Information Acquisition and Liquidity Fluctuations

Preliminary and Incomplete. Do Not Circulate.

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Abstract

This paper studies how sellers’ incentives to acquire private information about asset returns can lead to self-fulfilling liquidity shortages in financial markets. Information acquisition generates an adverse selection problem by increasing the degree of asymmetric information among traders. Market prices decrease as the fraction of informed sellers increases, since buyers become unable to determine whether sellers are trading because of liquidity needs or because they are trying to pass on lemons. We identify a new feedback from market prices to information acquisition incentives that can lead to multiple equilibria if sufficiently many traders face sudden liquidity needs. These include inefficient belief-driven liquidity dry-ups characterized by low market prices and low trading volumes.

Keywords: Information Acquisition, Market Liquidity, Financial Crises

JEL Classifications: D82, G01, G12

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1 Introduction

A distinctive feature of the 2007-09 financial crisis was the pronounced shortage of liquidity in financial markets (Brunnermeier 2009). Interbank and other funding markets that had been supplying liquidity to financial institutions came under extreme strain. As a result, many financial institutions turned to central banks to meet their liquidity shortfall, and others had to be directly recapitalized by government treasuries in order to avoid bankruptcy.

What explains the sudden dry-up of liquidity in financial markets? Gorton (2010) points the finger at recent changes in the funding structure of banks and bank-like financial institutions, which led retail deposits to be progressively replaced by money market funds, short-term repurchase agreements, and other forms of secured and unsecured lending as a main source of funding. This made financial institutions increasingly reliant on wholesale funding markets to manage their liquidity needs. Information frictions tend to be very prevalent in these markets, and such frictions are well-known to be a key impediment to trade (Akerlof 1970). Thus, the securities that are traded on these markets must be sufficiently informationally-insensitive in order for them to function properly (Gorton and Pennacchi 1990). According to Gorton, the liquidity crisis that started in the summer of 2007 should be thought of as a run on the “securitized-banking” system, caused by widespread concern about the value of the securities being traded in secondary funding markets.

This compelling view of the financial crisis of 2007-09 leaves a number of questions unanswered. In particular: What explains the sudden change in the “information sensitivity” of assets traded in secondary markets? More specifically: What determines agents’ incentives to acquire private information about the assets on their balance sheet? And how
do fluctuations in market liquidity affect these incentives? This paper proposes a theoretical framework that provides answers to these questions. It identifies a new feedback between market liquidity and information acquisition that helps rationalize the self-fulfilling nature of sudden liquidity shortages.

We consider a simple three-period exchange economy in which agents trade assets in order to meet unexpected liquidity needs. Agents enter the economy with a large storable endowment and a long-term asset that matures in the final period. Assets differ in terms of their payoff at maturity: some yield a high return (good), while others yield a low return (bad). Agents are initially uninformed about the quality of their assets, but have the option to acquire information and become informed at a cost. Incurring this cost allows agents to privately observe the future payoff of their assets. In the interim period, some agents are subject to an idiosyncratic liquidity shock that reduces their utility from consumption in the final period. This leads to gains from trading assets before they mature. To realize these gains from trade, agents can access a secondary market in the interim period that allows them to trade assets among themselves.

By acquiring information about their assets, agents necessarily introduce a degree of asymmetric information into the economy. As in standard models, the presence of asymmetric information can prevent mutually-beneficial trades from taking place due to a standard adverse selection problem: i.e. buyers of assets may be unable to determine whether sellers are trading because of liquidity needs or because they are trying to offload bad assets. Information acquisition exacerbates this adverse selection problem, which leads the secondary market price to decrease as more agents become informed. Hence, contrary to classic “lemons” models à la Akerlof, the degree of asymmetric information is endogenous as it is determined by agents’ information acquisition decisions. As argued below, agents’ information acquisition incentives themselves critically depend on the price
at which assets trade in the secondary market. This leads to a novel feedback from market prices to information acquisition which, to the best of our knowledge, has not yet been studied in the existing literature.

Agents’ incentives to acquire information depend on the profits they make by selling assets in the secondary market. The value from acquiring information in this environment stems from two actions that informed agents can take, but that uninformed agents cannot. The first is to offload assets that are found to be bad. The second is to hold assets that are found to be good rather than selling them at a relatively low (pooling) price. Importantly, we show that the former is decreasing in the secondary market price, while the latter increases as the price falls. Given this, agents’ incentives to acquire information can be either increasing or decreasing in the secondary market price, depending on two key parameters of the model: (i) the probability of a liquidity shock, and (ii) the fraction of bad assets in the economy.

Strategic complementarities in information acquisition arise if the probability of a liquidity shock is sufficiently high (see Figure ??). In this case, increases in the fraction of informed agents lowers secondary market prices due to distortions caused by asymmetric information, which in turn increases agents’ incentives to become informed in order to avoid selling good assets at a low price. This feedback effect can lead to multiple Pareto-ranked equilibria. Equilibria without information acquisition are characterized by high secondary market prices and high trading volumes (measured in terms of the fraction of impatient agents that trade in the market). Equilibria with information acquisition are instead characterized by low secondary market prices and low trading volumes where not all gains from trade are realized. In this sense, our model rationalizes the possibility of self-fulfilling liquidity dry-ups sparked by agents’ sudden decision to acquire information about the quality of their assets.
More generally, our model identifies a new channel by which to understand the fragility of wholesale funding markets. As mentioned above, a key feature of the 2007-09 financial crisis was the pronounced fall in prices and trading volumes in these markets. This affected a wide-class of assets; i.e. the liquidity dry-up was not limited to mortgage-backed securities whose value was directly affected by the bursting of the US housing bubble. Several other theories have been cited to explain why other asset classes far less exposed to the US housing market also experienced a dramatic deterioration in liquidity conditions. These include network contagion, balance-sheet effects, and counterparty risk. This paper studies a different mechanism, based on financial institutions’ strategic incentives to acquire private information about asset returns, that can explain the fragility of wholesale funding markets even in the absence of these other, complementary channels.

Related Literature

Our paper builds on two distinct, but related strands of the literature. The first studies how information frictions affect the fragility of secondary asset markets and can lead to self-fulfilling liquidity freezes. Malherbe (2014) shows how liquidity hoarding imposes a negative pecuniary externality on secondary asset traders by exacerbating adverse selection frictions, and can lead to self-fulfilling dry-ups in secondary market liquidity. The channel through which this occurs is noticeably different than in our model, however, as it depends on the return agents enjoy from holding cash. Kuong (2015) develops a model of self-fulfilling fire sales in collateralized short-term debt markets. His paper is based on contracting frictions caused by borrower moral hazard, and focuses on the feedback between initial margins and the market value of collateral. More closely related to our paper, Feijer (2015) offers a theory of liquidity crises based on self-fulfilling fears of asymmetric

\footnote{See Allen and Gale (2000), Brunnermeier and Pedersen (2009) and Heider et. al. (2015).}
information. Similarly to Kuong (2015), equilibrium multiplicity in his framework results from contracting frictions caused by a risk-shifting problem. The feedback from market prices to information acquisition incentives is therefore distinct from the one emphasized here as it operates through borrowing costs rather than secondary market prices.

Our paper also draws from a growing body of literature studying agents’ incentives to acquire private information about asset quality and how this affects market prices and credit supply. Bolton et. al. (2015) analyze investors’ incentives to acquire information about assets that trade in over-the-counter (OTC) markets. They show that information acquisition generates a negative externality: investors that acquire information “cream-skim” good assets from the market, thereby worsening the residual pool of assets that are offered to uninformed investors. Relatedly, Fishman and Parker (2015) study how investors’ decision to perform valuation – i.e. expend resources to acquire information about project payoffs – affects market prices. As in Bolton et. al. (2015), valuation lowers market prices due to adverse selection, which increases investors’ return from acquiring information and can lead to multiple equilibria. The mechanism responsible for this multiplicity is conceptually very different from the one studied in our paper. In particular, they focus on how information acquisition affects agents’ profits from buying assets, while we study how it affects agents’ profits from selling assets. Hence, we consider their models to be better suited to study how information acquisition interacts with primary market liquidity rather than liquidity conditions in secondary markets.

2Other notable papers that model endogenous liquidity fluctuations caused by asymmetric information include Eisfeldt (2004), Kurlat (2013) and Parlour and Plantin (2008).

3See also Glode et. al. (2012) and Biais et. al. (2015). Another related strand of literature focuses on how information acquisition affects security design, including Dang et. al. (2013) and Vanasco (2013).
2 Model

2.1 Description of the Economy

We consider a three period exchange economy, with time indexed by $t \in \{0, 1, 2\}$. The economy is populated by a continuum of risk-neutral agents. Each agent is endowed with a large storable endowment (e.g. cash) and a risky asset that returns $\tilde{R} \in \{R_h, R_l\}$ in $t = 2$, where $R_h > R_l \geq 0$ and $\pi \equiv \Pr(\tilde{R} = R_h)$. The ex ante expected return of the asset is thus $E_0[\tilde{R}] = \pi R_h + (1 - \pi)R_l$.

Preferences. Agents are potentially subject to an idiosyncratic liquidity shock at the beginning of $t = 1$, which lowers their valuation for $t = 2$ consumption. Formally, agents’ preferences are given by

$$U(c_1, c_2) = c_1 + \tilde{\beta}c_2$$

where $\tilde{\beta} \in \{1, \beta\}$, with $\beta < 1$ and $\lambda \equiv \Pr(\tilde{\beta} = \beta)$. Liquidity shocks realize at the beginning of $t = 1$ and are assumed to be unverifiable. Agents subject to a liquidity shock are called “impatient” while agents not facing a liquidity shock are referred to as “patient.”

This preference structure implies that there are gains from trading assets in $t = 1$ since impatient agents value consumption in $t = 1$ strictly more than consumption in $t = 2$. In particular, the first-best trading mechanism requires that patient agents purchase assets in $t = 1$ from impatient agents and hold them to maturity, thereby allowing impatient agents to consume in $t = 1$.

Market Structure. Assets can be traded on a competitive market that opens in $t = 1$. We assume that patient agents who purchase assets in $t = 1$ have sufficient funds so that assets trade at the “fundamental price” $p = E_1[\tilde{R}]$, where $E_1[\cdot]$ denotes buyers’
expectations about asset returns based on the available information at the beginning of \( t = 1 \). In particular, the price \( p \) will depend on the share of good assets \( \tau \in [0, 1] \) that are supplied to the market.

**Information Structure.** In \( t = 0 \) agents have the option to acquire information at a fixed cost \( \psi \geq 0 \). For simplicity, we assume that agents who incur this cost perfectly observe the future return of their asset. Let \( \sigma_j \in [0, 1] \) denote the probability with which agent \( j \) chooses to acquire information and denote by \( \sigma \in [0, 1] \) the fraction of agents choosing to acquire information in \( t = 0 \).

Agents’ valuation of the asset depends on the price at which assets trade in the secondary market, and also varies depending on whether or not they choose to acquire information. We denote by \( \Omega_j \in \{n, h, l\} \) an agent’s information set conditional on not acquiring information (\( n \)), or acquiring information and verifying the asset to be good (\( h \)) or bad (\( l \)). Let \( E[\tilde{R}\mid\Omega_j] \in \{E_0[\tilde{R}], R_h, R_l\} \) denote agents’ beliefs about their asset’s payoff at maturity, depending on their information at \( t = 0 \). A typical agent’s value function conditional on his information set can then be written as

\[
V(\Omega_j; p) = (1 - \lambda) \max\{p, E[\tilde{R}\mid\Omega_j]\} + \lambda \max\{p, \beta E[\tilde{R}\mid\Omega_j]\}
\]

We impose the following structure on the ordering of agents’ valuations:

**Assumption 1.** Asset returns are such that

\[
\rho \equiv \frac{R_l}{R_h} \in \left( \frac{\beta \pi}{1 - \beta(1 - \pi)}, \frac{\beta - \pi}{1 - \pi} \right) \quad \Leftrightarrow \quad \beta R_h > E_0[\tilde{R}] \quad \text{and} \quad \beta E_0[\tilde{R}] < R_l
\]

The upper bound on \( \rho \) implies that impatient agents that have verified their asset to be good value it more than patient uninformed agents: i.e. \( \beta R_h > E_0[\tilde{R}] \). The lower
Agents decide about probability $\sigma_j$ of acquiring information at cost $\psi$.

Agents who acquire information observe whether return is $R_h$ or $R_l$.

Liquidity shocks occur.

Secondary market opens and assets trade at price $p$.

Asset returns are realized.

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2$</th>
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Figure 1: Sequence of Events.

bound on $\rho$ implies that uninformed impatient agents value their asset less than patient agents that have verified their asset to be bad: i.e. $\beta E_0(\tilde{R}) < R_l$. These admittedly strong restrictions on agents’ valuations are made in order to simplify the exposition of the model. As we discuss in Section 3.4 below, the qualitative nature of the results remain unchanged if we relax either of these two bounds.

2.2 Equilibrium

Equilibrium Definition. Agents’ information acquisition decision in $t = 0$ induces a distribution of informed and uninformed agents in the economy. Given the aggregate value of $\sigma$, patient agents update their beliefs about the share of good assets supplied to the secondary market. In equilibrium, agents’ information choices in $t = 0$ and the resulting equilibrium market price in $t = 1$ have to be mutually consistent. Formally, the equilibrium is defined as follows.

Definition 1. An equilibrium is described by an aggregate information acquisition choice $\sigma^* \in [0, 1]$ and a price $p^* \in \mathbb{R}_{++}$ such that

\[ \beta E_0(\tilde{R}) < R_l. \]
1. Individual information acquisition choices, \( \sigma_j \in [0, 1] \), maximize individual agent’s expected utility, given the secondary market price \( p(\sigma) \).

2. Agents update their beliefs about the share of good assets supplied to the secondary market using Bayes’ Rule.

3. The secondary market price \( p(\sigma) \) is such that the buyers of assets break even in expectation, i.e. \( p(\sigma) \leq E_1[\tilde{R}] \).

4. In any equilibrium, \( \sigma^* = \int \sigma_j^*(\sigma^*) dj \).

**Secondary Market Price.** We begin the characterization of the equilibrium by deriving the secondary market price, given agents’ aggregate information choice. In equilibrium, patient agents that purchase assets in the secondary market must break-even. Hence, the participation constraint of buyers in \( t = 1 \) is given by

\[
p \leq E_1[\tilde{R}] \equiv \tau R_h + (1 - \tau) R_l
\]  

(2)

Competition among buyers ensures that condition (2) holds with equality in equilibrium. Consequently, patient agents who have verified their asset to be good never sell it in \( t = 1 \) since \( p < R_h \). Agents who have verified their asset to be bad, however, always want to sell since \( p \geq R_l \). Hence, whenever a positive fraction of agents acquire information, the share of good assets traded in the secondary market will be strictly less than the share of good assets in the economy: i.e. \( \tau < \pi \). This implies that patient agents who have not acquired information also never sell their asset in \( t = 1 \), since \( p < E_0[\tilde{R}] \).

Condition (2) clearly shows that the price at which assets trade on the secondary market depends on the share of good assets being traded. The unobservability of agents’
information acquisition creates asymmetric information about asset returns so that this share itself depends on the market price. Given Assumption 1, the share of good assets that are offered for sale in \( t = 1 \) is given by

\[
\tau(\sigma) = \frac{\lambda(1-\sigma)\pi}{\lambda(1-\sigma) + (1-\pi)\sigma}
\]

(3)

This follows from the fact that good assets are only supplied by uninformed impatient agents, while informed agents holding bad assets want to sell these regardless of whether they are patient or impatient. Notice that \( \tau'(\sigma) < 0 \), meaning that the share of good assets traded in the secondary market is strictly decreasing in the fraction of agents acquiring information in \( t = 0 \). The secondary market price can then be written as

\[
p^*(\sigma) = E_1(\tilde{R}) = E_0[\tilde{R}] - (\pi - \tau(\sigma))(R_h - R_l)
\]

(4)

Lemma 1. Under Assumption 1, the secondary market price is strictly decreasing in the fraction of informed agents: i.e. \( p'(\sigma) < 0 \).

Information acquisition leads assets supplied to the secondary market to trade at a discount. More specifically, information acquisition introduces a degree of asymmetric information into the economy, and the resulting adverse selection problem tends to depress the price at which assets trade. This is because agents verifying assets to be bad always want to sell in \( t = 1 \), while impatient agents verifying assets to be good opt to hold them since the market price is always below their reservation value.

Information Acquisition. Given the price in the secondary market, we now characterize agents’ information acquisition choice. Let \( U(\sigma_j; \sigma) \) denote an agent’s expected utility in terms of his information acquisition choice \( \sigma_j \) and the aggregate information choice \( \sigma \).
If an agent does not acquire information, his expected utility is given by

$$U(0; \sigma) = (1 - \lambda)E_0[\tilde{R}] + \lambda \max\{p(\sigma), \beta E_0[\tilde{R}]\}$$

while if he acquires information with probability one, it becomes

$$U(1; \sigma) = \pi ((1 - \lambda)R_h + \lambda \max\{p(\sigma), \beta R_h\}) + (1 - \pi)p(\sigma) - \psi$$

Notice that Assumption $\Box$ implies that $p(\sigma) < \beta R_h$ and $p(\sigma) > \beta E_0[\tilde{R}]$, so that informed agents holding good assets never trade while impatient uninformed agents always trade in the secondary market. In what follows, it will be useful to express agent $j$'s best response, $\sigma_j^*(\sigma)$, in terms of the net expected surplus from acquiring information $S(\sigma) \equiv U(1; \sigma) - U(0; \sigma) + \psi$. Profit maximization implies that

$$\sigma_j^*(\sigma) = \begin{cases} 
0 & \text{if } S(\sigma) < \psi \\
\in (0, 1) & \text{if } S(\sigma) = \psi \\
1 & \text{if } S(\sigma) > \psi 
\end{cases} \quad (5)$$

Given the payoff functions derived above, the expected surplus is given by

$$S(\sigma) = \lambda \pi (\beta R_h - p(\sigma)) + (1 - \pi)(1 - \lambda)(p(\sigma) - R_l) \quad (6)$$

Equation (6) shows that the surplus from acquiring information consists of two parts: (i) the option value of an impatient agent from holding good assets rather than selling them, given by $\lambda \pi (\beta R_h - p)$; and (ii) the information rent of a patient agent from offloading bad assets, given by $(1 - \pi)(1 - \lambda)(p - R_l)$. Note that the market price affects the two
parts of the surplus in opposite ways. An increase in the price lowers impatient agents’ value of holding a good asset to maturity, while it raises the rent from offloading a bad asset. If \( \lambda > (1 - \pi) \), the surplus from information acquisition is decreasing in \( p \) as it allows agents to hedge against the risk of selling good assets at a price below their true value. Conversely, if \( \lambda < (1 - \pi) \), the surplus is increasing in \( p \) as it becomes less attractive to acquire information in order to offload bad assets when the price falls.

\textbf{Assumption 2.} \textit{The probability of a liquidity shock is such that }\( \lambda > 1 - \pi \).

Recall from Lemma 1 that the market price falls when the share of informed agents increases. Hence, if \( \lambda > 1 - \pi \), agents’ decisions to acquire information are \textit{strategic complements}: more information acquisition in the aggregate lowers the market price and raises agents’ incentives to acquire information. Conversely, if \( \lambda < 1 - \pi \), agents’ information acquisition decisions are \textit{strategic substitutes}: more aggregate information acquisition makes agents less inclined to become informed since the information rent from offloading bad assets falls. Our benchmark model focuses on the case of strategic complementarities. We discuss the strategic substitutes case in Section 3.1 below.

Under Assumption 2, the net expected surplus \( S(\sigma) - \psi \) is strictly increasing in \( \sigma \). Increases in the cost parameter \( \psi \) shift the net surplus curve down, implying that information acquisition becomes unattractive for a larger range of values of \( \sigma \) (see Figure 2). To understand the intuition behind agents’ information acquisition decision, consider the interaction between changes in the secondary market price \( p(\sigma) \) and the cost parameter \( \psi \). For example, agents find it individually optimal to abstain from information acquisition if assets trade at their \textit{ex ante} fundamental value and information costs are sufficiently high. However, even at these high costs, they may begin to acquire information if the price falls sufficiently below the \textit{ex ante} fundamental value as this raises the option value from
holding good assets by more than it decreases the information rents obtained by selling bad assets at the pooling price.

Equilibrium. Having characterized the secondary market price for a given aggregate information choice \( \sigma \) and agents’ incentives to acquire information, we can now characterize the equilibria of the model in terms of the cost parameter, \( \psi \).

**Lemma 2.** Under Assumption 1, there exists at least one equilibrium \((\sigma^*, p^*(\sigma^*)) \in [0, 1] \times \mathbb{R}_{++}^+ \) for all \( \psi \geq 0 \).

We focus on symmetric equilibria whereby all agents adopt the same verification strategy in \( t = 0 \). Solving for the equilibrium therefore reduces to solving the fixed point problem \( \sigma_j^* (\sigma) = \sigma \).

**Proposition 1.** Under Assumptions 1 and 2, there exist threshold costs \( \underline{\psi} \) and \( \overline{\psi} \) such that \( \underline{\psi} < \overline{\psi} \). The equilibria are characterized by:

1. No information acquisition, \( \sigma^* = 0 \), where assets trade at the ex ante fundamental price \( p^*(0) = E_0(\bar{R}) \) if and only if \( \psi \geq \underline{\psi} \);
2. Partial information acquisition $\sigma^* \in (0, 1)$, such that

$$p^*(\sigma^*) = \frac{\lambda \pi \beta R_h - (1 - \lambda)(1 - \pi)R_l - \psi}{\lambda - (1 - \pi)} < E_0(\tilde{R})$$ (7)

if and only if $\psi \in (\underline{\psi}, \overline{\psi})$;

3. Full information acquisition, $\sigma^* = 1$, where the asset price collapses to $p^*(1) = R_l$ if and only if $\psi \leq \underline{\psi}$.

There exist multiple equilibria for values of $\psi \in (\underline{\psi}, \overline{\psi})$.

Figure 3 shows the equilibrium price correspondence in terms of $\psi$. For $\psi > \overline{\psi}$, information costs are so high that agents will never acquire information, implying that assets trade at the ex ante fundamental price and all impatient agents trade. For $\psi < \underline{\psi}$, information costs are so low that agents always want to acquire information, leading the price to fall to $R_l$. For $\psi \in (\underline{\psi}, \overline{\psi})$, there are multiple equilibria that arise due to the existence of strategic complementarities in information acquisition. In particular, if agents believe that others acquire information, they expect the price to fall below $E_0(\tilde{R})$. As long as the costs are sufficiently small, i.e. below $\overline{\psi}$, agents find it individually optimal to acquire information, which lowers the price and thus vindicates their initial belief. In what follows, we refer to this type of phenomena as a self-fulfilling liquidity dry-up.

3 Discussion

This section provides a brief overview of some key properties of the equilibria derived above, as well as a discussion of the key conditions needed in order for belief-driven liquidity dry-ups to occur.
3.1 Liquidity Shocks and Strategic Complementarities

A key assumption of the model analyzed above is that the probability of a liquidity shock is sufficiently high (Assumption 2). This guaranteed the existence of strategic complementarities, since agents’ surplus from acquiring information in this case increases as the secondary market price falls. Importantly, the existence of self-fulfilling liquidity dry-ups fundamentally depends on this feedback between secondary market prices and information acquisition incentives.

When the probability of liquidity shocks is relatively low, $\lambda < 1 - \pi$, agents’ incentives to acquire information are primarily determined by the rents obtained from offloading bad assets at the pooling price rather than the option value of keeping good assets. This implies that the surplus function $S(\sigma)$ is strictly decreasing in $\sigma$, and that agents’ information acquisition incentives become strategic substitutes. More specifically, increasing the fraction of informed agents lowers secondary market prices due to adverse selection,
but this fall in the price leads to a reduction in agents’ incentives to acquire information since it reduces the information rents obtained from selling bad assets.

**Proposition 2.** *Liquidity dry-ups sustained by self-fulfilling beliefs about information acquisition cannot arise if* $\lambda < 1 - \pi$.

Our model therefore indicates that specific conditions must be met in order for self-fulfilling liquidity dry-ups to be possible. Our preferred interpretation is thinking of the probability of a liquidity shock ($\lambda$) as a proxy for the stability of financial institutions’ liabilities (e.g., leverage, debt maturity structure). Indeed, a distinctive feature of the 2007-09 financial crisis was financial institutions’ dependency on short-term debt instruments to finance their operations. Proposition suggests that the collapse in the “informational insensitivity” of wholesale funding markets cannot be explained without taking into account the fragility of financial institutions’ funding sources. In this sense, our model complements an important body of literature showing that liquidity risk in financial markets tends to be inefficiently high due to financial firms’ excessive reliance on short-term debt financing. More specifically, we find that this widely-cited fragility stemming from the liability-side of financial institutions’ balance sheets can be amplified by agents’ sudden decision to acquire information about the quality of their assets.

### 3.2 Welfare

Welfare in this economy depends on whether or not all gains from trade are realized. Agents’ preferences and the return structure of assets implies that there are gains from

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4 The absence of strategic complementarities leads to the equilibrium being *unique* in this case.

5 This finding mirrors closely the result of Gorton and Pennacchi (1990), who show that “informationally insensitive” securities can be issued only if not too many liquidity traders are in the market.

6 See Stein (2012) and Brunnermeier and Oehmke (2013), among others.
trading both good and bad assets. Hence, welfare is unambiguously reduced whenever impatient agents withhold their assets from the secondary market.

**Definition 2.** Given some value \( \sigma \in [0, 1] \), aggregate utility from consumption is given by

\[
W(\sigma) = \sigma U(1; \sigma) + (1 - \sigma) U(0; \sigma)
\]

There exists a strict Pareto ranking of the equilibria characterized above. Without information acquisition, impatient agents always trade, and aggregate utility from consumption is equal to the fundamental value of the asset, \( E_0[\tilde{R}] \). With positive information acquisition, aggregate utility from consumption (net of verification costs) is equal to

\[
W(\sigma^*) + \sigma^* \psi = E_0[\tilde{R}] - \sigma^* \lambda (1 - \beta) \pi R_h < E_0[\tilde{R}]
\]

Even without taking into account the cost of acquiring information, aggregate welfare when \( \sigma^* > 0 \) is always strictly less than when \( \sigma^* = 0 \). Indeed, it is immediate to see that aggregate utility from consumption is strictly decreasing in the fraction of informed agents. This is because information acquisition leads impatient agents with good assets to drop out of the secondary market. Since they value the asset at maturity strictly less than patient agents, this leads to an unambiguous reduction in welfare.

**Proposition 3.** For \( \psi \in [\underline{\psi}, \overline{\psi}] \), the equilibrium without information acquisition (\( \sigma^* = 0 \)) Pareto dominates the equilibria with information acquisition (\( \sigma^* > 0 \)).

As shown by equation (8), the magnitude of the welfare loss caused by information acquisition critically depends on agents’ discount factor, \( \beta \). This parameter effectively measures the gains from trading assets in the secondary market: high values of \( \beta \) indicate low gains from trade, while low values of \( \beta \) indicate high gains from trade. How does
welfare vary with $\beta$? Define the welfare loss relative to the first best as follows:

$$L(\sigma^*, \beta) = W(0) - W(\sigma^*) = \sigma^*(\lambda(1 - \beta)\pi R_h + \psi)$$

**Corollary 1.** For $\psi \in [\underline{\psi}, \bar{\psi}]$ and $\sigma^* > 0$, the degree of information acquisition and associated welfare loss are such that

$$\frac{d\sigma^*}{d\beta} \leq 0 \quad \text{and} \quad \frac{dL(\sigma^*, \beta)}{d\beta} < 0$$

In the equilibrium with full information acquisition ($\sigma^* = 1$), the reduction in welfare follows directly from the fact that the utility loss implied by unrealized gains from trade is decreasing in $\beta$. Importantly, the degree of information acquisition in the mixed strategy equilibrium, $\sigma^* \in (0, 1)$, also depends on the value of $\beta$. As shown by Corollary 1, the welfare loss is amplified when we take into account the endogenous response of $\sigma^*$ to changes in agents’ discount factor. A fall in $\beta$ lowers the rents informed agents’ enjoy from holding good assets rather than selling them at the pooling price. Hence, the secondary market price must fall in order for agents to remain indifferent in equilibrium, thereby leading the fraction of informed agents to increase. As shown below, this “counter-intuitive” comparative static reflects the instability of the mixed strategy equilibrium.

### 3.3 Stability

A distinctive feature of the model analyzed above is that it exhibits multiple equilibria. In this sense, it rationalizes the possibility of self-fulfilling liquidity dry-ups sparked by agents’ information acquisition decisions. An interesting question that deserves answering is whether one should expect such “panic” episodes to be stable. In other words, to
what extent is it possible for the economy to remain trapped in an equilibrium with high information acquisition and low market liquidity? To answer this question, we rely on a standard myopic stability concept. In particular, we reinterpret the strategic game described above as a dynamic adjustment process where agents myopically adjust their strategies in directions that increase their expected utility. Here, we assume that agents myopically adjust their strategies according to the following simple revision protocol:

\[ \dot{\sigma} = F(\sigma) \equiv \kappa \sigma (1 - \sigma)(S(\sigma) - \psi) \]

where \( \kappa > 0 \) denotes some exogenous adjustment parameter. It is immediate to verify that this dynamic system has three stationary points that correspond to the three equilibria characterized above.

**Definition 3.** Given some revision protocol \( F(\sigma) \in \mathbb{R} \), an equilibrium \((\sigma^*, p^*(\sigma^*))\) is said to be stable if and only if

\[ F'(\sigma^*) < 0 \]

The revision protocol specifies in what direction agents are expected to adjust their strategies if \( \sigma \) is not an equilibrium. For example, \( F(\sigma) > 0 \) implies that uninformed agents can increase their expected utility by acquiring information, and that we should consequently expect the fraction of informed agents in the economy to increase. The stability concept builds on this simple intuition. In particular, an equilibrium is said to be stable if all paths in the neighborhood of a stationary point lead the system back to this stationary point.

**Proposition 4.** For \( \psi \in [\underline{\psi}, \bar{\psi}] \), the equilibria without information acquisition \((\sigma^* = 0)\)

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7See Sandholm (2010) for more details.
and full information acquisition ($\sigma^* = 1$) are stable, while the equilibrium with partial information acquisition ($\sigma^* \in (0, 1)$) is not.

This result suggests that liquidity scares caused by information acquisition need not always lead to persistent dry-ups in market liquidity. “Informationally insensitive” regimes in which agents operate under conditions of blissful ignorance are generally stable, vindicating market participants’ beliefs that secondary markets constitute a reliable source of liquidity. Small deviations from this equilibrium are unlikely to lead to long-lasting disruptions in liquidity supply, as local deviations from this regime are not profitable. However, the model also suggests that if sufficiently many agents simultaneously choose to acquire information, the economy can become trapped in an “informationally sensitive” regime characterized by low market prices and low trading volumes. Once a certain threshold level of information acquisition is reached, any remaining uninformed agents will find it profitable to acquire information, further depressing market prices and trading volumes.

### 3.4 Robustness to Ordering of Agents’ Valuations

The model analyzed above assumed a particular ordering of agents’ valuations, depending on their information set and their idiosyncratic discount factor (see Assumption 1). In particular, we considered an environment where impatient informed agents holding good assets never supply these to the market, while impatient uninformed agents always trade. That is, we assumed

$$\beta R_h < p(\sigma) \quad \text{and} \quad \beta E_0[R] > p(\sigma), \quad \forall \sigma \in [0, 1]$$

We argue here that self-fulfilling liquidity dry-ups can arise even if we relax this particular ordering of agents’ valuations.
Allowing All Informed Impatient Agents To Trade. We begin by relaxing the assumption that impatient good agents never enter the secondary market. In particular, we now assume \( \beta R_h < E_0[R] \), so that informed agents with good assets may want to trade if the secondary market price is sufficiently high. We show in Appendix B that in order for impatient good agents to be willing to trade, the fraction of informed agents cannot be too large: i.e. \( \sigma \leq \hat{\sigma} \). Agents’ surplus from acquiring information then becomes

\[
S(\sigma) = \begin{cases} 
(1 - \pi)(1 - \lambda)(p(\sigma) - R_i) & \text{if } \sigma \leq \hat{\sigma} \\
\lambda \pi (\beta R_h - p(\sigma)) + (1 - \pi)(1 - \lambda)(p(\sigma) - R_i) & \text{if } \sigma > \hat{\sigma}
\end{cases}
\]

Notice that if impatient agents who have verified their assets to be good trade in the secondary market, the value of acquiring information reduces to the information rent obtained from offloading bad assets at the pooling price. As this information rent is decreasing in the secondary market price, agent’s information acquisition incentives are no longer global but rather only partial strategic complements since the surplus function is decreasing for values of \( \sigma \leq \hat{\sigma} \) (see Figure 4). Notwithstanding the absence of global strategic complementarities, self-fulfilling liquidity dry-ups can still occur providing that \( \lambda > 1 - \pi \).

Corollary 2. Under Assumption 2, liquidity dry-ups sustained by self-fulfilling beliefs about information acquisition can arise even if \( \beta R_h < E_0[R] \).

In order for multiple equilibria to arise in this case requires that informed impatient agents holding good assets eventually drop out of the secondary market. Indeed, self-fulfilling liquidity dry-ups fundamentally depend on informed agents preferring to hold good assets even if they are hit by a liquidity shock. This channel distinguishes our model from other recent models of information acquisition in financial markets, including Gorton
and Ordonez (2014). They show how agents’ decision to acquire private information about asset values leads to a decline in output. Contrary to our model, liquidity fluctuations in their environment require there to be exogenous shocks to asset values. This is because agents’ information acquisition incentives consist only of the information rents obtained from reselling bad assets at a price that exceeds their fundamental value.\footnote{More precisely, the value from acquiring information in their framework stems from the ability to resell good assets at a price that strictly exceeds the pooling price at which they were purchased.} The absence of the “option value” motive stressed above thus implies that their framework cannot account for self-fulfilling liquidity freezes driven by agents’ fears about others’ information acquisition behavior.

**Allowing Uninformed Impatient Agents Not To Trade.** Similarly, we can relax the assumption that uninformed impatient agents always want trade in the secondary market regardless of the price, so that $\beta E_0[\tilde{R}] > R_t$. As above, we show in Appendix B
that we must have $\sigma \leq \overline{\sigma}$ in order for uninformed impatient agents to be willing to trade. The expected surplus from acquiring information is then

$$S(\sigma) = \begin{cases} 
\lambda \pi (\beta R_h - p(\sigma)) + (1 - \pi)(1 - \lambda) (p(\sigma) - R_l) & \text{if } \sigma \leq \overline{\sigma} \\
(1 - \pi)(1 - \beta)\lambda R_l & \text{if } \sigma > \overline{\sigma}
\end{cases} \quad (10)$$

If the share of informed agents increases to $\sigma > \overline{\sigma}$, uninformed agents prefer to hold their assets and the market price collapses to $R_l$. Although both the option value from holding on to good assets and the information rent from offloading bad assets are zero at this price, the expected surplus from acquiring information may still be positive. This is because information acquisition allows impatient agents holding bad assets to sell these and realize the associated gains from trade. Consequently, agents may still acquire information even if only bad assets are traded as long as information costs are sufficiently small\footnote{As long as $\sigma \leq \overline{\sigma}$, these gains from trade are enjoyed by both informed and uninformed agents, so that they do not effect agents’ information acquisition decision.}

As in the previous case, it can be shown that agents’ information acquisition incentives no longer exhibit global strategic complementarities since the gains from trading only bad assets are always strictly less than the information rents that otherwise accrue to informed agents. This leads the surplus function $S(\sigma)$ to jump down discontinuously at $\overline{\sigma}$ when uninformed impatient agents exit the market (see Figure 5). However, self-fulfilling liquidity dry-ups are again feasible providing that the probability of being hit by a liquidity shock is high enough.

\textbf{Corollary 3.} Under Assumption 2, liquidity dry-ups sustained by self-fulfilling beliefs about information acquisition can arise even if $\beta E_{0}[\overline{R}] > R_l$.

Interestingly, the absence of global strategic complementarities in this case implies that
Figure 5: Surplus function if $\beta \mathbb{E}_0[\tilde{R}] > R_l$.

an equilibrium is no longer guaranteed to exist. In particular, an equilibrium may fail to exist for low values of $\psi$ if the gains from trading only bad assets is sufficiently low. Barring this technical detail, however, the model still admits multiple equilibria driven by agents’ sudden decision to acquire information about asset returns.

4 Conclusion

This paper shows that belief-driven liquidity shortages can arise due to agents’ strategic incentives to acquire private information about the quality of their assets. Information acquisition engenders a classic adverse selection problem because the buyers of assets can no longer determine whether agents are selling because of liquidity needs or because they are trying to offload bad assets. Importantly, the degree of asymmetric information in the economy is endogenously determined through agents’ information acquisition decisions. We identify a new feedback between market prices and information acquisition that helps

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10 Formally, the existence result established in Lemma 2 no longer holds for all values of $\psi$.

11 See Appendix B for a more detailed explanation of this potential non-existence result, and a more precise characterization of the conditions under which it can arise.
rationalize self-fulfilling liquidity dry-ups. A necessary condition for these dry-ups to occur is that the probability of facing a liquidity shock be sufficiently high. In this case, a fall in secondary market prices increases agents’ incentives to acquire information in order to avoid selling good assets at a low price. We show that the model supports multiple Pareto-ranked equilibria, including inefficient liquidity dry-ups characterized by low market prices and low trading volumes.

As in most models where multiple equilibria arise due to agents coordinating on a particular set of beliefs, an outside agent can always force agents to coordinate on the Pareto optimal outcome. For example, in the context of our model, inefficient liquidity dry-ups can be avoided if a central bank stands ready to purchase assets at their ex ante fundamental value.\textsuperscript{12} Because of this, the model is ill-suited to study optimal policy design (e.g. asset purchase programs) given the absence of an underlying policy trade-off. A prevalent concern about anticipated public liquidity support is that it may lead to excessive risk-taking by financial institutions due to standard moral hazard channels.\textsuperscript{13} Addressing these concerns in the context of our model would require endogenizing the risk-profile of agents’ assets. Studying these issues in greater detail is fruitful ground for future research.

\textsuperscript{12}NB: Such “market maker of last resort” operations can only work if $\psi \geq \bar{\psi}$, since otherwise the equilibrium with full information acquisition is unique.

\textsuperscript{13}See Farhi and Tirole (2012) and Philippon and Schnabl (2013).
References


Parlour, Christine and Guillaume Plantin, “Loan sales and relationship banking,” 


Appendix

Appendix A: Main Proofs

Proof of Lemma 2 Under Assumption 2, the surplus function $S(\sigma)$ is continuous for all values of $\sigma \in [0, 1]$. It follows that agents' best response correspondence, $\sigma_j^*(\sigma)$, is convex valued and has a closed graph, so that Kakutani’s fixed point theorem applies.

Proof of Proposition 1 Threshold costs are defined as

$$\psi \equiv S(0) \quad \text{and} \quad \tilde{\psi} \equiv S(1)$$

It follows immediately from Assumption 2 that $\psi < \tilde{\psi}$ since $S'(\sigma) > 0$ for all $\sigma$.

1. Suppose $\psi > \tilde{\psi}$. Then $S(0) - \psi < 0$ and $\sigma_j^*(0) = 0$, implying that there exists an equilibrium where $\sigma^* = 0$ and $p^*(\sigma^*) = E_0(\tilde{R})$. For the converse direction, suppose that $\sigma^* = 0$ and therefore $p^*(0) = E_0(\tilde{R})$. But for this to be an equilibrium, $\sigma_j^*(\sigma^*) = 0$ must be a best response implying that $\psi > S(0) \equiv \psi$.

2. Suppose $\psi \in [\tilde{\psi}, \tilde{\psi}]$. From the proof of Lemma 2 there exists a unique value $\sigma_\psi \in (0, \sigma)$ such that $S(\sigma_\psi) = \psi$ implying that $\sigma_j^* = \sigma_\psi > 0$, so that the equilibrium price is given by $p(\sigma_\psi)$. Since $p'(\sigma) \leq 0$ from Lemma 1 it must be that $p(\sigma_\psi) \in (R_l, E_0(\tilde{R}))$. Conversely, suppose there exists $\sigma_\psi \in (0, 1)$ such that $S(\sigma_\psi) = \psi$. Since $\sigma_\psi < \sigma$, equation (4) implies $p^*(\sigma_\psi) = R_l + \tau(\sigma_\psi)(R_h - R_l) \in (R_l, E_0(\tilde{R}))$.

3. Suppose $\psi < \tilde{\psi}$. Then, $S(\sigma) > \psi$ for all $\sigma$ implying that $\sigma_j^*(1) = 1$, and hence $\sigma^* = 1$. From equation (4) this implies $p^* = R_l$. For the converse direction, suppose that $p^* = R_l$. Equation (1) implies that $\tau(\sigma^*) = 0$, so that $\sigma^* = 1$. Hence, for $\sigma_j^* > \sigma$ to be a best reply it must be that $\psi < S(1) \equiv \tilde{\psi}$.

Given the ordering of the thresholds, $\psi < \tilde{\psi}$, it follows immediately that there are three equilibria if and only if $\psi \in [\tilde{\psi}, \psi]$. For values of $\psi \notin [\tilde{\psi}, \psi]$ there is a unique equilibrium.

Proof of Proposition 3 Given some value of $\sigma \in [0, 1]$, the aggregate utility from consumption (net of
verification costs) is defined as

\[ W(\sigma) + \sigma\psi \equiv \sigma\pi((1 - \lambda)R + \lambda\beta R) + \sigma(1 - \pi)p(\sigma) + (1 - \sigma)((1 - \lambda)E_0[\tilde{R}] + \lambda p(\sigma)) \]

Rearranging, we obtain

\[ W(\sigma^*) + \sigma^*\psi = \sigma^*(1 - \lambda(1 - \beta))\pi R_h + (1 - \sigma^*)(1 - \lambda)E_0[\tilde{R}] + (\lambda(1 - \sigma^*) + (1 - \pi)\sigma^*)p(\sigma^*) \]

Substituting for \( E_0[\tilde{R}] \) and \( p(\sigma^*) \) yields

\[ W(\sigma^*) + \sigma^*\psi = (1 - \sigma^*\lambda(1 - \beta))\pi R_h + (1 - \pi)R_l \]

Given the possible equilibrium values of \( \sigma^* \in [0, 1] \), we therefore have that

\[ W(0) = E_0[\tilde{R}] > W(\sigma^*) + \sigma^*\psi = E_0[\tilde{R}] - \sigma^*\lambda(1 - \beta), \quad \forall \sigma^* \in (0, 1] \]

It is immediate to verify that \( W'(\sigma^*) < 0 \).

\[ \square \]

Proof of Proposition 4. In order for there to be multiple equilibria, we must have \( \psi \in [\psi, \overline{\psi}] \). Differentiating the revision protocol \( F(\sigma) \) with respect to \( \sigma \), we obtain

\[ F'(\sigma) = \kappa(1 - \sigma)(S'(\sigma)) + \kappa(1 - 2\sigma)(S(\sigma) - \psi) \]

Evaluating this expression at the three stationary points, \( \sigma^* = 0, \sigma^* = 1 \) and \( S(\sigma^*) = \psi \), yields

\[ F'(0) = \kappa(S(0) - \psi) < 0, \quad \forall \psi \in [\psi, \overline{\psi}] \]
\[ F'(1) = -\kappa(S(1) - \psi) < 0, \quad \forall \psi \in [\psi, \overline{\psi}] \]
\[ F'(S^{-1}(\psi)) = \kappa\sigma^*(1 - \sigma^*)S'(S^{-1}(\psi)) > 0 \]

where the last inequality follows from Assumption 2 and Lemma 2. Given Definition 3, it follows that the two pure strategy equilibria are stable, while the mixed strategy equilibria is not. \[ \square \]
Appendix B: Reordering of Agents’ Valuations

This Appendix shows how the key multiplicity result of the model does not depend on particular ordering of agents’ valuations imposed by Assumption 1.

Case 1 \((\beta_R h < E_0[\tilde{R}])\): We begin by considering the case where

\[
\rho > \max \left\{ \frac{\beta - \pi}{1 - \pi}, \frac{\beta \pi}{1 - \beta(1 - \pi)} \right\}
\]

Under this assumption, the fraction of good assets supplied to the secondary market is given by

\[
\tau(\sigma) = \begin{cases} 
\frac{\lambda \pi}{\lambda + (1 - \lambda)(1 - \sigma)} \sigma & \text{if } p \geq \beta_R h \\
\frac{\lambda(1 - \sigma) \pi}{\lambda(1 - \sigma) + (1 - \sigma)} & \text{if } p < \beta_R h
\end{cases}
\]

In order for impatient informed agents with good assets to trade in the market, the following condition must hold:

\[
\sigma \leq \hat{\sigma} \equiv \frac{\lambda(\pi - \Gamma_1)}{\Gamma_1(1 - \lambda)(1 - \pi)}, \quad \text{where } \Gamma_1 \equiv \frac{\beta - \rho}{1 - \rho} < \pi
\]

By definition, we must have \(p(\sigma) \geq \beta_R h\) for values of \(\sigma \leq \hat{\sigma}\). Given agents’ payoff functions, this implies that the surplus from acquiring information satisfies equation (9).

Proof of Corollary 2. Contrary to the benchmark model considered in the text, the surplus function \(S(\sigma)\) is no longer continuous for all values of \(\sigma \in [0,1]\). We begin by showing that the surplus function jumps up discontinuously at \(\sigma = \hat{\sigma}\).

Claim A1. The surplus function \(9\) is such that \(S(\hat{\sigma}) < \lim_{\sigma \to \hat{\sigma}} - S(\sigma)\).

Proof of Claim A1. Using the fact that \(p(\hat{\sigma}) = \beta_R h\), we have

\[
S(\hat{\sigma}) = (1 - \pi)(1 - \lambda)(\beta_R h - R_t)
\]

By the definition of the surplus function (9), we then have

\[
\lim_{\sigma \to \hat{\sigma}} S(\sigma) - S(\hat{\sigma}) = \lambda \pi (\beta R_h - p(\sigma)) + (1 - \pi)(1 - \lambda)(p(\sigma) - R_t) - (1 - \pi)(1 - \lambda)(\beta R_h - R_t)
\]
which simplifies to
\[
\lim_{\sigma \to \hat{\sigma}} S(\sigma) - S(\hat{\sigma}) = (1 - \pi - \lambda)(p(\sigma) - \beta R_h) > 0
\]
where the inequality follows from Assumption 2 and the fact that \( p(\sigma) < \beta R_h \) for all \( \sigma > \hat{\sigma} \).

We show next that even if the surplus function is discontinuous at \( \hat{\sigma} \), an equilibrium always exists.

Claim A2. Under Assumption 2, there exists at least one equilibrium \((\sigma^*, p^*(\sigma^*)) \in [0, 1] \times \mathbb{R}^+ \) for all \( \psi \geq 0 \), even if \( \beta R_h < E_0[\tilde{R}] \).

Proof of Claim A2. Notice that since \( S(\sigma) \) is discontinuous at \( \hat{\sigma} \), the best response correspondence \( \sigma^*_j(\sigma) \) does not have a closed graph, so that Kakutani’s fixed point theorem no longer applies. Nonetheless, we proceed to show that a pure strategy equilibrium always exists by application of Tarski’s fixed point theorem.

Theorem 1 (Tarski’s Fixed Point Theorem). Let \( X \) be a non-empty complete lattice. If \( f : X \to X \) is weakly increasing, then the set of fixed points of \( f(\cdot) \) is a non-empty complete lattice.

Notice that the set of candidate pure strategy equilibria \( \sigma \in \{0, 1\} \) and corresponding best responses \( \sigma^*_j(\sigma) \in \{0, 1\} \) constitute a lattice \( X = \{(0,0), (0,1), (1,0), (1,1)\} \). This lattice is complete since it is partially ordered and contains both its supremum and infimum. Finally, from Claim A1 it follows that the best response correspondence in pure strategies, \( \sigma^*_j(\sigma) : \{0,1\} \to \{0,1\} \), is weakly increasing in \( \sigma \). Hence, there always exists at least one pure strategy equilibrium for all \( \psi \geq 0 \).

As in the benchmark model, proving Corollary requires us to show that there exist parameter values such that \( S(0) < S(1) \). Given equation (9), it is immediate to verify that this inequality is satisfied if
\[
\rho < \frac{\lambda \beta - (1 - \pi)(1 - \lambda)}{\lambda - (1 - \pi)(1 - \lambda)} \tag{A1}
\]
where the right hand side of this inequality is positive as long as \( \beta > (1 - \pi)(1 - \lambda)/\lambda \). Hence, as long as condition (A1) is satisfied, any value of \( \psi \in [S(0), S(1)] \) admits multiple equilibria in pure strategies.

Case 2 \((\beta E_0[\tilde{R}] > R_t)\): We now consider the case where:
\[
\rho < \min \left\{ \frac{\beta - \pi}{1 - \pi}, \frac{\beta \pi}{1 - \beta(1 - \pi)} \right\}
\]
Under this assumption, the fraction of good assets traded in the secondary market satisfies
\[
\tau(\sigma) = \begin{cases} 
\frac{\lambda(1-\sigma)p}{\lambda(1-\sigma)+(1-\sigma)\beta} & \text{if } p \geq \beta E_0[\bar{R}] \\
0 & \text{if } p < \beta E_0[\bar{R}]
\end{cases}
\]

In order for uninformed impatient agents to trade, we must have:
\[
\sigma \leq \bar{\sigma} = \frac{\lambda(\pi - \Gamma_2)}{\lambda(\pi - \Gamma_2) + (1-\pi)\Gamma_2}, \quad \text{where} \quad \Gamma_2 \equiv \frac{\pi \beta - (1-\beta)(1-\beta)\rho}{(1-\beta)\rho} < \pi
\]

Again, by definition we must have \(p(\sigma) < \beta E_0[\bar{R}]\) for values of \(\sigma > \bar{\sigma}\). This implies that the surplus from acquiring information is given by equation (10).

**Proof of Corollary 3.** As in Case 1, the surplus function is no longer continuous for all values of \(\sigma \in [0,1]\). We show that in Case 2 the surplus function jumps discontinuously down at \(\sigma = \bar{\sigma}\).

**Claim A3.** The surplus function (10) is such that \(S(\bar{\sigma}) > \lim_{\sigma \to \bar{\sigma}^-} S(\sigma)\).

**Proof of Claim A3.** Again, by definition, we must have \(p(\bar{\sigma}) = \beta E_0[\bar{R}]\). Evaluating the surplus function (10) at this price and simplifying, we obtain
\[
S(\bar{\sigma}) = (1 - \pi)(\beta \pi R_h - (\beta \pi + (1-\lambda)(1-\beta))R_t)
\]
This implies that
\[
S(\bar{\sigma}) - \lim_{\sigma \to \bar{\sigma}^-} S(\sigma) = (1 - \pi)(\beta \pi R_h - (\beta \pi + (1-\lambda)(1-\beta))R_t) - (1 - \pi)(1-\beta)\lambda R_t
\]
which simplifies to
\[
(1 - \pi)(\beta \pi - (1 - \beta(1-\pi))\rho)R_h > 0
\]
where the inequality follows from the assumption that \(\rho < (\beta \pi)/(1-\beta(1-\pi))\).

Contrary to Case 1, we now show that the discontinuity of the surplus function at \(\bar{\sigma}\) implies that an equilibrium need not always exist.

**Claim A4.** There exist values of \(\psi\) such that no equilibrium exists if \(\beta E_0[\bar{R}] > R_t\).
Proof of Claim \[ A4 \]

Begin by noticing we can have \( \lim_{\sigma \to \sigma^-} S(\sigma) < S(0) \) if

\[
\rho < \frac{\pi((1 - \pi) - \lambda(1 - \beta))}{(1 - \pi)(\pi + \lambda(1 - \beta))}
\]  

(A2)

The right hand side of this inequality is positive as long as \( \beta > (\lambda - (1 - \pi))/\lambda \). In this case, we must have \( S(1) < S(0) \) since the surplus function \[10\] is constant for values of \( \sigma > \sigma^- \). Hence, agents’ best response correspondence in pure strategies is weakly decreasing in \( \sigma \) and Tarski’s fixed point theorem does not apply. Also notice that, as in Case 1, the discontinuity of \( S(\sigma) \) at \( \sigma^- \) implies that the best response correspondence \( \sigma_j(\sigma) \) does not have a closed graph. Hence, Kakutani’s fixed point theorem does not apply.

We proceed to prove the claim by construction. Assume that condition \[ A2 \] holds and consider values of \( \psi \in [S(1), S(0)] \). Then \( S(0) - \psi > 0 \) and \( \sigma_j^*(0) = 1 \), implying that agents’ best response to nobody acquiring information is to acquire information. Similarly, \( S(1) - \psi < 0 \) and \( \sigma_j^*(1) = 0 \), implying that agents’ best response to everybody acquiring information is to not acquire information. Hence, for these particular values of \( \psi \) no pure strategy equilibria exist. To see that no equilibrium exists in mixed strategies either notice that, by Assumption \[2\], \( S'(\sigma) > 0 \) for all \( \sigma \leq \sigma^- \). Hence, we must also have \( S(\sigma) > \psi \) for all values of \( \sigma \leq \sigma^- \).

Notwithstanding this potential non-existence problem, we can show that multiple equilibria can also arise in this case. In particular, given Assumption \[2\] there always exist values of \( \psi \in [S(0), S(\sigma^-)] \) such that the model supports one pure strategy equilibrium and one mixed strategy equilibrium.