ACTIVISM, MOBILIZATION AND POLARIZATION

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Abstract. I develop a theory of activism and mobilization in the context of electoral competition. Two candidates simultaneously announce policy platforms and seek the support of ideologically inclined party activists. Activists compete to influence electoral outcomes by mobilizing support for their respective candidate, and trade-off expected ideological loss and marginal costs of mobilization. Under this setup, I establish that the relationship between ideological polarization of activists and political polarization depends critically on the mobilization aversion parameter (elasticity of marginal costs of mobilization). Specifically, the main finding of my model is the following counter-intuitive result: when mobilization aversion falls within a certain interval, as activists become more ideologically polarized, the equilibrium political polarization of candidates decreases, rather than increasing. Specifically, a more polarized electorate results in more convergent platforms. Mobilization aversion is interpreted to be intricately connected to the supply and demand for activism. In interpreting the results, I provide a rationale for publicly funded campaigns and limits to independent expenditure as useful institutional reforms to reduce political polarization. Furthermore, I provide an endogenous characterization of the mobilization function for activism. In particular, mobilization under a symmetric equilibrium is affected positively by ideological polarization of activists and mobilization aversion.

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1. Introduction

"People at the top might devote time and resources to supporting a political party strongly opposed to redistribution. People at the bottom would have an opposite response." - McCarty, Poole and Rosenthal, Polarized America

An important component of a representative democracy is electoral competition between political parties and effective participation by the electorate. Parties compete by providing a choice of platforms with an aim to capture power, and implement the policy once in office. The success of a party in this process of political competition therefore relies on its ability to influence and mobilize support among the citizens. To this effect, political parties rely heavily on party activists\(^1\) to persuade voters in favor of their chosen platforms. In some sense, party activism could be viewed as an active, but more costly form of engagement and participation in representative democracies\(^2\). Party activists, on the other hand, are themselves driven by an ideological inclination to a set of policies, besides a sense of identification with the political party\(^3\).

Therefore, the benefits of such costly participation to activists is ideological in that the activists seek to influence electoral outcomes in favour of their identified party. Since political parties, and candidates, rely on party activists during an electoral campaign, it is in their interest to propagate policy platforms keeping in mind the prospect of subsequent support from the activists. This, invariably, creates a cycle of dependence - one in which activists trade-off costly campaign effort for a more relevant ideology, and candidates trade-off ideology for greater political support from activists\(^4\).

Aldrich [1] [2], in his seminal work, develops a proximity model of activism, and provides a framework

\(^1\)Norris [33] describes the role of an activist as the following: "..... donating money, signing petitions, passing motions... helping with door-to-door canvassing and leafleting... attending the national party convention, and assisting with community fund-raising events - in short, making tea and licking envelops". In essence, any form of contribution-time or money - towards a political party or its candidate is loosely defined in my paper as activism.

\(^2\)Activists contribute time and effort in party work and during elections, aim to persuade voters through a long-drawn political campaign. Whiteley and Seyd [38] refer to such activism as high-intensity political participation.

\(^3\)The critical role played by party activists have been well documented in the field of political science. For more on the role of political parties and party activists, read Aldrich [3], Hershey [37], Norris [34].

\(^4\)Obama's 2008 and -to a lesser extent- 2012 campaigns were propelled by grass roots mobilization, heretofore unseen in US presidential elections. A more recent example is the New Delhi state elections of 2015, in which the AAP party won 95% of the seats in the legislative assembly by promising a platform of ending governmental corruption and nepotism. The campaign witnessed mass mobilization by grass roots volunteers and activists who were able to influence voters to believe in a fledgling party and its promised "change".
for studying rational choice motivations for becoming an activist. How activism influences political platforms, and what affects extent of activism in electoral competition are important questions that remain pertinent and unanswered, especially so in the current context of the polarization debate surrounding US politics. In this paper, I attempt to reconcile these questions in an intuitive way.

I develop a theory of activism and mobilization to analyze the precise nature of candidate platforms, and crucially, study the effect of partisan activism on polarization, in a simple model of two party political competition. Some important features distinguish my analysis from previous work. Firstly, political candidates have mixed motivations and depend upon activists to mobilize support for their stated policy platform. The candidates themselves do not incur any costs on campaigning. Secondly, activists act as price-takers, implying that they do not set the political agenda of the candidate, but merely decide on the level of mobilization contingent on the platforms announced by the two candidates. Finally, activists face a cost of mobilization, and weigh the costs and benefits of activism in a strategic sense.

The model proceeds as follows: candidates simultaneously announce platforms, activists mobilize support for either candidate, and (median) voter decides whom to vote for; in that order. Candidates, when announcing platforms, and activists, when deciding on levels of mobilization, are unaware of the voter’s preferred policy, which is determined from an uniform distribution. Activist mobilization plays a ‘persuasive role’, in the sense that mobilization directly affects median voter’s utility, by shifting their preferences towards a candidate. There are two important trade-off’s induced by the setup of my model. Candidates trade-off their ideology to elicit greater mobilization from activists. Activists trade-off benefits of mobilization (improving winnability of candidate) and the costs of mobilization. Together, these twin trade-off’s are sufficient to study how political polarization reacts to various parameters of the model.

As McCarty, Poole and Rosenthal [32] (henceforth MPR) have argued recently, political polarization in the American political system has increased in recent times. MPR attribute polarization and the political gridlock to social and economic inequality, which they argue has divided society and created a deeply polarized electorate.

Median voter convergence with office-motivated candidates, as propagated by Downs [21] and Black [17], has been a benchmark result in the analysis of spatial models of political competition. Several subsequent works have established platform divergence under varying setup’s. Most prominent among them include Calvert [18], Bernhardt et al. [12], Besley and Coate [15] Osborne and Slivinski [35], and Wittman [40].

We provide an extension in which activism plays a directly informative role. See Austen-Smith ([6]) for more.
The main result of my paper is the finding that when activists are more polarized, and the mobilization aversion parameter is low enough, equilibrium platforms converge\textsuperscript{8}, reducing polarization of platforms. In fact, my finding implies that when mobilization aversion is below a threshold, greater activist polarization results in moderation of candidate platforms. Increasing activist polarization causes divergence in candidate platforms only when mobilization aversion parameter exceeds a threshold. This counter-intuitive result is the main contribution of our paper, since it implies that activist polarization is necessary for increasing divergence of political platforms, but is not sufficient. Another important finding is that when activist preferences are homothetic, equilibrium polarization is independent of their ideological preference. Polarization is purely driven by candidate preferences and uncertainty about median voter’s preference, even though activists’ role in the process is unchanged, in the sense that mobilization in equilibrium is non-zero. Combined with the earlier finding, this illustrates the criticality of the interaction between ideology and participation aversion of activists.

The second significant finding of our analysis is the behaviour of the mobilization function of activists. To state precisely, mobilization in my model is generated endogenously and is a function of two parameters - the ideological polarization and mobilization aversion of activists. I find that mobilization is greater in elections where activists are either more polarized or mobilization aversion parameter is greater. The characterization of the mobilization function is of significance since it helps explain some of the differences in mobilization across elections\textsuperscript{9}. The model is an earnest attempt to shed light on some of the factors that could potentially affect activist mobilization in electoral competition. The analysis shows that what is crucial is not only the ideological inclinations

\textsuperscript{8}Convergence in our paper does not imply Downsian convergence. We refer to platforms that move closer, but not necessarily to ex-ante median, as convergence.

\textsuperscript{9}Consider the following statement published in the Time magazine:

"The outfit that put upwards of 8 million volunteers on the street in 2008 — known as Organizing for America — is a ghost of its former self. Its staff has shrunk from 6,000 to 300, and its donors are depressed: receipts are a fraction of what they were in 2008. Virtually no one in politics believes it will turn many contests this fall." - September, 2010
of polarized groups, but the trade-off between their ideological polarization and participation costs of mobilization\textsuperscript{10} during campaigns.

To my knowledge, this is the first attempt to build a simple model of activism, and in doing so, bridge the gap between theoretical work and empirical findings on activism and mobilization. Using a simple set-up, the model pins down the relation between a polarized electorate (that manifests through activism) and political polarization. Specifically, the relationship depends critically on mobilization (or participation) aversion of activists. My analysis suggests that political polarization could be explained better by understanding and interpreting the aversion coefficient. I offer two ways to look at mobilization aversion. From the perspective of the supply side of activism, a higher aversion to mobilize could be attributed to general apathy of voters in democracies to actively engage in the political process. For example, constraints on time and resources may have pushed ordinary Americans from active engagement\textsuperscript{11}, putting the onus of activism on small elite groups (labor unions, tea-party, or the top 1%). On the demand side, candidates - more than ever - rely heavily on television and other digital campaigns, and less so on grass-roots campaigns. This creates a greater dependence on money and diminishes the value of door-to-door local campaigns (to both the voter and the politician). The importance of money may in turn be interpreted as an increase in the elasticity of marginal costs of mobilization, which reflects the mobilization aversion\textsuperscript{12}.

My analysis suggests a possible institutional reform that could potentially address the issue of mobilization aversion and inclusion of moderate supporters to campaigns. Specifically, I argue for publicly funded elections and curbing independent expenditures as possible solutions to bring down

\textsuperscript{10}Traditional theories have mostly concentrated on voter turnout as the main mode, and goal, of activism and political participation. Gerber and Green [24] argue that political mobilization (as measured by canvassing, phone calls etc.) increases voter turnout in elections. McClurg and Holbrook [30] investigate the role of campaign mobilization on driving core partisan groups to participate in voting. Recently, Madestam et al. [31] find evidence for tea party protests affecting the Republicans’ vote share during the mid-term elections of 2010.

\textsuperscript{11}Norris [33] [34], Dalton and Wattenberg [20] have studied the phenomenon of decreasing political engagement and weakening party structures in a more detailed way. They argue that weaker parties explain much of the reduced voter turnout in advanced economies. A consequence of this could be the capture of parties by a small, but effectively active, group with rather extreme agendas.

\textsuperscript{12}Polarization could be viewed as demanding a bigger bang for the buck by those who finance the campaigns of candidates, be it unions or billionaires.
political polarization. The reforms, as my analysis predicts, reduce the aversion on the supply-side and simultaneously increase dependence on more moderate and traditional supporters, both of which may in turn lead to decreased political polarization.

In Section 5, two extensions to the baseline model are presented. The first considers a noisy campaign with activism in the sense of Austen-Smith [6]. Activism, instead of affecting voter utility by shifting preferences, plays an informative role. Voter only observes an imperfect (noisy) signal of the actual platform, and greater levels of activism reduce the variance of this noise, rendering platforms more informative. In this setting, the unique symmetric equilibrium is established. I find that the relationship between equilibrium polarization and activist polarization depends critically on two parameters - namely, elasticity of mobilization and efficiency of activism. When the elasticity of mobilization is low, and activism is highly ineffective (or inefficient in reducing marginal variance of noise), activist polarization results in greater moderation of candidate platforms, analogous to the baseline result. Higher efficiency of activism combined with increased activist polarization results in greater political polarization.

In the second extension, we change the assumption of an unitary representative activist to include multiple activist groups. We identify a free-rider problem in activism. When number of activists increase, with activists ideologically ordered on the policy space, mobilization by each of the activist group is increasing in their ideology. This means more extreme groups contribute a greater share of the total mobilization. However, we find that since the marginal importance of extra mobilization for a candidate decreases with more activists, equilibrium polarization decreases. For equilibrium polarization to increase, we predict one of the following two should happen: either the within-party ideological cohesiveness of activists increases, or the number of activist groups within a party decreases.

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13 We define this as the elasticity of marginal variance of noise.
14 We refer to this scenario as collective action in electoral activism. See the social psychology literature for more on collective action (Friedman and McAdams [22], Bimber et al. [16], and Garrison [23]).
15 Think of this as being the ideological distance (or "spread") between the most extreme, and least extreme activist. Therefore activists are more cohesive if this spread is smaller.
Conversely, whenever the number of activist groups increases, ideological polarization of candidates in equilibrium decreases. This, we posit, is akin to a failure of collective-action in political activism. Collective-action fails to achieve divergence in platforms because of the free-riding nature of mobilization. Our finding provides a rationale for why activist groups should consolidate their efforts, rather than campaigning as disparate groups with diverging interests\textsuperscript{16}. This, in political terms, implies an identification with a broad ideological cause, instead of behaving as factions within the same party\textsuperscript{17}.

The rest of the paper is organised as follows. Section 2 briefly discusses related literature, relating it to the microfoundations of my model. Section 3 presents the benchmark model, and Section 4 discusses the main results from equilibrium analysis. Section 5 contains two extensions to the model. Concluding remarks are provided in Section 6. All proofs are confined to the appendix.

2. Related Literature

My paper, at its core, is a model of electoral competition which induces platform separation in equilibrium. Several models have explored the idea of platform divergence\textsuperscript{18}. My analysis sheds light on electoral activism as a possible channel for divergence, and more importantly, the impact of changes in activists’ parameters on equilibrium polarization. Earlier work of Aldrich \cite{1} on rational-choice motivations for party activism is a natural starting point for my analysis. Aldrich derives two important results with respect to the existence of "party cleavages": firstly, within a party, the distribution of activists is cohesive in ideology; and secondly, across party lines, ideologies are distinctly polarized in terms of their distributions. I use the modeling assumption of the existence

\textsuperscript{16}Tea party Activists were seen as pioneers of a revitalized US conservatism movement (Williamson et al. \cite{39}). They stood for an unified agenda, namely - fiscal conservatism, less government, lower taxes and regulation, and mobilized support in favor of candidates that were deemed to represent this conservative ideology.

\textsuperscript{17}Take the example of the Aam Aadmi Party (AAP) in the recent New Delhi state elections in 2013. Activists belonging to the party campaigned on the promise of ending bureaucratic and executive corruption, and provide transparent governance - both of which the major political parties were accused of turning a blind eye to. The AAP activists managed to influence voter preferences towards this anti-corruption cause, and enabled them to form a government.

\textsuperscript{18}See Bernhardt et al. \cite{12} \cite{13}, Osborne and Slivinski \cite{35}, Besley and Coate \cite{15}, Gul and Pesendorfer \cite{28}, Ashworth and Mesquita \cite{5}, Aragones and Palfrey \cite{4}
of partisan cleavages\textsuperscript{19} to analyze the extent of mobilization and the nature of political competition. I specifically concentrate my analysis on the relationship between activist polarization as envisaged by Aldrich, and political polarization.

The role of activism in my analysis differs starkly from traditional theories, in which activists contributions affect voter turnout - either as a result of direct (face-to-face) canvassing, or by encouraging participation of extreme partisan voters\textsuperscript{20}. More recently, Madestam et al. [31] investigated whether political protests (or activism) alter voter preferences and impact political outcomes, in the context of Tea-party activism in US during the 2010 midterm elections. They find evidence\textsuperscript{21} for a "persuasive role" of activism, in that it directly influences voters' ideology, and increases political participation as a result. This is precisely the mechanism I study wherein there is a persuasive component of activism, and candidates respond to this by either polarizing or moderating their platforms.

It is important to differentiate activism from interest groups or lobbies. The literature on campaign contributions and influence seeking by interest groups or lobbies is vast and has been extensively studied. Baron ([9],[8]), Grossman and Helpman ([27], [26],[25]), Bernheim and Whinston [14], Austen-Smith [7] investigate various aspects of influence seeking by interest groups. The distinctions between these models and mine is twofold. Firstly, activists are quite different in their objectives, in the sense that their support is ideologically partisan and is not driven by any form of non-partisan or economic considerations. Further, the commitment of activists towards a candidate is temporary and short-term, unlike lobbies or interest groups that have interests once a candidate gets elected. Interest groups and lobbies seek influence of incumbents to get favorable legislations or policies passed, and in this sense, their relationship is not short term. The focus of my work,

\textsuperscript{19}The preferences of candidates in my model are derived from Bernhardt et al. [13], who make a case for responsible parties. See Calvert [18], and Wittman [40] for candidate motivations.

\textsuperscript{20}Gerber and Green [24] argue that political mobilization (as measured by canvassing, phone calls etc.) increases voter turnout in elections. McClurg and Holbrook [30] investigate the role of campaign mobilization on driving core partisan groups to participate in voting.

\textsuperscript{21}Their main finding is that political activism had significant multiplier effects in terms of affecting the number of votes secured by Republican candidates, and also resulted in more conservative stances by policymakers in congress. They conclude "...these results are consistent with larger political protests creating a stronger political movement that is able to more effectively persuade the populace about its policy agenda come election time, which ultimately affects both incumbent behavior and election outcomes".
alternatively, is purely on electoral campaigns and the direct role of activists in influencing political platforms of candidates.

My model is also closely related to the work on direct informative role of campaign spending, notably, by Austen-Smith [6]. In Austen-Smith [6], candidates simultaneously announce policy, and elicit contributions from two firms. Campaign contributions help reduce the variance of noise in pronounced policy of candidates. Our electoral game follows the same sequence as the one described by Austen-Smith, but activist contributions play a "persuasive role", in the sense that they affect median voter’s ideological preferences. This apart, the contribution decision of firms in Austen-Smith are not constrained in that there is no party affiliation (firms can choose to contribute to either candidate). The motivations for activists to mobilize is quite different from Austen-Smith, and level of mobilization is captured by the elasticity parameter and ideological inclinations.

Lastly, another strand of relevant literature is the work by Baron [11] on protest based activism and private politics. They focus on activist groups that target - through boycotts - private firms, outside the purview of public institutions (hence private politics). Their model is essentially a bargaining game between a activist group and the private firm over the final policy outcome. My paper leans more towards a model of influence with activists, and is not a bargaining game between candidates and activists. The focus in this paper is also quite different. Party activists work within the confines of a party framework (price-takers in the sense of Aldrich), and hence different from activist groups in Baron (setting agenda). Also, the nature of competition is between competing activists belonging to different parties, and the goal is to ensure that their candidate wins the election. Hence, in both intent and the mechanism involved, my model of party activism differs considerably from Baron’s theory of private politics.

\[22\] Perhaps, the quid pro quo that results from activist support and its effect on incumbent decisions once elected, is an interesting direction to pursue in the future.

\[23\] Coate [19] presents an alternate model of informative campaign spending.

\[24\] We extend our model to include noisy campaigns, and characterize equilibrium polarization and activist mobilization. See Section 4.
3. Model

Two candidates, who care about ideology and benefits of office, contest elections on an uni-
dimensional policy space $[-1, 1]$. Candidate L has an ideal point $p^L_C = -\alpha$ and Candidate R has
an ideal point $p^R_C = \alpha$, where $\alpha \in (0, 1)$. The candidates simultaneously announce policy $X_i$ (where
$i \in \{L, R\}$), and the winning candidate enjoys benefits from office, $b$. The winner implements the
ex-ante chosen policy. Candidate utilities are symmetric.

$$U^C_i = \begin{cases} -(X_i - p^C_i)^2 + b & \text{if } i \text{ wins} \\ -(X_{-i} - p^C_i)^2 & \text{otherwise} \end{cases}$$

There is one representative activist supporting each party - L and R, with bliss points $p^A_L = -\beta$
and $p^A_R = \beta$ respectively. Activists observe the announced platforms by candidates, and simulta-
neously mobilize support to their favored candidate. Mobilization is simply the absolute levels of
contribution, given by $c_i$ (where $i \in \{L, R\}$), from either activists. Activists incur a convex cost of
mobilization $c_i^\gamma$ ($\gamma > 1$), where $\gamma$ is a measure of the mobilization aversion.\footnote{Let $m(c) = c^\gamma$ be the mobilization cost function. Then, the mobilization aversion parameter is related to the elasticity of marginal costs of mobilization given by $e_m = \left| \frac{\text{d}m}{\text{d}c} \right| = c \cdot \frac{\gamma c^{\gamma-1}}{m} = \gamma - 1$. Thus, $\gamma = 1 + e_m$ determines the mobilization aversion of activists.}

Note that electorate polarization is defined in a very elegant way by taking the ideological distance
between activists, $2\beta$. So, an increase in $\beta$ is a reflection of more partisan or extreme activists.
Greater $\beta$ could be thought of as more extreme views (to the right) on tax policy, gay rights,
regulations, or minimum wages, and so on. I deliberately place no restrictions on $\alpha$ and $\beta$, the
ideological inclinations of the candidate and the activist, respectively. Of course, it is straightforward
to glean that the more interesting case is when $\beta > \alpha$.

The mobilization term is very loosely defined to capture any form of contribution by activists.
This could be either direct donations to candidates or indirect ones to super-PAC’s, conducting
door-to-door or telephonic campaigns, talking to potential supporters in local districts\footnote{Take the case of Kshama Sawant, during the Seattle City Council elections in 2013. Sawant was a candidate of
the Socialist Alternative party; a party which espoused unabashed socialist policies, and competed against Richard
Conlin - an entrenched Democrat incumbent, in a city controlled by the Democratic establishment. Sawant stood for
$15$ a hour minimum wage, a millionaires tax, rent control, amongst other perceived leftist platforms. The campaign}

\footnote{Since the model is symmetric the interpretation of $-\beta$ is similar and the opposite of the one at $\beta$.}
on. Broadly, any measure of time, effort, or money spent on endorsing and campaigning for the candidate affiliated to a party could be accounted for by the variable $c_i$. The utility for the activist is given by,

$$U^A_i = \begin{cases} 
-(X_L - p^A_i)^2 - c_i^2 & \text{if } L \text{ wins} \\
-(X_R - p^A_i)^2 - c_i^2 & \text{if } R \text{ wins}
\end{cases}$$

Finally, the model assumes a single median voter, whose ideal point $\mu$ is unknown to candidates and activists, when they choose platforms and level of mobilization, respectively. All other parameters - $\alpha$, $\beta$, and $\gamma$ - are common knowledge. We assume that the median voter's bliss point is distributed uniformly on $[-1, 1]$.

$$U_m = \begin{cases} 
-(X_L - \mu)^2 + c_L - c_R & \text{if } L \text{ wins} \\
-(X_R - \mu)^2 + c_R - c_L & \text{if } R \text{ wins}
\end{cases}$$

This is a reduced form utility function\(^{28}\), to capture the fact that increased mobilization by activists persuades the voter by shifting preferences towards their favored candidate. Notice that activists have an incentive to contribute since increased mobilization directly affects the utility of the median voter, and thereby, the winnability of their respective candidates. This is the persuasive role of activism that I use in the benchmark case. Contributions reflect the extent of mobilization by activists during the election cycle. Note that only the net mobilization matters, not the absolute levels from either activist. One possible way of looking at this is if both candidates choose the same platforms, and furthermore, $c_L > c_R$, then $c_L - c_R$ is the least additional mobilization required to tilt the election in favour of candidate $R$, and vice-versa.

The timing of the game is summarised as follows:

1. Candidates $L$ and $R$ simultaneously announce policy platforms $X_L, X_R$
2. Activists observe platforms, and simultaneously choose contributions $c_L$ and $c_R$
3. Nature draws the median voter’s bliss point $\mu$ from an uniform distribution $[-1, 1]$

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\(^{28}\)Using such an utility form for the voter provides a tractable equilibrium solution to the electoral framework. In the extension presented in Section 4.2, we change this assumption to include noisy campaigns.

Witnessed a high level of political activism from faithful volunteers, who mobilized support behind Sawant’s choice of policies, and the Democratic incumbent was viewed as pandering to the wishes of the corporates. The most noticeable aspect of the elections was the non-reliance on 'big (corporate) money', and the use of ground level mobilization by activists belonging to a wide gamut of social backgrounds.
(4) The median voter observes policy platforms of candidates, contribution of activists, and decides the winner.

Notice that elections involve many other facets which have been ignored in this set-up. Instead, by isolating and concentrating on the channel of electoral activism, I intend to draw critical insights on the effects of activism on political polarization. The equilibrium concept is sub-game perfect Nash equilibria (SPNE) in pure strategies.

3.1. **Median Voter Subgame.** The (median) voter chooses the party which gives an higher utility, ie, the voter prefers candidate $L$ over candidate $R$ iff,

$$-(X_L - \mu)^2 + c_L - c_R \geq -(X_R - \mu)^2 + c_R - c_L$$

Therefore the cutoff $\mu$, below which the median voter will vote for party $L$ is,

$$\hat{\mu} = \frac{(X_R + X_L)}{2} + \frac{(c_L - c_R)}{(X_R - X_L)}$$

Let $\lambda$ denote the probability with which candidate $L$ wins. Given the distribution of $\mu$, the probability\(^{29}\) of candidate $L$ winning is, therefore,

$$\lambda = \frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{(c_L - c_R)}{2 (X_R - X_L)} \tag{3.1}$$

The win-probability of candidate $L$ is increasing in the contributions from activist $L$, and decreasing in the contributions from activist $R$. Therefore, the level of mobilization directly affects the win-probability of each candidate, after candidate platforms are announced. This reflects the motivation of activists - and of activism. Fixing one of the mobilization constant (say $c_L$) and increasing the other ($c_R$) reduces candidate $L$’s winnability, and vice-versa \((\frac{\partial \lambda}{\partial c_R} < 0, \frac{\partial \lambda}{\partial c_L} > 0)\). Therefore, there is always an incentive to mobilize more support, but equally, activists also face a convex cost of mobilizing support. Hence, mobilization decision balances these two opposing forces.

We proceed to look at the activists’ mobilization decision to understand this trade-off better.

\(^{29}\)For all $\mu < \hat{\mu}$, candidate $L$ wins the election. That is, $\lambda = Pr(\mu \leq \hat{\mu}) = \frac{\hat{\mu} + 1}{2}$.
3.2. Activist Contribution Subgame. Consider the contribution decision of the activist. The activist $L(R)$ can evaluate the winning probability of candidate $L(R)$, which in turn is dependent, among other variables, on the difference in contributions of the two activists. In fact, smaller the divergence in platforms, greater is the effect of the difference in relative contributions.

**Lemma 1.** When $X_L \neq X_R \neq 0$, each activist chooses a level of contribution given by,

$$c_L = \left( \frac{\beta}{\gamma} + \frac{X_L + X_R}{2\gamma} \right)^{\frac{1}{\gamma-1}}$$

and

$$c_R = \left( \frac{\beta}{\gamma} - \frac{X_L + X_R}{2\gamma} \right)^{\frac{1}{\gamma-1}}$$

**Proof.** See Appendix A.1

The contribution function represents the extent of mobilization by activists, and is dependent on activists’ ideology, the platforms announced by the candidates, and the mobilization aversion ($\gamma$). Observe that the contributions are an increasing function of $2\beta$, the divergence in activist bliss points. This suggests that there is greater level of mobilization in elections when the activists are more polarized. This is fairly intuitive to think about. Any electorate that is polarized care more about the issues and face a higher loss when the other side wins, prompting greater involvement and mobilization in the campaign\textsuperscript{30}. To understand the forces more deeply, take the problem of the activists. More mobilization affects the candidate’s chances of winning, reducing the ideological losses (in expected terms) of the activist. When activists are more polarized and the stakes are higher, it becomes more imperative for activists to support their candidate, since not doing so would result in the opposite side winning and implementing a more extreme policy.

The mobilization function exhibits some interesting properties. When a candidate becomes more extreme, the activist supporting the other candidate mobilizes more ($\frac{\partial c_R}{\partial X_L} < 0$ and $\frac{\partial c_L}{\partial X_R} > 0$).

\textsuperscript{30}MPR make this point rather succintly, pointing out the role of 'big soft money' from organized groups and the rapidly increasing campaign spending. I quote them: "It is not unreasonable to speculate that the impetus for the greater financial involvement was the increase in polarization and the increased ideological stakes of who wins elections." In US politics, policies related to taxes, minimum wages, health care, and gay rights could be perceived as polarizing and high-stakes issues.
since the expected ideological loss is now higher, and hence, the marginal benefit of mobilization is greater. This shows that activists not only care about whether their candidate moves closer, but equally care about whether the other side becomes more extreme. In a way, this incentive to mobilize comes from the fact that when one side adopts a more extreme stance, the expected ideological loss increases, thereby prompting greater countervailing (reactionary) mobilization.

Why this counter-mobilization happens is fairly clear from the incentives faced by the activists. Given two platforms, when one candidate deviates to a more extreme platform, two things change. First, the activist supporting the other candidate would face a greater ideological loss in the case when the extreme candidate wins. The activist’s incentive prompts greater mobilization so as to compensate for this loss by increasing the winnability of their candidate, or decrease the extreme candidate’s chances of winning. Second, the activist supporting the extreme candidate now faces a greater loss because an extreme platform, ceteris paribus, reduces the candidate’s winnability. This decrease in probability of winning the election reduces the incentive to mobilize support and therefore the activist supporting the extreme candidate now decreases participation, in order to compensate for the perceived higher expected ideological losses.

To understand the incentives, fix the initial platforms to be symmetric \( X_L = -X_R = -x, X_L + X_R = 0 \) and \( c_i = c^s \). Without loss of generality, suppose candidate \( R \) decides to deviate from the symmetric platform, and move closer to the ex-ante median \( X_R = x - \varepsilon \). The mobilization functions are no longer symmetric since \( \frac{\partial c_R}{\partial X_R} > 0 \) and \( \frac{\partial c_L}{\partial X_R} > 0 \). There are essentially two forces at play. When candidate \( R \) becomes less extreme, activist \( R \) increases mobilization, while activist \( L \), simultaneously, reduces contributions to the other candidate \( L \). Therefore, by becoming more moderate (or less extreme), candidate \( R \) could benefit since the win-probability of \( L \) decreases as \( \varepsilon \) increases, since the mobilization by activists seizes to be equal, creating a difference in their winnability.

Therefore, one immediate inference from the incentive structure is that candidate platforms would have to be symmetric in equilibrium. Of course, what the platform is depends crucially on the
candidate parameters ($\alpha$ and $b$), activist ideologies ($\beta$) and mobilization aversion ($\gamma$). I analyze the case of mobilization under symmetric platforms to make these points clearer.

3.3. **Case of symmetric platforms.** Since we restrict attention to symmetric candidate platforms, it is imperative to analyze the form and structure of mobilization function under this formulation. Moreover, as will be evident later, the main result of the paper hinges crucially on the form of the mobilization function under symmetric platforms.

**Lemma 2.** When candidate platforms are symmetric, i.e., $X_R = -X_L$, mobilization by activists are equal and given by, $c_{L,R} = c^e = (\frac{\beta}{\gamma})^{\frac{1}{\gamma-1}}$. Moreover, the following holds: $\frac{\partial c^e}{\partial \beta} > 0$, $\frac{\partial c^e}{\partial \gamma} > 0$ and $\frac{\partial^2 c^e}{\partial \beta \partial \gamma} < 0$ as long as $\beta > \gamma e^{2-\gamma}$.

**Proof.** See Appendix A.2

It is quite trivial to observe the form of the mobilization function\(^{31}\) under symmetric candidate platforms. What is more important is the marginal impact of ideology on mobilization, given by $\frac{\partial c^e}{\partial \beta} = \frac{1}{(\gamma-1)\beta} (\frac{\beta}{\gamma})^{\frac{1}{\gamma-1}}$, which determines how activists trade-off ideology and mobilization. Whether this marginal impact of ideology on mobilization is increasing or decreasing, depends on the mobilization aversion parameter.

To illustrate this, we analyze the marginal impact of ideology on mobilization, $\frac{\partial c^e}{\partial \beta}$. Notice that $\frac{\partial^2 c^e}{\partial \beta^2} = \frac{2-\gamma}{\gamma-1} \frac{1}{\beta} \frac{\partial c^e}{\partial \beta}$ is positive only when $\gamma < 2$. The mobilization aversion can then be evaluated as,

$$-\beta \frac{\partial^2 c^e}{\partial \beta^2} = \frac{\gamma - 2}{\gamma - 1}$$

This means that as activists become more polarized, the impact of polarization on mobilization depends critically on whether $\gamma$ is less than, equal to, or greater than 2. As long as $\gamma < 2$, mobilization function is convex in ideological separation of activists, or analogously, the marginal impact of ideology on mobilization is increasing. When $\gamma$ is greater than two, impact of ideology on mobilization under symmetric platforms is wasteful, since both sides contribute the same levels thereby having no relative impact on the winnability of the candidate. However, the reason why they are positive is precisely because if one activist were to reduce mobilization, it decreases the corresponding candidate’s winnability. The other activist, as a consequence, has a greater incentive to contribute, since the marginal benefits of mobilization exceeds the marginal cost of doing so.

\(^{31}\)Note that mobilization under symmetric platforms is wasteful, since both sides contribute the same levels thereby having no relative impact on the winnability of the candidate. However, the reason why they are positive is precisely because if one activist were to reduce mobilization, it decreases the corresponding candidate’s winnability. The other activist, as a consequence, has a greater incentive to contribute, since the marginal benefits of mobilization exceeds the marginal cost of doing so.
mobilization is concave, and it is independent of ideology when $\gamma = 2 \left( \frac{\partial^2 c^s}{\partial \beta^2} = 0 \right)$. This apart, the dependence of mobilization on ideological polarization also affects the marginal costs of activists. This, ceteris paribus, changes the incentives of candidates, who choose platforms such that the marginal benefits to activists equals their marginal costs of mobilization. I will refer to this whilst analyzing the main results of the paper, presented in the next section.

4. Candidate Equilibrium and Comparative statics

Candidates anticipate contributions and the winning probability as a function of their chosen platforms. Given complete information on $\beta$ and $\gamma$, each candidate’s strategy is therefore to commit to a platform, given the other candidate’s platform. We restrict attention to symmetric pure strategies.

**Proposition 1.** The electoral game with activism has an unique symmetric pure strategy equilibrium in candidate platforms, $(X_L, X_R) = (-x, x)$. Furthermore, the equilibrium level of divergence increases when (a) ideological polarization of the candidate $\alpha$ increases, or (b) the benefits of office $b$ decreases. That is, $\frac{\partial x}{\partial \alpha} > 0$ and $\frac{\partial x}{\partial b} < 0$

**Proof.** See Appendix A.3

From Proposition 1, it is established that when two candidates who belong to responsible parties contest for elections in the presence of activists, there is divergence of platforms in equilibrium. This result has been already shown by Bernhardt et al. [13], in the absence of activism. The comparative statics with respect to $\alpha$ and $b$ are straightforward and intuitive.

The main contribution, and finding, of my model is the comparative statics result with respect to the activists’ ideology. From the above discussion, it follows that the relationship between $x$ and $\beta$ hinges on the mobilization aversion parameter. My main finding reflects this dependency.

**Proposition 2.** (i) For $\gamma \in (1, 2)$, as activists become more extreme, equilibrium platforms are less polarized, i.e., $\frac{\partial x}{\partial \beta} < 0$. 
(ii) For all $\gamma \in (2, \infty)$, as activists become more extreme, equilibrium platforms are more polarized, ie, $\frac{\partial x}{\partial j} > 0$.

(iii) For $\gamma = 2$, the equilibrium polarization is independent of the ideological preferences of the activist.

Proof. See Appendix A.3

Proposition 2(i) and 2(ii) are the main contributions of this paper - that there is a non-monotonic relationship between activist polarization and political polarization of candidates, and the nature of this relation is captured by the mobilization aversion parameter. To understand why this happens, it is important to understand the relationship between marginal costs and benefits of activists. When $\gamma < 2$, as discussed earlier, the dependence of mobilization on ideology of activists is convex, or the marginal impact of ideology on mobilization is increasing. This has two effects. Firstly, this directly translates into increasing and convex marginal costs of ideological polarization for the activist\(^{32}\). Secondly, a convex change in marginal costs has to be matched by a corresponding increase in marginal benefit, in terms of reduced expected ideological loss. The marginal benefit for activist $R$ is just $\frac{1}{2}[\beta - x \frac{\partial c_s}{\partial j}]$. This is probability that candidate $R$ wins times the benefit of a closer (relatively) ideology. This has to be equal to the activist’s marginal cost\(^{33}\), $\beta \frac{\partial c_s}{\partial j}$. Candidate $R$ would have to choose platforms such that this trade-off of activist $R$ is satisfied.

\(^{32}\)The relationship to marginal cost function can be seen by some simple algebra. Take the mobilization cost function under symmetric platforms, as defined earlier:

$m(c^s) = (c^s)^\gamma$

$m'(c^s) = \gamma(c^s)^{\gamma-1} \frac{\partial c^s}{\partial j} = \beta \frac{\partial c^s}{\partial j}$

$m''(c^s) = [\beta \frac{\partial c^s}{\partial j} + \frac{\partial^2 c^s}{\partial j^2}] = (\frac{1}{1-1}) \frac{\partial c^s}{\partial j}$

$m'''(c^s) = (\frac{1}{1-1}) \frac{\partial c^s}{\partial j}$

The sign of the third derivative depends on $\frac{\partial^2 c^s}{\partial j^2}$, which in turn depends on $\gamma$. I have established earlier that $\frac{\partial^2 c^s}{\partial j^2} = \frac{2-\gamma}{\gamma-1} \frac{\partial c^s}{\partial j} > 0$ iff $\gamma < 2$. Hence, marginal cost function of activists is convex or concave depending on whether $\gamma < 2$ or $\gamma > 2$, respectively.

\(^{33}\)Note that the ratio $-\beta \frac{m''(c^s)}{m'(c^s)} = -\beta \frac{\partial c^s}{\partial j} = \frac{\gamma-2}{\gamma-1}$. Rather surprisingly, this is similar to the measure of prudence in the context of precautionary savings that Kimball [?] had proposed. This seems a rather intriguing and unforeseen similarity, one that I shall avoid delving or commenting further upon, in this paper.
Since activists are ideologically risk averse, platforms that reduce variance (more convergent platforms) provide them with marginal benefits that equate marginal costs of increased polarization. As activists continue to become more polarized, and $\gamma < 2$, the marginal costs to the activist is increasing at a convex rate. To compensate for this increase, candidate $R$ chooses a platform closer to ex-ante median ($\frac{\partial x}{\partial \beta} < 0$). Simultaneously, candidate $L$ does the same, and the overall platform polarization ($2x$) reduces, leading to greater convergence.

The opposite effects are at play when $\gamma > 2$, and the arguments are reversed. The right side of equation 4.1 is increasing but concave, as $\beta$ increases. That is, eventhough impact of ideology on mobilization is increasing, it is now doing so at a decreasing rate. Hence, the compensation for activists (in terms of ideology) decreases too, which leads to shift in the sign of $\frac{\partial x}{\partial \beta}$.

Proposition 2(iii) states that activist polarization does not affect equilibrium platform polarization. Consider what happens to $\frac{\partial c_s}{\partial x}$ when $\gamma = 2$. It is trivial to see that impact of polarization on mobilization is independent of $\beta$, and equal to $\frac{1}{2}$. That implies the right hand side of equation 4.1 is constant and is equal to $\frac{\beta}{2}$. This immediately translates to $\frac{\partial x}{\partial \beta} = 0$. Therefore, whenever $\gamma = 2$, activist preferences are homothetic, and their marginal rate of substitution between ideological loss and mobilization is constant, thereby making political polarization independent of the activists’ ideology. Candidates, therefore, need not compensate activists for greater polarization and mobilization, since their marginal costs remain unchanged. Therefore, this is a case where electoral competition is unaffected by activism.

Before proceeding to the extensions, I must discuss the implications of this finding, for policy and otherwise. From my analysis, political polarization can increase because of two reasons: first, when $\gamma > 2$ and activists become more polarized; second, when $\gamma < 2$ and activists become less polarized. The latter of the two is counter-intuitive and is perhaps the more interesting case. The technical argument for why this happens has been explored in detail previously. I will now try
to explain possible mechanisms to reflect on this finding, and the implications of these results for policy changes with respect to electoral funding, campaign finance laws and so on.

A direct implication of the result is that a polarized electorate need not result in more polarized candidates. For example, income inequality, cited by MPR as one of the drivers of political polarization, reflects only the ideological polarization of activists in my model. Subsequently, for candidate platforms to polarize, mobilization aversion parameter plays a critical role. This dependence of platforms on both ideology and participation costs of activists naturally leads us to the question of what this mobilization aversion is, in the real world.

I argue that the level of polarization and the mobilization aversion can be reconciled with by looking at both the supply side and the demand for activism. On the supply side, one possible interpretation of the mobilization aversion parameter is that it captures the electoral process, in which mobilization happens over a period of time. Electoral cycles are a continuous and arduous process, taking up a lot of time and resources on the part of candidates and activists, starting with the announcement of platforms up until election day. In some sense, the aversion parameter, therefore, seems to reflect the technology of the underlying process of political competition, or length of the electoral campaign cycle. If this is indeed the case, then local (municipal bodies or city councils) elections where stakes are lower and length of campaign is substantially shorter, may have lower levels of polarization compared to state or federal elections, controlling for activist ideologies.

A lot recent research\textsuperscript{34} have shown that participation levels of the general electorate has decreased over time in industrialized nations. That is, irrespective of the length of campaign cycle, participation in advanced countries seems to have fallen. In the case of US, it may be that the decline of traditional participation is closely linked to the demand for such activism. Modern communication and messaging techniques employed by candidates preclude the need for a more grass-roots campaign, crowding out moderate supporters in favor of "big money" donors. This is fairly evident in the case of US politics, where candidates have increasingly relied on professional managers to

\textsuperscript{34}Refer to Dalton and Wattenberg [20] for a comprehensive account of the decline in partisanship in Western Democracies.
run campaigns; campaigns that are more digital in nature. The obvious consequence of this shift is the bigger role of money and the diminishing role of smaller - in terms of contributions - but more moderate activists or volunteers in campaigns. These changes on the demand side has indeed affects who participates, and the propensity of diverse groups to contribute.

The polarization in American politics, therefore, according to my model, can be pinned down to a mix of supply-side and demand-side considerations on activism. Political polarization is as much a result of crowding out of more moderate supporters as it is about a polarized elite that supports campaigns. An obvious part of the remedy is to curb excessive campaign spending by introducing stricter laws, akin to ones previously articulated by Prat [36] and Gul and Pesendorfer [29].

I also propose steps must be taken in order to incentivize participation by a wider gamut of the electorate. As my analysis has shown, it is important to address the issue of aversion to participate, since it determines the nature of relationship between political polarization and electorate polarization. For example, publicly funded campaigns could tilt the balance of demand towards smaller volunteers and activists, and possible inclusion of moderate supporters to the campaign. Public funding\textsuperscript{35} could also automatically reduce the length of the campaign cycle, thereby making it more even in terms of participation, encouraging traditional support groups to get involved in campaigns. My theoretical findings suggest that there could be a possible link between publicly funded elections and political polarization. I therefore propose that curbing independent expenditures (through restrictions on PACs and super-PACs spending) and simultaneously introducing publicly funded campaigns would be an useful institutional reform to encourage wider participation and bringing down political polarization.

\textsuperscript{35}Some other prominent advantages of public funding is that is reduces candidate dependence on pernicious interest group considerations, allowing for more electoral competition, and possibly decreasing incumbency advantage. In U.S, the states of Maine and Arizona have had publicly funded state legislature elections. Look at the report by Government Accountability Office (GAO) for more on this: http://www.gao.gov/assets/310/305079.pdf
5. Extensions

Thus far, we have assumed that mobilization has a direct persuasive role, and that there is a single representative activist (group). We relax each of these assumptions, separately, and analyze the behaviour of equilibrium platforms.

In Section 5.1, we model noisy campaigns, where the role of activism is to reduce the variance of noise in candidate platforms. Voter only observes a noisy signal of the candidates’ platforms, and more mobilization by activists helps reduce the variance of this noise. We derive the unique symmetric equilibrium candidate platform and activist mobilization. Further, we derive conditions for comparative statics of the baseline model to hold.

Section 5.2 extends the model to include multiple activist groups - with different ideological preferences - to campaign for either candidate. Again, we assume symmetry in the number of activists supporting either candidate. As opposed to the single activist case, when multiple activists are involved in a campaign, there is a free-riding effect and convergence in platforms, when political participation increases, providing a rationale for failure of “collective action” in electoral activism.

5.1. Activism in Noisy campaigns. Suppose the policy platform of candidates are observed with noise by median voter, and activists’ role is to inform the median voter of the precise position. If $X_i$ is the true position of the candidate, the policy observed by the median voter is $\tilde{X}_i = X_i + \eta_i$, where $\eta_i$ is a random variable (noise term) with expectation zero and variance $\sigma_i^2$. Further, contribution from activists reduces the variance of the noise term. If $c_i$ is the contribution from the activist, then $\sigma_i^2 = a(c_i)$. The following assumptions are made on the functional form of $a(\cdot)$\textsuperscript{37}: $a'(\cdot) < 0$, $a''(\cdot) > 0$, $a'''(\cdot) < 0$ and $a(0) > 0$. First two conditions ensure that as activists contribute more, the variance of noise is decreasing, and convex in mobilization. The subsequent condition implies that the concavity of marginal variance, and the last condition states that, in the absence of

\textsuperscript{36}By collective action, we mean different groups with non-aligned ideologies mobilizing support to a candidate. When the distance between their ideologies goes to zero, we end up with the representative activist argument, presented in our baseline set-up.

\textsuperscript{37}We additionally assume that the noise reduction mechanism $a(\cdot)$, is the same for both candidates.
activism, there is a positive level of noise in platforms, meaning voters imperfectly observe platform of candidates.

This formulation naturally implies that greater activist mobilization is beneficial for candidates since it reduces the variance of the platforms, and since the voter is risk-averse, less variance is preferred. Modelling campaigns this way is very useful. Imagine a candidate standing for elections, and the opposite party runs a negative campaign\(^\text{38}\), twisting the policy platform of the candidate. Activists or volunteers, then, have an important role in conveying - through door-to-door canvassing or phone calls - the true policy stance of their candidate.

Before presenting the results, it is important to glean the role of noise reduction function. Remember, activism is now not a persuasive tool, but restricted to only reducing the variance of the noisy platform. We introduce the parameter \(e_n\) to define the efficiency of activism in reducing the noisiness of platforms. That is, Efficiency of Marginal Noise Reduction, \(e_n = -c\frac{\alpha''}{a}\).

In the next proposition, we present the conditions for the main result of "activist polarization-candidate convergence" to hold. The comparative statics of equilibrium platforms with respect to \(b\) and \(\alpha\), and of mobilization with respect to \(\beta\) and \(\gamma\), remain unchanged, and confined to the appendix.

**Proposition 3.** *In the case of activism in a noisy electoral campaign, there exists an unique symmetric equilibrium in candidate platforms. Furthermore,*

i. \(\frac{\partial x}{\partial \beta} < 0 \) if \(e_n < \frac{1-e_m}{2}\); ii. \(\frac{\partial x}{\partial \beta} > 0 \) if \(e_n > \frac{1-e_m}{2}\)

**Proof.** See Appendix A.5

From Proposition 4, it is clear that when the role of activism changes from 'persuasive' to 'informative', the efficiency of activism plays a critical role in deciding the nature of candidate platforms. Activist divergence leads to candidate divergence only when, apart from elasticity of mobilization remaining low, the efficiency of activism satisfies a simple condition, \(e_n < \frac{1-e_m}{2}\). The intuition is simple - when activism is efficient in reducing the variance of noise in policy, marginal costs of activism is decreasing, leading to the same argument as before, where candidates diverge to balance

\(^{38}\)See Polborn et al. (2008) for a model of negative campaigning in elections.
marginal costs and benefits. In some ways, higher efficiency of activism However, when activism remains inefficient, the nature of campaign is such that marginal costs of activists is increasing as they become more polarized. As in the benchmark model, this is compensated by increasing marginal benefits, by moving platforms closer, reducing overall polarization.

The results obtained from analyzing noisy campaigns only reiterates our earlier finding - that a polarized electorate does not necessarily increase platform polarization. In the case of activism with noisy campaigns, this relationship depends, apart from the elasticity parameter, on efficiency of activism in reducing variance of noise. In democratic countries with freedom of press, and no curbs on mobilization, we should anticipate higher efficiency of activism, since the information generated by activism is not censored in any kind. Hence, the case of polarization in US could be seen through the prism of efficient activism in a noisy campaign setting. This interpretation takes into account the inherent nature of noisy campaigns and the informative role of door-to-door activism, which could be perceived as providing information to the public about policies of a candidate.

Thus, two different interpretations of what factors - apart from extreme groups - cause polarization emerges from our analysis. The first is that if activism is persuasive, then a higher elasticity of mobilization induces separation of platforms in equilibrium. Alternatively, if elasticity is low, but the electoral campaigns are noisy, informative activism that is efficient beyond a threshold could result in polarization of platforms. Essentially, the mechanism underlying the two methods are complementary, and our theory sheds light on the similarities between persuasive and informative activism. This, we believe, is an important contribution of our analysis.

5.2. Case of multiple activists. Instead of a single representative activist mobilizing support, we now consider a scenario where multiple activists (simultaneously) participate in electoral mobilization for their respective candidates. Suppose there are \( N \) activists on either side of the political spectrum. Let \( A_j = \{A_{1j}, A_{2j}, \ldots, A_{Nj}\} \), where \( j = L, R \) represent the set of \( N \) activists supporting candidate \( L \) and \( R \). Further, ideological preferences of the activists are ordered in the following way: \( A_{pL} = -1 + (p - 1)\varepsilon \), \( A_{pR} = 1 - (p - 1)\varepsilon \). This immediately implies that the most extreme activists supporting candidates \( L \) and \( R \), are located at \(-1\) and \(1\) respectively; and the least extreme
at $-1 + (N - 1)\varepsilon$ and $1 - (N - 1)\varepsilon$, respectively. We will refer to the distance between the most extreme and least extreme activist (given by $(N - 1)\varepsilon$), as the "ideological spread" of activists.

This representation has two advantages. Firstly, it provides a simplified framework in which every activist is at an ideological distance $\varepsilon$ from a neighboring activist. Secondly, such a formulation allows for studying the relationship between political (electoral) participation and polarization.

We provide two variants of political participation. In the first, $N$ is fixed but $\varepsilon$ changes, affecting the ideological spread. In the second interpretation, the ideological spread remains constant, but $N$ changes (by adjusting $\varepsilon$). The former allows for asking the question, "What would happen to polarization in elections if political participation is confined to a narrow ideological spread of the electorate?", while the latter addresses the following question: "How does polarization react to decreasing the density of electoral participation within a narrow ideological (possibly extreme) band?". The result below sheds light on these questions.

**Proposition 4.** In the presence of multiple activists, polarization in candidate platforms increases when

i) fixing $N$, the ideological spread becomes more extreme;

ii) fixing ideological spread, the number of activists within this decreases (less dense).

**Proof.** See Appendix A.7

The first part of Proposition 4 argues for a wider ideological spread to participate in the electoral process. If political activism is confined to a tiny ideologically extreme "elite", then our model predicts that polarization of platforms would increase starkly. This reaffirms the stand of MPR on the exclusive nature of financial contributions in US elections. They find that financial participation in electoral process is restricted to a very small number of powerful people, who also happen to be ideologically extreme.\footnote{See Chapter 5 of MPR, for more on inequality in financial contributions, in US politics.}

The intuition provided by our model is that when activists belong to an extreme ideological spread, the ability to pull policy towards their own ideology increases, since candidates depend on them for electoral mobilization. However, when the ideological spread is wider, then moderate
elements of the electorate participate in activism. This has two effects. Firstly, compared to an extreme elite, moderate groups mobilize less (free-riding effect). Secondly, the presence of moderate groups reduces the influence of extremist groups, by increasing the marginal costs of their contribution. This reduces overall polarization of candidate platforms.

We find that, despite the polarization effected by an extreme elite, the effect of this outward pull in policy is much weaker than in the case of a single activist. This follows directly from the fact that as the number of activists increase, their marginal importance to the candidate decreases, which makes candidates pander less towards their ideology.

Proposition 4, therefore, explains two types of free-riding problem in activism. In $i$), free-riding arises because a significant portion of the electorate that are ideologically moderate, do not actively participate in the electoral process. This leaves campaign contributions to ideologically extreme groups, which leads to greater polarization. In Proposition 4 $ii$), we witness the classical free-riding problem. Suppose, unlike $i$), moderate groups also participate. We argue that even if this were to happen, mobilization is still increasing in ideological extremity of activist groups ($\frac{\partial c_i}{\partial z} < 0$). Decreasing the density of the groups, therefore, increases the relative importance of each group within the spread, and causes platforms to diverge. Extending the above argument, increasing the density of activists by reducing $\varepsilon$ leads to greater policy convergence. Greater density of participation implies the value of any single activists’ contribution is decreasing to the candidate.

In a way, Proposition 4 highlights the problem of approaching activism as "collective action". As the number of activist groups increase, more people participate in elections, and since aggregate mobilization is what matters to the candidate, the relevance of individual activists diminishes, leading to convergence in platforms. From the point of view of extreme groups, this is not ideal. As far as extreme ideological groups are concerned, more exclusion in the political process is better. Encouraging political participation of moderate groups, as our analysis has shown, has a moderating influence on platforms, in equilibrium.

Thus, to ensure that voters are offered a choice by candidates (through divergent platforms), and at the same time, that these choices aren’t too extreme, what is required is a combination of
low $N$, and high $\varepsilon$. Reducing the number of activists increases the importance of their marginal contribution to the candidate, and a higher $\varepsilon$ forces candidates to react to more moderate forces in campaigns.

6. Conclusion

I have analysed a model of activism and mobilization to address the question of whether and how the participation of a more polarized electorate in the electoral process affects political polarization. An important feature of my model is that activists mobilize support to candidates but face a convex cost of mobilization. The costs of mobilization is captured by the mobilization aversion parameter. Further, I assume that activism plays a persuasive role (in the benchmark case) or an informative role (in the extension), and activists trade off marginal costs of mobilization and ideological benefits of campaigning for a candidate.

In this setting, I have established the existence of an unique equilibrium in symmetric candidate platforms. This is the equilibrium polarization induced by activism. The main finding of my model is the following comparative statics result - when mobilization aversion is below a threshold and activists become more polarized, the equilibrium platforms of candidates tend to converge, or in other words, political polarization decreases. The implication of this result is that electorate polarization, on its own, is insufficient to explain political polarization. The combination of activist ideology and aversion towards participation together determine the extent of political polarization. In the context of US politics, I have argued that this aversion to participate is driven by supply side and demand side considerations of activism. To state more precisely, activism is costly when campaign cycles are longer, since it entails greater resources in terms of time or money. This could potentially reduce the propensity to participate in the political process. Simultaneously, the demand for traditional activism has diminished and candidates seek more money to run a personalized campaign. This, I argue, has crowded out moderate supporters and replaced them with big-money interests. My model’s main finding, therefore, provides a rationale for introducing publicly funded elections and curbing independent expenditures (by PACs and super-PACs for example) as an institutional reform to tackle political polarization.
Furthermore, I also characterize equilibrium mobilization, under symmetric platforms. I establish that mobilization increases when activists become more polarized and mobilization aversion increases. This paper, to the best of my knowledge, is the first attempt to endogenously characterize mobilization, and provides a framework to answer the question of why some elections witness greater mobilization than others. I predict that, all else equal, greater electorate polarization leads to more activist mobilization. This implies that campaigns that are more polarized would generate higher levels of mobilization since the stakes are higher.

Finally, I consider the possibility of multiple activist groups, with varying ideological preferences, to mobilize support to candidates. In this scenario, I find that polarization increases when participation is confined to elite groups which are ideologically extreme, or when the number of activist groups decreases. Both these results stem from free-riding, albeit of different kinds. We illustrate that free-riding problems in the case of multiple activists is why collective-action in activism may not be successful in offering a choice to the electorate. However, on the other hand, restricting political participation to extreme groups is also detrimental, since it tends to polarize platforms.

The results of my theoretical analysis provides for some interesting and significant testable implications on empirical front. Firstly, my model predicts a possible negative relation between activist polarization and candidate polarization. In order to test this result, it would be necessary to estimate the mobilization aversion parameter, which determines elasticity of the marginal costs of mobilization. Estimating and providing a measure of the mobilization aversion parameter from donor contributions in election campaigns, would be an important avenue for future empirical research. Once an estimate is developed, it would definitely be possible to test the relationship between polarization of various groups supporting the different candidates, and the candidate platforms themselves. Secondly, empirical work on studying the relationship between levels of mobilization and the activist polarization, as predicted by my model, would help shed light on the puzzle of why certain electoral campaigns witness large scale mobilization.
Appendix A. Proofs

All the proofs are carried out for candidate/activist R, and are symmetric for candidate/activist L.

A.1. Proof of Lemma 1. Consider activist R and their contribution decision. Given candidate R’s win probability from the voting stage subgame, the expected utility of activist R is,

\[ EU_A^R(\beta, c_R) = -\lambda(X_L - \beta)^2 - (1 - \lambda)(X_R - \beta)^2 - c_R^\gamma \]

\[ = -\left(\frac{1}{2} + \frac{(X_R + X_L)}{4} \right) - \frac{(c_L - c_R)}{2(X_R - X_L)} \left( X_L - \beta \right)^2 - \frac{(c_L - c_R)}{2(X_R - X_L)} \left( X_R - \beta \right)^2 - c_R^\gamma \]

The activist chooses a contribution, given the ideal point \(\beta\), to maximise his expected utility. The FOC for a maximum is therefore,

\[ \frac{\partial EU_A^R(\beta, c_R)}{\partial c_R} = \frac{\partial \left(-\frac{1}{2} + \frac{(X_R + X_L)}{4} \right)}{\partial c_R} - \frac{(c_L - c_R)}{2(X_R - X_L)} \left( X_L - \beta \right)^2 - \frac{(c_L - c_R)}{2(X_R - X_L)} \left( X_R - \beta \right)^2 - c_R^\gamma \]

Solving for \(c_R\),

\[ 2\gamma c_R^{\gamma-1} - 2\beta + X_L + X_R = 0 \Rightarrow c_R = \left( \frac{\beta}{2} + \frac{X_R + X_L}{2\gamma} \right)^{\frac{1}{\gamma-1}} \]

By symmetry,

\[ c_L = \left( \frac{\beta}{2} + \frac{X_L + X_R}{2\gamma} \right)^{\frac{1}{\gamma-1}} \]

A.2. Proof of Lemma 2. Under symmetric platforms, the mobilization by Activists R and L are equal.

That is, \(c_R = c_L = c^* = \left( \frac{\beta}{2} \right)^{\frac{1}{\gamma-1}}\).

\[ \frac{\partial c^*}{\partial \gamma} = -\frac{1}{\beta(1-\gamma)} \left( \frac{\beta}{\gamma} \right)^{\frac{1}{\gamma-1}} > 0 \]

This proves the first part of the lemma.

\[ \frac{\partial c^*}{\partial \gamma} = \frac{\partial \left( \frac{\beta}{\gamma} \right)^{\frac{1}{\gamma-1}}}{\partial \gamma} = -\frac{1}{\gamma(\gamma-1)} \left( \frac{\beta}{\gamma} \right)^{\frac{1}{\gamma-1}} \left( 1 - \gamma - \gamma \ln \left( \frac{\beta}{\gamma} \right) \right) \]

\[ \frac{\partial c^*}{\partial \gamma} > 0 \text{ iff } \left( 1 - \gamma - \gamma \ln \left( \frac{\beta}{\gamma} \right) \right) > 0 \]

But \(1 - \gamma - \gamma \ln \left( \frac{\beta}{\gamma} \right) \geq 0\) only when \(\beta \leq \gamma e^{-\frac{2-1}{\gamma}}\)

Consider the expression \(\gamma e^{-\frac{2-1}{\gamma}}\): If \(\gamma = 1\), then \(\gamma e^{-\frac{2-1}{\gamma}} = 1\) and \(\frac{\partial (\gamma e^{-\frac{2-1}{\gamma}})}{\partial \gamma} > 0\). This implies that for any \(\gamma > 1\), the expression \(\gamma e^{-\frac{2-1}{\gamma}} > 1\). Since the ideology of the activists, \(\beta \in (0, 1]\), the condition \(\beta \leq \gamma e^{-\frac{2-1}{\gamma}}\) is always satisfied. Hence \(\frac{\partial c^*}{\partial \gamma} > 0\), proving the second part of the lemma. Consider the last part of the proposition,
\[
\frac{\partial^2 c'}{\partial \beta \partial \gamma} = \frac{\partial \left( \frac{\beta}{\beta(\gamma - 1)} \gamma^{-1} \right)}{\partial \gamma} = \frac{1}{\beta \gamma(\gamma - 1)} \left( \frac{\beta}{\gamma} \right)^{1-\gamma} (1 - \gamma^2 - \gamma \ln \frac{\beta}{\gamma})
\]

This expression is less than zero iff \((1 - \gamma^2 - \gamma \ln \frac{\beta}{\gamma}) < 0\). This happens when \(\frac{\beta}{\gamma} > e^{-\frac{(\gamma^2 - 1)}{\gamma}}\), or \(\beta > \gamma e^{-\frac{(\gamma^2 - 1)}{\gamma}}\).

This expression encapsulates the fact that activists have to be sufficiently polarized, so that the impact of ideology on mobilization is diminishing as the elasticity of mobilization (\(\gamma\)) increases. This concludes proof of the lemma.

A.3. Proof of Proposition 1. Substituting the expressions \(c_L\) and \(c_R\) into 3.1,
\[
\lambda = \frac{1}{2} + \left( \frac{X_R + X_L}{4} \right) + \frac{\left( \frac{\beta}{\gamma} + \frac{X_L + X_R}{2} \right)^{1-\gamma} - \left( \frac{\beta}{\gamma} - \frac{X_L + X_R}{2} \right)^{1-\gamma}}{2(X_R - X_L)}
\]

In the initial period, the candidates have to simultaneously choose platforms. By simple backward induction, the candidate subgame involves a decision by each candidate to locate on the policy space, given the win-probability and the contribution functions that they are likely to face in the subsequent stages. Again, each candidate chooses a platform to maximize their payoffs, taking as given the platform chosen by the other candidate.

The SPNE is such a pair of platform choices that maximizes the expected utility of both candidates in the first stage. Let us consider candidate R.

\[
EU^C_R(X_L, X_R) = -\lambda (X_L - \alpha)^2 - (1 - \lambda) (X_R - \alpha)^2 + (1 - \lambda) b
\]

where \(\lambda = \frac{1}{2} + \left( \frac{X_R + X_L}{4} \right) + \frac{\left( \frac{\beta}{\gamma} + \frac{X_L + X_R}{2} \right)^{1-\gamma} - \left( \frac{\beta}{\gamma} - \frac{X_L + X_R}{2} \right)^{1-\gamma}}{2(X_R - X_L)}\)

Supposing that candidate L chooses \(X_L = -x\). The FOC is given by,
\[
\frac{dEU^C_R(-x, X_R)}{dX_R} = \lambda' X_R \left[ (X_R - \alpha)^2 - (x + \alpha)^2 - b \right] - 2(1 - \lambda)(X_R - \alpha) = 0
\]

where,
\[
\lambda' = \frac{\partial \lambda}{\partial X_R} = \frac{\partial \left( \frac{1}{2} + \left( \frac{X_R + X_L}{4} \right) + \frac{\left( \frac{\beta}{\gamma} + \frac{X_L + X_R}{2} \right)^{1-\gamma} - \left( \frac{\beta}{\gamma} - \frac{X_L + X_R}{2} \right)^{1-\gamma}}{2(X_R - X_L)} \right)}{\partial X_R}
\]

(A.1) \[
\lambda' = \frac{1}{4} + \frac{1}{2} \left[ \left( \frac{\beta}{\gamma} + \frac{X_R + X_L}{2} \right)^{1-\gamma} - \left( \frac{\beta}{\gamma} - \frac{X_R + X_L}{2} \right)^{1-\gamma} \right] \frac{1}{(x_R - x_L)^2} (x_R - x_L)
\]

\[
- \left[ \left( \frac{\beta}{\gamma} + \frac{X_R + X_L}{2} \right)^{1/(\gamma - 1)} - \left( \frac{\beta}{\gamma} - \frac{X_R + X_L}{2} \right)^{1/(\gamma - 1)} \right] \frac{1}{(x_R - x_L)^2}
\]
Since we are interested in symmetric equilibria,

\[(A.2)\]

\[\lambda'_{X_R} \mid (-x,x) = \frac{1}{4} + \frac{1}{4\gamma (\gamma - 1) \left(\frac{\beta}{\gamma}\right)^{\frac{\gamma-2}{\gamma-1}} x}\]

FOC evaluated at \((-x, x),\)

\[\frac{dEU_C (-x, X_R)}{dX_R} \big|_{X_R=x} = \frac{1}{4\gamma(\gamma-1)x^2} \left((\gamma^2 - \gamma)x^2 + \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\gamma-1}} (\frac{\beta}{\gamma})x[-4\alpha x - b] + (\alpha - x) = 0\right)\]

\[\Rightarrow \frac{1}{4\gamma(\gamma-1)x^2} \left(\gamma^2 - \gamma\right)x^2 + \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\gamma-1}} (\frac{\beta}{\gamma})x[4\alpha x + b] - (\alpha - x) = 0\]

\[\Rightarrow 8(\alpha + 1)x^2 - (4\alpha - 2b + 4\alpha - \frac{8\alpha}{(\gamma-1)\beta^{\frac{1}{\gamma-1}} \beta^{\frac{1}{\gamma-1}}})x + \frac{2b}{(\gamma-1)\beta^{\frac{1}{\gamma-1}} \beta^{\frac{1}{\gamma-1}}} = 0\]

Letting \(M \approx M(\gamma; \beta) = \frac{1}{(\gamma-1)\beta^{\frac{1}{\gamma-1}} \beta^{\frac{1}{\gamma-1}}} = \frac{1}{(\gamma-1)\beta^{\frac{1}{\gamma-1}} \beta^{\frac{1}{\gamma-1}}},\) the equation that solves for equilibrium platform is,

\[4(\alpha + 1)x^2 - (4\alpha (1 - M) - b)x + Mb = 0\]

Let \(B = (4\alpha (1 - M) - b); A = 4(\alpha + 1); C = bM\)

The solutions to the equation are,

\[x = \frac{1}{8(\alpha + 1)} \left[(4\alpha (1 - M) - b) \pm \sqrt{(4\alpha (1 - M) - b)^2 - 16b(\alpha + 1)M}\right]\]

Let the solutions be,

\[x^+ = \frac{1}{8(\alpha + 1)} \left[(4\alpha (1 - M) - b) + \sqrt{(4\alpha (1 - M) - b)^2 - 16b(\alpha + 1)M}\right]\]

and,

\[x^- = \frac{1}{8(\alpha + 1)} \left[(4\alpha (1 - M) - b) - \sqrt{(4\alpha (1 - M) - b)^2 - 16b(\alpha + 1)M}\right]\]

To ensure that only \(x^+\) is a maximum, we need the second order derivative to be negative. That is,

\[\frac{\partial}{\partial x} \frac{1}{4\gamma(\gamma-1)x^2} \left((\gamma^2 - \gamma)x^2 + \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\gamma-1}} (\frac{\beta}{\gamma})x[-4\alpha x - b] + (\alpha - x)\right)\]

\[= \frac{1}{4\gamma(\gamma-1)} \left(4x^2\beta + b \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\gamma-1}} + 4x^2\alpha\beta - 4x^2\beta\gamma - 4x^2\alpha\beta\gamma\right) < 0\]

\[\Rightarrow \left(\frac{b}{M}\right) \frac{1}{\gamma^2} - (\alpha + 1) < 0\]

\[\Rightarrow \frac{b}{M} < 4(\alpha + 1)x^2\]

From the expressions for \(B, A\) and \(C\), the above inequality is \(C < Ax^2\)

But \(Ax^2 = Bx - C\), from the equation that solves for equilibrium.
\[ \Rightarrow 2C < Bx \]
\[ \Rightarrow 4AC < (2Ax)B \]
Take \( x^+ = \frac{B + \sqrt{B^2 - 4AC}}{2A} \);

Then, the following has to be valid for \( x^+ \) to be a maximum,
\[ 4AC < (2Ax^+)B \]
\[ 4AC < (B + \sqrt{B^2 - 4AC})B \]
\[ 4AC < (B + \sqrt{B^2 - 4AC})B \]
\[ B^2 - 4AC + B\sqrt{B^2 - 4AC} > 0 \], which is valid. Therefore, \( x^+ \) is a maximum.

Take \( x^- = \frac{B - \sqrt{B^2 - 4AC}}{2A} \);

Then, for \( x^- \) to be a maximum,
\[ 4AC < B^2 - B\sqrt{B^2 - 4AC} \]
\[ B\sqrt{B^2 - 4AC} < B^2 - 4AC \] or, \( B < \sqrt{B^2 - 4AC} \), which is not valid.

Therefore \( x^- \) is not an equilibrium. Hence, there is an unique equilibrium platform given by,

\[ x^e = \frac{1}{8(\alpha + 1)}[(4\alpha(1 - M) - b) + \sqrt{(4\alpha(1 - M) - b)^2 - 16b(\alpha + 1)M}] \]

Let \( \phi(\alpha, M(\gamma, \beta), b; x) = 4(\alpha + 1)x^2 - (4\alpha(1 - M) - b)x + bM \). Then, the following holds: \( \frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x) = 0 \).

Using Implicit function theorem,
\[ \frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x) = 8x(1 + \alpha) - (4\alpha - b - 4\alpha M) \]
But \( x^e = \frac{1}{8(\alpha + 1)}[(4\alpha(1 - M) - b) + \sqrt{(4\alpha(1 - M) - b)^2 - 16b(\alpha + 1)M}] \)

Substituting the equilibrium \( x \) expression into above equation and cancelling terms, we get,
\[ \frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x) = \sqrt{(4\alpha(1 - M) - b)^2 - 16b(\alpha + 1)M}] \]
When we impose the regularity condition of real roots, and this establishes the claim that \( \frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x) > 0 \)

Continuing with the earlier results and applying implicit function theorem,
\[
\frac{\partial x}{\partial b} = -\frac{\frac{\partial}{\partial b} \phi(\alpha, M(\gamma, \beta), b; x)}{\frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x)} = -\frac{\frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x)}{(+ve)}
\]

Where \( \frac{\partial}{\partial b} \phi(\alpha, M(\gamma, \beta), b; x) = x + M > 0 \). Therefore, \( \frac{\partial x}{\partial b} < 0 \).

\[
\frac{\partial x}{\partial \alpha} = -\frac{\frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x)}{\frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x)} = -\frac{\frac{\partial}{\partial x} \phi(\alpha, M(\gamma, \beta), b; x)}{(+ve)}
\]

The sign of \( \frac{\partial x}{\partial \alpha} \) is the opposite of \( \frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x) \). Consider the expression \( \frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x) \),
\[
\frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x) = 8x(x - 1 + M)
\]
\[
\frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x) < 0 \text{ iff } x < 1 - M
\]

When \( b = 0 \), the equilibrium platforms are always such that \( x = \frac{1}{(\alpha+1)} \frac{4\alpha}{1 - M} < (1 - M) \). But, given that \( \frac{\partial x}{\partial b} < 0 \), it is straightforward that with responsible candidates (\( \beta > 0 \)), the equilibrium platforms are going to be even more convergent and since \( M < 1 \) and \( x \) is always non-negative, the inequality \( x < 1 - M \) always holds. Therefore, \( \frac{\partial}{\partial \alpha} \phi(\alpha, M(\gamma, \beta), b; x) < 0 \), and hence, \( \frac{\partial x}{\partial \alpha} > 0 \).

This concludes proof of Proposition 1.

A.4. **Proof of Proposition 2.** \( \frac{\partial x}{\partial \beta} = \frac{\partial}{\partial \beta} \phi(\alpha, M(\gamma, \beta), b; x) = \frac{\partial \phi(\alpha, M(\gamma, \beta), b; x)}{\partial M} \frac{\partial M(\gamma, \beta)}{\partial \beta} \)

Take the expression \( \frac{\partial}{\partial \beta} \phi(\alpha, M(\gamma, \beta), b; x) = \frac{\partial \phi(\alpha, M(\gamma, \beta), b; x)}{\partial M} \frac{\partial M(\gamma, \beta)}{\partial \beta} \)

But,
\[
\frac{\partial \phi(\alpha, M(\gamma, \beta), b; x)}{\partial M} = b + 4x\alpha > 0
\]
\[
\Rightarrow \frac{\partial x}{\partial \beta} = -\frac{\frac{\partial \phi(\alpha, M(\gamma, \beta), b; x)}{\partial M}}{\frac{\partial M(\gamma, \beta)}{\partial \beta}} = -\left( \frac{(+ve)}{(+ve)} \right) \frac{\partial M(\gamma, \beta)}{\partial \beta}
\]

Therefore the sign of the expression \( \frac{\partial x}{\partial \beta} \) depends critically on the sign of \( \frac{\partial M(\gamma, \beta)}{\partial \beta} \).

\[
\frac{\partial M}{\partial \beta} = \frac{1}{(\gamma - 1)^2 \beta^2} \left( \frac{\beta}{\gamma} \right)^{\frac{1}{\gamma-1}} (2 - \gamma) > 0, \text{ if } 1 < \gamma < 2
\]
\[
\frac{\partial M}{\partial \beta} = \frac{1}{(\gamma - 1)^2 \beta^2} \left( \frac{\beta}{\gamma} \right)^{\frac{1}{\gamma-1}} (2 - \gamma) < 0, \text{ if } \gamma > 2
\]
This implies that whenever $\gamma \in (1, 2)$, $\frac{\partial x}{\partial \beta} < 0$. This means that equilibrium platforms tend to move closer when activism becomes more extreme. Similarly, when $\gamma \in (2, \infty)$, $\frac{\partial x}{\partial \beta} > 0$. This implies that for any value of the parameter $\gamma > 2$, we observe divergence in platforms when activism is more extreme. When $\gamma = 2$, the expression $M(\gamma, \beta) = \frac{1}{(\gamma-1)\beta}(\frac{1}{\gamma}) = \frac{1}{2}$. Therefore the equilibrium platform is,

$$x^* = \frac{1}{8(\alpha + 1)}[(2\alpha - b) + \sqrt{(2\alpha - b)^2 - 8b(\alpha + 1)}]$$

This expression is independent of $\beta$. This means that when costs of contribution are also quadratic, equilibrium platforms reach a steady state value and are not affected by any change in $\beta$ parameter. This completes proof of Proposition 2.

A.5. **Proof of Proposition 3.** The solution to the electoral model is solved backwards, as previously.

As before, we present the results for candidate (and activist) $R$.

The median voter observes the platforms with a noise: $\tilde{X}_L = X_L + \eta_L$, $\tilde{X}_R = X_R + \eta_R$.

Voter prefers candidate $L$ if, $E[-(\tilde{X}_L - \mu)^2] \geq E[-(\tilde{X}_R - \mu)^2]$

$\iff E[(X_L - \mu)^2 + \eta_L^2 + 2\eta_L(X_L - \mu)] \leq E[(X_R - \mu)^2 + \eta_R^2 + 2\eta_R(X_R - \mu)]$

$\iff E(\eta_L^2) - E(\eta_R^2) \leq (X_R - \mu)^2 - (X_L - \mu)^2$

$\iff a(c_L) - a(c_R) \leq (X_R - \mu)^2 - (X_L - \mu)^2$

$\mu \leq \frac{(X_R + X_L)}{2} + \frac{a(c_R) - a(c_L)}{2(X_R - X_L)}$

The win-probability for Candidate $L$ is $\lambda = \frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{a(c_R) - a(c_L)}{4(X_R - X_L)}$. Working backwards, the activist $R$ faces the following expected utility,

$$EU_{R}^{A}(\beta, c_R) = -\lambda(X_L - \beta)^2 - (1 - \lambda)(X_R - \beta)^2 - c_{R}^\gamma$$

$$= -\frac{1}{2} + \frac{(X_R + X_L)}{4} + \frac{a(c_R) - a(c_L)}{4(X_R - X_L)}(X_L - \beta)^2 - (1 - \frac{1}{2} + \frac{(X_R + X_L)}{4} - \frac{a(c_R) - a(c_L)}{4(X_R - X_L)})(X_R - \beta)^2 - c_{R}^\gamma$$

Solving the FOC gives,

$$\frac{\partial EU_{R}^{A}(\beta, c_R)}{\partial c_R} = 0 \Rightarrow \lambda'_{c_R}[(X_R - \beta)^2 - (X_L - \beta)^2] = \gamma c_{R}^{\gamma - 1}$$

$$\lambda'_{c_R} = \frac{a'(c_R)}{4(X_R - X_L)} \Rightarrow \frac{a'(c_R)}{4(X_R - X_L)}[(X_R^2 - X_L^2) - 2\beta(X_R - X_L)] = \gamma c_{R}^{\gamma - 1}$$
The equilibrium condition for mobilization is

\[(A.3) \quad \frac{c_{R}^{\gamma-1}}{a'(c_R)} = \frac{[X_R+X_L-2\beta]}{4\gamma}\]

By symmetry, the equivalent condition for activist \(L\) is,

\[(A.4a) \quad \frac{c_{L}^{\gamma-1}}{a'(c_L)} = -\frac{[X_R+X_L+2\beta]}{4\gamma}\]

The candidate \(R\) solves the following expected utility,

\[EU_{R}^{c}(X_L, X_R) = -\lambda(X_L-\alpha)^2 - (1-\lambda)(X_R-\alpha)^2 + (1-\lambda)b\]

\[
\frac{dEU_{R}^{c}(-x,X_R)}{dX_R} = \lambda'_{X_R}[(X_R-\alpha)^2 - (x+\alpha)^2 - b] - 2(1-\lambda)(X_R-\alpha) = 0
\]

where, \(\lambda = \frac{1}{2} + \frac{(X_R+X_L)}{4} + \frac{a(c_R)}{4(X_R-X_L)} - \frac{a(c_L)}{4(X_R-X_L)}\)

This implies \(\lambda'_{X_R} = \frac{3}{4} + \frac{(X_R-X_L)}{4}a'(c_R) - \frac{a'(c_L)}{4(X_R-X_L)} - \frac{a(c_R)}{4(X_R-X_L)}\)

\[\lambda'_{X_R} = \frac{1}{4} [1 + \frac{a(c_L)-a(c_R)}{(X_R-X_L)^2} + \frac{a'(c_R)-a'(c_L)}{(X_R-X_L)^2}]\]

At the symmetric equilibrium \((-x,x)\), equations A.3 and A.4a reduce to \(\frac{c_{R}^{\gamma-1}}{a'(c_R)} = -\frac{\beta}{2\gamma}\) and \(\frac{c_{L}^{\gamma-1}}{a'(c_L)} = -\frac{\beta}{2\gamma}\), respectively. The functional assumptions on \(a(\cdot)\) imply that in the symmetric equilibrium, activist mobilizations are equal: \(c_L = c_R = c\). The equation that solves for the symmetric equilibrium \((-x,x)\) is,

\[\lambda'_{X_R}[-4\alpha x - b] + (\alpha - x) = 0\]

Substituting for \(\lambda'_{X_R}\) and simplifying yields,

\[4(\alpha+1)x^2 - (4\alpha - b - \frac{2a\alpha a'}{\beta[\frac{a}{\alpha}-(\gamma-1)]})x + \frac{c\alpha a'}{\beta[\frac{a}{\alpha}-(\gamma-1)]} = 0\]

Let \(M(c(\gamma, \beta), \gamma, \beta) = \frac{c\alpha a'}{\beta[\frac{a}{\alpha}-(\gamma-1)]}\approx M > 0\). Then, the equation that solves for the symmetric equilibrium is given by the following:

\[(A.5) \quad 4(\alpha+1)x^2 - (4\alpha - b - 2\alpha M)x + M\frac{b}{2} = 0\]

\[(A.6) \quad \frac{c^{\gamma-1}}{a'(c)} = -\frac{\beta}{2\gamma}\]
Together, the above two equations determine the symmetric equilibrium platform of candidates, and mobilization by activists. The argument for existence and uniqueness of the symmetric equilibrium follows from Bernhardt et al. [13]. Proceeding, the dependence of equilibrium platform \( x \) on \( \alpha \) and \( b \) are along the lines of Proposition 1.

Equilibrium analysis wrt \( \alpha, b \):

Let \( \Phi(x; c(\gamma, \beta), \alpha, b, \gamma, \beta) = 4(\alpha + 1)x^2 - (4\alpha - b - 2\alpha M)x + \frac{Mb}{2} \)

Using Implicit function theorem,

\[
\frac{\partial x}{\partial \alpha} = -\frac{\frac{\partial \Phi}{\partial \alpha}}{\frac{\partial \Phi}{\partial x}} = -\frac{4x^2 - (4-2M)x}{8(\alpha+1)x-(4\alpha-b-2\alpha M)}
\]

Multiply and divide the numerator by \( \alpha \):

\[
\frac{\partial x}{\partial \alpha} = -\frac{1}{\alpha} \frac{4\alpha x^2 - (4\alpha - 2\alpha M)x}{8(\alpha+1)x-(4\alpha-b-2\alpha M)}
\]

Re-arranging the numerator from equation A.5,

\[
4\alpha x^2 - (4\alpha - 2\alpha M)x + \frac{Mb}{2} + 4x^2 + bx = 0
\]

This straightaway implies that \( 4\alpha x^2 - (4\alpha - 2\alpha M)x = -(\frac{Mb}{2} + 4x^2 + bx) < 0 \)

\[
\Rightarrow \frac{\partial x}{\partial \alpha} = -\frac{1}{\alpha} \frac{8(\alpha+1)x-(4\alpha-b-2\alpha M)}{8(\alpha+1)Mb_2(\alpha+1)} < 0
\]

The denominator is greater than zero iff,

\[
8(\alpha + 1)x - (4\alpha - b - 2\alpha M) > 0 \text{ or if } x > \frac{(4\alpha-b-2\alpha M)}{8(\alpha+1)}.
\]

To check if this is satisfied, we take the root of the equation,

\[
x = \frac{(4\alpha-b-2\alpha M) + \sqrt{(4\alpha-b-2\alpha M)^2 - 8(\alpha+1)Mb}}{8(\alpha+1)}
\]

The regularity condition \( \sqrt{(4\alpha-b-2\alpha M)^2 - 8(\alpha+1)Mb} > 0 \) implies,

\[
x = \frac{(4\alpha-b-2\alpha M)}{8(\alpha+1)} + \frac{\sqrt{(4\alpha-b-2\alpha M)^2 - 8(\alpha+1)Mb}}{8(\alpha+1)} > \frac{(4\alpha-b-2\alpha M)}{8(\alpha+1)}
\]

Therefore, \( \frac{\partial x}{\partial \alpha} = \frac{-ve + ve}{+ve} > 0 \)

By similar argument,

\[
\frac{\partial x}{\partial b} = -\frac{\frac{\partial \Phi}{\partial b}}{\frac{\partial \Phi}{\partial x}} = -\frac{8(\alpha+1)x-(4\alpha-b-2\alpha M)}{\alpha+1} < 0
\]

We now show that \( \frac{\partial x}{\partial \beta} > 0 \):

\[
\frac{\partial x}{\partial \beta} = -\frac{\frac{\partial \Phi}{\partial \beta}}{\frac{\partial \Phi}{\partial x}} = \frac{\frac{\partial \Phi}{\partial \beta}}{+ve}
\]

Therefore, the sign of \( \frac{\partial x}{\partial \beta} \) depends crucially on \( \frac{\partial \Phi}{\partial \beta} \).
Before proceeding further, it is useful to recollect the elasticity and efficiency parameters, since their interaction determines the sign of \( \frac{\partial \Phi}{\partial \beta} \). Recall that, \( e_m = (\gamma - 1) \) and \( e_n = -c \frac{a''}{a} \). Substituting the expression into \( M(c(\gamma, \beta), \gamma, \beta) \),
\[
M(c(\gamma, \beta), \gamma, \beta) = \frac{c.a'}{\beta(\frac{c-a}{a}-(\gamma-1))} = \frac{2\gamma c'}{\beta^2(e_m+e_n)^2},
\] from equation A.6, \( e_m \) and \( e_n \), we obtain:
\[
\frac{\partial M}{\partial \beta} + \frac{\partial M}{\partial c} \frac{\partial c}{\partial \beta} = -\frac{4\gamma c'}{\beta^2(e_m+e_n)} + \frac{\partial c}{\partial \beta} \frac{2\gamma c'}{\beta^2(e_m+e_n)}.
\]

Applying Implicit function theorem to equation A.6, we get \( \frac{\partial c}{\partial \beta} = -\frac{\frac{2\gamma c'}{\beta^2}}{a'(\frac{c-a}{a}-(\gamma-1))} \). Since \( c = -\frac{2\gamma c'}{\beta^2 a} \), the expression \( \beta \frac{\partial c}{\partial \beta} \) depends on the expression \(-2+\frac{e_m}{e_m+e_n}\), which is greater than zero when \( e_n < \frac{1-e_m}{2} \). When this is satisfied, \( \frac{\partial \Phi}{\partial \beta} > 0 \) and further, \( \frac{\partial x}{\partial \beta} < 0 \). When the sign of this inequality is reversed, that is \( e_n > \frac{1-e_m}{2} \), it is trivial to observe that \( \frac{\partial x}{\partial \beta} > 0 \). This concludes the proof of Proposition 3.

A.6. Proof of Proposition 4. Solving the model backwards is identical to the previous subsections.

Proceeding as before, the final equation that solves for the equilibrium platform is given by,
\[
4(\alpha + 1)x^2 - (4\alpha(1-M) - b)x + Mb = 0
\]
where \( M = \sum_{p=1}^{N} \frac{1}{(1-(p-1)\varepsilon)^{\gamma-2}/(\gamma-1)\gamma^{\gamma-2}} \).

We restrict our attention to \( \gamma \) such that \( M < 1 \), and the equilibrium platform \( x \) is greater than zero. Whenever \( x < 0 \), we impose \( x = 0 \), without loss of generality. Note that as \( M \) increases, \( x \) decreases.

Therefore, to analyze the effect of changing \( \varepsilon \) or \( N \), we only need to study their relationship to \( M \).
\[
M = \sum_{p=1}^{N} \frac{1}{(1-(p-1)\varepsilon)^{\gamma-2}/(\gamma-1)\gamma^{\gamma-2}} = \frac{1}{(\gamma-1)\gamma^{\gamma-2}} \left[ 1 + \frac{1}{(1-\varepsilon)^{\gamma-2}} + \frac{1}{(1-2\varepsilon)^{\gamma-2}} + \ldots + \frac{1}{(1-(N-1)\varepsilon)^{\gamma-2}} \right]
\]

\[
\frac{\partial}{\partial \varepsilon} \frac{1}{(1-(p-1)\varepsilon)^{\gamma-2}/(\gamma-1)\gamma^{\gamma-2}} = \frac{1}{\gamma-1} (\gamma - 2) \frac{p-1}{(\varepsilon-p\varepsilon+1)^{\gamma-1}(2\gamma-3)} > 0 \text{ for all } \gamma > 2.
\]

This implies that \( \frac{\partial M}{\partial \varepsilon} > 0 \) and since \( M \) and \( x \) are inversely related, \( \frac{\partial x}{\partial \varepsilon} < 0 \). This proves the first part of the proposition.
To prove the second statement of the proposition, let's assume that the initial ideological band is \((N - 1)\epsilon\). Suppose we increase \(N\) to \(N'\), by decreasing \(\epsilon\) to \(\epsilon'\): that is, \(N' > N, \epsilon' < \epsilon\) such that the equality \((N - 1)\epsilon = (N' - 1)\epsilon'\) is satisfied.

The corresponding \(M' = \frac{1}{(\gamma - 1)^{1/\gamma}} \left[ 1 + \frac{1}{(1 - \epsilon')^{\frac{\gamma-2}{\gamma-1}}} + \frac{1}{(1 - 2\epsilon')^{\frac{\gamma-2}{\gamma-1}}} + \ldots + \frac{1}{(1 - (N' - 1)\epsilon')^{\frac{\gamma-2}{\gamma-1}}} \right]\). Now take the following expressions that are part of \(M\) and \(M'\),

(A.7) \[\Psi(N, \epsilon) = \left[ 1 + \frac{1}{(1 - \epsilon)^{\frac{\gamma-2}{\gamma-1}}} + \frac{1}{(1 - 2\epsilon)^{\frac{\gamma-2}{\gamma-1}}} + \ldots + \frac{1}{(1 - (N - 1)\epsilon)^{\frac{\gamma-2}{\gamma-1}}} \right]\]

(A.8) \[\Psi'(N', \epsilon') = \left[ 1 + \frac{1}{(1 - \epsilon')^{\frac{\gamma-2}{\gamma-1}}} + \frac{1}{(1 - 2\epsilon')^{\frac{\gamma-2}{\gamma-1}}} + \ldots + \frac{1}{(1 - (N' - 1)\epsilon')^{\frac{\gamma-2}{\gamma-1}}} \right]\]

We will proceed to establish that \(\Psi'(N', \epsilon') > \Psi(N, \epsilon)\). Both the above equations have \(N\) terms each, and the first and last \((N_{th})\) terms are equal. Compare the \((N - 1)th\) term of equation A.7 with the corresponding \((N' - 1)th\) term of equation A.8. Since activists are equidistant, \(\frac{\gamma-2}{\gamma-1} > 1\) and \(\epsilon' < \epsilon\), it is true that \(\frac{1}{(1 - (N' - 2)\epsilon')^{\frac{\gamma-2}{\gamma-1}}} > \frac{1}{(1 - (N - 2)\epsilon)^{\frac{\gamma-2}{\gamma-1}}}\). Repeat the procedure, similarly, comparing terms \((N - 2)\) and \((N' - 2)\), \((N - 3)\) and \((N' - 3)\), and so on. For every such comparison, the terms belonging to equation A.8 are greater than those belonging to equation A.7. This implies that \(\Psi'(N', \epsilon') > \Psi(N, \epsilon)\), and ipso facto, \(M' > M\). This further implies that the equilibrium platforms in the case of \((N', \epsilon')\) is less polarized. This proves the second part.

References


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