Informal Insurance and Endogenous Poverty Traps

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Abstract

Households in developing countries appear to invest too little in ‘lumpy’ capital goods, which from asset transfer programs are observed to have high returns. To understand why households are not able to use their own resources towards such investment, I construct a dynamic risk sharing model which incorporates limited commitment and lumpy investment. These two features interact to produce four key results: at low levels of initial capital there is a poverty trap; valuable risk sharing can crowd out lumpy investment; inequality has an inverse-U effect on growth; and larger risk-sharing groups will find it easier to invest. I use data from an asset transfer program in Bangladesh to test the predictions of the model. I then estimate structurally the parameters of the model, and I use this to understand how much the inability to commit reduces household welfare, and to quantify the gains from restructuring the asset transfer program to take into account the spillover effects through the poverty trap. The results of this paper have important implications for development policy more generally: whilst it is already known that the provision of public insurance can crowd out private insurance, it has not previously been recognised that this reduction in private insurance can be partly beneficial by making investment easier.

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1 Introduction

An old literature going back to Rosenstein-Rodan (1943) suggests that the failure of poor economies to develop comes from coordination failures, where multiple simultaneous investments could be profitable, but alone none of these investments will be.\(^1\) However, there are many investments which are profitable even without others’ investment, and yet do not take place. For example the purchase of small capital good such as livestock is typically profitable, and yet most households are unable to buy these (Bandiera et al., 2016; Banerjee et al., 2015; see also de Mel et al., 2008 for similar results in the context of small businesses). Poorly functioning credit markets can explain why households are unable to borrow to purchase livestock, but this raises a question: why can’t they use their own resources? The observed returns are often too high to be consistent with standard discounting, given the estimated discount rates. A recent literature (Banerjee and Mullainathan, 2010; Bernheim et al., 2015) suggests instead that temptation and self-control issues can explain the lack of saving towards investment. In this paper I develop an alternative explanation: informal insurance can, in certain circumstances, ‘crowd out’ lumpy investment.

I develop a dynamic risk sharing model which incorporates limited commitment and lumpy investment. Whilst both these features have been considered separately, and are known to be important in explaining the decisions of households in developing countries, I show that there are important interactions between them. With limited commitment, insurance arrangements are endogenously incomplete: the value of autarky restricts the ability of households to completely smooth shocks (Kocherlakota, 1996; Ligon et al., 2002). Adding the option to invest weakly improves households’ outside options, reducing the extent of insurance. Conversely, the presence of insurance affects incentives to invest, as insurance transfers provide an alternative means of smoothing shocks.\(^2\)

Analysis of the model provides three main findings. First, I show that a ‘poverty trap’ naturally arises in this model: the long run equilibrium income distribution depends on the

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\(^1\)For a more modern treatment of this model, see Murphy et al. (1989).

\(^2\)Investment allows the smoothing of shocks by transferring resources from periods where income is high to future periods, while insurance allows smoothing against idiosyncratic shocks across states of the world. Other work studying the interaction between risk sharing and some alternative form of smoothing include Attanasio and Rios-Rull (2000), Attanasio and Pavoni (2011), Krueger and Perri (2011), Ábrahám and Laczó (2014), and Morten (2015).
initial level of capital invested. Second, I demonstrate how limited commitment can increase or reduces investment, relative to a full commitment benchmark. Third, I provide testable comparative static predictions on how income inequality and group size affect investment. I next test the predictions of the model empirically, using an asset transfer program in Bangladesh to show empirical evidence and to provide support for the predictions of the model. I find that in the context of rural Bangladesh, when the asset transfer program makes transfers to more than eight households in a village, it allows additional investment to start taking place, allowing the village economy to grow faster and escape the poverty trap. Finally, in ongoing work I estimate the model structurally, and use it to answer two questions. I first study how costly is limited commitment in reducing income growth and welfare, relative to a full commitment benchmark. I then use the model to determine how best to target the asset transfer program, given the spillover effects that occur via risk sharing, and to quantify the welfare gain from the restructuring.

I show that there is a threshold level of aggregate income needed in order for any investment to take place, given the level of development, and I show that this threshold is declining in the level of development (existing level of capital). Consequently, at low levels of development the village economy remains trapped: aggregate income will never exceed the threshold needed for even a single investment. However, with only a ‘small push’ that provides some initial capital, the economy can be set on a path of further investment and income growth. Unlike so-called ‘big push’ models (Rosenstein-Rodan, 1943, Murphy et al., 1989), here it is not necessary for all households to be simultaneously coordinated in investment, nor is coordination alone – without the provision of assets – sufficient to generate further investment.

Next I show that the effect of limited commitment on the number of investments depends on properties of the income distribution. If incomes are highly positively correlated, then insurance allows households to pool their incomes to make investments. Limited commitment weakens the ability of households to commit to making repayments, reducing the number of investments. Conversely, if aggregate income is close to constant, then insurance is highly valuable as most of

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3There is a sequence of thresholds, one for each possible level of investment that is possible currently. I focus on the lowest threshold, needed for any investment at all to occur, as a decline in this makes it more likely that any investment can take place this period. Then, since the stock of capital has risen, this reduces the threshold further making investment easier in the future, if all else is equal. However, as I note below, once some households invest, this also changes the distribution of income in the economy, which will have additional effects under limited commitment.
the income shock is idiosyncratic. Limited commitment reduces the available level of insurance, increasing the demand for investment as an alternative hedge.

I then derive further comparative static results, showing how the threshold level of income varies with other parameters of the model. In particular I study the effect of inequality in income, inequality in consumption, and group size on the income threshold. Under full commitment the distribution of income would have no bearing, and the distribution of consumption (in particular the ratio of marginal utilities across households) would remain fixed in all periods. With limited commitment, a rise in inequality has an inverse-U shaped effect on investment. When the income distribution is relatively equal, a mean preserving spread first increases the amount of additional investment, as some currently richer households’ incentive constraints will begin to bind. For those households, complete smoothing with other households is no longer incentive compatible, as they cannot expect a large enough compensation for it in the future. But if the rich households’ incomes are high enough, investment becomes attractive as an additional way to smooth. However, if there is a limit to the total amount of investment such households can manage, for example a maximum number of livestock they can manage (and if they are not able to hire others to manage the livestock for them, perhaps due to moral hazard issues), then further increases in inequality that come from increasing the share of income to these households do not result in additional investment. Eventually, as inequality is increased even by concentrating income further within a subset of the rich households, the remaining richer households are also unable to afford to invest, reducing total investment, giving the inverse-U shape. A similar result holds for inequality in consumption.

Increasing group size, holding mean income constant, increases the mean and variance of total income, whilst lowering the variance of mean income.\(^4\) The latter increases the value of the insurance arrangement, reducing the incentive to move to autarky and thus making investment easier, as future repayments become more credible. Simultaneously, the former effect also makes investment easier, even under full commitment, by making it more likely that aggregate income exceeds the threshold needed for investment.

The key prediction of the model is that there should be a non-linear relationship between the existing level of development and further investment. In particular, at low levels of existing

\(^4\)This assumes any additional households’ incomes are neither perfectly positively or negatively correlated with the existing aggregate income.
capital, additional investment should be flat and close to zero. Once the capital stock is above some unknown threshold, additional investment should be increasing in the level of existing capital. I test this prediction using data from an asset transfer program in rural Bangladesh conducted by the NGO BRAC. The program made asset transfers to eligible households in a random subset of villages. I test formally the prediction of the model using econometric methods developed for identifying the location and significance of an unknown threshold point, and find evidence of a significant threshold at eight households. Hence, on average, once more than eight transfers are provided to a village, this is enough to allow households in the village to begin their own investment, leading to strong positive growth in future income and additional investment. I also verify that the comparative static predictions of the model hold, providing further supportive evidence for the model.

This paper contributes to four areas of literature. Firstly, it contributes directly to the recent and growing work on asset transfer programs (Bandiera et al., 2016; Banerjee et al., 2015; de Mel et al., 2008; Morduch et al., 2015). These papers finds that in many (though not all) cases, across a range of countries and contexts, asset transfer programs are very successful in increasing incomes. My paper provides a possible explanation for why such one-off transfers of assets appear to have larger effects on income growth than smaller, but longer term, cash transfer programs such as Progresa. Small increases in income will still be partly smoothed away, rather than providing the basis needed for additional investment. They will still be important in improving consumption, which determines current welfare, but they will help less with achieving the household income growth that is essential for long term welfare improvements.

Secondly, I contribute to the large literature on poverty traps. It is well-known that non-convexity in production, when combined with some financial friction, can produce a poverty trap (see for example Banerjee and Newman, 1993; and Aghion and Bolton, 1997). The novel aspect of my model is that it introduces a financial friction (limited commitment), and then solves for the optimal contract. This creates a trade-off between the value of insurance – which allows households to smooth consumption across states – and the value of investment – which improves long term income and consumption. Suspending the insurance arrangement would reduce expected household welfare, but might increase investment, highlighting the key result: insurance with limited commitment can crowd out investment. This result is distinct from the

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pure coordination failure result of Rosenstein-Rodan (1943) and Murphy et al. (1989), since households lack the real resources needed to invest, rather than simply failing to coordinate. However, if improvements could be made to the contract enforcement technology, then this alone could bring some economies out of the poverty trap.\(^5\)

A third literature to which I contribute is the literature on risk sharing with frictions (Kocherlakota, 1996; Ligon et al., 2002; among others). In particular, there is a growing literature examining how endogenously incomplete insurance affects and is affected by opportunities in other markets (Attanasio and Rios-Rull, 2000; Attanasio and Pavoni, 2011; Ábrahám and Cárceles-Poveda, 2009; Ábrahám and Laczo, 2014; Morten, 2015). Attanasio and Pavoni (2011) highlight an important trade-off between using insurance and using (continuous) investment to provide consumption smoothing. A similar trade-off is present in my model, but the ‘lumpiness’ of investment in my context (mirrored by many development applications) changes the nature of the decision-making, and creates the possibility of a poverty trap. Morten (2015) also considers a model with risk sharing and a binary decision, but in her case the decision only directly affects payoffs today (and any indirect effect is wrapped into the households’ promised utilities). By contrast, in my model investment has permanent effects on the distribution of income, allowing me to study questions of longer term development and growth. Also, an important result from this literature is that the value of public insurance programs is overstated if one doesn’t consider the implications this has for the private insurance market. This paper suggests that whilst this is true, the crowding out of private insurance may be less costly than one might first think, because this allows more investment to take place.

Finally, I contribute to the literature on the impact of ‘networks’ and network taxes on decision-making in developing countries (see for example Anderson and Baland, 2002; Baland et al., 2011; Attanasio et al., 2012; and Jakiela and Ozier, 2016). Jakiela and Ozier (2016) in particular describe how individuals make decisions that lower their expected income (in a lab-in-the-field experiment) in order to hide that income when other network members are present. They suggest this is indicative of a network tax. This explanation is consistent with my findings: households engage in the provision of mutually beneficial informal insurance, but this arrangement distorts investment decisions.

\(^5\)In practice, making contract enforcement cheap and accessible is likely to be more expensive than simply making asset transfers, although contract enforcement is likely to also benefit other areas of economic activity.
The next Section develops the model formally, and provides the theoretical results. Section 3 tests a number of key predictions of the model. Section 4 discusses the structural estimation of the model, while Section 5 assesses the implications of these estimates by performing counterfactual simulations. The final Section concludes.

2 A Model of Insurance, Investment, and an Endogenous Poverty Trap

Consider an infinite-horizon village economy composed of \( N \) households. Households have log preferences over their consumption, and an common discount rate, \( \beta \). Each period the village experiences one of finitely many events \( s \in S \). The probability of event \( s \) is denoted \( \pi(s) \), and events are independent over time.

Each period household \( j \in \{1,\ldots,N\} \) receives endowment income \( y^j = y^j_s \) in state \( s \).\(^6\) Borrowing and saving are not possible, however households may invest in a binary investment \( I \) at cost \( d \) today, which pays a guaranteed return of \( R \) in all future periods. Investment is an absorbing state, and households may hold at most one investment. Hence, in the absence of any insurance arrangement, the only choice for non-invested households is, given their realised income that period, whether or not to invest. Households who already had an investment have total income \( y^j_s + R \) at the end of the current period, and consume this in its entirety.

Once risk sharing is possible, households have two decisions to make on receiving their total income \( y^j_s + \mathbb{P}R \): how much (if anything) to transfer to other households \( \tau^j \), and (if they have not already invested) whether or not to invest.\(^7\) After they have made and received any transfers, and done any desired investment, they consume the cash-on-hand they hold. The same problem, with updated investment state, occurs next period.

I define \( Y_s := \sum_{j=1}^N y^j_s \) as the aggregate endowment income across the \( N \) households, and \( t^i_s := y^i_s / Y_s \) as household \( i \)'s share of aggregate endowment income in state of the world \( s \).

\(^6\)It will later be useful to think about the set of households \( \{2,\ldots,N\} \) as a group. For clarity, when I am indexing over all households I will use the index \( j \), and when I am excluding household 1 I use the index \( i \).

\(^7\)Whilst in principle each household could choose how much to transfer to each other household, I model all transfers as going to or from household 1 ("the planner").
2.1 Risk Sharing under Limited Commitment without Investment

I first consider the limited commitment problem when all households have already invested. In this case there is no investment decision to make, and the problem has the same form as Ligon et al. (2002). As noted above, I model the problem as though a single household (WLOG household 1, ‘the planner’) makes and receives all the insurance transfers. Other households are willing to make transfers to the planner in exchange for being promised a certain level of expected utility in the future.

Let $V(\omega, 1)$ denote the planner’s value function when it has promised utility $\omega = \{\omega_2^s, \ldots, \omega_N^s\}$ to the other households in realised state of the world $s$, and when all households already have one unit of capital, $\mathbb{1} = 1$. The planners’ problem is to choose transfers today and contingent utility promises (for all possible future states of the world $r$) $\{\tau^i_s, \omega^i_r\}_{i=2}^N$ to maximise:

$$u \left( t_s^1 Y_s + R + \sum_{i=2}^N \tau^i_s \right) + \beta \mathbb{E} \left[ V(\omega', 1) \right]$$

s.t.

$$\lambda_i \left[ u(t_i^s Y_s + R - \tau_i^s) + \beta \mathbb{E} [\omega_i^r] \right] \geq \omega_s^i \quad \forall i \in \{2, \ldots, N\}$$

$$\phi_i \left[ u(t_i^s Y_s + R + \sum_{i=2}^N \tau^i_s) + \beta \mathbb{E} [V(\omega'_r, 1)] \right] \geq u(t_i^s Y_s + R) + \frac{\beta}{1 - \beta} \mathbb{E} \left[ u(y_i^r + R) \right]$$

$$\phi_i \left[ u(t_i^s Y_s + R - \tau_i^s) + \beta \mathbb{E} [\omega_i^r] \right] \geq u(t_i^s Y_s + R) + \frac{\beta}{1 - \beta} \mathbb{E} \left[ u(y_i^r + R) \right]$$

where the promise-keeping constraints, indexed by $\lambda_i$, ensure utility promises to households are met; whilst the limited commitment constraints, indexed by $\phi_i$, require that households must be at least as well off within the insurance agreement as they would be if they left it. Henceforth where there is no confusion I will suppress the dependence on $s$ and $r$ for notational clarity. Also to simplify notation I denote by $c^j = t^j Y + \mathbb{1} R - \tau^j \forall j \in \{2, \ldots, N\}$ the consumption of household $j$. 

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Taking FOCs (and using the envelope theorem for \( \omega^i \)), I get (for \( i \in \{2, \ldots, N\} \)):

\[
[\text{FOC}(\tau^i)] \quad \frac{u'(c^1)}{u'(c^i)} = \frac{\lambda^i + \phi^i}{1 + \phi^1}
\]

\[
[\text{FOC}(\omega^i)] \quad \mathbb{E} \left[ \frac{\partial V'}{\partial \omega^i} \right] = -\frac{\lambda^i + \phi^i}{1 + \phi^1}
\]

\[
[\text{ET}(\omega^i)] \quad \frac{\partial V}{\partial \omega^i} = -\lambda^i
\]

Hence

\[
\mathbb{E} \left[ \frac{\partial V'}{\partial \omega^i} \right] = \frac{\partial V}{\partial \omega^i} + \phi^i
\]

From the envelope theorem it can be seen that the value function is decreasing in the promised utility \( \omega^i \) to each household \( i \). When neither the limited commitment constraint for the planner (household 1) or household \( i \) binds, \( \phi^1 = \phi^i = 0 \), the slope of the value function (the ratio of marginal utilities for \( i \) and 1) remains constant. When a household’s LC constraint binds, the ratio of marginal utilities in that period and all future periods (until another constraint binds), is adjusted so that they receive an increased share of consumption.

2.2 Risk Sharing under Limited Commitment with Investment

Next I consider the case where \( K < N \) investments have already been made, \( \sum_{j=1}^{N} I_j = K \). The planner now has to choose the optimal number (and allocation) of investments \( k \in \{1, \ldots, N - K\} \), as well as transfers and utility promises. I now write the planner’s value function, \( V(\omega, I_K) \), as the maximum over a set of conditional value functions each for a different fixed number of investments (where \( I_K 1 := K \)). So

\[
V(\omega, I_K) = \max_{k \in \{0, 1, \ldots, N - K\}} \{ V_k(\omega, I_K) \}
\]
where $V_k$ is the conditional value function when $k$ investments are made. These conditional value functions are given by

$$V_k(\omega, \Pi_K) = \max_{(\tau_k^i, \omega_k^i)_{i=2}^N} u \left( \left( t^iY + \Pi_k^1R - \Delta \Pi_k^1d + \sum_{i=2}^N \tau_k^i \right) + \beta \mathbb{E} \left[ V(\omega_k^i, \Pi_{K+k}^i) \right] \right)$$

subject to

$$\begin{align*}
[\lambda_k^i] & \quad u(t^iY + \Pi_k^1R - \Delta \Pi_k^1d - \tau_k^i) + \beta \mathbb{E} \left[ \omega_k^i \right] \geq \omega^i \\
[\phi_k^i] & \quad u(t^iY + \Pi_k^1R - \Delta \Pi_k^1d + \sum_{i=2}^N \tau_k^i) + \beta \mathbb{E} \left[ V(\omega_k^i, \Pi_{K+k}^i) \right] \geq \Omega^i(t^iY, \Pi_k^i) \\
[\phi_k^i] & \quad u(t^iY + \Pi_k^1R - \Delta \Pi_k^1d - \tau_k^i) + \beta \mathbb{E} \left[ \omega_k^i \right] \geq \Omega^i(t^iY, \Pi_k^i)
\end{align*}$$

where $i \in \{1, \ldots, N\}$,

$$\Omega^i(t^iY, \Pi) := u(t^iY + \Pi R - \Delta \Pi_{aut}d) + \beta \mathbb{E} \left[ \Omega^i(y^i, \Pi_{aut}) \right]$$

is the best outside option for household $j \in \{1, \ldots, N\}$, and the investment state is updated as:

$$\Pi_{K+k}^i = \Pi_K^i + \Delta \Pi_k^i$$

where

$$\sum_{i=1}^N \Delta \Pi_k^i = k; \quad \Pi_K^i, \Delta \Pi_k^i \in \{0, 1\}$$

The main differences between these conditional value functions and the case without investment are that (i) some households will potentially now invest, adding the $-\Delta \Pi d$ terms to household utility; (ii) this requires the investment state $\Pi$ to be updated; and (iii) the outside option for household $j$ now allows for the option of future investment (if the household has not already invested).

As before I get first order conditions for $i \in \{2, \ldots, N\}$, now conditional on the investment

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8In a full commitment setting it would not matter which households 'held' the investments, since the planner could always require them to make arbitrary transfers. With limited commitment this is no longer the case: if the planner requires too high a transfer, the household may prefer autarky. Hence the planner should assign any investments to minimise the expected transfers the investing households will have to make. In practice this means that if $k$ investments are to occur, they should be performed by the $k$ uninvested households to which the planner must provide the most utility: those with the highest promised utilities or highest outside options (if their LC constraints bind). Those households have the highest expected consumption and therefore, since households draw from the same income distribution, make the lowest expected net transfers.
decision \( (k) \):

\[
\frac{u'(c^1_k)}{u'(c^i_k)} = \frac{\lambda_i + \phi_i}{1 + \phi_i}
\]

\[
E \left[ \frac{\partial V'}{\partial \omega^i_k} \right] = \frac{\lambda_i + \phi_i}{1 + \phi_i}
\]

The first order conditions take the same form as without the investment decision, with the value function decreasing in promised utility, and the ratio of marginal utilities updated when a limited commitment constraint binds. However, the planner’s overall value function is now the upper envelope of the conditional value functions, so the slope of the value function at a point will be the slope of the highest conditional value function at that point, and there will be kinks where value functions cross.

2.3 Poverty Trap

The first result from the model is that it naturally gives rise to the possibility of a poverty trap. I will first show that under full commitment there exists a sequence of aggregate income thresholds \( \hat{Y}_k \), one between each possible level of investment and the level above it, such that if \( \hat{Y}_k < Y < \hat{Y}_{k+1} \) then it will be optimal to make \( k \) investments this period. I will then show that this leads to the possibility of a poverty trap: if an economy never receives a large enough level of aggregate income to reach \( \hat{Y}_1 \) then it will forever remain with the current income distribution (absent external shocks), whilst if an external shock is provided to produce a ‘small push’ then further investment will be able to occur over time.\(^9\)

**Proposition 1.** There exists a unique threshold \( \hat{Y}_k = \hat{Y}_k(\mathbb{I}_K, N) \) such that with full commitment:

1. \( \forall Y < \hat{Y}_k \), the optimal number of investments is no greater than \( k - 1 \);

2. at \( Y = \hat{Y}_k \), \( V_{k-1}(\cdot) = V_k(\cdot) \geq V_{k'}(\cdot) \ \forall k' \) i.e. the planner is indifferent between making \( k - 1 \) and \( k \) investments and does not strictly prefer any other level of investment to these;

\(^9\)In contrast with ‘big push’ models (Rosenstein-Rodan, 1943; Nurkse, 1953; Murphy et al., 1989), here no coordination is needed between agents: an initial push that is large enough to allow one investment to occur will then automatically spillover, allowing further investments.
3. \( \forall Y > \hat{Y}_k \), the optimal number of investments is no fewer than \( k \).

There are \( N - K \) such thresholds, with \( Y_{k-1} < Y_k \), each implicitly defined by \( \Gamma_k(Y_k; \Pi_K, N) \equiv 0 \) where \( \Gamma_k(\cdot) := V_{k-1}(\cdot) - V_k(\cdot) \).

Proof. See Appendix B.

Proposition 1 states in an \( N \) household economy in which \( K = \sum_j I_j \) investments have already been made, there are \( N - K \) income thresholds, whose level depends on the number of existing investments, and the group size. Intuitively, when aggregate income is very low, it will be optimal to consume it all today, potentially after some redistribution. At higher levels of aggregate income, the utility cost of reducing total consumption by \( d \) today (the cost of an investment) is sufficiently low compared with the expected improvement in future utility, so it will become optimal to make one investment. At yet higher levels of aggregate income, additional investments become worthwhile. Group size scales down the cost (and return) of each investment.

This threshold result leads naturally to the possibility of a poverty trap: a situation in which the long run equilibrium of the economy depends on its initial state. When an economy has only a small number of initial investments (low level of capital), the highest possible aggregate income may be lower than \( \hat{Y}_1(\cdot) \), the level needed to make the first additional investment worthwhile. However, at a higher level of initial capital stock the maximum level of aggregate income is higher, allowing further investments to take place in some states of the world.

Lemma 1. The threshold level of income needed to make \( k \) additional investments, \( \hat{Y}_k \), is decreasing in the existing level of capital \( K \), i.e. \( \Delta \hat{Y}/\Delta K < 0 \).

Proof. I first show that under full commitment, the value function \( V_k(\Pi_K) \) has increasing differences in \((k, K)\):

\[
V_{k+1}(\Pi_K) - V_{k+1}(\Pi_{K-1}) > V_k(\Pi_K) - V_k(\Pi_{K-1})
\]

To see this I expand the conditional value functions

\[
V_{k+1}(\Pi_K) - V_{k+1}(\Pi_{K-1}) = u(c_{k+1,K}^1) - u(c_{k+1,K-1}^1) + \beta E[V(\Pi_{K+k+1}) - V(\Pi_{K+k})]
\]
and

\[ V_k(\mathbb{I}_K) - V_k(\mathbb{I}_{K-1}) = u(c^1_{k+1,K}) - u(c^1_{k+1,K-1}) + \beta \mathbb{E}[V(\mathbb{I}_{K+k}) - V(\mathbb{I}_{K+k-1})] \]

Hence the double difference, \([V_{k+1}(\mathbb{I}_K) - V_{k+1}(\mathbb{I}_{K-1})] - [V_k(\mathbb{I}_K) - V_k(\mathbb{I}_{K-1})]\) gives

\[ [u(c^1_{k+1,K}) - u(c^1_{k+1,K-1})] - [u(c^1_{k,K}) - u(c^1_{k,K-1})] + \beta \mathbb{E}[V(K + k + 1) - V(K + k - 1)] \]

Letting \(C := [Y + KR - kd]\) denote aggregate consumption, I note that household consumption is proportion to aggregate consumption. The increase in aggregate consumption when initial capital increases from \(K - 1\) to \(K\) is independent of the number of investments made today, i.e. \(C_{k+1,K} - C_{k+1,K-1} = C_{k,K} - C_{k,K-1} = R\). Then the difference in consumption is the same in both the first and second set of square brackets above, but by concavity of the utility function \(u(\cdot)\) the gain in utility from this increase is higher at lower levels of consumption i.e. when investment is higher. Hence \(u(c^1_{k+1,K}) - u(c^1_{k+1,K-1}) > u(c^1_{k,K}) - u(c^1_{k,K-1}) > 0\), so the first two terms are (together) strictly positive. Since value functions are increasing in the level of capital, the final term is also strictly positive, so the value function exhibits increasing differences in \((k, K)\). Then, since investment and capital are positive integers, \(k, K \in \mathbb{Z}_+\), the set of possible values for \((k, K)\) form a lattice.\(^{10}\) Finally, as \(V_k(\mathbb{I}_K)\) has increasing differences in \((k, K)\), and the set of possible \((k, K)\) form a lattice, \(V_k(\mathbb{I}_K)\) is supermodular in \((k, K)\). Hence by application of Topkis’ Theorem (Topkis, 1978), the optimal choice of \(k\) is non-decreasing in \(K\) i.e. the threshold level of income needed for \(k\) investments to be optimal, \(\hat{Y}_k\), is weakly lower.

Under full commitment the poverty trap result has very stark predictions: there are only two possible long run equilibria, \(\mathbb{I} = 0\) or \(1\). This is because under full commitment only the level of aggregate income matters for whether investment takes place. Suppose there exists a state of the world in which, from a base of zero capital, making at least one investment is optimal for the planner. Then making an investment in the same state of the world, when the same combinations of endowment incomes are realised but when some investments have already

\(^{10}\)A lattice is a partially-ordered set where for any pair of elements in the set, the least upper bound and greatest lower bound of the elements are also in the set. For more details see Milgrom and Shannon (1994).
occurred, must also be optimal (by Lemma 1). Hence either the economy will remain with zero capital or will converge to a state in which all households are invested.

**Limited Commitment**

I next consider how the above results are changed by limited commitment. I first show that limited commitment can change the ‘depth’ of the poverty trap: the threshold level of income needed such that doing some investment becomes optimal in equilibrium. To do this I consider how the investment threshold under autarky compares to that with full commitment. The results under limited commitment will fall somewhere between these, depending on the extent of limited commitment. I then show that with limited commitment, a wider range of equilibrium levels of investment are possible.\(^{11}\) This is important since in practice one observes intermediate levels of investment, which would never be a long run equilibrium with full commitment.

Under autarky, there will be an income threshold \(\hat{y}\) such that if an (uninvested) individual household’s income exceeds \(\hat{y}\) it will invest, else it will not, and all non-investment income will be consumed.\(^{12}\) To see how this threshold compares to \(\hat{Y}_1\), the threshold for a single investment to occur under full commitment, note that a move from autarky to full commitment insurance has two effects.

First, it effectively scales the cost and return of the investment, as these can now be shared across households. Under full commitment, household \(j\) pays only \(\alpha^j d\) per investment, where 
\[
\alpha^j = \alpha(\lambda^j, \lambda^{-j})
\]
is household \(j\)’s share of aggregate consumption. In the limit as \(N \rightarrow \infty\), holding the distribution of individual income constant \(\alpha^j \rightarrow 0\) so collectively investment becomes infinitely divisible, and the problems of ‘lumpiness’ go away. Doing at least one investment (collectively) therefore becomes increasingly attractive relative to zero investments, reducing the threshold level of income needed for a single investment to occur.

Second, it reduces the variance of future consumption. Part of the value of the investment is that it is not perfectly correlated with households’ endowment income, so provides some

---

\(^{11}\)Adding heterogeneity in investment returns to the model would also allow intermediate equilibria, where only some households were invested. However, as I show in the next subsection, limited commitment has implications for how the distribution of income and risk sharing group size should matter for investment, which would not hold in a full commitment model with heterogeneity. I will show that these implications are borne out in the data, so that limited commitment is an important reason why such intermediate cases may be observed, although heterogeneity is certainly also present.

\(^{12}\)To see that such a threshold exists, the same lines of reasoning used in the full commitment case can be replicated. The threshold is implicitly defined as 
\[
u(\hat{y}) + \beta\mathbb{E}[\Omega(g', 0)] = u(\hat{y} - \hat{d}) + \beta\mathbb{E}[\Omega(g', 1)].
\]
partial insurance. Consequently, insurance from other households will reduce the demand for investment relative to current consumption. This effect will increase the threshold level of aggregate income needed for an investment to occur. Hence the overall effect of limited commitment may be to increase or decrease investment relative to full commitment insurance: determining the effect in a particular context will be an empirical question, and this provides one motivation for estimating the model structurally.

To see how the relative magnitudes of these effects depend on the distribution of income, consider two polar cases: the case where all risk is aggregate (household incomes are perfectly positively correlated) and the case where all risk is idiosyncratic (aggregate income is fixed). In the former case, ‘full commitment insurance’ actually provides no insurance at all. However, it does allow households to use transfers to share the costs and returns of investment, so only the first of the above effects exists here. If individual households’ incomes are not already high enough to make investment worthwhile, then pooling income may allow households to invest. In this case it is clear that as soon as limited commitment is introduced, the households are effectively in autarky. Without any idiosyncratic variation in income, there is no value to an invested household in remaining in the arrangement. Hence there is an immediate unravelling, with no households being willing to make transfers that support another household’s investment, since they know that repayment is not credible.

Conversely, when aggregate income is fixed, and all variation is idiosyncratic, and hence insurable, the insurance arrangement has its maximum value. However, it is now possible that the availability of insurance can ‘crowd out’ investment: households with relatively high income shocks would invest in autarky, but with insurance they are required to instead make transfers. If the fixed level of aggregate income is below $\hat{Y}_1$, then with full commitment insurance no investments will ever take place. In this case limited commitment weakens the insurance arrangement, which might allow more investment to take place. Households who receive relatively good shocks might not be able to commit to providing full smoothing to those who were unlucky: instead they may also invest. Since insurance is here at its most valuable, this is the case where households are most willing to forgo investment to ensure continued access to the insurance arrangement. At intermediate levels of correlation, the effect of the LC constraint is

\[\text{In fact I model the return from investment as non-stochastic and hence entirely independent of the endowment income process, but this is not necessary.}\]
in between these extreme cases.

As well as changing the level of the lowest threshold, \( \hat{Y}_1 \), limited commitment can change the distance between the thresholds. This is important, as it can create long run equilibria where the economy has an intermediate level of capital, rather than the all or nothing result seen under full commitment.

To see this, consider the situation where one household has invested. Under full commitment, I showed that the threshold level of income needed to do one additional investment has now fallen. Under limited commitment there is an additional effect: relative to the case where no-one has invested, the household with an investment has an improved outside option. This endogenously restricts the set of possible equilibrium transfers. Since household consumption will no longer be a constant share of aggregate consumption, and since owning an investment increases the consumption share for a household, there will no longer necessarily be increasing differences in the planner’s utility when another household invests. The limited commitment analogue of Lemma 1 may therefore not hold: an increase in the level of capital will not necessarily reduce the income thresholds for investment. Instead there are now parameters which can support ‘intermediate’ equilibria, where the long run share of households who are invested is strictly between zero and one.\(^{14}\)

### 2.4 Comparative Statics

I now consider two additional testable predictions of the model, which I will verify directly from the data (see Section 3.3 below). In particular, I study the effect of changes in the distributions of income, and in the size of the risk-sharing group. These predictions are important as they arise specifically from the interaction of investment with limited commitment. Neither a model with full commitment, nor a model with a single alternative source of insurance market incompleteness (e.g. moral hazard, hidden income) would give the results I describe below, so these predictions are an important test of the specific mechanism central to the model.\(^{15}\)

\(^{14}\)Hence although all households are ex ante identical, there are long run equilibria where they necessarily have different levels of expected utility. Matsuyama (2002, 2004, 2011) provides other examples of models which have this ‘symmetry-breaking’ property.

\(^{15}\)In Appendix C I consider in more details some possible alternative explanations. I test the predictions made by these alternative mechanisms, and show that none of these are sufficient to explain the patterns observed in the data.
2.4.1 How does inequality in initial income affect investment?

In the presence of limited commitment, income inequality affects the decision to invest. The intuition of this result is straightforward: increased income inequality affects the set of limited commitment constraints which are binding, by changing the outside options for households. As shown above, limited commitment has a direct impact on investment decisions.

More concretely, consider a transfer of endowment income from a (poorer) uninvested household, whose limited commitment constraint does not bind, towards a (richer) uninvested household whose constraint is binding. The increase in income for the richer household improves that household’s outside option, making the limited commitment constraint for that household more binding. If the arrangement previously required the household to invest, this will remain unchanged (see below), whilst if the household was not previously asked to invest, the planner may now find this an optimal way to provide utility to the household.

If the transfer had been from a poorer to richer household where both households had binding limited commitment constraints, the same argument would hold for the richer household, but now the reverse may occur for the poorer household: since the planner need to transfer less utility to this household, it may no longer provide the household with investment as a way to transfer some of this utility. Depending on which household is at the margin of investment, an increase in inequality can therefore increase or decrease total investment. For a given level of aggregate income, a small increase in inequality (starting from a very equal initial distribution) will lead to some LC constraints starting to bind. As this inequality increases and these constraints become increasingly binding, this increases the number of investments that occur. Eventually, further increases in income inequality lead to a reduction in investments, as they are effectively transfers from one constrained household to another, reducing the need to provide the poorer of these households with an investment. Hence there will be an ‘inverse-U’ shape effect of inequality, where initial increase in inequality will increase investment, but too much concentration in just a few hands will again reduce the level of investment.

**Proposition 2.** Consider an initial distribution of income, \( t \). Let \( t' \) be an alternative, more unequal distribution, such that second order stochastically dominates \( t \). There is some \( h \) s.t. \( t^{(i)}(i) \geq t^{(i)}(i) \forall i < h \) and \( t^{(i)}(i) \leq t^{(i)}(i) \forall i > h \). Then \( \hat{Y}'_k \leq \hat{Y}_k \forall k < h \) and \( \hat{Y}'_k \geq \hat{Y}_k \forall k > h \) i.e. the threshold level of aggregate income needed to do \( k \) investments falls for all \( k < h \), and rises for
all \( k > h \).

Proof. See Appendix B.

### 2.4.2 How does group size affect investment?

Given a fixed distribution for individual income, increasing group size has two complementary effects on investment. First, it raises expected aggregate income. Second, it reduces the variance of average income (assuming that incomes are not perfectly correlated) and increases the variance of aggregate income. Even if the mean of aggregate income were fixed, a mean preserving spread of aggregate income would increase investment, since there would be more extreme high income shocks. Under full commitment these periods provide a large incentive to invest to smooth income across time. Under limited commitment there is an additional effect that, with a lower variance for mean income, the value of the insurance arrangement is improved, so autarky is less tempting. This makes it easier to sustain investment.

**Proposition 3.** An increase in the number of households reduces the threshold level of aggregate income needed for investment by improving the value of the insurance arrangement. It also increases the likelihood of aggregate income exceeding even the initial threshold.

Proof. See Appendix B.

As with the prediction on income inequality, this prediction is important because it would not be true if moral hazard or hidden information were the friction driving incomplete insurance.

### 2.5 Key Predictions

The model makes four predictions:

1. At low initial levels of investment, the economy will be in a ‘poverty trap’, and there will be no further investment. Above some threshold, additional investment will be increasing in the stock of investment.

2. Limited commitment will increase the threshold level of income needed to make investment possible.
3. Inequality in realised income, *ceteris paribus*, reduces the threshold level of income at which investment is optimal.

4. Increases in the size of the risk-sharing group will increase investment, both by lowering the threshold level of income needed for investment to take place and by increasing the (maximum) level of aggregate income.

Predictions 1, 2, and 4 can be tested directly, since there is directly observed variation in initial levels of investment, inequality, and group size. Without observed variation in the extent of limited commitment, Prediction 3 cannot be tested in a ‘reduced form’ way: this will provide one motivation for the structural work in Section ?? . By estimating the parameters of the model I am able to then simulate how changes in limited commitment affect the ‘depth’ of the poverty trap i.e. how the threshold level of income for the first investment varies with limited commitment.

3 Testing predictions of the theory

3.1 Data Sources

I use data from an asset transfer program in 1409 villages in rural Bangladesh, collected in partnership with microfinance institute BRAC. Households in these villages typically receive variable incomes, engage in some forms of risk-sharing behaviour, and have potentially productive investments they are able to engage in.

I observe a census of all households in these villages at baseline, in 2007, and then have responses to a detailed questionnaire for one-quarter of these households at baseline and at two follow ups (2009, 2011). Crucially for my question, I have a large sample of village economies, and for the 23,000 households included in the main questionnaire I have information on household income, assets, consumption, and investment.

The asset transfer program provided assets (almost always livestock, in particular cows) to targeted households in half of the villages. Household eligibility for transfers was based on the household’s initial wealth and some demographic characteristics.\textsuperscript{16} After the baseline

\textsuperscript{16}For more details about the program, see Bandiera et al. (2016).
survey had occurred, half of the villages were randomly selected to be ‘treated’, and eligible households in those villages received asset transfers.\textsuperscript{17} The asset transfer program provides exogenous variation in both the initial level of investment, and the distribution of income, by shifting both of these. These data allow me to test the predictions of the model.

### 3.2 Verifying Model Assumptions

I first verify four features of the model:

1. households have variable incomes;
2. household consumption is less variable than income, meaning that some smoothing is occurring;
3. household savings are small relative to income;
4. households have potentially productive investments available.

[THESE FEATURES HAVE BEEN VERIFIED, BUT ARE YET TO BE WRITTEN UP]

### 3.3 Testing Model Predictions

#### 3.3.1 Poverty Trap

I first provide evidence for a poverty trap. The prediction of the model is that for otherwise similar village economies, there is a range of aggregate incomes over which no further investment takes place ($Y < \hat{Y}_1$), and above this investment is increasing in aggregate income (albeit in a lumpy way).

[FIGURE A1 ABOUT HERE]

Figure A1 shows non-parametrically the relationship between the number of households in a village that were given asset transfers, and the number of further investments they made. Visually, there appears to be a threshold at around eight households. This suggests that, in this context, when fewer than eight households received transfers, it had little effect on additional investments.

\textsuperscript{17}Eligible households in control villages received transfers after the endline survey in 2011.
investment uptake, whilst above this point additional investment was increasing in the number of households who received transfers, consistent with the prediction of the model.

To test this prediction formally I use a regression of the form

$$\Delta I_v = a_0 + a_1 K_v \cdot 1\{K_v < \bar{K}\} + a_2 K_v \cdot 1\{K_v \geq \bar{K}\} + a_3 X_v + \epsilon_v$$  \hspace{1cm} (1)$$

where $\Delta I_v$ is the increase in cow investment between 2009 and 2011, $K_v$ is the number of households who were provided asset transfers in 2007, $\bar{K}$ is a proposed threshold number of asset transfers, and $X$ are any controls. Since theory does not provide any guide to the location of $\bar{K}$, I use an iterative regression procedure. This involves run a sequence of such regressions over a prespecified range of possible values for $\bar{K}$, and then testing for significance using a method designed for this type of ‘unknown threshold’ problem. I use two different statistics, both the Quandt Likelihood Ratio test (see Quandt, 1960; Andrews, 1993) and the Hansen test (Hansen, 1999). The former selects as the threshold location the point which maximises the absolute value of the t-statistic on $a_2$, whilst the latter uses a criterion based on the residual sum of squares (so accounting more directly for the relative explanatory power of the regression as a whole).

Precisely, for the Quandt Likelihood Ratio, I calculate for each possible threshold the F-statistic, comparing the model with the threshold to the same model but without any threshold. We then select from among these regressions, the one with the highest F-statistic. The corresponding threshold in that regression is then taken as the estimated location of the threshold. To test whether this threshold value is ‘significant’, I compare the F-statistic to the limiting distribution for this statistic under the null (Andrews, 1993), thus correcting for the multiple testing.

[TABLE A1 ABOUT HERE]

Table A1 shows the results of this test. I find that the most likely location for a threshold is at eight households, consistent with the level suggested by visual inspection of Figure A1. Testing whether this potential threshold is itself statistically significant, I can reject at the 1% level the hypothesis that there is no threshold effect. This is true with and without additional controls. Studying the regression results, one can see that before the threshold the level of
investment is close to zero, and after the threshold it is increasing, consistent with the model predictions.\footnote{Note however that since this regression is chosen using the iterative procedure described above, it would not be correct to use the standard errors provided directly for inference.}

For the Hansen test, I calculate for each possible threshold the residual sum of squares (RSS). We select among the regressions the one with the lowest RSS. The corresponding threshold in that regression is the estimated location using this method. To test whether the threshold is significant, I test construct the Hansen statistic.\footnote{In particular, the test statistic at any possible threshold is the difference between the RSS at that threshold and the minimum RSS from all thresholds considered, divided by the minimum RSS and multiplied by the sample size. This is necessarily equal to zero at the proposed threshold.}

Figure A2 shows again that the most likely location for the threshold is at eight households, and that no other possible thresholds are at all close as alternative locations.

3.3.2 Comparative Statics

Next I test the additional comparative static predictions of the model. Specifically I test for an inverse-U shape in inequality and the investment is increasing in group size.

To test for the effects of inequality in expected consumption (i.e. in promised utility), I use the standard deviation of total assets as a proxy for inequality in expected consumption. Table A2 Column (1) shows that, unconditionally, additional investments are indeed first increasing and then declining in the standard deviation of assets. When additional controls are included (Column 3), the relationship becomes less clear: investment appears to be first flat and then declining, rather than rising before declining.

I next test the prediction on group size. Table A2 Columns (2) and (3) show the unconditional and conditional relationships respectively. I find that for each additional household in the risk sharing group, there is a $30 increase in investment, around 8pp of the base figure.

4 Simulating Counterfactual Policies

[THIS SECTION STILL TO COME]
5 Conclusion and Discussion

Understanding why households remain poor is at the heart of development economics. A recent strand of empirical work has noted that providing asset transfers to households and small businesses in developing countries can have very high returns (Bandiera et al., 2016; Banerjee et al., 2015; de Mel et al., 2008; Morduch et al., 2015). Notably, these experiments do not introduce any new technologies, but simply provide additional productive capital. It is also important to note that these positive returns are seen across the distribution, with households making a profit even at the 10th percentile of returns (Banerjee et al., 2015, lower percentiles are not provided.) This raises the question: why are households unable to find the resources to make these investments themselves?

One feature of many of these programs is that the technologies considered are often non-convex: a cow can provide a regular supply of milk, half a cow cannot. Hence decisions to invest cannot be made in an incremental way, but are rather an all-or-nothing decision. In this paper I examine how the presence of informal insurance with limited commitment affects, and is affected by, the opportunity to engage in lumpy investment.

I showed three main results: valuable risk sharing can crowd out lumpy investment; at low levels of initial capital there is a poverty trap; and inequality has an inverse-U effect on growth. Using data from an asset transfer program to households in rural Bangladesh, I verified the latter finding (as well as other comparative static predictions) in the data.

In ongoing work I am estimating structurally the parameters of the model. This will allow me to quantify how much the inability to commit reduces household welfare, and also to measure the gains from restructuring the asset transfer program to take into account the spillover effects through the poverty trap.

The results of this paper have important implications for development policy more generally. It is already known that the provision of public insurance can crowd out private insurance, so that only looking at the direct effects of public insurance overstates the gains (see for example Attanasio and Rios-Rull, 2000; and Morten, 2015). However, it has not previously been recognised that private insurance can reduce investment and hence income growth. Not considering this effect also would lead to the benefits of public insurance programs being understated.
References


Figure A1: Increase in cow ownership between 2009 and 2011, against number of treated households

*Notes.* The figure shows non-parametrically the relationship between the increase in the aggregate value of cows in a village between two years and four years after transfers were made, and the number of treated households in the same wealth class in the village. A clear break is visible at around eight households: below this point investment in cows does not vary with the number of households receiving assets, after this point there is a clear increasing trend. The shaded region provides the 95% confidence interval.
Table A1: Testing for a threshold effect in Cow Investment

<table>
<thead>
<tr>
<th></th>
<th>(1) Unconditional</th>
<th>(2) With Income Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope below threshold ( (a_1) )</td>
<td>23</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>(174)</td>
<td>(176)</td>
</tr>
<tr>
<td>Slope above threshold ( (a_2) )</td>
<td>453***</td>
<td>450***</td>
</tr>
<tr>
<td></td>
<td>(90)</td>
<td>(90)</td>
</tr>
<tr>
<td>Constant ( (a_0) )</td>
<td>605*</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td>(336)</td>
<td>(699)</td>
</tr>
<tr>
<td>Optimal Threshold Level ( (\bar{K}) )</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>F-statistic</td>
<td>25.2</td>
<td>24.9</td>
</tr>
<tr>
<td>1% critical value for F-statistic</td>
<td>13.2</td>
<td>13.2</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>140</td>
<td>140</td>
</tr>
</tbody>
</table>

Notes. *** denotes significance at 1%, ** at 5%, and * at 10% level, when treated as a standalone regression. Standard errors clustered at the village level. The outcome measures the increase in the aggregate value of cows owned by household in a village-class group, in 2007 USD at purchasing power parity. The wealthiest class is excluded, since they are a very small share of each village, and look very different to the other households. Column (1) performs the estimation unconditionally, whilst Column (2) also includes aggregate income as a control. These specifications are run sequentially at different potential values for the threshold, varying the threshold between 3 and 13 households, at unit intervals. I provide the results for the threshold among those which produced a regression with the highest F-statistic when tested against the null of no threshold. The location of this threshold, the estimated F-statistic, and the 1% critical value (which corrects for the repeated testing, see Andrews, 1993) against which this statistic should be compared are provided at the bottom of the table. The parameters needed to choose this critical value are the degrees of freedom of the test (1), and a statistic calculated from the share of observations above and below the threshold (\( \lambda \) in Andrews’ notation, equal to 204 here).
Figure A2: Hansen test

Notes. The figure shows, for each assumed threshold, the likelihood ratio (LR) statistic. This statistic is the difference in residual sum of squares (RSS) from the assumed threshold regression relative to the RSS from the regression for which the lowest RSS was achieved, divided by that minimum RSS and multiplied by the sample size. Any possible thresholds for which the LR is below .05 cannot be rejected as possible values for the threshold. The graph show the range of LRs both for the unconditional case and when aggregate income is controlled for (corresponding to Columns 1 and 2 of Table A1 respectively). In both cases it is clear that a threshold value of eight is by far the most likely.
**Table A2: Affect of Inequality and Group Size on Cow Investment**

Dependent Variable: Increase in Total Cow Assets (2007 USD PPP)
Clustered Standard Errors in Parentheses

<table>
<thead>
<tr>
<th></th>
<th>(1) Unconditional</th>
<th>(2) Unconditional</th>
<th>(3) With Controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance group size</td>
<td>28.8***</td>
<td>28.9***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.25)</td>
<td>(15.2)</td>
<td></td>
</tr>
<tr>
<td>S.D. of asset distribution</td>
<td>.148***</td>
<td>.008</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.046)</td>
<td>(.076)</td>
<td></td>
</tr>
<tr>
<td>Var. of asset distribution (×10^{-6})</td>
<td>-5.29***</td>
<td>-4.38***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(1.66)</td>
<td></td>
</tr>
<tr>
<td>Constant ($\beta_0$)</td>
<td>417***</td>
<td>351***</td>
<td>369***</td>
</tr>
<tr>
<td></td>
<td>(37.2)</td>
<td>(29.5)</td>
<td>(34.4)</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>1840</td>
<td>2398</td>
<td>1840</td>
</tr>
</tbody>
</table>

*Notes.*** denotes significance at 1%, ** at 5%, and * at 10% level, when treated as a standalone regression. Standard errors clustered at the village level. The outcome measures the increase in the aggregate value of cows owned by household in a village-class group, in 2007 USD at purchasing power parity. The wealthiest class is excluded, since they are a very small share of each village, and look very different to the other households.

Columns (1) shows the unconditional regression of additional investment on the standard deviation of the asset distribution and the standard deviation squared (the variance), to allow for an inverse-U shaped relationship, as predicted by the model. Column (2) shows the unconditional regression of additional investment on group size, measured by the number of household in the same wealth class in the village. Column (3) combines (1) and (2), and additionally adds aggregate income and aggregate assets as additional controls.
Appendix B  Theoretical Appendix

Appendix B.1  Proof of Proposition 1: Threshold income level for investment

The proof of Proposition 1 involves three steps. I first show that when the planner is choosing $k$ optimally, $\frac{\partial V_k}{\partial Y} > 0$. Next I show that $\frac{\partial V_k}{\partial Y} > \frac{\partial V_{k-1}}{\partial Y} > 0$, so that $V_{k-1}$ and $V_k$ cross at most once. Finally I show that $V_{k-1}$ and $V_k$ do cross at least once, and hence there is a unique $\hat{Y}_k$ s.t. $\Gamma_k(\hat{Y}_k) \equiv V_{k-1}(\hat{Y}_k) - V_k(\hat{Y}_k) = 0$.

Conditional value functions are increasing in $Y$

I want to show that $\frac{\partial V_k}{\partial Y} > 0$. Taking the derivative of the conditional value function $V_k$ wrt $Y$, and applying the envelope theorem, I get:

$$\frac{\partial V_k}{\partial Y} = t^1 u'(c^1) + \sum_{i=2}^{N} t^i \lambda^i u'(c^i)$$

$$= u'(c^1)$$

where for notational convenience I define $t^1 = \left(1 - \sum_{i=2}^{N} t^i\right)$, and the second equality comes from use of the FOCs wrt $\tau^i$. Hence from the properties of $u(\cdot)$, $\frac{\partial V_k}{\partial Y} > 0$.

Slopes of conditional value functions are increasing in $k$

By the budget constraint, aggregate consumption $C = \sum_{j=1}^{N} c^j$ is total income less spending on investment: $C = Y + KR - kd$. From the first order conditions wrt $\tau^i$, $c^i$ is strictly increasing in $c^1$, so all households’ consumptions must be strictly increasing in aggregate consumption. Hence, since aggregate consumption is strictly decreasing in $k$, the number of investments, $c^1_k < c^1_{k-1}$. Then by concavity of $u(\cdot)$, $\frac{\partial V_{k-1}}{\partial Y} - \frac{\partial V_k}{\partial Y} = u'(c^1_{k-1}) - u'(c^1_k) < 0$. As the conditional value function when there are $k$ investments is always strictly steeper than the value function associated with $k - 1$ investments, and the value functions are continuous (again inherited from properties of $u(\cdot)$) they can cross at most once.
Conditional value functions cross at least once

To see that the conditional value functions do have at least one crossing, I show the limits of their difference as aggregate income falls and rises.

As aggregate income falls towards $kd$, the cost of making $k$ investments, the value of making $k - 1$ investments today remains positive. However the value of making $k$ investments goes to negative infinity as there are no resources left for consumption (since $u(\cdot)$ satisfies the Inada conditions). Hence the difference in value becomes infinite:

$$\lim_{Y \to d} V_{k-1} - V_k = \infty$$

Conversely, as aggregate income this period rises towards infinity, the difference in utility today between investing and not investing goes to zero. Hence the difference in the conditional value functions is just the difference in the value between having $K + k - 1$ or having $K + k$ investments (collectively), where $K$ is the number of existing investments. Since the value is increasing in the number of investments, the difference in values is negative.

$$\lim_{Y \to \infty} V_{k-1} - V_k = \beta \mathbb{E} \left[ V(\tilde{\omega}', I_{K+k-1}) - V(\tilde{\omega}', I_{K+k}) \right] < 0$$

Hence since the conditional value functions are continuous in $Y$, they must cross. Then, since I showed earlier that $\frac{\partial V_k}{\partial Y} > \frac{\partial V_{k-1}}{\partial Y} > 0$, there can be at most one crossing of the value functions.
Appendix C  Alternative Explanations

In this appendix I consider a number of possible alternative explanations for the low levels of investment by households in developing countries. I show that none of these are alone sufficient to explain the observed patterns of behaviour in the data.

The alternative hypotheses I consider are:

1. Economies of scale
2. Learning across individuals
3. High discount rates
4. Aspiration failures
5. Quality-Quantity Tradeoff (e.g. Rosenzweig Wolpin 1980 Ecta; Becker Murphy Tamura 1990; Moav 2002 EJ)
6. Heterogeneity in returns

Appendix C.1  Economies of scale

If there are economies of scale in investment, this might also provide a coordination result. For example, suppose that households investing in cows want to sell the milk they produce in some nearby market. There is a fixed cost of having someone transport the milk to market. Then we would see that the return from investing in cows has a discontinuous jump, once the number of cows in the village exceeded the fixed cost of travel to the nearby market. Hence one should see that the average return on cows is increasing in the total number of cows in the village. Since the cost of travelling to the nearest major market will likely be different across villages, one would not expect to see empirically a sharp threshold, but still the increasing pattern should hold.

Figure C3: No evidence of increasing returns

Figure C3 shows a scatter plot (and local polynomial line) of how the average return on a cow varies with the number of cows in the village. If economies of scale are relevant at the
margin, then the return on an additional cow should be at least as high as the return on the last inframarginal cow, and hence average returns should also be non-decreasing in the number of cows. This is not consistent with the pattern seen in Figure C3.

Appendix C.2 Learning across individuals

Another source of coordination in investment decisions would come about if there were learning across individuals. The predictions of this mechanism are, however, the same as for economies of scale: the marginal (and hence average) return should be increasing in the number of cows. But I have already shown this pattern is not present in these data.

Appendix C.3 High discount rates

Perhaps the most obvious reason why households might not choose to invest is that the future returns are not valuable enough. If households discount the future enough, then the future return from investment will not be enough to compensate for the reduction in current consumption needed to invest.

In the midline survey, households were asked how many months they would be willing to wait to receive a transfer of $12, rather than receiving $10 today. In principle one could back out an estimate of the households’ discount rates using this information. However, there is a worry that many households might ask to receive the money today because they wouldn’t trust that the surveyors would return in the future.\(^\text{20}\) There are also possible issues over seasonality, as households choosing the $12 might interpret the question as when in the future they would most like to receive the $12 (consistent with the finding that many of them choose a number of months that coincides with receiving the money in the ‘monga’ hungry season).

To circumvent these issues, I instead compare a measure of discount rates across households who have and have not chosen to invest in cows. To avoid contamination due to the asset transfer program, I use data only from the 700 control villages (giving a sample of 12,000 households). Households who own cows must clearly have discount rates such that investment was worthwhile for them. If household who currently do not own cows have the same distribution of choices

\(^{20}\)This was in fact a purely hypothetical question, and it was made clear that no money would actually be provided.
over when to receive the money, then there is no reason to think that these households are any
different in terms of their discounting, suggesting that discount rates are not enough to explain
their lack of investment.

![Figure C4: No evidence of differences in discounting](image)

**Notes.** The figure shows the share of households willing to wait a particular number of months to receive a transfer of $12, rather than receiving $10 today, split according to whether the household does or does not own any cows.

Figure C4 shows how the declared willingness to wait for a higher income transfer varies by cow ownership status. I cannot reject that the two distributions are identical, despite a reasonably large sample size, suggesting that discount rates are not the explanation for the lack of investment.

### Appendix C.4 Aspiration failures

Another possible explanation for the low levels of investment is that individuals are caught in an ‘aspiration trap’, where they simply cannot conceive of being in a position to be able to invest in cows, and hence do not take the actions that might allow them to do this. Table C3 shows that, at baseline more than a third of households would like to start a new business activity. Among the poorer households (‘STUPS’) who are targeted by the asset transfer program, and
who are less likely to already own cows, this rises to almost half. Among those households who
would like to start a new activity, around 80% would like to invest in cows. Hence even if some
households might be trapped in an aspirations trap, there is a significant number of households
who would like to be able to invest and are thinking about it.

Table C3: Households’ business plans

<table>
<thead>
<tr>
<th>Would like to start new activity</th>
<th>(1) Mean</th>
<th>(2) S.D.</th>
<th>(3) Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole sample</td>
<td>.384</td>
<td>.483</td>
<td>26,481</td>
</tr>
<tr>
<td>STUPS</td>
<td>.477</td>
<td>.499</td>
<td>7,878</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Desired new activity = livestock husbandry</th>
<th>(1) Mean</th>
<th>(2) S.D.</th>
<th>(3) Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole sample</td>
<td>.790</td>
<td>.408</td>
<td>10,172</td>
</tr>
<tr>
<td>STUPS</td>
<td>.838</td>
<td>.368</td>
<td>3,756</td>
</tr>
</tbody>
</table>

Notes. Observations are at the household level. Data are from the baseline, some time before households were informed about the asset transfer program.

Appendix C.5 Heterogeneity in returns

Another possible worry is that there is heterogeneity in households’ returns from cows. If households have uncertainty about the return they will receive, perhaps because they don’t know the cows’ types, or because they don’t have information about their own abilities with cows, then this may dissuade them from investing by lowering the expected utility under investment.

Figure C5 shows the distribution of returns from owning two cows, both for households that received cows as part of the asset transfer program (‘Ultrapoor’) and households in the next lowest wealth class who already owned two cows (‘other poor’). The figure shows that the distributions of returns are very similar for the two groups, and using a Kolmogorov-Smirnov test for differences in distribution I cannot reject that the distributions are the same. Since the other poor chose already to buy cows, and they have the same return from cows as the ultrapoor, this suggests the heterogeneity in returns is not sufficient to explain the lack of investment.
Figure C5: No evidence of differences in distribution of returns from cows

Notes. The figure shows the distribution of income earned from cows by a household over a one year period. The distributions are plotted for the ultrapoor households, who received two cows as part of the asset transfer program, and for the ‘other poor’ households (the next lowest wealth class) who have precisely two cows.