

Identification and Inference of Network Formation Games with Misclassified Links

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Abstract

This paper considers a network formation model when links are measured with error. We focus on a game-theoretical model of strategic network formation with incomplete information, in which the linking decisions depend on agents' exogenous attributes and endogenous positions in the network. In the presence of link misclassification, we derive moment conditions that characterize the identified set of the preference parameters associated with homophily and network externalities. Based on the moment equality conditions, we provide an inference method that is asymptotically valid with a single network of many agents.

Keywords: Misclassification, Network formation models, Strategic interactions, Incomplete information

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1 Introduction

Researchers across different disciplines have recorded that measurement error of links is a pervasive problem in network data (e.g., Holland and Leinhardt 1973, Moffitt 2001, Kossinets 2006, Ammermueller and Pischke 2009, Wang, Shi, McFarland, and Leskovec 2012, Angrist 2014, de Paula 2017, Advani and Malde 2018). Although strategic network formation models are essential for learning about the creation of linking connections and peer effects with an endogenous network structure, to the best of our knowledge, there has been no work addressing the misclassification problem in strategic network formation models. In this paper, we consider identification and inference in a game-theoretical model of strategic network formation with potentially misclassified links.

We focus on a simultaneous game with imperfect information in which agents decide to form connections to maximize their expected utility (cf. Leung 2015, and Ridder and Sheng 2015). The agents' decisions are interdependent since the utility attached to establishing a specific link depends on the agents' observed attributes and positions in the network through link externalities (such as reciprocity, in-degree, and out-degree statistics). The misclassification problem will affect the decisions to forming network links in two different ways. First, the binary outcome variable of link, which represents an individual's optimal decision, is misclassified. Second, the link misclassification problem prevents us from directly identifying the belief system of an agent about others' linking decisions, which the agent's decision relies on. In this sense, the measurement error problem occurs on the left- and right-hand side of the equation describing the linking decisions as in Lemma 1.

We propose a novel approach for analyzing network formation models, which is robust to misclassification. Specifically, we characterize the identified set for the structural parameters, including the preference parameters concerning homophily and network externalities. A notable innovation from our approach is that we derive the relationship between the choice probabilities of observed network connections and the belief system (Lemma 2). This result is crucial to control for the endogeneity of the equilibrium beliefs and reduce the model to a single agent decision model in the presence of misspecification.

We also introduce an inference method that is asymptotically valid as long as we observe one network with a large number of agents. Given a finite support of the exogenous attribute, the identified set is characterized by a finite number of unconditional moment equalities. Based on these moment equalities, we construct a test statistic whose asymptotic null distribution is the χ^2 distribution with known degrees of freedom.

Our methodology contributes to the growing econometric literature that studies strategic formation of networks. See Graham (2015), Chandrasekhar (2016), and de Paula (2017) for a recent survey. The network formation model considered in this paper builds on the framework of strategic interactions with incomplete information introduced by Leung (2015) and extended by Ridder and Sheng (2015). The analysis in our paper addresses the problems arising due to link misclassification in their models.

This paper is also related to the literature of mismeasured discrete variables, e.g., misclassified binary outcome variable (Hausman, Abrevaya, and Scott-Morton, 1998), and misclassified discrete treatment variable (Mahajan, 2006; Lewbel, 2007; Chen, Hu, and Lewbel, 2008; Hu, 2008). Specifically, our approach to misclassified links is based on Molinari (2008), which offers a general bounding strategy with misclassified discrete variables.

There are a few papers in the literature of social interactions that have examined the presence of measurement error in network data (Chandrasekhar and Lewis, 2014; Kline, 2015; Lewbel, Norris, Pendakur, and Qu, 2017). However, the results in these papers cannot be applied directly to our framework since they have a different object of interest. In particular, they have primarily focused on studying peer effects given a

network of interactions, and do not investigate directly the underlying process that drives the formation of the network connections. In contrast, our paper studies the effects of link misclassification in a model of strategic network formation.

The remainder of this paper is organized as follows. Section 2 describes the network formation model as a game of incomplete information. Section 3 characterizes the identified set of the structural parameters. Section 4 introduces an inference method based on the representation of the identified set.

2 Network formation game with misclassification

We extend Leung (2015) and Ridder and Sheng (2015) to model the formation of a directed network with misclassified links. Particularly, our approach follows Leung (2015) for simplicity.

The network consists of a set of n agents, which we denote by $\mathcal{N}_n = \{1, \dots, n\}$. We assume that each pair of agents (i, j) with $i, j \in \mathcal{N}_n$ is endowed with a vector of exogenous attributes $X_{ij} \in \mathbb{R}^d$ and an idiosyncratic shock $\varepsilon_{ij} \in \mathbb{R}$. Let $X = \{X_{ij} : i \in \mathcal{N}_n\} \in \mathcal{X}^n$ be a profile of attributes that is common knowledge to all the agents in the network, and $\varepsilon_i = \{\varepsilon_{ij} : j \in \mathcal{N}_n\}$ is a profile of idiosyncratic shocks that is agent i 's private information. Let $\varepsilon = \{\varepsilon_i : i \in \mathcal{N}_n\}$.

The network is represented by a $n \times n$ adjacency matrix G_n^* , where the ij th element $G_{ij,n}^* = 1$ if agent i forms a direct link to agent j and $G_{ij,n}^* = 0$ otherwise. We assume that the network is directed, i.e., $G_{ij,n}^*$ and $G_{ji,n}^*$ may be different. The diagonal elements are normalized to be equal to zero, i.e., $G_{ii,n}^* = 0$. The network G_n^* is potentially misclassified and the researcher observes G_n , which is a proxy for G_n^* .

Given the network G_n^* and information (X, ε_i) , agent i has utility

$$U_i(G_{i,n}^*, G_{-i,n}^*, X, \varepsilon_i) = \frac{1}{n} \sum_{j=1}^n G_{ij,n}^* \left[\left(G_{ji,n}^*, \frac{1}{n} \sum_{k \neq i} G_{kj,n}^*, \frac{1}{n} \sum_{k \neq i} G_{ki,n}^* G_{kj,n}^*, X'_{ij} \right) \beta_0 + \varepsilon_{ij} \right],$$

where $G_{i,n}^* = \{G_{ij,n}^* : j \in \mathcal{N}_n\}$, $G_{-i,n}^* = \{G_{j,n}^* : j \neq i\}$, and β_0 is an unknown finite dimensional vector in a parameter space \mathcal{B} .

Agent i 's marginal utility of forming the link $G_{ij,n}^*$ depends on a vector of network statistics, the profile of exogenous attributes, and the link-specific idiosyncratic component. The first component in the vector of network statistics capture the utility obtained from a reciprocate link with agent j , $G_{ij,n}^*$. The second network statistic is the weighted in-degree of agent j , $\frac{1}{n} \sum_{k \neq i} G_{kj,n}^*$, captures the utility of connecting with agents of high centrality in the network. The last network statistic capture the utility of being connected to the same agents, $\frac{1}{n} \sum_{k \neq i} G_{ki,n}^* G_{kj,n}^*$. The profile of exogenous attributes captures the preferences for homophily on observed characteristics. Finally, ε_{ij} is an unobserved link-specific component affecting agent i 's decision of linking with agent j .

Let $\delta_{i,n}(X, \varepsilon_i)$ denote a generic agent i 's pure strategy, which maps the information (X, ε_i) to an action in $\mathcal{G}^n = \{0, 1\}^n$. Let $\sigma_{i,n}(g_{i,n}^* | X) = Pr(\delta_{i,n}(X, \varepsilon_i) = g_{i,n}^* | X)$ be the probability that agent i chooses $g_{i,n}^* \in \mathcal{G}^n$ given X , and let $\sigma_n(X) = \{\sigma_{i,n}(g_{i,n}^* | X), i \in \mathcal{N}_n, g_{i,n}^* \in \mathcal{G}^n\}$. We call $\sigma_n(X)$ a belief profile. Given a belief profile σ_n and the information (X, ε_i) , the agent i chooses $g_{i,n}^*$ from \mathcal{G}^n to maximize the expected utility of $U_i(g_{i,n}^*, \delta_{-i,n}(X, \varepsilon_{-i}), X, \varepsilon_i)$ given $(X, \varepsilon_i, \sigma_n)$.

In an n -player game, a Bayesian Nash Equilibrium $\sigma_n(X)$ is a belief profile that satisfies

$$\sigma_{i,n}(g_{i,n}^* | X) = Pr(\delta_{i,n}(X, \varepsilon_i) = g_{i,n}^* | X, \sigma_n)$$

for all attribute profiles $X \in \mathcal{X}^n$, actions $g_{i,n}^* \in \mathcal{G}^n$, and agents $i \in \mathcal{N}_n$, where

$$\delta_{i,n}(X, \varepsilon_i) = \arg \max_{g_{i,n}^* \in \mathcal{G}^n} E [U_i(g_{i,n}^*, \delta_{-i,n}(X, \varepsilon_{-i}), X, \varepsilon_i) \mid X, \varepsilon_i, \sigma_n].$$

We impose the following assumptions, which are also used by Leung (2015) and Ridder and Sheng (2015).

Assumption 1. *The following hold for any n ,*

1. *For any $A_1, A_2 \subset \mathcal{N}_n$ disjoint, $\{X_{ij} : i, j \in A_1\}$ and $\{X_{kl} : k, l \in A_2\}$ are independent.*
2. *$\{\varepsilon_{ij} : i, j \in \mathcal{N}_n\}$ are marginally distributed with the standard normal distribution with the cdf Φ and the pdf ϕ . Further, $\{\varepsilon_i : i \in \mathcal{N}_n\}$ are mutually independent.*
3. *ε and X are independent.*
4. *Attributes $\{X_{ij} : i, j \in \mathcal{N}_n\}$ are identically distributed with a probability mass function bounded away from zero.*

We focus on a symmetric equilibrium, where an equilibrium profile σ_n is symmetric if $\sigma_{i,n}(g_{i,n}^* \mid X) = \sigma_{\pi(i),n}(\pi(g_{\pi(i),n}^*) \mid \pi(X))$ for any $i \in \mathcal{N}_n$, $g_{i,n}^* \in \mathcal{G}^n$, and permutation $\pi \in \Pi$.¹ Given Assumption 1, Leung (2015, Theorem 1) and Ridder and Sheng (2015, Proposition 1) show the existence of a symmetric equilibrium.

Assumption 2. *For any n , the agents play a symmetric equilibrium σ_n , i.e., there exists $\{\delta_{i,n} : i \in \mathcal{N}_n\}$ such that for any $i \in \mathcal{N}_n$ the following holds: (i) $G_{i,n}^* = \delta_{i,n}(X, \varepsilon_i)$, (ii) $\sigma_{i,n}(g_{i,n}^* \mid X) = \Pr(\delta_{i,n}(X, \varepsilon_i) = g_{i,n}^* \mid X, \sigma_n)$, (iii) $\delta_{i,n}(X, \varepsilon_i) = \arg \max_{g_{i,n}^* \in \mathcal{G}^n} E [U_i(g_{i,n}^*, \delta_{-i,n}(X, \varepsilon_{-i}), X, \varepsilon_i) \mid X, \varepsilon_i, \sigma_n]$, and (iv) σ_n is symmetric.*

The next lemma characterizes the optimal decision rule for the formation of each link in the network.

Lemma 1. *Under Assumption 1 and 2, $G_{ij,n}^* = \mathbf{1} \{(Z_{ij,n}^*)' \beta_0 + \varepsilon_{ij} \geq 0\}$, where*

$$\gamma_{ij,n}^* = E \left[\left(G_{ji,n}^*, \frac{1}{n} \sum_{k \neq i} G_{kj,n}^*, \frac{1}{n} \sum_{k \neq i} G_{ki,n}^* G_{kj,n}^* \right)' \mid X, \sigma_n \right]$$

and

$$Z_{ij,n}^* = \begin{pmatrix} \gamma_{ij,n}^* \\ X_{ij} \end{pmatrix}.$$

Notice that given the misclassification problem, both the optimal action $G_{ij,n}^*$ and the equilibrium beliefs about the network statistics $\gamma_{ij,n}^*$ in the optimal decision rule will be misclassified.

We assume that the conditional distribution of the observed network G_n is related to that of the true state of network, G_n^* , as follows.

¹Define permutation functions as follows. Fix any $k, l \in \mathcal{N}_n$, and let $g_{i,n}^* \in \mathcal{G}^n$. Define $\pi_{kl} : \mathcal{N}_n \mapsto \mathcal{N}_n$ as a permutations of the indices k and l . Specifically, it maps the index k to the index l , l to k , and i to itself for any $i \neq k, l$. Define π_{kl}^X as a function that maps each component $X_{ij} \in \mathbb{R}^d$ to $X_{\pi_{kl}(i)\pi_{kl}(j)}$; π_{kl}^a as a function that permutes the k th and l th elements of any $g_{i,n}^* \in \mathcal{G}^n$. Hence, π_{kl}^X swaps the attributes of agents k and l ; and π_{kl}^a swaps the links $G_{ik,n}^*$ and $G_{il,n}^*$ for any i . $\pi(\cdot)$ denote a generic element of $\Pi = \{(\pi_{kl}, \pi_{kl}^X, \pi_{kl}^a); k, l \in \mathcal{N}_n\}$. In this paper, we abuse the notation $\pi(\cdot)$ so that it denotes any of the three components of an element in Π .

Assumption 3. *There are two non-negative real numbers ρ_0 and ρ_1 with $\rho_0 + \rho_1 < 1$ such that the following two statements hold for every n and every $i, j, k \in \mathcal{N}_n$. (i) $G_{ki,n}$ and $G_{kj,n}$ are independent given $(G_{ki,n}^*, G_{kj,n}^*, X, \sigma_n)$. (ii) $Pr(G_{ij,n} \neq G_{ij,n}^* | G_{ij,n}^*, X, \sigma_n) = \rho_0 1\{G_{ij,n}^* = 0\} + \rho_1 1\{G_{ij,n}^* = 1\}$.*

Condition (i) in Assumption 3 requires that the observed linking decision $G_{ki,n}$ and $G_{kj,n}$ are conditional independent given the true state of the links $G_{ki,n}^*$ and $G_{kj,n}^*$, and information X, σ_n . Condition (i) in Assumption 3 characterizes the misclassification probabilities.

The following statement is a key observation in our analysis, which relates the observed network statistics, $\gamma_{ij,n}$, to the payoff relevant network statistics, $\gamma_{ij,n}^*$.

Lemma 2. *If Assumptions 1-3 hold, then $\gamma_{ij,n}^* = c(\rho_0, \rho_1) + C(\rho_0, \rho_1)\gamma_{ij,n}$ for every i, j , where*

$$\gamma_{ij,n} = E \left[\left(G_{ji,n}, \frac{1}{n} \sum_{k \neq i} G_{kj,n}, \frac{1}{n} \sum_{k \neq i} G_{ki,n} G_{kj,n}, \frac{1}{n} \sum_{k \neq i} (G_{ki,n} + G_{kj,n}) \right)' \mid X, \sigma_n \right]$$

$$c(r_0, r_1) = - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 - r_0 - r_1 & 0 & 0 & 0 \\ 0 & 1 - r_0 - r_1 & 0 & 0 \\ 0 & 0 & (1 - r_0 - r_1)^2 & r_0(1 - r_0 - r_1) \\ 0 & 0 & 0 & 1 - r_0 - r_1 \end{pmatrix}^{-1} \begin{pmatrix} r_0 \\ r_0 \\ r_0^2 \\ r_0 \end{pmatrix}$$

$$C(r_0, r_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 - r_0 - r_1 & 0 & 0 & 0 \\ 0 & 1 - r_0 - r_1 & 0 & 0 \\ 0 & 0 & (1 - r_0 - r_1)^2 & r_0(1 - r_0 - r_1) \\ 0 & 0 & 0 & 1 - r_0 - r_1 \end{pmatrix}^{-1}.$$

Notice that the first three components in $\gamma_{ij,n}$ are the observed analog to the statistics in $\gamma_{ij,n}^*$ since they determined by the observed network G_n . The last component in $\gamma_{ij,n}$ is the sum of the in-degrees of agents i and j , and it is the result of controlling for the unobserved network statistics $\frac{1}{n} \sum_{k \neq i} G_{ki,n}^* G_{kj,n}^*$. Precisely, the last two statistics in $\gamma_{ij,n}$ control for the beliefs about the unobserved network statistic $\frac{1}{n} \sum_{k \neq i} G_{ki,n}^* G_{kj,n}^*$. The intuition behind this result is similar to the one found in polynomial regression models with mismeasured continuous covariates (Hausman, Newey, Ichimura, and Powell, 1991).

Assumptions 1-3 imply the following relationship between the distributions of $G_{ij,n}$ and $G_{ij,n}^*$, which will be used in our identification analysis. Since we observe $G_{ij,n}$ in the dataset but the outcome of interest is $G_{ij,n}^*$, it is crucial to connect these two objects.

Lemma 3. *Under Assumptions 1-3, $Pr(G_{ij,n} = 1 \mid X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*) = (1 - \rho_0)Pr(G_{ij,n}^* = 0 \mid X_{ij}, \gamma_{ij,n}^*) + \rho_1 Pr(G_{ij,n}^* = 1 \mid X_{ij}, \gamma_{ij,n}^*)$.*

3 Identification Analysis

We characterize the identified set based on the joint distribution $P_{0,n}$ of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$.² In this section, we treat $\gamma_{ij,n}$ as observed because we can estimate it as follows. For a generic value x in the support of X_{ij} ,

²We could aim to characterize the identified set based on the joint distribution of $\{(G_{ij,n}, X_{ij}, \gamma_{ij,n}) : i, j \in \mathcal{N}_n\}$. Since we cannot estimate the joint distribution about n agents from a sample of n agents, however, the identified set based on $\{(G_{ij,n}, X_{ij}, \gamma_{ij,n}) : i, j \in \mathcal{N}_n\}$ is not immediately useful for an inference. In contrast, $P_{0,n}$ can be estimated.

we can define

$$\hat{p}(x) = \frac{1}{n^2} \sum_{i,j} 1\{X_{ij} = x\}$$

$$\hat{\gamma}(x) = \frac{\frac{1}{n^2} \sum_{i,j} (G_{ji,n}, \frac{1}{n} \sum_k G_{kj,n}, \frac{1}{n} \sum_k G_{ki,n} G_{kj,n}, \frac{1}{n} \sum_k (G_{ki,n} + G_{kj,n}))' 1\{X_{ij} = x\}}{\hat{p}(x)},$$

$\hat{p}(x)$ is an estimator for $Pr(X_{ij} = x)$ and $\hat{\gamma}_{ij} = \hat{\gamma}(X_{ij})$ is an estimator for $\gamma_{ij,n}$. Then we can estimate the distribution of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$ using the empirical distribution of $(G_{ij,n}, X_{ij}, \hat{\gamma}_{ij})$.

To formalize our identification analysis, we introduce several notations. Denote by \mathcal{P}^* the set of joint distributions of $(G_{ij,n}, G_{ij,n}^*, X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*, \varepsilon_{ij})$. Define the parameter space $\Theta = \mathcal{B} \times \{(r_0, r_1) : r_0, r_1 \geq 0, r_0 + r_1 < 1\}$, where \mathcal{B} is the parameter space for β_0 . Denote by \mathcal{P} the set of joint distributions of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$.

Based on Assumptions 1-3 and Lemmas 1-3, we impose the following three conditions on the true joint distribution $P_{0,n}^*$ of the variables $(G_{ij,n}, G_{ij,n}^*, X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*, \varepsilon_{ij})$ and the true parameter value $\theta_0 = (\beta, \rho_0, \rho_1)$.

Condition 1. Under P^* the following holds: (i) ε_{ij} is normally distributed with mean zero and variance one. (ii) ε_{ij} and $(X_{ij}, \gamma_{ij,n}^*)$ are independent.

Condition 2. $G_{ij,n}^* = \mathbf{1}\{(Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0\}$ a.s. P^* , where

$$Z_{ij,n}^* = \begin{pmatrix} \gamma_{ij,n}^* \\ X_{ij} \end{pmatrix}$$

Condition 3. (i) $P^*(G_{ij,n} = 1 \mid X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*) = (1 - r_0)P^*(G_{ij,n}^* = 0 \mid X_{ij}, \gamma_{ij,n}^*) + r_1P^*(G_{ij,n}^* = 1 \mid X_{ij}, \gamma_{ij,n}^*)$. (ii) $\gamma_{ij,n}^* = c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n}$ a.s. P^* .

For each element P of \mathcal{P} , we are going to define the identified set based on the above three conditions.

Definition 1. For each distribution $P \in \mathcal{P}$, the identified set $\Theta_I(P)$ is defined as the set of all $\theta = (b, r_0, r_1)$ in Θ for which there is some joint distribution $P^* \in \mathcal{P}^*$ such that Condition 1, 2, and 3 hold, and that the distribution of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to P .

Note that $\Theta_I(P)$ does not depend on the sample size n , but the identified set $\Theta_I(P_{0,n})$ based on the data distribution $P_{0,n}$ does.

The identified set is characterized as follows.

Theorem 1. Given a joint distribution $P \in \mathcal{P}$, $\Theta_I(P)$ is equal to the set of $\theta \in \Theta$ satisfying

$$E_P[G_{ij,n} - r_0 - (1 - r_0 - r_1)\Phi((c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n})'b_1 + X'_{ij}b_2) \mid X_{ij}, \gamma_{ij,n}] = 0. \quad (1)$$

If the links were measured without error, the moment equation in Eq. (1) would become $E_P[G_{ij,n} - \Phi([\gamma_{ij,n}]'_{123}b_1 + X'_{ij}b_2) \mid X_{ij}, \gamma_{ij,n}] = 0$, where $[\gamma_{ij,n}]_{123}$ is a vector composed by the first three components of $\gamma_{ij,n}$. The specification without measurement error is used as the basis for the maximum likelihood estimation in Leung (2015).

The identified set in Theorem 1 relies on the assumption that we know the distribution of ε_{ij} . In Appendix B, we characterize the identified set in a semiparametric framework.

4 Inference

In this section, we propose a confidence interval for θ based on the identification analysis in Theorem 1 and derive its asymptotic coverage when we observe one single network with many agents. As in Leung (2015) and Ridder and Sheng (2015), we use the asymptotic arguments based on a symmetric equilibrium.

Consider the unconditional sample analog of the moment condition in Eq. (1) is

$$\hat{m}_n(\theta) = \frac{1}{n^2} \sum_{i,j} (G_{ij,n} - r_0 - (1 - r_0 - r_1)\Phi((c(r_0, r_1) + C(r_0, r_1)\hat{\gamma}_{ij})'b_1 + X'_{ij}b_2)) \mathbf{1}_{ij},$$

where x_1, \dots, x_J are all the support points for X_{ij} and $\mathbf{1}_{ij} = (1\{X_{ij} = x_1\}, \dots, 1\{X_{ij} = x_J\})'$. We estimate the variance of $\hat{m}_n(\theta)$ by

$$\hat{S}(\theta) = \left(\frac{1}{n} \sum_{i=1}^n \hat{\psi}_i(\theta) \hat{\psi}_i(\theta)' - \left(\frac{1}{n} \sum_{i=1}^n \hat{\psi}_i(\theta) \right) \left(\frac{1}{n} \sum_{i=1}^n \hat{\psi}_i(\theta) \right)' \right).$$

where

$$\begin{aligned} \hat{\psi}_i(\theta) &= \frac{1}{n} \sum_{j \neq i} G_{ij,n} \mathbf{1}_{ij} \\ &\quad - (1 - r_0 - r_1) \frac{1}{n^2} \sum_{l,j} \left(\phi((c(r_0, r_1) + C(r_0, r_1)\hat{\gamma}_{lj,n})'b_1 + X'_{lj}b_2)b'_1 C(r_0, r_1) \hat{\psi}_{\gamma,i,n}(X_{lj})) \right) \mathbf{1}_{lj} \end{aligned}$$

and

$$\hat{\psi}_{\gamma,k,n}(x) = \frac{1}{n^2} \sum_{i_1, j_1} \frac{1\{X_{i_1, j_1} = x\}}{\hat{p}(x)} \begin{pmatrix} 0 \\ G_{kj_1} \\ G_{ki_1} G_{kj_1} \\ G_{ki_1} + G_{kj_1} \end{pmatrix} + \frac{1}{n} \sum_{i_1} \frac{1\{X_{i_1, k} = x\}}{\hat{p}(x)} \begin{pmatrix} G_{ki_1} \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

For a size $\alpha \in (0, 1)$, a confidence interval for θ is constructed as

$$CI_n(\alpha) = \{\theta \in \Theta : n\hat{m}_n(\theta)' \hat{S}(\theta)^{-1} \hat{m}_n(\theta) \leq c_\alpha\},$$

where c_α is the $1 - \alpha$ quantile of χ_J^2 . The following theorem demonstrates the asymptotic coverage for the confidence interval $CI_n(\alpha)$.

Theorem 2. *Suppose that the minimum eigenvalue of $\text{Var}(\psi_i(\theta_0) | X, \sigma_n)$ is bounded away from zero, and that $\liminf \min_x \hat{p}(x) > 0$. Under Assumptions 1-3,*

$$\liminf_{n \rightarrow \infty} Pr(\theta_0 \in CI_n(\alpha) | X, \sigma_n) \geq 1 - \alpha.$$

5 Conclusion

We study a network formation models with potentially misclassified links. Specifically, we focus on a strategic game of network formation with incomplete information. In the presence of network misclassification, we derive the moment equality conditions which characterize the identified set of the preference parameters

associated with homophily and network spillovers. Based on the moment equality conditions, we provide an inference method which is asymptotically valid when a single large network is available.

A Proofs

A.1 Proof of Lemmas in Section 2

Proof of Lemma 1. By Assumption 2,

$$\begin{aligned} G_{i,n}^* &= \arg \max_{g_{i,n}^* \in \mathcal{G}^n} E [U_i(g_{i,n}^*, G_{-i,n}^*, X, \varepsilon_i) \mid X, \varepsilon_i, \sigma_n] \\ &= \arg \max_{g_{i,n}^* \in \mathcal{G}^n} \frac{1}{n} \sum_{j=1}^n g_{ij,n}^* [(Z_{ij,n}^*)' \beta_0 + \varepsilon_{ij}]. \end{aligned}$$

Therefore, $G_{ij,n}^* = \mathbf{1} \{(Z_{ij,n}^*)' \beta_0 + \varepsilon_{ij} \geq 0\}$. □

Proof of Lemma 2. Define

$$D(r_0, r_1) = \begin{pmatrix} 1 - r_0 - r_1 & 0 & 0 & 0 \\ 0 & 1 - r_0 - r_1 & 0 & 0 \\ 0 & 0 & (1 - r_0 - r_1)^2 & r_0(1 - r_0 - r_1) \\ 0 & 0 & 0 & 1 - r_0 - r_1 \end{pmatrix}.$$

By Assumption 3, we can derive

$$E [G_{ki,n} G_{kj,n} \mid X, \sigma_n] = \rho_0^2 + (1 - \rho_0 - \rho_1)^2 E [G_{ki,n}^* G_{kj,n}^* \mid X, \sigma_n] + \rho_0(1 - \rho_0 - \rho_1) E [G_{ki,n}^* + G_{kj,n}^* \mid X, \sigma_n].$$

Therefore,

$$\begin{aligned} \gamma_{ij,n} &= \begin{pmatrix} E [G_{ji,n} \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{kj,n} \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n} G_{kj,n} \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n} + G_{kj,n} \mid X, \sigma_n] \end{pmatrix} \\ &= \begin{pmatrix} \rho_0 \\ \rho_0 \\ \rho_0^2 \\ \rho_0 \end{pmatrix} + D(\rho_0, \rho_1) \begin{pmatrix} E [G_{ji,n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{kj,n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n}^* G_{kj,n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n}^* + G_{kj,n}^* \mid X, \sigma_n] \end{pmatrix}. \end{aligned}$$

Since $D(\rho_0, \rho_1)$ is invertible given $1 - \rho_0 - \rho_1 \neq 0$, it follows that

$$\begin{pmatrix} E [G_{ji,n}^* \mid X] \\ \frac{1}{n} \sum_k E [G_{kj,n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n}^* G_{kj,n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k E [G_{ki,n}^* + G_{kj,n}^* \mid X, \sigma_n] \end{pmatrix} = D(\rho_0, \rho_1)^{-1} \begin{pmatrix} \gamma_{ij,n} - \begin{pmatrix} \rho_0 \\ \rho_0 \\ \rho_0^2 \\ \rho_0 \end{pmatrix} \end{pmatrix}.$$

The first three component of the right hand side on the above equation is $\gamma_{ij,n}^*$, so

$$\begin{aligned}\gamma_{ij,n}^* &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} D(\rho_0, \rho_1)^{-1} \begin{pmatrix} \gamma_{ij,n} - \begin{pmatrix} \rho_0 \\ \rho_0 \\ \rho_0^2 \\ \rho_0 \end{pmatrix} \end{pmatrix} \\ &= c(\rho_0, \rho_1) + C(\rho_0, \rho_1)\gamma_{ij,n}.\end{aligned}$$

□

Proof of Lemma 3. It suffices to show that $Pr(G_{ij,n} = 1 | X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*, X, \sigma_n) = (1 - \rho_0)Pr(G_{ij,n}^* = 0 | X_{ij}, \gamma_{ij,n}^*) + \rho_1 Pr(G_{ij,n}^* = 1 | X_{ij}, \gamma_{ij,n}^*)$. Since $(X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*)$ are a function of X, σ_n , it follows that

$$Pr(G_{ij,n} = 1 | X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*, X, \sigma_n) = Pr(G_{ij,n} = 1 | X, \sigma_n).$$

Using Assumptions 1-3,

$$\begin{aligned}Pr(G_{ij,n} = 1 | X, \sigma_n) &= \rho_0 Pr(G_{ij,n}^* = 0 | X, \sigma_n) + (1 - \rho_1) Pr(G_{ij,n}^* = 1 | X, \sigma_n) \\ &= \rho_0 Pr((Z_{ij,n}^*)'b + \varepsilon_{ij} < 0 | X, \sigma_n) + (1 - \rho_1) Pr((Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0 | X, \sigma_n) \\ &= \rho_0 Pr((Z_{ij,n}^*)'b + \varepsilon_{ij} < 0 | Z_{ij,n}^*) + (1 - \rho_1) Pr((Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0 | Z_{ij,n}^*),\end{aligned}$$

where the first equality follows from Assumption 3, the second follows from Lemma 1, and the last follows from the independence between ε and X . □

A.2 Proof of Theorem 1

Proof. To show that every element θ of $\Theta_I(P)$ satisfies Eq. (1), we can see the following equalities

$$\begin{aligned}P(G_{ij,n} = 1 | X_{ij}, \gamma_{ij,n}) &= P^*(G_{ij,n} = 1 | X_{ij}, \gamma_{ij,n}) \\ &= P^*(G_{ij,n} = 1 | X_{ij}, \gamma_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1) P^*(G_{ij,n}^* = 1 | X_{ij}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1) P^*((Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0 | X_{ij}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1) \Phi((\gamma_{ij,n}^*)'b_1 + X_{ij}'b_2) \\ &= r_0 + (1 - r_0 - r_1) \Phi((c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n})'b_1 + X_{ij}'b_2),\end{aligned}$$

where the first equality follows from $P = P^*$ for the observables $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$, the second equality follows because $\gamma_{ij,n}^*$ is a function of $\gamma_{ij,n}$ in Condition 3(ii), the third equality follows from Condition 3(i), the fourth equality follows from Condition 2, the fifth equality follows from Condition 1, and the last equality follows from Condition 3(ii). The rest of the proof is going to show that every element θ of Θ satisfying Eq. (1) belongs to $\Theta_I(P)$.

Define the joint distribution P^* in the following way. The marginal distribution of ε_{ij} is standard normal. The conditional distribution of $(\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij})$ given ε_{ij} is

$$P^*((\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}) \in B | \varepsilon_{ij}) = P((\gamma_{ij,n}, c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n}, X_{ij}) \in B) \quad (2)$$

for all the measurable sets B . The conditional distribution of $G_{ij,n}^*$ given $(\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij})$ is

$$P^*(G_{ij,n}^* = 1 \mid \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij}) = 1\{(Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0\}. \quad (3)$$

The conditional distribution of $G_{ij,n}$ given $(G_{ij,n}^*, \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij})$ is

$$P^*(G_{ij,n} = 1 \mid G_{ij,n}^*, \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij}) = \begin{cases} 1 - r_0 & \text{if } G_{ij,n}^* = 0 \\ r_1 & \text{if } G_{ij,n}^* = 1. \end{cases} \quad (4)$$

Note that (P^*, θ) satisfies Conditions 1-3, because Condition 1(i) follows because ε_{ij} is normally distributed under P^* , Condition 1(ii) follows from Eq. (2). Condition 2 follows from Eq. (3). Condition 3(i) follows from Eq. (3) and (4), and Condition 3(ii) follows from Eq. (2).

The distribution of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to P . The distribution of $(X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to that from P , by the construction of $P^*((\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}) \in B \mid \varepsilon_{ij})$. The equality of $P^*(G_{ij,n} = 1 \mid Z_{ij,n}) = P(G_{ij,n} = 1 \mid Z_{ij,n})$ a.s. under P^* is shown as follows. Note that

$$\gamma_{ij,n}^* = c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n} \text{ a.s. under } P^*. \quad (5)$$

Then

$$\begin{aligned} P^*(G_{ij,n} = 1 \mid Z_{ij,n}) &= P^*(G_{ij,n} = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= (1 - r_0)P^*(G_{ij,n}^* = 0 \mid Z_{ij,n}, \gamma_{ij,n}^*) + r_1P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)E_{P^*}[P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*, \varepsilon_{ij}) \mid Z_{ij,n}, \gamma_{ij,n}^*] \\ &= r_0 + (1 - r_0 - r_1)P^*((Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)\Phi((Z_{ij,n}^*)'b) \\ &= r_0 + (1 - r_0 - r_1)\Phi((c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n})'b_1 + X_{ij}'b_2) \\ &= P(G_{ij,n} = 1 \mid Z_{ij,n}), \end{aligned}$$

where the first and seventh equalities follow from Eq. (5), the second follows from Eq. (4), the fifth follows from Eq. (3), and the last follows from Eq. (1). \square

A.3 Proof of Theorem 2

In the proof of this theorem, all the statements are conditional on X and σ_n . We use the norm for matrices and vectors. For any vector, the norm is understood as the Euclidean norm, and for any matrix the norm is induced by the Euclidean norm. Theorem 2 follows from Lemma 12.

Define

$$\begin{aligned} u_{ij}(\theta_0) &= (c(\rho_0, \rho_1) + C(\rho_0, \rho_1)\gamma_{ij,n})'\beta_1 + X_{ij}'\beta_2 \\ \hat{u}_{ij}(\theta_0) &= (c(\rho_0, \rho_1) + C(\rho_0, \rho_1)\hat{\gamma}_{ij})'\beta_1 + X_{ij}'\beta_2. \end{aligned}$$

For a generic random variable RV, define

$$RV^\dagger = RV - E[RV \mid X, \sigma_n],$$

and note that $E[RV^\dagger \mid X, \sigma_n] = 0$. Define

$$\psi_{\gamma,k,n}(x) = \frac{1}{n^2} \sum_{i,j} \left(\frac{\mathbf{1}\{X_{i,j} = x\}}{\hat{p}(x)} \right) \begin{pmatrix} 0 \\ G_{kj,n}^\dagger \\ (G_{ki,n} G_{kj,n})^\dagger \\ (G_{ki,n} + G_{kj,n})^\dagger \end{pmatrix} + \frac{1}{n} \sum_i \left(\frac{\mathbf{1}\{X_{i,k} = x\}}{\hat{p}(x)} \right) \begin{pmatrix} G_{ki,n}^\dagger \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \psi_k(\theta_0) &= \frac{1}{n} \sum_{j \neq k} (G_{kj,n} - \rho_0 - (1 - \rho_0 - \rho_1)\Phi(u_{kj}(\theta_0))) \mathbf{1}_{kj} \\ &\quad - (1 - \rho_0 - \rho_1) \frac{1}{n^2} \sum_{i,j} (\phi(u_{kj}(\theta_0))\beta_1' C(\rho_0, \rho_1) \psi_{\gamma,k,n}(X_{ij})) \mathbf{1}_{ij} \end{aligned}$$

$$\tilde{\psi}_k(\theta_0) = \frac{1}{n} \sum_{j \neq k} G_{kj,n} \mathbf{1}_{kj} - (1 - \rho_0 - \rho_1) \frac{1}{n^2} \sum_{i,j} (\phi(u_{ij}(\theta_0))\beta_1' C(\rho_0, \rho_1) \hat{\psi}_{\gamma,k,n}(X_{ij})) \mathbf{1}_{ij}.$$

Lemma 4.

$$\mathbf{1}\{X_{i_1, j_1} = X_{ij}\} \begin{pmatrix} E[G_{j_1 i_1, n}^* \mid X, \sigma_n] - E[G_{ji, n}^* \mid X, \sigma_n] \\ \frac{1}{n} \sum_k (E[G_{kj_1, n}^* \mid X, \sigma_n] - E[G_{kj, n}^* \mid X, \sigma_n]) \\ \frac{1}{n} \sum_k (E[G_{ki_1, n}^* G_{kj_1, n}^* \mid X, \sigma_n] - E[G_{ki, n}^* G_{kj, n}^* \mid X, \sigma_n]) \\ \frac{1}{n} \sum_k (E[G_{ki_1, n}^* + G_{kj_1, n}^* \mid X, \sigma_n] - E[G_{ki, n}^* + G_{kj, n}^* \mid X, \sigma_n]) \end{pmatrix} = 0. \quad (6)$$

Proof. This result follows from symmetry of the equilibrium and it is shown in a similar way to Lemma 1 in Leung (2015). \square

Lemma 5.

$$\max\{\|\hat{\psi}_{\gamma,k,n}(X_{ij})\|, \|\psi_{\gamma,k,n}(X_{ij})\|\} \leq \frac{\sqrt{7}}{\min_x \hat{p}(x)}$$

$$\max_i \{\|\tilde{\psi}_i(\theta_0)\|, \|\hat{\psi}_i(\theta_0)\|, \|\psi_i(\theta_0)\|\} \leq 1 + (1 - \rho_0 - \rho_1)\phi(0)\|\beta_1' C(\rho_0, \rho_1)\| \frac{\sqrt{7}}{\min_x \hat{p}(x)}.$$

Proof. The bound for $\|\hat{\psi}_{\gamma,k,n}(X_{ij})\|$ is derived as follows. (The proof for $\|\psi_{\gamma,k,n}(X_{ij})\|$ is similar.)

$$\|\hat{\psi}_{\gamma,k,n}(X_{ij})\| \leq \frac{\sqrt{7}}{\min_x \hat{p}(x)}.$$

The bound for $\|\tilde{\psi}_i(\theta)\|$ is derived as follows. (The proof for $\|\hat{\psi}_i(\theta)\|$ is similar.)

$$\begin{aligned} \|\tilde{\psi}_i(\theta)\| &\leq \max_{j \neq i} |G_{ij,n}| + \max_{l,j} \left| \phi(u_{lj}(\theta))\beta_1' C(\rho_0, \rho_1) \hat{\psi}_{\gamma,i,n}(X_{lj}) \right| \\ &\leq 1 + (1 - \rho_0 - \rho_1)\phi(0)\|\beta_1' C(\rho_0, \rho_1)\| \frac{\sqrt{7}}{\min_x \hat{p}(x)}. \end{aligned}$$

The bound for $\|\psi_i(\theta)\|$ is derived as follows.

$$\begin{aligned}\|\psi_i(\theta_0)\| &\leq \max_{j \neq i} |G_{ij,n} - \rho_0 - (1 - \rho_0 - \rho_1)\Phi(u_{ij}(\theta_0))| \\ &\quad + (1 - \rho_0 - \rho_1) \max_{l,j} \|\phi(u_{ij}(\theta_0))\beta'_1 C(\rho_0, \rho_1)\| \|\psi_{\gamma,i,n}(X_{lj})\| \\ &\leq 1 + (1 - \rho_0 - \rho_1)\phi(0)\|\beta'_1 C(\rho_0, \rho_1)\| \frac{\sqrt{7}}{\min_x \hat{p}(x)}.\end{aligned}$$

□

Lemma 6.

$$\hat{\gamma}_{ij} - \gamma_{ij,n} = \frac{1}{n} \sum_k \psi_{\gamma,k,n}(X_{ij})$$

and

$$\hat{\gamma}_{ij} - \gamma_{ij,n} = O_p(n^{-1/2}) \text{ given } X \text{ and } \sigma_n.$$

Proof. First, we are going to show

$$1\{X_{i_1,j_1} = X_{ij}\} \begin{pmatrix} E[G_{j_1 i_1,n} | X, \sigma_n] - E[G_{ji,n} | X, \sigma_n] \\ \frac{1}{n} \sum_k (E[G_{kj_1,n} | X, \sigma_n] - E[G_{kj,n} | X, \sigma_n]) \\ \frac{1}{n} \sum_k (E[G_{ki_1,n} G_{kj_1,n} | X, \sigma_n] - E[G_{ki,n} G_{kj,n} | X, \sigma_n]) \\ \frac{1}{n} \sum_k (E[(G_{ki_1,n} + G_{kj_1,n}) | X, \sigma_n] - E[(G_{ki,n} + G_{kj,n}) | X, \sigma_n]) \end{pmatrix} = 0. \quad (7)$$

It follows from Lemma 4 and Assumption 3.

Using Eq. (7), we have

$$\begin{aligned}\hat{\gamma}_{ij} - \gamma_{ij,n} &= \frac{1}{n^2} \sum_{i_1,j_1} \frac{1\{X_{i_1,j_1} = X_{ij}\}}{\frac{1}{n^2} \sum_{i_1,j_1} 1\{X_{i_1,j_1} = X_{ij}\}} \begin{pmatrix} G_{j_1 i_1,n} - E[G_{ji,n} | X, \sigma_n] \\ \frac{1}{n} \sum_k (G_{kj_1,n} - E[G_{kj,n} | X, \sigma_n]) \\ \frac{1}{n} \sum_k (G_{ki_1,n} G_{kj_1,n} - E[G_{ki,n} G_{kj,n} | X, \sigma_n]) \\ \frac{1}{n} \sum_k ((G_{ki_1,n} + G_{kj_1,n}) - E[(G_{ki,n} + G_{kj,n}) | X, \sigma_n]) \end{pmatrix} \\ &= \frac{1}{n^2} \sum_{i_1,j_1} \frac{1\{X_{i_1,j_1} = X_{ij}\}}{\frac{1}{n^2} \sum_{i_1,j_1} 1\{X_{i_1,j_1} = X_{ij}\}} \begin{pmatrix} G_{j_1 i_1,n}^\dagger \\ \frac{1}{n} \sum_k G_{kj_1,n}^\dagger \\ \frac{1}{n} \sum_k (G_{ki_1,n} G_{kj_1,n})^\dagger \\ \frac{1}{n} \sum_k ((G_{ki_1,n} + G_{kj_1,n}))^\dagger \end{pmatrix} \\ &= \frac{1}{n} \sum_k \psi_{\gamma,k,n}(X_{ij}).\end{aligned}$$

By Lyapunov's central limit theorem, it suffices to show that $E[\psi_{\gamma,k,n}(X_{ij}) | X, \sigma_n] = 0$ and that

$\psi_{\gamma,k,n}(X_{ij})$ is independent across k given X and σ_n . The equality $E[\psi_{\gamma,k,n}(X_{ij}) | X, \sigma_n] = 0$ follows from

$$\begin{aligned} E[\psi_{\gamma,k,n}(X_{ij}) | X, \sigma_n] &= \frac{1}{n^2} \sum_{i_1, j_1} \left(\frac{1\{X_{i_1, j_1} = X_{ij}\}}{\hat{p}(X_{ij})} \right) \begin{pmatrix} 0 \\ E[G_{kj_1, n}^\dagger | X, \sigma_n] \\ E[(G_{ki_1, n} G_{kj_1, n})^\dagger | X, \sigma_n] \\ E[(G_{ki_1, n} + G_{kj_1, n})^\dagger | X, \sigma_n] \end{pmatrix} \\ &\quad + \frac{1}{n} \sum_{i_1} \left(\frac{1\{X_{i_1, k} = X_{ij}\}}{\hat{p}(X_{ij})} \right) \begin{pmatrix} E[G_{ki_1, n}^\dagger | X, \sigma_n] \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ &= 0 \end{aligned}$$

since $E[RV^\dagger | X, \sigma_n] = 0$ by definition of RV^\dagger . The conditional independence of $\psi_{\gamma,k,n}(X_{ij})$ across k is shown as follows. Note that $\psi_{\gamma,k,n}(X_{ij})$ does not depend on $G_{-k,n}$, so it is a function of ε_k , X and σ_n . Therefore, it follows from Assumptions 1 that $\psi_{\gamma,k,n}(X_{ij})$ is independent across k given X and σ_n . \square

Lemma 7. $\max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| = o_p(1)$.

Proof. Note that

$$\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0) = -(1 - \rho_0 - \rho_1) \frac{1}{n^2} \sum_{l, j} (\phi(\hat{u}_{lj}(\theta_0)) - \phi(u_{lj}(\theta_0))) \beta_1' C(\rho_0, \rho_1) \hat{\psi}_{\gamma, i, n}(X_{lj}) \mathbf{1}_{lj}.$$

Then

$$\begin{aligned} \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| &\leq \|\beta_1' C(\rho_0, \rho_1)\| \max_{l, j} |\phi(\hat{u}_{lj}(\theta_0)) - \phi(u_{lj}(\theta_0))| \|\hat{\psi}_{\gamma, i, n}(X_{lj})\| \\ &\leq \phi(0) \|\beta_1' C(\rho_0, \rho_1)\| \max_{l, j} \max\{|\hat{u}_{lj}(\theta_0)|, |u_{lj}(\theta_0)|\} |\hat{u}_{lj}(\theta_0) - u_{lj}(\theta_0)| \|\hat{\psi}_{\gamma, i, n}(X_{lj})\|, \end{aligned}$$

where the last inequality follows from the mean value expansion of the normal pdf ϕ : $|\phi(u_1) - \phi(u_2)| \leq \max_{u_1 \leq u \leq u_2} |\phi'(u)| |u_1 - u_2| \leq \phi(0) \max\{|u_1|, |u_2|\} |u_1 - u_2|$. Since

$$\begin{aligned} |u_{lj}(\theta_0)| &\leq (\|c(\rho_0, \rho_1)\| + \|C(\rho_0, \rho_1)\| \|\gamma_{lj, n}\|) \|\beta_1\| + \|X_{lj}\| \|\beta_2\| \\ &\leq (\|c(\rho_0, \rho_1)\| + \sqrt{7} \|C(\rho_0, \rho_1)\|) \|\beta_1\| + \max_x \|x\| \|\beta_2\| \\ |\hat{u}_{lj}(\theta_0)| &\leq (\|c(\rho_0, \rho_1)\| + \sqrt{7} \|C(\rho_0, \rho_1)\|) \|\beta_1\| + \max_x \|x\| \|\beta_2\| \\ |\hat{u}_{lj}(\theta_0) - u_{lj}(\theta_0)| &= |C(\rho_0, \rho_1)(\hat{\gamma}_{lj} - \gamma_{lj, n})' \beta_1| \\ &\leq \|C(\rho_0, \rho_1)\| \|\beta_1\| \max_{l, j} \|\hat{\gamma}_{lj} - \gamma_{lj, n}\|, \end{aligned}$$

it follows that

$$\max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| = O_p(\max_{l, j} \|\hat{\gamma}_{lj} - \gamma_{lj, n}\|) = o_p(1).$$

\square

Lemma 8. $\psi_i(\theta_0)$ is independent across i given X and σ_n .

Proof. $\psi_i(\theta_0)$ does not depend on $G_{-i,n}$, so it is a function of ε_i , X and σ_n . It implies the statement of this lemma. \square

Lemma 9. *Conditional on X and σ_n ,*

$$\hat{m}_n(\theta_0) = \frac{1}{n} \sum_{i=1}^n \psi_i(\theta_0) + o_p(n^{-1/2}).$$

Proof. Note that

$$\hat{m}_n(\theta_0) - \frac{1}{n} \sum_{i=1}^n \psi_i(\theta_0) = (1 - \rho_0 - \rho_1) \frac{1}{n^2} \sum_{i,j} (\Phi(\hat{u}_{ij}(\theta_0)) - \Phi(u_{ij}(\theta_0)) - \phi(u_{ij}(\theta_0)) \beta_1' C(\rho_0, \rho_1) (\hat{\gamma}_{ij} - \gamma_{ij,n})) \mathbf{1}_{ij}$$

By the second-order Taylor expansion of the normal cdf Φ ,

$$\Phi(u_1) = \Phi(u_2) + \phi(u_2)(u_2 - u_1) + R(u_1, u_2)$$

where

$$|R_{ij}| \leq \frac{1}{2} \max_{u_1 \leq u \leq u_2} \phi'(u) |u_1 - u_2|^2 \leq \frac{1}{2} \phi(0) \max\{|u_1|, |u_2|\} |u_1 - u_2|^2.$$

Since

$$\begin{aligned} \max\{|u_{ij}(\theta_0)|, |\hat{u}_{ij}(\theta_0)|\} &\leq (\|c(\rho_0, \rho_1)\| + \sqrt{7} \|C(\rho_0, \rho_1)\|) \|\beta_1\| + \max_x \|x\| \|\beta_2\| \\ |\hat{u}_{ij}(\theta_0) - u_{ij}(\theta_0)| &\leq \|C(\rho_0, \rho_1)\| \|\beta_1\| \max_{ij} \|\hat{\gamma}_{ij} - \gamma_{ij,n}\|, \end{aligned}$$

it follows that

$$\|\hat{m}_n(\theta_0) - \frac{1}{n} \sum_{i=1}^n \psi_i(\theta_0)\| = O_p(\max_{ij} \|\hat{\gamma}_{ij} - \gamma_{ij,n}\|^2) = O_p(n^{-1}).$$

\square

Lemma 10. *Conditional on X and σ_n ,*

$$\hat{m}_n(\theta_0) = o_P(1)$$

and

$$\text{Var}(\psi_i(\theta_0) \mid X, \sigma_n)^{-1/2} \sqrt{n} \hat{m}_n(\theta_0) \rightarrow_d N(0, I).$$

Proof. By Lemmas 5 and 8 and Lyapunov's central limit theorem, it suffices to show $E[\psi_i(\theta_0) \mid X, \sigma_n] = 0$.

It follows from

$$\begin{aligned} E[\psi_i(\theta_0) \mid X, \sigma_n] &= \frac{1}{n} \sum_{j \neq i} (E[G_{ij,n} \mid X, \sigma_n] - \rho_0 - (1 - \rho_0 - \rho_1) \Phi(u_{ij}(\theta_0))) \mathbf{1}_{ij} \\ &\quad - (1 - \rho_0 - \rho_1) \frac{1}{n^2} \sum_{l,j} (\phi(u_{lj}(\theta_0)) \beta_1' C(\rho_0, \rho_1) E[\psi_{\gamma,i,n}(X_{lj}) \mid X, \sigma_n]) \mathbf{1}_{lj} \\ &= 0, \end{aligned}$$

because

$$\begin{aligned}
E[G_{ij,n} | X, \sigma_n] &= \rho_0 + (1 - \rho_0 - \rho_1)\Phi(u_{ij}(\theta_0)) \\
E[\psi_{\gamma,i,n}(X_{lj}) | X, \sigma_n] &= \frac{1}{n^2} \sum_{i_1, j_1} \left(\frac{1\{X_{i_1, j_1} = X_{ij}\}}{\hat{p}(X_{ij})} \right) \begin{pmatrix} 0 \\ E[G_{kj_1,n}^\dagger | X, \sigma_n] \\ E[(G_{ki_1,n} G_{kj_1,n})^\dagger | X, \sigma_n] \\ E[(G_{ki_1,n} + G_{kj_1,n})^\dagger | X, \sigma_n] \end{pmatrix} \\
&\quad + \frac{1}{n} \sum_{i_1} \left(\frac{1\{X_{i_1, k} = X_{ij}\}}{\hat{p}(X_{ij})} \right) \begin{pmatrix} E[G_{ki_1,n}^\dagger | X, \sigma_n] \\ 0 \\ 0 \\ 0 \end{pmatrix} \\
&= 0.
\end{aligned}$$

Note that $E[RV^\dagger | X, \sigma_n] = 0$ by the definition of RV^\dagger . □

Lemma 11. *Conditional on X and σ_n ,*

$$\hat{S}(\theta_0) = \text{Var}(\psi_i(\theta_0) | X, \sigma_n) + o_p(1).$$

Proof. First, we are going to show $\hat{S}(\theta_0) = \frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \tilde{\psi}_i(\theta_0)' - \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right) \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right)' + o_p(1)$. Since

$$\begin{aligned}
&\hat{S}(\theta_0) - \frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \tilde{\psi}_i(\theta_0)' + \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right) \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right)' \\
&= \frac{1}{n} \sum_{i=1}^n (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)) (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0))' \\
&\quad + \frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0))' \\
&\quad + \frac{1}{n} \sum_{i=1}^n (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)) \tilde{\psi}_i(\theta_0)' \\
&\quad - \left(\frac{1}{n} \sum_{i=1}^n (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)) \right) \left(\frac{1}{n} \sum_{i=1}^n (\hat{\psi}_i(\theta_0)) \right)' \\
&\quad - \frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \left(\frac{1}{n} \sum_{i=1}^n (\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)) \right)'
\end{aligned}$$

it follows that

$$\begin{aligned}
&\left\| \hat{S}(\theta_0) - \frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \tilde{\psi}_i(\theta_0)' + \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right) \left(\frac{1}{n} \sum_{i=1}^n \tilde{\psi}_i(\theta_0) \right)' \right\| \\
&\leq \max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\|^2 \\
&\quad + 3 \max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| \max_i \|\tilde{\psi}_i(\theta_0)\| \\
&\quad + \max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| \max_i \|\hat{\psi}_i(\theta_0)\|.
\end{aligned}$$

Thus it suffices to show $\max_i \|\hat{\psi}_i(\theta_0) - \tilde{\psi}_i(\theta_0)\| = o_p(1)$ and $\max_i \{\|\tilde{\psi}_i(\theta_0)\|, \|\hat{\psi}_i(\theta_0)\|\} = O_p(1)$. They are shown in Lemmas 5 and 7.

Second, we are going to show $\hat{S}(\theta_0) = \text{Var}(\tilde{\psi}_i(\theta_0) \mid X, \sigma_n) + o_p(1)$. It suffices to show $E[\|\tilde{\psi}_i(\theta_0)\|^4 \mid X, \sigma_n] < \infty$. By the triangle inequality,

$$\begin{aligned}
E[\|\tilde{\psi}_i(\theta_0)\|^4 \mid X, \sigma_n]^{1/4} &\leq \frac{1}{n} \sum_{j \neq i} E[\|G_{ij,n}\|^4 \mid X, \sigma_n]^{1/4} \\
&\quad + \frac{1}{n^2} \sum_{l,j} E\left[\left\|\phi(u_{ij}(\theta_0))\beta_1' C(\rho_0, \rho_1)\hat{\psi}_{\gamma,i,n}(X_{lj})\right\|^4 \mid X, \sigma_n\right]^{1/4} \\
&\leq \frac{1}{n} \sum_{j \neq i} \left(E[\|G_{ij,n}\|^4 \mid X, \sigma_n]^{1/4}\right) \\
&\quad + \frac{1}{n^2} \sum_{l,j} \phi(u_{ij}(\theta_0))\beta_1' C(\rho_0, \rho_1) E\left[\left\|\hat{\psi}_{\gamma,i,n}(X_{lj})\right\|^4 \mid X, \sigma_n\right]^{1/4} \\
&\leq 1 + \frac{1}{n^2} \sum_{l,j} \phi(u_{ij}(\theta_0))\beta_1' C(\rho_0, \rho_1) E\left[\left\|\hat{\psi}_{\gamma,i,n}(X_{lj})\right\|^4 \mid X, \sigma_n\right]^{1/4} \\
&< \infty,
\end{aligned}$$

where the last inequality follows from Lemma 5.

Third, we are going to show that $\text{Var}(\tilde{\psi}_i(\theta_0) \mid X, \sigma_n) = \text{Var}(\psi_i(\theta_0) \mid X, \sigma_n)$. Note that $\tilde{\psi}_i(\theta_0) - \psi_i(\theta_0)$ is a function of X and σ_n , so the conditional variances are the same. \square

Lemma 12. *Conditional on X and σ_n ,*

$$n\hat{m}_n(\theta)' \hat{S}(\theta)^{-1} \hat{m}_n(\theta) \rightarrow_d \chi^2.$$

Proof. It follows from Lemma 10 and 11. \square

B Semiparametric Identification Analysis

Given $P \in \mathcal{P}$, we are going to characterize the identified set in the semiparametric model.

Definition 2. *For each distribution $P \in \mathcal{P}$, the identified set $\Theta_{I,SP}(P)$ is defined as the set of all $\theta = (b, r_0, r_1)$ in Θ for which there is some joint distribution $P^* \in \mathcal{P}^*$ such that Condition 1, 2(ii), and 3 holds, and that the distribution of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to P .*

Theorem 3. *Given $P \in \mathcal{P}$, $\Theta_{I,SP}(P)$ is equal to the set of $\theta \in \Theta$ satisfying the following statements a.s.:*

$$r_0 \leq E_P[G_{ij,n} \mid Z_{ij,n}] \tag{8}$$

$$r_1 \leq E_P[1 - G_{ij,n} \mid Z_{ij,n}] \tag{9}$$

$$E_P[G_{ij,n} \mid Z_{ij,n}] = \Lambda((c(r_0, r_1) + \gamma'_{ij,n} C(r_0, r_1))' b_1 + X'_{ij} b_2) \tag{10}$$

for some weakly increasing and right-continuous function Λ .

Proof. First, we are going to show that every element θ of $\Theta_{I,SP}(P)$ satisfies the conditions in (8)-(10).

Denote by Λ^* the cdf of $-\varepsilon_{ij}$. Based on the assumptions,

$$\begin{aligned} E_{P^*} [G_{ij,n} | Z_{ij,n}] &= r_0 + (1 - r_0 - r_1) E_{P^*} [G_{ij,n}^* | Z_{ij,n}] \\ &= r_0 + (1 - r_0 - r_1) \Lambda^* ((c(r_0, r_1) + \gamma'_{ij,n} C(r_0, r_1))' b_1 + X'_{ij} b_2). \end{aligned}$$

Define $\Lambda(v) = r_0 + (1 - r_0 - r_1) \Lambda^*(c(r_0, r_1)' b_1 + v)$ and we have

$$E_{P^*} [G_{ij,n} | Z_{ij,n}] = \Lambda(\gamma'_{ij,n} C(r_0, r_1)' b_1 + X'_{ij} b_2).$$

Since Λ^* is strictly increasing, Λ is also strictly increasing. Therefore,

$$E_{P^*} [G_{i_1 j_1} | Z_{i_1 j_1}] \geq E_{P^*} [G_{i_2 j_2} | Z_{i_2 j_2}] \iff \gamma'_{i_1 j_1} C(r_0, r_1)' b_1 + X'_{i_1 j_1} b_2 \geq \gamma'_{i_2 j_2} C(r_0, r_1)' b_1 + X'_{i_2 j_2} b_2,$$

which implies the condition (10). The two inequalities in (8) and (9) are shown as follows:

$$\begin{aligned} E_{P^*} [G_{ij,n} | Z_{ij,n}] &= r_0 + (1 - r_0 - r_1) E_{P^*} [G_{ij,n}^* | Z_{ij,n}] \geq r_0 \\ E_{P^*} [1 - G_{ij,n} | Z_{ij,n}] &= r_1 + (1 - r_0 - r_1) E_{P^*} [1 - G_{ij,n}^* | Z_{ij,n}] \geq r_1. \end{aligned}$$

where the inequalities follow from $1 - r_0 - r_1 \geq 0$.

Next, we are going to show that every element $\theta \in \Theta$ satisfying (8)-(10), belongs to $\Theta_{I,SP}(P)$. By the condition (10) as well as Conditions (8) and (9), there is a weakly increasing and right-continuous function $\Lambda : \mathbb{R} \rightarrow [r_0, 1 - r_1]$ such that

$$E_P [G_{ij,n} | (c(r_0, r_1) + \gamma'_{ij,n} C(r_0, r_1))' b_1 + X'_{ij} b_2] = \Lambda ((c(r_0, r_1) + \gamma'_{ij,n} C(r_0, r_1))' b_1 + X'_{ij} b_2). \quad (11)$$

Denote by Λ^* the cdf satisfying $\Lambda(v) = r_0 + (1 - r_0 - r_1) \Lambda^*(c(r_0, r_1)' b_1 + v)$.

Define the joint distribution P^* in the following way. Define the cdf of ε_{ij} such that Λ^* is the cdf of $-\varepsilon_{ij}$. The conditional distribution of $(\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij})$ given ε_{ij} is

$$P^*((\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}) \in B | \varepsilon_{ij}) = P((\gamma_{ij,n}, c(r_0, r_1) + C(r_0, r_1) \gamma_{ij,n}, X_{ij}) \in B) \quad (12)$$

for all the measurable sets B . The conditional distribution of $G_{ij,n}^*$ given $(\gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij})$ is

$$P^*(G_{ij,n}^* = 1 | \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij}) = 1\{(Z_{ij,n}^*)' b + \varepsilon_{ij} \geq 0\}. \quad (13)$$

The conditional distribution of $G_{ij,n}$ given $(G_{ij,n}^*, \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij})$ is

$$P^*(G_{ij,n} = 1 | G_{ij,n}^*, \gamma_{ij,n}, \gamma_{ij,n}^*, X_{ij}, \varepsilon_{ij}) = \begin{cases} 1 - r_0 & \text{if } G_{ij,n}^* = 0 \\ r_1 & \text{if } G_{ij,n}^* = 1. \end{cases} \quad (14)$$

Note that (P^*, θ) satisfies Conditions 1(ii), 2 and 3, because Condition 1(ii) follows from Eq. (12), Condition 2 follows from Eq. (13), Condition 3(i) follows from Eq. (13) and (14), and Condition 3(ii) follows from Eq. (12).

The distribution of $(G_{ij,n}, X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to P . The distribution of $(X_{ij}, \gamma_{ij,n})$ induced from P^* is equal to that from P , by Eq. (12). The equality of $P^*(G_{ij,n} = 1 | Z_{ij,n}) = P(G_{ij,n} = 1 |$

$Z_{ij,n}$) a.s. under P^* is shown as follows. Note that

$$\gamma_{ij,n}^* = c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n} \text{ a.s. under } P^* \quad (15)$$

Then

$$\begin{aligned} P^*(G_{ij,n} = 1 \mid Z_{ij,n}) &= P^*(G_{ij,n} = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= (1 - r_0)P^*(G_{ij,n}^* = 0 \mid Z_{ij,n}, \gamma_{ij,n}^*) + r_1P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)E_{P^*}[P^*(G_{ij,n}^* = 1 \mid Z_{ij,n}, \gamma_{ij,n}^*, \varepsilon_{ij}) \mid Z_{ij,n}, \gamma_{ij,n}^*] \\ &= r_0 + (1 - r_0 - r_1)P^*((Z_{ij,n}^*)'b + \varepsilon_{ij} \geq 0 \mid Z_{ij,n}, \gamma_{ij,n}^*) \\ &= r_0 + (1 - r_0 - r_1)\Lambda^*((Z_{ij,n}^*)'b) \\ &= r_0 + (1 - r_0 - r_1)\Lambda^*((c(r_0, r_1) + C(r_0, r_1)\gamma_{ij,n})'b_1 + X'_{ij}b_2) \\ &= P(G_{ij,n} = 1 \mid Z_{ij,n}), \end{aligned}$$

where the first and seventh equalities follow from Eq. (15), the second follows from Eq. (14), the fifth follows from Eq. (13), and the last follows from Eq. (11). \square

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