“WAIT AND SEE” OR “FEAR OF FLOATING”?

XIAOWEN LEI, DONG LU AND KENNETH KASA

Abstract. This paper studies the evolution of China's exchange rate policy using real options theory. With intervention costs and ongoing uncertainty, intervention involves the exercise of an option. As always, increased uncertainty increases the value of this option. This “wait and see effect” leads the Central Bank to widen its intervention band. However, increased volatility also produces larger fluctuations in welfare, which creates a “fear of floating”. This induces the Central Bank to set a tighter band. To study this trade-off, our paper incorporates stochastic volatility into a New Keynesian target zone model, and then calibrates it to daily data from China. We find that increased uncertainty leads to a tighter intervention band, both in the data and in the model. Hence, in China, “fear of floating” dominates the “wait and see” effect.

Keywords: Target Zone, Option Value, Stochastic Volatility

JEL Classification Numbers: F31, E58

China's exchange rate policy will consistently follow the principles of automony, controllability and gradualism.
—Zhou Xiaochuan, Former Governor of China's Central Bank, Feb 13, 2016

1. Introduction

Exchange rate policy is often portrayed as a choice between fixed or flexible exchange rates. Reality, of course, is in between.[1] During the late 1980s and early 1990s, the target zone literature pioneered by Krugman (1991) seemed to provide a useful way to model this middle ground. In a target zone, the Central Bank allows the exchange rate to fluctuate within a predetermined band, but then intervenes at the boundary to keep the exchange rate from moving outside of the band. Target zone models were widely applied to the European Monetary System (EMS) in the years leading up to the euro.[2]

After the euro was introduced, the target zone literature largely died out. These days interest focuses on China's exchange rate, given that China's currency (RMB) is becoming

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[1]See, e.g., Ilzetzki, Reinhart, and Rogoff (2017) and Han and Wei (2018).

[2]Krugman and Miller (1992) surveys this literature.

ever more important in world trade and payments. In some respects, China's recent exchange rate policy seems to resemble that of the EMS. Since 2005 the People's Bank of China (PBOC) has allowed market forces to move the exchange rate, but only within limits. Hence, it is tempting to dust off a target zone model and see whether it can explain the recent behavior of the RMB/USD exchange rate.

Even a cursory look at the data suggests that European-inspired target zone models might have trouble explaining China's recent exchange rate policy. Panel A of Figure 1 plots daily data since 2003 on the RMB central parity and closing rate, while Panel B plots percentage deviations of the closing rate from the central parity, along with the trading band set and maintained by the PBOC.

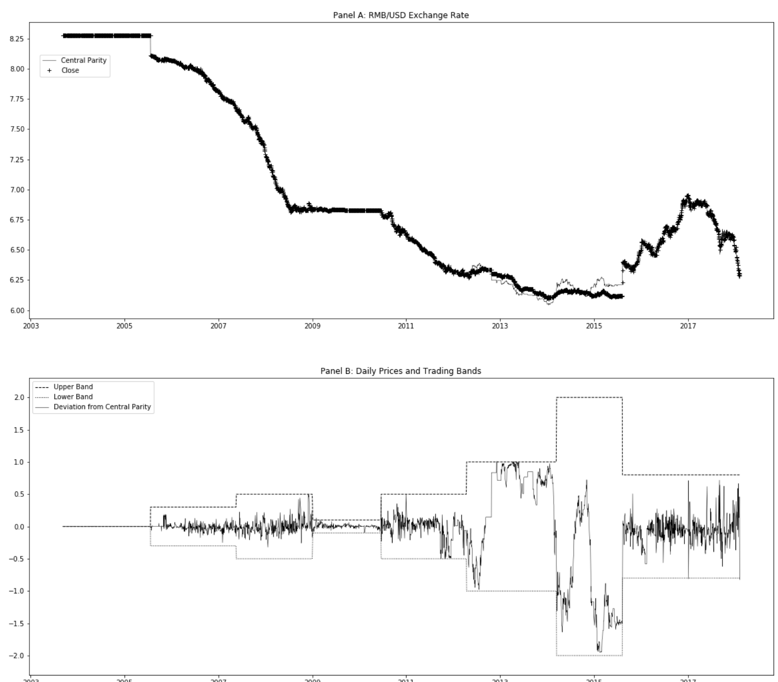

FIGURE 1. RMB/USD exchange rate and trading bands

Two discrepancies stand out. First, it is clear that the RMB does not fluctuate within a single, time-invariant band. From 2005-2015, the RMB steadily appreciated, with a brief respite during the financial crisis. Then, from August 2015 until 2017 it reversed course, and began to steadily depreciate. Second, Panel B shows that the *width* of the band has evolved over time. For example, it tightened during the financial crisis, and then widened after 2012, only to be tightened again in 2016. In contrast, the band width in a traditional target zone model is time invariant.

Perhaps we should not be too surprised that off-the-shelf target zone models don't work. After all, they failed to explain EMS data as well (Bertola and Caballero (1992)). However, the extensions that were developed to fit the EMS data seem ill-suited to China. In particular, there is little evidence of discrete realignments of the RMB. Instead, Figure 1 suggests that the PBOC controls the RMB's rate of drift, rather than its level. This suggests the need for a different sort of extension.

We argue that *stochastic volatility* is the key to understanding China's recent exchange rate policy. In particular, we show that the width of the PBOCs trading band evolves in response to changes in volatility. For example, Figure 2 plots the width of the trading band against the VIX index, a common measure of aggregate uncertainty.

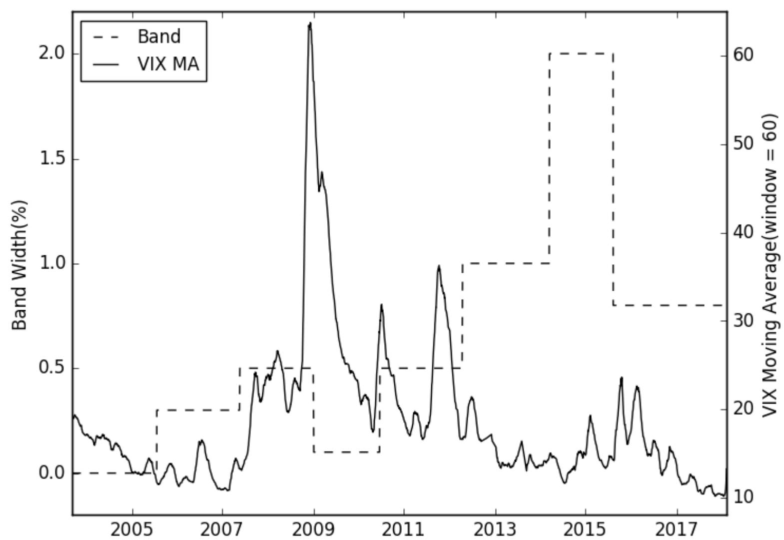

FIGURE 2. VIX and China's exchange rate band

A negative correlation is apparent. We show that this same negative correlation is present when other uncertainty measures are used, e.g., terms of trade volatility, spreads on China's Credit Default Swaps (CDS) and implied volatility from offshore RMB option prices.[3]

Adding stochastic volatility to a target zone model is straightforward in principle, but technically challenging in practice. When both the level and volatility of the exchange rate vary, the optimal band is characterized by a *partial* differential equation (PDE). This PDE does not have a closed-form solution. However, since the constant volatility case has a well know analytical solution, we can obtain a first-order perturbation approximation. Using this approximation, we show that volatility has countervailing effects on the band width. On the one hand, a higher *mean* level of volatility widens the band, for the usual

[3]Marconi (2018) also finds that the width of RMB/USD band is negatively related to the volatility of the RMB/USD exchange rate, but no formal theory is provided to explain it.

option value reasons. Hence, if we compare across economies, those with higher mean volatility will have wider average band widths. However, if we look at band width over time within a given economy, holding mean volatility constant, then band width increases in response to (temporary) increases in volatility. When volatility is stochastic, it creates additional flow costs that increase the benefits of regulation.

Although stochastic volatility explains why the PBOC adjusts the width of its trading band, it does not explain the drift of the central parity that is apparent in Figure 1. The question of why and how the PBOC adjusts the central parity has recently been addressed by Jermann, Wei, and Yue (2017). They show that choice of the central parity after August 2015 adheres to a so-called "two pillars approach", which strikes a balance between stability and adjustment to market forces.[4] In particular, today's central parity is assumed to be a weighted average of yesterday's closing price (reflecting adjustment to market forces), and a rate that lends stability to a predefined currency basket. Although their model is successful at tracking changes in the central parity, they do not address the question of why the band width changes. Hence, we view our paper as complementary to theirs. Their paper focuses on panel A of Figure 1, while ours focuses on panel B.[5]

The remainder of the paper is organized as follows. Section 2 presents the model. We start with the case of constant volatility. We show that a permanent one-time increase in volatility produces a "wait-and-see" effect, which (permanently) widens the band. We then augment the model by incorporating stochastic volatility, using a Cox-Ingersoll-Ross (CIR) specification. Now the trading band responds continuously to ongoing changes in volatility. We show that stochastic fluctuations in volatility produce a "fear of floating", which narrows the band in response to increased uncertainty. Section 3 uses Doob's optional stopping theorem to verify that our solution constitutes a rational expectations equilibrium. Section 4 studies the model's quantitative implications by calibrating it to data from China. Following Reinhart and Rogoff (2004), we distinguish between *De Facto* and *De Jure* bands, and find that our model fits best when using a 6-month rolling window to compute the *De Facto* band. Section 5 provides more formal and comprehensive empirical tests of the model's predictions, using a variety of volatility measures. Section 6 briefly discusses related literature. Finally, Section 7 summarizes our conclusions and discusses possible extensions. A supplementary Appendix provides institutional background on China's exchange rate policy, derives technical results on the implementation of the target zone, and fills in details of our New Keynesian target zone model.

2. The Model

The original target zone literature was based on the monetary model of exchange rates, in which the central bank is assumed to influence the exchange rate by changing the

[4]Most of the discrete bands here are the same as in Figure 1 in Jermann, Wei, and Yue (2017). Two notable differences are: (1) From the end of 2008 to June 2010, China repegged to the dollar with a very narrow band of less than 0.1%, as announced by former Governor Zhou Xiaochuan. (2) After August 11 2015, and the reform of the central parity, the PBOC implemented an effective band of 0.8% based on our calculation, similar in magnitude to Jermann, Wei, and Yue (2017)'s band of 0.5%. The difference arises because they use a shorter sampling period. Appendix 8.1 provides further details.

[5]Of course, it is possible that choice of the central parity interacts with choice of the trading band. Hence, it might be useful to consider an extension that models both choices simultaneously.

money supply. It was never clear in these models why the central bank wanted to limit exchange rate fluctuations. These days, monetary policy is typically studied using the general equilibrium New Keynesian framework, in which central banks set interest rates to limit fluctuations in the output gap and inflation. They do this in order to maximize household welfare.[6] Besides incorporating stochastic volatility, another contribution of our paper is to study target zone dynamics using this more modern framework.

Our model is based closely on the prior work of Gali and Monacelli (2005) and Clarida, Gali, and Gertler (2001) (henceforth CGG). We study a small open economy operating in a world of complete financial markets.[7] Goods are freely traded, and the Law of One Price holds. The key friction in the model, as in all New Keynesian models, is that firms cannot continuously adjust their prices. In an open economy, sluggish price adjustment implies that exchange rate fluctuations induce terms of trade fluctuations, which then produce output and inflation fluctuations. As a result, the Central Bank cares about exchange rate fluctuations.

The model is summarized by the following five log-linearized equations. Appendix 8.4 provides a more detailed presentation of the model.

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma_\gamma}\left(i_t - \mathbb{E}_t \pi_{H,t+1} - rr^0\right) \tag{2.1}$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \kappa_\gamma x_t \tag{2.2}$$

$$s_t = \sigma_\gamma x_t \tag{2.3}$$

$$s_t = e_t + p_t^* - p_{H,t} \tag{2.4}$$

$$i_t = i_t^* + E_t(\Delta e_{t+1}) \tag{2.5}$$

As usual, x_t denotes the output gap, $\pi_{H,t}$ denotes *domestic* inflation (as opposed to CPI inflation), rr^0 denotes the natural interest rate, s_t represents the terms of trade (defined as the relative price of foreign goods), e_t represents the nominal exchange rate (defined as the price of foreign currency), and p_t^* and i_t^* are the foreign price level and foreign nominal interest rate. Except for nominal interest rates and natural interest rate, all variables are in units of percent deviation from the steady state. Equation (2.1) is the household's Euler equation, which plays the role of a dynamic IS curve. Equation (2.2) is the New Keynesian Phillips curve, describing optimal price adjustment in the presence of Calvo price setting. The remaining three equations represent goods market clearing, the Law of One Price, and Uncovered Interest Parity, respectively.

As emphasized by Gali and Monacelli (2005) and CGG (2001), the great virtue of this particular model is that it remains quite similar to the canonical closed-economy New Keynesian model. All that really changes is parameter values. For example, $\sigma_\gamma = \frac{\sigma}{1+\gamma(\omega-1)}$, where γ is the share of foreign goods in domestic consumption. ω is an elasticity parameter that exceeds unity for plausible parameter values. Hence, output tends to respond more

[6] Galí (2015) provides a textbook review.

[7] Obviously, China is *not* a small open economy. However, relaxing this assumption raises substantial complications. For example, in the presence of other large economies (e.g., the USA), it would require analysis of strategic interaction. Still, the domestic economy remains a monopolistic supplier of its own domestic good, which has non-negligible weight in domestic utility. Hence, the domestic Central Bank has an incentive to manipulate its own terms of trade, as in Costinot, Lorenzoni, and Werning (2014).

strongly to interest rate fluctuations in an open-economy, because interest rate fluctuations produce exchange rate fluctuations, which then trigger expenditure switching effects. Similarly, the slope of the Phillips Curve, $\kappa_\gamma = \lambda(\sigma_\gamma + \phi)$, depends on the degree of openness as well. Since $\sigma_\gamma \leq \sigma$, domestic inflation is less responsive to output gap fluctuations in an open economy.

Another feature of this model is perhaps less well suited to study optimal target zone policy. Without Phillips Curve shocks, this model features a well known 'divine coincidence', in which optimal interest rate policy can fully stabilize the output gap and inflation, and reproduce the flexible price equilibrium.[8] In an open-economy, it is then optimal to allow the exchange rate to fluctuate in response to fluctuations in the flexible price terms of trade. Hence, a target zone would not be optimal.

We argue that two key ingredients can be added to explain why a Central Bank might be led to pursue a target zone policy. First, divine coincidence assumes the Central Bank adheres to the Taylor Principle, with sufficiently strong reactions to the output gap and inflation. These reactions are required to eliminate multiple equilibria. Since in equilibrium the output gap and inflation never change, these strong reactions are off-the-equilibrium path, and so are unobservable. We instead suppose the Central Bank simply targets the natural real interest rate, which is assumed to be constant. Although this policy also supports the flexible price equilibrium, it opens the door to sunspot fluctuations. The second key ingredient is to suppose that the Central Bank confronts a cost to changing either the interest rate or exchange rate. Lei and Tseng (2018) show that a fixed cost can explain the optimal timing of interest rate changes at both the Fed and the Bank of Canada. Here we adopt a similar assumption, although to be consistent with the absence of jumps in the RMB exchange rate, we instead suppose the intervention cost is linear. This produces an optimal barrier control policy. Hence, in our model a target zone is a substitute for the Taylor Principle. Without the Taylor Principle, the economy is exposed to sunspot fluctuations. The target zone contains these fluctuations. Although it might appear to be optimal to drive the fluctuations to zero by adopting a very narrow band, intervention costs make this prohibitively expensive.[9]

To simplify the mathematics, we make one additional parametric assumption. We assume that $\kappa_\gamma = 0$. This is the case when the Calvo fairy never visits, and firms never get the chance to adjust their prices. Obviously then, $\pi_{H,t} = 0 \ \forall t$. The Central Bank still has a job to do, however, since output still fluctuates.[10] By inspection of equations

[8]See Gali (Chapt. 4) for a discussion. As usual, the result is premised on the existence of lump-sum taxes that can offset monopoly price distortions. In an open-economy, there are additional elasticity assumptions required. However, Faia and Monacelli (2008) show that even without these assumptions domestic inflation and output gap stabilization remains approximately optimal.

[9]A large literature attempts to measure the 'cost of foreign exchange intervention'. However, this literature is based on UIP deviations. (Chang, Liu, and Spiegel (2015), Fanelli and Straub (2016), and Adler and Mano (2016) provide recent examples). Here there are no UIP deviations, and interventions take the form of (high frequency) unsterilized changes in domestic monetary policy. Hence, the costs here are more accurately interpreted as microstructural transaction costs, like bid/ask spreads. Not surprisingly then, our calibrated costs turn out to be much smaller than UIP-based intervention costs.

[10]Without this assumption, the dynamics within the target zone become somewhat more complicated, and the resulting differential equations that characterize the optimal band would be harder to solve. However, the essential economics of the model would be the same.

(2.1)-(2.5), it is clear that one equilibrium of the resulting model is to have x_t follow a martingale. Then since s_t is proportional to x_t, it too follows a martingale. If we assume the foreign price level is constant, and without loss of generality is equal to the domestic price level, then the martingale in s_t produces a martingale in e_t as well. Finally, this will be consistent with Uncovered Interest Parity if the domestic and foreign real interest rates are constant and equal to each other.[11]

2.1. **Constant Volatility.** Since we want to solve an optimal timing problem, it pays to work in continuous-time. The continuous-time limit of a discrete-time martingale is a Brownian motion. Uncertainty is represented by a filtered probability space $(\Omega, P, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F})$, induced by an observable one-dimensional standard Brownian motion $B(t)$, which satisfies the usual conditions. In the absence of any control, the exchange rate $e = \{e_t, t \geq 0\}$ fluctuates as a standard Brownian Motion with standard deviation σ,

$$de = \sigma dB \tag{2.6}$$

.

The Central Bank faces two costs that it would like to minimize. First, there is the flow welfare cost $ax^2 + b\pi^2$, where a and b are the weights on output gap and inflation deviation respectively, which occurs whenever the exchange rate fluctuates flexibly. Given the above assumptions, we know that $x = \alpha_x e$ and $\pi = 0$. [12] Therefore, the instantaneous loss function can be simplified to αe^2, where $\alpha = a\alpha_x^2$. Second, there is an adjustment cost whenever the central bank wants to "push up" e_t at a cost of m times the size of the adjustment. Similarly, it also pays m times the size of the adjustment whenever it wants to "push down" e_t. One interpretation of this cost is the change of foreign reserves for the central bank in order to defend the target zone. This will be clearer in the calibration section.

The Central Bank's objective is to find a policy that minimizes the expected discounted value of these two types of costs over an infinite planning horizon, when future costs are discounted at the rate of $\rho > 0$. A policy is defined as a pair of non-negative processes $L = \{L_t, t \geq 0\}$ and $U = \{U_t, t \geq 0\}$ that are non-decreasing and non-anticipating w.r.t e_t. One can interpret L_t as the cumulative upward adjustment of e_t, and U_t as the cumulative downward adjustment to e_t. Therefore, under policy (U, L), the controlled process can be defined as

$$z_t = e_t - U_t + L_t \tag{2.7}$$

This implies the regulated process z_t follows [13]

$$dz = \sigma dB \tag{2.8}$$

[11]Another function of the target zone is to maintain validity of the model's log-linear approximation. Untethered martingale dynamics would eventually drive the system far from its steady state.

[12]With price being fixed, assuming that domestic and foreign countries all start at the same price level at time 0, we have $p_t^* = p_{H,t}, \forall t$. Therefore, $s_t = e_t$. Since $x_t = \frac{1}{\sigma_\gamma} s_t$ from equation (2.3), we have that $x_t = \frac{1}{\sigma_\gamma} e_t$, where $\sigma_\gamma = \frac{\sigma}{1+\gamma(w-1)}$. Therefore, $\alpha_x = \frac{\sigma}{1+\gamma(w-1)}$.

[13]The time script of variables are eliminated for simplicity later on unless necessary.

within the inaction region. The Central Bank's control problem can now be stated as

$$V(z_0) = \inf_{L,U} \mathbb{E}_{z_0} \left[\int_0^\infty e^{-\rho t} \alpha z^2 dt + m \int_0^\infty e^{-\rho t} dL + m \int_0^\infty e^{-\rho t} dU \right] \tag{2.9}$$

for a given initial exchange rate z_0.[14] Given the assumption of quadratic flow "carrying cost", it can be shown that the value function V is twice continuously differentiable, and that it satisfies the following Hamilton-Jacobian-Bellman (HJB) equation within the inaction region,

$$\rho V = \alpha z^2 + \frac{1}{2} V_{zz} \sigma^2 \tag{2.10}$$

The general solution is

$$V(z) = Az^2 + K_1 e^{\beta_1 z} + K_2 e^{\beta_2 z} + C \tag{2.11}$$

Due to symmetry, we have $K_1 = K_2 = K$, which simplifies the solution to

$$V(z) = Az^2 + K(e^{\beta_1 z} + e^{\beta_2 z}) + C \tag{2.12}$$

where $A = \frac{\alpha}{\rho}$, $\beta_1 = \frac{\sqrt{2\rho}}{\sigma}$,$\beta_2 = -\frac{\sqrt{2\rho}}{\sigma}$, $C = \frac{\alpha\sigma^2}{\rho^2}$. The constants of integration K, and the optimal barriers, $(\bar{z}, -\underline{z})$, are determined by four boundary conditions: a pair of smooth-pasting/value-matching conditions, and a pair of higher order contact conditions

$$V'(\bar{z}) = -m; \quad V'(\underline{z}) = m; \quad V''(\bar{z}) = V''(\underline{z}) = 0 \tag{2.13}$$

Applied optimally, the marginal cost of control should be the same as the marginal present value benefit at both barriers.[15] Using the general solution, one can approximate the threshold with a fourth-order expansion in z. It is interesting to inspect how the target zone band depends on volatility.

Proposition 1. *The Central Bank's optimal exchange rate policy features barrier control. The width of the target zone around the normalized central parity $z = 0$ is approximately*

$$\bar{z} = \left(\frac{3m\sigma^2}{4\alpha} \right)^{1/3} \tag{2.14}$$

To keep the exchange rate within the target zone, the Central Bank exerts instantaneous control, lowering the domestic interest rate when z hits $\underline{z}$, and raising the domestic interest rate when z_t hits $\bar{z}$.

Proof. See Appendix. □

Evidently, with constant volatility, a one-time permanent increase in uncertainty increases the value of "wait and see". In response, the Central Bank should optimally adopt a wider (time invariant) target zone. This seems contradictory at first glance. However, there are two opposing forces underlying the determination of the target zone. On one hand, increased uncertainty raises the option value of waiting, so that the Central Bank would favor a wider band in order to avoid constantly paying the regulation cost. On the

[14] Our formulation of the problem is identical to an inventory or cash management problem. For a detailed derivation of the optimality conditions, see Harrison and Taksar (1983).

[15] The smooth pasting and higher order contact conditions for the infinitesimal control problem are derived in Dumas (1991)

other hand, increased uncertainty also makes domestic variables more volatile, which is something the Central Bank wants to minimize. It turns out that with constant volatility, the former effect dominates. This is opposite to what we observe in the data. In the following section, we resolve this puzzle by incorporating stochastic volatility.

2.2. **Stochastic Volatility.** In this section, we develop and solve a model of an exchange rate target zone under stochastic volatility. This allows us to examine the dynamics of the optimal band width. Now the volatility of the exchange rate is itself random. Note, this does not violate the martingale property of the the *level* of the exchange rate. Exchange rate changes within the band remain unpredictable. Exchange rate dynamics are now governed by

$$dz = \sigma dB \tag{2.15}$$

$$d\sigma = \kappa(\theta - \sigma) + \sqrt{\xi}\sqrt{\sigma}dB_\sigma \tag{2.16}$$

The Cox-Ingersoll-Ross specification for volatility is attractive, since it constrains volatility to be non-negative. For simplicity, we assume that $corr(dB, dB_\sigma) = 0$.[16] The long-run mean of the volatility is θ and the speed of mean reversion is equal to κ.

Now the Central Bank's barrier control problem can be stated as

$$V(z_0, \sigma_0) = \inf_{L,U} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\alpha z^2 dt + m\int_0^\infty e^{-\rho t}dL + \int_0^\infty e^{-\rho t}dU)\right] \tag{2.17}$$

Once again, within the inaction region the Central Bank's policy must respect the HJB equation, which is now a *partial* differential equation (PDE)

$$\rho V = \alpha z^2 + \frac{1}{2}V_{zz}\sigma^2 + V_\sigma\kappa(\theta - \sigma) + \frac{1}{2}V_{\sigma\sigma}\xi\sigma \tag{2.18}$$

The particular solution becomes

$$V^p(z, \sigma) = \frac{\alpha}{\rho}z^2 + A_1\sigma^2 + A_2\sigma + B_1 \tag{2.19}$$

where $A_1 = \frac{\alpha}{\rho(\rho+2\kappa)}$; $A_2 = \frac{2\kappa\theta+\xi}{\rho+\kappa}A_1$; $B_1 = \frac{\kappa\theta}{\rho}A_2$. The homogeneous part of the solution must satisfy the following PDE

$$\rho V = \frac{1}{2}V_{zz}\sigma^2 + V_\sigma\kappa(\theta - \sigma) + \frac{1}{2}V_{\sigma\sigma}\xi\sigma \tag{2.20}$$

One can guess and verify the following functional form for the solution

$$V^h(z, \sigma) = e^{f(z,\sigma)} \tag{2.21}$$

where $f(z, \sigma)$ must satisfy the following nonlinear PDE

$$\rho = \frac{1}{2}(f_{zz} + f_z^2)\sigma^2 + f_\sigma\kappa(\theta - \sigma) + \frac{1}{2}(f_{\sigma\sigma} + f_\sigma^2)\xi\sigma \tag{2.22}$$

We solve this PDE using a perturbation approximation around the benchmark case where $\kappa = \xi = 0$,

$$f(z, \sigma) = f^0(z, \sigma) + \kappa f^\kappa(z, \sigma) + \xi f^\xi(z, \sigma) \tag{2.23}$$

[16]The original idea of embedding CIR stochastic volatility into asset pricing models comes from Heston (1993).

Taking derivatives of the PDE, and evaluating at the perturbation expansion point, one gets

$$f^0(z,\sigma) = \beta_{1,2} z; \quad \beta_{1,2} = \pm\frac{\sqrt{2\rho}}{\sigma} \tag{2.24}$$

$$f^\kappa(z,\sigma) = \frac{1}{2}p_1 z^2 + q_1 z; \quad p_1 = -\frac{\beta_\sigma(\theta-\sigma)}{\beta\sigma^2}; \quad q_1 = \frac{\beta_\sigma(\theta-\sigma)}{2\beta^2\sigma^2} \tag{2.25}$$

$$f^\xi = -\frac{1}{6}z^3 + (\frac{1}{4\beta} - \frac{1}{2})z^2 - (\frac{1}{4\beta^2} - \frac{1}{2\beta})z \tag{2.26}$$

Therefore, the general solution of the value function is

$$V(z,\sigma) = \frac{\alpha}{\rho}z^2 + A_1\sigma^2 + A_2\sigma + B_1 + K_1 e^{f(z,\sigma|\beta_1)} + K_2 e^{f(z,\sigma|\beta_2)} \tag{2.27}$$

As before, to pin down $(K_1, K_2, \bar{z}, \underline{z})$ we need two smooth-pasting and two higher order contact conditions

$$V_z(\bar{z}) = -m; \quad V_z(\underline{z}) = m \tag{2.28}$$

$$V_{zz}(\bar{z}) = V_{zz}(\underline{z}) = 0 \tag{2.29}$$

A detailed derivation of these boundary conditions under stochastic volatility is provided in the Appendix. The key difference now is that the optimal barriers, $(\bar{z}(\sigma), \underline{z}(\sigma))$, become functions of σ. When volatility changes, so does the target zone. Due to symmetry, we know again that $K_1 = K_2 = K$. To an $\mathcal{O}(z^2)$ approximation, we know that the threshold $\bar{z}(\sigma)$, and its mirror image $-\bar{z}(\sigma)$ are characterized by the two following two conditions[17], and the subscripts $(1,2)$ denote the values corresponding to the positive and negative root $\beta_{1,2}$ respectively.

$$V_z = -\frac{2\alpha}{\rho}z + K(a_1 + a_2 + 2z(\frac{a_1^2 + a_2^2}{2} + b_1 + b_2) + \frac{3}{2}z^2(a_1 b_1 + a_2 b_2)) = -m \tag{2.30}$$

$$V_{zz} = -\frac{2\alpha}{\rho} + K(2(\frac{a_1^2 + a_2^2}{2} + b_1 + b_2) + 3z(a_1 b_1 + a_2 b_2)) = 0 \tag{2.31}$$

Figure 3 plots the model implied band width as a function of the currently prevailing level of volatility.

We use the benchmark parameters from Table 1 to compute optimal target zone barrier as a function of underlying volatility. The values of these parameters will be discussed in detail in the next section. From the graph, we can see that an increase in volatility now *reduces* the bandwidth. Stochastic volatility serves as an extra source of risk, which induces the Central Bank to tighten the band.

TABLE 1. Benchmark Parameter Values

Parameters	α	ρ	m	κ	ξ	θ
Value	0.5006	0.34%	0.0014	0.40091	0.0193	0.07993

[17]Where $a_i = \beta_i + \kappa q_i - \xi(\frac{1}{4\beta_i^2} - \frac{1}{2\beta})$ and $b_i = \frac{\kappa}{2}p_i + \xi(\frac{1}{4\beta} - \frac{1}{2})$.

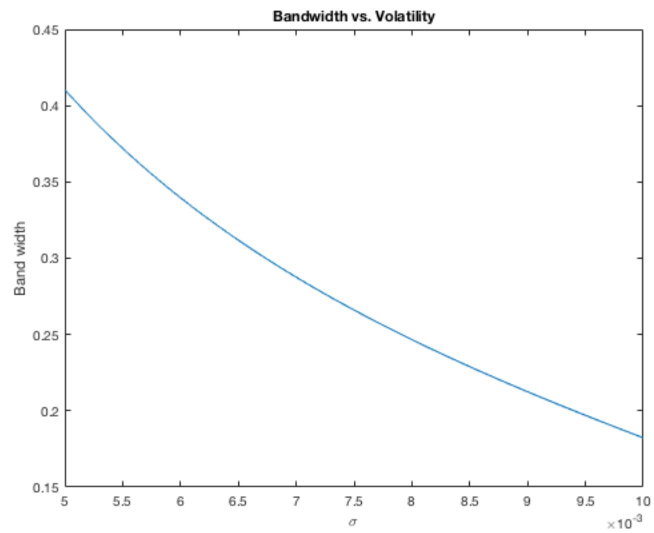

FIGURE 3. Bandwidth vs. Volatility

2.3. **Comments on stochastic volatility.** One can see that by considering stochastic volatility, the relation between bandwidth and uncertainty is reversed. This result is not generated by adding any additional assumptions other than letting uncertainty evolve stochastically. "Fear of floating", in the words of Calvo and Reinhart (2002),is generated endogenously. To understand this result, recall that the central bank is trading off two type of costs: the flow welfare cost and the regulation cost. In the absence of stochastic volatility, the weight on welfare cost comes from structural parameters derived from the underlying micro-founded New-Keynesian model. With stochastic volatility, the expected quadratic deviation of exchange rate now involves a term with the variance of volatility, which produces an endogenous higher variance of the unregulated exchange rate that the central bank is attempting to minimize. This can be shown more rigorously below.

Proof. One can show that the weight on the "welfare cost" part of the objective function is higher under stochastic volatility. Let σ_0 be drawn from its long-run stationary distribution, so that the long-run mean of σ_t is equal to θ, which eases our comparison of the constant volatility economy with the stochastic volatility economy. Note that $E(\sigma_t^2|\sigma_0) = var(\sigma_t|\sigma_0) + E^2(\sigma_t^2|\sigma_0) = var(\sigma_t|\sigma_0) + \theta^2$.

Let $e_0 = 0$. Since $e_t = e_0 + \int_0^t \sigma_s dB_s$, we have $E(e_t^2) = E[(\int_0^t \sigma_s dB_s)^2|\sigma_0] = E\left[\int_0^t (\sigma_s^2|\sigma_0)ds\right] = E\left[\int_0^t E(\sigma_s^2|\sigma_0)ds\right]$ where the second equality comes from Ito Isometry. Therefore, $E(e_t^2|\sigma_0) = E\int_0^t E(\sigma_s^2|\sigma_0)ds = E[\int_0^t var(\sigma_t|\sigma_0)ds] + \int_0^t \theta^2 ds \geq \int_0^\infty \theta^2 ds$. Therefore, $E^{stochastic}\int_0^\infty e^{-\rho t}e_t^2 dt \geq E^{constant}\int_0^\infty e^{-\rho t}e_t^2 dt$. Note that this result does not rely on any specific structure of stochastic volatility, and holds more generally than the CIR process specification. □

2.3.1. *Comments on Regulation Costs.* As noted earlier, the Central Bank maintains the target zone by using high frequency changes in the instantaneous interest rate. Appendix

8.6 provides the details. Each time the Central Bank changes the interest rate, it must pay a linear cost of m. How big are the resulting costs? Given the nature of Brownian motion, it pays the cost 'many' times, but each time it does so, the adjustment is infinitesimal, so it is not at all obvious how big the product of the two turns out to be over a finite interval of time. Appendix 8.7 uses results from Stokey (2008) to compute the regulation cost as a percentage of total welfare cost, using our benchmark parameter values. It turns out that regulation costs are very small, only about 0.053% of long-run total welfare cost.

3. Rational Expectations

By exploiting the fact that martingales behave in a nice way when it comes to stopping times, we prove here that the above solution constitutes a rational expectations equilibrium.[18] Define the stopping times $\tau_k < \tau < \tau' < \tau_{k+1}$, where τ_k is the k'th intervention time. From Doob's optional stopping theorem,

$$E(v_{\tau'}|\mathcal{F}_\tau, z_k) = v_\tau \tag{3.32}$$

by assumption. We need to show that the continuous time limit of the martingale x_τ

$$E(x_{\tau+\epsilon}|\mathcal{F}_\tau, z_k) = x_\tau \tag{3.33}$$

where $\epsilon \to 0$, indeed satisfies rational expectations for any stopping time τ. We proceed by showing that this local martingale process is consistent with the private sector's expectations. We know from equations (3.34) that

$$E(x_{\tau'}|\mathcal{F}_\tau, z_k) = x_\tau \tag{3.34}$$

Recall that the private sector's behavior links z_τ and x_τ in the following way

$$z_\tau = \lambda_s x_\tau \tag{3.35}$$

which can be written into

$$x_\tau = \frac{1}{\lambda_s} z_\tau; \quad x_{\tau'} = \frac{1}{\lambda_s} z_{\tau'} \tag{3.36}$$

and that $z_{\tau'} = z_\tau + v_\tau$, so we have $E(z_{\tau'}|\mathcal{F}_\tau, z_k) = z_\tau$. Therefore, we have

$$E(x_{\tau'}|\mathcal{F}_\tau, z_k) = \frac{1}{\lambda_s} E(z'_\tau|\mathcal{F}_\tau, z_k) = \frac{1}{\lambda_s} z_\tau = x_\tau \tag{3.37}$$

Therefore, expectations of the output gap are confirmed.

4. Calibration

In this section, we take the above model to Chinese data, and examine its quantitative implications for the evolution of the RMB exchange rate trading band. Since there is a one to one translation from the nominal exchange rate to the terms of trade in the model, we use monthly data for export and import prices from China's National Bureau of Statistics to estimate the parameters governing the dynamics of stochastic volatility for China's terms of trade. The base year is 2000, and the terms of trade is normalized to 100 in 2000. To deal with seasonality, we follow the literature to consider year-over-year

[18]We consider an equilibrium featuring martingale dynamics for the exchange rate as a natural and empirically plausible equilibrium, since exchange rates tend to follow random walks in high frequency. However, as noted above, we are not claiming uniqueness of the equilibrium.

(YoY) changes.[19] The data span from January 2001 to December 2017. A graph depicting changes in the terms of trade is in Figure 9 in the Appendix 8.2. We estimate the volatility of terms of trade (σ) using an IGARCH(1,1) model.[20] We then use maximum likelihood estimation (MLE) to estimate the CIR parameters for the volatility process using equation (2.16), and then convert to their continuous time counterparts. We find the mean reversion parameter $\kappa = 0.40091$, the long-run mean of volatility $\theta = 0.07993$ and the volatility of stochastic volatility parameter $\xi = 0.0193$.

For the other parameters, the discount rate ρ is set to be 0.0034, so that the steady state annual risk free interest rate is about 4%. To calibrate the weight on "carrying cost", α, we use the structural parameters of the New Keynesian model in the Appendix 8.4. We select the "openness" parameter $\hat{\alpha}$ to be 0.2171, which matches China's average import/GDP ratio from 2006-2016. All other structural parameters assume the same values as in Galí (2015).[21]), the discrete time monthly counterpart of the discount rate $\delta = 0.034$. Finally, the labor supply elasticity is set to 5, which implies that the inverse of it being $\phi = 0.2$. Using these parameters, our calibrated $\alpha_x = 1$. Following the standard Taylor Rule literature, we set the weight on output gap deviation to be as half as important as inflation deviation, such that $a = 0.5, b = 1$. This yields the weight on "carrying cost" $\alpha = a\alpha_x^2 = 0.5$. This gives us $\alpha = 0.5006$. Finally, the intervention cost parameter, m, is calibrated to match the long run average target zone width of $\pm 0.4334\%$ from our sample period using the long-run mean volatility. This implies that $m = 0.0014$. Finally, we use the terms of trade shock estimated from the IGARCH model from April 2006 to February 2018 to generate a model-predicted target zone, and then compare it with the de facto target zone.

When comparing the model-implied band with the data, it is better to measure a *De Facto* continuous band, rather than the discretely adjusted *De Jure* band depicted earlier in Figure 1. It is widely believed that the announced (*De Jure*) band is much wider than the *De Facto* band. Therefore, we compute the de facto bands using different time windows, and check which one provides the best fit. In particular, we compute the de facto band by selecting the percentage deviation from the daily central parity that is effectively hit for at least 80% of the time during the past 3 months, 6 months and 1 year respectively.[22]

More specifically, based on the methodology used in the de facto exchange rate regime study such as Reinhart and Rogoff (2004) and Ilzetzki, Reinhart, and Rogoff (2017), we get the continuous *De Facto* band using the following algorithm:

$$P(\epsilon < b\%) \geq 80\%$$

where ϵ is the daily absolute percentage change of the nominal exchange rate from the central parity. This probabilistic approach has been widely used in the exchange rate

[19]This means we lose one year of data so the starting month is January 2001.

[20]We use IGARCH because it fits the data better than the more conventional GARCH(1,1) model.

[21]Following Gali and Monacelli (2005), we consider the case of log utility ($\sigma = 1$), unit elasticity of substitution between domestic and foreign goods ($\eta = 1$), unit elasticity of substitution between domestic and foreign goods ($\gamma = 1$

[22]The same methodology of computing a de facto continuous band is used in Reinhart and Rogoff (2004).

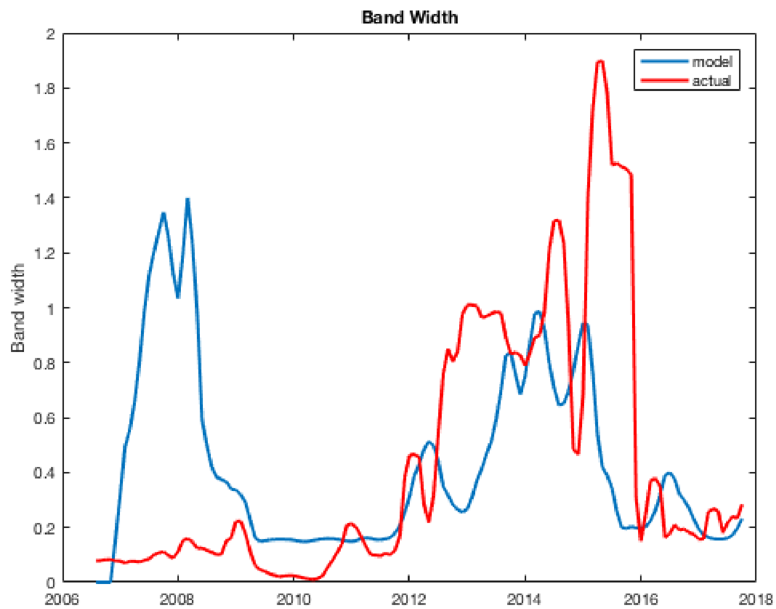

FIGURE 4. Band width: Model vs. Actual

classification literature.[23] We also experiment with a higher probability of 95% and the qualitative results are robust.

We then compare that with the model-implied 3 months, 6 months and 1 year moving average level of the trading band. Since the *De Facto* band is taken at the level in which at least 80% of the time the nominal exchange rate within a certain level, we also need to revise the model implied band by taking its historical average. Except before the financial crisis, one can see that the model captures time-variation in the band quite well. The results are shown in Table 2.

The mean bandwidth is calibrated to be the same across the different model horizons. As one can see, the model implied variation of the band is closest to the data when using a 6-month window. The model also generates a reasonable amount of negative correlation between the width of the band and terms of trade volatility, with the exception of the 1-year model. One can see that both the 3-month and the 6-month models seem to account for around half of the variation of the band size, with the 6 months model better at explaining its correlation with the underlying uncertainty. The 1 year model seems to overshoot in terms of variance, and the implied correlation with uncertainty is actually positive, which could be a reflection of too much averaging.

One thing to note is that our model predicts a much wider band than the actual band before the 2008-2009 financial crisis. Given the low uncertainty environment in that period, it makes sense that our model predicts a large bandwidth. One plausible explanation for why the PBOC adopted a narrower band is policy inertia. Given that China first started

[23]Klein and Shambaugh (2012) survey this literature.

TABLE 2. Calibration Result

Measure	3 months data	3 months model
mean	0.43%	0.43%
variance	0.2089	0.1111
corr(Band,uncertainty)	-0.2270	-0.5258
Measure	6 months data	6 months model
mean	0.43%	0.43%
variance	0.2328	0.1053
corr(Band,uncertainty)	-0.2278	-0.3076
Measure	1 year data	1 year model
mean	0.43%	0.43%
variance	0.2412	1.7195
corr(Band,uncertainty)	-0.2468	0.3854

to increase exchange rate flexibility in July 2005 after more than a decade with a fixed exchange rate, the PBOC might have been initially reluctant to set a wide band, even though the uncertainty was low. Another explanation might be related financial market underdevelopment: during that period, China was in the process of introducing various FX financial products to help investors hedge exchange rate risk, such as FX forward contracts, FX swaps, and options. Thus, they might have hesitated to embrace a wider band before these important FX financial markets were established. [24]

5. Empirical Evidence

The goal of this section is to systematically assess the empirical validity of the model's main testable prediction: the negative correlation between the exchange rate bandwidth and the underlying uncertainty in the economy. We use five different uncertainty measures. As a robustness check, we also construct various continuous *De Facto* bands following the algorithm in Reinhart and Rogoff (2004), as well as a currency basket band following Frankel and Wei (2007).*In summary, the empirical patterns are: (a) First, we find strong empirical evidence that the bandwidth in China narrows when uncertainty increases. (b) Second, central parity changes do not seem to affect the bandwidth. (c) Third, uncertainty's effect on the bandwidth is more pronounced when there is currency depreciation.*

5.1. **Uncertainty Measures.**

To empirically examine the correlation between bandwidth and uncertainty, we look at five different uncertainty measures: terms of trade uncertainty, exchange rate volatility from the Hongkong offshore RMB market, global uncertainty, China-specific macroeconomic uncertainty, and RMB option price implied volatility. For the dependent variable, we focus on the discrete daily band as used in Figure 1. A summary of variable construction and data sources can be found in Table 5 in the Appendix 8.2.

[24]See Obstfeld (2006) and Prasad, Wang, and Rumbaugh (2005) for more detailed discussion on PBOC's policy stance in that early-reform period.

First, we look at terms of trade uncertainty. Term of trade shocks have been shown in the literature to be important for emerging market economies.[25] As illustrated in Section 5, we estimate the volatility of terms of trade shocks using an IGARCH(1,1) model. A plot of the optimal exchange rate band and terms of trade volatility is shown in Figure 5. We can see that when terms of trade volatility increases, the trading band tends to narrow. The negative correlation between the bandwidth and terms of trade uncertainty is statistically significant.

Second, we consider the parallel foreign exchange market in Hong Kong, or the so-called CNH market, where the RMB is traded offshore against many other currencies without any regulatory restrictions. For instance, there are neither daily trading bands nor central parities in the CNH market. One can therefore regard it as a hypothetical RMB exchange rate if the PBOC's band restrictions were lifted. Specifically, we use the one year RMB/USD NDF (non deliverable forwards) rate in the Hong Kong offshore market, which is among the most active forward contracts. The data is from Thomson Reuters and spans from September 2003 to February 2018. After taking the daily percentage change, we fit it into a GARCH(1,1) model and predict the variance. We smooth it using a 60-day rolling window. We find that the negative correlation between CNH volatility and bandwidth is statistically significant. The graph is shown in Figure 6.

Third, we consider a measure of global uncertainty, measured by the VIX index of the SP500. It is daily data from the FRED St.Louis dataset and spans from January 2003 to February 2018. VIX is the implied volatility from S&P500 option prices. We use rolling windows of 30/60/90 days to compute moving averages of daily VIX-implied volatility. Figure 2 illustrates the relationship between the target zone band and VIX volatility (smoothed using a 60-day rolling window). We also find statistically significant negative correlation between the bandwidth and VIX volatility.

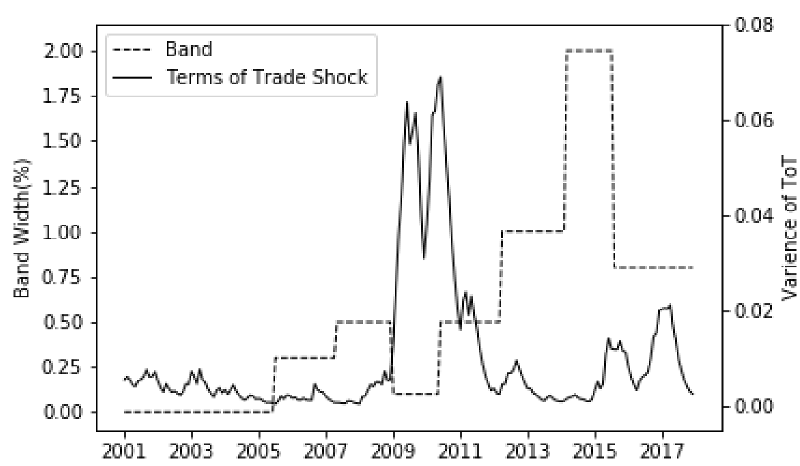

FIGURE 5. Terms of trade volatility and China's exchange rate band

[25]See Schmitt-Grohé and Uribe (2018) and the references therein.

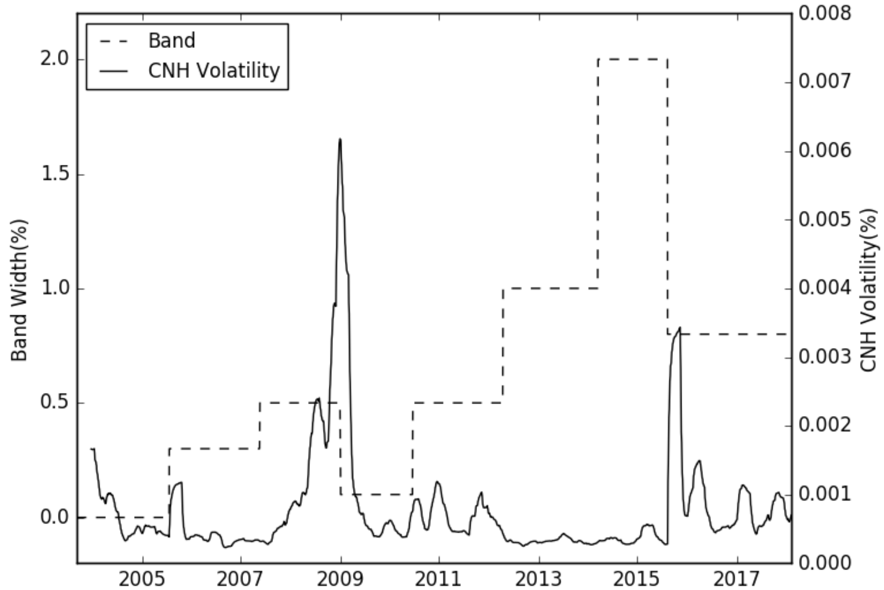

FIGURE 6. CNH Volatility and China's exchange rate band

Fourth, we use China's sovereign credit default swap (CDS) spread, which is a proxy for China-specific macroeconomic uncertainty. We get daily time-series data from Bloomberg which spans from January 2003 to February 2018. We calculate percentage changes of the CDS spreads and fit them into a GARCH(1,1) model, then predict the variance. We calculate the 60-day moving average of the volatility of CDS spreads. The correlation between the bandwidth and the volatility of CDS spreads is also negative and statistically significant. See Figure 10 in Appendix 8.2.

Lastly, we use the implied volatility of 3 month RMB/USD option prices (at the money options) in the Hongkong offshore market following the method in Jermann, Wei, and Yue (2017). The data are from Bloomberg. As evident from Figure 11 in the Appendix 8.2, there is also a negative correlation between bandwidth and the implied volatility from RMB option prices.

As all uncertainty measures show, the two obvious and well-known spikes of uncertainty are the 2008-2009 global financial crisis and the 2015 turmoil in China's financial markets. However there are other variations in uncertainty measures, e.g. uncertainty measures jumped in 2011-2013 due to heightened European sovereign debt default risk.

5.2. **Empirical Estimation.** We now empirically test to what extent the exchange rate bandwidth is affected by various measures of uncertainty. We use daily data for most regressions, but use monthly data for terms of trade uncertainty. Denote the dependent variable, bandwidth, as w_t. Denote the key explanatory variable, the uncertainty measure, as $Uncertainty_t$. For the control variables, we consider policy inertia, which is captured by 12-month moving average of bandwidth, denoted as $BandMA_t$. The percentage change of the central parity, e.g. the central parity's appreciation or depreciation, is denoted as $CentralParity_t$. We also add an interaction term between uncertainty and exchange rate changes, which is denoted as $Uncertainty_t * ExRate_t$. We want to test whether uncertainty's effect on the bandwidth is more pronounced when the exchange rate is depreciating. More precisely, we perform linear regressions of bandwidth on the uncertainty measures as follows:

$$w_t = \alpha + \beta_0 Uncertainty_t + \beta_1 BandMA_t + \beta_2 CentralParity_t + \beta_3 Uncertainty_t * ExRate_t + \epsilon_t \tag{5.38}$$

Columns (1) and (2) in Table 3 show a strong negative correlation between bandwidth and uncertainty. For the five different measures of uncertainty, the coefficients are all negative and statistically significant. This finding is consistent with the recent empirical work in Marconi (2018), which shows that the width of RMB/USD band is negatively related to the volatility of RMB/USD exchange rate. Furthermore, the bandwidth has strong inertia. The change of the level of RMB central parity does not have significant effects on the bandwidth. More interestingly, we use two different RMB exchange rates to interact with the uncertainty measures: the Hongkong offshore RMB exchange rate and the central parity rate. We find that if we use the central parity's change to measure the currency depreciation/appreciation, the coefficient of the interaction term is not statistically significant. However, if we use the exchange rate in the Hong Kong offshore market, the coefficient of the interaction term becomes negative and statistically significant. One possible explanation is that while the central parity is tightly controlled by the PBOC, the exchange rate in the Hong Kong offshore market is less manipulated and can therefore serve as a better measure for currency depreciation/appreciation.

For comparison with different uncertainty measures, we also compute the economic significance of uncertainty's effect on the *De Facto* bandwidth. Since our measures of uncertainty have different units, we calculate the one standard deviation increase in these measures to see how that affects the central bank's bandwidth choice. The results are shown in Table 4: it reports the percentage change of the bandwidth, which are calculated using the discrete bandwidth, as well as the continuous bands from 3 months and 6 months rolling windows. We put them in the order of economic significance. Option price implied RMB exchange rate volatility and terms of trade volatility seem to the be most important uncertainty measures. For example, one standard deviation increase in terms of trade shock is associated with about 15.64% and 13.57% decrease in the bandwidth for the 3M and 6M measure of *De Facto* continuous bands respectively. On the other hand, China's CDS volatility seems to have a much smaller effect on the bandwidth choices.

5.3. **Robustness Checks: Continuous Band and Uncertainty Measures.** We also calculate the continuous *De Facto* band that central bank allows the exchange rate to float. While there is no consensus on how to compute a *De Facto* continuous band, we follow Reinhart and Rogoff (2004)'s method to estimate the band using a rolling window test. More specifically, let the band size be denoted as s, if 80% of the daily absolute changes of the RMB exchange rate during the past two years lie within the band, we will use that band as our *De Facto* continuous exchange rate bandwidth. Since we have daily data, we try different rolling windows, such as 3-months and 6-months.

Dependent Variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(a) Terms of Trade Uncertainty						
Terms of Trade Volatility	-1.13***	-1.04***	-0.81***	-0.60***	-0.74***	-0.46***
BandMA	1.03***	0.93***	0.82***	0.72***	0.91***	0.81***
CentralParity	-0.88	0.83	-0.32	0.23	1.04**	0.96
Uncertainty*CNH Exchange Rate Change	-34.60***		-34.32***		-34.42***	
Uncertainty*Central Parity Change		-23.76**		-8.92		0.98
R^2	0.8144	0.8048	0.7241	0.7040	0.8420	0.8214
N	169	169	166	166	163	164
(b) Exchange Rate Uncertainty						
CNH Volatility	-92.41***	-108.02**	-27.67***	-32.75***	-22.23***	-28.65***
BandMA	0.98***	0.96***	0.95***	0.94***	0.96***	0.95***
CentralParity	-0.08	-0.13	-0.08**	0.03	-0.03	-0.03
Uncertainty*CNH Exchange Rate Change	-7.83***		-2.29***		-3.11***	
Uncertainty*Central Parity Change		65.17		-143.53*		-0.11
R^2	0.8587	0.8567	0.7523	0.7526	0.8241	0.8237
N	3013	3013	3013	3013	2989	2989
(c) Global Uncertainty						
VIX	-0.005***	-0.005***	-0.001***	-0.002***	-0.0027***	-0.002***
BandMA	0.97***	0.95***	0.94***	0.93***	0.95***	0.94***
CentralParity	-0.07	-0.12	-0.08**	-0.02	-0.02	-0.04
Uncertainty*CNH Exchange Rate Change	-0.0003***		-0.0001***		-0.0001***	
Uncertainty*Central Parity Change		0.002		-0.004		0.001
R^2	0.8381	0.8368	0.7485	0.7484	0.8228	0.8225
N	2978	2978	2933	2933	2904	2904
(d) China Macroeconomic Uncertainty						
China CDS Volatility	-0.06***	-0.01***	-0.01***	-0.01***	-0.02***	-0.01***
BandMA	0.96***	0.96***	0.94***	0.94***	0.95***	0.95***
CentralParity	-0.09	-0.09	-0.08**	-0.08**	-0.03	-0.03
Uncertainty*CNH Exchange Rate Change	-0.007***		-0.001**		-0.003***	
Uncertainty*Central Parity Change		-0.01		-0.003		-0.02
R^2	0.8320	0.8310	0.7484	0.7483	0.8211	0.8209
N	3063	3063	3018	3018	2989	2989
(e) Option Price Implied RMB volatility						
3M Option Implied Volatility	-0.03**	-0.08***	-0.02	-0.08***	-0.06***	-0.10***
BandMA	0.96***	0.92***	0.95***	0.88***	0.98***	0.94***
CentralParity	0.25**	0.02	0.01	0.24	0.03	-0.34
Uncertainty*CNH Exchange Rate Change	-0.01***		-0.01***		-0.01**	
Uncertainty*Central Parity Change		0.03		-0.06		0.06
R^2	0.7527	0.7444	0.6831	0.6694	0.8061	0.8020
N	259	259	259	259	259	259

TABLE 3. Regression of Exchange Rate Bandwidth on Various Uncertainty Measures

Columns (3) (4) (5) and (6) in Table 3 report regression results using 3-month and 6-month rolling windows for continuous bands. In general, the coefficients on uncertainty are all negative and statistically significant. The bandwidth has strong inertia in all samples.

Table 4. One standard deviation increase in uncertainty measures

	Bandwidth	Band (3M)	Band (6M)
Option price implied volatility	-9.76%	-9.07%	-33.63%
Terms of trade uncertainty	-11.58%	-15.64%	-13.57%
CNH uncertainty	-13.07%	-7.51%	-5.60%
VIX	-7.12%	-4.07%	-4.72%
China CDS volatility	-5.79%	-2.50%	-4.13%

The change of RMB central parity does not have significant effects on the bandwidth. For the interaction terms we also get robust results: if we use the exchange rate in the Hong Kong market, the coefficient on the interaction term becomes negative and statistically significant. This shows that uncertainty will have a larger effect on the bandwidth when there is currency depreciation.

5.4. **Comments on currency basket.** The current paper focuses on the daily trading band imposed by the central bank. In China's case, it is the trading band of RMB/USD. The US dollar is the single most important currency for China in terms of international trade and finance. Ilzetzki, Reinhart, and Rogoff (2017) recently finds China uses the US dollar as the anchoring currency. However it is interesting to check the band for a currency basket. Frankel and Wei (2007) has the detailed discussion where he estimated both the basket and flexibility of Chinese exchange rate regime. We follow Frankel and Wei (2007)'s method to get the floating band to the major currency basket (see Appendixe 8.3) and the qualitative results are robust.

6. Literature Review

Our paper contributes to three strands of the existing literature.

First, this paper is related to the literature on exchange rate target zones (e.g.,Krugman (1991), Bertola and Caballero (1992), Bertola and Svensson (1993), Miller and Zhang (1996), etc.). However, the existing literature does not allow the bandwidth to respond to changes in uncertainty. Our paper extends the classic target zone literature (summarized in Krugman and Miller (1992)) by incorporating stochastic volatility within a New Keynesian general equilibrium model. These extensions are crucial in helping us understand how uncertainty influences the choice of target zone bandwidth.

Our paper also contributes to the understanding of China's monetary and exchange rate policy, and more broadly, the choice of band in the intermediate exchange rate regimes that are widely used in developing countries.[26] Obstfeld (2006) studies the RMB exchange rate's exit strategy from pegging to US dollar and propose to establish a limited trading band for the RMB relative to a basket of major trading partner currencies. Cheng (2015) and Yu, Zhang, and Zhang (2017) both argue for a limited target zone for RMB exchange rate, but none of them study the optimal bandwidth choice when Central Bank faces

[26]As Frankel and Wei (2007) forcefully argued, "The issue of the regime governing the Chinese exchange rate and specifically whether the currency is moving away from the *De Facto* peg that for ten years had tied it to the US dollar is much more than just another application, to a particular country, of the long-time question of fixed versus floating exchange rates. It is a key global monetary issue."

uncertainty. Our work is also related to the recent work by Jermann, Wei, and Yue (2017) which studies the formation of RMB/USD central parity. Our paper complements theirs by looking at the width of target zone band.

Finally, at a technical level, our paper is closely related to the real options literature, summarized by Dixit and Pindyck (1994) and Stokey (2008). Recently, real options theory has been successfully applied to macroeconomic theory where discrete actions and inertia are present. For example, Alvarez and Dixit (2014) uses the techniques to answer what is the optimal timing of a potential eurozone breakdown; Stokey (2016) studies investment options under policy uncertainties; Lei and Tseng (2017) study the optimal timing and size of interest rate adjustment, etc. Recent advances in bringing stochastic volatility into the discussion of option exercising also shed lights on our analysis. Fouque, Papanicolaou, and Sircar (2000) studies the pricing of an American option under stochastic volatility. Such technique was later introduced into the real options literature by Sarkar (2000) and Tsekrekos and Yannacopoulos (2016). Our paper applies such techniques to the target zone model.

7. Conclusion

This paper has shown that uncertainty has vital implications for exchange rate target zone choices. We first document the stylized facts for China's daily exchange rate target zone. We show that increased uncertainty causes the PBOC to narrow the daily RMB/USD exchange rate band. Using the theory of real options, we find that two opposing effects influence the choice of bandwidth: (1) a permanent increase in the level of uncertainty increases the value of the "wait and see" component, which widens the target zone band, and (2) When volatility is stochastic, temporary increases in uncertainty cause the Central Bank to *narrow* the band. Our calibration results show that a stochastic volatility target zone model matches the dynamics of China's exchange rate trading band well, except during the initial experimental period when the PBOC just started liberalizing its exchange rate regime. We provide empirical evidence that terms of trade shocks correlate negatively with the size of daily target zone choice. We also systematically test the relationship between various uncertainty measures and the bandwidth. A fruitful direction for future research would be to study how the dynamics of the central parity, studied by Jermann, Wei, and Yue (2017), interact with the dynamics of the bandwidth. This could be done by introducing dynamics into the mean of the uncontrolled exchange rate process.

8. Appendix

8.1. **Institutional background of China's FX regimes and its evolutions.** China's government has a long tradition of inward-looking and believes in pragmatism. Thus, China only gradually added market force into the determination of its exchange rate and changed the width of the limited trading band . Over the past two decades, China took several reforms and experiments on its exchange rate policy. In this section, we lay out the brief history of RMB exchange rate policy, with the focus on the justification of the bandwidth we use in Figure 1. Figure 7 plots the times series of bilateral RMB/USD exchange rate with the majoy reforms. For more details of China's foreign exchange regime, see Sun (2016) and Yu, Zhang, and Zhang (2017).

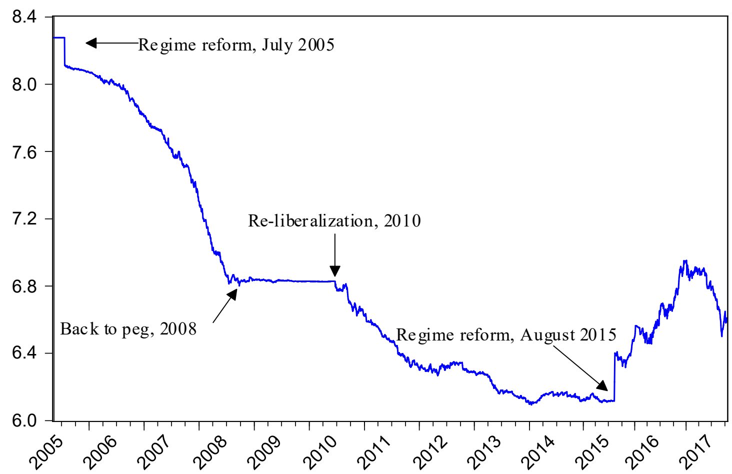

Figure 7. China's exchange rate regime reforms

Period 1: July 2005 to End of 2008

The RMB had been effectively pegged to the US dollar at the rate of 8.28 from 1997 until 2005. The exchange rate reform on July 21, 2005 aimed to increase the flexibility of RMB exchange rate but at a tightly controlled pace. The PBOC set the daily *De Jure* target band of ±0.3%. In the official announcement, "the daily trading price of the US dollar against the RMB in the interbank foreign exchange market will be allowed to float within a band of ±0.3% around the central parity published by PBOC". And the PBOC "will make adjustment of the RMB exchange rate band when necessary according to market development as well as the economic and financial situation" and "maintain the RMB exchange rate basically stable at an adaptive and equilibrium level so as to promote the basic equilibrium of balance of payment and safeguard macroeconomic and financial stability." On May 21 2007, PBOC further expanded the daily *De Jure* target band from ±0.3% to ±0.5%.

Period 2: End of 2008 to June 2010

As the global financial crisis peaked in the summer of 2008, China became worried about the volatile exchange rate and its potential negative effects on the economy. At the end of 2008, China started to *De Facto* repeg to the US dollar at 6.82 within a very narrow band less than ±0.1%. Based on Hu and Zhang (2012) and Shuo, Jiwei, and Kan (2016), governor Zhou Xiaochuan said in public in early 2009 that "the choice of RMB's repeg to US dollar is a response to the financial crisis which generated large increase in the uncertainty". Therefore, the effective trading band set by PBOC is much narrower than the *De Jure* band of ±0.5%. Many studies have found that RMB pegged to US dollar at that period, including Klein and Shambaugh (2012) and Marconi (2018).

Period 3: June 2010 to August 2015

The global financial crisis was over in 2009, but the fixed exchange rate was maintained until the June of 2010 as more information have arrived showing that the uncertainty has faded. On Jun 19 2010, China let RMB exchange rate to regain some flexibility with the daily target band 0.5%, back to the precrisis level. But during that time, European sovereign debt crisis was still looming and China did not further expand the band when they still face significant uncertainty. In 2012, market forces of demand and supply were more balanced and the RMB exchange rate started to two-way fluctuate, so PBOC expanded the target band to 1%. In 2014 it further widened the target zone from 1% to 2%.

Period 4: August 2015 to current

On August 11 2015, PBOC implemented a new reform to improve the mechanism to set the central parity. Around the same period, Fed started to quit quantitative easing and raised the interest rate. The central bank of China intervened the market forcefully from 2015 to 2016 to prevent large and persistent depreciation pressure.(Yu, Zhang, and Zhang (2017)) The *De Facto* target zone shrinked dramatically. Jermann, Wei, and Yue (2017) finds the effective band is as narrow as 0.5%, while others get similar band of 0.75%.[27]

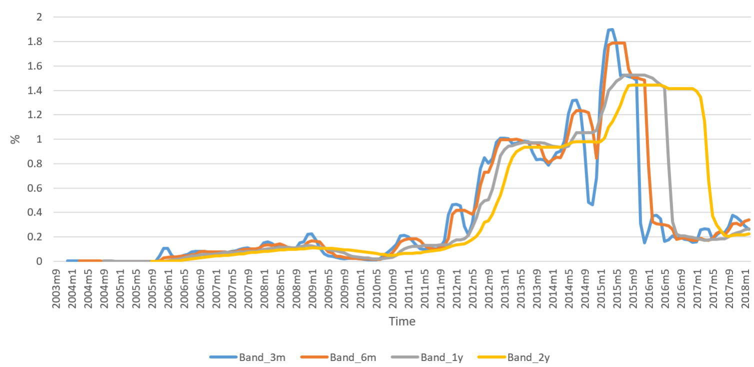

FIGURE 8. *De Facto* continuous Band using different rolling windows

8.2. **Data Source and Variable Construction.** The following table shows the description of data and their sources.

[27]Tao Guan, a former senior officer in PBOC, finds the effective band is 0.75% after August 2015. We use the more updated data to find the band is 0.8%, narrower than the officially announced band of 2%.

Table 5. Variable construction and Data Source

Name	Description	Source
Discrete Band	The upper and lower band set and maintained by PBOC	PBOC and author's calculation
De Facto continuous band	Use different rolling windows as Reinhart and Rogoff (2004)	Author's calculation
Terms of trade	Ratio of Export vs Import Prices	China National Bureau of Statistics
VIX	Implied volatility from Option price of SP500	FRED St.Louis FED
RMB Hongkong Exchange Rate	1 year RMB NDF prices	Thomson Reuters
China CDS Spread	5 year Credit Default Swap Spreads	Bloomberg
RMB option prices	3 month at the money option	Bloomberg

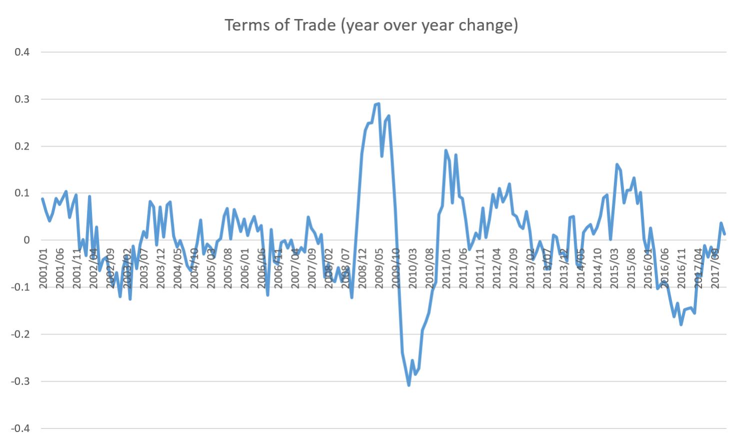

Figure 9. China's Terms of Trades

8.3. **The band for a currency basket.** In the main text we consider the RMB/USD exchange rate. US dollar is the most important currency from the trade and financial perspective. The anchoring currency of China's exchange rate policy is still the US dollar as argued in Ilzetzki, Reinhart, and Rogoff (2017). While it makes a lot of sense to focus on the RMB/USD exchange rate, it is also important and interesting to look at the basket of currencies that RMB is pegged to. As the officially announced in PBOC's statement, it will try to stabilized the RMB exchange rate against a basket of currencies. Therefore, in this section we calculate the band of RMB exchange rate to the basket of currencies.

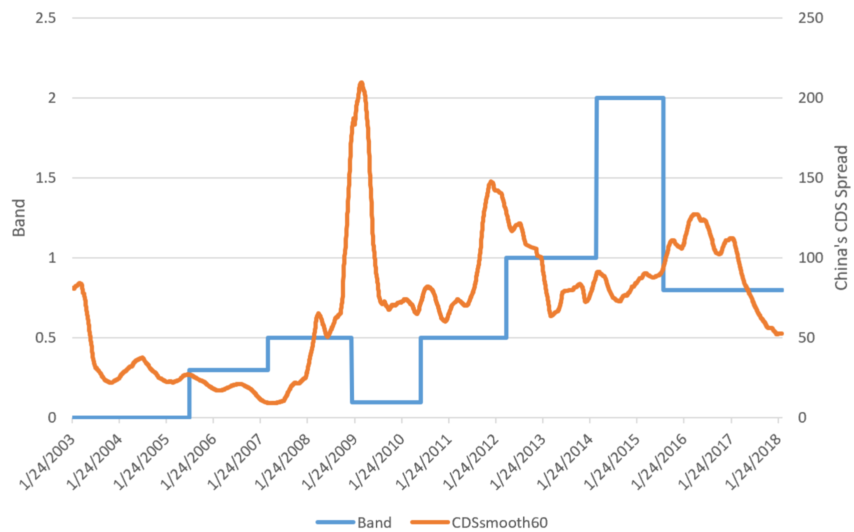

FIGURE 10. Bandwidth and China's CDS spreads

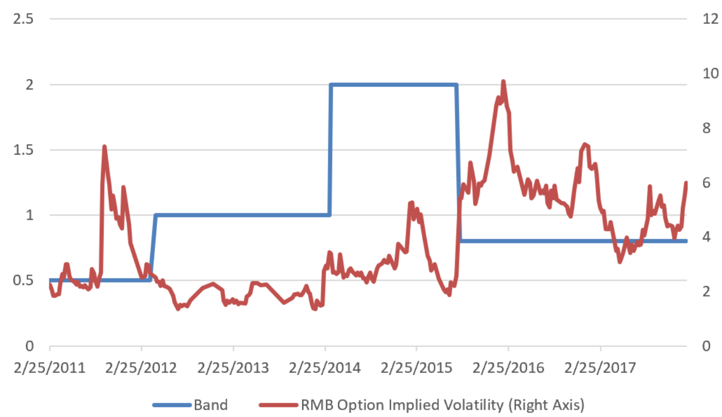

FIGURE 11. Bandwidth and RMB Option Implied Volatility

The calculation is in two steps: in the first step, since the weight of China's currency basket is opaque, we need to estimate the weights. In the second step, we calculate the distance between RMB exchange rate relative to this currency basket.

We estimate the weight of currency basket following Frankel and Wei (2007) and Frankel (2009). The four currencies are US dollar, Euro, Japanese Yen and Korea Won. We estimate the following equation:

$$dlog(CNY) = \alpha_1 dlog(USD) + \alpha_2 dlog(EUR) + \alpha_3 dlog(JPY) + \alpha_4 dlog(KRW) \qquad (8.39)$$

We focus on the data period from September 2003 to August 2015. In this period, China announced that its exchange rate will be kept stable relative to a basket of currencies. For the baseline currency, we follow the literature to use the SDR. Based on the institutional background, we decide into four sub-periods: Sep 2003 to July 2005 when RMB is strictly pegged to USD, so we are interested in the horizontal band; July 2005 to Oct 2008 when PBOC started to use a wider band and let the central parity to gradually appreciated, so we need to calculate the crawling bands; Oct 2008 to June 2010 when China narrow the band and got back to pegging to USD; June 2010 to Aug 2015 when China enlarge the trading band and let the central parity to crawl. We get different weights in different sub-periods. We also detrended the central parity's change. The band to a basket of currencies is shown in the following figure:

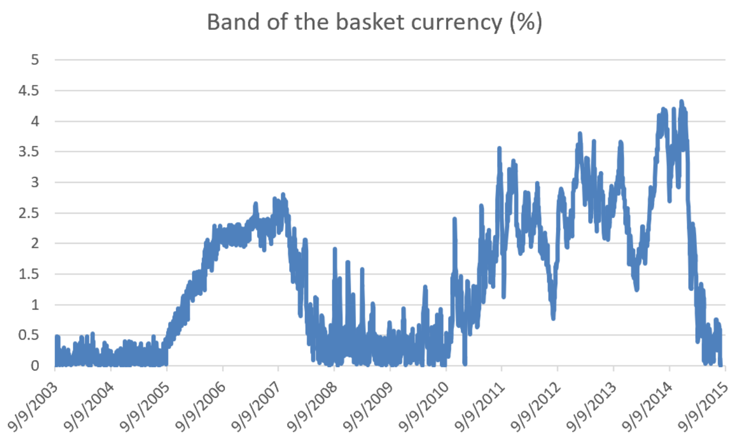

FIGURE 12. RMB band to the basket of currency

The above floating band around the major currency basket is similar to Sun (2010). By stabilizing RMB/USD exchange rate, PBOC will indirectly stabilized RMB exchange relative to the currency basket. Obstfeld (2006) points out this policy objective of China's monetary authority.

8.4. **An Open Economy New Keynesian Setting.** Let's consider a small open economy model with money, imperfect competition, and nominal price rigidity. Consumption goods are consumed and traded across countries as in Galí (2015) and Clarida, Gali, and Gertler (2001). Households consume domestic and foreign goods that are imperfect substitutes. The consumption goods are a composite of a continuum of differentiated goods, each produced by an associated monopolistic-ally competitive firms. The home economy is small, in the sense that it doesn't affect world output, world price, and world interest rate. There is perfect risk sharing in consumption risk internationally.

The log linear form of consumption is denoted by [28]

$$c_t = (1-\hat{\alpha})c_t^h + \hat{\alpha}c_t^f \tag{8.40}$$

where the parameter $\hat{\alpha} \in [0,1]$ is (inversely) related to the degree of home bias of consumption goods preference. Higher α implies a higher degree of openness. Now define the effective terms of trade S_t as a composite of a continuum of import goods to export goods price ratio. Let i be the index of countries, i.e:

$$S_t = \frac{P_{F,t}}{P_{H,t}} = (\int_0^1 S_{i,t}^{1-\gamma} di)^{\frac{1}{1-\gamma}} \tag{8.41}$$

where γ governs the degree of substition across goods from different countries, which can be approximated up to first order by the log linear expression

$$s_t = \int_0^1 s_{i,t} di = p_{F,t} - p_{H,t} \tag{8.42}$$

where s_t becomes the log of the effective terms of trade, where $p_{F,t} = \log P_{F,t}, p_{H,t} = \log P_{H,t}$. With law of one price, we have

$$s_t = e_t + p_t^* - p_{H,t} \tag{8.43}$$

[28]Variables denoted in small letters below all implies percentage deviation from the steady state, except interest.

where e_t is the log of a composite of bilateral nominal exchange rate, p_t^* denotes the log of foreign price index. Note that in our fixed price economy, assuming that domestic and foreign prices start at the same level, there is then no distinction between domestic, foreign or world price index That is, overall inflation level in the home country equals domestic inflation, adjusted by "imported inflation". This allows to write down the consumption Euler equation as follows

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}_t(\pi_{H,t} + \alpha \mathbb{E}_t \Delta s_{t+1})] \tag{8.44}$$

where σ is the coefficient of relative risk aversion, $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. With imports and exports, we need to adjust the relation between aggregate domestic output with aggregate domestic consumption. As in Galí (2015), we have $x_t = c_t + \frac{\hat{\alpha} w}{\sigma} s_t$, where x_t denotes log output, $w = \sigma\gamma + (1-\hat{\alpha})(\sigma\eta - 1)$. Using the same parameters as in Galí (2015), we have $w = 1$..

With complete international financial markets, uncovered interest parity (UIP) holds, i.e:

$$i_t - i_t^* = \mathbb{E}_t \Delta(e_{t+1}) \tag{8.45}$$

Expanding this with the definition of nominal exchange rate and terms of trade, one get

$$s_t = (i_t^* - \mathbb{E}_t \pi_{t+1}^*) - (i_t - \mathbb{E}_t \pi_{H,t+1}) + \mathbb{E}_t(s_{t+1}) \tag{8.46}$$

Combining this with the consumption euler equation,we get essentially equation (2.1).

Under complete markets, one can get the relation between domestic and world consumption, i.e:

$$c_t = c_t^* + (\frac{1-\hat{\alpha}}{\sigma}) s_t \tag{8.47}$$

Again, combining this with $x_t = c_t + \frac{\hat{\alpha} w}{\sigma} s_t$, we get $s_t = \sigma_\gamma x_t$ as in equation (2.3), where $\sigma_\gamma = \frac{\sigma}{1+\gamma(w-1)}$. Finally, the Phillips curve takes the usual form

$$\pi_{H,t} = \beta \mathbb{E}_t(\pi_{H,t+1}) + \kappa_\gamma x_t \tag{8.48}$$

where β is the discrete time discount factor, $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ is related to the frequency of price adjustment, and ψ denotes the inverse of Fischer elasticity. With $\theta = 1$ (fixed price), we have $\lambda = \kappa_\gamma = 0$.

8.5. **Derivation of boundary conditions.** We prove in this section that the following two sets of boundary conditions

$$V_z(\bar{z}) = -m; V_z(\bar{z}) = m \tag{8.49}$$

$$V_{zz}(\bar{z}) = V_{zz}(\underline{z}) = 0 \tag{8.50}$$

hold using discrete approximations of the continuous time process. Recall that the two stochastic process we have are

$$dz = \sqrt{\sigma} dB \tag{8.51}$$

$$d\sigma = \kappa(\theta - \sigma)dt + \sqrt{\xi}\sqrt{\sigma} dB_\sigma \tag{8.52}$$

It can be shown that the first process can be approximated using the discretization of a binomial tree with step size Δh_0, with half half probability of either moving up or down, where $\Delta h_0 = \sqrt{\sigma}\sqrt{\Delta t}$. The second process of stochastic volatility can be approximated with probability of p_1 of moving up, and $(1-p_1)$ probability of moving down, with step size Δh_1, where

$$p_1 = \frac{\kappa(\theta - \sigma)\sqrt{\Delta t} + \sqrt{\xi}\sqrt{\sigma}}{2\sqrt{\xi}\sqrt{\sigma}} \tag{8.53}$$

and

$$\Delta h_1 = \sqrt{\xi}\sqrt{\sigma}\sqrt{\Delta t} \tag{8.54}$$

If we use $e^{-\rho t} \approx 1 - \rho \Delta t$ as an approximation of time discrete, we know that at the threshold of FX intervention [29], the value of z has to go one step down. However, the volatility, which evolve exogenously,

[29]We derive proof on the upper barrier, but the argument applies to the lower barrier too.

can either go up or down. Therefore, the one step ahead discrete approximation of the value function can be approximated using

$$\begin{aligned} V(\bar{z},\sigma) &= -\alpha z^2\Delta t + (1-\rho\Delta t)\left[p_1 V(\bar{z}-\Delta h_0, \sigma+\Delta h_1) + (1-p_1)V(\bar{z}-\Delta h_0,\sigma-\Delta h_1)\right] - m\Delta h_0 \\ &= -\alpha z^2\Delta t + (1-\rho\Delta t)\left[V(\bar{z},\sigma) - \Delta h_0 V_z(\bar{z},\sigma) + (2p_1-1)\Delta h_1 V_\sigma(\bar{z},\sigma)\right] - m\Delta h_0 \end{aligned} \tag{8.55}$$

Substituting the value of discrete approximation, one get

$$0 = -\alpha z^2\Delta t - \rho\Delta t V(\bar{z},\sigma) + (1-\rho\Delta t)\left[\kappa(\theta-\sigma)\Delta t V_\sigma(\bar{z},\sigma) - \sqrt{\sigma}\sqrt{\Delta t}\right] - m\sqrt{\sigma}\sqrt{\Delta t} \tag{8.56}$$

At the limit, we have $\sqrt{\Delta t} >> \Delta t >> \Delta t^{\frac{3}{2}} >> \Delta t^2$. So the order of $\sqrt{\Delta t}$ dominates. Collecting terms with $\sqrt{\Delta t}$, one get

$$V_z(\bar{z},\sigma) = -m \tag{8.57}$$

This is the smooth pasting condition, which only uses the continuity of the value function. To pin down the solution, one needs an optimality condition. That is, for any given σ, the marginal utility before and after intervention should be the same, i.e:

$$V_z(\bar{z},\sigma) = V_z(\bar{z}-\Delta h_0,\sigma) \tag{8.58}$$

Expand the above, one get

$$V_z(\bar{z},\sigma) = V_z(\bar{z},\sigma) - \sqrt{\sigma}\sqrt{\Delta t}V_{zz}(\bar{z},\sigma) \tag{8.59}$$

So that $V_{zz}(\bar{z},\sigma) = 0$ holds as the higher order contact condition.

8.6. **Implied interest rate policy.** The discrete time UIP condition says that at each time t,

$$i_t = i_t^* + E_t(\Delta z_{t+1}) \tag{8.60}$$

must hold, where z_t is the controlled process for the exchange rate. Let's work out the implied interest rate policy to make this equation hold at the upper barrier (the same logic applies at the lower barrier). Let $\Delta i = i - i^*$, and take the continuous time limit. Let τ be a stopping time where z_τ hit the upper boundary $\bar{z}$, one then gets

$$\Delta i_\tau = \lim_{\epsilon\to 0}\mathbb{E}_\tau(dz_{\tau+\epsilon}|\mathcal{F}_\tau) \tag{8.61}$$

where $dz_{\tau+\epsilon}$ is the truncated incremental Brownian motion, s.t: $dz_{\tau+\epsilon} = \sqrt{\sigma}dB_{\tau+\epsilon}$ whenever $-2\bar{z} \leq dz_{\tau+\epsilon} < 0$, and takes the value of zero otherwise. Since for any $x \sim \mathcal{N}(\mu,\sigma)$ truncated at (a,b) has

$$\mathbb{E}(x|a<x<b) = \mu + \sqrt{\sigma}\frac{\phi(\alpha)-\phi(\beta)}{\Phi(\beta)-\Phi(\alpha)} \tag{8.62}$$

where $\alpha = \frac{a-\mu}{\sqrt{\sigma}}, \beta = \frac{b-\mu}{\sqrt{\sigma}}$, and that $\phi(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right)$, $\Phi(x) = \frac{1}{2}(1+erf(\frac{x}{\sqrt{2}}))$.

Therefore, we have

$$i_\tau = i_\tau^* - \sqrt{\sigma}\frac{\phi(-2\bar{z}/\sqrt{\sigma})-\phi(0)}{\Phi(0)-\Phi(-2\bar{z}/\sqrt{\sigma})}d\tau \tag{8.63}$$

at the upper barrier $\bar{z}$, implying the Central Bank raises the interest rate when its currency threatens to depreciate beyond the upper barrier. Similarly, we have

$$i_\tau = i_\tau^* + \sqrt{\sigma}\frac{\phi(-2\bar{z}/\sqrt{\sigma})-\phi(0)}{\Phi(0)-\Phi(-2\bar{z}/\sqrt{\sigma})}d\tau \tag{8.64}$$

at the lower barrier.

8.7. **Implied share of regulation cost.** How quantitatively important is the regulation cost relative to the welfare cost? To answer this question, let

$$\alpha(z) = E_z[\int_0^\infty e^{-\rho t} dL] \tag{8.65}$$

$$\beta(z) = E_z[\int_0^\infty e^{-\rho t} dU] \tag{8.66}$$

be the long run average regulation costs at the lower and upper barriers, respectively. Also, let

$$f(z) = \alpha z^2 \tag{8.67}$$

denote the flow welfare cost. Therefore, assuming that $z_0 = 0$ as we assume that the process z starts from the central parity, the discounted welfare cost can be expressed by

$$E_z\left[\int_0^\infty e^{-\rho s}\left(\rho f(z) - \frac{1}{2}\sigma^2 f"(z)\right)ds\right] = f'(\underline{z})E_z\left[\int_0^\infty e^{-\rho s}dL\right] - f'(\bar{z})E_z\left[\int_0^\infty e^{-\rho s}dU\right] \tag{8.68}$$

Using a well known theorem on local times (Stokey (2008)), one can change the integration w.r.t time to an integration w.r.t occupancy measure on the left hand side, i.e:

$$E_z\left[\int_0^\infty e^{-\rho s}\left(\rho f(z) - \frac{1}{2}\sigma^2 f"(z)\right)ds\right] = \int_{\underline{z}}^{\bar{z}}\left(\rho f(z) - \frac{\sigma^2}{2}f"(z)\right)\pi(z;0)dz \tag{8.69}$$

where $\pi(z;0)$ is the expected discounted local time associated with the occupancy measure $\Pi(A;z,\underline{z},\bar{z};\rho) = E_z\left[\int_0^\infty e^{-\rho s}1_A(z(s))ds\right], A \in \mathcal{B}_{[\underline{z},\bar{z}]}$. Combining the above two equations, and plug in the definition of $\alpha(z), \beta(z)$, one get

$$\int_{\underline{z}}^{\bar{z}}\left(\rho f(z) - \frac{\sigma^2}{2}f"(z)\right)\pi(z;0)dz = f'(z)\alpha(z) - f'(\bar{z})\beta(z) \tag{8.70}$$

Plugging in the cost function, and integrating out, we have

$$2\alpha\underline{z}\alpha(z) - 2\alpha\bar{z}\beta(z) = \int_{\underline{z}}^{\bar{z}}(\alpha\rho z^2 - \alpha\sigma^2)\pi(z;0)dz \tag{8.71}$$

Due to symmetry, we know that $\alpha(z) = \beta(z)$. If we use the change of measure again $\int_{\underline{z}}^{\bar{z}} z^2\pi(z;0)dz = \int_0^t e^{-\rho t}z^2dt$, this reduces to

$$4\bar{z}\alpha(z) = \sigma^2 - \rho\int_0^t e^{-\rho t}z^2dt \tag{8.72}$$

with the right integral we know the value again where

$$\int_0^t e^{-\rho t}z^2dt = f'(\underline{z})\alpha(z) - f'(\bar{z})\beta(z) = -4\alpha\bar{z}\alpha(z) \tag{8.73}$$

Therefore, we get

$$\alpha(z) = \frac{\sigma^2}{4(1-\alpha\rho)\bar{z}} \tag{8.74}$$

Finally, one can compare the regulation cost to the welfare cost by defining the ratio of long run expected discounted value of regulation cost on the two sides to the unregulated welfare cost counterpart, i.e:

$$\begin{aligned} share &= \frac{m\left(E_z[\int_0^\infty e^{-\rho t}dL] + E_z[\int_0^\infty e^{-\rho t}dU]\right)}{E_e[\int_0^\infty e^{-\rho t}e^2dt]} \\ &= \frac{m\frac{\sigma^2}{4(1-\alpha\rho)\bar{z}}}{\frac{\alpha\sigma^2}{\rho}} = \frac{m\rho}{4(1-\alpha\rho)\alpha(\frac{3m\sigma^2}{4\alpha})^{1/3}} \end{aligned} \tag{8.75}$$

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