The Equity Premium, Long-Run Risk, & Optimal Monetary Policy

Anthony M. Diercks *†

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Abstract

In this study I examine the welfare implications of monetary policy by constructing a novel production-based asset pricing New Keynesian model. I find that the Ramsey optimal monetary policy yields an inflation rate above 3.5% and inflation volatility close to 1.5%. The same model calibrated to a counterfactually low equity premium implies an optimal inflation rate close to zero and inflation volatility less than 10 basis points, consistent with much of the existing literature. Relatively higher optimal inflation is due to the greater welfare costs of recessions associated with matching the equity premium. The standard optimal policy that focuses on stabilizing inflation tends to amplify long-run risk. Furthermore, the interest rate rule that comes closest to matching the dynamics of the optimal Ramsey policy puts a sizable weight on capital growth along with the price of capital, as it emphasizes reducing long-run risk.

Keywords: Asset Pricing, Long-Run Risk, Monetary Policy


*Federal Reserve Board, Monetary and Financial Market Analysis, 20th Street and Constitution Avenue N.W., Washington, D.C. 20551 Anthony.M.Diercks@frb.gov. The analysis and conclusions set forth in this paper are those of the author and do not indicate concurrence by other members of the research staff or the Board of Governors of the Federal Reserve System.

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1 Introduction

As we continue to recover from the Great Recession, monetary policy makers are confronted with the decision of when to start raising interest rates and by how much. Underlying these choices is a concern about the trade-off between inflation and output. On one side there are “inflation hawks” who suggest that the primary goal of monetary policy should be price stability, with overwhelming concern for the stabilization of inflation—potentially at the expense of output. On the other side, “inflation doves” suggest that monetary policy should place a greater emphasis on stabilizing output to decrease the severity of recessions, potentially at the expense of increased inflation. For policy makers, determining the proper emphasis on inflation versus output is a high-stakes decision that impacts everyone in the economy.

Existing studies of monetary policy predominantly find that stabilizing inflation is strongly preferred to stabilizing output. However, these studies suffer from the equity premium puzzle (Mehra and Prescott, 1985) and the risk-free rate puzzle (Weil, 1989), and thus they ignore key characteristics of financial data. The present study addresses this issue by incorporating financial data and solving the Ramsey optimal policy in addition to evaluating the welfare implications of simple monetary policy rules that are functions of inflation and output. Specifically, I construct a model that is consistent with the historical equity premium, the historical risk-free rate, and the presence of long-run risk in productivity. Each of these features helps capture key aspects of the macroeconomy. The equity premium captures the welfare costs of recessions (Tallarini, 2000); the risk-free rate dictates the extent to which households and firms are patient and forward looking; and long-run productivity risk is crucial because of its major impact on the pricing decisions of forward-looking firms.

I find that the combination of the high equity premium and long-run risk in productivity yields a Ramsey optimal inflation rate greater than 3.5% with a volatility close to 1.5%. The interest rate rule that comes closest to matching the dynamics of the optimal Ramsey policy puts a sizable weight on capital growth along with the price of capital, as it emphasizes stabilizing the medium to long term over the very short run. In terms of exclusively inflation and output-based rules, I find that policymakers

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1 See Table 1 in the following section for an overview of over 70 existing studies.
should place significantly greater weight on output and less weight on inflation than many of the studies in the existing literature.

To explain these results, I investigate the trade-offs among three sources of welfare losses in the New Keynesian model: inflation volatility, output volatility, and the average markup. Output volatility and the average markup have greater impact within my model relative to previous studies because households and firms are assumed to have recursive preferences. Unlike constant relative risk aversion (CRRA) preferences, recursive preferences break the inverse relationship between risk aversion and the intertemporal elasticity of substitution (IES). This allows my model to replicate both the high equity premium and low risk-free rate observed in financial data. High levels of risk aversion combined with a high IES causes households to strongly dislike recessions while being much more patient and forward looking.

The introduction of long-run productivity shocks dramatically alters the influence of monetary policy on firms’ price setting and the average markup. The reduction in the average markup, a key determinant of welfare, provides substantial welfare benefits. To fully understand the mechanism that drives the importance of the markup, it is first necessary to recognize that in the context of my fully specified nonlinear model, risk-adjusted measures are what matter for pricing and consumer decisions. The likelihood of positive shocks is down-weighted, while negative shocks receive greater weight; this creates an asymmetry that is present in any standard macro model that does not linearize first-order conditions. This second-order-based asymmetry is crucial for risk characterization, the determination of patience, and ultimately the determination of optimal price setting.

This asymmetry also generates important implications for optimal monetary policy when bad news for long-run growth is realized. Specifically, negative long-run news shocks to productivity lead forward-looking firms to choose substantially higher prices, which pushes inflation persistently higher. Higher inflation in turn mechanically lowers relative prices set by firms in previous periods and erodes the real value of the average markup. The persistent reduction of the average markup works like a hedge when the household receives negative long-run news shocks. By allowing inflation to rise, monetary policy provides good long-run news thanks to the reduction in the average markup.

Reducing the markup by pushing output and labor higher during down times has increased relevance
for today’s economic environment. As pointed out by Bernstein (2013), “... in the largely deunionized American job market, full employment is the way workers get bargaining clout.” He goes on to note that the last time the labor share of income significantly rose was the late 1990’s, a period with lower unemployment. His discussion implies that monetary policy can influence the labor share of income by pushing for higher employment, which increases the bargaining power of workers. In standard models, monetary policy cannot persistently influence the labor share of income and the markup. In contrast, my model with the second order approximation can affect the steady-state distance between actual and efficient output. Specifically, my model suggests it would be optimal for monetary policy to push output and labor higher in response to a persistently negative shock in order to (1) stabilize both output and consumption, and (2) decrease the steady state markup while increasing the steady state labor share of income.\(^2\) While this policy leads to higher and more volatile inflation, these costs do not dominate as they do in the standard macroeconomic model due to the higher welfare costs of recessions coming from matching the high equity premium.

With regards to simple rules and the coefficient on inflation, my model yields an interior solution of 3.05, where the benefits and costs to stabilizing inflation perfectly balance each other. With higher values on the coefficient of inflation in the monetary policy rule, inflation does not rise as much in response to a negative shock, which implies a smaller reduction in the markup. As a result of this smaller reduction, the steady-state markup rises with the inflation coefficient, placing a greater implicit tax on labor and capital as monetary policy increasingly stabilizes inflation. At values greater than 3.05 for the inflation coefficient, I find that the costs of the higher markup outweigh the benefits associated with lower inflation volatility.

This result stands in contrast to the conclusions of a number of previous studies, which suggest that the reduction of inflation volatility should be the primary focus of monetary policy. I replicate this finding in my model with CRRA utility, in which the weight placed on fluctuations of inflation in the monetary policy rule is set to infinity. Results show that the markup channel is insignificant in the standard (second-order CRRA) setting because its movements are not as asymmetric and persistent as in

\(^2\)Decreasing the markup during bad states is important for the stochastic steady state because the dynamics that occur in bad times dominate.
the recursive preferences setting. In the CRRA utility setting, both the representative agent and firms are relatively impatient, and monetary policy is unable to persistently influence the price setting and average markup. Furthermore, not as much weight is placed on negative shocks, which reduces the asymmetry compared to the setting with recursive preferences.

Another justification for the low value of the inflation coefficient is the trade-off between real and nominal uncertainty. In my model, monetary policy can lower nominal uncertainty by placing a greater weight on inflation fluctuations, but this comes at the expense of the greater volatility of real variables such as output. This makes intuitive sense, because greater stabilization of inflation is achieved through greater changes in real interest rates. The higher IES by definition makes households more willing to substitute consumption intertemporally due to changes in the real interest rates, which implies that monetary policy is more effective in altering real quantities. This channel is also present in a standard model but is much smaller (due to the restricted, lower IES) and is dominated in terms of welfare by the price dispersion channel. Holding all else constant, higher output and consumption volatility reduce welfare. Thus, this channel also contributes to the finding of an interior solution of 3.05 for the coefficient on inflation.

In addition to the lower value for the coefficient on inflation, my proposed model also yields a high value on the coefficient for output. Placing a higher weight on the output growth benefits welfare because it reduces fluctuations in consumption. Moreover, it also pushes up inflation during negative long-run productivity shocks, which effectively lowers the markup. Matching the high equity premium in the data implies that households strongly dislike the recessions associated with output fluctuations, so that the costs and benefits equal at a coefficient value of 1.55 for output growth in the interest rate rule. This weight is three times greater than the optimal weight on the output growth of Sims (2013), who does not include long-run productivity shocks and instead uses habits. Results from my model with CRRA utility show that zero weight should be placed on the output growth; this is because the welfare costs of recessions are significantly lower and the costs coming from inflation volatility dominate.
1.1 Related Literature

This paper contributes to the sizable literature on monetary policy as well as a growing body of work on production-based asset pricing. A benchmark result in the monetary policy literature is that attention should be completely focused on inflation stabilization (see Table 1, which shows 35 studies with an optimal inflation volatility of less than 20 basis points). However, my asset pricing–driven approach suggests that strict inflation stabilization is far from optimal.

Previous studies have outlined a deviation from strict inflation stabilization for some of the following reasons: (1) money (opportunity cost of positive interest rate), (2) distortionary taxes (inflation can be shock absorber to reduce tax volatility), (3) govt. transfers (represent pure rents that inflation can confiscate), (4) sticky wages (stabilize wages rather than inflation), (5) price and wage markup shocks (cost push shocks), (6) zero lower bound (inflation reduces chances of reaching ZLB), (7) capital accumulation (composition of demand between investment and consumption matters), (8) flexible prices (no costs associated with inflation), (9) foreigners demand for domestic currency (inflation generates seignorage), (10) price indexation (reduces costs of inflation), (11) collateral constraints (prevent borrowers from smoothing the way savers do), and (12) endogenous firm entry (higher entry costs reduce number of firms but increase desired markups, inflation lowers markup and discourages welfare-inefficient entry).\(^3\)

However, the above mechanisms typically suggest optimal inflation rates of zero or below and optimal inflation volatilities less than one percent. The exception to this are flexible price models, which often suggest an optimal inflation volatility greater than five percent due to the absence of associated welfare costs of inflation.

To better isolate the welfare effects of asset pricing data, I do not include any of the above mechanisms except for capital accumulation. Rather, I simply use a second-order approximation around a distorted steady state and assume Epstein and Zin (1989) preferences for households. I resolve both the equity premium puzzle (Mehra and Prescott, 1985) and the risk-free rate puzzle (Weil, 1989) by introducing long-run risk in productivity, in the spirit of Croce (2014b) and more broadly of Bansal and Yaron (2004). This setting has not been explored to date in the monetary policy literature.

\(^3\)For other reasons, see the Comments column in Table 1
Long-run risk is a key driver of my qualitative results. Croce (2014b), Beaudry and Portier (2004), Schmitt-Grohé and Uribe (2012c), Kurmann and Otrok (2010), and Barsky and Sims (2011) have all found evidence of long-run news shocks to productivity, which explain a large fraction of business cycle fluctuations. Endogenous growth models have also been used to generate long-run consumption risk, as in Kung and Schmid (2011). The connection between endogenous growth, monetary policy and the term structure of interest rates has also been explored by Kung (2014). None of these studies has explored the welfare implications of long-run news for the trade-off between inflation and output stabilization.

To be clear, my study is not the first to incorporate recursive preferences when evaluating the welfare effects of monetary policy. Levin et al. (2008) use a very stylized, one-shock New Keynesian model with no capital in order to study the linearized Ramsey planner problem. They find that the planner is more risk averse and permits fewer fluctuations in output with more volatile inflation. In contrast to my study, there is no long-run risk and no quantification of optimal inflation volatility or the optimal coefficients on an interest rate rule.
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<td>Cash in advance, one period ahead price setting</td>
</tr>
<tr>
<td>Schmitt-Grohé and Uribe (2012b)</td>
<td>–</td>
<td>-0.6</td>
<td>–</td>
<td>–</td>
<td>On Quality Bias and Inflation Targets</td>
</tr>
<tr>
<td>Adam and Billi (2004)</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>Optimal Policy under commitment with ZLB</td>
</tr>
<tr>
<td>Annicchiarico and Rossi (2013)</td>
<td>–</td>
<td>0</td>
<td>0.9375</td>
<td>1.3125</td>
<td>Endogenous Growth</td>
</tr>
<tr>
<td>Schmitt-Grohé and Uribe (2010)</td>
<td>–</td>
<td>0</td>
<td>–</td>
<td>–</td>
<td>Review paper shows all their papers: zero inflation</td>
</tr>
<tr>
<td>An (2010)</td>
<td>–</td>
<td>0.003</td>
<td>0</td>
<td>3</td>
<td>Uses Recursive Preferences, no equity premium</td>
</tr>
<tr>
<td>Kim and Ruge-Murcia (2009)</td>
<td>–</td>
<td>0.35</td>
<td>–</td>
<td>–</td>
<td>Asymmetric wage adjustment costs</td>
</tr>
<tr>
<td>Blanco (2015)</td>
<td>–</td>
<td>1.0 (5.0)</td>
<td>–</td>
<td>–</td>
<td>Calvo (Menu Costs), Idios. Vol., hetero.</td>
</tr>
<tr>
<td>Schmitt-Grohé and Uribe (2012a)</td>
<td>–</td>
<td>6</td>
<td>–</td>
<td>–</td>
<td>(Flex) Frgn Demand Dom. Currency (Seignorage)</td>
</tr>
<tr>
<td>Ascarì et al. (2011)</td>
<td>–</td>
<td>–</td>
<td>0.07</td>
<td>5.59</td>
<td>Limited Asset Market Part., Sticky Wages</td>
</tr>
<tr>
<td>Debortoli et al. (2015)</td>
<td>–</td>
<td>–</td>
<td>54.81</td>
<td>29.28</td>
<td>Smet-Wouters with price &amp; wage-markup shocks</td>
</tr>
<tr>
<td>Erceg and Levin (2006)</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>54.5</td>
<td>Taylor price, ord. of magn. ↓ $\pi$ costs. Durable C.</td>
</tr>
<tr>
<td>Giannoni and Woodford (2003)</td>
<td>–</td>
<td>–</td>
<td>0.0026</td>
<td>0.996</td>
<td>Incorporates Habits, focuses on loss functions</td>
</tr>
</tbody>
</table>

List sorted by $\sigma(\pi)^* \& E(\pi)^*$. $\alpha_\pi$ and $\alpha_y$ are optimal weights on inflation and output (gap, level, growth, etc.) in rule or loss function.
An (2010) shows how to use perturbation methods to solve models with recursive preferences and simple monetary policy rules. Long-run risk is not incorporated, and his optimal policy coefficients and inflation volatility are low and almost identical to those in the setting of Schmitt-Grohé and Uribe (2007) with CRRA preferences.

Darracq Paries and Loublier (2010b) come closest to my study by moving beyond a highly stylized setting to look at the effects of recursive preferences. Their study differs from mine in a number of ways: (1) habits are incorporated, (2) the IES is less than one, (3) wages are rigid, and (4) there is no growth and no long-run risk. They find that the optimal inflation volatility is less than 0.25 percent after including Epstein-Zin preferences. Finally, Benigno and Paciello (2014) incorporate doubts and ambiguity aversion and find that as doubts rise, monetary policy finds it optimal to further deviate from strict inflation targeting. However, while they target a high equity premium, they do not match the low risk-free rate or include long run risk. In fact, none of the studies that incorporate recursive preference when evaluating the effects of monetary policy includes long-run risk shocks or attempts to match both the equity premium and the risk-free rate.

Certain studies, such as Gavin et al. (2009), do look at the effects of monetary policy with permanent changes in the growth rate of productivity. However, they use log utility and do not match financial data. In their model, the central bank’s optimal policy is to fully stabilize the inflation rate at its steady state in order to completely eliminate the sticky price distortion. Other studies such as Gust and López-Salido (2014) incorporate portfolio re-balancing costs and examine the effects of monetary policy. They find that a counter-cyclical policy can generate higher welfare than a zero inflation policy, but there is no discussion of optimal Ramsey policy or quantification of optimal inflation moments and output is assumed to be exogenous.

Similar studies of optimal fiscal policy with recursive preferences have been conducted in Karantounias (2013) and Karantounias (2014). These studies conclude that recursive preferences along with robustness can play important roles in altering the optimal policy prescriptions with regard
to taxation and debt.

The rest of the paper is organized as follows. Section 2 discusses the model and empirical motivation. Section 3 describes the optimal Ramsey monetary policy and the rule that comes close to mimicking its dynamics. Section 4 analyzes simple interest rate rules and explains the welfare channels driving the results and Section 5 concludes.

2 Model and Empirical Motivation

In the discussion below I focus on two key differences of my model with respect to existing monetary policy analysis: the preferences and the productivity process. The economy consists of a continuum of identical households, a continuum of intermediate-goods firms, and a government that conducts monetary and fiscal policy. The structure of the model is a standard neoclassical growth model augmented with real and nominal frictions. The nominal friction is sticky prices. The real friction is monopolistic competition, which results in a markup of price over marginal costs. Monetary policy assumes full commitment to an interest rate rule that is a function of inflation, output growth, and the previous period’s interest rate. Fiscal policy raises lump-sum taxes to pay for exogenous expenditures.

Preferences. The households have Epstein-Zin preferences defined over consumption goods, \( c_t \), and leisure, \( 1 - h_t \). These preferences exhibit a CES aggregate of current and future utility certainty equivalent weighted by \((1-\beta)\) and \( \beta \), respectively.

\[
v_t = \left\{ (1 - \beta)(c_t^\gamma(1 - h_t)^{1-\gamma})^{1-\frac{1}{\psi}} + \beta(E_t[v_{t+1}]^{\frac{1-\gamma}{\psi}}) \right\}^{\frac{1}{1-\psi}}
\]

s.t.

\[
b_t + c_t + i_t + \tau_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + w_t h_t + u_t k_t + \phi_t
\]

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The real value of debt is $b_t$; $c_t$ is consumption; $i_t$ is investment; $R_{t-1}$ is the risk-free rate; $\pi_t$ is the inflation rate $\frac{P_t}{P_{t-1}}$; $\tau_t$ is the lump-sum tax; $w_t$ is the real wage; $h_t$ is labor hours; $u_t$ is the rental rate of capital; $k_t$ is capital; and $\phi_t$ is profits.

Unlike CRRA preferences, Epstein-Zin preferences allow for the disentanglement of $\gamma$, the coefficient of relative risk aversion, and $\psi$, the elasticity of intertemporal substitution. When $\frac{1}{\psi} = \gamma$, the utility collapses to CRRA preferences, with additively separable expected utility both in time and state. When $\gamma > \frac{1}{\psi}$, the agent prefers early resolution of uncertainty, so the agent dislikes shocks to long-run expected growth rates.

**Stochastic Discount Factor.** The stochastic discount factor (SDF) represents the intertemporal marginal rate of substitution for consumption. It is the major focal point in the forward-looking New Keynesian model, as it translates the value of future income/profits to the present. Given the fact that monetary policy relies on long-term expectations to influence the economy, the functional form is very relevant for all of the household’s and firm’s intertemporal maximization decisions:

$$M_{t,t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\nu(1-\frac{1}{\psi})-1} \left( 1 - h_{t+1} \right) \left( 1 - \frac{h_t}{1-h_t} \right)^{(1-\nu)(1-\frac{1}{\psi})} \frac{P_t}{P_{t+1}} \left[ \frac{u_{t+1}}{E_t[u_{t+1}^{1-\gamma}]} \right]^{\frac{1}{\psi} - \gamma}$$

The last part of equation 1 is unique to recursive preferences. This factor captures news regarding the continuation value of the representative agent. Future utility, as represented by the continuation value, is very sensitive to long-run news and this allows for greater variation of the stochastic discount factor without the need for excessive levels of risk aversion. Compared to CRRA preferences, the last factor implies a significantly higher weight on negative outcomes as marginal utility rises. This results in endogenous asymmetric responses to shocks because agents are more concerned with negative long-run news.
**Productivity.** The law of motion of the productivity process captures both short-run and long-run productivity risks:

\[
\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \varepsilon_{a,t+1},
\]

\[
x_{t+1} = \rho x_t + \sigma_x \varepsilon_{x,t+1},
\]

\[
\begin{bmatrix}
\varepsilon_{a,t+1} \\
\varepsilon_{x,t+1}
\end{bmatrix}
\sim i.i.d. N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \quad t = 0, 1, 2, \ldots
\]

According to the above specification, short-run productivity shocks, \( \varepsilon_{a,t+1} \), affect contemporaneous output directly, but they have no effect on future productivity growth. Shocks to long-run productivity, represented by \( \varepsilon_{x,t+1} \), carry news about future productivity growth rates but do not affect current output.

### 2.1 Welfare Analysis: Three Major Channels

In this section I describe the three key sources of welfare losses in the New Keynesian model: inflation volatility, the average markup, and output volatility. In short, inflation volatility causes relative price dispersion and inefficient production; the average markup acts as an implicit tax on factors of production; and output volatility reduces concave utility. The average markup and output volatility will have a greater impact within my model relative to previous studies. The reasons for this will be discussed in the following section.

#### 2.1.1 Inflation Volatility: Price Dispersion

From a welfare perspective, price dispersion is a crucial characteristic of the Calvo-based time-dependent sticky price models. Each period, a fraction \( \alpha \in [0, 1] \) of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. The remaining \( 1 - \alpha \) firms choose \( P_{i,t} \) to maximize the expected present discounted value of profits.
Price dispersion arises when a subset of firms finds it optimal to choose a different price relative to the remaining firms who are unable to update. Differences in pricing across firms are important because it is assumed that monopolistically competitive firms choose a price and agree to supply the quantity demanded. The quantity demanded for each firm \( Y_{i,t} \) follows a downward-sloping demand schedule based on the firm’s price \( P_{i,t} \) and the elasticity of substitution across goods, \( \eta \):

\[
Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t
\]

Given that supply must equal demand at the firm level,

\[
K^\theta_{i,t}(A_t N_{i,t})^{1-\theta} = \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} Y_t
\]

One can then aggregate across all firms to arrive at

\[
K^\theta_t(A_t N_t)^{1-\theta} = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} d_i \cdot Y_t.
\]

Defining the resource cost of price dispersion as \( s_t = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\eta} d_i \), one can see that its effect is similar to a negative aggregate productivity shock, as it increases the amount of labor and capital required to produce a given level of output.

\[
\frac{1}{s_t} K^\theta_t(A_t N_t)^{1-\theta} = Y_t
\]

This happens because firms with relatively low prices produce an inefficiently high quantity to meet relatively high demand, while the opposite occurs for the high-price firms. As shown by Schmitt-Grohé and Uribe (2007), \( s_t \geq 1 \), and \( s_t \) is equal to one when there is no price dispersion.
Furthermore, price dispersion can be recursively decomposed into the following equation:

\[ s_t = (1 - \alpha)(p^*_t)^{-\eta} + \alpha \pi^\eta_t s_{t-1} \]  

where \( p^*_t = \frac{P^*_t}{P^*_t} \) represents the relative price of the optimizing firm at time \( t \). Price dispersion is an increasing function of \( \alpha \), the probability that firms are unable to update their prices; \( \eta \), the elasticity of substitution across goods; and \( \pi \), the steady-state rate of inflation.\(^5\)

The main driver of price dispersion within the model is inflation volatility. Given the elasticity of substitution (\( \eta \geq 1 \)), more volatile inflation will lead to higher price dispersion on average due to the Jensens Inequality (\( \pi^\eta_t \)) in equation 2. This effect is absent in a first-order approximation.

2.1.2 The Markup: Implicit Tax

Monopolistic competition allows firms to charge a price that is higher than their marginal costs. The difference between the price and the marginal costs is known as the average markup. From a welfare perspective, the average markup is very important because it acts as a wedge between factor prices and marginal products, which causes inefficiently low levels of labor, capital, and output. Hence, the average markup is akin to an implicit tax on capital and labor.

The implicit tax can be seen in the firm’s demand for labor and capital:

\[ MPL_t = \mu_t w_t \]
\[ MPK_t = \mu_t u_t \]

where \( \mu_t \) is the markup. Higher markups imply lower real wages and rental rates of capital, and this affects both labor and capital supply. For example, combining the labor supply equation with

---

\(^5\)If prices were flexible (\( \alpha = 0 \)), all firms would be able re-optimize every period, and there would be no price dispersion (\( s = 1 \)). A higher elasticity of substitution across intermediate goods moves the economy closer to perfect competition (where \( \eta = \infty \)). This results in greater costs of price dispersion because households are more willing to switch from the high-price good to the low-price good, causing aggregate production to become more inefficient because the low-price firm must meet the higher demand. Lastly, price dispersion increases with steady-state inflation because firms that are resetting their prices optimally choose a higher price than the existing price level.
labor demand yields

\[
\frac{1 - \iota}{\iota} \frac{c_t}{1 - h_t} = w_t = \frac{MPL_t}{\mu_t}
\]

where \(\iota\) is the share of consumption in the consumption-leisure bundle. This shows that a higher markup reduces the amount of labor supplied. Moreover, given that the slope of the labor supply is both positive and increasing, increases in the markup result in greater and greater decreases in the quantity of labor supplied. This fact is important in explaining the reasons that the costs of higher markups dominate the benefits of reduced price dispersion as monetary policy increasingly stabilizes inflation.

**Markup decomposition.** As outlined in King and Wolman (1996), the average markup of price over marginal cost can be decomposed into two components:

\[
\mu_t = \left( \frac{P_t}{P_t^*} \right) \left( \frac{P_t^*}{MC_t} \right)
\]

where \(\left( \frac{P_t}{P_t^*} \right)\) is defined as the price adjustment gap and \(\left( \frac{P_t^*}{MC_t} \right)\) is defined as the marginal markup. The price adjustment gap is just the inverse of the relative price of the optimizing firm at time \(t\) and the marginal markup is the ratio of price to marginal cost for firms allowed to adjust their price in period \(t\).

If inflation increases, it must be that \(P_t^*\) is greater than \(P_t\), and the price adjustment gap falls. The price adjustment gap captures the notion that higher inflation automatically decreases relative prices set by firms in previous periods and decreases the real value of the average markup. King and Wolman (1996) find that if the average markup only consisted of the price adjustment gap, an increase in inflation from 5 to 10 percent would raise output permanently by 7 percent.

**Marginal markup.** The marginal markup captures the markup that re-optimizing firms are able to charge. This can be shown to depend positively on expected future inflation as firms
choose a higher price when they are allowed to update their prices. Firms will choose a higher price because they are concerned that they will get “stuck” (i.e., be unable to re-optimize) and that inflation will erode their relative price. Erosion of a firm’s relative price causes households to substitute toward their good as it becomes relatively less expensive, and this can be problematic as the firm is required to meet demand by securing more labor at higher costs. Inflation will also erode the real value of any markup established at time $t$, so that per-unit profits will decline for as long as the firm is stuck.

Ascari and Sbordone (2013) show that as trend inflation increases, the increase in the marginal markup dominates so that the overall average markup also rises. As the average markup rises, output declines along with welfare. This raises the question of which channel dominates in a setting with zero trend inflation or a zero inflation target. In a comparison across different policy rules, I find that the price adjustment gap channel dominates the marginal markup for negative productivity shocks, so that the average markup falls as inflation volatility increases. This is not surprising because under the benchmark calibration, most firms are unable to update their prices each period.\footnote{Under the benchmark calibration, 75\% of the firms are unable to update in any given period, which is equivalent to firms getting stuck on average for 12 months. This duration is in the middle of empirical estimates and will be further discussed in the calibration section.}

### 2.1.3 The Output Gap: Consumption-Leisure Volatility

The output gap is defined as the deviation between the actual level of output and its natural level (the level of output in the absence of nominal rigidities). In the typical New Keynesian setup, a second-order approximation to household welfare gives rise to a loss function in the variances of the output gap and inflation. The loss function is then used to evaluate various monetary policy rules. However, the reason the output gap enters the loss function is due to the simplifying assumption that consumption is equal to output.

In my model, output of final goods and services goes toward not only consumption but also...
investment and government expenditures. In addition, agents care not about consumption but rather a consumption-leisure bundle. Therefore, I focus on the bundle rather than output when evaluating the effects of policy on welfare. With regard to the welfare costs of recessions, the magnitude is much greater in the asset pricing–oriented New Keynesian model, as indicated by the higher equity premium. The higher welfare costs suggest that greater weight should be placed on stabilizing output and the consumption-leisure bundle, in contrast to what is the case in existing studies that do not match the equity premium.\footnote{The procedure for computing welfare is based on An (2010). Welfare costs in consumption equivalent units is defined as $\chi_c = 1 - \left( \frac{v_{i|ss}^i}{\pi_{i|ss}} \right)^\iota$ where $\iota$ is the share of consumption in the consumption-leisure bundle and $v_{i|ss}^k$ represents the lifetime welfare based on policy $k = i, j$.}

2.2 Calibration

In the proposed model, I calibrate the time period to a quarterly frequency. I then annualize the moments and focus on matching the behavior of macroeconomic variables over the long sample of US data from 1929–2008. Data on consumption and investment are from the Bureau of Economic Analysis (BEA). Fiscal policy variables such as government spending and steady-state debt are taken from Schmitt-Grohé and Uribe (2007). The parameters described below are listed in table 2.

For the New Keynesian parameters, the markup due to monopolistic competition is set to 15%, which is in line with previous studies (Bils and Klenow, 2002). Firms are assumed to re-optimize their prices every 12 months, which is in the middle of empirical estimates that range from 6 to 18 months (see Altig et al. (2011)). With regard to fiscal policy, steady-state government purchases make up 17% of GDP, and the steady-state debt-GDP ratio is set to the historical average of 44%, following Schmitt-Grohé and Uribe (2007). Taxes are collected by lump sum to pay for an exogenous expenditure stream. The persistence and standard deviation of the government spending shocks follow Croce et al. (2012), but given the assumed nondistortionary nature of taxes, they essentially have a nil effect on the results.
Table 2: Model Features and Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Recursive</th>
<th>CRRA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High IES</td>
<td>Low IES</td>
</tr>
<tr>
<td></td>
<td>High RA</td>
<td>High RA</td>
</tr>
<tr>
<td><strong>Preference parameters</strong></td>
<td></td>
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<tr>
<td>Discount factor, $\beta$</td>
<td>0.9955</td>
<td>0.999</td>
</tr>
<tr>
<td>Effective risk aversion, $\gamma$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Intertemporal elasticity of subs., $\psi$</td>
<td>2.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Leisure weight, $\phi$</td>
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<td>0.35</td>
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<tr>
<td><strong>Technology parameters</strong></td>
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<td></td>
</tr>
<tr>
<td>Capital share, $\theta$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Quarterly depreciation rate, $\delta$</td>
<td>1.725%</td>
<td>3%</td>
</tr>
<tr>
<td><strong>Productivity parameters</strong></td>
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<tr>
<td>Risk exposure of new investment, $\phi_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Average quarterly growth rate, $\mu$</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Volatility of short-run risk, $\sigma_a$</td>
<td>0.0055</td>
<td>0.015</td>
</tr>
<tr>
<td>Volatility of long-run risk, $\sigma_x$</td>
<td>$0.15 \cdot \sigma_a$</td>
<td>$0.15 \cdot \sigma_a$</td>
</tr>
<tr>
<td>AR(1) of expected growth, $\rho$</td>
<td>0.98</td>
<td>0.92</td>
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<tr>
<td><strong>New Keynesian parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price elasticity of demand, $\eta$</td>
<td>7</td>
<td>7</td>
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<tr>
<td>Probability firm cannot change P, $\alpha$</td>
<td>75%</td>
<td>75%</td>
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<tr>
<td><strong>Policy parameters</strong></td>
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<tr>
<td>Steady-state debt to GDP, $S_B$</td>
<td>0.44</td>
<td>0.44</td>
</tr>
<tr>
<td>Steady-state $\xi$</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Monetary policy inflation coeff., $\alpha_\pi$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Output growth gap coeff., $\alpha_{\Delta y}$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Inertia coeff., $\alpha_r$</td>
<td>0.8</td>
<td>0.8</td>
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</table>

The four models were calibrated to minimize the distance between the moments implied by the model and the moments taken from the data. The moments to match were the volatilities of consumption and investment growth, the level of the risk-free rate, and if possible the equity premium.

For monetary policy, the default interest rate rule used for calibration includes an inertia coefficient of 0.8 and an output growth coefficient of 0.75. The parameters chosen for these coefficients are set to match inflation dynamics and the interest rate serial correlation observed in the data. An inertia coefficient of 0.8 is consistent with empirical quarterly estimates, which set it as high as 0.9. Without the output growth, inflation in the model is positively correlated with permanent productivity shocks, which is counterfactual. The incorporation of the output growth causes inflation to fall with good productivity shocks and rise with bad productivity shocks. The
inflation coefficient is set to 1.5 to match the original work of Taylor (1993).

**Production and Preference Parameters.** The parameters for effective risk aversion ($\gamma = 10$) and intertemporal elasticity of substitution ($\psi = 2$) are consistent with the estimates of Bansal et al. (2007), Bansal et al. (2010), and Colacito and Croce (2011).\(^8\) The capital share ($\alpha = 0.25$) and the quarterly depreciation rate of physical capital ($\delta = 0.01725\%$) are consistent with the labor share of income in the data and with the quarterly depreciation rate of 1.5 to 4% observed in the real business cycle (RBC) literature. I calibrate $\mu$ at 2% per year, consistent with the average annual real growth rate of the US economy. I set the persistence parameter on long run risk at $\rho = 0.98$, which is close to the point estimate of Croce (2014b).

The subjective discount factor is set to be consistent with the low risk-free rate observed in the data. Labor is endogenous and set to match the level of hours in the data, $\iota = 0.35$. I set $\sigma_a$ to match the standard deviation of consumption. The smaller long-run shock is set to $0.15 \cdot \sigma_a$ as estimated by Croce (2014b).

**Free Parameters and Moments Matched.** The free parameters above are the subjective discount factor ($\beta$), the depreciation rate for capital ($\delta$), the volatility of short-run risk ($\sigma_a$), and the persistence parameter on long-run risk ($\rho$). These parameters are chosen within a grid of values accepted by the literature in order to maximize the model’s ability to reproduce the moments of interest, which are the mean of the real risk-free rate, the levered excess return, and the volatilities of consumption and investment growth. I minimize the distance between the moments implied by the model and the moments taken from the data as shown in table 3. The distance is measured by

$$\min_a \zeta(a) = [\hat{F}_T - f(a)]'[\hat{F}_T - f(a)]$$

\(^8\)See Swanson (2012) for a discussion of risk aversion when leisure is present.
The four models are calibrated to minimize the distance between the moments implied by the model and the moments taken from the data. The minimization is done by searching over a grid of values: $\beta = [0.995, 1]$, $\delta = [.015,.04]$, $\sigma_a = [0.005,0.02]$, and $\rho = [0.92, 0.98]$. The moments to match were the volatilities of consumption and investment growth, the level of the risk-free rate, and the equity premium, if possible. For recursive preferences, settings tested were High RA ($\gamma = 10$), High IES ($\psi = 2$), and Low IES ($\psi = 0.2$). For CRRA preferences, settings tested were High RA ($\gamma = 10$) and High IES ($\psi = 2$). All entries for the models are obtained from repetitions of small samples. Data refer to the US and include pre–World War II observations (1930–2012). Quarterly calibrations are reported in table 2. Excess returns are levered by a factor of three, consistent with García-Feijóo and Jorgensen (2010). Note that with CRRA preferences, a low risk free rate and high equity premium are not possible given the inverse link of RA and IES.

where $f(a)$ is the vector of moments generated by the model, and $\hat{F}_T$ is the moments in the data.

The minimization is done by searching over a grid of values for $a$: $\beta = [0.995, 1]$, $\delta = [.015,.04]$, $\sigma_a = [0.005,0.02]$, and $\rho = [0.92, 0.98]$. Note that in the data, excess returns are levered. Hence, I use the following excess return: $R^{LEV}_{ex,t} = \chi_{LEV}(R^K_t - R^f_t)$. The calibration of $\chi_{LEV}$ is set to 3 to match the debt-equity ratio in the data along with the degree of operating leverage as estimated in García-Feijóo and Jorgensen (2010).

**Simulation.** The policy rules are numerically computed using second-order approximations from Dynare++. The simulations consist of random draws of the two productivity shocks (short-run and long-run) and a fiscal shock (government spending). The number of periods is 200 (the first 100 are discarded) and the number of simulations is 100. The moments for the models are listed in table 3. The preferred High IES ($\psi = 2$), High RA ($\gamma = 10$) model comes closest to matching
the low risk-free rate, the high equity premium, the smooth volatility of consumption growth, and volatile investment growth. Matching data on both macroeconomic aggregates and asset pricing facts imposes joint structural restrictions on both the quantity and price of risk in the data.

3 Ramsey Optimal Monetary Policy

In this section, I characterize optimal monetary policy under commitment in the spirit of Ramsey (1927). I focus on optimality from the “timeless perspective,” as described in Woodford (2003). This simplifies the analysis (same first order conditions for \( t = 0 \) and \( t > 0 \)) and implies that the economy has been operating for an infinite number of periods. I assume that the Ramsey planner commits to decisions from the past and its objective is to maximize the lifetime utility of the representative agent.

The optimal monetary policy is based on the behavior of the nominal interest rate \( \{R_t\}_{t=0}^{\infty} \) associated with the competitive equilibrium that maximizes the utility of the agent. To accomplish this, the Ramsey planner takes the \( N \) equilibrium conditions for optimal behavior as given but excludes the interest rate rule. The Ramsey planner attaches a lagrange multiplier to each equilibrium condition and then takes first order conditions with respect to the \( N \) endogenous variables and \( N - 1 \) multipliers. The solution yields \( 2 \cdot N - 1 \) Ramsey equilibrium conditions and \( 2 \cdot N - 1 \) endogenous variables, as the Ramsey planner effectively pins down the behavior of the nominal interest rate.

The equilibrium dynamics are computed by taking second-order approximations around the deterministic Ramsey steady state. Following Kamenik (2011), I set backward looking multipliers to zero at \( t = 0 \) for all simulations in order to ensure that policy is not time-inconsistent. This ensures that the first order conditions for \( t = 0 \) are identical to the first order conditions for \( t > 0 \), making it consistent with the “timeless perspective.”

Note that stability conditions that apply under the interest rate rule do not necessarily hold
Fig. 1: Determinacy Region for Ramsey Optimal Policy

This figure shows all the configurations in which there exists a unique and stable equilibrium, which are indicated by blue crosses. As the intertemporal elasticity of substitution rises to greater than one, the quantity of risk aversion that allows for a unique and stable equilibrium also declines. The coefficients are changed while keeping everything else parameterized as in Table 2. Recalibrating does not lead to a unique equilibrium for any IES > 1.

under the Ramsey optimal policy. For instance, I find that any IES greater than one leads to a violation of the Blanchard-Kahn conditions. This can be seen in Figure 3, which shows the levels of risk aversion and the IES that yield a unique and stable equilibrium. The intuition for why a higher IES may lead to instability is directly related to its definition. With a higher IES, consumption growth responds to a greater extent to changes in interest rates and this leads to stability issues for the Ramsey solution that are not present with simple interest rate rules. While using a lower IES becomes an issue with regards to matching the real risk free rate, it does not preclude the model from matching the high equity premium observed in the data. For this reason, the equity premium and its impact on Ramsey optimal monetary policy will be the focus of the following section.
3.1 Ramsey Optimal Moments

This section focuses on characterizing the optimal moments associated with the Ramsey policy. Panel A of Table 4 shows the model properties when risk aversion is high and all shocks (government spending, short run and long run productivity) are present. The only difference between the six models is the role of Monetary Policy: (1) “Ramsey Optimal” chooses the nominal interest rate to maximize lifetime utility, (2) “Inflation Targeting” keeps inflation constant, and (3) “\( R_t = 3.0 \cdot \pi_t \)” allows inflation to fluctuate but doesn’t completely stabilize inflation.

There are many interesting components of Table 4 that will be further explained in the impulse response functions below. First, both the average and volatility of the inflation rate for the Ramsey policy are much higher than previous Calvo-based studies of optimal monetary policy (see Table 1). When the model does not match the equity premium, as in Panels B, C, D, E, and F, the optimal first and second moment of inflation fall by orders of magnitude and become consistent with the previous literature. This is graphically demonstrated in Figure 2, which shows that the optimal moments of inflation rise as the model reaches an equity premium that is consistent with the data.
Table 4: Dynamic Properties of Optimal Ramsey Policy and Various Rules

|                      | $E(rp)$ | $E(\pi)$ | $\sigma(\pi)$ | $\sigma(\Delta y)$ | $\sigma(CU)$ | $\sigma(V)$ | $\sigma(EV)$ | $\sigma(SDF)$ | $\rho_{y,\pi}$ | $E(\mu)$ | $\rho_{\Delta y, \mu}$ | Welfare | CE % | Perp. ($) |
|----------------------|---------|-----------|---------------|---------------------|--------------|-------------|--------------|---------------|----------------|-----------|----------------—|----------|------|-----------|
| **Panel A: RA = 5.5** |         |           |               |                     |              |             |              |               |                |           |                  |          |      |           |
| Ramsey Optimal       | 4.58    | 3.61      | 1.38          | 3.20                | 1.35         | 3.95        | 58.34        | 29.18         | −0.4449        | 0.09      | 0.44             | 1.4342   | 0    | −         |
| Inflation Targeting  | 1.34    | 0.00      | 0.00          | 3.06                | 1.49         | 3.98        | 58.65        | 29.44         | 0              | 0.15      | 0                | 1.4311   | −0.61| $10,593$ |
| $R_t = 3.0 \cdot \pi_t$ | 1.33    | −0.62     | 0.46          | 3.18                | 1.52         | 3.97        | 58.62        | 29.48         | 0.5057         | 0.16      | −0.46            | 1.4309   | −0.65| $11,277$ |
| **Panel B: RA = 6.5** |         |           |               |                     |              |             |              |               |                |           |                  |          |      |           |
| Ramsey Optimal       | 6.36    | 4.93      | 1.97          | 3.63                | 1.36         | 3.79        | 67.41        | 35.11         | −0.22          | 0.066     | 0.23             | 1.4095   | 0    | −         |
| Inflation Targeting  | 1.64    | 0         | 0             | 3.07                | 1.49         | 3.98        | 70.61        | 35.38         | 0              | 0.154     | 0                | 1.4026   | −1.40| $24,025$ |
| $R_t = 3.0 \cdot \pi_t$ | 1.63    | −0.74     | 0.45          | 3.18                | 1.52         | 3.97        | 70.57        | 35.41         | 0.51           | 0.156     | −0.46            | 1.4024   | −1.44| $24,723$ |
| **Panel C: RA = 2**  |         |           |               |                     |              |             |              |               |                |           |                  |          |      |           |
| Ramsey Optimal       | 0.44    | 0.09      | 0.06          | 3.04                | 1.46         | 3.98        | 18.79        | 9.7           | 0.03           | 0.153     | −0.03             | 1.5304   | 0    | −         |
| Inflation Targeting  | 0.38    | 0         | 0             | 3.05                | 1.48         | 3.98        | 18.76        | 9.71          | 0              | 0.154     | 0                | 1.5302   | −0.03| $639$     |
| $R_t = 3.0 \cdot \pi_t$ | 0.38    | −0.23     | 0.47          | 3.17                | 1.51         | 3.98        | 18.75        | 9.74          | 0.51           | 0.155     | −0.45            | 1.53     | −0.07| $1,279$  |

Panel D: RA = 5.5, No LRR

| Ramsey Optimal       | 0.23    | 0.23      | 0.16          | 2.52                | 0.60         | 0.68        | 9.97         | 15.18         | 0.39           | 0.159     | −0.38             | 1.5414   | 0    | −         |
| Inflation Targeting  | 0.05    | 0         | 0             | 2.31                | 0.65         | 0.67        | 9.82         | 15.17         | 0              | 0.153     | 0                | 1.54119  | −0.03| $638$     |
| $R_t = 3.0 \cdot \pi_t$ | 0.05    | −0.19     | 0.17          | 2.33                | 0.64         | 0.67        | 9.81         | 15.17         | 0.50           | 0.154     | −0.51            | 1.54117  | −0.04| $717$     |

Panel E: RA = 6.5, No LRR

| Ramsey Optimal       | 0.34    | 0.34      | 0.21          | 2.59                | 0.59         | 0.69        | 12.05        | 18.16         | 0.41           | 0.148     | −0.4              | 1.5337   | 0    | −         |
| Inflation Targeting  | 0.06    | 0         | 0             | 2.32                | 0.65         | 0.67        | 11.82        | 18.16         | 0              | 0.154     | 0                | 1.5335   | −0.03| $638$     |
| $R_t = 3.0 \cdot \pi_t$ | 0.07    | −0.23     | 0.17          | 2.34                | 0.65         | 0.67        | 11.81        | 18.15         | 0.51           | 0.154     | −0.51            | 1.5334   | −0.05| $957$     |

Panel F: RA = 2, No LRR

| Ramsey Optimal       | 0.03    | 0.05      | 0.04          | 2.36                | 0.64         | 0.68        | 3.15         | 5.27          | 0.09           | 0.153     | −0.04             | 1.5673   | 0    | −         |
| Inflation Targeting  | 0.02    | 0         | 0             | 2.32                | 0.65         | 0.67        | 3.14         | 5.27          | 0              | 0.154     | 0                | 1.5673   | −0.003| $62$      |
| $R_t = 3.0 \cdot \pi_t$ | 0.02    | −0.07     | 0.18          | 2.34                | 0.65         | 0.67        | 3.14         | 5.26          | 0.51           | 0.154     | −0.51            | 1.5672   | −0.009| $156$    |

This table is divided into six panels. The first panel contains results for the model with risk aversion of 5.5 and all three shocks (government spending, short run and long run productivity). The second panel is the same except with a higher level of risk aversion (6.5), while the third panel has lower risk aversion set to 2. The bottom three panels turn off the long run productivity shocks. $E(rp)$ is the equity risk premium, $E(\pi)$ is the average inflation rate, $\sigma(\pi)$ is the volatility of inflation, $\sigma(\Delta y)$ is the volatility of output growth, $\sigma(CU)$ is the volatility of the consumption-leisure bundle, $\sigma(V)$ is the volatility of lifetime welfare, $\sigma(EV)$ is the volatility of the expectation of the continuation value, $\sigma(SDF)$ is the volatility of the stochastic discount factor, $\rho_{y,\pi}$ is the correlation of output and inflation, $E(\mu)$ is the average markup, $\rho_{\Delta y, \mu}$ is the correlation of output growth and the markup, Welfare is $E(V)$ and CE% is the consumption equivalent welfare difference compared to the Ramsey policy. The perpetuity of the welfare gain is computed by assuming an interest rate of 2% and real consumption expenditures per capita of $34,270 per year. The consumption equivalent unit percentage is multiplied by $34,270 and divided by the interest rate to arrive at the value of the perpetuity. Moments are calculated from simulations that are 400 periods and repeated 100 times.
In addition to greater volatility of inflation, the Ramsey planner also imposes greater volatility of output growth compared to the other rules in Panel A. However, given that output is not equal to consumption, the more relevant measure in explaining the higher welfare is the volatility of the consumption-leisure bundle, $\sigma(CU)$. This measure, along with volatility of lifetime welfare ($\sigma(V)$), the volatility of the continuation value ($\sigma(EV)$), and the volatility of the stochastic discount factor are all the lowest for the Ramsey optimal policy.

Second, in terms of comovements, the optimal Ramsey Policy yields a negative correlation between output and inflation and a positive correlation between labor and inflation. As will be discussed more in the following section, the Ramsey Planner finds it optimal to push inflation up in bad times, which drives down the markup. The Ramsey Planner uses the markup as a hedge, as it has a positive correlation with output growth.

Third, the optimal Ramsey Policy is able to considerably lower the steady state average markup. This is graphically demonstrated in Figure 3. By lowering the average markup, the optimal Ramsey policy is pushing income away from profits (which reflect pure rents) to labor and capital income. This reduction in the inefficiency associated with monopolistic competition is completely missing in previous studies that (1) assume a counterfactual subsidy to eliminate the distortion, (2) use a first-order approximation, which does not allow for monetary policy to have any effect on the steady state mean of the markup, or (3) does not use a level of risk aversion that is consistent with the high equity premium.

The difference in welfare in consumption equivalent units between the optimal Ramsey policy and the inflation targeting rule is -0.61%, which is over 200 times greater than the typical setting that uses low risk aversion and has no long-run productivity shocks, as shown in Panel F. This difference between the optimal Ramsey policy and the inflation targeting rule is equivalent to a perpetuity worth over $10,000. In other words, the greater emphasis on stabilizing output would be worth a one-time payment of over $10,000 to each individual in the economy.\footnote{Base year 2009 dollars.} In contrast,
Fig. 2: Ramsey Optimal Policy: Moments for Increasing Risk Aversion

This figure shows the optimal inflation rate, optimal inflation volatility, and equity premium on the vertical axis. On the horizontal axis is the effective risk aversion. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA” case but with the Ramsey optimal policy. As the equity premium reaches higher levels, the optimal inflation rate and inflation volatility also significantly rise. This suggests a decline in the relative costs of inflation as the model becomes more consistent with financial data.

with a low equity premium, inflation targeting is very close to the best that monetary policy can do because the costs of inflation dominate the welfare costs of recessions. The difference between the optimal policy and inflation targeting for this setting is only equivalent to a perpetuity worth $62.

Finally, note that excluding the long run productivity shocks (as in Panels D, E, and F) leads to a lower equity premium and as a result, lower optimal inflation moments that are more in line with the existing literature. This illustrates that higher risk aversion is not enough in a production-based general equilibrium model to match the high equity premium. Time variation in the conditional expectation of productivity growth (as empirically documented in Croce (2014a)) is an important feature that dramatically alters the quantitative implications of optimal monetary
Fig. 3: Ramsey Optimal Policy: Markup and Factor Shares

This figure shows the labor share of income, capital share of income, and the markup on the vertical axis. On the horizontal axis is the effective risk aversion. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA” case but with the Ramsey optimal policy. As the equity premium reaches higher levels, the Ramsey optimal policy uses inflation to increase the labor share of income and pushes down the average markup.

3.2 Ramsey Optimal Dynamics

The previous section documented that the Ramsey optimal monetary policy was able to engineer a significant decline in the steady state markup. To understand this, observe the dynamics associated with the negative short-run productivity shock in Figure 4. The “High Equity Premium” and “Low Equity Premium” optimal monetary policy responses are plotted with the only difference being the level of risk aversion.

In the first period of the shock, output, consumption, investment, and labor all decline for both
models. Interestingly, these variables along with inflation decline to a greater extent in the initial period for the “High Equity Premium” model. The optimal monetary policy induces a positive (or less negative) wealth effect in response to the negative productivity shock with the promise of greater accommodation going forward, causing leisure to rise in the first period. This causes output to be lower than the natural level of output which results in lower inflation and an initial rise in the markup.

The following period, capital adjusts and productivity growth reverts back to zero and the accommodation of monetary policy induces greater labor and investment demand, pushing output above the natural level and causing the markup to decline while inflation rises. The exact same logic applies to the long run shock, shown in Figure 5. By allowing inflation to rise, monetary policy provides good long-run news thanks to the reduction in the average markup.

More importantly, the second order approximation indicates an asymmetry with respect to positive and negative shocks, as the model is no longer linear. The dynamics that occur in the aftermath of the negative shocks appear to dominate in terms of influencing the average markup and average inflation rate. In a first order approximation or linear setting, there would be symmetrical responses and thus the dynamics would have no effect on steady state levels.

Figure 6 is included to show the difference in dynamics between the strict inflation targeting policy and the Ramsey optimal policy. Compared to strict inflation stabilization, labor, output, consumption, and investment are all higher for the Ramsey optimal policy. With a high equity premium, the Ramsey optimal monetary policy advocates for pushing inflation higher in the aftermath of negative productivity shocks in order to stabilize output and place downward pressure on the markup.

The responses to the government spending shock in Figure 7 show a similar pattern. Excluding government shocks from the analysis has little effect on the optimal average and volatility of the inflation rate. A variance decomposition (not shown) suggests the permanent productivity shocks are the major driver of fluctuations within the model.
Fig. 4: Ramsey Optimal Monetary Policy: Negative Short-Run Shock

This figure shows the effects of a negative short-run productivity shock. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent with the equity premium under the Ramsey policy. As the equity premium reaches higher levels, the Ramsey optimal policy allows for persistently higher inflation in the medium to long-term to stabilize output, consumption, and investment.
This figure shows the effects of a negative long-run productivity shock. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent with the equity premium under the Ramsey policy. As the equity premium reaches higher levels, the Ramsey optimal policy allows for persistently higher inflation in the medium to long-term to stabilize output, consumption, and investment.
Fig. 6: Ramsey Optimal Monetary Policy vs Inflation Targeting: Negative Long-Run Shock
This figure shows the effects of a negative long-run productivity shock for the Ramsey optimal policy versus the strict inflation targeting rule. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent with the equity premium under the Ramsey policy. The Ramsey optimal policy stabilizes output, consumption and investment to a greater extent over the medium to long term, at the expense of higher inflation.
Fig. 7: Ramsey Optimal Monetary Policy: Negative Government Spending Shock

This figure shows the effects of negative government spending shock. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent with the equity premium under the Ramsey policy. The Ramsey optimal policy pushes up output, consumption and investment to a greater extent over the medium to long term, at the expense of higher inflation.
3.3 Can Any Rule Replicate the Ramsey Optimal Policy?

The goal of this section is to better understand which rule (if any) is capable of replicating the Ramsey optimal policy. The issue with the Ramsey optimal policy is it provides no guidance as to how the optimal policy can be implemented. To conduct this analysis, I do not restrict attention to just measures of inflation or output, as the Ramsey planner is unconstrained in choosing the optimal interest rate rule that maximizes household utility. I split the analysis into settings with long run shocks only and short run shocks only to better isolate the different responses for optimal monetary policy.

3.3.1 Long Run Shocks Only

I use impulse response functions (IRFs) in response to a small but persistent productivity shock in order to determine the rule that comes close to matching the optimal monetary policy. Following the method used in Schmitt-Grohe and Uribe (2005), I was able to find a rule that closely matches the optimal responses of output, consumption, investment, labor, average markup, real wage, inflation rate, and real return of capital. The rule is

\[
\hat{R}_t = 0.95 \cdot \hat{R}_{t-1} + (1 - 0.95) \cdot (5 \cdot \hat{\pi}_t + 9 \cdot \Delta k_t + 1 \cdot q_t)
\]

The above rule puts weight on the previous period’s interest rate, inflation, growth in capital, and the price of capital.\(^{10}\) The dynamics are shown below in figure 8. Shutting off any of these variables in the rule leads to responses that are less able to match the dynamics of the optimal Ramsey policy. For instance, shutting down the response to the price of capital holding all else constant leads to a substantial decline in output, consumption, labor, and inflation over the first ten periods compared to the Ramsey optimal policy. The price of capital falls immediately in response to the news of a small but persistent decline in productivity growth, which is why

\(^{10}\)Note there may be multiple rules that come close to replicating the Ramsey optimal policy and this is just one example.
putting a positive weight on it helps push up output and consumption towards the optimal policy.

Responding to capital growth is close to isomorphic to responding to the level of investment. Not responding to capital growth results in higher output, consumption, labor, and inflation in the short run but these variables persistently fall below the Ramsey optimal responses after five periods. In other words, responding to capital growth stabilizes output and consumption to a greater extent in the medium to long term. Stabilizing the medium to long term levels of output and consumption appears to be a priority of the Ramsey planner when the equity premium is high. This is accomplished at the expense of higher inflation.

Finally, lowering the weight on the previous period’s nominal interest rate tends to amplify output, consumption, and labor in the first period but these variables decline to a much greater extent every period there after. Similar to the weight on capital growth, placing weight on the previous period’s nominal interest rate stabilizes output and consumption more in the medium to long-term, bringing the rule closer to the Ramsey optimal policy.

\[11\text{Note that within the model, responding to simply the level of investment is not possible as it is normalized by the level of productivity.}\]
Fig. 8: Ramsey Optimal Monetary Policy and Implied Rule: Negative Long-Run Shock

This figure shows the effects of a negative long-run productivity shock for the Ramsey optimal policy and the implied rule. The implied rule is defined as

\[ \hat{R}_t = 0.95 \cdot \hat{R}_{t-1} + (1 - 0.95) \cdot (5 \cdot \hat{\pi}_t + 9 \cdot \Delta k_t + 1 \cdot q_t). \]

The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent with the equity premium under the Ramsey policy.
**Fig. 9 - Ramsey Optimal Monetary Policy and Implied Rule: Negative Short-Run Shock**

This figure shows the effects of a negative short-run productivity shock for the Ramsey optimal policy and the implied rule. The implied rule is defined as $\hat{R}_t = 0.95 \cdot \hat{R}_{t-1} + (1 - 0.95) \cdot (5 \cdot \hat{\pi}_t + 9 \cdot \Delta k_t + 0.15 \Delta i_t)$. The parameterization is based on the calibration in table 2 for the “Low IES=0.2, High RA=5.5” case with risk aversion set to be consistent the equity premium under the Ramsey policy.

### 3.3.2 Short Run Shocks Only

With short run shocks only, a similar rule comes close to matching the optimal policy. The following rule, $\hat{R}_t = 0.95 \cdot \hat{R}_{t-1} + (1 - 0.95) \cdot (5 \cdot \hat{\pi}_t + 9 \cdot \Delta k_t + 0.15 \Delta i_t)$, also responds to investment growth and does not respond to the price of capital. The price of capital for short run shocks is not volatile because there are no adjustment costs. Weight is placed on investment growth because it reduces the volatility of consumption and output growth and becomes closer to the optimal policy.

The takeaway from this exercise is that when the model exhibits a high equity premium, the Ramsey planner’s actions resemble a rule that responds to capital growth, the previous period’s interest rate, the price of capital, and inflation. The Ramsey planner wishes to stabilize consumption and output over the medium to long run, compared to the very short run. In contrast, when the model exhibits a low equity premium, the major focus of optimal monetary policy is stabilizing inflation only. This section focused on rules that were based on all variables within the economy. The following section focuses on simple rules that are functions only of observable variables.
4 Simple Interest Rate Rules

In the following section, I describe the characteristics of the interest rate rules that yield the highest welfare. I then restrict the model to a setting with only long-run shocks and a setting with only short-run shocks to provide a deeper understanding of the underlying dynamics. Following this, I examine the effect of the inflation coefficient and output growth coefficient on the three major channels of welfare.

To obtain the simple monetary policy rule that yields the highest welfare, I search across a grid for the inflation coefficient, $\alpha_\pi$, the output growth coefficient, $\alpha_\Delta y$, and the inertia coefficient, $\alpha_r$. Their respective grids are $(1, \infty)$, $(-\infty, \infty)$, and $(-\infty, \infty)$. I constrain $\alpha_\pi > 1$ in order to be consistent with the Taylor principle. With $\alpha_\pi > 1$, the real interest rate rises with inflation, and this ensures determinacy. The rule is formulated as follows:

$$\hat{R}_t = \alpha_r \cdot \hat{R}_{t-1} + (1 - \alpha_r)(\alpha_\pi \cdot \hat{\pi}_t + \alpha_\Delta y \cdot \Delta \hat{y}_t)$$

(3)

The variables denoted with a hat are log deviations from steady state. The first two panels of table 5 list the optimal inflation volatility and inflation coefficient for each model, and the third panel shows the optimal output growth coefficient. The bottom panel shows the relative welfare gain (in the model that matches asset prices) for the optimal policy compared to the policy that completely stabilizes inflation. The columns are split into settings based on the sources of the shocks in order to capture the role of long-run risk.

Inflation Volatility. The setting with recursive preferences, high risk aversion ($\gamma = 10$), and a high IES ($\psi = 2$) yields the highest optimal inflation volatility of 0.42% with all shocks. Compared to results in extant studies with similar model features, this value is relatively high and represents close to 20% of the observed historical inflation volatility. Most of the inflation volatility can

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12 Inflation is defined as the annualized percentage change in the quarterly GDP price deflator dating back to 1947.
Table 5: Optimal Coefficients, Volatility, and Welfare within Class of Simple Rules

<table>
<thead>
<tr>
<th>Preferences</th>
<th>RA</th>
<th>IES</th>
<th>Optimal Inflation Volatility</th>
<th>Optimal Inflation Coefficient</th>
<th>Optimal Output Growth Coefficient</th>
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<td></td>
<td></td>
<td></td>
<td>All Shocks</td>
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<td>SRR only</td>
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<td>High</td>
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<td>0.39%</td>
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<td>Low</td>
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<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
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<td>0</td>
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<tr>
<td>CRRA</td>
<td>Low</td>
<td>High</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Rec.Pref., High RA, High IES:

- Relative Welfare Gain: 0.07% | 0.07% | 0.00037%
- Consumption Equivalent Units: 0.19% | 0.20% | 0.001%
- Perpetuity of Welfare Gain: $3,255 | $3,427 | $17.13

This table shows the optimal inflation volatility, optimal inflation coefficient, and optimal output growth coefficient across different shocks and different utility specifications. RA is risk aversion, IES is intertemporal elasticity of substitution, and CRRA is constant relative risk aversion. For recursive preferences, settings tested were High RA ($\gamma = 10$), High IES ($\psi = 2$), and Low IES ($\psi = 0.2$). For CRRA preferences, settings tested were High RA ($\gamma = 10$) and High IES ($\psi = 2$). Quarterly calibrations are reported in table 2. The bottom panel shows the relative welfare gain compared to the policy that places infinite weight on inflation. The consumption equivalent units are computed as in An (2010). The perpetuity of the welfare gain is computed by assuming an interest rate of 2% and real consumption expenditures per capita of $34,270 per year. The consumption equivalent unit percentage is multiplied by $34,270 and divided by the interest rate to arrive at the value of the perpetuity.
be attributed to the long-run shocks. The higher optimal inflation volatility is due to the rise in the average markup as monetary policy increasingly stabilizes inflation. These dynamics are explained in greater detail in section 4.1.

**Output Growth.** Under recursive preferences, I find that the output growth coefficient is dramatically larger for the setting with long-run shocks compared to that with only short-run shocks. Increasing the weight on the output growth reduces the markup and lowers the volatility of the consumption-leisure bundle. For CRRA preferences, I find it optimal to completely eliminate price dispersion and place an infinite coefficient on inflation with no weight on output growth. The average markup and the volatility of the consumption-leisure bundle stay relatively constant as monetary policy increasingly stabilizes inflation, which is counterfactual to empirical evidence.

**Welfare Gains.** For the benchmark setting with a low risk free rate and high equity premium (table 5, last panel), I find the optimal policy provides a welfare gain of 0.07% compared to the policy that places an infinite weight on inflation. This translates into about 0.20 additional percentage of consumption units every quarter. While this may be small compared to the gains found in the policy literature that incorporates endogenous growth, a perpetuity that pays this amount each year from now on is worth over $3,000, assuming an interest rate of 2%. In other words, a greater response to output growth and less to inflation (relative to that advocated in the existing literature) would be worth a one-time payment of over $3,000 to each individual.\(^\text{13}\) This is in contrast to results from the setting with only short-run shocks, in which an equivalent calculation would yield a perpetuity worth only $17. This small magnitude is in line with the existing literature (see Schmitt-Grohé and Uribe (2007)).

To explain the low inflation coefficient and high weight on the output growth for the asset pricing-oriented model, I focus on the setting in which there are only long-run shocks. Although the long-run shock is one-seventh the size of the short-run shock, its persistence (combined with

\(^{13}\)Base year 2009 dollars.
the very forward-looking nature of agents) dominates the other shocks.

4.1 Long-Run Productivity Shocks Only

The small but persistent news shock to productivity growth has been empirically documented by Croce (2014b), Beaudry and Portier (2004), Schmitt-Grohé and Uribe (2007), Barsky and Sims (2011), and Kurmann and Otrok (2010). Specifically, Croce (2014b) finds that the conditional mean of productivity growth is extremely persistent and time-varying. In a setting with forward-looking monopolistically competitive firms, a low-frequency predictable fluctuation in productivity can have significant ramifications for the prices chosen by firms, which reverberates through the demand-driven New Keynesian model. Below, I examine the channels that drive the finding of a low optimal inflation coefficient and relatively high weight on output growth.

Optimal Inflation Coefficient and Three Welfare Channels. In this section I examine the reasons the optimal inflation coefficient is low for the recursive preferences asset pricing-oriented model and high for the CRRA preferences model. In figures 10–11, I compare two different models: (1) recursive preferences with high risk aversion ($\gamma = 10$) and high IES ($\psi = 2$) and (2) CRRA preferences with high risk aversion ($\gamma = 10$) and low IES ($\psi = 1/10$). Figure 10 depicts the effects of increasing the inflation coefficient and figure 11 shows the welfare. The increase in $\alpha_\pi$ can be thought of as moving from a dove regime to a hawk regime. The level of price dispersion decreases in all settings due to the lower inflation volatility (far right panel of figure 10). Monetary policy, by increasingly stabilizing inflation, reduces nominal uncertainty.

Major differences across the two settings can be seen in the left two panels of figure 10. The far left panel shows the volatility of the consumption-leisure bundle. While volatility is increasing with the degree of inflation stabilization for all two settings, the increase is lowest for the CRRA preferences. This makes sense because of the inverse nature of CRRA preferences, in which the IES is lower due to the high risk aversion. With a lower IES, by definition agents will react less
Fig. 10: Effects of Increasing the Inflation Coefficient ($\alpha_\pi$)

This figure shows the three key channels for welfare (volatility of consumption-leisure bundle, average markup, price dispersion) and the effects of greater inflation stabilization. The setting is a world with only long-run news shocks to productivity. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case, as in equation 3 and table 2. The recursive preference setting exhibits a strong increase in the volatility of the consumption-leisure bundle compared to the CRRA preferences. The average markup rises the most for the high IES and high risk aversion setting. All of the settings see a reduction in price dispersion as the inflation coefficient rises.

to monetary policy and so the volatility does not rise as much for a given change in policy. This is also true for the recursive preferences as one moves from a low IES to a high IES. The increase in volatility of the consumption-leisure bundle is greatest for the high IES case.

The average markup rises the most for the model with the high IES. The markup falls more in response to negative productivity shocks, and the more that monetary policy stabilizes inflation, the less the markup will be allowed to decline, resulting in a higher average markup. Note that with less asymmetry, the low IES markup does not rise as much with the inflation coefficient. Both models converge to the average markup that would occur in the nonstochastic steady-state, in which the variance of the markup is zero. In other words, they all converge to the level of the markup that would occur under a first-order approximation.

In terms of overall welfare, the optimal inflation coefficient is lowest for the model with high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$), as shown in figure 11. Beyond the value of 3 for the inflation coefficient, the benefits of reducing price dispersion are outweighed by the costs of the higher markup and greater volatility of the consumption-leisure bundle. For the low IES case,
Fig. 11: Effects of Increasing the Inflation Coefficient: Welfare ($\alpha_{\pi}$)

This figure shows the effects on welfare of increasing the inflation coefficient in a world with only long-run productivity shocks. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case as in equation 3 and table 2. The greatest fall in welfare comes with the High IES, High Risk Aversion on the far left due to the higher markup from stabilizing inflation and the increased volatility of the consumption-leisure bundle. This is in contrast to the CRRA preferences setting in which completely stabilizing inflation is optimal.

the optimal inflation coefficient is higher with lesser increases in the markup. Finally, the CRRA preferences setting has a value on the inflation coefficient of infinity as the reduction in price dispersion dominates the markup channel. Monetary policy has very little effect on the volatility of the consumption-leisure bundle and the markup, which means the costs of stabilizing inflation are essentially nil.

Output Growth Coefficient and Three Welfare Channels. In this section I examine the reasons the optimal output growth coefficient is high for the asset pricing-oriented model. As shown in figure 12, placing weight on the output growth reduces the variability of the consumption-leisure bundle and also provides a greater anchor for inflation expectations. This is because when output falls below potential, monetary policy lowers interest rates with the implicit assurance that interest rates will rise as output increases back to potential. Anchoring inflation expectations is imperative because current inflation depends on expectations of future inflation, and in the model firms are very patient and forward looking.\footnote{This is in contrast to reacting to simply the level of output, which destabilizes inflation expectations. I find that a zero weight on the level of output is optimal as the negative effects of price dispersion dominate.}
Fig. 12: Effects of Increasing the Output Growth Coefficient ($\alpha \Delta y$)

This figure shows the effect of placing higher weight on the output growth in a world with only long-run productivity shocks. The other coefficients are set to match the rule that yields the highest welfare for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case, as in equation 3 and table 2. The volatility of the consumption-leisure bundle and the average markup decline, while price dispersion is increasing due to higher inflation volatility, which causes a hump-shape for welfare.

In addition, the markup monotonically decreases with the output growth coefficient. Greater weight on the output growth means relatively less weight on inflation, so that inflation volatility and price dispersion increase. Thus, after the negative long-run productivity shock shown in Figure 13, the average markup is persistently lower in the medium to long term, while inflation is higher. Furthermore, the decline in the response of the consumption-leisure bundle is also beneficial to welfare. Eventually, the consumption-leisure bundle volatility increases due to monetary policy overcompensation. This, combined with the higher price dispersion, leads to the decline in welfare when moving beyond an output growth coefficient of 1.5.

4.2 CRRA Preferences

This section shows how agents and firms with CRRA preferences react to negative long-run productivity shocks. Figure 14 shows the real stochastic discount factor in the upper left panel. Note the rise for recursive preferences is eight times greater in magnitude due to the continuation value that captures news. The higher real SDF combined with the higher IES makes firms more forward looking, which causes them to raise prices to a greater extent because they are concerned with higher future marginal costs.
Fig. 13: Higher Output Growth Response: Negative Long-Run Shock ($\alpha_\Delta y$)

This figure shows the response to a negative long-run productivity shock. The parameterization is based on the calibration in table 2 for the high IES ($\psi = 2$) and high risk aversion ($\gamma = 10$) case and focuses on the policy that yields the highest welfare in equation 3. The volatility of the consumption-leisure bundle declines and the average markup falls more over the medium to long term due to the higher inflation.

The higher inflation directly translates into a lower real markup, which acts as a hedge and provides good long-run news. The low IES with CRRA preferences reduces the effectiveness of monetary policy so that the consumption-leisure bundle and average markup quickly converge to the steady state. This analysis concurs with the evidence in figure 10, which shows that for CRRA preferences, monetary policy is less effective in reducing the average markup and the volatility of the consumption-leisure bundle.

4.3 Short-Run Shocks Only

In a setting with only short-run shocks, I find that the differences in welfare across policies are negligible, as shown in figure 15. Under CRRA preferences, the policy that yields the highest welfare places an infinite value on the inflation coefficient. Under recursive preferences, the following rule yields the highest welfare: $\hat{R}_t = 0.9 \cdot \hat{R}_{t-1} + 1.75 \cdot (1 - 0.9) \cdot \hat{\pi}_t + 1 \cdot (1 - 0.9) \cdot \Delta \hat{y}_t$. However, in contrast to the policy that places an infinite weight on inflation, the welfare benefit is only
This figure shows the response to a negative long-run productivity shock. The parameterization is based on the calibration in table 2 for the low IES ($\psi = 1/10$) and high risk aversion ($\gamma = 10$) case and around the policy that yields the highest welfare in equation 3. The real stochastic discount factor (SDF) in the top-left panel shows the impact of the continuation utility and the higher IES for recursive preferences. The negative news leads to a greater rise in the real SDF. This translates into a greater rise of inflation and much lower markup compared to the CRRA setting, as firms place greater weight on the future. In addition, the low IES for CRRA preferences reduces the effectiveness of monetary policy so that the consumption-leisure bundle and average markup quickly converge to steady state.

0.000367%. This is 200 times smaller than a similar comparison with a setting comprising only long-run shocks, where the welfare benefit of the best policy is 0.07% compared to the one that places an infinite weight on inflation.

The differences in welfare across policies are small due to the largely symmetric responses of the average markup and inflation to short-run shocks. Moreover, these variables revert to the steady state very quickly compared to those in the setting with long-run shocks. In summary, the lack of persistence and asymmetry in response to short-run shocks combined with the near-zero equity premium makes all three welfare channels insignificant. Note that the small differences in welfare across policies in this setting are consistent with the findings of Schmitt-Grohé and Uribe (2007), who also reach this conclusion in a setting without long-run risk.
Volatility of Consumption−Leisure Bundle

Average Markup ($\mu$)
(Stochastic Steady State)

Price Dispersion
(Stochastic Steady State)

EZ: High IES, High Risk Aversion
CRRA: Low IES, High Risk Aversion

With short run shocks: flat, negligible effect

5 Conclusion

Asset pricing is important for monetary policy analysis because it reveals how much agents dislike recessions and the extent to which they are forward looking and patient. I have shown in this study that the combination of these characteristics, along with the presence of long-run risk, leads to policy recommendations that are very different from those of prior studies. Specifically, in my asset pricing-oriented New Keynesian model, the optimal Ramsey policy yields an inflation rate greater than 3.5% and inflation volatility close to 1.5%. These values are orders of magnitude higher than similar studies that do not match the equity premium.

The implied rule that provides similar dynamics to the Ramsey optimal policy puts sizable weights on capital growth and the price of capital. This acts to stabilize output and consumption to a greater extent in the medium to long term. In addition to the greater stabilization, the Ramsey optimal policy also pushes for higher inflation in the aftermath of negative shocks, which effectively reduces the markup. In terms of simple rules, much greater weight is placed on output
growth, and a much smaller weight is placed on inflation.

I find that the welfare gain of moving away from a policy that completely stabilizes inflation is 200 times greater for settings with long-run shocks relative to those with short-run shocks. This translates into the equivalent of a one-time benefit of over $10,000 for every individual for the optimal Ramsey policy. In terms of the current economic environment and in contrast to many of the previous studies, my findings suggest the potential welfare costs associated with raising rates too soon are greater than those associated with overshooting inflation. Future research will be targeted towards implementing the findings of this paper in a richer and more empirically driven model similar to Justiniano et al. (2011).
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For Online Publication: Appendix

A Model

The economy consists of a continuum of identical households, a continuum of intermediate-goods firms, and a government that conducts monetary and fiscal policy. The structure of the model is the standard neoclassical growth model augmented with real and nominal frictions. The nominal friction is sticky prices. The real friction is monopolistic competition, which results in a markup of price over marginal costs. Monetary policy assumes full commitment to an interest rate rule that is a function of inflation and output growth. Fiscal policy raises lump-sum taxes to pay for exogenous expenditures.

Preferences. The households have Epstein-Zin preferences defined over consumption goods, \( c_t \), and leisure, \( 1 - h_t \). These preferences exhibit a CES aggregate of current and future utility certainty equivalent weighted by \((1-\beta)\) and \(\beta\), respectively.

\[
v_t = \max_{\{c_j, h_j, i_j, b_j, k_j + 1\}} \left\{ (1 - \beta)(c_t (1 - h_t)^{1-\gamma})^{\frac{1-\psi}{1-\gamma}} + \beta(E_t[v_{t+1}^{1-\gamma}])^{\frac{1-\psi}{1-\gamma}} \right\}
\]

s.t.

\[
b_t + c_t + i_t + \tau_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + w_t h_t + u_t k_t + \tilde{\phi}_t
\]

The real value of debt is \( b_t \); \( c_t \) is consumption; \( i_t \) is investment; \( R_{t-1} \) is the risk-free rate; \( \pi_t \) is the inflation rate \( \frac{P_t}{P_{t-1}} \); \( \tau_t \) is the lump-sum tax; \( w_t \) is the real wage; \( h_t \) is labor hours; \( u_t \) is the rental rate of capital; \( k_t \) is capital; and \( \tilde{\phi}_t \) is profits.

Unlike standard preferences, Epstein-Zin preferences allow for the disentanglement of \( \gamma \), the coefficient of relative risk aversion, and \( \psi \), the elasticity of intertemporal substitution. When \( \frac{1}{\psi} = \gamma \), the utility collapses to standard preferences with additively separable expected utility both in time and state. When \( \gamma > \frac{1}{\psi} \), the agent prefers early resolution of uncertainty, so the
agent dislikes shocks to long-run expected growth rates.

**Intermediate good bundling.** The consumption good is assumed to be a composite made of a continuum of differentiated goods $c_{it}$ indexed by $i \in [0, 1]$ via the aggregator:

$$c_t = \left[ \int_0^1 c_{it}^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}}$$

The elasticity of substitution across different varieties of consumption goods is $\eta > 1$ (also the price elasticity of demand for good $j$). As $\eta \to \infty$, the goods become closer and closer substitutes, so that individual firms have less market power.

The household minimizes total expenditures subject to an aggregation constraint, where $P_{jt}$ is price of intermediate good $j$:

$$\min_{c_{jt}} \int_0^1 P_{jt} c_{jt} dj$$

s.t.

$$\left[ \int_0^1 c_{it}^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}} \geq c_t$$

The optimal demand for the level of intermediate consumption good $c_{jt}$ is given by

$$c_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\eta} c_t$$

where $P_t$ is the nominal price index

$$P_t \equiv \left[ \int_0^1 P_{jt}^{1-\eta} dj \right]^{\frac{1}{\eta-1}}$$
**Productivity.** The law of motion of the productivity process captures both short-run and long-run productivity risks:

\[
\log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} = \mu + x_t + \sigma_a \varepsilon_{a,t+1}, \tag{4}
\]

\[
x_{t+1} = \rho x_t + \sigma_x \varepsilon_{x,t+1}, \tag{5}
\]

\[
\begin{bmatrix}
\varepsilon_{a,t+1} \\
\varepsilon_{x,t+1}
\end{bmatrix}
\sim i.i.d. N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \quad t = 0, 1, 2, \ldots . \tag{6}
\]

According to the above specification, short-run productivity shocks, \( \varepsilon_{a,t+1} \), affect contemporaneous output directly but have no effect on future productivity growth. Shocks to long-run productivity, represented by \( \varepsilon_{x,t+1} \), carry news about future productivity growth rates but do not affect current output.

**Capital Accumulation Technology.** Assume that investments in different vintages of capital have heterogeneous exposure to aggregate productivity shocks.\(^{15}\) In other words, there will be vintage-specific productivity growth that is going to depend on the age \( j = 0, 1, \ldots, t - 1 \) of the vintage of capital

\[
\frac{A_{t+1}^{t-j}}{A_{t}^{t-j}} = e^{\mu + \phi_j (\Delta a_{t+1} - \mu)}
\]

Under the above specification, production units of all generations have the same unconditional expected growth rate. Also, \( A_t^{t-0} = A_t \) is set to ensure that new production units are on average as productive as older ones. The log growth rate of the productivity process for the initial generation of production units, \( \Delta a_{t+1} \), is given by equation (4). Heterogeneity is driven solely by differences in aggregate productivity risk exposure, \( \phi_j \).

\(^{15}\)Multiple frictions for capital accumulation have been tested, and this friction was chosen because standard capital adjustment costs result in counterfactually low investment growth volatility. The friction in this paper does not suffer from this issue.
The empirical findings in Ai et al. (2012) suggests that older production units are more exposed to aggregate productivity shocks than younger ones, i.e., the exposure $\phi_j$ is increasing in $j$. To capture this fact, a parsimonious specification for $\phi_j$ is adopted:

$$\phi_j = \begin{cases} 0 & j = 0 \\ 1 & j = 1, \ldots \end{cases}$$

New production units have zero exposure to aggregate productivity shocks in the first period of life. Every period thereafter, they have 100% exposure to aggregate productivity shocks as do all other existing vintages.

Let $K_t$ denote the productivity-adjusted physical capital stock. Despite the heterogeneity in productivity, aggregate production can be represented as a function of $K_t$ and $N_t$. The law of motion of the productivity-adjusted physical capital stock $K_t$, takes the following form:

$$K_1 = I_0, \quad K_{t+1} = (1 - \delta)K_t + \omega_{t+1}I_t$$

$$\omega_{t+1} = \left( \frac{A_{t+1}^{t-0}}{A_t^{t+1}} \right)^{\frac{1-\alpha}{\alpha}} e^{-\frac{1-\alpha}{\alpha} (x_t + \sigma_a \epsilon_{a,t+1})(1 - \phi_0)}$$

where $I_t$ is the total mass of new vintage capital produced at time $t$, and $\omega_{t+1}$ is an endogenous process that accounts for the productivity gap between the newest vintage of capital and all older vintages. Note that when $\phi_0 = 1$, the new capital vintage has the same exposure to aggregate productivity shocks as older ones. In this case, $\omega_{t+1} = 1$ for all $t$ and capital of all generations are identical.
The Government. The government issues one-period nominal risk-free bonds, $b_t$, collects taxes in the amount of $\tau_t$, and faces an exogenous expenditure stream, $g_t$. Its period by period budget constraint is given by

$$b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} + g_t - \tau_t$$

The exogenous expenditure streams are formulated as in Croce et al. (2012)

$$G = \frac{1}{1 + e^{-g_y}}$$

$$g_y t = (1 - \rho_y) \bar{g} y + \rho_y g_y t-1 + \epsilon_{G,t}, \quad \epsilon_{G,t} \sim N(0, \sigma_{g_y}^2)$$

Total tax revenues, $\tau_t$, consist of lump-sum tax revenues. Tax smoothing by the government consists of tax revenues rising whenever the previous period’s debt rises

$$\tau_t - \tau^* = \gamma_1 (b_{t-1} - b^*)$$

where $\gamma_1 > 0$ ensures that the debt-GDP ratio is bounded and there is a unique solution.

The monetary authority sets short-term nominal interest rate according to a simple Taylor rule

$$\ln(R_t/R^*) = \alpha_r \ln(R_{t-1}/R^*) + \alpha_{\pi} ln(\pi_t/\pi^*) + \alpha_{\Delta y} ln(\Delta y_t/\Delta y^*)$$

where $\alpha_r$ is the inertia coefficient, $\alpha_{\pi}$ is the inflation coefficient, $\alpha_{\Delta y}$ is the output growth gap coefficient, $\pi^*$ is the inflation target, and $\Delta y^*$ is the output growth target.

Firms. Each variety $i \in [0,1]$ is produced by a single firm in a monopolistically competitive environment. Each firm $i$ produces output using as factor inputs capital services, $k_{it}$, and labor services, $h_{it}$. It is assumed that the firm must satisfy demand at the posted price. Formally,
\[ k_{it}^\theta (A_{it} h_{it})^{1-\theta} \geq \left( \frac{P_{it}}{P_t} \right)^{-\eta} y_t \]

The objective of the firm is to choose \( P_{it}, h_{it}, k_{it} \) to maximize the present discounted value of profits, given by

\[ E_t \sum_{s=t}^{\infty} m_{t,s} P_s \phi_{is} \]

where real profits of firm \( i \) are

\[ \phi_{it} \equiv \frac{P_{it}}{P_t} y_{it} - u_{it} k_{it} - w_{it} h_{it} \]

Prices are assumed to be sticky as in Calvo (1983). Each period, a fraction \( \alpha \in [0,1) \) of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. Instead, these firms index their prices to past inflation according to the equation

\[ P_{it} = P_{it-1} \pi_{t-1}^\chi. \]

Note, that in all settings \( \chi = 0 \), which implies there is no price indexation. The remaining \( 1 - \alpha \) firms choose \( \tilde{P}_t \) to maximize the expected present discounted value of profits:

\[ E_t \sum_{s=0}^{\infty} d_{t,s} P_{t+s} \phi_{is} \]

The firm’s first-order conditions for labor, capital, and optimal price are

\[ mc_t (1 - \theta) \left( \frac{k_t}{h_t} \right)^\theta = \tilde{w}_t \]

\[ mc_t \theta \left( \frac{k_t}{h_t} \right)^{\theta - 1} = u_t \]
The firm’s optimal price is set such that marginal revenues are equal to some markup over marginal costs

$$\eta \frac{x_1^1}{x_1^2} = x_1^2$$

(10)

where

$$\tilde{x}_1^1 = p_t^{*} - \eta \tilde{y}_t m c_t + \alpha E_t D_t, t+1 \pi_{t+1}^{\eta+1} \pi_t^{1-\eta} \left( \frac{p_t^*}{p_{t+1}} \right)^{1-\eta} \tilde{x}_1^1 e^{\Delta \alpha t+1}$$

(11)

$$\tilde{x}_1^2 = p_t^{*} \tilde{y}_t + \alpha E_t D_t, t+1 \pi_{t+1}^{\eta} \pi_t^{1-\eta} \left( \frac{p_t^*}{p_{t+1}} \right)^{1-\eta} \tilde{x}_2^2 e^{\Delta \alpha t+1}$$

(12)

**Aggregation and Equilibrium.** This period’s price level is a weighted average of the firm’s optimal price and the previous period’s price level:

$$1 = \alpha \pi_t^{-1+\eta} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha) p_t^*(1-\eta)$$

(13)

It can be shown that the resource costs of inefficient price dispersion are characterized as follows

$$s_t = (1-\alpha)p_t^{*-\eta} + \alpha \left( \frac{\pi_t}{\pi_{t-1}^{\chi}} \right)^{\eta} s_{t-1}$$

(14)

Given the price dispersion, output is described by

$$\tilde{y}_t = \frac{1}{s_t} [\tilde{k}^\theta (A_t h_t)^{1-\theta}]$$

(15)

and aggregate demand is the following sum

$$\tilde{y}_t = \tilde{c}_t + \tilde{i}_t + \tilde{g}_t$$

(16)
1.1 Empirical Motivation for Welfare Channels

The existing literature on optimal monetary policy finds that movement toward a hawk policy has no effect on the steady-state markup. However, I show below that the theoretical prediction of a nil effect is not supported by the data. In other words, CRRA preferences are not only inconsistent with asset pricing facts but also with the empirical evidence on the average markup when monetary policy increasingly stabilizes inflation. In addition, CRRA models do not predict a meaningful trade-off between reducing nominal uncertainty at the expense of higher real uncertainty. This lack of a trade-off is also inconsistent with empirical evidence discussed in more detail below.

Compared to empirical estimates of the Taylor rule, the optimal coefficients on inflation in the interest rate rule as determined by the existing literature are often orders of magnitude higher. This is because the CRRA models are not fully taking into account the explicit costs of the higher markup and greater consumption volatility implied by the data, so the benefits of reducing inflation volatility dominate all other channels. In contrast, my novel asset pricing-oriented New Keynesian model is consistent with the empirical evidence on the average markup and the real-nominal trade-off. As this model endogenously captures these channels as reflected in the data, the optimal policy coefficients are much closer to empirical estimates for the interest rate rule.

In the sections that follow, I provide a discussion along with empirical evidence on the markup and the trade-off involved in reducing nominal volatility at the expense of increased consumption volatility. I then estimate interest rate rules over the pre-Volcker and post-Volcker periods to connect the observed behavior of the Federal Reserve with the markup channel and real-nominal trade-off.

The Markup. The empirical evidence of the association of a higher markup with greater inflation stabilization was first found by Benabou (1992). Using US data on the retail trade sector, he finds that inflation has significantly negative effects on the markup. Other studies have focused on the inverse of the labor share, as this can be shown to be theoretically proportional to the
markup if the production function is Cobb-Douglas. Nekarda and Ramey (2013) show that an upward trend of the markup began in the early 1980s. The rise in the markup coincides with the more aggressive role of monetary policy in stabilizing inflation. This can be seen in figure A1, which plots data on the average markup according to the Bureau of Labor Statistics records dating back to 1950. During the post-Volcker period (1980–2007), the mean of the markup increased as inflation was stabilized to a greater extent. Other studies such as Alcala and Sancho (2000) and Raurich et al. (2012) have shown using various measures that the markup has risen since 1980.¹⁶

The rise in the markup due to greater inflation stabilization is consistent with the theoretical dynamics of my asset pricing-oriented New Keynesian model. Specifically, figure A1 shows that the mean of the markup rises between 2.5–3% when moving from pre-Volcker to post-Volcker. Similarly, my general equilibrium model predicts that the average markup will also rise 3%, from 12% to 15%, as monetary policy increasingly stabilizes inflation.¹⁷ Therefore, the theoretical effect of monetary policy on the markup lines up with the empirical evidence. Moreover, the volatility of the markup is 3% under the optimal rule within my model, while in the data, the markup volatility is historically between 5% and 6%. This suggests that my model is conservative with respect to the observed volatility of the markup and that the relative importance of the markup channel for welfare is not driven by exaggerated or implausible dynamics.

Real-Nominal Trade-Off. The real-nominal trade-off suggests that as monetary policy increasingly stabilizes inflation, nominal uncertainty falls while real uncertainty increases. This trade-off arises as an endogenous outcome of my general equilibrium model. The intuition is that as the central bank increasingly targets inflation, this induces greater changes in both nominal rates and

¹⁶The mechanism for the relationship between inflation and labor share is simple. As pointed out by Alcala and Sancho (2000), accelerated inflation is correlated with higher employment, higher employment leads to greater bargaining power, and greater bargaining power is correlated with lower markups. In my New Keynesian model inflation is largely due to output being pushed above the natural level, which reduces the real average markup because labor costs rise but most firms are unable to update their prices.

¹⁷The rise in the markup within the asset pricing-oriented New Keynesian model is further demonstrated in section 4, figure 10.
Fig. A1: Historical Average Markup: Higher Markup Since 1980

The data depicted in this figure are based on the inverse of the labor share of income, as computed by the Bureau of Labor Statistics. The inverse of the labor share of income can be shown to be theoretically equal to the average markup when the production function is Cobb-Douglas. Splitting the time series into 1950–1980 and 1980–2007, the solid red line reflects the increase in the mean of the markup for the post-Volcker period (1980-2007), a period in which inflation was stabilized to a much greater extent. This figure provides empirical evidence for the theoretical result that increasingly stabilizing inflation is associated with a higher markup, which is relevant for my welfare analysis.

real rates (due to sticky prices), which causes higher real consumption volatility. This channel is also present in the endogenous growth model of Kung (2014). Bansal and Shaliastovich (2013) provide empirical evidence that real uncertainty is higher relative to nominal uncertainty for the post-Volcker period.

**Empirical Interest Rate Rules.** A number of studies suggest that simple interest rate rules can characterize the behavior of the Federal Reserve over various time periods. Taylor (1993) proposes a simple rule of the Federal Funds Rate as a function of inflation and the output gap around a trend. Weights of 1.5 and 0.5 are assumed on inflation and output, respectively, and seem to roughly capture the behavior of monetary policy. Other papers have since evaluated the
empirical fit of simple interest rate rules, including Judd and Rudebusch (1998), Taylor (1999), Clarida et al. (1998) and Orphanides (2003). In contrast to many of these studies, I choose to focus on output growth rather than the output gap because there is greater consensus in terms of its measurement.\footnote{Other studies have estimated interest rate rules that include output growth, including Ireland (2004), Carlstrom and Fuerst (2012), and Coibion and Gorodnichenko (2011).}

I conduct my own estimation of the interest rate reaction function using data from 1983Q1 to 2002Q4. The starting and ending dates are chosen to match Coibion and Gorodnichenko (2011). I run a GMM estimation in which the dependent variable is the Federal Funds Rate, and the independent variables are one lag of the Federal Funds Rate, the expected inflation rate one year ahead, and the real output growth one quarter ahead. Using expectations for inflation and real output growth with their respective horizons is consistent with Clarida et al. (1998) and the choice of horizon has little effect on the ensuing analysis. The instruments are three lags each of real GDP growth, inflation, the federal funds rate, and oil prices.\footnote{Tests based on the C-Stat and J-Stat fail to reject the null that these are valid instruments.}

\[
R_t = \alpha_r \cdot R_{t-1} + (1 - \alpha_r) \left( \alpha_\pi \cdot (E_t[\pi_{t+k}] - \pi^*) + \alpha_\Delta y \cdot (E_t[\Delta y_{t+k}] - \Delta y^*) \right) + \epsilon_t
\]

where \(\alpha_r\) captures partial adjustment or inertia, \(\alpha_\pi\) is the weight on inflation, \(\alpha_\Delta y\) is the weight on output growth, and the starred variables indicate means that act as proxies for the targets.

The results of my GMM estimation for post-1983 are in line with Coibion and Gorodnichenko (2011), as shown in table 6. Coibion and Gorodnichenko (2011) solely focus on characterizing Federal Reserve policy, which leads them to use a more richly parameterized empirical model. Here, I characterize this behavior in a more parsimonious and straightforward manner that follows Clarida et al. (1998). The small difference in results can be attributed to these modeling differences, along with Coibion and Gorodnichenko (2011)’s use of expected inflation and expected output growth based on the Greenbook forecasts from the Federal Open Market Committee (FOMC).
Table 6: GMM Estimated Interest Rate Reaction Function

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_r$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_{\Delta y}$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GMM Estimation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1955:1–1982:4</td>
<td>0.82</td>
<td>0.97</td>
<td>-0.51</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.19)</td>
<td>(0.20)</td>
<td></td>
</tr>
<tr>
<td>1983:1–2002:4</td>
<td>0.89</td>
<td>3.02</td>
<td>1.01</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.90)</td>
<td>(0.60)</td>
<td></td>
</tr>
<tr>
<td><strong>Coibion &amp; Gorodnichenko (2012)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983:1–2002:4</td>
<td>0.93</td>
<td>2.20</td>
<td>1.56</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.40)</td>
<td>(0.39)</td>
<td></td>
</tr>
</tbody>
</table>

This table shows results of GMM estimations of the Federal Funds Rate on the lagged rate ($\alpha_r$), inflation based on the CPI ($\alpha_\pi$), and the growth of real GDP ($\alpha_{\Delta y}$). Results from Coibion and Gorodnichenko (2011) over a similar time period are shown for sake of comparison. Their method uses real-time measures of expected inflation and expected output growth based on Greenbook forecasts from the Federal Open Market Committee (FOMC). Standard errors and covariance are computed with a HAC weighting matrix with a Bartlett kernel and Newey-West fixed bandwidth of 4. The instruments are three lags each of real GDP growth, inflation, the federal funds rate, and oil prices. Standard errors are listed in parentheses and the p-value for J Statistics are listed in the last column.

In both instances, the results indicate that the Federal Reserve did not target inflation as much for the pre-Volcker period. With less weight placed on inflation, higher inflation volatility was associated with lower average markups, which is an important channel for my welfare analysis. Most importantly, the values on the inflation coefficient for the post-Volcker regressions are much lower than the optimal monetary policy literature frequently suggests. For example, the model of Schmitt-Grohé and Uribe (2007) in a similar setting with no long-run risk and CRRA preferences yields an optimal inflation coefficient of 332 with no weight on output. Likewise, Kollmann (2008) finds an optimal inflation coefficient of 8,660.

The optimal policy in my asset pricing-oriented New Keynesian model places a weight on inflation ($\alpha_{\pi}^{OPT} = 3.05$) that is within the standard error of the point estimate based on my post-Volcker GMM estimation. Also, the optimal weight placed on the output growth is higher ($\alpha_{\Delta y}^{OPT} = 1.55$) than that estimated in both of my regressions. Thus, this model, with its high welfare costs of recessions, suggests that it would have been optimal for the Federal Reserve to respond more to output growth over the estimated time period.

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Overall, CRRA preferences are inconsistent not only with asset pricing facts but also with the empirical evidence described in this section. Once Epstein-Zin preferences are introduced, the empirical evidence on the markup and consumption volatility is reproduced in a general equilibrium setting and the implications for monetary policy change dramatically.

B Further Inspection of Dynamics

In this section I directly compare the low IES ($\psi = 0.2$) and high IES ($\psi = 2$) settings to further emphasize the importance of making firms and agents more patient. Following that, I show the reasons the average markup rises as monetary policy increasingly stabilizes inflation. Further sensitivity analysis with respect to changes in the risk aversion and intertemporal elasticity of substitution can be found in the online appendix.\(^{20}\)

**High vs. Low IES.** The high IES model better captures the low risk-free rate in the data and makes forward-looking agents and firms more patient, as shown in the left panel of figure A2. With greater precautionary savings, the consumption-leisure bundle growth declines more, and this results in the doubling of the stochastic discount factor when moving from the low to high IES model. In this context, forward-looking firms place greater weight on future marginal costs, so that the decrease in inflation in the initial period is five times less in the high IES model than the low IES model. The higher inflation pushes down the relative prices set by firms in previous periods and decreases the real value of the average markup. All of the above results suggest that the high IES model has a greater negative effect on the average markup.

\(^{20}\)In the online appendix, I also show why the markup channel is largely not present in other studies—namely, due to the absence of both a high IES and high risk aversion. When the IES is relatively low (which is typically the case for CRRA preferences), moving from a first-order to second-order approximation barely changes the dynamics for simple rules.
High vs. Low IES ($\psi = 2$ vs. $\psi = 0.2$)

Fig. A2: Negative Long-Run Shock

This figure shows the effects of a negative long-run productivity shock on the three key channels for welfare: inflation volatility, markup, and consumption-leisure volatility. The parameterization is based on the calibration in table 2 and focuses on the policy that yields the highest welfare in equation 3. The left panel shows that moving from a low IES to a high IES leads to significantly different implications for each channel. The right panel shows that moving from the low inflation coefficient of 2.25 to the high inflation coefficient of 5, there is a decrease in inflation volatility which means the markup does not fall by as much in recessions. This suggests the average markup will be higher as monetary policy increasingly stabilizes inflation.
High IES: Low vs. High $\alpha_{\pi}$ (Dove vs. Hawk). As shown in the right panel of figure A2, a higher inflation coefficient combined with a high coefficient on the lagged interest rate imply a higher real interest rate for many periods in response to inflation. Forward-looking firms take this into account when setting prices, knowing that monetary policy is actively attempting to stabilize inflation. Therefore, firms choose lower prices due to the lower expected inflation, and this causes the initial inflation to be lower.

However, the decrease of inflation volatility lowers the inefficient allocation coming from price dispersion. Therefore, a trade-off exists between increasing the average markup and decreasing price dispersion as monetary policy increasingly stabilizes inflation. Note that this markup channel is nonexistent for first-order approximations and is not as large for the low IES model. Since the decrease in inflation volatility means the markup does not decrease by as much in recessions, this implies the average markup will be higher as monetary policy increasingly stabilizes inflation.