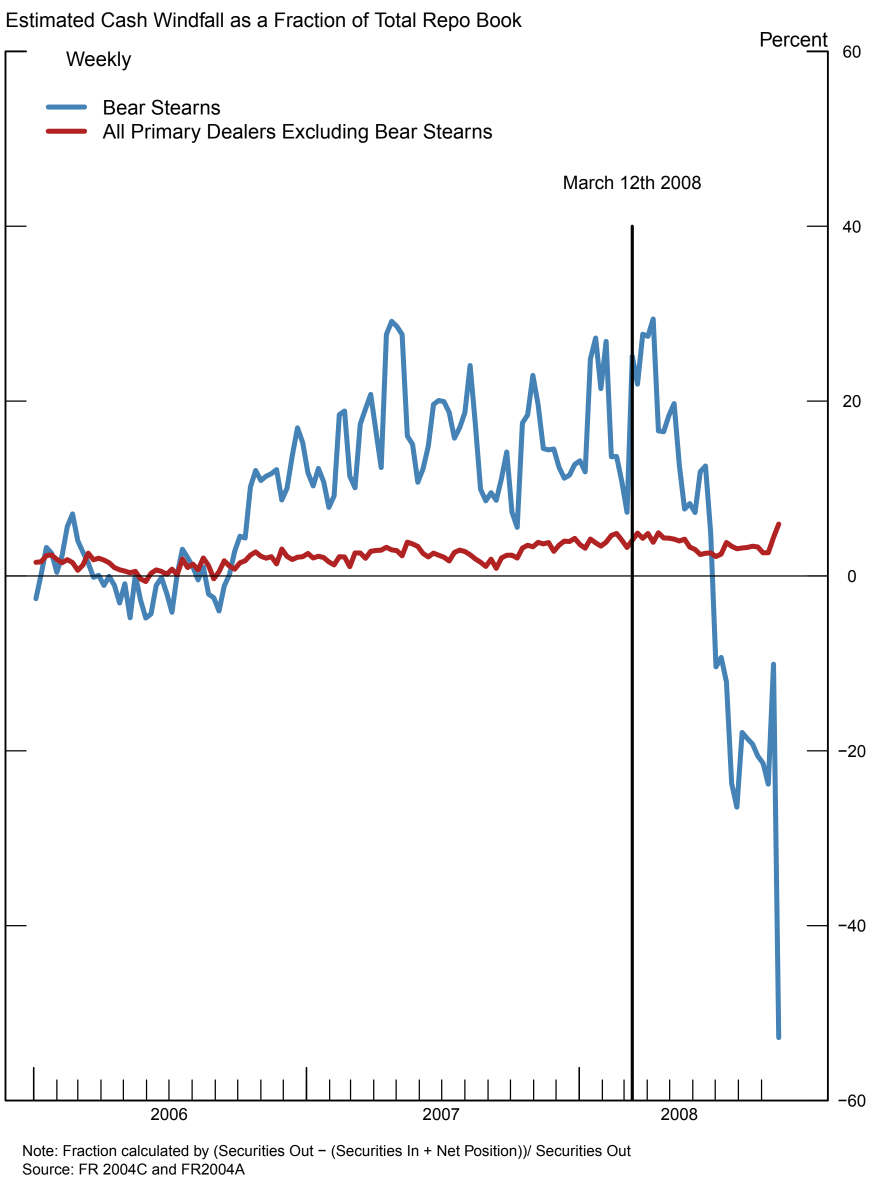


Figure 2: Liquidity from Rehypothecation as a Fraction of Total Repo Activity
Figure shows the lower-bound estimate of the liquidity sourced through dealers' repo activity (securities out minus securities in and net position) as a fraction of their total secured financing (securities out) for both Bear Stearns and average fraction for the rest of the primary dealer community. Source: FR 2004.

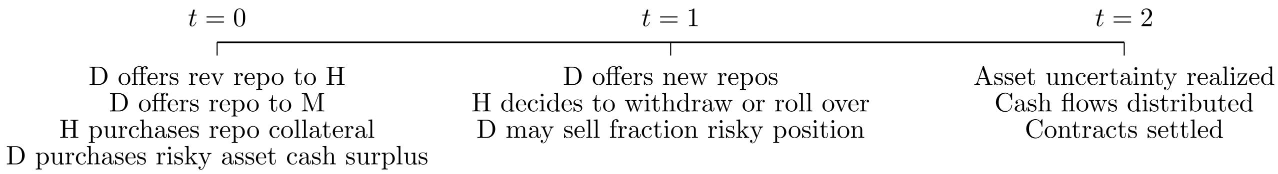


Figure 3: Model Timeline

(M). The money fund sector is assumed to be competitive. The dealer is (potentially) risk averse, with a payoff function u that satisfies $u(0) = 0$, hedge funds are risk-neutral, and money market funds are "very" risk averse.[8] All agents discount the future the same way. The timeline is presented in Figure 3.

Hedge funds would like to borrow to invest in a (safe) asset T, which is in perfect elastic supply and is worth 1 in every period. Abusing notation, T will also denote the amount of the asset purchased. Each hedge fund borrows money from the dealer at $t = 0$ (a reverse repo from the dealer's point of view), purchases the asset, and pledges it as collateral. Simultaneously, the dealer enters into a repo contract with money market funds, whereby she rehypothecates the pledged collateral. We assume that the "number" of reverse repo contracts is equal to the "number" of repo contracts, or in other words, an individual money market fund is anonymously funding an individual hedge fund through dealer intermediation. Repo contracts are short term, i.e., they mature after one period, and can be rolled over at $t = 1$ as we describe further below.

Apart from intermediating between hedge funds and money funds, the dealer can also invest at $t = 0$ in a risky asset $\tilde{R}$ which pays off R^U with probability θ and R^D otherwise in $t = 2$ per unit purchased, where $R^U > 1 > R^D \geq 0$. The state of the world θ is realized at $t = 1$ and follows a uniform distribution $\theta \sim U[0, 1]$. The risky asset has price of 1, is in perfect elastic supply, and its expected value, conditional on θ, is denoted by $\mathbb{E}_\theta(\tilde{R}) = \overline{R}_\theta$. Although the risky asset fully pays the random return if it is allowed to mature, it can be liquidated at $t = 1$ for a discount. The liquidation value is a fraction, $\lambda \in (0, 1)$, of $\overline{R}_\theta$. We will assume that the unconditional expected liquidation value is higher than the initial price of the asset, i.e., $\lambda(R^U + R^D)/2 > 1$. Thus, there is liquidity risk because liquidation is

[8]The assumption that the dealer's payoff function has $u(0) = 0$ is merely for simplicity. "Very" risk averse money funds will be useful to focus on the collateral channel, rather than the traditional repo-run channel.

generally inefficient, but unconditionally the project has a positive net present value even if liquidated. In particular, we impose a stricter version of this condition—$\lambda R^U > 2$—to also allow R^D to go arbitrarily close to 0 without altering the other parameters.

At $t = 1$, the dealer offers new repo contracts to counterparties, and both hedge funds and money funds decide whether to roll over their positions. Given our assumptions, described in detail later, money funds will always roll over their repos as along as the dealer rehypothecates the safe asset. In other words, the collateral from hedge fund repos that are rolled over are then used to issue repos to money funds.[9] If the repo is rolled over, we assume that the closing leg of existing repos (morning) and the opening leg of new repos (evening) happen simultaneously and, thus, we focus on net flows of funds.

However, an individual hedge fund may decide against rolling over its repo and rather withdraw its collateral at $t = 1$. If enough hedge funds withdraw their collateral, the dealer must sell a fraction of its risky asset position at its liquidation value in order to collect the collateral from money funds, which are the property of the withdrawing hedge funds. If asset sales are not enough to recuperate withdrawing hedge funds' collateral, the dealer is liquidated. Upon the dealer's liquidation, money funds that were not repaid keep the collateral, and hedge funds that were not served receive nothing.

At $t = 2$, conditional that the dealer survives, the final payoff on the risky investment is realized, cash flows are distributed, and contracts are settled. The payoffs accruing to the three agents will not only depend on the repo contract terms, but also on the realization of θ and the portion of hedge funds that withdraw their collateral at $t = 1$, which we denote by $\mu \in [0, 1]$. In section 2.1-2.3 we present the payoffs to the dealer, the hedge funds, and the money funds as function of the contract terms, as well as the level of fundamentals, θ, and the portion of hedge funds withdrawing, μ.

[9] We intentionally abstract from the dynamics governing the roll over decision of cash providers (money funds), which have received ample attention in the literature, in order to focus on the dynamics governing the roll-over decision of the providers of collateral (hedge funds).

2.1 Dealer

The dealer offers repo contracts to hedge funds and money funds.[10] The repo contracts issued at time t, to/from counterparty $j \in \{H, M\}$,[11] have two terms: haircut m_t^j and repurchase price F_t^j.[12] It will be useful to introduce some additional notation. Let $\Delta m_t = (T - m_t^M) - (T - m_t^H) = m_t^H - m_t^M$ be the incremental cash flow the dealer receives from intermediating the initial leg of the repo at t. Moreover, denote by $\Delta F_t = F_t^H - F_t^M$ the incremental cash flow that the dealer receives from the closing leg of the repo at t, paid at $t + 1$.[13]

The net cash flow to the dealer in $t = 0$, i.e., Δm_0, is used to purchase the risky asset. That is, the dealer will use the funds stemming from the difference in haircuts to invest in a risky position.

At $t = 1$, the dealer needs to unwind the repos for the μ hedge funds withdrawing, which is achieved by repurchasing the collateral from money fund at a price F_0^M and returning it to hedge funds at a price F_0^H. The net cash flow from these operations is $\mu \Delta F_0 < 0$. The available resources to meet this negative cash flow can come either from collecting additional cash from hedge funds that roll over their repos or from liquidating (part of) the risky asset. The former yields $(1 - \mu)((F_0^H - F_0^M) + (T - m_1^M) - (T - m_1^H)) = (1 - \mu)(\Delta F_0 + \Delta m_1)$, i.e., the sum of cash owed from repos in $t = 0$ and cash received from repos in $t = 1$ can be positive or negative. The latter yields $\xi(\mu, \theta)\lambda \overline{R}_\theta \Delta m_0$, where $\xi(\mu, \theta) \in [0, 1]$ is the fraction of the risky asset the dealer liquidated as a function of the portion μ of hedge fund withdrawing their collateral. For the subsequent analysis, we shall consider the case in which the dealer will have a positive net cash flow in the interim period from hedge funds rolling over their position. That is, the positive cash flow from the rehypothecation of collateral at $t = 1$ is

[10] For simplicity, we will assume that the dealer has offers take-it-or-leave it contracting terms, allowing her to extract all the surplus from her counterparties. The results are qualitatively similar if the dealer has some say over hedge funds' contracting terms.

[11] Note that repo counterparties are with respect to the dealer.

[12] Hence, $F_t^H/(T - m_t^H) - 1$ and $F_t^M/(T - m_t^M) - 1$ are the implied interest rates promised to D from H and to M from D, when the reverse repo and repo contracts mature, respectively. In addition, the market practice to quote haircuts is 1 minus the loan amount over the collateral value, which in the model translates to m_t^j/T.

[13] Throughout the paper we will assume that in equilibrium $\Delta F_t \leq 0$ and $\Delta m_t \geq 0$. We prove that this is, indeed, the case in section 4.

higher than the outflow from closing the existing repo contracts:

$$\text{C0:} \qquad \Delta m_1 + \Delta F_0 \geq 0. \tag{1}$$

Depending on the number of hedge funds withdrawing for given θ, three outcomes are possible at $t = 1$. First, for $\mu \in [0, \mu_S]$, the dealer can raise additional funds at $t = 1$ to meet the withdrawals and refrain from selling a fraction of the risky asset. Second, for $\mu \in (\mu_S, \mu_R]$ the dealer need to additionally liquidate part of the risky asset to meet the withdrawals of collateral, i.e. $\xi \in (0, 1)$. Third, for $\mu \in (\mu_R, 1]$ the dealer cannot meet all the withdrawals even if she liquidates all of the risky assets. We have implicitly considered that the dealer will first use all the excess cash in hand to meet outflows before liquidating the risky asset.

The threshold μ_S is the maximum number of withdrawals that can be fulfilled by the additional cash collected, that is,

$$\begin{aligned} &\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) > 0 \\ \Rightarrow &\mu < \mu_S \equiv 1 + \frac{\Delta F_0}{\Delta m_1}. \end{aligned} \tag{2}$$

Given that $\Delta m_1 > 0$ and $\Delta F_0 < 0$, μ_S is less than one but is strictly positive only if hedge funds that roll over contribute additional cash, i.e., $\Delta F_0 + \Delta m_1 > 0$.

The threshold μ_R is the maximum number of withdrawals that can be fulfilled by the additional cash collected plus the liquidation of all the risky holdings, that is,

$$\begin{aligned} &\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \lambda \overline{R}_\theta \Delta m_0 > 0 \\ \Rightarrow &\mu < \mu_R \equiv 1 + \frac{\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0}{\Delta m_1}. \end{aligned} \tag{3}$$

For $\mu \in [\mu_R, 1]$, only a fraction of the hedge funds withdrawing will collect their collateral. That is, a fraction $f(\mu, \theta)\mu$ of money funds get their repayment back and deliver the collateral to the dealer, which is routed back to the hedge funds that decided to withdraw, following a sequential service constraint. The fraction that gets repaid, whenever all risky assets are sold, is given by

$$
\begin{aligned}
& f(\mu,\theta)\mu\Delta F_0 + (1-\mu)(\Delta F_0 + \Delta m_1) + \lambda \overline{R}_\theta \Delta m_0 = 0 \\
\Rightarrow & f(\mu,\theta) = -\frac{\lambda \overline{R}_\theta \Delta m_0 + (1-\mu)(\Delta F_0 + \Delta m_1)}{\mu \Delta F_0}.
\end{aligned} \tag{4}
$$

Comparing (2)) and (3) it is clear that $\mu_S < \mu_R$. Consequently, the fraction of the risky asset holdings that are liquidated for $\mu \in (\mu_S, \mu_R)$ is

$$
\xi(\mu,\theta) = -\frac{\Delta F_0 + (1-\mu)\Delta m_1}{\lambda \overline{R}_\theta \Delta m_0}. \tag{5}
$$

For $\mu < \mu_R$, the dealer survives the collateral withdrawals and can continue to the final period. However, depending on the level of μ and the realization of $\tilde{R}$, the available resources may not be enough to guarantee repayment of the money funds that rolled over their repos. In that case, the dealer defaults, money funds seize and sell the collateral, and any remaining resources are distributed pro rata to the hedge funds that rolled over their repos at $t = 1$.

First, consider the case that the dealer has enough money to serve early withdrawals without liquidating any assets, i.e., $\mu \in [0, \mu_S)$. The cash flow to the dealer in the final period is equal to $\tilde{R}\Delta m_0 + \Delta F_0 + (1-\mu)\Delta m_1 + (1-\mu)\Delta F_1$. When there is no selling at $t = 1$, dealer optimization should result in positive cash flow if R^U realizes. However, if R^D realizes, the cash flow may be negative, resulting in dealer default. In that case, the available resources are distributed pro rata to the $1-\mu$ hedge funds that rolled over at $t = 1$, and each individual hedge fund receives:

$$
G_S^D(\mu,\theta) = \frac{R^D \Delta m_0 + \Delta F_0 + (1-\mu)\Delta m_1}{1-\mu}. \tag{6}
$$

To keep the model interesting, we ensure that after a bad outcome the amount raised in the interim period is not enough to make all money funds whole in the final one; that is,

$$
\text{C1:} \qquad R^D \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 \leq 0. \tag{7}
$$

Condition (7) implies that even if all hedge funds roll over in the interim period (i.e., $\mu = 0$), there is not enough wealth to payoff cash lenders' entire claim if R^D realizes. This restriction is important to guarantee the existence of a region for fundamentals where an individual hedge fund withdraws its collateral independent of their beliefs about the actions of other hedge funds, i.e., the lower dominance region (see section 3.2 for details).

Similarly, condition (8) below implies that the dealer is solvent at $t = 2$ if R^U realizes and all funds decide to roll over at $t = 1$. This condition will always hold in equilibrium, because the dealer would optimally choose contract terms yielding positive profit in the good state, i.e., $R^U \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 > 0$, even if condition (1) binds.

$$\text{C2:} \qquad R^U \Delta m_0 + \Delta F_1 > 0. \tag{8}$$

Second, consider the case that the dealer has to liquidate some, but not all, of her assets to serve early withdrawals, i.e., $\mu \in [\mu_S, \mu_R)$. The cash flow to the dealer in the final period is equal to $\tilde{R}\Delta m_0(1 - \xi(\mu, \theta)) + (1 - \mu)\Delta F_1$, where $\xi(\mu, \theta)$ is given by (5). For realization $\tilde{R} = R^D$ it is obvious, given condition (7), that the dealer defaults for all $\mu \in [\mu_S, \mu_R)$. Hence, what is left in the dealer's portfolio is distributed pro rata to hedge funds that rolled over at $t = 1$, and each individual hedge fund receives

$$G_I^D(\mu, \theta) = \frac{R^D \Delta m_0 + \frac{R^D}{\lambda \bar{R}_\theta} (\Delta F_0 + (1 - \mu)\Delta m_1)}{1 - \mu}, \tag{9}$$

or, in other words, they collect the payoff on the risky assets not liquidated at $t = 1$, since $\Delta F_0 + (1 - \mu)\Delta m_1 \le 0$ for $\mu \in [\mu_S, \mu_R)$.

Yet, the dealer may also default when $\tilde{R} = R^U$. That is, if a large fraction of the asset has to be liquidated, the portfolio payoff may not cover the costs of returning the collateral to hedge funds that rolled over. Denote by μ_I the maximum number of withdrawals after which the dealer default. Then, for $\mu \in [0, \mu_I)$ the dealer is solvent if R^U realizes, while for $\mu \in [\mu_I, \mu_R)$ she defaults. In the latter case, what is left in the dealer's portfolio is distributed pro-rate to hedge funds that rolled over at $t = 1$, and each individual hedge fund receives

$$G_I^U(\mu, \theta) = \frac{R^U \Delta m_0 + \frac{R^U}{\lambda \bar{R}_\theta} (\Delta F_0 + (1 - \mu)\Delta m_1)}{1 - \mu}. \tag{10}$$

Therefore, the threshold μ_I is determined at the largest $\mu \in [\mu_S, \mu_R)$, such that the dealer

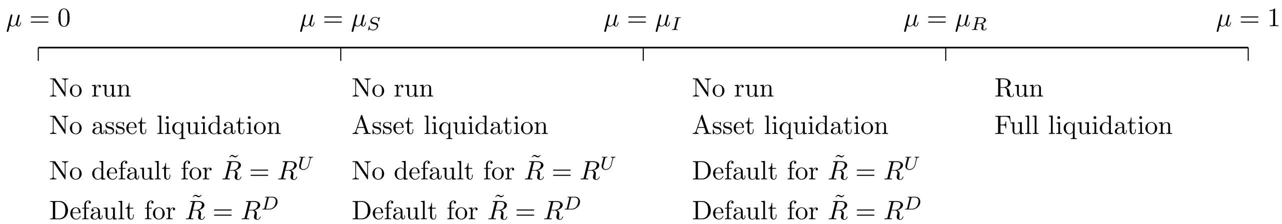


Figure 4: Outcomes as μ Varies for Zero to One for Given Fundamental θ.

is just solvent at $t = 2$ if R^U realizes, i.e.,

$$R^U \Delta m_0 \left(1 + \frac{\Delta F_0 + (1-\mu)\Delta m_1}{\lambda \overline{R}_\theta \Delta m_0}\right) + (1-\mu)\Delta F_1 \geq 0$$

$$\Rightarrow \mu \leq \mu_I \equiv 1 + \frac{R^U \left(\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0\right)}{\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1}. \tag{11}$$

Lemma 1. *The maximum level of withdrawals that the dealer is solvent in the good state at $t = 2$ is above the level that she starts liquidating assets and below the level that she is fully liquidated at $t = 1$, i.e., $\mu_S < \mu_I < \mu_R$.*

Lemma 1 will be useful to show the existence and uniqueness of a run equilibrium in section 3.2.

Figure 4 summarizes the different outcomes for given fundamental θ depending on the number of hedge funds withdrawing at $t = 1$.

2.2 Hedge Funds

There are a continuum of hedge funds, each of which approaches the dealer to finance the purchase of the riskless asset T, such as Treasuries. Hedge funds are ex ante identical and value holding T above and beyond its fair value. That is, the hedge fund receives non-pecuniary benefits for holding the asset, which potentially accrue from hedging motives, demand for safe assets or other reasons, which magnify its value by $\eta > 1$. Without loss of generality, we assume that hedge funds get the extra valuation for assets held at the final period.[14] Hedge funds start out with an initial endowment of W_0 which, along with the repo issued to the dealer, allows them to purchase T.

The payoff to an individual hedge fund depends on the realization of θ, the number of hedge funds that withdraw, μ, and the action that it takes in the roll-over stage. Denote by $\alpha = \{0, 1\}$ the strategy set of a hedge fund, where $\alpha = 0$ stands for withdrawing and $\alpha = 1$ for rolling over. The utility that a hedge fund receives can be expressed by $U^H(\mu, \theta; \alpha)$. Note that in equilibrium either all hedge funds will roll over, i.e., $\alpha = 1$ implying $\mu = 0$, or all hedge funds will withdraw, i.e., $\alpha = 0$ implying $\mu = 1$. But, in writing the utility payoff for out-of-equilibrium paths, it is important to determine the threshold strategy in the incomplete information game described in section 3.2.

First, consider the case that a hedge fund rolls over at $t = 1$, i.e., $\alpha = 1$. The available cash at the end of $t = 1$ after rolling over, which can be invested in additional Treasuries, is equal to $W_0 + (T - m_0^H) - F_0^H + (T - m_1^H) - T$, i.e., what is left of the initial wealth after receiving cash from the starting leg of both repos $(T - m_0^H) + (T - m_1^H)$, paying the closing leg of the initial repo F_0^H, and purchasing the collateral at the onset of the game T.[15] Note that the available cash at $t = 1$ is independent of the realization of fundamentals, θ, and the portion of hedge funds withdrawing, μ. However, the final payoff at $t = 2$ will depend on θ and μ as they determine whether a run occurs and the probability that the dealer defaults at $t = 2$.

For a given realization of fundamentals θ and $\mu < \mu_S$, a hedge fund that rolls over can repurchase its collateral at price F_1^H and enjoy a utility payoff ηT if the dealer does not default at $t = 2$. This occurs with probability θ. On the other hand, if R^D realizes, the dealer defaults and the hedge fund is repaid its share of the dealer's remaining portfolio: $G_S^D(\mu, \theta)$ in cash which does not yield the utility benefit η. The expected utility of an individual hedge fund that rolls over is, then,

$$U^H(\mu < \mu_S, \theta; 1) = \theta(\eta T - F_1^H) + (1 - \theta) G_S^D(\mu, \theta) + \eta \left(W_0 - m_0^H + T - F_0^H - m_1^H \right). \quad (12)$$

[14]Alternatively, hedge funds could also receive these non-pecuniary benefits for holding the asset between $t = 0$ and 1. This assumption unnecessarily complicates the model without providing any meaningful insights.

[15]Recall that Treasury holdings at $t = 2$ yield a utility payoff $\eta > 1$, thus a hedge fund will invest all available cash at $t = 1$ in Treasuries.

If $\mu \in [\mu_S, \mu_I)$, a hedge fund that rolls over can still repurchase its collateral if R^U realizes at $t = 2$, but it receives a cash payment $G_I^D(\mu, \theta)$ otherwise, which is different than $G_S^D(\mu, \theta)$ because the dealer had to liquidate some assets at $t = 1$ to serve the higher early withdrawals. The expected utility is, then,

$$U^H(\mu_S \leq \mu < \mu_I, \theta; 1) = \theta(\eta T - F_1^H) + (1-\theta)G_I^D(\mu, \theta) + \eta \left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{13}$$

If more hedge funds withdraw, the dealer will default in the good state as well, for $\mu \in [\mu_I, \mu_R)$, receiving its share of the dealer's remaining portfolio in either state,

$$U^H(\mu_I \leq \mu < \mu_R, \theta; 1) = \theta G_I^U(\mu, \theta) + (1-\theta)G_I^D(\mu, \theta) + \eta \left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{14}$$

If the withdrawals continue, the dealer will eventually run out of money and will be fully liquidated at $t = 1$ for $\mu \in [\mu_R, 1]$. In this case, a hedge fund that rolled over at $t = 1$ will receive utility

$$U^H(\mu_R \leq \mu \leq 1, \theta; 1) = \eta \left(W_0 - m_0^H + T - F_0^H - m_1^H\right). \tag{15}$$

In these last two cases, the hedge fund cannot repurchase back its collateral and the utility benefit η applies only to the additional Treasuries purchased with the remaining cash at $t = 1$.

On the other hand, a hedge fund that does not roll over at $t = 1$, i.e., $\alpha = 0$, is able to invest $W_0 - m_0^H$ in Treasuries, plus any incremental cash from closing the initial repo position. The latter will depend on whether the dealer is fully liquidated at $t = 1$. If the dealer has enough resources to serve all early withdrawals, a hedge fund that does not roll over can repurchase its collateral at $t = 1$ at price F_0^H, receiving a net cash flow $T - F_0^H$ and final utility equal to

$$U^H(\mu < \mu_R, \theta; 0) = \eta \left(W_0 - m_0^H + T - F_0^H\right). \tag{16}$$

If the dealer cannot serve all of the early withdrawals, then a hedge fund that does not roll over will only be able to repurchase its collateral with probability $f(\mu, \theta)$ given by (4), and the expected utility is equal to

$$U^H(\mu_R \leq \mu \leq 1, \theta; 0) = \eta \left(W_0 - m_0^H + f(\mu, \theta) \cdot (T - F_0^H) \right). \tag{17}$$

It is important to note that η plays a dual role. First, it generates gains from trade to induce the hedge fund to participate. But, it also creates incentives for the hedge fund to roll over. Specifically, in equation (12), when the hedge fund rolls over it enjoys the entire benefit of having T, that is, in the good state it receives $\eta T - F_1^H$. From equation (16), if the hedge fund withdraws it uses the net amount of funds to purchase Treasuries, that is, it earns $\eta(T - F_0^H)$. This is the effect of leverage: allowing investors to increase their exposure, which in the model creates incentives to roll over.

As derived later in section 3.2, hedge funds will follow a strategy such that all roll over at $t = 1$ if the realization of fundamentals, θ, is above a threshold θ^*, and all withdraw their collateral if θ is below θ^*. Moreover, every individual hedge fund should be willing to enter into a repo contract both at $t = 0$ and $t = 1$ given that all other hedge funds follow the equilibrium strategy. An individual hedge fund would not choose to deviate and will enter the repo contract at $t = 0$ if the following participation constraint is satisfied:

$$PC_0: \quad \int_{\theta^*}^{1} \eta \cdot (T - F_0^H) d\theta + \int_{0}^{\theta^*} \eta \cdot f(1, \theta) \cdot (T - F_0^H) d\theta - \eta \cdot m_0^H \geq 0, \tag{18}$$

where $f(1)$ is given by (4) for $\mu = 1$. In other words, an individual hedge fund will not deviate from the equilibrium strategy at $t = 0$ if the expected cash flow at $t = 1$ is higher than the original margin contribution. Cash flows are scaled by η because the hedge fund can use the cash to invest in Treasuries at $t = 1$ and, thus, receive the utility benefit by holding them until the final period.

Moreover, an individual hedge fund will not deviate from the equilibrium strategy at $t = 1$ if for every $\theta \geq \theta^*$ the following participation constraint is satisfied:

$$PC_1: \quad \theta(\eta \cdot T - F_1^H) + (1 - \theta) G_S^D(0, \theta) - \eta \cdot m_1^H \geq 0, \tag{19}$$

where $G_S^D(0, \theta)$ is given by (6) for $\mu = 0$. In other words, an individual hedge fund will only roll over at $t = 1$ for $\theta \geq \theta^*$ if the expected benefit is higher than the outside option of investing the margin in Treasuries, which is equal to $\eta \cdot m_1^H$. The former is equal to the utility benefit of repurchasing the collateral, $\eta \cdot T$, minus the repurchase price, F_1^H occurring with probability θ plus the cash flow received when the dealer defaults, $G_S^D(0,\theta)$, occurring with probability $1-\theta$. Given that the right-hand side in (19) is increasing in θ, it suffices that the participation constraint is satisfied for θ^*. We establish this in Corollary 1 in section 3.2.

Note that the decision to enter a repo at $t=0$ is independent of the decision to enter a repo at $t=1$. If PC_0 is not satisfied, but PC_1 is, then an individual hedge fund will deviate from the equilibrium strategy at $t=0$, and vice versa. Hence, both (18) and (19) need to hold in equilibrium, which restricts the ability of the dealer to extract all surplus from hedge fund. As we discuss later, (19) will not be binding in equilibrium because hedge funds need to have the proper incentives to roll over in the incomplete information game, while a binding (18) restricts the ability of the dealer to set a very high margin, m_0^H, or repurchase price, F_0^H. Finally, integrating (19) over $[\theta^*, 1]$, and adding (18) as well as $\eta \cdot W_0$ on both sides yields $\int_{\theta^*}^1 U^H(0,\theta;1)d\theta + \int_0^{\theta^*} U^H(1,\theta;0)d\theta \geq \eta \cdot W_0$. Hence, the overall utility of a hedge fund playing the equilibrium strategy is higher than the utility in autarky. $U^H(\mu=0,\theta;1)$ and $U^H(\mu=1,\theta;0)$ are given by (12) through to (17).

Finally, using the period 0 participation constraint and condition (1), we can prove the following Lemma, which will be useful in later analysis.

Lemma 2. *The contract terms are such that:*

1. *The dealer's liabilities at $t=0$ are higher than the cash inflow from the rehypothecation of collateral, i.e., $-\Delta F_0 > \Delta m_0$.*
2. *The cash inflow from the rehypothecation of collateral at $t=1$ is higher than at $t=0$, i.e., $\Delta m_1 > \Delta m_0$.*

As discussed in section 4, the dealer will choose contract terms that push hedge funds to their period 0 participation constraint *in equilibrium*. Hence, we can rewrite (18) as $-\Delta F_0 \geq g(\theta^*)\Delta m_0$, where $g(\theta^*) > 1$ from Lemma 2 and given by

$$g(\theta^*) = \frac{1 - \lambda \left[\left(R^U - R^D \right) \frac{\theta^{*2}}{2} + \theta^* R^D \right]}{1 - \theta^*}. \tag{20}$$

2.3 Money Funds

Money funds are the providers of cash and enter into reverse repo contracts with the dealer at $t = 0$ and $t = 1$. There is a continuum of identical money funds, each providing the dealer with $T - m_t^M$ at t, where m_t^M is the margin that the dealer has to contribute and T is the value of the Treasuries pledged as collateral. Denote by F_t^M the repurchase price agreed at t.

Our focus is on the incentives of collateral providers (cash borrowers) to withdraw their collateral rather than on the incentive of cash lenders to withdraw their funding, which has received a lot of attention in the literature. Hence, we make assumptions such that the cash lenders do not face a coordination problem which prompts a run on the dealer. This will allow us to isolate our mechanism and focus on the run dynamics stemming, instead, from a coordination problem among the providers of collateral. As we will discuss in detail, a run by collateral providers can occur even when repo contracts are over-collateralized and the dealer does not face any funding risk. Combining the two sources of run risk could open interesting avenues for future research.

Specifically, we assume that money funds are "very risk averse" (i.e., infinitely risk averse) such that they will not tolerate a loss. Thus, they must be covered even if there is a run or dealer default. In other words, the repo contracts between the dealer and money funds are over-collateralized or, equivalently, $F_t^M \leq T$. All contracts that satisfy this condition are acceptable because they completely eliminate the money fund's exposure to the dealer. In case of a run or insolvency of the dealer, the money fund would have immediate access to the collateral (because repo are exempt from automatic stay), selling it onto the market for a value of T—and possibly returning any surplus above and beyond it was owed. It is in the dealer's interest to maximize the funds she obtains from money funds at $t = 0$ and $t = 1$ rather than receiving some residual cash at $t = 2$ when she is insolvent and protected by limited liability. In other words, using the safe asset as collateral, the dealer can borrow (from a competitive money fund market) at zero haircuts, i.e., $m_t^M = 0$, and at repurchase prices $F_t^M = T$, which implies that the recovery value from the sale of Treasuries upon a dealer default is zero.

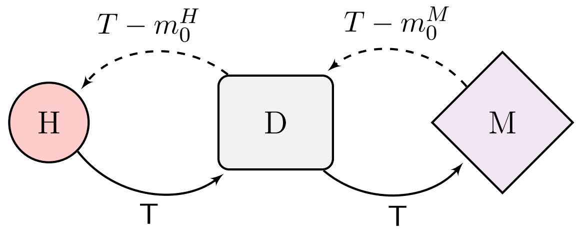


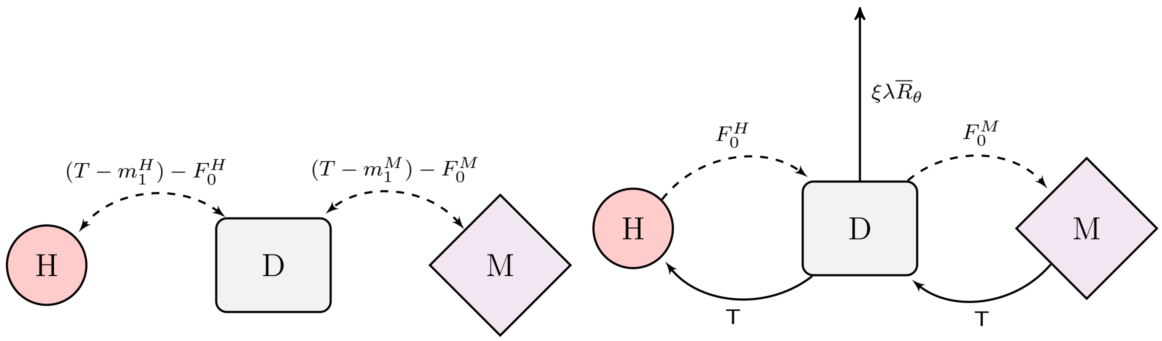
Figure 5: Open Leg of Repo in $t = 0$

2.4 Illustration of Rehypothecation and the Dealer's Balance Sheet

In order to summarize the model setup, we provide visual representations of the rehypothecation process and how the dealer's balance sheet can change. Figures 5–7 illustrate how the intermediation process evolves over time when the dealer meets all its obligations. Figure 5 shows the initial leg of the rehypothecation process, with cash coming into the dealer $T - m_0^M$, a portion of which is then distributed to the hedge fund $T - m_0^H$. Simultaneously, the hedge fund delivers the collateral T to the dealer, which then passes it on to the money fund. Figure 6 shows an individual hedge fund's choice between rolling over (left panel) and withdrawing (right panel). If the hedge fund decides to roll over, the dealer settles the net contracting difference between the hedge fund $(T - m_1^H) - F_0^H$ and the money fund $(T - m_1^M) - F_0^M$. In this case there is only cash settlement, that is, the collateral stays with the money fund. If the hedge fund decides to withdraw, the hedge fund pays its repurchase price F_0^H while the dealer pays the corresponding repurchase price F_0^M, with the collateral begin returned to its original owner. In order to repay F_0^M the dealer may have to sell a fraction of her risky portfolio at a discount $\lambda \overline{R}_\theta$. Finally, for completeness, Figure 7 illustrates the final closing leg of the $t = 1$ repo.

To understand how the dealer might default in $t = 1$, and the nature of hedge funds' coordination problem, it is useful to illustrate the composition of her balance sheet at $t = 0$ and $t = 1$. For simplicity, payables and receivables are marked at book value, and we omit changes in equity because of mark-to-market accounting. Figure 8 shows the dealer's balance sheet at the end of $t = 0$. The liability side contains the dealer's total obligations to money funds, F_0^M. The asset side contains the dealer's total holdings of hedge fund obligations, F_0^H, plus her investment in the risky portfolio, $R\Delta m_0$.

Figure 9 illustrates how the dealer's balance sheet can change for different fractions of



(a) Hedge Fund Rolls Over **(b)** Hedge Fund Withdraws

Figure 6: Close Leg of Repo in $t = 0$/Open Leg of Repo in $t = 1$

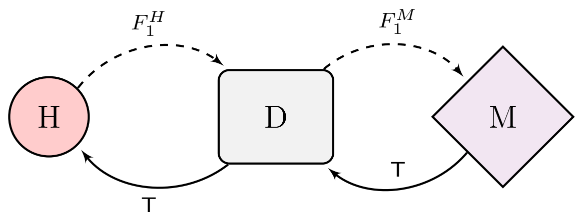


Figure 7: Closing Leg of Repo in $t = 2$

hedge fund withdrawals. In the left panel, only a small fraction of hedge fund withdraw ($\mu < \mu_S$), swapping $t = 0$'s outstanding payables and receivables to $t = 1$ payables and receivables. In this case, contracting terms are such that there is no need to liquidate any of the dealer's risky portfolio. In the middle panel, an intermediate amount of hedge funds withdraw ($\mu \in (\mu_S, \mu_R)$), implying a reduction in the dealer's balance sheet illustrated by the purple boxes. In this case, the dealer is forced to sell a fraction ξ of its risky asset position in order to repay money funds that hold the withdrawing hedge funds' collateral. Note that the reduction assets equals the reduction in liabilities. In the right panel, a large amount of hedge funds withdraw ($\mu > \mu_R$), implying a severe reduction in the dealer's balance sheet, illustrated by the red boxes. In this case the dealer is forced to sell the entire risky asset position, the proceeds of which are not enough to repay the money funds that hold the withdrawing hedge funds' collateral, implying a run on the dealer. Note that in this final case, the cash the dealer is able to raise is not enough to recuperate the collateral from money funds. Thus, the dealer is in default.

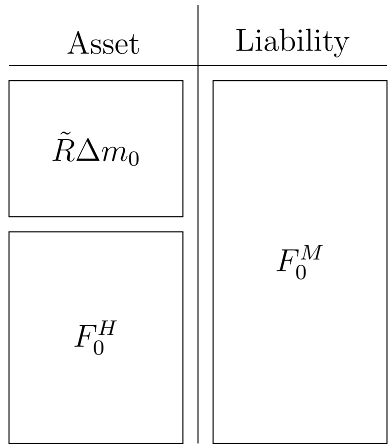
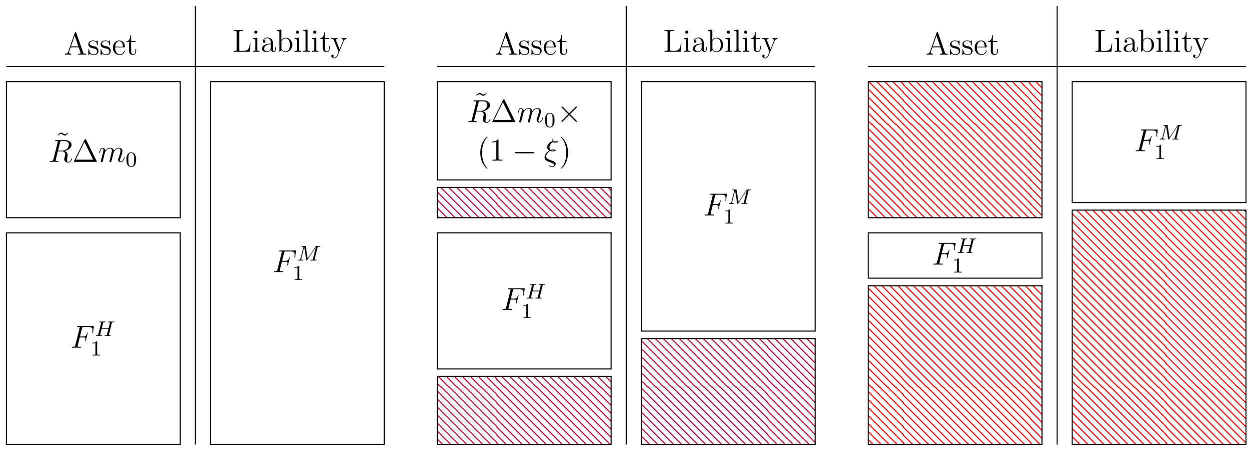


Figure 8: Dealer Balance Sheet at End of $t = 0$
The dealer's liabilities are all the claims she issued to money funds F_0^M, and the dealer's assets are all the claims she purchased from hedge funds F_0^H, plus the risky investment $\tilde{R}\Delta m_0$.

3 Collateral Runs and Coordination Failure

This section examines the decision of an individual hedge fund to withdraw its collateral or roll over its repo contract in the intermediate period. The decision not only depends on the hedge fund's belief of the dealer's solvency, but also on its beliefs of other hedge funds' actions/beliefs. Section 3.1 discusses the case that all hedge funds have full knowledge of the fundamental value θ and shows how the coordination problem arises for certain regions of the parameter space, giving rise to multiple equilibria. Section 3.2 introduces incomplete information whereby each hedge fund receives a private noisy signal about the realization of θ. These signals do not only provide information about θ, but also about other hedge funds signals, allowing an inference about their actions. The higher the signal, the higher the posterior belief about θ and the smaller is the likelihood that other hedge funds receives low signals urging them to withdraw. Both effects reduce the incentive to withdraw. As a result, incomplete information forces hedge funds to coordinate their actions such that they withdraw only for one range of fundamentals in what is known in the literature as a Global Game (see also Carlsson and van Damme, 1993).

In sections 3.1 and 3.2 we derive results assuming that contract terms are pre-determined. Section 4 derives the optimal contract terms in equilibrium and shows the conditions under



(a) $\mu < \mu_S$ **(b)** $\mu \in (\mu_S, \mu_R)$ **(c)** $\mu > \mu_R$

Figure 9: Dealer Balance Sheet at End of $t = 1$
The figures shows the dealer's new balance sheet in the refinancing period. Left-hand panel is after the dealer does not sell any risky assets and is able to roll over her repos. Middle panel is after the dealer sells a fraction of her risky assets and returns the collateral to withdrawing hedge funds (purple boxes). Right-hand panel is after the dealer sells all of her risky assets and cannot return all the collateral to withdrawing hedge funds (red boxes). In this final case, the dealer is said to be run on.

which they are consistent with the existence of a coordination problem.

3.1 Complete Information

Assume that all hedge funds receive a signal that fully reveals the realization of θ at $t = 1$.[16] The signal provides information about dealer's insolvency at $t = 2$, i.e., the probability that R^D realizes, but also about the liquidation value of the dealer's risky investment, $\lambda \overline{R}_\theta \Delta m_0$.

We can establish two regions for fundamentals where the actions of an individual hedge funds are independent of the actions of other hedge funds. The one region, dubbed "lower dominance region" in the literature, is defined by a threshold θ^{LD} such that an individual hedge fund withdraws even if no other hedge funds withdraw because it learns that the

[16] Recall that contract terms m_t^j and F_t^j, for both $t = 0, 1$ and $j = H, M$ are set before the signals arrive and cannot be made contingent of the realization of θ. As a result, the decision to withdraw based on the information received takes the contract terms as pre-determined. The dealer and money funds do not adjust their behavior after information arrives, so we focus the analysis on hedge funds and abstract from any information the dealer may receive.

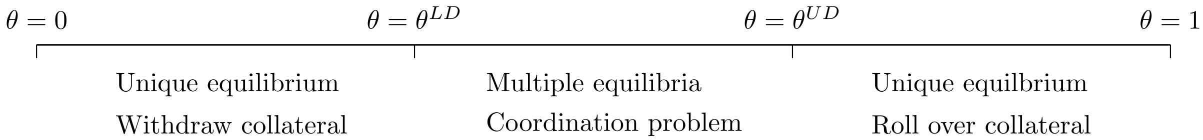


Figure 10: Unique and Multiple Equilibria Under Complete Information on θ.

fundamentals are very bad. The other region, dubbed "upper dominance region," is defined by a threshold θ^{UD}, such that an individual hedge fund will not withdraw its collateral even if all other hedge funds withdraw because it learns that fundamentals are very good to support a high liquidation value of dealer's assets. Lemma 3 below derives the lower and upper dominance thresholds.

Lemma 3. *There are two regions of fundamentals defined by thresholds θ^{LD} and θ^{UD} where the decision of an individual hedge fund to withdraw its collateral is independent of the decisions of other hedge funds. A hedge fund will always withdraw its collateral for $\theta \leq \theta^{LD} = ((\eta-1)\Delta m_1 - R^D\Delta m_0 - \Delta F_0)/((\eta-1)T - R^D\Delta m_0 - \Delta F_0 - \Delta m_1 - \Delta F_1)$, and will always roll over for $\theta \geq \theta^{UD} = -(\Delta F_0 + \lambda R^D \Delta m_0)/(\lambda(R^U - R^D)\Delta m_0)$. Moreover, θ^{LD} and θ^{UD} lie in the support of θ, i.e., $\theta^{LD}, \theta^{UD} \in (0,1)$.*

Given that all hedge funds are fully informed about θ, they will all withdraw for $\theta \leq \theta^{LD}$ and all rollover for $\theta \geq \theta^{UD}$ in equilibrium. However, for intermediate values of fundamental, $\theta \in (\theta^{LD}, \theta^{UD})$ multiple equilibria are possible, whereby an individual hedge fund's actions depends on its beliefs about the actions of other hedge funds (Figure 10). The intermediate region has positive mass as long as $\theta^{LD} < \theta^{UD}$, which implies that $\lambda\overline{R}_\theta\Delta m_0 + \Delta F_0 < 0$ for $\theta \in (\theta^{LD}, \theta^{UD})$, i.e., the dealer runs out of money if all (or a sufficient portion) of hedge funds withdraw. This is the source of the coordination problem and the reason why multiple equilibria exist under complete information.

3.2 Incomplete Information

In order to resolve the coordination problem described in section 3.1, we introduce incomplete information such that at $t = 1$, each hedge fund i receives a private noisy signal of the state of nature $x_i = \theta + \epsilon_i$ where the error terms ϵ_i are independently and uniformly distributed over $[-\epsilon, \epsilon]$. An individual hedge fund's decision to roll over depends on the signal it receives. The signal provides information regarding the quality of the risky asset in the dealer's balance sheet. In other words it helps inform the probability that the dealer will eventually default at $t = 2$ and the hedge fund will forfeit its collateral. The signal also provides information about other hedge funds' signals, which allows an inference regarding their actions. An individual hedge fund may decide to withdraw its collateral not only because it believes that fundamentals are bad, but also because the conjectured portion of hedge funds withdrawing is high enough to push the dealer into illiquidity.

We seek a symmetric equilibrium characterized by two thresholds (x^*, θ^*) such that an individual hedge fund will withdraw its collateral if its private signal realization x_i is lower than a threshold x^* and the dealer will be fully liquidated at $t = 1$ if the fundamentals realization θ is lower than a threshold θ^*.

Under such a threshold strategy, the portion of hedge funds that withdraw their collateral at a given level of fundamentals θ is

$$\mu(\theta, x^*) = \begin{cases} 1 & \text{if } \theta < x^* - \epsilon \\ Prob(x_i \leq x^*|\theta) & \text{if } x^* - \epsilon \leq \theta \leq x^* + \epsilon \\ 0 & \text{if } \theta > x^* + \epsilon. \end{cases} \tag{21}$$

If the fundamental value θ is lower than $x^* - \epsilon$, then all hedge funds receives signals $x_i < x^*$. Hence, all hedge funds, following a threshold strategy, withdraw and $\mu(\theta, x^*) = 1$. The opposite is true for $\theta > x^* + \epsilon$, whereby all hedge funds roll over and $\mu(\theta, x^*) = 0$. Finally, if fundamentals are not sufficiently higher or lower than x^*, i.e., $\theta \in [x^* - \epsilon, x^* + \epsilon]$, some hedge funds will receive signals that are lower than x^* and, thus, will withdraw their collateral. Given that private noise, ϵ_i, is independently and identically distributed, from the law of large numbers the portion of hedge funds withdrawing for a given level of θ in the intermediate region is $\mu(\theta, x^*) = Prob(x_i \leq x^*|\theta) = (x^* - \theta + \epsilon)/2\epsilon$.

The signal and fundamentals thresholds are derived in two steps as follows. First, given the threshold strategy x^*, we can derive the threshold for fundamentals, θ^*, which determines whether the dealer is fully liquidated at $t = 1$ or survives to $t = 2$. Because the portion of hedge funds withdrawing is decreasing in θ from (21), the dealer is fully liquidated only if

$\theta < \theta^*$ where θ^* is the unique solution to $f(\mu(\theta^*, x^*), \theta^*) = 1$ given by equation (4):

$$\theta^* = x^* - \epsilon \frac{\Delta m_1 + 2\Delta F_0 + 2\lambda \overline{R}_{\theta^*} \Delta m_0}{\Delta m_1}. \tag{22}$$

In other words, for threshold strategy x^*, if θ is lower than θ^*, then the portion of hedge funds withdrawing is higher than what the dealer can serve by liquidating her assets and $f(\mu(\theta, x^*), \theta) < 1$. On the contrary, if θ is higher than θ^*, fewer hedge funds withdraw, allowing the dealer to decrease asset liquidations and survive to $t = 2$.[17]

Second, given the fundamentals threshold θ^*, an individual hedge fund can compute the signal threshold x^*, below which it is optimal to withdraw conditional on its expectation over the portion of hedge funds withdrawing and the private signal it receives. This signal threshold depends on the utility differential between rolling over and withdrawing for a given level of θ and μ. The difference in expected payoff is given by $U^H(\mu, \theta; a = 1)$-$U^H(\mu, \theta; a = 0)$ derived from (12)-(17). Given that in equilibrium $F_t^M = T$ and $m_t^M = 0$, the utility differential $\nu(\mu, \theta)$ is given by the following piecewise function:

$$\nu(\mu, \theta) = \begin{cases} \theta\left[(\eta - 1)T - \Delta F_1\right] + (1 - \theta) G_S^D(\mu, \theta) - \eta \Delta m_1 & \mu \in [0, \mu_S) \\ \theta\left[(\eta - 1)T - \Delta F_1\right] + (1 - \theta) G_I^D(\mu, \theta) - \eta \Delta m_1 & \mu \in [\mu_S, \mu_I) \\ \theta G_I^U(\mu, \theta) + (1 - \theta) G_I^D(\mu, \theta) - \eta \Delta m_1 & \mu \in [\mu_I, \mu_R) \\ -\eta \frac{\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 + \Delta m_1}{\mu} & \mu \in [\mu_R, 1] \end{cases} \tag{23}$$

where $G_S^D(\mu, \theta) = \left(R^D \Delta m_0 + \Delta F_0 + (1 - \mu)\Delta m_1\right) / (1 - \mu)$, $G_I^D(\mu, \theta) = (R^D \Delta m_0 + R^D / \lambda \overline{R}_\theta \cdot (\Delta F_0 + (1 - \mu)\Delta m_1))/(1 - \mu)$, and $\theta G_I^U(\mu, \theta) + (1 - \theta) G_I^D(\mu, \theta) = (\overline{R}_\theta \Delta m_0 + 1/\lambda \cdot (\Delta F_0 + (1 - \mu)\Delta m_1))/(1 - \mu)$ from (6), (9) and (10).

We plot the utility differential $\nu(\mu, \theta)$ for a certain level of θ in Figure 11. Looking at the two first legs, the payment in the bad state for a hedge fund that rolled over is decreasing in μ and $\lim_{\mu \to \mu_S^-} G_S^D(\mu, \theta) = \lim_{\mu \to \mu_S^+} G_I^D(\mu, \theta)$. [18] The third leg is also decreasing in μ for $\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 < 0$, i.e., for $\theta < \theta^{UD}$. The utility differential is discontinuous at μ_I because the hedge fund fails to receive the non-pecuniary benefit η if the dealer defaults. However,

[17] The fraction of assets distributed is strictly decreasing in μ, i.e., $\partial f(\mu, \theta)/\partial \mu = (\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 + \Delta m_1)/(\mu^2 \Delta F_0) < 0$ given condition in equation (1).

[18] $\partial G_S^D(\mu, \theta)/\partial \mu = (R^D \Delta m_0 + \Delta F_0)/(1 - \mu)^2 < 0$, because $R^D < 1$ and $\Delta m_0 + \Delta F_0 < 0$ from Lemma 2, while $\partial G_I^D(\mu, \theta)/\partial \mu = R^D / (\lambda \overline{R}_\theta (\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0)/(1 - \mu)^2 < 0$ for $\theta < \theta^{UD}$.

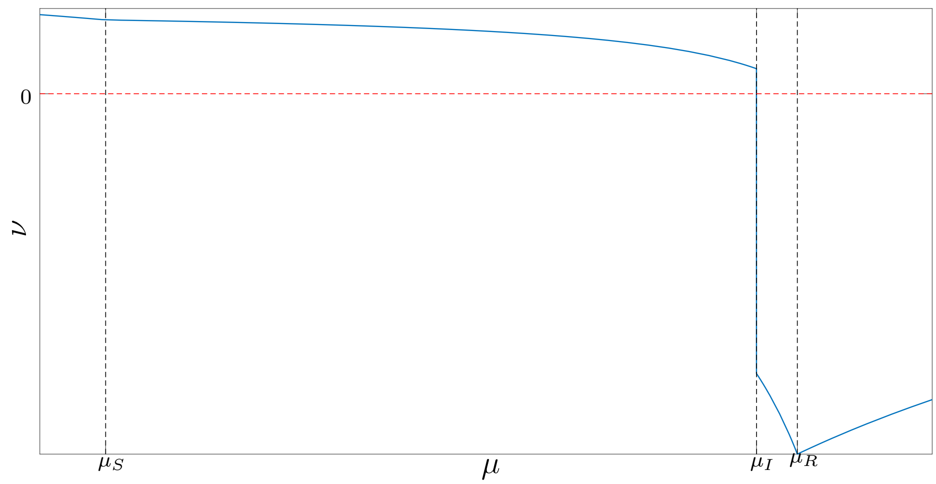


Figure 11: Indifference Function $\nu(\mu, \theta)$ as a Function of μ for Arbitrary $\theta \in (\theta^{LD}, \theta^{UD})$ and Arbitrary Contract Terms.

$\lim_{\mu \to \mu_I^-} \nu(\mu, \theta) - \lim_{\mu \to \mu_I^+} \nu(\mu, \theta) = \theta(\eta - 1)T > 0$, and, thus, $\nu(\mu, \theta)$ is strictly decreasing in $\mu \in [0, \mu_R)$ for $\theta < \theta^{UD}$ which is the relevant region where a coordination problem may occur. The final leg in (23) is increasing in μ given that $\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 + \Delta m_1 > 0$ from the condition in equation (1). Thus, the model features one-sided, rather than global, strategic complementarities as in Goldstein and Pauzner (2005). Once the dealer is fully liquidated, a hedge fund has fewer incentives to withdraw as withdrawals increase. Note that ν "crosses" zero as μ increases from above. That is, depending on the contract terms, this can happen within any of the first two legs or at the jump, but not within the third and fourth legs where $\nu(\mu, \theta)$ always takes negative values.[19]

Consider an individual hedge fund that receives signal x_i. The hedge fund will use the signal to update its beliefs about the realization of θ. Given that both θ and ϵ_i are uniformly distributed, the posterior distribution of θ given x_i is $\theta | x_i \sim U[x_i - \epsilon, x_i + \epsilon]$. This implies that the utility differential between rolling over and withdrawing for a hedge fund that receives signal x_i as a function of the cutoff value is

[19] $\nu(\mu, \theta) < 0$ for $\mu \in [\mu_I, \mu_R)$ —third leg— requires $\mu > 1 + (\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0)/\Delta m_1 > \mu_I$, which is always true.

$$\Delta(x_i, x^*) = \frac{1}{2\epsilon} \int_{x_i-\epsilon}^{x_i+\epsilon} \nu(\mu(\theta, x^*), \theta) d\theta. \tag{24}$$

In a threshold equilibrium, a hedge fund prefers to withdraw, i.e., $\Delta(x_i, x^*) < 0$, for all $x_i < x*$, and prefers to roll over, i.e., $\Delta(x_i, x^*) > 0$, for all $x_i > x*$. $\Delta(x_i, x^*)$ is continuous in x_i because a change in the signal only changes the limits of integration $[x_i - \epsilon, x_i + \epsilon]$ and the integrand is bounded. Hence, a hedge fund that receives signal $x_i = x^*$ is indifferent between rolling over and withdrawing, i.e.,

$$\Delta(x^*, x^*) = \frac{1}{2\epsilon} \int_{x^*-\epsilon}^{x^*+\epsilon} \nu(\mu(\theta, x^*), \theta) d\theta = 0. \tag{25}$$

Equations (22) and (25) jointly determine the threshold for fundamentals θ^* and the threshold strategy x^*. Proposition 1 establishes the existence and uniqueness of a threshold equilibrium.[20]

Proposition 1. *Given contract terms satisfying $\theta^{LD} < \theta^{UD}$ in Lemma 3, there exist a threshold, x^*, such that a hedge fund rolls over if $x_i > x^*$ and withdraws if $x_i < x^*$, and a threshold θ^*, such that the dealer does not experience any withdraws if $\theta \geq \theta^*$ and is fully liquidated if $\theta < \theta^*$. Moreover, the thresholds are unique if noise is not too large.*

Hereafter, we focus on the case that the noise goes arbitrarily close to zero. Taking the limit $\epsilon \to 0$ implies that $x^* \to \theta^*$ from (22). A hedge fund that receives signal x^*, the posterior distribution of θ is uniform over the interval $[x^* - \epsilon, x^* + \epsilon]$. Thus, that hedge fund's belief of the portion of hedge funds withdrawing as a function of θ, $\mu(\theta, x^*)$, is uniform over $[0, 1]$.[21] In other words, as θ decreases from $x^* + \epsilon$ to $x^* - \epsilon$, μ increases from 0 to 1.

[20]As already mentioned, the model features one-sided strategic complementarities. Hence, we follow the steps in Goldstein-Pauzner and make the same assumptions, most importantly that noise is uniformly distributed, to show the uniqueness of a threshold equilibrium. However, our framework features two additional complications. First, due to limited liability, the dealer's default threshold is endogenous. Second, the liquidation value matters for the payoff in a run and, thus, state monotonicity of $\nu(\mu, \theta)$ is not straightforward. We explicitly address these complications in the proof of Proposition 1. But to simplify the analysis we restrict attention to a uniformly distributed probability of a good realization, while Goldstein-Pauzner allow for more general distributions of the probability of a good realization, $p(\theta)$, where θ is uniformly distributed and $p'(\theta) > 0$. In our case, $p(\theta) = \theta$ and $p'(\theta) = 1$.

[21]This is true because $\lim_{x^* \to \theta^*} Prob(\mu(\theta, x^*) \leq N) = Prob(\mu(\theta, \theta^*) \leq N) = 1 - Prob(\theta \leq \theta^* + \epsilon - 2\epsilon N) = 1 - (\theta^* + \epsilon - 2\epsilon N - \theta^* + \epsilon)/(2\epsilon) = N$. Hence, $\mu(\theta, \theta^*) \sim U[0, 1]$.

Changing variables in $\Delta(x^*, x^*) = 0$ provides the indifference condition that determines the unique value θ^*:

$$V(\theta^*) = \int_0^1 \nu(\mu, \theta^*) d\mu = 0. \tag{26}$$

The detailed expression for $V(\theta^*)$, with its derivatives with respect to θ^* and the contract terms, are shown in equation (B.28) in Appendix B. Moreover, (26) implies that $\nu(0, \theta^*) > 0$ given that ν is decreasing in μ when positive, which can be used to establish the following Corollary.

Corollary 1. *The period 1 participation constraint* (19) *is always slack for all* $\theta \geq \theta^*$*.*

Contrary to the complete information case, the introduction of noisy private signals eliminates the possibility of multiple equilibria in the intermediate region of fundamentals, i.e., $\theta \in (\theta^{LD}, \theta^{UD})$. The dealer is fully liquidated for $\theta < \theta^*$. Figure 12 shows the regions where full liquidation (a "run") occurs. The region of θ below the threshold θ^{LD} corresponds to a fundamental run similar to the complete information case. The region between θ^{LD} and θ^* corresponds to a panic-based run due to dealer illiquidity. The overall run probability is equal to $Prob(\theta < \theta^*)$, and can be split between a fundamental run probability, $Prob(\theta < \theta^{LD})$, and a panic-based run probability, $Prob(\theta^{LD} \leq \theta < \theta^*)$.

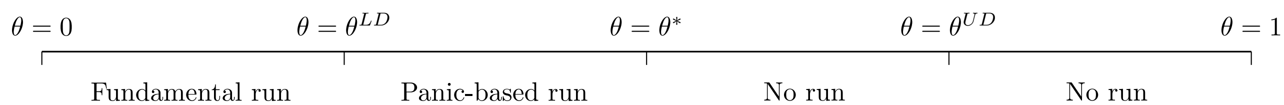


Figure 12: Unique equilibrium under Incomplete Information on θ.

4 Threshold Equilibrium

Having characterized hedge funds' threshold strategy under incomplete information, we turn to see the effective take-it-or-leave-it contracting terms the dealer chooses, anticipating hedge funds' optimal strategy. Because a hedge fund's problem is scalable, we normalize $T = 1$ for simplicity. This implies that feasible contract terms should satisfy $\Delta m_t \in [0, 1]$ and $\Delta F_t \in [-1, 0]$.

Given threshold for fundamentals θ^* defined in (26), all hedge funds withdraw their collateral for $\theta < \theta^*$, inducing the dealer to default and receive zero profits. Conversely, if the realization of θ is above θ^* all hedge funds roll over their repos to period $t = 2$ and the dealer is exposed to the risky asset's payoff. With probability θ, the good state realizes and the dealer enjoys positive profits. Otherwise, the bad state realized and the dealer defaults receiving nothing. Her expected utility is, then, given by:

$$\begin{aligned} U^D &= \int_0^{\theta^*} \theta u(0) d\theta + \int_{\theta^*}^1 \left[\theta u(R^U \Delta m_0 + \Delta m_1 + \Delta F_0 + \Delta F_1) + (1-\theta) u(0)\right] d\theta \\ &= \frac{(1-\theta^{*2})}{2} u(R^U \Delta m_0 + \Delta m_0 + \Delta F_0 + \Delta F_1), \end{aligned} \quad (27)$$

where $u(\cdot)$ is a concave utility function–not excluding linear utility–with $u(0) = 0$.[22].

The dealer will internalize that changing the contracting terms directly affects hedge funds' threshold strategy θ^* through the "global game" condition (26), and, thus, the probability of a collateral run. In many global games applications, the run threshold can derived in closed-form using a condition similar to (26). Given the complexity herein, we are not able to solve for the threshold in closed-form to substitute it in the dealer's problem.[23] Instead, we will explicitly impose (26) as a constraint that the dealer faces and have her optimize also over θ^* respecting its relationship with the other contract terms.

Hence, the dealer chooses $\{\Delta m_0, \Delta m_1, \Delta F_0, \Delta F_1, \theta^*\}$ to maximize (27) subject to the period-0 hedge funds participation constraint (18), the global game constraint (26), the positive liquidity injection constraint (1), and the bad-state default constraint (7).[24] We have imposed the last constraint in the dealer's problem to guarantee the existence of a

[22]It is important to note that a coordination problem can exist only if $\Delta m_0 > 0$. If hedge funds do not have an unsecured claim on the dealer, i.e., $\Delta m_0 = 0$, there cannot be an advantage to withdraw early. This situation can be appreciated graphically through Figure 8 and 9: if there is nothing hedge funds' can claim, beyond their collateral, there is no reason to withdraw. In this case the dealer would not invest in the risky asset, and the only feasible contracting terms that give dealer non-negative profits are $\Delta m_0 = \Delta m_1 = \Delta F_0 = \Delta F_1 = 0$, i.e., $u(0) = 0$. This implies that the dealer is better off setting contracting terms that expose her to a run as long as the profits in the good state are positive and $\theta^* < 1$

[23]A closed-form solution is attainable in Goldstein and Pauzner (2005) because the liquidation value does not depend on fundamentals. In Rochet and Vives (2004), the liquidation value depends on fundamentals, but the payoff structure does not, allowing for a closed-form solution.

[24]Alternatively, the problem can be stated with θ^* determined implicitly, and the dealer internalizing how contracting terms change the threshold. These approaches are mathematically equivalent.

lower dominance region, which is essential for the existence of a threshold equilibrium in the incomplete information game. We will elaborate further on the presence of these four constraints below, after we have established the existence of contract terms that give rise to a coordination problem and, hence, the possibility of collateral runs.[25]

Proposition 2. *For $\lambda R^U > 2$, $R^D < \eta R^U/(\eta + R^U)$, and dealer's risk-aversion not sufficiently low, there exist optimal contracting terms $\Delta m_t(\theta^*)$ and $\Delta F_t(\theta^*)$ under which hedge funds adopt a threshold strategy θ^*.*

Note that the existence result in Proposition 2 requires a high degree of dealer risk aversion so that that the marginal utility of the dealer is low enough to push θ^* to its upper bound θ^{UD}. In other words, conditional on survival at θ^{UD} the dealer would prefer a lower run probability over higher profits in the good state. As we will see in the following corollary, this can be true when the dealer is risk-neutral, but under stricter parameter conditions.

The optimal contracting terms of Proposition 2 are only a function of the threshold θ^* and given by $\Delta m_0(\theta^*) = -\theta^*(\eta - 1)\mu_I/f(\theta^*)$, $\Delta m_1(\theta^*) = g(\theta^*)\Delta m_0(\theta^*)$, $\Delta F_0(\theta^*) = -g(\theta^*)$, $\Delta m_0(\theta^*)$, and $\Delta F_1(\theta^*) = -R^D \Delta m_0(\theta^*)$, where $g(\theta^*)$ and $f(\theta^*)$ are given by (20) and (A.25), respectively. Finally, the run threshold is the solution to the following optimization condition:

$$\frac{1}{2}(1 - \theta^{*2})u'((R^U - R^D)\Delta m_0(\theta^*))(R^U - R^D) + \frac{\theta^* u\left((R^U - R^D)\Delta m_0(\theta^*)\right) f(\theta^*)}{\frac{\partial V}{\partial \theta^*} - \Delta m_0(\theta^*) g'(\theta^*)\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right)} = 0, \tag{28}$$

which can be easily interpreted. The first term captures the incremental utility to the dealer from an increase in the initial risky investment while keeping the run probability unchanged. The second term captures the effect on the probability that the dealer suffers a run, and, hence, forfeits any profits in the final period (note that the term is negative). So, the dealer balances the higher profits conditional on a run not occurring, with the associated increase in the probability of a run.

The optimal contracting terms of Proposition 2 are characterized by four binding constraints: the global game constraint, the initial participation constraint, the positive liquidity

[25] The participation constraint in period one is always slack from Corollary 1, and the contract terms will be interior. Thus, for the sake of conciseness, we do not include (19), $0 \leq \Delta m_t \leq 1$ and $-1 \leq \Delta F_t \leq 0$ in the dealer's optimization problem. The Lagrangian and the first-order optimization conditions are reported in (A.18)-(A.23) in Appendix A.

injection constraint, and the dealer default constraint.[26] The last three constraints allow us to write the contract terms ΔF_0, Δm_1, and ΔF_1 as function of Δm_0 and θ^*. The first one gives Δm_0 as a function of θ^*. As already mentioned, the global game and initial participation constraints will always be binding. We, now, discuss in more detail the intuition behind the other two binding constraints.

The binding liquidity injection constraint implies that the dealer does not collect any net funds in $t = 1$. The intuition behind the tightness of this restriction stems from inspecting θ^*'s sensitivity to both Δm_1 and ΔF_0. To see this, consider contracting terms which do not have a binding PC_0 constraint. In that case, the only difference between Δm_1 and ΔF_0 is how those variables affect θ^*. In the proposed equilibrium we have, $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0 > 0$.[27] Intuitively this is because a hedge fund loss due to an increase in Δm_1 only affects hedge funds that roll over. A hedge fund loss due to an increase in ΔF_0 is mutualized between those that roll over and those that do not. That is, the dealer gets more "bang for her buck" by altering the roll over haircut. Thus, to increase the possibility for hedge funds to rollover (i.e., lower θ^*), it is optimal to reduce hedge funds' loss via a reduction in Δm_1 rather than ΔF_0, which is pinned down by the initial participation constraint.

Interestingly, the choice of Δm_1 versus ΔF_0 highlights a tradeoff which can be lost without considering hedge fund's rollover decision. Given that the sum of these two contracting terms results in the effective net payment from hedge funds that roll over, their direct marginal impact on the dealer's utility is identical. The difference stems from considering how these contracting terms affect the marginal hedge fund's decision to roll over or not. In this regard, the contracting terms are very different, because ΔF_0 is paid by all whereas Δm_1 is only paid by those who continue.

The final active constraint is the dealer's default condition. The intuition behind this active constraint is that an increase in ΔF_1 affects the dealer's payoff directly with a minimal impact on the sensitivity of θ^*: It has a small impact on μ_I, and only affects the final repayment of hedge funds' that roll over directly, with no effect on the incentives *between*

[26] In a model extension where the dealer can default for reasons outside of the rehypothecation process, a threshold equilibrium exists even when the default condition is slack. The characterization and proof of this extension is available upon request.

[27] From the implicit function theorem $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0 = (\partial V/\partial\Delta F_0 - \partial V/\partial\Delta m_1)(\partial V/\partial\theta^*)^{-1}$. $\partial V/\partial\Delta F_0 - \partial V/\partial\Delta m_1$ pins down the value of the multiplier on the participation constraint (see (A.26)), and thus is positive. Given that $\partial V/\partial\theta^* > 0$ from the proof of Proposition 1, $\partial\theta^*/\partial\Delta m_1 - \partial\theta^*/\partial\Delta F_0$ is positive as well.

rolling over and withdrawing. This is why in equilibrium the dealer decides to set contracting terms in which she is on the verge of defaulting in the bad state.

Having a general characterization of the equilibrium in Proposition 2 we focus on a specific case that gives a more precise characterization of the equilibrium outcome, and also allows us to do comparative statics. Specifically, we shall assume that the risky asset payoff is zero in the down state $R^D = 0$ and the dealer is risk neutral. In this case, we have the following result,

Corollary 2. *For $R^D = 0$, $\lambda R^U \in \left(2, \frac{4+8\sqrt{2}}{7}\right)$, and risk neutral dealer, there exist optimal contracting terms*

$$\Delta m_0(\theta^*) = \frac{\theta^*(\eta-1)}{\eta g(\theta^*)\left(1-\ln\left(\frac{\lambda\overline{R}_\theta}{g(\theta)}\right)\right)}, \quad \Delta m_1(\theta^*) = g(\theta^*)\Delta m_0(\theta^*)$$
$$\Delta F_0(\theta^*) = -g(\theta^*)\Delta m_0(\theta^*), \qquad \Delta F_1(\theta^*) = 0$$

under which hedge funds adopt a threshold strategy θ^* that solves,

$$2\left(1-\theta^*-3\theta^{*2}+\theta^*(1+\theta^*)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right) = \ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\left(1-\theta^*-4\theta^{*2}+\theta^*(1+\theta^*)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)$$

with $\frac{\partial\theta^*}{\partial R^U} < 0$.

The optimal contracting terms have the same functional form as in Proposition 2 with many of the expressions simplified, because in this case $\mu_I = \mu_R$. Since $R^D = 0$, the no default condition can be replaced by the reasonable restriction of having the $\Delta F_1 \leq 0$. If this were not the case, the dealer would never default.

Note that we present the comparative statics with respect to R^U, but these are identical to the ones with respect to λ.[28] As the asset becomes more valuable at liquidation, either due to a higher payment on the good state or a lower liquidity discount, the possibility of a run becomes smaller. That is, the dealer would have more funds to compensate hedge funds in case they were to withdraw, reducing their incentives to do so.

Changes in η do not affect the equilibrium threshold, again because of the dealer's risk neutrality. But changes in η do materially change the optimal contrasting terms. As hedge

[28] With dealer risk aversion this would not be the case, because the dealer's marginal utility is affected by the final payoff (see equation (28)).

funds have higher profits through their preference for holding safe assets, the initial repo margin Δm_0 can be higher.

The following subsection presents a numerical example of the model's comparative statics whenever the dealer is risk averse and the asset value is positive in the down state.

4.1 Numerical Example

Proposition 2 characterizes equilibria which we can calculate numerically. Figure 13 shows the hedge fund's utility differential under the optimal strategies for one particular set of parameters. As stated in the equilibrium of Proposition 2, it's optimal for the dealer not to collect any additional payments in the interim period $\Delta m_0 + \Delta F_1 = 0 \Longrightarrow \mu_S = 0$. As with all threshold equilibria, we have that $\mu_I < \mu_R < 1$.

Figure 14 shows how the equilibrium threshold θ^*, and its associated upper and lower dominance, change with model parameters. The interpretation of the top two and bottom right panels shows that as the assets unconditional liquidation value increases, the possibility of a threshold strategy is smaller. In particular, the interval with panic-based runs (i.e., (θ^{LD}, θ^*) shrinks. This might seem like a beneficial outcome: it is less likely to have a panic based equilibrium. In addition, the range where both panic-based and fundamental runs is also smaller (i.e., $(0, \theta^*)$).

But changes in η shows that there are cases where the panic-based equilibrium interval is larger, while both panic-based and fundamental runs are overall smaller. That is, in these cases, the larger coordination problem in fact *decreases* the probability of a run, highlighting the case where coordination can be helpful.

The numerical exercise also highlights how contracting terms change as the equilibrium changes with underlying parameters. For illustration purposes, Figure 15 shows the equilibrium haircuts (top panel) and equilibrium interest rates (bottom panel) in t=0 and t=1 as a function of the equilibrium θ^* for different values of R^D. The top panel shows that the haircut in $t = 0$ is lower than the haircut in $t = 1$. This is merely a graphical representation of Lemma 2 and stems from the fact that $-\Delta F_0 > \Delta m_0$, so as to compensate hedge funds to

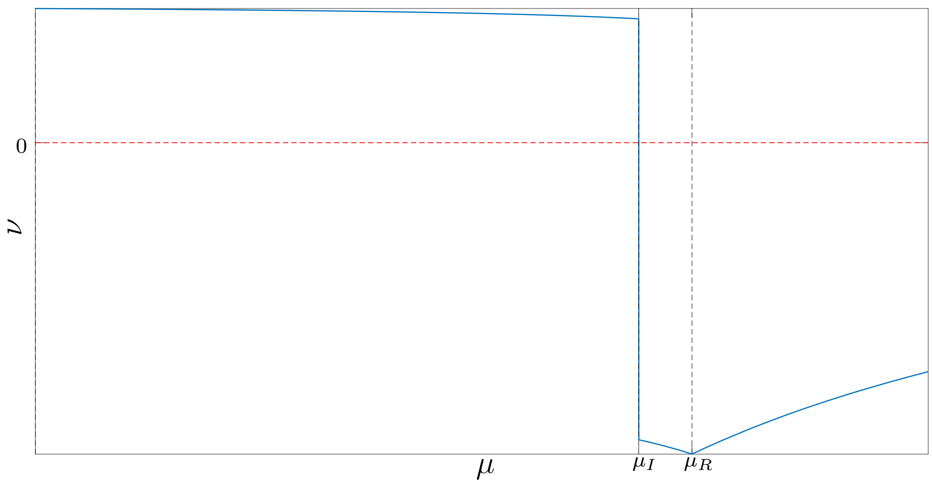


Figure 13: Marginal Hedge Funds Indifference Function ν
Figure shows the indifference function ν of the optimal contract for a marginal Hedge Fund with a safe asset valuation of $\eta = 5$, asset parameters $R^U = 2$, $R^D = 0.5$, $\lambda = 0.8$, and the Dealer with with CARA utility, risk aversion $\gamma = 5$. Dashed red line highlights where ν crosses zero.

participate initially, and in equilibrium there is no liquidity injection $\Delta m_1 = -\Delta F_0$. That is, even though there is no transfer of payments in $t = 1$, observing haircuts in isolation gives an increase in haircuts from the initial to rollover period. This can change the interpretation of an increase in haircuts: dealers may not be putting more onerous contracting terms to their counterparties, but rather they just reviving enough funds to pay off existing contracts.

The bottom panel shows that the interest rate in $t = 0$ is lower that the interest rate in $t = 1$. Moreover, the interest rate in $t = 0$ is negative, indicating dealers are willing to get a negative return when contracting with hedge funds to incentivize them to participate. This is somewhat similar to a well-known market outcome when there is high demand for a particular asset, known as *repo specialness*. In this context, the safe asset is not *trading special* because counterparties need to source collateral for short positions, but rather because the dealer can make a profit by purchasing a risky asset position with liquidity stemming from rehypothecation.

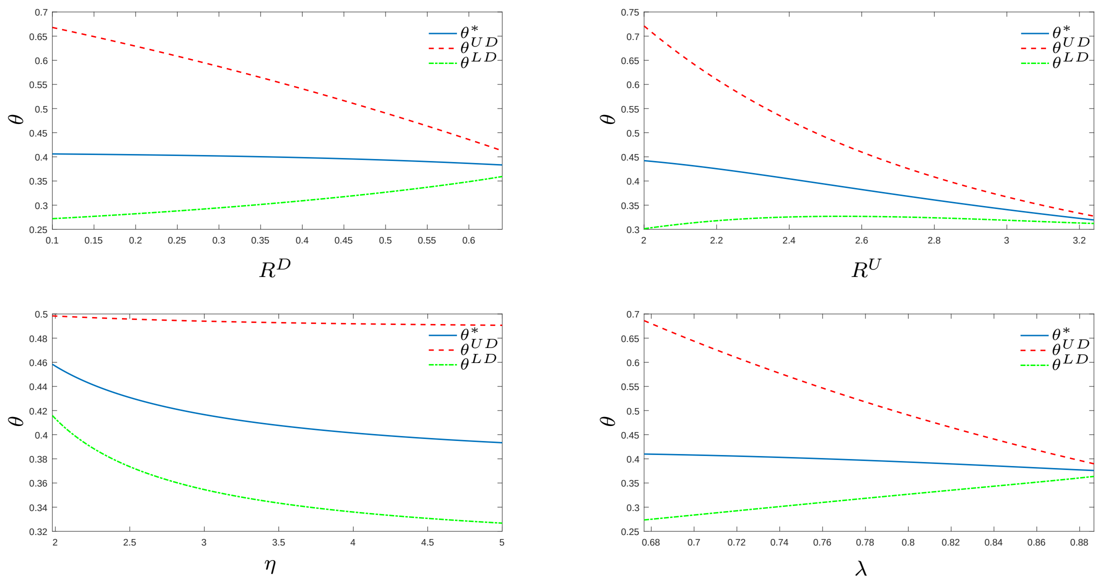


Figure 14: Changes in θ^* for Different Model Parameters
Figure shows how the threshold strategy changes with different underlying parameters (solid blue line). The figure also shows how the upper dominance (dashed red line) and lower dominance (dotted green line) thresholds change. Equilibria are around the initial parameter valuations of $\eta = 5$, $R^U = 2.5$, $R^D = 0.5$, $\lambda = 0.8$, and the dealer with CARA utility, risk aversion $\gamma = 5$.

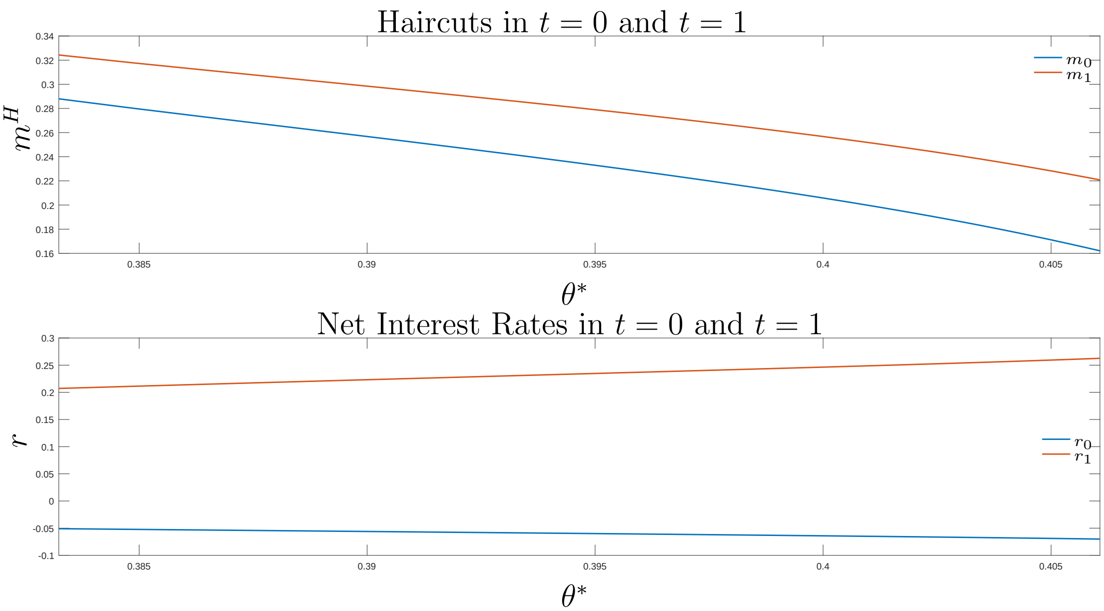


Figure 15: Changes in Contracting Terms as Equilibrium Changes with R^D
Figure shows how contracting terms change as the equilibrium changes for different values of R^D. Equilibria are with a safe asset valuation of $\eta = 5$, asset parameters $R^U = 2.5$, $R^D \in [0.1, 0.64]$, $\lambda = 0.8$, and the Dealer with CARA utility, risk aversion $\gamma = 5$.

5 Conclusion

This paper presents a model which highlights fragility that can arise from the re-use of collateral in a short-term collateralized lending context. Specifically, the paper characterizes a coordination failure that can arise amongst cash borrowers, inducing a panic-based default on an intermediary. In contrast to traditional wholesale funding runs, the model shows that when intermediating secured financing fragility can materialize on the asset side of an intermediary's balance sheet. The model delivers a unique threshold equilibria in which panic-based runs can ensue. In addition, the paper shows how different repo contracting terms, specifically the haircut and repurchase price, can have differential effects on collateral providers' incentives to run. This provides another mechanism in which to disentangle different repo contracting terms.

The results in this paper also pose a challenge for regulators concerned with the fragility of large broker dealers. Much of the regulation introduced since the 2007–09 is designed to monitor and restrict the composition of banks liabilities. This paper cautions that this focus is too narrow. Given that the total amount of collateral received is an important source of liquidity, fragility can present itself on the asset side of bank's balance sheet, as well.

Policy prescriptions that could address the source of fragility studied in this paper might be to restrict the amount of over-collateralization in dealers' rehypothecation activity, which effectively limits the cash windfall dealers are able to extract, or to restrict dealers' reinvestment of said cash windfall. These are areas of future research.

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Appendices

A Proofs

Proof of Lemma 1

For values of θ such that a run of the dealer is possible, i.e., $\mu_R < 1$, or equivalently $\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0 < 0$, the default threshold, μ_I, is always lower than the run threshold, μ_R, if $\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1 > 0$. The latter expression can be written as $\lambda \overline{R}_\theta \Delta F_1 + R^U \Delta m_1 > (R^U \Delta m_0 \Delta m_1 - \Delta F_0 \Delta F_1)/\Delta m_0$ using the fact that $\Delta F_0 + \lambda \overline{R}_\theta \Delta m_0 < 0$ and $\Delta F_0, \Delta F_1 < 0$. Given that $\Delta m_1 \geq -\Delta F_0$ from condition $C0$ in (1), $R^U \Delta m_0 \Delta m_1 - \Delta F_0 \Delta F_1 > -\Delta F_0 (R^U \Delta m_0 + \Delta F_1) > 0$, using condition $C2$ in (8).

Finally, to ensure that the dealer will begin to sell before it comes insolvent, i.e., $\mu_S < \mu_I$, we also need $R^U \Delta m_1 \Delta m_0 - \Delta F_1 \Delta F_0 > 0$, which we proved above.

Proof of Lemma 2

From (18) we get that:

$$-\Delta F_0 \left[\int_{\theta^*}^{1} d\theta + \int_{0}^{\theta^*} f(1,\theta) d\theta \right] \geq \Delta m_0 \quad \Rightarrow \quad -\Delta F_0 > \Delta m_0, \tag{A.1}$$

because $\int_{\theta^*}^{1} d\theta + \int_{0}^{\theta^*} f(1,\theta) d\theta < 1$. This proves claim 1.

Combining (A.1) and (1), we get that $\Delta m_1 > \Delta m_0$. This proves claim 2.

Proof of Lemma 3

The lower dominance region is defined by the values of θ for which an individual hedge fund chooses to withdraw even if other hedge funds do not. The utility differential between rolling over and withdrawing when $\mu = 0$ from (12) and (16) is $U^H(0,\theta;1) - U^H(0,\theta;0) = \theta[(\eta-1)T - \Delta F_1] + (1-\theta)[R^D \Delta m_0 + \Delta F_0 + \Delta m_1] - \eta \Delta m_1$, where we have substituted the equilibrium conditions $F_t^M = T$ and $m_t^M = 0$ derived in section 2.3. Given that the differential is increasing in θ the lower dominance region comprises of values for $\theta \leq \theta^{LD}$, where θ^{LD} is the solution to $U^H(0,\theta^{LD};1) - U^H(0,\theta^{LD};0) = 0$, i.e.,

$$\theta^{LD} = \frac{(\eta-1)\Delta m_1 - R^D \Delta m_0 - \Delta F_0}{(\eta-1)T - R^D \Delta m_0 - \Delta F_0 - \Delta m_1 - \Delta F_1}.$$

The lower dominance threshold θ^{LD} is greater than zero because, from Lemma 2 and $R^D < 1$, $R^D \Delta m_0 + \Delta F_0 < (R^D - 1)\Delta m_0 < 0$, and from condition (7) $R^D \Delta m_0 + \Delta F_0 + \Delta m_1 + \Delta F_1 < 0$. It is also lower than one because $(\eta-1)T - \Delta F_1 > \eta \Delta m_1$ from the period 1 participation constraint (19).

The upper dominance region is defined by the values of θ for which an individual hedge fund rolls over even if all other hedge funds withdraw. First, we need to guarantee that for $\theta \geq \theta^{UD}$, the dealer has enough liquidity to serve all early withdraws, i.e., $\lambda \overline{R}_\theta \Delta m_0 + \Delta F_0 \geq 0$. Given that the last expression is increasing in θ, the upper dominance threshold is the root, i.e.,

$$\theta^{UD} = -\frac{\Delta F_0 + \lambda R^D \Delta m_0}{\lambda(R^U - R^D)\Delta m_0}.$$

The upper dominance threshold θ^{UD} is greater than zero because, from Lemma 2 and $\lambda R^D < 1$, $\Delta F_0 + \lambda R^D \Delta m_0 < (\lambda R^D - 1)\Delta m_0 < 0$. To show that θ^{UD} is lower than one, we impose a binding participation constraint (18), which is always the case *under the equilibrium contract terms*. Then, $\theta^{UD} = (g(\theta^*) - \lambda R^D)/(\lambda R^U - \lambda R^D)$, where $g(\theta^*)$ is given by (20), and is smaller than one if $g(\theta^*) < \lambda R^U$. Substituting the expression for $g(\theta^*)$, the latter condition can be written as $\mathcal{X}(\theta^*) \equiv \lambda(R^U - R^D)\theta^{*2} - 2\lambda(R^U - R^D)\theta^* + 2(\lambda R^U - 1) > 0$. $\mathcal{X}$ does not have real roots because the discriminant $4\lambda(R^U - R^D)\left[\lambda(R^U - R^D) - 2(\lambda R^U - 1)\right] < 0$ given the restriction $\lambda(R^U + R^D)/2 > 1$.

We have shown that for $\theta \geq \theta UD$, the hedge fund believes the liquidation price is high enough to avoid a liquidity default. Now we must ensure that the hedge fund in fact wants to roll over. To conclude the proof, we need to show that for $\theta \geq \theta^{UD}$ and $\mu \to 1$, the utility differential between rolling and withdrawing for an individual hedge fund is positive. Technically, we need to check whether an infinitesimal hedge fund with mass ε that deviates from the strategy of other fund that withdraw can repurchase its collateral at $t = 2$. In other words, we need to check whether the dealer default on the remaining $-\varepsilon\Delta F_1$ obligations when $\varepsilon \to 0$. The dealer will not default if the value of her remaining asset is higher that her remaining obligations, i.e., $\lim_{\mu\to 1} G_I^U(\mu,\theta)/(-\Delta F_1) > 1$ with $G_I^U(\mu,\theta)$ given by (10). Changing variables such that $\mu = 1 - \varepsilon$ and substituting $\lambda\overline{R}_\theta \Delta m_0 = \lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Omega(\theta)\Delta m_0$, where $\Omega(\theta) \geq 0$ because $\theta \geq \theta^{UD}$, we get that:

$$\begin{aligned}\lim_{\varepsilon\to 0}\frac{G_I^U(1-\varepsilon,\theta)}{-\Delta F_1} &= -\frac{R^U}{\lambda\overline{R}_\theta}\lim_{\varepsilon\to 0}\frac{\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Omega(\theta)\Delta m_0 + \Delta F_0 + \varepsilon\Delta m_1}{\varepsilon\Delta F_1}\\ &= -\frac{R^U}{\lambda\overline{R}_\theta}\frac{\Delta m_1}{\Delta F_1} + \frac{\Omega(\theta)\Delta m_0}{\Delta F_1}\lim_{\varepsilon\to 0}\frac{1}{\varepsilon},\end{aligned}$$

where we used the fact that $\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Delta F_0 = 0$ from the definition of θ^{UD}. Given that $lim_{\varepsilon\to 0}\frac{\Omega(\theta)\Delta m_0}{\varepsilon\Delta F_1} \to \infty$ for $\theta > \theta^{UD}$, it suffices to show that for $\theta = \theta^{UD}$, i.e., $\Omega(\theta) = 0$ irrespective of the value of ε, the limit converges to a value higher than 1. Using $\lambda\overline{R}_{\theta^{UD}}\Delta m_0 + \Delta F_0 = 0$, we get that the limit converges to $-(R^U\Delta m_1/(\lambda\overline{R}_{\theta^{UD}}\Delta F_1) = R^U\Delta m_0\Delta m_1/(\Delta F_0\Delta F_1)$, which is greater than 1 because, as proved in Lemma 1, $R^U\Delta m_0\Delta m_1 - \Delta F_0\Delta F_1 > 0$. Hence, the dealer will not default at $t = 2$ if R^U realizes and the hedge fund will be able to repurchase its collateral.

If, instead R^D realizes at $t = 2$, the limit is

$$\lim_{\varepsilon\to 0}\frac{G_I^U(1-\varepsilon,\theta)}{-\Delta F_1} = -\frac{R^D}{\lambda\overline{R}_\theta}\frac{\Delta m_1}{\Delta F_1} + \frac{\Omega(\theta)\Delta m_0}{\Delta F_1}\lim_{\varepsilon\to 0}\frac{1}{\varepsilon}.$$

Again, for the $\theta > \theta^{UD}$, the second term goes to infinity and the dealer does not default. But, for $\theta = \theta^{UD}$, the limit goes to $-(R^D\Delta m_1/(\lambda\overline{R}_{\theta^{UD}}\Delta F_1) = R^D\Delta m_0\Delta m_1/(\Delta F_0\Delta F_1)$. Using conditions (1) and (7) we get that $R^D\Delta m_0 + \Delta F_1 < 0$ and, hence, the limit is higher than 1 for $\Delta m_1 > \Delta F_0\Delta F_1/(R^D\Delta m_0) > -\Delta F_0$

and less than one otherwise. If the former case, the dealer does not default and the hedge fund is able to repurchase its collateral. In the latter case, the dealer defaults and the hedge fund gets $-R^D\Delta m_1\Delta m_0/\Delta F_0$.

In most cases described above the dealer does not default either in the good or in the bad state and it is straightforward that the hedge fund's period 1 participation constraint (19) is satisfied. Thus, the hedge fund rolls over its collateral. However, for the special cases that $\theta = \theta^{UD}$ and $\Delta m_1 \in [-\Delta F_0, \Delta F_0\Delta F_1/(R^D\Delta m_0)]$, the dealer defaults in the bad state. A sufficient condition such that the period 1 participation constraint is satisfied is $-R^D\Delta m_1\Delta m_0/\Delta F_0 \geq G_S^D(0,\theta) = R^D\Delta m_0 + \Delta F_0 + \Delta m_1$. For the lower bound on $\Delta m_1 = -\Delta F_0$ the latter relationship becomes $R^D\Delta m_0 \geq R^D\Delta m_0$, while for the upper bound on $\Delta m_1 = \Delta F_0\Delta F_1/(R^D\Delta m_0)$ it becomes $\Delta F_1 < R^D\Delta m_0 + \Delta F_0 + \Delta F_1$, which is always true because of condition (7). Given that both sides of the inequality are monotonically increasing in Δm_1, we conclude that the repayment is the case of default is higher than $G_S^D(0,\theta)$ and, thus, the participation constraint is satisfied.

Note that the participation constraint (19) holds for $\theta \geq \theta^*$, while in all the aforementioned cases the utility differential is computed for $\theta^{UD} > \theta^*$. Hence, the participation constraint is easily satisfied for $\theta \geq \theta^{UD}$ and an individual hedge fund will always roll over even if all other hedge fund withdraw.

Proof of Proposition 1

The proof follows Goldstein and Pauzner (2005), but introduces additional steps and derivations due to the complexity accruing from the limited liability of the dealer and the fact that the liquidation value of the risky asset depends on θ.

An equilibrium with threshold x^* exists only if $\Delta(x^*, x^*) = 0$ given by (25). Consider a potential threshold x'. We will show that x' exists and it satisfies (25) at exactly one point, $\xi' = \xi^*$.

By the existence of θ^{LD} and θ^{UD} in Lemma 3, $\Delta(x', x')$ is negative for $x' < \theta^{LD} - \epsilon$ and positive for $x' > \theta^{UD} + \epsilon$. Thus, it suffices to show that $\Delta(x', x')$ is continuous is x' to establish that a threshold equilibrium exists. It is convenient to write the utility differential $\Delta(x', x')$ as $\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x)$ for some $\hat{x}$ such that Δx is the change in both the signal that the marginal hedge fund receives and the threshold strategy. Then,

$$\begin{aligned}\Delta(\hat{x} + \Delta x, \hat{x} + \Delta x) &= \frac{1}{2\epsilon}\int_{\hat{x}+\Delta x-\epsilon}^{\hat{x}+\Delta x+\epsilon} \nu(\mu(\theta, \hat{x} + \Delta x), \theta)d\theta \\ &= \frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\hat{x}+\epsilon} \nu(\mu(\theta + \Delta x, \hat{x} + \Delta x), \theta + \Delta x)d\theta \\ &= \frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\hat{x}+\epsilon} \nu(\mu(\theta, \hat{x}), \theta + \Delta x)d\theta, \qquad \text{(A.2)}\end{aligned}$$

because $\mu(\theta + \Delta x, \hat{x} + \Delta x) = \mu(\theta, \hat{x})$ from (21). In other words, the marginal hedge fund's belief about how many other hedge funds withdraw is unchanged when its private signal and the threshold strategy change by the same amount. Yet, it expects the θ to be higher for $\Delta x > 0$ and lower for $\Delta x < 0$ which is reflected in the calculation of $\nu(\mu(\theta, \hat{x}), x + \Delta x)$. Thus, we need to show that for a given distribution of $\mu' s$ the integral in (A.2) is continuous in Δx.

The integrand $\nu(\mu(\theta,\hat{x}),\theta+\Delta x)$ in (A.2) is a piecewise function such that each sub-function is computed over a distribution of μ unaffected by Δx, but the interval for each sub-function depends on Δx apart from $\mu \in [0,\mu_S)$ in (2). In other words, μ_I and μ_R given by (11) and (3), respectively, move with $\theta+\Delta x$. Note that θ always lies between $\hat{x}-\epsilon$ and $\hat{x}+\epsilon$, hence only Δx will matter. Given that the distribution of μ is unchanged, we can compute the levels of threshold $\theta's$ as functions of Δx, such that $\mu(\theta_{\mu_S}(\Delta x),\hat{x}) = \mu_S(\theta_{\mu_S}(\Delta x)+\Delta x)$, $\mu(\theta_{\mu_I}(\Delta x),\hat{x}) = \mu_I(\theta_{\mu_I}(\Delta x)+\Delta x)$ and $\mu(\theta_{\mu_R}(\Delta x),\hat{x}) = \mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)$ as follows:[29]

$$\begin{aligned}&\mu(\theta_{\mu_S}(\Delta x),\hat{x}) = \mu_S(\theta_{\mu_S}(\Delta x)+\Delta x)\\ \Rightarrow &\frac{\hat{x}-\theta_{\mu_S}(\Delta x)+\epsilon}{2\epsilon} = 1+\frac{\Delta F_0}{\Delta m_1},\end{aligned}\tag{A.3}$$

$$\begin{aligned}&\mu(\theta_{\mu_I}(\Delta x),\hat{x}) = \mu_I(\theta_{\mu_I}(\Delta x)+\Delta x)\\ \Rightarrow &\frac{\hat{x}-\theta_{\mu_I}(\Delta x)+\epsilon}{2\epsilon} = 1+\frac{R^U(\Delta F_0+\lambda\overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x}\Delta m_0)}{\lambda\overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x}\Delta F_1+R^U\Delta m_1},\end{aligned}\tag{A.4}$$

$$\begin{aligned}&\mu(\theta_{\mu_R}(\Delta x),\hat{x}) = \mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)\\ \Rightarrow &\frac{\hat{x}-\theta_{\mu_R}(\Delta x)+\epsilon}{2\epsilon} = 1+\frac{\Delta F_0+\lambda\overline{R}_{\theta_{\mu_R}(\Delta x)+\Delta x}\Delta m_0}{\Delta m_1}.\end{aligned}\tag{A.5}$$

Because the number of hedge funds withdrawing decreases as fundamentals improve for given strategy threshold (see equation (21)), we have $\theta_{\mu_R}(\Delta x) < \theta_{\mu_I}(\Delta x) < \theta_{\mu_S}(\Delta x)$, which is the reverse ordering of μ_S, μ_I and μ_R from Lemma 1. Thus, using (23), (A.2) can be written as:

$$\begin{aligned}&\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x) =\\ &\quad\frac{1}{2\epsilon}\int_{\hat{\mathrm{x}}-\epsilon}^{\theta_{\mu_{\mathrm{R}}}(\Delta \mathrm{x})} -\eta\frac{\lambda\overline{\mathrm{R}}_{\theta+\Delta\mathrm{x}}\Delta\mathrm{m}_0+\Delta\mathrm{F}_0+\Delta\mathrm{m}_1}{\mu(\theta,\hat{\mathrm{x}})}\mathrm{d}\theta\\ &+\frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)}\left[(\theta+\Delta x)G_I^U(\mu(\theta,\hat{x}),\theta+\Delta x)+(1-(\theta+\Delta x))G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)-\eta\Delta m_1\right]d\theta\\ &+\frac{1}{2\epsilon}\int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)}\left[(\theta+\Delta x)[(\eta-1)T-\Delta F_1]+(1-(\theta+\Delta x))G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)-\eta\Delta m_1\right]d\theta\\ &+\frac{1}{2\epsilon}\int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon}\left[(\theta+\Delta x)[(\eta-1)T-\Delta F_1]+(1-(\theta+\Delta x))G_S^D(\mu(\theta,\hat{x}),\theta+\Delta x)-\eta\Delta m_1\right]d\theta.\end{aligned}\tag{A.6}$$

[29]Note that for θ^* and x^*, then $\mu(\theta^*,x^*) = \mu_R(\theta^*)$, which yields (22).

Then, $\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)$ in (A.6) is continuous in Δx, because all the integrand are bounded and continuous, θ_{μ_I} and θ_{μ_R} change continuously with Δx from (A.4) and (A.5) (θ_{μ_S} doesn't move from (A.3)), and the discontinuity in ν occurs only at one discrete point, θ_{μ_I}. Hence, a threshold equilibrium exists.

We will now establish that the threshold equilibrium is unique. By implicitly differentiating (A.4) and (A.5) we get:

$$\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x} = -\frac{2\epsilon\Gamma_{\theta_{\mu_I}}(R^U - R^D)}{1 + 2\epsilon\Gamma_{\theta_{\mu_I}}(R^U - R^D)} < 0 \tag{A.7}$$

because $\Gamma_{\theta_{\mu_I}} \equiv \lambda R^U(R^U\Delta m_0\Delta m_1 - \Delta F_0\Delta F_1)/(\lambda\overline{R}_{\theta_{\mu_I}(\Delta x)+\Delta x}\Delta F_1 + R^U\Delta m_1)^2 > 0$ from Lemma 1, and

$$\frac{d\theta_{\mu_R}(\Delta x)}{d\Delta x} = -\frac{2\epsilon\Gamma_{\theta_{\mu_R}}(R^U - R^D)}{1 + 2\epsilon\Gamma_{\theta_{\mu_R}}(R^U - R^D)} < 0, \tag{A.8}$$

because $\Gamma_{\theta_{\mu_R}} \equiv \lambda\Delta m_0/\Delta m_1 > 0$

The derivative of (A.6) with respect to Δx is:

$$\begin{aligned}
&\frac{\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)}{\Delta x} = \\
&-\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)} \eta\frac{\lambda(R^U - R^D)}{\mu(\theta,\hat{x})}d\theta - \frac{1}{2\epsilon}\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x}(\theta_{\mu_I}(\Delta x) + \Delta x)(\eta - 1)T + \frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)} \frac{R^U - R^D}{1-\mu(\theta,\hat{x})}\Delta m_0 d\theta \\
&+\frac{1}{2\epsilon}\int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)} \left[(\eta-1)T - \Delta F_1 - G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x) + (1-\theta-\Delta x)\frac{G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)}{d\Delta x}\right] d\theta \\
&+\frac{1}{2\epsilon}\int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon} \left[(\eta-1)T - \Delta F_1 - G_S^D(\mu(\theta,\hat{x}),\theta+\Delta x)\right] d\theta.
\end{aligned} \tag{A.9}$$

The third, fourth and fifth terms in (A.9) are positive – $dG_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)/d\Delta x = -(R^D(R^U - R^D)(\Delta F_0 + (1-\mu)\Delta m_1)/(\lambda(1-\mu)\overline{R}^2_{\theta+\Delta x}) > 0$ in the respective region of fundamentals. The second term in (A.9) represents the utility change from changing the threshold $\theta_{\mu_I}(\Delta x)$ where default occurs and hedge funds forfeit the extra benefit $\eta - 1$ and is positive due to (A.7). However, the first term, which correspond to the change in the range that the run occurs, is negative. In order to establish that the positive terms outweigh the negative term we will evaluate (A.9) at a candidate threshold $\hat{x}$, which we know that exists. If the derivative is positive at candidate threshold, we can conclude that (A.6) does not cross zero from above and, given continuity, the threshold is unique. Using (A.6), we can derive the following lower bound for

(A.9):

$$\frac{\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)}{\Delta x} >$$

$$\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)} \eta\frac{\lambda\overline{R}_{\theta+\Delta x}\Delta m_0+\Delta F_0+\Delta m_1-\lambda(R^U-R^D)\Delta m_0}{\mu(\theta,\hat{x})}d\theta$$

$$-\frac{1}{2\epsilon}\frac{d\theta_{\mu_I}(\Delta x)}{d\Delta x}(\theta_{\mu_I}(\Delta x)+\Delta x)(\eta-1)T$$

$$+\frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)}\left[\eta\Delta m_1+\frac{R^U-R^D}{1-\mu(\theta,\hat{x})}\Delta m_0-\frac{\overline{R}_{\theta+\Delta x}\Delta m_0+1/\lambda[\Delta F_0+(1-\mu(\theta,\hat{x}))\Delta m_1]}{1-\mu(\theta,\hat{x})}\right]d\theta$$

$$+\frac{1}{2\epsilon}\int_{\theta_{\mu_I}(\Delta x)}^{\theta_{\mu_S}(\Delta x)}\left[\eta\Delta m_1-G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)+(1-\theta-\Delta x)\frac{G_I^D(\mu(\theta,\hat{x}),\theta+\Delta x)}{d\Delta x}\right]d\theta$$

$$+\frac{1}{2\epsilon}\int_{\theta_{\mu_S}(\Delta x)}^{\hat{x}+\epsilon}\left[\eta\Delta m_1-G_S^D(\mu(\theta,\hat{x}),\theta+\Delta x)\right]d\theta. \tag{A.10}$$

From the lower dominance region, the last three terms on the right-hand side are positive. The second term in also positive as mentioned above. The first term is always positive if $(\hat{x}-\epsilon-1)(R^U-R^D)+R^D>0$, which in general would not be true for low values of R^D and certainly not true for $R^D=0$. Thus, we consider that $(\hat{x}-\epsilon-1)(R^U-R^D)+R^D<0$ and show that the negative part is outweighed by the other positive terms given that noise is not too big. Taking the first and third terms in (A.10) in isolation we obtain:

$$\frac{1}{2\epsilon}\left[\left(\theta_{\mu_I(\Delta x)}-\theta_{\mu_R(\Delta x)}\right)\eta\Delta m_1-2\epsilon(\theta_{\mu_R(\Delta x)}-\hat{x}+\epsilon)\eta\lambda\Delta m_0(R^U-R^D)\right]+\Omega$$

$$=\eta\left[\left(\mu(\theta_{\mu_R}(\Delta x),\hat{x})-\mu(\theta_{\mu_I}(\Delta x),\hat{x})\right)\Delta m_1-2\epsilon\lambda\Delta m_0(R^U-R^D)\mu(\theta_{\mu_R}(\Delta x),\hat{x})\right]+\Omega, \tag{A.11}$$

where

$$\Omega=\frac{1}{2\epsilon}\int_{\theta_{\mu_R}(\Delta x)}^{\theta_{\mu_I}(\Delta x)}\left[\frac{(R^U-R^D)\Delta m_0-\overline{R}_{\theta+\Delta x}\Delta m_0-1/\lambda[\Delta F_0+(1-\mu(\theta,\hat{x}))\Delta m_1]}{1-\mu(\theta,\hat{x})}\right]d\theta$$

$$+\frac{1}{2\epsilon}\int_{\hat{x}-\epsilon}^{\theta_{\mu_R}(\Delta x)}\eta\frac{\lambda\Delta m_0R^D+\Delta F_0+\Delta m_1}{\mu(\theta,\hat{x})}d\theta$$

$$-\eta\lambda\Delta m_0(R^U-R^D)(1-\hat{x}-\epsilon)\left[1-ln\left(\mu(\theta_{\mu_R}(\Delta x),\hat{x})\right)\right]ln(2\epsilon)>0. \tag{A.12}$$

(A.11) is positive for small enough noise, satisfying

$$\epsilon<\epsilon^A\equiv\frac{\left(\mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)-\mu_I(\theta_{\mu_I}(\Delta x)+\Delta x)\right)\Delta m_1}{2\lambda\Delta m_0(R^U-R^D)\mu_R(\theta_{\mu_R}(\Delta x)+\Delta x)}, \tag{A.13}$$

using (A.4) and (A.5).

The second and third terms in (A.12) are positive given that $\epsilon < 1/2$. The first term is unambiguously positive if $(1-\theta)(R^U - R^D) - R^D > 0$. Thus, θ should be lower than $(R^U - 2R^D)/(R^U - R^D)$. Given that a threshold equilibrium exists, the threshold for fundamentals θ^* is between the upper and lower dominance regions and, thus, bounded above by a hypothetical threshold $\bar{\theta}$, which solves $\lambda \overline{R}_{\bar{\theta}} - g(\bar{\theta}) = 0$, where $g(\cdot)$ is given by (20). This threshold is given by:

$$\bar{\theta} = 1 - \sqrt{1 - \frac{2(1-\lambda R^D)}{\lambda(R^U - R^D)}}. \tag{A.14}$$

From (22) the upper bound for the signal threshold is $\bar{\theta} + \epsilon$, and a hedge fund that receives threshold signals believes that θ^* can be at most $\bar{\theta}$. Hence, the first term in (A.12) is positive if

$$\epsilon < \epsilon^B \equiv \frac{1}{2}\left(\frac{R^U - 2R^D}{R^U - R^D} - \bar{\theta}\right), \tag{A.15}$$

where $\epsilon^B > 0$ because $\lambda R^U > 2$. In sum, the threshold equilibrium is uniqueness for $\epsilon < \min(\epsilon^A, \epsilon^B)$ given by (A.13) and (A.13).

To conclude the proof we need to show that the threshold equilibrium is indeed an equilibrium, i.e., $\Delta(x_i, x^*)$ in (24) is positive for all $x_i > x^*$, and negative for all $x_i < x^*$. The steps (and notation) below are the same as in Goldstein and Pauzner (2005).

First, consider that $x_i < x^*$. Then we can decompose the intervals $[x_i - \epsilon, x_i + \epsilon]$ and $[x^* - \epsilon, x^* + \epsilon]$ into a common part $c = [x_i - \epsilon, x_i + \epsilon] \cap [x^* - \epsilon, x^* + \epsilon]$, and two disjoint parts $d^i = [x_i - \epsilon, x_i + \epsilon] \backslash c$ and $d^* = [x^* - \epsilon, x^* + \epsilon] \backslash c$. Thus, (24) and (25) can be written as:

$$\Delta(x_i, x^*) = \frac{1}{2\epsilon}\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta + \frac{1}{2\epsilon}\int_{\theta \in d^i} \nu(\mu(\theta, x^*), \theta) d\theta, \tag{A.16}$$

$$\Delta(x^*, x^*) = \frac{1}{2\epsilon}\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta + \frac{1}{2\epsilon}\int_{\theta \in d^*} \nu(\mu(\theta, x^*), \theta) d\theta. \tag{A.17}$$

From (23), ν is always one over d^i, thus $\int_{\theta \in d^i} \nu(\mu(\theta, x^*), \theta) d\theta < 0$. As a result, it suffices to show that $\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta < 0$. ν only crosses zero once and it is positive for higher values of θ and negative for lower values of θ in the interval $[x^* - \epsilon, x^* + \epsilon]$. Hence, given that (A.17) is zero, we get that $\int_{\theta \in d^*} \nu(\mu(\theta, x^*) > 0$ and $\int_{\theta \in c} \nu(\mu(\theta, x^*), \theta) d\theta < 0$, since the fundamentals are higher over d^* than c. Essentially, observing a signal x_i below x^* shifts probability from positive values of ν to negative values of ν because noise is uniformly distributed. Note that the argument goes through even if the interval c is empty. The proof for $x_i > x^*$ is similar.

Proof of Proposition 2 In a threshold strategy equilibrium, the dealer's optimization problem has the following

Lagrangian,

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(1-\theta^{*2})u(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)\\ &\quad -\xi_0(\Delta F_0+g(\theta^*)\Delta m_0)+\xi_{PL}(\Delta m_1+\Delta F_0)-\xi_{DD}(R^D\Delta m_0+\Delta F_0+\Delta m_1+\Delta F_1)+\xi_V V(\theta^*).\end{aligned} \tag{A.18}$$

Taking the first order conditions with respect to the contract terms and the run threshold gives:

$$\frac{\partial\mathcal{L}}{\partial\Delta m_0}=\frac{1}{2}(1-\theta^{*2})u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)R^U-\xi_0 g(\theta^*)-\xi_{DD}R^D+\xi_V\frac{\partial V}{\partial\Delta m_0}=0, \tag{A.19}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta m_1}=\frac{1}{2}(1-\theta^{*2})u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)+\xi_{PL}-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta m_1}=0, \tag{A.20}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta F_0}=\frac{1}{2}(1-\theta^{*2})u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_0+\xi_{PL}-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta F_0}=0, \tag{A.21}$$

$$\frac{\partial\mathcal{L}}{\partial\Delta F_1}=\frac{1}{2}(1-\theta^{*2})u'(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_{DD}+\xi_V\frac{\partial V}{\partial\Delta F_1}=0, \tag{A.22}$$

$$\frac{\partial\mathcal{L}}{\partial\theta^*}=-\theta^* u(R^U\Delta m_0+\Delta F_1+\Delta F_0+\Delta m_1)-\xi_0 g'(\theta^*)\Delta m_0+\xi_V\frac{\partial V}{\partial\theta^*}=0, \tag{A.23}$$

where $g(\theta^*)$ is given by (20) and $g'(\theta^*)=(g(\theta^*)-\lambda\overline{R}_{\theta^*})/(1-\theta^*)$.

The global games expression holds always will equality in equilibrium, i.e., $\xi_V\neq 0$. Moreover, we conjecture that the participation constraint in $t=0$, the positive liquidity injection constraint, and the dealer default constraint are all binding in equilibrium, i.e., $\xi_0,\xi_{PL},\xi_{DD}>0$.

The last three binding constraints pin down the optimal ΔF_0, Δm_1 and ΔF_1 as functions of θ^* and Δm_0 such that $\Delta F_0=-g(\theta^*)\Delta m_0$, $\Delta m_1=-\Delta F_0=g(\theta^*)\Delta m_0$ and $\Delta F_1=-R^D\Delta m_0$. Substituting in the conjectured contract terms, the global game expression $V(\theta^*)=0$ (the detailed expression is reported in (B.28) in Appendix B) becomes:

$$\theta^*(\eta-1)\mu_I+f(\theta^*)\Delta m_0=0, \tag{A.24}$$

where $\mu_I=\lambda\overline{R}_{\theta^*}(R^U-R^D)/(g(\theta^*)R^U-\lambda\overline{R}_{\theta^*}R^D)$, $\mu_R=\lambda\overline{R}_{\theta^*}/g(\theta^*)$, and

$$\begin{aligned}f(\theta^*)&=\theta^*R^D\mu_I-\eta g(\theta^*)\mu_R+\frac{g(\theta^*)}{\lambda}(\mu_R-\mu_I)+\frac{(\lambda\overline{R}_{\theta^*}-g(\theta^*))}{\lambda}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right)+\eta\lambda\overline{R}_{\theta^*}\ln(\mu_R)\\ &\quad+(1-\theta^*)\frac{R^D}{\lambda\overline{R}_{\theta^*}}\left[g(\theta^*)\mu_I-(\lambda\overline{R}_{\theta^*}-g(\theta^*))\ln(1-\mu_I)\right].\end{aligned} \tag{A.25}$$

Finally, evaluating the first-order conditions (A.20)-(A.23) at the conjectured equilibrium we obtain the following expressions for the Lagrange multipliers:

$$\xi_V = \frac{\theta^* u\left((R^U - R^D)\Delta m_0\right)}{\frac{\partial V}{\partial \theta^*} - \Delta m_0 g'(\theta^*)\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right)}, \qquad \xi_0 = \xi_V\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right),$$

$$\xi_{DD} = \frac{(1-\theta^{*2})}{2}u'\left((R^U - R^D)\Delta m_0\right) + \xi_V \frac{\partial V}{\partial \Delta F_1}, \qquad \xi_{PL} = \xi_V\left(\frac{\partial V}{\partial \Delta F_1} - \frac{\partial V}{\partial \Delta m_1}\right). \tag{A.26}$$

Using the derived Lagrange multipliers and the conjectured contract terms the first-order condition (A.19) can be written as:

$$\frac{1}{2}(1-\theta^{*2})u'((R^U - R^D)\Delta m_0(\theta^*))(R^U - R^D) + \frac{\theta^* u\left((R^U - R^D)\Delta m_0(\theta^*)\right) f(\theta^*)}{\frac{\partial V}{\partial \theta^*} - \Delta m_0(\theta^*) g'(\theta^*)\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right)} = 0, \tag{A.27}$$

where we used the fact the $f(\theta^*) = \frac{\partial V}{\partial \Delta m_0} - g(\theta^*)\left(\frac{\partial V}{\partial \Delta F_0} - \frac{\partial V}{\partial \Delta m_1}\right) - \frac{\partial V}{\partial \Delta F_1} R^D$ when the partial derivatives of $V(\theta^*)$ are evaluated at the conjectured contract terms (the detailed expressions for these derivatives are reported in (B.29)-(B.33) in Appendix B). $\Delta m_0(\theta^*) = -\theta^*(\eta - 1)\mu_I / f(\theta^*)$ is given by (A.24).

We have reduced the problem down to one equation (A.27) and one unknown θ^*. We proceed to show that a solution $\theta^* \in (\theta^{LD}, \theta^{UD})$ exists. Recall that $0 < \theta^{LD}, \theta^{UD} < 1$ from Lemma 3.

As a first step, consider the hypothetical upper bound $\bar{\theta} = 1 - \sqrt{(\lambda R^U + \lambda R^D - 2)/(\lambda(R^U - R^D))}$. Recall that we utilized this threshold to show the uniqueness of threshold strategy and derived it in (A.14). Then, $\Delta m_0(\bar{\theta}) = \bar{\theta}(\eta - 1)/(\eta\lambda \overline{R}_{\bar{\theta}} - R^D)$, which is strictly positive if $R^D < \eta R^U/(\eta + R^U)$.

Next, consider a θ' relatively close to $\bar{\theta}$. In that case, because $\Delta m_0(\theta)$ is continuous and $\Delta m_0(\bar{\theta}) = \bar{\theta}(\eta - 1)/(\eta\lambda \overline{R}_{\bar{\theta}} - R^D)$ is strictly greater than zero, $\Delta m_0(\theta')$ is strictly positive. In addition, $(\lambda \overline{R}_{\theta'} - g(\theta'))$ is very close to zero, $\partial V/\partial \theta$ is finite and positive, and $f(\theta')$ is negative. Thus for a dealer sufficiently risk averse, i.e., u' small enough, the left-hand side of equation (A.27) is negative. Moreover, for $\theta'' = 0$, $f(0)$ is finite, because $\lambda R^D < 1$ implying that $\mu_I, \mu_R \in (0, 1)$, and therefore $\Delta m_0(0) = 0$. Hence, for $\theta'' = 0$, the left-hand side of (A.27) becomes $1/2u'(0)(R^U - R^D) > 0$. Given that the left-hand side of (A.27) is continuous, there exists θ^* between zero and $\bar{\theta}$. Given that $\lambda \bar{R}_\theta - g(\theta)$ is an increasing function in θ, then $\lambda \bar{R}_{\theta^*} - g(\theta^*) < 0$, and hence lower than θ^{UD}. To finalize the proof, we need to show that $\theta^{LD} < \theta^*$. For any $\Delta m_0(\theta^*)$ solving (A.24), we know from Corollary 1 that the participation constraint (19) in period 1 is not binding for θ^*. Rearranging (19), we get that $\eta - 1 - \Delta F_1 > (\eta \Delta m_1 - (1-\theta^*)(R^D \Delta m_0 + \Delta F_0 + \Delta F_1))/\theta^*$, which implies that $\theta^{LD} < \theta^*$ by substituting the latter expression in the definition of θ^{LD} in Lemma 3.

Proof of Corollary 2

With $R^D = 0$ we have that the proposed equilibrium $\Delta F_1 = 0$, therefore $\mu_I = \mu_R = \frac{\lambda \overline{R}_{\theta^*}}{g(\theta^*)}$ reducing

$f(\theta^*)$ and $\Delta m_0(\theta^*)$ to,

$$\begin{aligned} f(\theta^*) &= -\eta\lambda\overline{R}_{\theta^*}\left(1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right) \\ \Delta m_0(\theta^*) &= \frac{\theta^*(\eta-1)}{\eta g(\theta^*)\left(1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right)} \end{aligned}$$

To find the expression for the first order condition characterized in equation (28), we first observe that,

$$\begin{aligned} \frac{\partial V}{\partial\theta^*} &= (\eta-1)\left(2+\frac{\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)}{1-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)}\right)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)} \\ \frac{\partial V}{\partial\Delta F_0}-\frac{\partial V}{\partial\Delta m_1} &= \eta\left(2-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\right)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)} \end{aligned}$$

reducing equation (28) to

$$T(\theta^*) := 2\left(1-\theta^*-3\theta^{*2}+\theta^*(1+\theta^*)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)-\ln\left(\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)\left(1-\theta^*-4\theta^{*2}+\theta^*(1+\theta^*)\frac{\lambda\overline{R}_{\theta^*}}{g(\theta^*)}\right)=0$$

To ensure the existence of an equilibrium we have to determine under what conditions $T(\theta^*)=0$ has a solution. Given that T is continuous, it suffices to show under what conditions T is positive and negative for the possible limits of θ^*. Recall that from the proof of Proposition 28 we know that θ^* must be below $\bar{\theta}$, defined by $\lambda\overline{R}_{\bar{\theta}}=g(\bar{\theta})$, which in this case equals $1-\sqrt{1-2/\lambda R^U}$.[30]

Note that,

$$\begin{aligned} \lim_{\theta\longrightarrow 0} T(\theta) &= \infty \\ \lim_{\theta\longrightarrow\bar{\theta}} T(\theta) &= 2(1-\bar{\theta}-3\bar{\theta}^2+\bar{\theta}(1+\bar{\theta})) = 2(1-2\bar{\theta}^2) \end{aligned}$$

therefore we have to ensure that $1<2\bar{\theta}^2$, that is, $7(\lambda R^U)^2-8\lambda R^U-16<0$ which holds for $\lambda R^U<\frac{4+8\sqrt{2}}{7}$. From Proposition 28 we also know that λR^U needs to be greater than 2. Therefore, an equilibrium exists if $\lambda R^U\in\left(2,\frac{4+8\sqrt{2}}{7}\right)$.

Finally, comparative statics of the equilibrium are given by taking the implicit derivative of T with respect to R^U. For notational purposes, consider $h(R^U,\theta)=\frac{\lambda R^U\theta}{g(\theta)}$, therefore we can write T as,

$$T(\theta)=(2-\ln(h(R^U,\theta)))\left(1-\theta-4\theta^2+\theta(1+\theta)h(R^U,\theta)\right)+2\theta^2$$

Taking the partial derivative of T with respect to R^U gives,

[30] θ^* must be below $\bar{\theta}$ because $\lambda\overline{R}_\theta-g(\theta)$ is increasing and any feasible threshold equilibria requires that the liquidation value $\lambda\overline{R}_\theta\Delta m_0$ be below ΔF_0.

$$\begin{aligned}\frac{\partial T}{\partial R^U} &= \left[(2-\ln(h(R^U,\theta)))\theta(1+\theta) - \frac{1}{h(R^U,\theta)}\left(1-\theta-4\theta^2+\theta(1+\theta)h(R^U,\theta)\right)\right]\frac{\partial h}{\partial R^U} \\ &= \underbrace{\left[(1-\ln(h(R^U,\theta)))\theta(1+\theta) - \frac{1}{h(R^U,\theta)}\left(1-\theta-4\theta^2\right)\right]}_{>0,\text{when } \theta=\theta^*}\frac{\partial h}{\partial R^U}\end{aligned}$$

where the term in brackets is positive in θ^* because in the equilibrium $1-\theta^*-4\theta^{*2} = \frac{-2\theta^{*2}}{2-\ln(h(R^U,\theta^*)} - \theta(1+\theta^*)h(R^U,\theta^*) < 0$. Taking the partial derivative of T with respect to θ gives,

$$\begin{aligned}\frac{\partial T}{\partial \theta} &= \underbrace{\left[(1-\ln(h(R^U,\theta)))\theta(1+\theta) - \frac{1}{h(R^U,\theta)}\left(1-\theta-4\theta^2\right)\right]}_{>0,\text{when } \theta=\theta^*}\frac{\partial h}{\partial \theta} + \\ &\quad \underbrace{(2-\ln(h(R^U,\theta)))(-1-8\theta+(1+2\theta)h(R^U,\theta))+4\theta}_{>0,\text{when } \theta=\theta^*}.\end{aligned}$$

The second line is positive in θ^* because in the equilibrium $2-\ln(h(R^U,\theta^*)) = \frac{-2\theta^{*2}}{1-\theta^*-4\theta^{*2}+\theta^*(1+\theta^*)h(R^U,\theta^*)}$, implying that the sign of the second line has the same sign as $-2\theta^{*2}(-1-8\theta^*+(1+2\theta^*)h(R^U,\theta^*))+4\theta^*(1-\theta^*-4\theta^{*2}+\theta^*(1+\theta^*)h(R^U,\theta^*)) = 2\theta^*(2-\theta^*)+2\theta^{*2}h(R^U,\theta^*) > 0$.

Finally, note that,

$$\begin{aligned}\frac{\partial h}{\partial \theta} &= \frac{\lambda R^U}{\left(1-\lambda R^U\frac{\theta^2}{2}\right)}\left[1-2\theta+\lambda R^U\frac{\theta^2}{2}\right] \\ &> \frac{\lambda R^U}{\left(1-\lambda R^U\frac{\theta^2}{2}\right)}(\theta-1)^2 > 0 \\ \frac{\partial h}{\partial \theta} &= \frac{\lambda}{\left(1-\lambda R^U\frac{\theta^2}{2}\right)}\theta(1-\theta) > 0\end{aligned}$$

where the first inequality holds because $\lambda R^U > 2$. Therefore, applying the implicit function theorem we have,

$$\frac{\partial \theta^*}{\partial R^U} = -\frac{\partial T}{\partial R^U}\bigg/\frac{\partial T}{\partial \theta^*} < 0$$

B Detailed expression for $V(\theta^*)$ and its derivatives

Expanding (26), the threshold θ^* is the solution to the $V(\theta^*) = 0$ shown in (B.28) below.

$$\begin{aligned}
V(\theta^*) =& \theta^* \left[(\eta-1) - \Delta F_1\right] \mu_I - \eta \Delta m_1 \mu_R + \frac{\Delta m_1}{\lambda}(\mu_R - \mu_I) + (1-\theta^*)\Delta m_1 \mu_S \\
& - (1-\theta^*)\Delta F_0 \ln(1-\mu_S) + \frac{\Delta F_0 + \lambda \overline{R}_{\theta^*} \Delta m_0}{\lambda} \ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta(\lambda \overline{R}_{\theta^*} \Delta m_0 + \Delta F_0 + \Delta m_1)\ln(\mu_R) \\
& + (1-\theta^*)\frac{R^D}{\lambda \overline{R}_{\theta^*}} \left[-\lambda \overline{R}_{\theta^*} \Delta m_0 \ln(1-\mu_S) + (\Delta F_0 + \lambda \overline{R}_{\theta^*} \Delta m_0) \ln\left(\frac{1-\mu_S}{1-\mu_I}\right) + \Delta m_1(\mu_I - \mu_S)\right].
\end{aligned} \tag{B.28}$$

The derivative of $V(\theta^*)$ with respect to the contract terms and θ^* are shown in (B.29)–(B.33) below.

$$\frac{\partial V}{\partial \Delta m_0} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta m_0} + \overline{R}_{\theta^*} \ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta \lambda \overline{R}_{\theta^*} \ln(\mu_R) + (1-\theta^*)R^D \left[-\ln(1-\mu_S) + \ln\left(\frac{1-\mu_S}{1-\mu_I}\right)\right], \tag{B.29}$$

$$\frac{\partial V}{\partial \Delta m_1} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta m_1} - \eta \mu_R + \frac{(\mu_R - \mu_I)}{\lambda} + (1-\theta^*)\mu_S + \eta \ln(\mu_R) + (1-\theta^*)\frac{R^D}{\lambda \overline{R}_{\theta^*}}(\mu_I - \mu_S), \tag{B.30}$$

$$\frac{\partial V}{\partial \Delta F_0} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta F_0} - (1-\theta^*)\ln(1-\mu_S) + \frac{1}{\lambda}\ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta \ln(\mu_R) + (1-\theta^*)\frac{R^D}{\lambda \overline{R}_{\theta^*}} \ln\left(\frac{1-\mu_S}{1-\mu_I}\right), \tag{B.31}$$

$$\frac{\partial V}{\partial \Delta F_1} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \Delta F_1} - \theta^* \mu_I, \tag{B.32}$$

$$\begin{aligned}
&\frac{\partial V}{\partial \theta^*} = \theta^*(\eta-1)\frac{\partial \mu_I}{\partial \theta^*} + \left[(\eta-1) - \Delta F_1\right]\mu_I - \Delta m_1 \mu_S \\
&+ \Delta F_0 \ln(1-\mu_S) + (R^U - R^D)\Delta m_0 \ln\left(\frac{1-\mu_I}{1-\mu_R}\right) + \eta \lambda (R^U - R^D)\Delta m_0 \ln(\mu_R) \\
&- \frac{R^D}{\lambda \overline{R}_{\theta^*}} \left[-\lambda \overline{R}_{\theta^*} \Delta m_0 \ln(1-\mu_S) + (\Delta F_0 + \lambda \overline{R}_{\theta^*} \Delta m_0)\ln\left(\frac{1-\mu_S}{1-\mu_I}\right) + \Delta m_1(\mu_I - \mu_S)\right] \\
&+ (1-\theta^*)R^D \left[-\Delta F_0 \frac{(R^U - R^D)}{\lambda \overline{R}^2_{\theta^*}} \ln\left(\frac{1-\mu_S}{1-\mu_I}\right) - \Delta m_1 \frac{(R^U - R^D)}{\lambda \overline{R}^2_{\theta^*}}(\mu_I - \mu_S)\right],
\end{aligned} \tag{B.33}$$

with

$$\frac{\partial \mu_I}{\partial \theta^*} = \frac{\lambda(R^U - R^D)R^U \left(R^U \Delta m_0 \Delta m_1 - \Delta F_0 \Delta F_1\right)}{(\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1)^2}, \quad \frac{\partial \mu_I}{\partial \Delta m_0} = \frac{R^U \lambda \overline{R}_{\theta^*}}{\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1},$$

$$\frac{\partial \mu_I}{\partial \Delta m_1} = -\frac{R^U(\Delta F_0 + \lambda \overline{R}_{\theta^*} \Delta m_0)R^U}{(\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1)^2} = \frac{R^U(1-\mu_I)}{\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1}, \quad \frac{\partial \mu_I}{\partial \Delta F_0} = \frac{R^U}{\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1},$$

$$\frac{\partial \mu_I}{\partial \Delta F_1} = -\frac{R^U(\Delta F_0 + \lambda \overline{R}_{\theta^*} \Delta m_0)\lambda \overline{R}_{\theta^*}}{(\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1)^2} = \frac{\lambda \overline{R}_{\theta^*}(1-\mu_I)}{\lambda \overline{R}_{\theta^*} \Delta F_1 + R^U \Delta m_1}.$$

C Interpretation of FRBNY's Primary Dealer Survey

The model's notation is useful to interpret the data from FRBNY's primary dealer survey. The total amount of funds distributed and collected (i.e., Securities In and Securities Out) can be interpreted as loans made to hedge funds $T_I - m^H$ and loans received from money funds $T_O - m^M$, respectively. In this case, T_I is the total amount of collateral received from hedge funds and T_O is the total amount of collateral posted with money funds. It is important to note that the total amount of collateral posted with money funds T_O may not necessarily come from hedge fund counterparties. That is, a fraction of collateral posted in Securities Out can be part of the dealer's own asset position. But when dealing in cash and secured financing markets, dealers have a natural collateral restriction to follow, known as the *box constraint*. This constraint forces dealers to have a non-negative stock of collateral. That is, denoting L and S the dealer's long and short position, respectively, the box constraint can be translated into

$$(L - S) + (T_I - T_O) \geq 0.$$

That is, the amount of collateral owned and sourced must be larger than the amount of collateral sold and posted.

In figure 1 we argue that the difference between Securities Out and Securities In plus net position is a lower bound for the amount of liquidity coming from different haircuts. In effect,

$$\underbrace{(T_O - m^M)}_{sec-out} - (\underbrace{(T_I - m^H)}_{sec-in} + (L - S)) \leq (T_O - T_I) + m^H - m^M + (T_I - T_O) = m^H - m^M$$

where the inequality comes from imposing the box constraint.

An important caveat to this lower bound is that survey asks respondents to also report the total amount of long and short positions in forward contracts.[31] Because forward contracts are derivatives, they do not enter into the box constraint, which is strictly a cash market restriction. Regrettably, we cannot tease out

[31] Forwards are the only derivative contracts that are reported in the FR 2004.

how much the lower bound is attributable to haircut differences and how much is due to large forward positions.

From Figures 1 and 2, we can see that in the last year of Bear Stearn's activity, the estimated amount of liquidity the firm captured through rehypothecation was at least between \$10 and \$50 billion, equivalent to 1/10 or 1/3 of its entire repo activity.