Expected Spot Prices and the Dynamics of Commodity Risk Premia

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Abstract

We analyse a novel time series of investor expectations of future commodity spot prices, and provide evidence that survey predictions are extrapolative and inconsistent with a strong form of rationality. We show that a model of adaptive expectations can replicate investor forecasts, and use this to back out the dynamics of the monthly (ex-ante) risk premia, as postulated by the theory of normal backwardation, for different commodities and maturities between 1995 and 2016. The empirical analysis demonstrates that commodity risk premia are time-varying and their dynamics is predominantly due to risk sharing channels and the changing demand for risk insurance and appetite, as proxied by open interest and hedging pressure. Time-series momentum and value factors also significantly generate time variation in commodity risk premia. In this respect, we provide evidence that the explanatory power of diverse factors is not constant over time, both across commodities and time horizons.

Keywords: Survey Expectations, Commodity Markets, Adaptive Learning, Ex-Ante Risk Premia, Empirical Asset Pricing

JEL codes: G12, G17, E44, C58

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1 Introduction

The way in which investors form expectations about future commodity prices is of great interest to economists as well as market participants, and forward prices have been used extensively in economic models as an approximation of market expectations.\(^1\) However, the forward curve shows the price at which it is possible to buy or sell futures contracts for a forward date at a price agreed today. It includes not only the expectation of future spot prices, but also a component reflecting the compensation required by market participants for bearing the risk of uncertain future fluctuations in the price of the spot commodity, i.e. a risk premium.\(^2\) Whether such risk premium is positive, negative, or time-varying and driven by changes in economic fundamentals has been controversial in the literature.\(^3\) This controversy stems from the fact that investors’ expectations are not directly observable.

In this paper, we address this issue using a novel survey provided by Bloomberg, which contains forecasts from professional analysts on future spot prices at multiple quarterly horizons and across diverse commodities. The survey includes analysts highly specialized in the commodity markets, and is quantitative in nature as participants are asked to provide point forecasts on the average quarterly commodity price for specified futures contracts. These are key features of the survey composition as a deep knowledge of the commodity markets peculiarities from the survey respondent, coupled with a clear objective of the survey, allows to reduce the effect of potential biases, quality homogeneity issues, and limited information available, which generally characterizes the expectations formation process of non-specialized, or retail, cross-markets investors (see, e.g. Cutler et al. 1990, Greenwood and Shleifer 2014 and Koijen et al. 2015). The first main finding of our study is that survey expectations of future spot prices tend to be extrapolative across time horizons. As a result, we reject the null hypothesis of a strong form of rationality for the average analysts’ forecasts with considerable confidence.

\(^{1}\)For instance, futures-based forecasts for the Oil price play a role in the policy decision making process at the ECB, see e.g. Svensson (2005), at the Federal Reserve Board, see e.g. Bernanke (2004), and at the International Monetary Fund, see e.g. IMF World Economic Outlook 2005.

\(^{2}\)Throughout the paper we use the terms risk premium and expected payoff interchangeably. In fact, all these terms identify a payoff expected at time \(t\) as a compensation for a risk which materializes at maturity \(t + h\). Differently, a realized payoff, or realized risk premium, couples the risk premium with any unanticipated deviation of the future spot price from the expected future spot price.

Based on this evidence, we hypothesize that investor expectations are the results of an adaptive learning scheme in which expected future spot prices are revised in line with past prediction errors and are affected by changes in aggregate demand. In order to test such hypothesis, we first postulate the perceived law of motion of commodity prices starting from an extended version of a market model with inventory speculations as in Muth (1961), with the addition of predictable aggregate demand shocks.\(^4\) In this setting, the learning dynamics implies that past forecast errors can potentially affect the extent to which expectations on future spot prices are revised. We compare the model-implied expectations with the average analysts' forecasts. Although with differences across commodities, our time series of adaptive expectations are consistent with the survey-based point forecasts from two to four quarters ahead.

The possibility to replicate observable investors' expectations through adaptive learning is key for our purpose, as it allows to approximate the time-varying (ex-ante) risk premia implied by investors' beliefs for a reasonably long sample period.\(^5\) In order to understand the driving factors for the dynamics of risk premia, we compute the expected payoffs by taking the difference between the futures price (as of date \(t\)) for delivery at time \(t + h\) and expectations at time \(t\) on future spot prices at time \(t + h\). Our adaptive forecasts generate time varying risk premia similar to the ones filtered by using the survey expectations across alternative horizons and different commodities, as indicated by an average correlation between the two approaches for the overlapping sample of 0.75.

Reconciling the evidence on time-varying risk premia and the potential underlying risk factors that drives such dynamics poses significant challenges. First, the sensitivity of risk premia with respect to each risk factor is not constant over time. For example, the relative impact of emerging markets in commodity risk increased in the recent past mainly driven by the increasing competition with traditional suppliers.

\(^4\)We assume that shocks to aggregate supply are conditionally i.i.d. This assumption can be relaxed at the cost of having some reliable empirical proxy for aggregate supply shocks for agriculturals, e.g. Corn, and precious metals, e.g. Silver, to be used in modeling the dynamics of expected spot prices. Also, while the i.i.d. assumption for supply shocks can be questionable for energy or industrial commodities such as oil and copper, the same assumption can be a fair approximation of supply shocks in agriculturals and precious metals, e.g. “harvest” of commodities can be thought as i.i.d. and storage of, say, corn is temporally limited. Recently, Gilje et al. (2015) proposed a framework to identify the effect of shocks in the supply of Shale Oil on the aggregate stock market. A similar procedure would be interestingly be implemented in other commodity markets although could be prohibitive for precious metals and agriculturals.

\(^5\)Moreover, relying on extrapolative models of expectations formation allows to relax the assumption of information efficiency in the beliefs updating process which follows orthogonality between prediction errors and predictors, i.e. the expectation error conditional on all available information equals zero, which is underneath the rational expectations hypothesis.
weight of China on global economic growth. Similarly, the current financialization of commodity markets and the current regime of zero nominal short term interest rates represent changes in the perception of risk for commodity investors which, by definition, were not considered ten years ago (see, e.g. Cheng and Xiong 2014). If so, it is fair to assume that the coefficients on the economic determinants of risk premia are changing over time. Second, the model relevant to understand the sources of time variation in commodity risk premia can be subject to structural changes. For instance, the set of economic variables that affect risk premia on WTI Oil is potentially different before and after periods of slowdown in the global economic growth or a radical change in the predominant monetary policy. This suggests that for \( m = 1, \ldots, M \) set of explanatory factors one have to consider all of the possible \( 2^M \) model specifications at each time \( t = 1, \ldots, T \). Even in relatively simple regression-based analysis with a limited number of variables it would be infeasible to investigate pricing determinants by simply going through all of \( K = 2^{MT} \) combinations.

To address these issues, we estimate a dynamic linear model for each of the \( K \) models and understand the relative importance of each risk factor based on marginal models probabilities (see, e.g. West and Harrison 1997 for more details). As far as the model estimation strategy is concerned, we opt for a conjugate Bayesian framework, which allows to obtain robust finite-sample estimates that flexibly and explicitly accounts for different sources of uncertainty: uncertainty in the relative importance of predictors, uncertainty in the estimated coefficients and their degree of time-variation, and idiosyncratic risks.

Our empirical results show that risk premia are time-varying, both across different horizons and commodities, and their dynamics is predominantly due to risk sharing channels and the changing nature of commodity market participants, as proxied by Open Interest (OI henceforth) and Hedging Pressure (HP henceforth), as well as by Value and time-series Momentum factors. The role of OI has been recently outlined by Hong and Yogo (2012). They show that the number of futures contract outstanding represents a reliable signal of future economic activity and therefore could predict futures movements in asset prices. On the other hand, the role of HP in commodity risk premia has been first introduced by Keynes (1930) and Hicks (1939) in their Theory of Normal Backwardation, where a risk premium is accrued to speculators as a reward for bearing the risk of fluctuations in the price of spot commodity which hedgers sought
to transfer. More recently, Carter et al. (1983), Bessembinder (1992), De Roon et al. (2000), Acharya et al. (2010) and Basu and Miffre 2013 show that HP can be effectively interpreted as a systematic risk factor in the cross-section of commodity risk premia. Hamilton and Wu (2014) show that risk sharing mechanism can give rise to an affine term structure model that explains the dynamics of futures prices. Our results from a full-scale dynamic regression setting confirm the importance of HP in understanding the nature of commodity risk. Similarly, evidences from the marginal posterior model probabilities show that a time-series Momentum factor plays a significant role in the dynamics of commodity risk premia, consistent with Asness et al. (2013) and Szymanowska et al. (2014).

Furthermore, we show that the economic importance of emerging economies, as proxied by the MSCI Emerging Market Index (MXEF), has sensibly increased over time for energy and industrial commodities such as WTI Crude Oil and Copper. A potential explanation is the presence of spillover effects due to the increasing weight of the emerging economies in the global economic growth, especially the Chinese economy. As a matter of fact, although the direct impact of Chinese equity valuations is relatively low due to moderate foreign investments, financial turbulence in this stock market tend to be associated with issues about the global economic slowdown as China itself is the second largest economy and the second largest importer of both goods and commercial services. This is consistent with our initial model with speculative inventories, as a negative (positive) shock in aggregate demand reflects in decreasing (increasing) commodity prices, which in turn increase (decrease) the premium speculators require to provide insurance for those who which to hedge, e.g. farmers. As a whole, we provide evidence on the heterogeneity in the relative importance of economic risk factors in the dynamics of risk premia throughout our sample.

This paper connects to a number of works in the expectations formation literature such as Nerlove (1958), Evans and Honkapohja (2001), Sargent (2002), Sargent and Williams (2005), Carceles-Poveda and Giannitsarou (2008), and Malmendier and Nagel (2015), who consider a model of adaptive learning to explain the dynamics of expectations on inflation and more general

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6Hirshleifer (1990) further extended the theory in a general equilibrium setting, linking backwardation to lower levels of HP and contango, which is the mainstay of the other competing theory of commodity risk premia proposed by Working (1949).

7Recently Gorton et al. (2013) argue that the null hypothesis that HP is an important determinant of commodity risk premia should be rejected. However, they effectively look at ex post risk premia on commodity futures.
macroeconomic outcomes. Unlike them, we focus on commodity markets and use adaptive expectations as a tool to extract the real quantity of interest, which is the ex-ante risk premia across diverse commodities and for different expectations horizons. Conceptually, our work is also related to recent research on the effect of trading activity of commodity spot prices, e.g. Singleton (2014), who argues that trading activity is the result of an adaptive process in which hedgers and speculators learns about economic fundamentals, both from public information and market prices, and generate drifting prices.

There is only a small, but growing literature that looks at survey forecasts as a way to approximate investors’ expectations formation and test economic hypothesis across asset classes. Greenwood and Shleifer (2014) show that investors’ expectations on future stock market returns are inconsistent with a rational expectations representative agent model of returns. In fact, survey-based forecasts tend to be, on average, negatively correlated with expectations postulated by standard equilibrium asset pricing models. Koijen et al. (2015) extended these results by investigating the implications for returns predictability and excess volatility across three asset classes: equity, fixed income and currency. They show that survey expectations tend to predict price changes but with the wrong sign and display a extrapolative features being influenced by past price levels and returns. Unlike them, we formally postulate an adaptive learning scheme for future commodity spot prices which is consistent with a “learning from past errors” extrapolative behaviour, as in fact suggested by a preliminary test of the rationale expectations hypothesis on our survey predictions. Gennaioli et al. (2015) show that expectations on earnings affect investment decisions by corporate decision makers.

The rest of the paper is organized as follows. Section 2 discusses the difference between ex-ante and realized risk premia and why approximating investors’ expectations is crucial to understand the dynamics expected payoffs. Then, Section 3 introduces the time series of investors’ expectations and the data used in the empirical analysis. Next, in Section 4 we test the null hypothesis of rational expectations vs the alternative of adaptive expectations for the average investors’ forecasts. The analytical framework with learning from past experience and the econometric methodology we use to understand the dynamics of the ex-ante risk premia are presented in Section 5. Finally, Section 6 reports our core set of empirical results and Section 7 concludes.
2 Ex-Ante vs. Realized Risk Premium

In this section we briefly review the theory of normal backwardation of Keynes (1930) and Hicks (1939) and motivate the importance of using investors’ expectations to identify commodity risk premia with a special emphasis on the dichotomy ex-ante vs. realized payoffs.

Let \( S_t \) and \( F_t^{(h)} \) denote the spot and futures prices of a given commodity for delivery at time \( t + h \). Define the basis to be the difference between the current futures and spot prices \( F_t^{(h)} - S_t \).

The theory of normal backwardation links the price of futures contracts and the commodity spot prices on the basis of risk insurance and appetite from hedgers and speculators. In this theory, the risk premium required by market participants adds up to unexpected price changes;

\[
F_t^{(h)} - S_t = \hat{S}_{t+h|t} - S_t + y_t^{(h)},
\]

with \( S_{t+h|t} = E_t [S_{t+h}] \) the market aggregate expected price at time \( t \) for the spot commodity at time \( t + h \), and \( y_t^{(h)} \) the expected payoff at time \( t \) for the futures position. According to Keynes (1930), at least unconditionally, the risk premium is negative, which implies that futures contracts are on average sold at a discount with respect to the future spot price. As such, the underlying idea is that hedgers are of net short and therefore willing to pay a premium to speculators that provide insurance against the risk of fluctuations in the spot market.\(^8\) Another interpretation of the basis is provided by the theory of storage of Kaldor (1939), Working (1949), and Brennan (1958). This theory derives a fundamental relationship between spot and futures prices, and posits that the basis consists of two components: (1) an opportunity costs of forgone returns from a risk-less security \( i_t \) and (2) a convenience yield which for simplicity is assumed to be net of storage costs \( C_t^{(h)} \);

\[
F_t^{(h)} - S_t = i_t S_t - C_t^{(h)}
\]

Equation (2) is often justified by the absence of arbitrage opportunities in commodity markets. The economic mechanism that makes the basis linked to convenience yield lies on the fact that the latter is tightly and inversely related to the level of inventories. Fama and French (1988)

\(^8\) Notice that equation (1) does not rule out the possibility of commodity markets to be in contango as current futures prices could still be lower that spot valuations.
pointed out these two theories are not mutually exclusive. Also, Szymanowska et al. (2014) showed that there is a mapping with $y_t^{(h)}$, which they call “spot premium”, and the one implied by the theory of storage.

Few comments are in order. First, to the extent to which one want to investigate risk sharing and insurance mechanism in commodity markets, the theory of normal backwardation offers the ideal setting as isolates the risk premium component in futures prices directly from investors’ expectations. Second, the risk premium isolated in equation (1) is really the \textit{ex-ante} payoff the average market participant expects from investing on a futures and holding the contract until maturity. This is fundamentally different from using the \textit{realized} payoff of the contract, which represents the sum of the risk premium and an unexpected price change. Figures 1 makes this case in point.

We consider the situation in which the price of the commodity at time $t$ is equal to 50$ and market expectations for the future spot price at time $t+h$ are equal to 47$, i.e $E_t[S_{t+h}] = 47$. Let us assume also that in order to make investors willing to enter the market the current price of a futures contract at time $t$ for delivery at time $t+h$ is equal to 43$, which means futures are sold at a discount with respect to expected future spot prices. The difference between the futures price and $E_t[S_{t+h}]$ at time $t$ implies that the expected payoff of a futures short position, i.e. the risk premium, is equal to 4$.

Top panel shows the case in which the commodity is effectively traded at 47$ at time $t+h$. In this case, given the no-arbitrage assumption that futures contracts at expiration trade at the spot price, and given unexpected price depreciations are zero, the ex-ante and the realized payoffs are equivalent. Now consider a situation in which investors systematically make errors in forecasting future spot price realizations (see, e.g. Alquist and Kilian 2010 for a complete discussion on the predictability of nominal spot prices). The bottom panel shows an example in which the commodity is traded at a lower price of 45$ at time $t+h$ on the spot market, which implies a forecast error $\hat{S}_{t+h|t} - S_{t+h} = 2$. Given that the value of the futures contract at maturity coincides with the spot at time $t+h$ the realized payoff is 2$. In this case, ex-ante and the realized risk premia differ by the amount of an unexpected price realization.

Figure 1 shows that ex-ante and realized risk premia differ to the extent that investors’
misjudge the level of future spot prices over time. Testing the presence of a systematic and time-varying error in investors’ expectations is an empirical question that can be verified by using the average survey forecasts. Figure 2 shows the time series of the expectations error $\hat{S}_{t+h|t} - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$ quarters ahead, and two alternative commodities, i.e. WTI Crude Oil and Silver. The aggregate forecast $\hat{S}_{t+h|t}$ is proxied by the average expected price from the Bloomberg survey of professional analysts. A complete discussion on how the survey is collected and structured, as well as a description of the data, is provided in Section 3 below.

Figure 2 makes clear the existence of a systematic and time-varying expectation error in predicting future spot prices up to four quarters ahead for both WTI Crude Oil and Silver. Unsurprisingly, unexpected depreciation for crude oil occurred over the great financial crisis of 2008/2009 and the recent collapse of late 2014/beginning of 2015. Throughout the sample, the null hypothesis of an exactly zero unexpected price change can be sensibly rejected with confidence. Similarly, unexpected appreciation of Silver occurred in the recovery of financial markets after 2009, consistent with the idea that the value of precious metals tend to be negatively correlated with the business cycle.

As a whole, Figure 2 provides evidence that investors’ predictions are conditionally biased. Therefore the assumption that realized risk premia can be used as a perfect substitute of ex-ante risk premia, which is the key component of the theory of normal backwardation, is not supported by the empirical evidence, especially in a dynamic context.

3 Data Description

In this section we describe our data on futures prices, the proxies for economic risk factors and the methodology for constructing the average survey expectations.

3.1 Spot and Futures Prices

We cover four main commodity futures which represent the energy, metals, and agricultural markets. We focus on these commodities as they are the most traded consumption commodities
with the most complete sample of survey data. In that respect, the choice of the commodity to be included in the analysis is mostly dictated by the length of the corresponding survey and the number of professional analysts responding. Including other commodities would come at a cost of using averages of few respondents or very short time series, e.g. from 2013.

Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. As in Szymanowska et al. (2014) the spot price for each commodity is approximated by using the nearest contract to maturity. We define the futures price at time $t$ with average quarterly time to maturity $h$ as $F_t^{(h)}$, where the definition of the average time to maturity is consistent with the average forecasting horizon for the survey expectations. More formally, the price of a future for delivery at quarter $h$ ahead is computed interpolating the prices of the contracts between 10 and 12 months ahead. The sample period is monthly 01:1993-01:2016.

3.2 Measuring Investors’ Expectations

We obtain individual price forecasts for different commodities and horizons from the Bloomberg’s commodity price forecasts database. This database contains analysts’ price expectations at multiple quarterly forecasting horizons and across diverse commodities from 2006 to 2016. This survey exclusively includes sophisticated operators highly specialized in the commodity markets mainly from banks and commodity consultancy companies. Yet, the survey is quantitative in nature as participants are asked to provide a point forecasts on the average quarterly commodity price for a specified futures contract. A deep knowledge of the commodity markets peculiarities from the survey respondent and the clarity of the prediction made allows to reduce the effect of potential biases, quality homogeneity issues, and limited information available, which generally characterizes the directional expectations formation process of non-specialized, or retail, cross-markets investors. In other words, the fact that only operators specialized in
commodity markets are being surveyed increase the proportion of truly “informed” agents in the survey population compared to a case in which cross-market analysts are being surveyed (see, e.g. Cutler et al. 1990). The database allows to retrieve for each analyst the historical price forecasts and the related publication date. The use of the survey for operational purposes involves some challenges as the quarterly analysts’ survey forecasts submission are recorded daily and not evenly spaced in time.

Analysts provides forecasts on spot commodity prices in different days for fixed common maturities that corresponds to calendar quarters, namely survey respondents provide discontinued fixed-calendar maturity quarterly expectations. In order to perform a standard time series analysis we need to transform analysts’ responses in continued constant-horizon price forecasts. Following Beber and Piana (2016) we aggregate responses at the monthly frequency in order to reduce the difference in the market information available between early and late submitters within a month. We then compute the forecasting horizon with respect to the end of the month of the last month in the quarter which is the object of the prediction. More specifically, at each point in time, we stack the forecasts that belong to the following groups: 4 to 6; 7 to 9 and 10 to 12 months, then we approximate the aggregate expectations as the cross-sectional average prediction across analysts and time-horizons. In order to reduce moving average effects in the synthetic average expectations we discard the horizon between one and three months as the analysts take into account what has been the realized average price over the first part of the quarter generating now-casting dynamics which makes hard to disentangle the role of expectations versus current information in the dynamics of the very short-term risk premia.

Other surveys can be used to approximate the average investors’ expectations, such as for instance the Energy & Metals Consensus Forecasts from Consensus Economics. However, Consensus forecasts do not count for agricultural and is lower frequency being collected every other month after April 2012 and on a quarterly basis before that date. Also, unlike other surveys on commodity prices, the Bloomberg’s survey of forecasts is entirely made by professional analysts’ affiliated with banks and commodity consultancy companies, which make the survey of great interest in approximating expectations by actual market participants.9

9For instance, Consensus forecasts are “contaminated” with forecasts from economists working in institutions that do not necessarily invest in the commodity market.
3.3 Explanatory Factors for Risk Premia

In order to study the sources of time variation in commodity risk premia, we collect diverse determinants that are considered to capture alternative sources of risk. As a proxy for the level of global commodity demand we use the index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis. This index aggregate information on industrial production from 81 countries worldwide, which account for about 97% of the global industrial production. The aggregate series starts in January 1991 and relate to import weighted, seasonally adjusted, industrial production.

Fluctuations in the global supply-demand imbalance for each commodity are captured by using inventory stocks. We collect data on Copper and Crude Oil inventories from the London Metal Exchange (LME) and Energy Information Administration (EIA) respectively. Copper inventory levels are recorded daily from June 1974 and relate the previous day closing stock of commodities held in LME. Crude Oil inventories are recorded weekly by the EIA and published monthly since January 1945. Stocks levels are measured in thousands of barrels and exclude strategic petroleum reserves.\textsuperscript{10} For Corn inventories, we use the U.S. ending stocks reported in thousands of metric tons. The time series is sampled at monthly frequency using the inventory level reported on the last business day of the month. Data are recorded from the United States Department of Agriculture (USDA) from January 1993. As far as Silver is concerned, we omit the inventory level variable as, similar to other precious metals, a considerable part of the existing reserves is privately held and therefore not reported in official statistics.

Exchange rates is also a relevant risk factor as commodity trading takes place usually in U.S. Dollars, making the exchange rate a key factor for both producers and consumers, as it directly affect profits and costs denominated in domestic currency. In order to account for the risk of appreciation and depreciation in the U.S. Dollar, we include as additional risk factor the Federal Reserve U.S. trade weighted exchange rate index, normalized to be equal to one hundred in March 1973. Together with exchange rates, interest rates represents a key determinants of the cost of inter-temporal arbitrage strategies. For the analysis on the two- and four-quarter ahead risk premia we use the monthly LIBOR rate with 6 and 12-month maturity, respectively.

\textsuperscript{10} We include in the level of inventories those domestic and Customs-cleared foreign stocks held at, or in transit to, refineries and bulk terminals, and stocks in pipelines. Stocks include an adjustment of 10,630 thousand barrels (constant since 1983) to account for incomplete survey reporting of stocks held on producing leases.
matching the horizon of the hypothetical investment in the commodity futures and the risk-free asset.

Gorton et al. (2013) show that time variation in the level of inventories is responsible for the presence of momentum in the realized returns of commodity futures. More specifically, they argue that deviations of inventories from equilibrium levels are generally persistent, generating persistent changes in spot prices. Because past unexpected changes in spot and futures prices can be interpreted as signals of past shocks to inventories, they are expected to be correlated with expected futures risk premiums. This induces a form of “momentum” in futures realized payoffs. Momentum in commodity futures has been widely documented in the empirical finance literature, e.g. Erb and Harvey (2006), Miffre and Rallis (2007), Asness et al. (2013) and Szymanowska et al. (2014) among others. We construct time-series Momentum as the return over the past 12 months skipping the most recent month on each commodity future. In addition, we include a Value factor which is assumed to be intimately interrelated to the dynamics of commodity risk premia, as affects the propensity of market participants to trade in backwardation or in contango. We follow Asness et al. (2013) and define Value as the average of the log spot price from 4.5 to 5.5 years ago, divided by the most recent spot price, which is essentially the negative of the spot return over the last 60 months.

In addition to time-series Value and Momentum, we also directly consider the Standard and Poor’s 500 and the MSCI Emerging Markets indexes as proxy for financial risk. Beyond direct effects on financial flows, we incorporate stock indexes as they likely capture spillover effects to the real economy. For instance, although shocks to equity valuations in emerging markets are possibly of moderate effect, financial turmoil in emerging economies are typically associated with uncertainty about global economic growth.

Finally, to capture risk sharing channels in the economic mechanism that drives commodity risk premia we consider OI and HP as variables to capture the changing nature of futures market participation, (see e.g. Baker and Routledge 2011 and Singleton 2014). OI is measured as the total number of outstanding contracts that are held by market participants at the end of the month. An outstanding contract is when a seller and a buyer combine to create a single contract. For each seller of a futures there must be a buyer of that contract, therefore to determine the total OI for any given market we need to know the totals from one side or the other, buyers or
sellers, not the sum of both. Increasing OI means that new cash is flowing into the marketplace, while declining activity means that the market is liquidating and implies that the prevailing price trend is coming to an end. HP represents a measure of net positions of hedgers in commodity futures markets which is the result of risks that market participants do not want, or cannot trade because of market frictions, information asymmetries and limited risk capacity (see, e.g. Hong and Yogo 2012). We compute the level of HP for different commodities as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding hedging contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC).

4 Rational Expectations vs Adaptive Learning

At the outset of the paper we argue that one of the key issue we face is outline how best to describe the beliefs formation mechanism about future spot prices as proxied by survey forecasts. Our aim is not to develop new hypothesis, but rather test the rational vs adaptive expectations hypothesis to understand how to model investor expectations. The rational expectations model as proposed by Muth (1961) has obtained general acceptance as the benchmark model of expectations formation. It implies that decision makers know the true underlying model such that subjective beliefs are set to be equal to their objective counterparts. The assumption of complete knowledge of the data generating process is however a fairly restrictive assumption. This has led many researchers to propose a number of simple formulations based on weaker forms of the rational expectations hypothesis that allow for model instability, uncertainty, and learning (see, e.g. Hsieh and Kulatilaka 1982, Frenkel and Froot 1987, Marce and Sargent 1989, Evans and Honkapohja 2001, and Sargent 2002 in a general framework, and Singleton 2014 and Sockin and Xiong 2015 relatively to commodity markets).

Let $E_t S_{t+h}$ represents the expectation at time $t$ for the spot price at time $t + h$, averaged across individuals. The associated prediction error can be defined as $\epsilon_{t+h} = S_{t+h} - E_t S_{t+h}$. Assuming such forecast error can be observed, strong rationality in the expectations formation mechanism can be tested by checking the orthogonality condition $E (\epsilon_{t+h} | X_t) = 0$, with $X_t$ the information available to investors at time $t$ (see, e.g. Pesaran and Weale 2006 for more details).
To test this rationality condition we estimate the following regression model:\(^{11}\)

\[
\epsilon_{t+h} = \alpha + \beta X_t + \eta_{t+h}, \quad \text{for} \quad h = 2, 3, 4, \quad \text{quarters},
\]

(3)

with \(X_t = (S_t - S_{t-h})\) representing past changes in spot prices. We use past spot prices since, once become observable, are assumed to summarize all the relevant current information which is readily available to professional analysts (see, e.g. Sockin and Xiong 2015). The null hypothesis of rational expectations implies that past information should not help improve future forecasts above and beyond what is already included in \(E_t S_{t+h}\), i.e. \(H_0 : \beta = 0\). Panel A of Table 1 shows the results.

[Insert Table 1 about here]

Except for the forecasting errors in Copper at a four-quarter forecasting horizon, the null hypothesis that current information does not improve forecast is rejected across alternative forecasting horizons. The slope coefficient is positive and statistically significant for eleven out of twelve cases. Also, Durbin-Watson statistics show that residuals \(\eta_{t+h}\) are mildly autocorrelated. The evidence suggest that prediction errors are elastic with respect to past changes in spot prices, meaning, expectations are non-explosive since a change in the current spot price induces a revision in the expected future spot rate which is, although positive, less than proportional. A further test for strong form of rational expectations entails regressing the prediction error on its lagged values for the \(h\) horizon;

\[
\epsilon_{t+h} = \rho \epsilon_{t-h} + u_{t+h}, \quad \text{for} \quad h = 2, 3, 4, \quad \text{quarters},
\]

(4)

This regression model represents a conventional way to test for informational efficiency in the beliefs updating process (see, e.g. Frenkel and Froot 1987). Panel B of Table 1 shows the results. The null hypothesis that there is no “learning” from past errors, i.e. \(H_0 : \rho = 0\) is strongly rejected across forecasting horizons and commodities. The slope coefficients are positive and significant meaning expectations are insufficiently adaptive; investors could avoid to make the same directional error by exploiting the information contained in past errors. As a whole, based on the evidence provided in panel A and B we can reject a strong form of rationality for survey

\(^{11}\) We estimate the model by OLS with GMM corrected standard errors to account for autocorrelation and heteroscedasticity in the residuals \(\eta_{t+h}\).
expectations with considerable confidence. One comment is in order; rejecting strong rationality does not necessarily mean survey participants are irrational. A potential explanation for the presence of a systematic bias lies in the career concerns analysts face in designing forecasts (see, e.g. Bernhardt and Kutsoati 1999, Hong et al. 2000, and Hong and Kubik 2003). We make clear that the object of this paper is not to take a stand on what is the reason of rejecting the null of strong rationality, but rather we build on this evidence and try to understand if instead adaptive learning represents an explanation more consistent with the survey data.

In that respect, we now test for a general version of extrapolative expectations. In their most general formulations, the model for adaptive expectations have a limited number of testable implications; the most important of which is the impact of past information on current forecasts (see, e.g. Pesaran and Weale 2006). We test for a general rule of updating by estimating the impact of current prices on expectations;

\[ E_t S_{t+h} - S_t = \alpha + \beta X_t + \epsilon_t, \quad \text{for} \quad h = 2, 3, 4, \text{ quarters}, \]  

with \( X_t = (S_t - S_{t-h}) \) as above. Eq. (5) states that if a commodity has been recently depreciated, then it will continue to depreciate in the near future as well. Strong rationality would imply the null hypothesis \( H_0 : \beta = 0 \). Panel A of Table 2 shows the results.

Interestingly, the slope coefficients are all negative and strongly significant meaning there is momentum in investors’ expectations, namely a recent depreciation of a commodity leads to a pessimistic view on future spot prices. Such “bandwagon” dynamics in investors’ expectations does not rule out the possibility of rational bubbles in commodity markets. However, such eventual bubbles could reinforce at a rate which is less than proportionate since the slope coefficient is less than unity in absolute value. Building on this result, we now test the further restriction that expectations are adaptive. Adaptive learning is the most prominent form of extrapolative expectations formation process (see, e.g. Evans and Honkapohja 2001, Cho et al. 2002, Sargent 2002, Williams 2003, Sargent et al. 2004, Sargent and Williams 2005 and Malmendier and Nagel 2015 to cite a few). Under this model investors revise their expectations in line with
past prediction errors. In general, adaptive expectations need not be informationally efficient, and forecast errors can be serially correlated. We test the adaptive expectations hypothesis by regressing the expected price change on the lagged survey prediction error:

\[ E_t S_{t+h} - S_t = \mu + \delta (E_{t-h} S_t - S_t) + \nu_t, \quad \text{for} \quad h = 2, 3, 4, \quad \text{quarters}, \quad (6) \]

Panel B of Table 2 shows the results. The slope coefficients are positive and statistically significant across forecasting horizons and commodity markets. This implies that investors, on average, place positive weight on previous predictions, confirming the “bandwagon” dynamics shown in Panel A. To summarize, investors’ expectations on future spot prices are not static; in fact, we show that the elasticity of the expected future spot commodity prices with respect to current information and forecasting error is positive and significant (see, e.g. Alquist and Kilian 2010). The nature of the rejection for a strong form of rational expectations hypothesis does not depend on the prediction horizon and the specific commodity market. Consistent with the extrapolative nature of survey expectations, in the next section we propose a model of adaptive learning in which current expectations are revised in line with past prediction errors.

5 Adaptive Learning and Time-Varying Risk Premia

Section 3 gives some insight on the nature of investors’ expectations on commodity future spot prices. The candidate forecasting rule we examine have close resemblance to an adaptive learning scheme, in particular to the recursive forecasting proposed in Cho et al. (2002), Sargent (2002), and Sargent and Williams (2005). More recently, Leduc et al. (2015) showed that a simple learning framework accounts remarkably well for the fluctuations in oil price futures. To set up an analytical framework we need to specify the perceived law of motion that investors are trying to recursively estimate. A natural starting point is a simple rational expectations model closely related to the Muth (1961) market model with inventory speculation. The market behaviour is characterized by an infinite horizon, discrete time model with a market clearing condition that holds in each time period. We depart from the original Muth (1961) model by allowing demand shocks to be predictable and potentially persistent.\textsuperscript{12} In this framework, a unique reduced-form

\textsuperscript{12}In the original Muth (1961) framework demand shocks that induce changes in inventories quickly revert to their long-run equilibrium values. In that respect, inventories adjustments are perceived to have a stabilizing
rational expectations equilibrium is defined as (see Appendix A)

\[ S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1}, \tag{7} \]

with \( S_{t+1} \) the market commodity price at date \( t + 1 \), \( z_t \) the level of aggregate demand at time \( t \), and \( \eta_{t+1} \) an unobservable random shock. This solution is the same as the original Muth (1961)’s model except that future commodity spot prices now depend also on aggregate demand as suggested by Figure 3. This figure shows the year-on-year changes in the (log of) commodity spot prices (blue line) and aggregate demand (magenta line).\(^{13}\)

[Insert Figure 3 about here]

With the only partial exception of Corn (bottom-left panel), which is an agricultural commodity and therefore less sensitive to business cycles, changes in spot commodity prices tend to be contemporaneously correlated with changes in aggregate demand as proxied by an index of world industrial production, especially after the beginning of 2000s. Although concerning exclusively the Oil market, Kilian and Hicks (2013) similarly showed that unexpected economic growth sensibly affect the dynamics of spot prices. As a whole, Figure 3 shows that changes in aggregate demand represent an important source of fluctuations in commodity prices. Thus, since expectations are adaptive, in the sense that are revised in line with past prediction errors, aggregate demand should affect investors’ expectations in the model as well.

The key assumption to introduce learning is that agents do not know true values of the parameters \( \phi = (\phi_0, \phi_1, \phi_2) \) and expectations are instead formed on the basis of current observations plus a constant \( X_t = (1, S_t, z_t) \), and predictions of the parameters are updated over time. We follow explicit agents’ recursive estimates in terms of a Bayesian prior that describes the effect on prices. However, as recently showed by Dvir and Rogoff (2010) quick adjustments in inventories to demand shocks cannot explain the persistence in the time series of commodity prices and volatilities.

\(^{13}\)One may also specify a model in which expectations of future aggregate demand rather than current levels enter in the equilibrium outcome. As far the unique reduced-form solution in Eq. (7) is concerned, the two things are essentially equivalent. Aggregate demand is specified as a potentially persistent AR(1), i.e. \( z_{t+1} = b z_t + \epsilon_{t+1} \). This implies that \( E_t z_{t+1} = b z_t \), which means that the structural coefficient \( b \) of the actual law of motion, although cannot be identified, is embedded in the reduced-form parameter \( \phi_2 \) of the perceived law of motion.
how coefficients drift at each time $t$: \(^{14}\)

\[
S_{t+1} = \phi_{t+1} X_t + \eta_{t+1}, \quad \text{with} \quad \eta_{t+1} \sim N(0, \sigma^2),
\]

\[
\phi_{t+1} = \phi_t + \xi_{t+1}, \quad \text{with} \quad \xi_{t+1} \sim N(0, \Omega),
\]

(8)

with $\phi_t = (\phi_{0,t}, \phi_{1,t}, \phi_{2,t})'$ and $X_t = (1, S_t, z_t)'$. The shock $\eta_{t+1}$ is uncorrelated with $\xi_{t+1}$, and $\Omega \ll \sigma^2 I$. The innovation covariance matrix $\Omega$ governs the perceived volatility of increments to the parameters (see, e.g. Sargent and Williams 2005). Agents’ recursive optimal estimate of $\phi_{t+1}$ conditional on information available up to time $t$, $\gamma_{t+1} = \hat{\phi}_{t+1|t}$ are provided by a Kalman filter recursion;

\[
\gamma_{t+1} = \gamma_t + K_t (S_{t+1} - \gamma_t' X_t),
\]

\[
R_{t+1} = R_t - \frac{R_t X_t' R_t}{X_t' R_t X_t + 1} + \sigma^{-2} \Omega,
\]

(9)

where $K_t = R_t X_t (X_t' R_t X_t + \sigma^2)^{-1}$ determines the degree of updating of agents’ beliefs when faced when an unexpected commodity spot price $S_t - \gamma_t' X_t$. The recursive learning dynamics (9) represents a generalization of adaptive learning with constant gain as specified for example in Evans and Honkapohja (2001), Sargent (2002), Cho et al. (2002), and Williams (2003), Sargent et al. (2004), Cogley and Sargent (2005), and Sargent and Williams (2005). Although learning is perpetual in our model, the recursive estimates (9) converge to a steady-state solution for a given initial condition of the state covariance matrix $\Omega$ (see, Hamilton 1994 Proposition 13.1, pag. 390). We use the subscript $t + h|t$ to indicate a forecast for the $h > 0$ horizon made using information available to agents’ at time $t$. The market price expected to prevail at time $t + 1$ given the information available through the $t$th period is obtained as

\[
\hat{S}_{t+1|t} = \gamma_{t+1}' X_t,
\]

(10)

Multi-period forecasts $\hat{S}_{t+h|t}$ are obtained by iterating forward the time-$t$ estimates of the model parameters. Relatively simple recursive learning schemes as (9) are widely motivated in the adaptive learning literature by the fact that agents face constraints in cognitive abilities that

\(^{14}\)This random walk specification for the evolution of the parameters is widely used in applied work in macroeconomics and finance, e.g. Frühwirth-Schnatter (1994), West and Harrison (1997), Stock and Watson (1998), Primiceri (2005), Hansen (2007), Dangl and Halling (2012) and Leduc et al. (2015).
limit their possibility to observe the true market spot equilibrium parameters and use optimal, e.g. perfect foresight, forecasting rules (see, e.g. Carceles-Poveda and Giannitsarou 2008 and Malmendier and Nagel 2015). As such, adaptive learning can be interpreted as an approximation of the cognitive rule investors might employ to form their expectations.

We use the learning scheme outlined above as a starting point for the empirical investigation of the risk premia across predictive horizons and commodities. Let \( \hat{S}_{t+h|t} \) be the expected future spot price of a given commodity at time \( t \) for the horizon \( t+h \). The risk premium can be extracted by subtracting the price of a future contract at time \( t \) for delivery at time \( t+h \), i.e. \( F_t^{(h)} \), as (see, e.g. Fama and French 1987):

\[
\gamma_t^{(h)} = F_t^{(h)} - \hat{S}_{t+h|t}
\]  

Eq. (11) implies that it is not necessary for the investors to have private information for their actions to affect commodity risk premia. As a consequence, the latter may depend on the nature of agents' learning mechanism based on common signals, e.g. past prices and aggregate demand. We show in Section 5 that the learning mechanism inherited in adaptive expectations closely resembles observable forecasts as proxied by the average survey prediction.

According to Keynes (1930) and Hicks (1939), when hedgers are net short, futures prices are set at a discount with respect to the future expected spot price at date \( t+h \), i.e. \( \gamma_t^{(h)} > 0 \). This is because hedgers are willing to pay a premium for risk transfer to speculators that provide insurance against the risk of fluctuations in the spot commodity. On the other hand, if hedgers are of net long, futures prices are set at a premium with respect to the expected spot commodity as speculators are willing to take into contract with slightly negative payoff provided there is expectations of increasing futures price, i.e. \( \gamma_t^{(h)} < 0 \). In reality, the structure of commodity markets and price fluctuations, as well as the heterogeneity of the factors that drives risk sharing mechanisms, make unlikely that hedgers are permanently either of net short or long, such that expected risk premia are inherently time-varying. The evidence for time-varying risk premia in commodity markets is compelling in the finance literature (see, e.g. Fama and French 1987, Alquist and Kilian 2010, Hong and Yogo 2012, Gorton et al. 2013, Basu and Miffre 2013, Baumeister and Kilian 2014, Singleton 2014 and Szymanowska et al. 2014 among others). Section 6 confirm these evidence across commodities and investment horizons.
5.1 Econometric Framework

Linking the time-variation of ex-ante risk premia to a set of observable risk factors poses two main challenges. First, the exposure of risk premia to a given economic variable is not necessarily constant over time. Consider for instance the increasing weight of China for the global economy. This possibly generates spillover effects due to shocks in Chinese stock valuations more sizeable. Second, the optimal set of economic risk factors is arguably unknown ex-ante and potentially changes over time. As a result, for \( m = 1, \ldots, M \) set of explanatory factors one have to consider all of the possible \( 2^M \) model specifications at each time \( t = 1, \ldots, T \). Even in a relatively simple regression-based analysis with a limited number of variables it would be infeasible to investigate pricing determinants by simply going through all of \( K = 2^{MT} \) combinations.

We use a dynamic modeling framework that explicitly allows for a time variation in the relationship between the risk premia \( y_{t+1}^{(h)} \) over the interval \((t, t+1] \) and the realizations of the explanatory factors observed at time \( t \), \( Z_t \). Also, we estimate a dynamics model for each of the \( K \) combinations and investigate ex-post the optimal set of factors.

In the following, for the ease of exposition we drop the superscript \( (h) \) that indicates the expectations horizon from the notation. Observable risk factors have a subscript that indicates the time at which they are known. Let \( m = 1, \ldots, M \) be the number of regressors, which implies \( K = 2^M \) models characterized by a different subset of economic risk factors. Denoting these by \( Z_{k,t} \) for \( k \in K \), our set of models can be written as:

\[
\begin{align*}
  y_{t+1} &= Z'_{k,t} \theta_{k,t} + v_{k,t+1}, \quad v_{k,t+1} \sim N(0, H_k), \\
  \theta_{k,t+1} &= \theta_{k,t} + \varepsilon_{k,t+1}, \quad \varepsilon_{k,t+1} \sim N(0, W_k).
\end{align*}
\]

The vector \( \theta_{k,t} \) consists of unobservable, time-varying, regression coefficients that are specific for the model \( k \in K \) (see West and Harrison 1997 for more details on dynamic linear models).\(^{15}\)

More prominently, the state-space formulation allows to explicitly account for different sources of uncertainty: uncertainty in the relative importance of predictors, uncertainty in the estimated

\(^{15}\)We specify the relationship between risk premia and economic risk factors in a predictive sense. However, our purpose is not to “predict” future expected payoffs, but rather to mitigate standard endogeneity issues that can arise estimating a contemporaneous regression. Indeed, risk premia can potentially feedback one-step ahead, for instance, in the level of inventories or trading based measures such as HP and OI. By imposing a causality from factors to expected premia we can disentangle the direction of the effects in a much clearer way.
coefficients and their degree of time-variation, and uncertainty on the “right” set of predictors. Also, the specification (12)-(13) allows to flexibly consider alternative model restrictions; for instance, if the state variance $W_k$ is set to zero, the regression coefficients are made constant over time, which in turn reflect a standard unconditional least squares regression analysis. In that respect, the magnitude of $W_k$ really gives the size of variability in the exposure of risk premia to factors.

A key advantage of our dynamics specification is that we can investigate the relative dynamic probability of inclusion for each risk factor in the time-series of ex-ante risk premia. Let $I_t = \{1, 2, \ldots, K\}$ denote which model applies at each time period and $y^t = (y_1, \ldots, y_t)$ the time series of expected risk premia for the maturity $h$. The posterior probability that the model $k \in K$ applies given the information on risk factors and premia can be computed as

$$
\pi_{t|t,k} = \frac{\pi_{t|t-1,k} p(y_t|y^{t-1}, I_t = k)}{\sum_{l=1}^{K} \pi_{t|t-1,l} p(y_t|y^{t-1}, I_t = l)},
$$

(14)

with $p(y_t|y^{t-1}, I_t = k)$ be the marginal predictive density for the model $I_t = k$ evaluated at $y_t$ and given past information $y^{t-1}$, and $\pi_{t|t-1,k} = p(I_t = k|y^{t-1})$ represents the conditional probability of model $k \in K$ (see Appendix B for details on how these quantities are computed).

Few comments are in order; first, in this framework we are interested in understanding the sources of risk that can explain risk premia rather than provide a framework to do optimal recursive forecast. In that respect, our approach is suboptimal for real-time forecasting purposes as one should also compute the transition mechanism across models in order to achieve recursive model averaging or selection in a pure out-of-sample fashion. Second, the observational $H_k$ and state variances $W_k$ are estimated using the whole sample of observations of risk premia and factors. As such, although model probabilities are estimated dynamically, the structural variances are considered constant over time.\footnote{However, the framework could be easily extended by using an exponential weighted moving average recursion to obtain dynamic estimates for $H_{k,t}$ and $W_{k,t}$. We leave this for future research.} Third, we acknowledge that the state dynamics (13) implies that the elasticity of risk premia to given factors follows a random walk and that, at least asymptotically, this causes drift to deterministic high or low values of $y_t$, hence generating non-stationary expected risk premia (see, e.g. Kilian and Taylor 2003).\footnote{In a separate robustness check we extend Eq. (13) to be a more general AR(1) for the simple case of a model with all regressors. The estimation results show that the state parameters are highly persistence with low conditional variance. In that respect, the random walk assumption represents an attractive approximation for.”}
Posterior model probabilities $\pi_{t|t,k}$ can be used to rank risk factors in terms of their relative contribution to explain the dynamics of commodity risk premia over time. More specifically, from $\pi_{t|t,k}$ we can compute the marginal probability of including a specific risk factor in the dynamic model at each point in time as;

$$\hat{\pi}_{m,t} = \sum_{k=1}^{K} \pi_{t|t,k} \mathbb{1}_{\{m \in k\}}, \quad t = 1, \ldots, T, \quad \text{and} \quad m = 1, \ldots, M, \quad (15)$$

with $\mathbb{1}_{\{m \in k\}}$ an indicator function that takes value one if a given risk factor is included in model $k = 1, \ldots, K$ and zero otherwise. In that matter, $\hat{\pi}_{m,t}$ measures the importance of the $m$th factor in the dynamics of commodity expected risk premia at time $t$.

The sequential model description in (12)-(13) requires that the defining quantities at time $t$ be known at that time. Let $D_{k,0}$ contains the initial prior information about the elasticities and structural variances for the model $I_t = k$. We assume prior information about $\theta_{k,0}$ is vague and centered around the initial hypothesis of no effect of risk factors on premia, i.e. $\theta_{k,0}|D_{k,0} \sim N(c_{k,0}, C_{k,0})$, with $c_{k,0} = 0$ and $C_{k,0} = 10,000$. Also, we assume that the impact of risk factors is highly uncertain and volatile, as capture by an Inverse-Wishart distribution with small degrees of freedom and large scale parameter, i.e. $W_k|D_{k,0} \sim IW(a_{k,0}, A_{k,0})$ with $a_{k,0} = 3$ and $A_{k,0} = 10,000$. As a result we assume that when no historical information on expected risk premia and factors is available, elasticities are mainly driven by idiosyncratic risk as proxied by an Inverse-Gamma distribution with uninformative hyper-parameters, i.e. $H_k|D_0 \sim IG(n_{k,0}/2, n_{k,0}N_{k,0}/2)$ with $n_{k,0} = 0.001$ and $N_{k,0} = 0.001$. Notice priors are constant for all maturities $h = 2, 3, 4$ quarters. In Appendix B we fully describe in full details how parameters are estimated through a Gibbs sampler once historical information on expected risk premia is available.

The marginal probability for each model $p(y_t|y^{t-1}, I_t = k)$ is found by integrating the conditional density $p(y_t|y^{t-1}, I_t = k, \Theta_k)$ over the range of parameters $\Theta_k = (\theta_{k}^{T}, W_k, H_k)$. The posterior probability of $I_t = k$ is then updated according to (14).

because of its parsimony, ease of computation and the smoothness it induces in the estimated sensitivities over time. We share this findings with a large literature on returns predictability that assumes time variation in the predictive coefficients (see, Kilian and Taylor 2003, Ferreira and Santa-Clara 2011 and Dangl and Halling 2012). Similar to our argument they find that assuming parameters are random walks in predicting excess returns we benefit from a substantial reduction of estimation error without effectively increasing the precision in the estimated dynamics in a finite sample.
6 Empirical Results

In this section, we first focus on the validation of our model-implied expected future spot prices and risk premia by using the available survey expectations as benchmark. Second, after documenting that ex-ante risk premia are time-varying, we investigate the driving factors that generate such in-sample variation by using first an unconditional regression analysis and second the dynamic model specification as outlined in Section 5.1.

6.1 Investor Expectations and Risk Premia

For the ease of exposition we report the results for dollar value expectations at maturities \( h = 2, 4 \) quarters.\(^\text{18}\) The sample period is from 12:2006 to 01:2016 for the survey, aggregated monthly, and ranged from 01:1995 to 01:2016 for the model-implied expected future spot prices. The first 24 months of the model-based expectations are cut as burn-in sample for adaptive learning. Figure 4 reports the results for the WTI Crude Oil and Copper.

[Insert Figure 4 about here]

As far as a two-quarter ahead prediction is concerned (top-left panels), the survey forecasts (red line with circles) and the adaptive expectations (light-blue line with diamond markers) line up fairly well across the overlapping sample for WTI Crude Oil. This holds also during the dramatic rise and subsequent sharp fall in crude oil prices during the period 2008/2009 and also during the decline in spot valuations occurred since 2014. Adaptive learning replicates survey expectations also over a four-quarter horizon (top right panel). Bottom panel compares the model and the survey expectations for Copper. Also, in the case of Copper, adaptive learning can mimic the drop in spot price expectations across the period 2008/2009, the subsequent rapid price recovery as well as the downward trend from 2011 until the end of our sample. The survey and the model tends to diverge both at the short-term (bottom left panel) and the intermediate term, i.e. \( h = 4 \) (bottom right panel) across the high valuations from end 2006 to the end of 2008. We show below that such difference in dollar value expectations translates in a modest

\(^{18}\)The empirical evidence for the intermediate horizon \( h = 3 \) are essentially redundant with respect to the dynamics shown in \( h \) equal to 2 and 4. Results are available upon request.

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difference in terms of percentage risk premia. Figure 5 shows the results for Corn and Silver,

[Insert Figure 5 about here]

A comparison with observable expectations for Corn (top panel) is limited by the few observations we have from the survey, which does not provide opinions from analysts in the period of increasing spot prices that characterized the years 2011-2013. However, throughout the last part of the sample the model and survey expectations line up remarkably. As far as Silver (bottom panel) is concerned, our adaptive learning model almost perfectly replicates the dynamics outlined by the survey forecasts, with the partial expectation of the period 2011-2013 for the four-quarter maturity.

After having shown that model and survey-based price expectations sensibly overlap, we now compare the risk premia obtained by directly using the survey forecast with the risk premia extracted by approximating $S_{t+h|h}$ iterating forward the one-step ahead forecast (10). Figure 7 shows the risk premia for WTI Crude Oil (top panel) and Copper (bottom panel). The average sample correlation between survey- and model-implied risk premia is 0.8 for $h = 2$ and $h = 4$. More prominently, our adaptive expectations model allows to replicate the large fluctuation in the risk premia that characterized the period 2008/2009 at both maturities. The expected risk premia for Copper are less correlated on average, i.e. 0.66, with the ones resulting from survey forecasts.

[Insert Figure 6 and 7 about here]

As far as Corn and Silver are concerned, the sample of risk premia extracted from surveys and the adaptive model are highly, although not perfectly, correlated. The consistency between survey and model predictions tends to slightly deteriorate as the expectations horizon increases. Risk premia fluctuates widely between positive and negative values, with an unconditional mean which is not statistically different from zero. This evidence point against the assumption of constant, non-zero risk premia, meaning that long-only investment strategies should not reward investors in the long run.

Generally speaking, the futures curve might not be an ideal forecasting (see, e.g. Kilian and Taylor 2003 and Alquist and Kilian 2010). However, one may argue that the fact that
commodity futures have not been reliable predictors of subsequent price movements does not necessarily imply that the forward curve represents a biased estimate for the expectations of market participants. Table 3 shows this case in point.

[Insert Table 3 about here]

The unconditional risk premium for WTI and longer maturity and Copper at all maturities is negative. This is consistent with the theory of Keynes (1930) and Hicks (1939), which emphasizes that producers, who are typically reducing pricing risk selling futures contracts are the predominant group of hedgers in the market and therefore, need to buy the excess demand for hedging from speculators paying a risk premium. Conversely, the unconditional risk premium for Corn and Silver is positive across all maturities. This evidence, according to the hedging pressure theory, should be the result of consumers being on average the predominant type of hedgers. Indeed, the excess demand for long futures positions is obtained from speculators buying at a higher price than the expected future spot price.

A great deal of empirical research show that commodity realized risk premia are time varying (see, e.g. Fama and French 1987, Alquist and Kilian 2010, Hong and Yogo 2012, Gorton et al. 2013, Basu and Miffre 2013, Baumeister and Kilian 2014, Singleton 2014 and Szymanowska et al. 2014 among others). Figures 6-7 confirms these results for the ex-ante risk premia, which show large fluctuations and periods in which they are persistently positive or negative. The fact that risk premia have their own dynamics implies that the group of traders driving prices at any time $t$ is given by the group with the strongest incentive to trade.\footnote{Time variation in commodity risk premia could be linked to time-varying risk risk-bearing capacity. Acharya et al. (2013), Etula (2013), and Cheng et al. (2015) emphasize that the amount of risk investors’ are willing to take varies over time. This implies that the degree of risk sharing, and thus the risk premium required by hedgers and speculators is not constant over time.} Top panels of Figure 8 show the conditional mean of commodity risk premia obtained from an exponential weighted moving average for different maturities.\footnote{One may argue we could use standard rolling window estimates for the conditional mean rather than an exponential weighted average scheme. However, rolling window averages are inefficient in capturing fluctuations in the time series of risk premia. Such inefficiency can be better understood assuming the risk premium at time $t$ for maturity $h$ is originated by two orthogonal components, $y_t^{(h)} = \mu_t^{(h)} + \psi_t^{(h)}$, with $\psi_t^{(h)} \sim N \left(0, \sigma_{\psi_t^{(h)}}^2 \right)$. Rolling window estimates $\hat{\mu}_t$ exploit a limited amount of information assigning equal weight to each observation from $y_{t-n}^{(h)}$ to $y_t^{(h)}$: $\hat{\mu}_t^{(h)} = \sum_{i=0}^{n-1} \omega_i L^i y_t^{(h)}$, with $\omega_i = \frac{1}{n} \mathbb{I} \{i < n\}$,} Few comments are in order; first recursive estimates
of the expected risk premia shows large fluctuations in Oil ex-ante payoffs across our sample. On average risk premia for WTI turned from negative to positive after the recent great financial crisis of 2008/2009 until the end of 2013, which means HP has been driven by consumers willing to hedge and financial traders who where expecting declining futures prices. The sign of the risk premium for Copper tends to show positive higher risk premia in expansions and suffer during economic slowdown. Most drawdowns coincide with periods of slow global growth, which could explain the incentive of commercial producers to hedge against decreasing spot prices.

Bottom panels of Figure 8 show that the risk premium for Corn and Silver are predominantly positive over time, although steadily declined and turned to negative over the last few years of the sample. Unlike Oil and Copper, the average risk premium for both Corn and Silver does not show a strong correlation with business cycle fluctuations, which is coherent with being an agricultural commodity and a precious metal, respectively. Interestingly, the risk premium for all commodities turned to negative in the very last part of the sample.

Finally, Table 4 shows that, unconditionally, risk premia across different commodities are significantly, although weakly, correlated in the cross-section. The correlation between Oil and Copper tend to be stronger (0.361 at $h = 2$) although decreasing as the expectations horizon increases, i.e. drops to 0.243 at $h = 4$ horizon. The unconditional dependence between the risk premia on Silver and the other commodities is stable across products and steadily declining as the expectations horizon $h$ increases.

As a whole, coupling the evidence in terms of the AR(1) coefficient provided in Table 3, we
can conclude that risk premia are significantly correlated both in the time series and in the cross section, although the size of cross-sectional (time-series) correlation tends to be small and decreasing (increasing) as the expectations horizon increases.

6.2 Dissecting Commodity Risk Premia

As a preliminary analysis on the origins of risk premia we estimate a static version of the observation equation (12) in which we consider all of the economic risk factors $Z_t$ outlined in Section 3.3. For the ease of interpretation, all of the economic predictors and risk premia are standardized by dividing by their respective sample standard deviation. Table 5 shows the coefficients and the corresponding robust standard errors in parenthesis. From this table it can be concluded that HP and time-series Momentum explain the majority of the sample variation of the risk premia across commodities and investment horizons (see, e.g. De Roon et al. 2000, Basu and Miffre 2013, and Szymanowska et al. 2014).

Except for futures for Corn at two-quarter horizon, for each contract OI is positively and significantly related to the ex-ante risk premia, after controlling for net supply-demand imbalances among hedgers. The positive effect of market activity on risk premia is consistent with the idea that increasing market activity signal better economic conditions, which in turns increases the marginal propensity of hedgers to take a net long position, generating price pressure on futures and the corresponding risk premia required by speculators to take the short side of the trade. This result is in line with Hong and Yogo (2012) who show that OI has a significant predictive power for realized payoffs in commodity markets in the presence of hedging demand and limited risk capacity. Similarly, a Value factor turns out to be negatively and significantly related to risk premia with the only exception of futures contracts on Copper. This result is in line with Asness et al. (2013) who find consistent value and momentum premia in commodity markets, among other asset classes. The negative effect of Value can be rationalized by mean-reversion

\[\text{Insert Table 5 about here}\]

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\[\text{21We estimate the model by OLS with GMM corrected standard errors to account for autocorrelation and heteroschedasticity in the residuals $\nu_{t+1}$. For the ease of exposition we report only the results for two- and four-quarter ahead ex-ante risk premia. The results for the three-quarter ahead expectations are similar and therefore are not reported separately.}\]
in future spot prices; when current spot prices are low with respect to their anchoring value, i.e. high value commodity, there are expectations of increasing future spot prices which rapidly reduces the risk premia paid by, for instance, net-short hedgers.

The unconditional regression results of Table 5 suggest that risk sharing mechanism can sensibly explain the in-sample variation of the ex-ante risk premia. However, Figures 6-7 made clear that risk premia are not constant over time and experience large swings in sign and magnitude. The fact that risk premia have their own dynamics could be the consequence of an heterogeneous, on a time scale, exposure to different risk factors. In that respect, the results of a static regression might be potentially incomplete, at best.

In the following, we exploit the dynamic linear model (12)-(13) and carefully investigate whether the time variation of expected risk premia is mainly due to random unpredictable shocks, the nature of commodity market participants, or is the result of changes to market and economic conditions. Figures 9-10 shed light on which predictors are important at each time $t$ by showing the factor-specific probability of inclusion in the dynamic regression from (15). For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time. Top panels of Figure 9 shows the results for WTI Crude Oil for both a two-quarter (top-left panel) and a four-quarter (top-right panel) horizon. The empirical evidence shows that the effect of financial risk in emerging markets, as proxied by the MSCI Emerging Market Index (MXEF), has become increasingly important especially in the aftermath of the great financial crisis of 2008/2009. A possible explanation is the presence of spillover effects due to the increasing weight of the emerging economies, e.g. China, in the global economic outlook, as growth in developing markets accounts for over 70 percent of global growth in 2016 (see, IMF Economic Outlook 2016). Indeed, although the direct impact of equity valuations in emerging markets is relatively low due to moderate foreign investments, financial turbulence in this area is often associated with issues about the global economic slowdown.

[Insert Figure 9 about here]

Trading activity proxied by OI, also explains a considerable fraction of in-sample variation of risk premia in the period that coincides with the dramatic rise in oil prices between 2003 to the end of 2008. The jump of the conditional explanatory power of OI at the beginning of
2003 coincides with the Iraq invasion of March 20th 2003 and the overall higher volatility that affected the stock market, which increased the propensity of hedgers and speculators to trade on the futures market. Generally speaking, increasing (decreasing) OI means that new cash is flowing into (out of) the marketplace and implies that the prevailing price trend is coming to an end. As shown by Cheng et al. 2015 and Hong and Yogo 2012, when there are limits to the risk-bearing capacity of investors, large changes in market liquidity, i.e. desired long or short positions, possibly affect prices both in the futures and spot markets and ultimately affect expectations of future payoffs.

A large explanatory power is also shown by a time-series Momentum factor, which generally aims at capture the short-term autocorrelation in commodity spot returns driven by psychological biases of market participants and their learning about commodity fundamentals (see, e.g. Cutler et al. 1990 and Greenwood and Shleifer 2014). Similarly, HP shows a strong explanatory power throughout the sample, confirming the initial intuition provided by the static regression results of Table 5. The strong importance of HP for the dynamics of expected payoffs confirms the primary relevance of futures as a risk insurance market place, as postulated by Keynes (1930) and Hicks (1939). Interestingly, the effect of a Value factor increases with the horizon of the investors expectations and becomes relevant especially after the financial crisis of 2008/2009. At its most basic, Value aims at capturing a premium from buying (selling) undervalued (overvalued) commodities, with the expectation that spot prices will increase (decrease) in the near future. As such, the increasing importance for the ex-ante risk premia four quarter ahead possibly capture the belief by market participants of a mean reversion in prices in the mid-to-long run.

Bottom panel of Figure 9 shows the results for Copper for both a two-quarter (top-left panel) and a four-quarter (top-right panel) horizon. Much of the results for Oil holds also for Copper, which is not surprising as metals industrial and energy commodities are commonly sensitive to fluctuations over the business cycle and share most of the risk factors exposures and similar storage costs (see, e.g. Bhardwaj et al. 2015). Indeed, similar to Oil, the price of Copper is primarily determined by demand for goods and services that require Copper as well as the ability of suppliers to extract and transport the product.

Moving to agricultural commodities, top panels of Figure 10 shows that again HP, Momen-
tum and Value capture and important part of the dynamics of risk premia for \( h = 2, 4 \) quarters ahead. Momentum in agricultural markets can be generated by irregular production. Taking Corn as our example, consumer demand remains fairly stable throughout the year whilst production is seasonal and can vary hugely. For instance, a bad harvest in October/November in the U.S. (which represents around 40% of the global production) cannot be rebalanced until a good harvest occurs in the south hemisphere the next production cycle or in the U.S. the next year, increasing prices and possibly generating positive momentum as supply expectations are revised downward and stockpiles decrease.

Unlike WTI Oil and Copper, the expected risk premia of Corn is increasingly related to the level of interest rates and the U.S dollar exchange rate. A possible explanation of the impact of interest rates could be related to the effect of the cost of credit for industrial market participants. As a matter of fact, interest rates could have significant effects on the agricultural industry by affecting the cost of holding inventories, borrowing money, and investment decisions such as land, equipment and input purchases. Another explanation relates to the relevance of interest rates for the profitability of inter-temporal arbitrage strategies. For instance, when a commodity market is in contango, arbitrageurs borrow money to buy the physical commodity, which will then to be delivered in the future after having locked in a risk-less profit by selling over-priced futures contract. This is consistent with the fact that on average the risk premium on Corn is positive unconditionally, as shown by Table 3. To summarize, unexpected and adverse movement of interest rates is a source of operating risk for farmers that may result in higher than planned interest expenses which reduce the profitability of farms and agri-businesses, discourage investment and decrease farmland values. All of this would result in a reduction of production which in absence of decreasing demand would inevitably makes investors’ expectations for future payoffs decreasing.

The U.S. dollar exchange rate also affects risk premia for Corn. Again, this is not surprising as the U.S. represents on itself 40% of the global production for Corn, and the U.S. dollar is the globally recognized currency upon which commodity trade is based. In that matter, a weak dollar generally leads to higher exports for the U.S. as a consequence of higher demand.
given more competitive prices, but it also means that the production of Corn will becomes less profitable (see, e.g. Hamilton 2009). Another possible explanation relies on the increasing financialization of the agricultural commodity markets. As shown by Tang and Xiong (2012), after 2004, agricultural commodities included in financial indexes such as the Goldman Sachs Commodity Index (GSCI) and the Dow Jones (DJ)-AIG, became much more responsive to shocks to the U.S. dollar exchange rate.

Similarly to Corn, bottom panels of Figure 10 show that in addition to time-series Momentum and HP, the risk premium on precious metals such as Silver is also explained by changes in the U.S. dollar exchange rate. In worldwide countries, the price of Silver is dollar-denominated; therefore, fluctuations of the U.S. dollar have a great impact on prices of precious metals in general. A falling dollar exchange rate makes Silver price in Euros and yen-denominated as relatively cheap in the domestic market, which attracts investors to buy to preserve and increase profits. These capital inflows naturally promote the rise of Silver expected future spot price. As a result exchange rates tend to weigh on the Silver risk premium, so the two typically move in opposite directions. Unlike energy and industrial-related commodities, such mechanism is exacerbated by the fact that precious metals are historically considered as safe assets, which are bought to protect against currency depreciation and corresponding increasing inflation. Finally, similar to WTI Crude Oil and Copper, trading activity as proxied by OI and a Value factor also explain the risk premium dynamics toward the end of the sample.

6.2.1 A Further Discussion on Hedging Pressure and Momentum

The dynamic nature of the model (12)-(13) together with the time-variation in both risk premia and economic risk factors make the interpretation of the time-varying parameters quite cumbersome. As a matter of fact, the size and magnitude of the risk premia do not have a perfect symmetric interpretation as it all depends on which type of investor is dominating the demand for futures at a given time $t$. However, we believe it is worth it to try to speculate a bit on the interpretation of the results concerning two main predictors which turned out to be significant in the dynamic regression results, i.e. HP and Momentum. Figure 11 shows the posterior median of the betas for HP, i.e. $\hat{\beta}_{HP}$ across commodities for $h = 2, 4$ quarter-ahead

32
There is a fair amount of heterogeneity in the dynamics of $\hat{\beta}_{HP}$, although some common feature emerge. Except for Copper in 2006, the sign of the estimated coefficient is always positive, meaning that, on average throughout the sample, increasing net-excess short positions tend to be associated with higher risk premia across commodities. Also, the effect of HP tends to increase relatively after 2005 and to decrease after 2011, i.e. during the boom and the bust of the commodity super-cycle. A possible explanation lies on the fact that HP directly depends on constraints on the amount of capital different investor categories are willing to commit (see, e.g. Acharya et al. 2013 and Etula 2013). In this respect, HP typically increases when there expectations of falling prices on the future spot market such as for instance during the crisis of 2008/2009. Therefore, $\hat{\beta}_{HP}$ signals that when prices go down the marginal propensity to take the long side of the trade decreases, namely, traders to take long positions require a much higher risk premium ex-ante. These results are also in line with the findings of Kang and Tang (2014), who show that the cost for speculators to provide liquidity increases when the positions of hedgers become more imbalanced.

Figure 12 shows the posterior median of the betas for Momentum, i.e. $\hat{\beta}_{Mom}$ across commodities for $h = 2, 4$ quarter-ahead expectations horizons. Momentum strategies are found to be related to changes in the price of futures contracts themselves and to the propensity of market participants to trade in backwardation or in contango. This implies that Momentum is intimately interrelated to hedging against sharp market corrections as postulated by the theory of normal backwardation of Keynes (1930) and Hicks (1939). In agricultural commodities, however, momentum Therefore, momentum arises not only because of trading behaviours but also because of the production environment and the limited storage availability.

The peculiarity of agricultural commodities is somehow confirmed in Figure 12. Except for Corn, the effect of Momentum decreases throughout the boom and bust of commodity prices over the period 2005-2011. On average over the sample and across commodities, increasing past returns tend to be associated with a lower ex-ante risk premia. The dynamics of $\hat{\beta}_{Mom}$ is
consistent with the idea that increasing prices make hedgers more willing to take a long position; on the contrary speculators believing in mean-reversion are more willing to short when prices are high. This supply-demand mechanism lowers the risk premia embedded in futures contracts.

6.3 Model Assessment

One may argue that a full model in which all of the risk factors are given equal weight can equivalently shed light on the dynamics of commodity risk premia. To address this concern, we compare the in-sample goodness-of-fit of our dynamic model specification in which economic risk factors are weighted according to their probability of inclusion (15), with respect to two alternative mainstream specifications. The first benchmark is a model in which none of the economic risk factors is included and the dynamics of the ex-ante risk premia is determined uniquely by a time-varying intercept in addition to unpredictable idiosyncratic shocks. This specification is suitable to investigate the trade-off between including potentially irrelevant factors as opposed to consider the most possible parsimonious specification. The second benchmark model is a dynamic linear model with all the economic risk factors included, which allows to investigate the contribution of the weighting scheme induced by (15) as opposed to considering all of the risk factors as equally relevant for the dynamics of risk premia.

We assess the models performance by computing an in-sample Relative Root Mean Squared Error (RMSE) and a Bayes predictive factor. Gneiting (2011) showed that RMSE is a consistent evaluation measure when the point estimates equals the mean of the posterior distribution. As such we compute point predictions for each model as the average of the corresponding marginal posterior distribution of the ex-ante risk premia across commodities and horizons integrating out parameter uncertainty, see Appendix C for a more detailed explanation. Panel A of Table 6 shows the relative RMSE computed by taking the ratio between the RMSE and the competing specification. A number lower than one implies a better performance of the weighting scheme;

[Insert Table 6 about here]

Panel A shows that there is value in our dynamic weighting scheme as suggested by a reduction of the squared loss in the order of 50% on average. More precisely, the RMSE of a model with only a time-varying intercept is more than double for eleven out of twelve cases. This implies
that the risk factors effectively convey information in explaining the time-series variation of the ex-ante risk premia, across commodities and for different expectation horizons. Including all of the factors with equal weight throughout the sample, although clearly improves the goodness-of-fit with respect to the time-varying intercept specification, does not change the ranking. Indeed, for all of the cases the RMSE for the weighted model is lower and is less than half for three out of twelve cases. The closest RMSE with respect to the weighted model is for the pair (Corn, \( h = 2 \)) which however still shows a RMSE which is 25% higher than our dynamic weighted model specification.

Although the RMSE reveals interesting aspects of the in-sample goodness-of-fit implied by the marginal posterior means, it cannot provide insight into the uncertainty that is associated with producing conditional mean estimates. In that respect, a direct evaluation of the marginal likelihood is a natural tool to assess the ability of a dynamic weighted average specification to explain unusual developments in commodity expected payoffs, such as the likelihood of large drops or jumps in risk premia given current information. We now couple the evidence from the RMSE with a formal comparison of the marginal densities based on a Bayes factor that directly compares the marginal likelihood across models. In our setting, an analytical evaluation of the marginal likelihood is not possible. Gelfand and Dey (1994) and Newton and Raftery (1994) showed that a simulation consistent estimate of the marginal likelihood for a model \( M_i \) is obtained by the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions (see Appendix C). Panel B of Table 6 shows the Bayes’ factors in \( \log_{10} \) scale; a value greater than .5 would represents decisive evidence in favour of our dynamic weighted specification (see, Kass and Raftery 1995). The empirical results show substantial evidence in favour of our model specification, with values in the range of 13.02 (Silver, \( h = 3 \)) to 50.59 (WTI, \( h = 4 \)). Similarly, the Bayes factors show that by including all regressors to explain the dynamics of risk premia reduces the possibility to capture efficiently anomalous realizations as suggested by a Bayes factor that ranges from 0.86 (Copper, \( h = 2 \)) to 8.62 (WTI, \( h = 4 \)).

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22 A potential issue in using the harmonic mean of posterior-implied conditional likelihoods is that the inverse likelihood does not have finite variance (see, e.g. Chib 1995 for a detailed discussion). However, in our setting, the (log of) marginal likelihood can be efficiently computed through the Kalman filter recursions, mitigating potential concerns in using an harmonic mean approximation. In that respect, when computing the log-marginal likelihood, we checked the stability of the conditional likelihood for each draw from the posterior distributions. Results of the draw specific likelihood evaluation are available upon request.
7 Concluding Remarks

Our empirical analysis shows that investor expectations of future commodity spot prices can be rationalized by an adaptive learning scheme in which expected future spot prices are affected by past prediction errors and changes in aggregate demand as proxied by an index of world industrial production. This expectations formation mechanism can replicate, for example, the time-variation of (ex-ante) risk premia across commodities and forecasting horizons. In that matter, adaptive learning provides a framework to extract time-varying risk premia.

By using a dynamic linear regression in which we accommodate uncertainty in: (1) the relative importance of alternative predictors, (2) the estimated coefficients and (3) their degree of time-variation, we show that time-variation in commodity risk premia is predominantly due to risk sharing channels and the changing nature of commodity market participants, as proxied by HP and partly by OI. In addition to trading activity, an important determinant for the dynamics of commodity risk premia is the persistence of past spot returns and their relationship with fundamental valuations, as proxied by time-series Momentum and Value factors.

References


Appendix

A A Simple Model of Adaptive Expectations

We start from a simple rational expectations model which is closely related to the Muth (1961) market model with inventory speculation except demand shocks are predictable and not i.i.d. The market behavior is characterized by an infinite horizon, discrete time model with a market clearing condition that holds in each period, $t+1$:

$$C_{t+1} + I_{t+1} = Q_{t+1} + I_t,$$

(A.1)

where $Q_{t+1}$ represents the output produced for a commodity in a period lasting as long as the production lag, $C_{t+1}$ is the amount of commodity consumed in the same time period, and $I_{t+1}$ the commodity inventories at the end of period $t+1$. The standard Muth (1961) market model posits there are three categories of economic agents active in the market for commodities; the buyers, the producers and the inventory holders. The latter can capture speculation effects. Their aggregate demand, supply and holding functions are

$$C_{t+1} = -\delta S_{t+1} + z_{t+1},$$

(A.2)

$$Q_{t+1} = \lambda E_t [S_{t+1}] + u_{t+1},$$

(A.3)

$$I_{t+1} = \nu (E_t [S_{t+1}] - S_{t+1}),$$

(A.4)

with $S_{t+1}$ is the market price at date $t + 1$, and $E_t [S_{t+1}]$ is the market price expected to prevail at time $t + 1$ given the information available through the $t - th$ period. We extend the standard market model with inventory speculation assuming exogenous factors that affect aggregate demand are predictable and potentially persistent;

$$z_{t+1} = b z_t + e_{t+1},$$

(A.5)

with $e_{t+1}$ and $u_{t+1}$ zero-mean i.i.d. disturbance terms. Storage costs are assumed to be zero to simplify the model. These equations and assumptions are the same of the original Muth (1961) model, except for the predictability of demand shocks. Substituting (A.2)-(A.5) in the equilibrium condition (A.1), the spot market equilibrium can be expressed in terms of prices, price expectations, demand shocks and disturbances;

$$- (\nu + \delta) S_{t+1} + b z_t + e_{t+1} = \lambda E_t [S_{t+1}] + u_{t+1} + \nu S_t,$$

$$(\nu + \delta) S_{t+1} = b z_t + e_{t+1} + \lambda E_t [S_{t+1}] - u_{t+1} + \nu S_t,$$

which can be rewritten as a simple linear model as follows

$$S_{t+1} = \mu + \beta E_t [S_{t+1}] + \theta S_t + \omega z_t + \eta_{t+1},$$

(A.6)

By taking expectations on both sides and substituting back in (A.6), we can obtain a unique reduced-form Rational Expectations Equilibrium (REE) as

$$S_{t+1} = \phi_0 + \phi_1 S_t + \phi_2 z_t + \eta_{t+1},$$

(A.7)

with $\phi_0 = (1 - \beta)^{-1} \mu$, $\phi_1 = (1 - \beta)^{-1} \theta$, $\phi_2 = (1 - \beta)^{-1} \omega$ and $\eta_{t+1} = e_{t+1} - u_{t+1}$. This solution is the same as the original Muth (1961)'s model except that future commodity spot prices now depends on aggregate demand. Notice that for a given level of commodity prices, Eq. (A.7) implies that a positive (negative) shock to aggregate demand increases (decreases) future prices, while a positive (negative) shock in aggregate supply decreases (increases) prices. The difference between shocks on aggregate demand and supply is that the former

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*We assume there is a period distance in the future where the forward expectations are equivalent, i.e. $E_t [S_{t+1}] \equiv E_{t+1} [S_{t+2}]$, (see, e.g. Beck 1993).*
cumulates and affects future prices through \( z_t \), while shocks to aggregate demand are short-living.\(^{24}\)

The key assumption to introduce learning is that the expectations of economic agents \( E_t[S_{t+1}] \) are not necessarily rational as agents do not know the structural parameters. Expectations are instead formed on the basis of current observations and predictions of parameters which are updated over time. There are two key building blocks to explicit the agents’ learning dynamics. First, agents beliefs are described by means of a basis of current observations and predictions of parameters which are updated over time. There are two key components of the model (see Sargent and Williams 2005). Agents’ recursive optimal estimate of \( \phi_{t+1} \) conditional on information available up to time \( t \). \( \gamma_{t+1} = \phi_{t+1} t \) are provided by the Kalman filter recursion;

\[
\begin{align*}
\gamma_{t+1} &= \gamma_t + K_t (S_{t+1} - \gamma_t X_t), \\
R_{t+1} &= R_t - R_t X_t R_t X_t' / R_t + 1 + \sigma^{-2} \Omega,
\end{align*}
\]

where \( K_t = R_t X_t [X_t' R_t X_t + \sigma^{-2}]^{-1} \) determines the degree of updating of agents’ beliefs when faced when an unexpected commodity spot price \( S_t - \gamma_t X_t \), i.e. Kalman gain. The recursive learning dynamics (A.8) represents a generalization of a recursive learning with constant gain as specified in Evans and Honkapohja (2001), Sargent (2002), Cho et al. (2002), and Williams (2003), among others.

**B Estimation Strategy**

In this section we provide details of the Gibbs sampler we use for the estimation of the dynamic linear model (12)-(13). For the ease of exposition, we report the updating scheme conditional on \( L_t = \{1, 2, \ldots, K\} \) and disregard the maturity super-script \( h \). Let us denote \( x_{s,t} = (x_s, \ldots, x_t) \), \( s \leq t \), the set of vectors \( x_u \). The collections of parameters is defined as \( \Theta = (\theta_{1:T}, W, H) \), respectively, where \( \theta_{1:T} \) represents the \((T \times N)\) matrix of state parameters. Let \( \theta_0 \) represents the initial value of the dynamic sensitivity to the \( k \)-dimensional vector of regressors. The complete likelihood function can defined as

\[
p(y_{1:T}, \theta_{1:T} | Z_{1:T}, W, H) = \prod_{t=1}^{T-1} p(y_{t+1} | Z^t_{\theta_t}, H) p(\theta_t | \theta_{t-1}, W),
\]

with \( p(y_{t+1} | Z^t_{\theta_t}, H) = N(Z_{\theta_t} H, N_k(\theta_t, W)) \) two univariate and multivariate Gaussian distributions, respectively. The sequential model description in (12)-(13) requires that the defining quantities at time \( t \) be know at that time. We assume prior information about \( \theta_0 \) is vague and centered around the initial hypothesis of no effect of risk factors on premia, i.e. \( \theta_0 | D_0 = N(c_0, C_0) \), with \( c_0 = 0 \) and \( C_0 = 10,000 \). Also, we assume that the impact of risk factors is highly uncertain and volatile, as capture by an Inverse-Wishart distribution with small degrees of freedom and large scale parameter, i.e. \( W | D_0 \sim IW (a_0, A_0) \) with \( a_0 = 3 \) and \( A_0 = 10,000 \). As a result we assume that when no historical information on expected risk premia and factors is available, elasticities are mainly driven by idiosyncratic risk as proxied by an Inverse-Gamma distribution with uninformative hyper-parameters, i.e. \( H | D_0 \sim IG (n_0/2, n_0 N_0/2) \) with \( n_0 = 0.001 \) and \( N_0 = 0.001 \). Conditional on the latent states \( \theta_{1:T} \) the complete likelihood can be factorized as the product of, as such combining the prior

\(^{24}\) The i.i.d. assumption of shocks to aggregate supply can be relaxed at the cost of having some reliable empirical proxy to be used in modeling the empirical dynamics of expected spot prices.
specification with the factorized completed likelihood (A.10), we obtain the posterior density

\[
p(\theta_{1:T}, W, H | y_{1:T}, Z_{1:T}) \propto p(y_{1:T}; \theta_{1:T}, Z_{1:T}, W, H) p(\theta_0, W, H),
\]

\[
= p(y_{1:T} | \theta_{1:T}, Z_{1:T}, H) p(\theta_{1:T} | W) p (\theta_0, W, H),
\]

The joint posterior distribution of the states and parameters is not tractable analytically such that the estimator for the parameters cannot be obtained in closed form. The latent variables \(\theta_{1:T}\) are simulated alongside the model parameters \(H\) and \(W\). At each iteration, the sampler sequentially cycles through the following steps:

1. Draw \(\theta_{1:T}\) conditional on \(H, W\) and the data \(y_{1:T}, Z_{1:T}\).
2. Draw \(W\) conditional on \(\theta_{1:T}\).
3. Draw \(H\) conditional on \(y_{1:T}, Z_{1:T}\), and \(\theta_{1:T}\).

In what follows we provide details of each step of the Gibbs sampler.

### B.1 Step 1. Sampling the Conditional Factor Sensitivities \(\theta_{1:T}\)

The full conditional posterior density for the time-varying factor loadings is computed using a Forward Filtering Backward Sampling (FFBS) approach as in Carter and Kohn (1994). The initial prior are sequentially updated via the Kalman filtering recursion. Conditionally on idiosyncratic risk \(H\), state variance \(W\), and assuming an initial distribution \(\theta_0 | y_0 \sim N(m_0, C_0)\), it is straightforward to show that the (see West and Harrison 1997 for more details)

\[
\begin{align*}
\theta_t | Z_{1:t-1}, W &\sim N(a_t, R_t) & \text{Propagation Density} \\
Y_t | Z_{1:t-1}, H &\sim N(f_t, Q_t) & \text{Predictive Density} \\
\theta_t | Z_{1:t} &\sim N(m_t, C_t) & \text{Filtering Density}
\end{align*}
\]

with

\[
\begin{align*}
a_t &= m_{t-1} \\
R_t &= C_{t-1} + W \\
f_t &= Z_t a_t \\
Q_t &= Z_t R_t Z_t' + H \\
m_t &= a_t + K_t e_t \\
C_t &= R_t - K_t Q_t K_t'
\end{align*}
\]

and \(K_t = R_t X_t Q_t^{-1}\) and \(e_t = y_t - f_t\). Conditional thetas are drawn from the posterior distribution which is generated by backward recursion (see Frühwirth-Schäffter 1994, Carter and Kohn 1994, and West and Harrison 1997), i.e. \(p(\theta_t | y_{1:T}) = N_k \left( m^b_t, C^b_t \right)\), with

\[
\begin{align*}
m^b_t &= (1 - B_t) m_t + B_t m^b_{t+1}, \\
C^b_t &= (1 - B_t) C_t + B_t^2 C^b_{t+1}, \quad \text{with} \quad B_t = \frac{C_t}{C_t + W}.
\end{align*}
\]

### B.2 Step 2. Sampling the State Variance Parameters \(W\)

Conditional on the risk exposures, the estimate of the state variance covariance matrix coincide with the update of an Inverse-Wishart distribution. Posterior estimates are obtained by updating the prior structure as

\[
W | \theta_{1:T} \sim IW (a_1, A_1)
\]

with

\[
\begin{align*}
a_1 &= a_0 + T \\
A_1 &= A_0 + \hat{\varepsilon}' \hat{\varepsilon}'
\end{align*}
\]

where \(\hat{\varepsilon}' = (\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_T)\) and \(\hat{\varepsilon}_t = \hat{\theta}_t - \hat{\theta}_{t-1}\) given \(\hat{\theta}_t = m^b_t\).
B.3 Step 3. Sampling the Idiosyncratic Risk $H$

For the posterior estimates of the idiosyncratic risk we exploit the fact that the prior and the likelihood are conjugate. The updating scheme is easily derived as

$$H|\theta_{1:T},Z_{1:T},y_{1:T} \sim IG(\nu_1/2,\nu_1N_1/2)$$ (A.13)

with

$$\nu_1 = \nu_0 + T$$
$$\nu_1N_1 = \nu_0N_0 + \hat{v}'\hat{v},$$

where $\hat{v}' = (\hat{v}_1,\ldots,\hat{v}_T)$ and $\hat{v}_t = y_t - Z_t\hat{\theta}_{t-1}$ given $\hat{\theta}_{t-1} = m_{t-1}^b$.

C Marginal Likelihood Approximation

In our setting, an analytical evaluation of the marginal likelihood is not possible. Gelfand and Dey (1994) and Newton and Raftery (1994) showed that a simulation consistent estimate of the marginal likelihood for a model $M_i$ is obtained by the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions. The marginal likelihood is the probability that the model gives to the observed data, averaging over values of its parameters with respect to their prior distribution. If $y' = (y_1,\ldots,y_T)$ is the time series of the risk premia and $\Theta = (\theta^T,W,H)$ is the entire set of parameters for the model $M_i$, then the marginal likelihood is

$$p(y' | M_i) = \int p(y' | \Theta,M_i) p(\Theta | y',M_i) d(\Theta)$$ (A.14)

The harmonic mean of the likelihood with respect to the posterior distribution, can be approximated by using the posterior draws from the Markov Chain Monte Carlo (MCMC) done to estimate parameters of each model;

$$p(y' | M_i) \approx \frac{1}{N} \sum_{n=1}^{N} \frac{1}{p(y' | \Theta^n,M_i)}$$ (A.15)

with $p(y' | \Theta^n,M_i)$ the likelihood evaluated at the $n$th draw of the parameters from the posterior distribution. The Law of Large Numbers guarantees that this estimator is consistent

$$\int \frac{1}{p(y' | \Theta,M_i)p(\Theta | y',M_i)} p(\Theta | y',M_i) d(\Theta) = \int \frac{1}{p(y' | \Theta,M_i)} p(y' | \Theta,M_i) d(\Theta)$$
$$= \frac{1}{p(y' | M_i)} \int p(\Theta | M_i) d(\Theta)$$
$$= \frac{1}{p(y' | M_i)}$$

C.1 Root Mean Squared Error and Predictive Bayes Factor

To compare the performance of alternative models we rely on two complementary measures of in-sample goodness-of-fit. We first compute a relative Root Mean Squared Error (RMSE) which allows to investigate the performance of alternative methodologies compared to our weighted dynamic linear model. For each alternative specification, we compute the relative value as the ratio of the RMSE implied by our model over the benchmark, so that values lower than one indicates that our model improves upon the alternative specification. More specifically, let us define the marginal distribution of the ex-ante risk premia at time $t$ on a given commodity and for a given horizon as

$$p(y_t | M_i) = \int p(y_t | \Theta,M_i)p(\Theta | y',M_i) d(\Theta),$$

43
with

\[ p(\theta \mid y^t, M_i) \propto p(y^t \mid \theta, M_i) \cdot p(\theta \mid M_i), \]

and \( p(y^t \mid \theta, M_i), p(\theta \mid M_i) \) representing the conditional likelihood and the marginal prior probabilities, respectively. The RMSE of the \( i \)th model is defined as

\[
RMSE_i = \sqrt{\frac{1}{T} \sum_{t=1}^{T} (y_t - E[y_t \mid M_i])^2},
\]

(A.16)

with

\[ E[y_t \mid M_i] = \int y_t p(y_t \mid M_i) dy_t. \]

The relative measure is computed as \( \frac{RMSE_i}{RMSE_{weighted}} \). Gneiting (2011) showed that RMSE is a consistent evaluation measure when the point estimate equals the mean of the posterior distribution. Although point estimates of ex-ante risk premia reveal interesting aspects of the explanatory power of our weighted dynamic linear model, such conditional means cannot provide insight into the uncertainty that is associated with producing these forecasts. In that respect, a direct evaluation of the marginal likelihood \( p(y^t \mid M_i) \) is a more natural tool to assess the ability of the weighted dynamic linear model to explain unusual developments in ex-ante risk premia, such as the likelihood of large drops or jumps in future realizations given current information. We therefore compare the alternative specifications as above, on the basis of a Bayes factor. The Bayes factor compares our model \( M_{weighted} \) against the alternative specifications and is defined as

\[
BF(M_{weighted} \text{ vs } M_i) = \log_{10} \left[ \frac{p(y^t \mid M_{weighted})}{p(y^t \mid M_i)} \right],
\]

(A.17)

The Bayes factor is computed in a \( \log_{10} \) scale for the ease of exposition, so that values higher than 0.5 indicates substantial evidence in favour of our benchmark (see Kass and Raftery 1995 for more details). The complete marginal likelihood \( p(y^t \mid M_i) \) for each specification is computed as explained above.
Table 1. Testing Strong Rationality in Survey Expectations

Strong form of rationality. This table shows the results of a test for the null hypothesis of strong rationality of the investor expectations as proxied by the average Bloomberg survey price forecasts database. The sample period for the survey is 12:2006-01:2016, aggregated monthly, and collected for alternative commodities and time-horizons. The commodities considered are WTI Crude Oil, Copper, Silver and Corn, which are representative of the energy, industrial, agricultural and precious metals commodity markets. We exclude from the analysis the survey for Corn as the survey comprises lots of missing data which would make the sample size subject to potentially relevant small-sample biases. Regressions are estimated by GMM correcting standard errors to account for autocorrelation and heteroscedasticity in the residuals. Panel A: shows the results for a test of strong form of rationality. Panel B: shows the results for a the null hypothesis that there is no revision from past errors in the investors’ beliefs, i.e. information efficiency. Robust standard errors are in parenthesis, *p < 0.10,** p < 0.05,*** p < 0.01.

<table>
<thead>
<tr>
<th>Horizon (Quarters)</th>
<th>Commodity</th>
<th>Panel A: Rationality</th>
<th>Panel B: Information Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>α</td>
<td>β</td>
</tr>
<tr>
<td>h = 2</td>
<td>Crude Oil (WTI)</td>
<td>0.004</td>
<td>0.819***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.038)</td>
<td>(0.253)</td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>0.015</td>
<td>0.895***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.041)</td>
<td>(0.252)</td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>-0.019</td>
<td>0.685***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.044)</td>
<td>(0.173)</td>
</tr>
<tr>
<td>h = 3</td>
<td>Crude Oil (WTI)</td>
<td>-0.029</td>
<td>0.716***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.067)</td>
<td>(0.317)</td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>-0.009</td>
<td>0.512***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.057)</td>
<td>(0.201)</td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>-0.023</td>
<td>0.801***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.062)</td>
<td>(0.216)</td>
</tr>
<tr>
<td>h = 4</td>
<td>Crude Oil (WTI)</td>
<td>-0.071</td>
<td>0.706</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.085)</td>
<td>(0.401)</td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>-0.027</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.071)</td>
<td>(0.504)</td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>-0.043</td>
<td>0.634***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.094)</td>
<td>(0.216)</td>
</tr>
</tbody>
</table>
Table 2. Testing the Extrapolative Behaviour on Survey Expectations

Adaptive expectations. This table shows the results of a test for the null hypothesis of extrapolative investor expectations as proxied by the average Bloomberg survey price forecasts database. The sample period for the survey is 12:2006-01:2016, aggregated monthly, and collected for alternative commodities and time-horizons. The commodities considered are WTI Crude Oil, Copper, Silver and Corn, which are representative of the energy, industrial, agricultural and precious metals commodity markets. We exclude from the analysis the survey for Corn as the survey comprises lots of missing data which would make the sample size subject to potentially relevant small-sample biases. Regressions are estimated by GMM correcting standard errors to account for autocorrelation and heteroscedasticity in the residuals. Panel A: shows the results for a the null hypothesis that expectations are extrapolative in its general form. Panel B: shows the results for a the null hypothesis that expectations are revised in line with past prediction errors on future spot prices, i.e. adaptive expectations. Robust standard errors are in parenthesis, "$p < 0.10,$" "$p < 0.05,$" "$p < 0.01.$"

<table>
<thead>
<tr>
<th>Horizon (Quarters)</th>
<th>Commodity</th>
<th>Panel A: Extrapolative Expectations</th>
<th></th>
<th>Panel B: Adaptive Expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\text{adj } R^2$</td>
</tr>
<tr>
<td>$h = 2$</td>
<td>Crude Oil (WTI)</td>
<td>-0.002</td>
<td>-0.668***</td>
<td>0.466</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.017)</td>
<td>(0.095)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>-0.019</td>
<td>-0.491***</td>
<td>0.186</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.018)</td>
<td>(0.193)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>0.027</td>
<td>-0.695***</td>
<td>0.513</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.012)</td>
<td>(0.069)</td>
<td></td>
</tr>
<tr>
<td>$h = 3$</td>
<td>Crude Oil (WTI)</td>
<td>0.020</td>
<td>-0.691***</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.023)</td>
<td>(0.127)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>0.002</td>
<td>-0.497***</td>
<td>0.166</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.019)</td>
<td>(0.199)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>0.041</td>
<td>-0.733***</td>
<td>0.487</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.013)</td>
<td>(0.079)</td>
<td></td>
</tr>
<tr>
<td>$h = 4$</td>
<td>Crude Oil (WTI)</td>
<td>0.041</td>
<td>-0.802***</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.027)</td>
<td>(0.131)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Copper</td>
<td>0.009</td>
<td>-0.537***</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.021)</td>
<td>(0.161)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Silver</td>
<td>0.059</td>
<td>-0.769***</td>
<td>0.465</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.015)</td>
<td>(0.059)</td>
<td></td>
</tr>
</tbody>
</table>
Descriptive statistics. This table reports the descriptive statistics for the risk premia and the commodity-specific factors for WTI Oil Crude, Copper, Corn and Silver. Risk premia are those obtained by filtering out the model-implied expected future spot prices for $h = 2, 3, 4$ quarters ahead from the corresponding futures prices. The blue colors indicates the sample averages that are statistically significant at the 95% confidence level. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). The data on Copper and Crude Oil inventories from the London Metal Exchange (LME) and Energy Information Administration (EIA) respectively. For Corn inventories, we use the U.S. ending stocks reported in thousands of metric tons. Time series momentum and value are constructed as the return over the past 12 months skipping the most recent month and the average of the log spot price from 4.5 to 5.5 years ago divided by the most recent spot price, respectively. Open Interest is measured as the total number of outstanding contracts that are held by market participants at the end of the month. Hedging Pressure is measured as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). The sample period is 01:1995-01:2016, monthly. The first 24 months of the model-implied risk premia are cut as burn-in sample to mitigate the effect of the initial conditions on the recursive learning scheme as outlined in Section 3 of the main text.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Expectations Horizon (Quarters)</th>
<th>Inventories</th>
<th>Open Interests</th>
<th>HP</th>
<th>Momentum</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 2$</td>
<td>$h = 3$</td>
<td>$h = 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>WTI Oil Crude</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.002</td>
<td>-0.013</td>
<td>-0.021</td>
<td>0.002</td>
<td>0.006</td>
<td>0.054</td>
</tr>
<tr>
<td>Median</td>
<td>0.001</td>
<td>-0.023</td>
<td>-0.032</td>
<td>0.002</td>
<td>0.004</td>
<td>0.045</td>
</tr>
<tr>
<td>St.Dev</td>
<td>0.091</td>
<td>0.111</td>
<td>0.122</td>
<td>0.029</td>
<td>0.056</td>
<td>0.092</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.281</td>
<td>0.036</td>
<td>0.058</td>
<td>0.075</td>
<td>-0.103</td>
<td>-0.048</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.501</td>
<td>3.359</td>
<td>3.051</td>
<td>2.906</td>
<td>5.476</td>
<td>2.789</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.284</td>
<td>0.595</td>
<td>0.763</td>
<td>0.179</td>
<td>0.027</td>
<td>0.797</td>
</tr>
<tr>
<td><strong>Copper</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.013</td>
<td>-0.017</td>
<td>-0.025</td>
<td>0.002</td>
<td>0.005</td>
<td>0.072</td>
</tr>
<tr>
<td>Median</td>
<td>-0.010</td>
<td>-0.009</td>
<td>-0.015</td>
<td>0.009</td>
<td>0.002</td>
<td>0.046</td>
</tr>
<tr>
<td>St.Dev</td>
<td>0.091</td>
<td>0.098</td>
<td>0.101</td>
<td>0.020</td>
<td>0.113</td>
<td>0.215</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.088</td>
<td>-0.450</td>
<td>-0.230</td>
<td>0.175</td>
<td>0.249</td>
<td>0.145</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.823</td>
<td>4.871</td>
<td>3.512</td>
<td>4.147</td>
<td>3.440</td>
<td>1.989</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.319</td>
<td>0.372</td>
<td>0.474</td>
<td>0.038</td>
<td>-0.312</td>
<td>0.823</td>
</tr>
<tr>
<td><strong>Corn</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.041</td>
<td>0.047</td>
<td>0.056</td>
<td>0.008</td>
<td>0.002</td>
<td>0.019</td>
</tr>
<tr>
<td>Median</td>
<td>0.064</td>
<td>0.078</td>
<td>0.085</td>
<td>0.002</td>
<td>0.003</td>
<td>0.031</td>
</tr>
<tr>
<td>St.Dev</td>
<td>0.121</td>
<td>0.129</td>
<td>0.139</td>
<td>0.147</td>
<td>0.146</td>
<td>0.136</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.931</td>
<td>-0.801</td>
<td>-0.951</td>
<td>-0.406</td>
<td>-6.984</td>
<td>-0.184</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.548</td>
<td>0.624</td>
<td>0.677</td>
<td>0.107</td>
<td>0.059</td>
<td>0.817</td>
</tr>
<tr>
<td><strong>Silver</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.021</td>
<td>0.023</td>
<td>0.027</td>
<td>0.002</td>
<td>0.397</td>
<td>0.054</td>
</tr>
<tr>
<td>Median</td>
<td>0.018</td>
<td>0.025</td>
<td>0.031</td>
<td>0.001</td>
<td>0.398</td>
<td>0.027</td>
</tr>
<tr>
<td>St.Dev</td>
<td>0.088</td>
<td>0.087</td>
<td>0.086</td>
<td>0.093</td>
<td>0.166</td>
<td>0.246</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.519</td>
<td>-0.501</td>
<td>-0.599</td>
<td>0.446</td>
<td>-0.031</td>
<td>0.516</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.506</td>
<td>4.251</td>
<td>4.456</td>
<td>3.515</td>
<td>2.201</td>
<td>3.526</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.072</td>
<td>0.123</td>
<td>0.180</td>
<td>-0.221</td>
<td>0.759</td>
<td>0.879</td>
</tr>
</tbody>
</table>

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Table 4. Cross-Sectional Correlations of Ex-Ante Risk Premia

Correlations Across Commodity Risk Premia. This table reports the cross-sectional correlations for the risk premia on WTI Oil Crude, Copper, Corn and Silver. Risk premia are those obtained by filtering out the model-implied expected future spot prices for \( h = 2, 3, 4 \) quarters ahead from the corresponding futures prices. The blue colors indicate the sample correlations are statistically significant at the 95% confidence level. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). The sample period is 01:1995-01:2016, monthly. The first 24 months of the model-implied risk premia are cut as burnin-in sample to mitigate the effect of the initial conditions on the recursive learning scheme as outlined in Section 3 of the main text.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Expectations Horizon (Quarters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h = 2 )</td>
</tr>
<tr>
<td>Crude Oil (WTI)</td>
<td>-</td>
</tr>
<tr>
<td>Copper</td>
<td>0.361</td>
</tr>
<tr>
<td>Corn</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>0.142</td>
</tr>
<tr>
<td>Silver</td>
<td>0.263</td>
</tr>
<tr>
<td></td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td>0.232</td>
</tr>
</tbody>
</table>
Static regressions. This table shows the estimates of a static version of the observation equation (12) in which we consider all of the economic risk factors $Z_t$ outlined in Section 3.3. For the ease of interpretation, all of the economic predictors and risk premia are standardized by dividing by their respective sample standard deviation. Risk premia are those obtained by filtering out the model-implied expected future spot prices for $h = 2, 4$ quarters ahead from the corresponding futures prices. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). WLD represents the index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis and relate to import weighted, seasonally adjusted, industrial production. USD TW stands for the Federal Reserve U.S. trade weighted exchange rate index, normalized to be equal to one hundred in March 1973. Time-series Momentum and Value are constructed as in Asness et al. (2013). SPX and MXEF represent the Standard and Poor’s 500 and the MSCI Emerging Markets indexes as proxy for financial risk. Open Interest (OIN) is defined as the total number of outstanding contracts that are held by market participants at the end of the month. Finally, hedging pressure (HP) is defined as the net excess in short futures positions by commercial traders, i.e. short minus long positions, divided by the amount of outstanding contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). The sample period is 01:1995-01:2016, monthly. The first 24 months of the model-implied risk premia are cut as burn-in sample to mitigate the effect of the initial conditions in filtering the ex-ante risk premia from adaptive expectations. Robust standard errors are in parenthesis, *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. 

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Inventories</th>
<th>USD TW</th>
<th>WLD</th>
<th>SPX</th>
<th>MXEF</th>
<th>OIN</th>
<th>Libor 6m</th>
<th>HP</th>
<th>Momentum</th>
<th>Value</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTI h=2</td>
<td>0.010</td>
<td>-0.078</td>
<td>0.103</td>
<td>-0.038</td>
<td>0.191**</td>
<td>0.182**</td>
<td>0.084</td>
<td>0.154**</td>
<td>-0.551***</td>
<td>-0.217*</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td>(0.062)</td>
<td>(0.073)</td>
<td>(0.081)</td>
<td>(0.099)</td>
<td>(0.050)</td>
<td>(0.113)</td>
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<td>0.164**</td>
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<td>0.010</td>
<td>0.216***</td>
<td>0.063</td>
<td>0.270***</td>
<td>-0.332***</td>
<td>0.119**</td>
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<td>(0.060)</td>
<td>(0.052)</td>
<td>(0.061)</td>
<td>(0.069)</td>
<td>(0.046)</td>
<td>(0.098)</td>
<td>(0.091)</td>
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<td>-0.090</td>
<td>0.014</td>
<td>0.071</td>
<td>0.115</td>
<td>0.153**</td>
<td>0.324***</td>
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<td>(0.081)</td>
<td>(0.090)</td>
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<td>(0.126)</td>
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<td>-0.057</td>
<td>0.103</td>
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<td>0.093**</td>
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<td>0.193***</td>
<td>0.094</td>
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<td>-0.149**</td>
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<td>(0.044)</td>
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<td>(0.074)</td>
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Table 6. In-Sample Diagnostics

Root Mean squared errors and predictive Bayes factors. This table shows the results of in-sample goodness-of-fit of a dynamic model specification in which risk factors are weighted according to their probability of inclusion, against a model in which none of the economic risk factors is significant and the dynamics of the ex-ante risk premia is determined by a time-varying intercept, as well as a model in which all the economic risk factors are included. **Panel A:** shows the relative Root Mean Squared Errors (RMSE) for the model-implied risk premia of WTI Oil Crude, Copper, Corn and Silver for $h = 2, 3, 4$ quarters ahead. We report the ratio of the RMSE for the No-Factors and All Factors specification with respect to our model. **Panel B:** shows the Bayes factors in log10 scale for the two competing models. Bayes factors are obtained from the models marginal likelihood computed as the harmonic mean of the likelihood values, evaluated at each draw of the parameters sampled from the corresponding full conditional distributions (see Appendix C). The sample period is 01:1995-01:2016, monthly. The first 24 months of the model-implied risk premia are cut as burn-in sample to mitigate the effect of the initial conditions on the recursive learning scheme as outlined in Section 3 of the main text.

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<td>0.447</td>
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<tr>
<td>Corn</td>
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<td>0.767</td>
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<tr>
<td>Silver</td>
<td>0.556</td>
<td>0.745</td>
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<table>
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<td>Crude Oil (WTI)</td>
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<tr>
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<td>Corn</td>
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<td>Silver</td>
<td>15.40</td>
<td>4.63</td>
<td>13.02</td>
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Figure 1. Expected vs Realized Risk Premia

Ex-ante risk premia. This figure shows the differences between the expected payoff, namely the ex-ante risk premium, and the realized payoff of a futures position. **Panel A:** shows the payoff structure of a futures position keeping the contract until maturity under no unexpected changes in spot prices. In this case the expected and the realized risk premia coincides. **Panel B:** shows the payoff structure of a futures position keeping the contract until maturity under a negative unexpected fluctuation in spot prices. In this case, the ex-ante and the realized risk premia diverge.
Figure 2. Expectations Error for Future Spot Prices

Unexpected movements in spot prices. This figure shows the unexpected price realizations with respect to the survey forecasts, i.e. $E_t[S_{t+h}] - S_{t+h}$ for two different horizons, i.e. $h = 2, 4$. Panel A: shows the unexpected price depreciations and appreciations for WTI Crude Oil (USD/Barrel). Panel B: shows the unexpected price depreciations and appreciations for Silver (USD/Ounce). Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and on Silver are obtained from the Commodity Exchange (COMEX). The sample period for the Survey is 12/2006-01/2016, aggregated monthly.
Changes in Spot Prices and World Industrial Production. This figure shows the year-on-year changes in the (log of) commodity spot prices and the (log of) index of world industrial production. Top panels compare the changes in world industrial production to the variation in the WTI Crude Oil (top-left) and Copper (top-right) spot prices. Bottom panels compare the changes in world industrial production to the variation in the Corn (top-left) and Silver (top-right) spot prices. Spot prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Spot prices on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver futures are quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. The index of world industrial production published by the Netherlands Bureau for Economic and Policy Analysis, and contains aggregate information on industrial production from 81 countries worldwide, which account for about 97% of the global industrial production. The sample period is 01:1995-01:2016.
Figure 4. Survey Expectations and Model-Implied Expected Future Spot Prices: Crude Oil (WTI) and Copper

Expectations Formation. This figure compares the expected future spot prices implied by the adaptive learning with the Survey Price Forecasts on Crude Oil (WTI) and Copper come from Bloomberg’s Commodity Price Forecasts Database for $h = 2, 4$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). The sample period for the Survey is 12:2006-01:2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the adaptive learning scheme.
Figure 5. Survey Expectations and Model-Implied Expected Future Spot Prices: Corn and Silver

Expectations Formation. This figure compares the expected future spot prices implied by the adaptive learning with the Survey Price Forecasts on Corn and Silver come from Bloomberg’s Commodity Price Forecasts Database for $h = 2, 4$ quarters ahead. Data on Silver are obtained from the Commodity Exchange (COMEX), and data for Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period for the Survey is 12/2000-01/2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01/1995-01/2016. The first 24 months are cut as burn-in sample for the adaptive learning scheme.
Figure 6. Survey vs. Adaptive Expectations Risk Premia: Crude Oil (WTI) and Copper

Commodity Risk Premia. This figure compares the commodity risk premia implied by our model of adaptive expectations with the risk premia obtained from the Bloomberg’s Commodity Price Forecasts Database for $h = 2, 4$ quarters ahead. WTI Crude Oil prices are in U.S. Dollars per barrel, whereas Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX) and Copper are obtained from the Commodity Exchange (COMEX). The sample period for the Survey is 12:2006-01:2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Figure 7. Survey vs. Adaptive Expectations Risk Premium: Corn and Silver

Commodity Risk Premium. This figure compares the commodity risk premium implied by our model of adaptive expectations with the risk premium obtained from the Bloomberg's Commodity Price Forecasts Database for $h = 2$ quarters ahead. Corn futures prices are from the Chicago Board of Trade (CBOT), with price quotation in USD per bushel. Silver futures are quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME) contract. The sample period for the Survey is 12/2006-01/2016, aggregated monthly. The sample period of the model-implied expected future spot price is 01/1995-01/2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Figure 8. Recursive Averages of Commodity Risk Premia

Exponential weighted moving average of commodity risk premia. This figure shows the recursive average of the model-implied risk premia for WTI Crude Oil, Copper, Corn, and Silver for $h = 2, 3, 4$ quarters ahead. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Data on Silver and Copper are obtained from the Commodity Exchange (COMEX). Copper prices are transformed from USD Cents/Pound USD/Tonne to match the measurement unit used in the survey forecasts. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Dynamic inclusion probabilities for each risk factor. This figure shows the posterior dynamic inclusion probabilities for the risk factors driving the dynamics of WTI Crude Oil and Copper risk premia for $h = 2, 4$ quarters ahead. For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time, that is, any predictor where the inclusion probability is never above 0.5 is not reported in the corresponding figure. Data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Data on Copper are obtained from the Commodity Exchange (COMEX), quoted in U.S. cents/pound, respectively. We convert the price of futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME) contract. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Figure 10. Probability of Inclusion of a Risk Factor in the Dynamics of Risk Premium: Corn and Silver

Dynamic inclusion probabilities for each risk factor. This figure shows the posterior dynamic inclusion probabilities for the risk factors driving the dynamics of WTI Crude Oil and Copper risk premia for $h = 2, 4$ quarters ahead. For the ease of exposition, we only show those posterior inclusion probabilities which exceeds a threshold value of 0.5 at least one point in time, that is, any predictor where the inclusion probability is never above 0.5 is not reported in the corresponding figure. Data on Silver are obtained from the Commodity Exchange (COMEX), quoted in U.S. dollars per troy ounce. Data on Corn are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period of the model-implied expected future spot price is 01:1995-01:2016. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Time-varying betas on Hedging Pressure. This figure shows the posterior median for the beta on Hedging Pressure, i.e., $\hat{\beta}_{HP}$, across commodities. The solid blue line represents the estimated beta for the two-quarter ahead expected payoff. We compute the level of HP for different commodities as the net excess in short futures positions by commercial traders, i.e., short minus long positions, divided by the amount of outstanding hedging contracts. The data on commercial traders futures positions are from the Commodity Futures Trading Commission (CFTC). The sample period is 1993:01-2016:01. The first 24 months are cut as burn-in sample for the sequential learning scheme.
Figure 12. Time-Series Momentum on Ex-Ante Risk Premia

Time-varying betas on Momentum. This figure shows the posterior median for the beta on Momentum, i.e. $\hat{\beta}_{Mom}$, across commodities. The solid blue line represents the estimated beta for the two-quarter ahead risk premia, and the solid red line shows the estimated beta for the four-quarter ahead expected payoff. We construct time-series Momentum as the return over the past 12 months skipping the most recent month on each commodity future (see Asness et al. 2013). Data are obtained from different resources. Futures prices data on Crude Oil (WTI) are from the New York Mercantile Exchange (NYMEX), prices are in U.S. dollars per barrel. Futures on Silver and Copper are obtained from the Commodity Exchange (COMEX). Silver is quoted in U.S. dollars per troy ounce while Copper is quoted in U.S. cents/pound. We convert the price of Copper futures contracts to USD/tonne to match the measurement unit of the survey forecasts, that instead refer to the London Metal Exchange (LME). Corn futures prices are from the Chicago Board of Trade (CBOT) with price quotation in USD cents per bushel. The sample period 1993:01-2016:01. The first 24 months are cut as burn-in sample for the sequential learning scheme.