The Importance of Being Prudent: Dividends, Signaling and Risk Shifting

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Abstract

Shareholders may exploit creditors by engaging in risk shifting. However, shareholders with favorable private information have an incentive to convey it to outside parties to avoid mispricing. I show that not engaging in risk shifting—“being prudent”—can be a credible signal of unobserved firm quality. Empirically, the model predicts that higher-quality firms pay lower dividends and invest in safer projects when franchise values are low. Normatively, signaling may raise welfare if risk shifting is socially costly. The results are useful for understanding aspects of bank dividend policy in 2007–8.

Keywords: signaling, risk shifting, dividends, banks.

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1 Introduction

Shareholders—or managers acting in the shareholders’ interest—can profit at the expense of creditors by engaging in various forms of risk shifting (Jensen and Meckling, 1976). Firms may pay excessive dividends, increase the risk of their assets or load up on debt. Since shareholders are protected by limited liability, such actions lead to wealth transfers between shareholders and creditors. While risk shifting is an important concern for many corporations, risk shifting incentives are especially strong in banking, in part because of the opaqueness of banks and in part because of government guarantees (Becht, Bolton, and Roell, 2011).

At the same time, firms with favorable private information about their prospects have an incentive to signal it to outside parties to avoid mispricing (Leland and Pyle, 1977; Ross, 1977). These two problems, risk shifting and signaling, are among the key determinants of corporate financial policy.

The main contribution of this paper is to point out an important interaction between risk shifting and signaling: not engaging in risk shifting—“being prudent”—can be a credible signal of firm quality. The basic insight is that risk shifting incentives are generally decreasing in firm quality. By acting prudently, high-quality firms can distinguish themselves from low-quality firms and avoid mispricing.

The result emerges in a number of natural settings. In the baseline model, a firm has private information about its assets-in-place and needs to raise an exogenously given amount of short-term debt. Before raising debt, the firm announces a dividend. Since the quality of the firm’s assets is subject to asymmetric information, the dividend policy has an informational role: firms that are perceived to have better assets refinance their debt at a lower cost. However, a higher dividend increases the probability of default by reducing the firm’s cash holdings. In turn, the greater probability of default is priced in by the credit market and, all else equal, a higher dividend raises funding costs. Finally, if the firm defaults on its obligations, it loses its franchise (continuation) value which represents the expected future profits of the firm.

The key result from the baseline model is that when the franchise value of the firm is sufficiently low, equilibrium dividends are negatively related to asset quality. In the model, the marginal cost of paying a higher dividend is that the franchise value is lost with a greater probability. The marginal benefit is that paying a dividend reduces the deadweight cost of holding cash. In the absence of asymmetric information, the optimal dividend policy balances the costs of holding cash against the probability of losing the franchise value.

With asymmetric information, the creditors cannot directly ascertain the qual-
ity of a firm’s assets. As a result, dividends play an additional informational role. When the franchise value is sufficiently small, paying a low dividend and leaving cash inside the firm is costly for all firms as the cost of holding cash outweighs the benefit of the lower default probability. In addition, the shareholders face a risk shifting incentive: one dollar paid out in dividends today is one dollar less for the creditors in the event of bankruptcy tomorrow. The risk shifting incentive is stronger for low-quality firms since they are more likely to default for the same level of dividend payments. When the franchise value is sufficiently low, therefore, equilibrium dividends are negatively related to asset quality.

A corollary of the result that equilibrium dividends are negatively related to asset quality is that dividends and the probability of default are positively related: higher-quality firms pay a lower dividend and have a lower probability of default for a given dividend. The result is an equilibrium outcome even though funding costs are sensitive to the dividend policy, all debt is fairly priced, and the creditors break even.

In contrast, when franchise values are sufficiently high, one recovers the classical positive relationship between dividends and asset quality, just as in the seminal dividend signaling models of Bhattacharya (1979), John and Williams (1985) and Miller and Rock (1985). The intuition as to why high dividends are a positive signal of quality is similar to Ross (1977) and Bhattacharya (1979). By paying a higher dividend, the firm puts its franchise value at risk. Paying a high dividend is less costly for higher-quality firms because it does not put the franchise value at risk to the same extent as it does for lower-quality firms.

Risk shifting incentives are especially strong in banking.\(^1\) Bank dividend payments, in particular, have received much scrutiny after the global financial crisis of 2007–8. During the crisis, many banks and securities firms paid high dividends in the face of mounting losses and worsening funding conditions.\(^2\) Dividend payments were substantial—large commercial banks in the United States paid out more than $60 billion in dividends from 2007Q3 to 2008Q4 (Hirtle, 2014)\(^3\)—and caught

\(^{1}\) In addition to bank opacity and government guarantees as potential sources of risk shifting incentives in banking (Becht et al., 2011), current bank regulation discourages the use of bond covenants. In the US, subordinated debt may not have restrictive covenants to qualify as Tier 2 capital, as discussed by Ashcraft (2008, p. 557; see especially Footnote 7). Basel Accords require subordinated debtholders not to have any “rights to accelerate the repayment of future scheduled payments (coupon or principal)” (Bank for International Settlements, 2010, p. 18), which, as pointed out by Bliss (2014, p. 574), effectively prevents the enforcement of other covenants.


\(^{3}\) These dividend payments constitute roughly a fourth of the $250 billion used by the Treasury to recapitalize the banking system in 2008–9 (Department of the Treasury, 2013, p. 4). The dividend figure from Hirtle (2014) does not include certain institutions, such as investment banks and insurance companies, which also received support via the recapitalization program.
the attention of policymakers (e.g., Rosengren, 2010). More recently, large banks in
the euro area have paid out more than half of their earnings amid what some com-
mentators have called significant capital shortfalls (Acharya, Pierret, and Steffen,
2016b).

The baseline model is useful for interpreting these observations, although cau-
tion is necessary given the starkness of the modeling assumptions. The model pre-
dicts that when franchise values are low, as was arguably the case during the global
financial crisis, dividends are positively correlated with the probability of default.
The pattern of dividend payments by investment banks seems consistent with this
prediction (Acharya, Le, and Shin, 2016a). The model also predicts a negative divi-
dend announcement effect on the stock price as well as proxies of the default prob-
ability such as credit default swap spreads. The former prediction finds support in
the recent empirical work of Cziraki, Laux, and Loranth (2016) who show that divi-
dend cuts in 2007–8 were not associated with a statistically significant stock market
reaction.

I next consider the normative implications of the model in a setting in which
default leads to social costs that are not internalized by the firm. Such externali-
ties are particularly important for banks but may also be present in other indus-
tries.4 When franchise values are low, the informational role of dividends leads to
surprising policy implications. In some cases, the equilibrium under asymmetric
information may be welfare-superior to the equilibrium under symmetric informa-
tion. With asymmetric information and low franchise values, firms signal quality to
the creditors by paying dividends that are too low from a private perspective (i.e.,
lower than under symmetric information). With default externalities, however, the
symmetric information equilibrium features dividends that are too large from a so-
cial perspective. Hence, signaling incentives may push the equilibrium dividend
policies closer to the social optimum. Government policy in the form of taxing or
restricting dividends may also be welfare-improving.

The main result of signaling incentives leading high-quality firms to behave pru-
dently is robust to changes in the modeling assumptions. The basic result remains
unchanged if (i) instead of signaling quality to the short-term creditors, the firm
wants to boost its stock price; (ii) asymmetric information concerns the amount of
cash the firm has; (iii) instead of dividend policy, the firm can choose the riskiness
of its investment; (iv) the firm has outstanding long-term debt; or (v) there is a pos-
itive bailout probability. The general message from these extensions is that since

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4 For recent studies documenting the negative effects of bank failures and credit disruptions on the
functioning of the real economy, see, among many others, Ashcraft (2005), Almeida, Campello,
Laranjeira, and Weisbenner (2012), and Chodorow-Reich (2014).
risk shifting incentives are typically decreasing in firm quality, prudent behavior is a credible signal of quality. In addition, I provide a numerical example in which higher-quality firms signal quality by paying lower dividends when both the dividend policy as well as the level of borrowing are endogenous.

**Outline.** In the next subsection I discuss how the main result relates to the existing literature. In Section 2, I present a simple example illustrating the key idea of the paper. Readers who would like to quickly get a sense of the paper may wish to read Section 2 and the discussion of the numerical example of the baseline model surrounding Figure 3. In Section 3, I present the baseline model in which dividends are used to signal quality to short-term creditors; Section 4 contains the analysis of the baseline model. The normative implications of the baseline model are considered in Section 5. I present a sequence of extensions to investigate the robustness of the main result in Section 6. Section 7 discusses the empirical predictions, and Section 8 concludes.

**Related Literature**

The paper builds on three branches of the literature in corporate finance and banking.

**Asymmetric Information and Signaling.** The paper contributes to a large literature on the informational role of dividends in particular and corporate financial policy more generally. Seminal papers in the literature on dividends as signals include Bhattacharya (1979), John and Williams (1985), Miller and Rock (1985), and Ambarish, John, and Williams (1987). In contrast to these classic papers, in my model, the single-crossing property need not hold (or the direction in which the single-crossing property holds may vary), similarly to Bernheim (1991) and Araujo, Moreira, and Tsuchida (2011). Existing papers, however, typically abstract from default and the existence of debt.

The observation that high dividends need not always be good news is, of course, not new. In a critique of signaling games in corporate finance, John and Sundaram (2010, p. 1402) point out that whether or not high dividends are good news in the model of Miller and Rock (1985) depends on how the marginal return to investment varies with firm quality. For plausible production functions, the marginal return to investment is greater for higher-quality firms and therefore equilibrium dividends and firm quality are negatively related. In the current paper, I show that high dividends are not always good news when firms have outstanding debt and show how the informational effect of dividends varies with a firm’s franchise value.

The literature on signaling information through changes in corporate financial
policy, including changes in capital structure, is too voluminous to summarize succinctly. Two papers that are particularly relevant are John (1987) and Brick, Frierman, and Kim (1998). John (1987) considers a model in which insiders receive information about the variance of cash flows after obtaining funds, leading to a risk shifting problem. However, once it has obtained the funds, the firm does not have any signaling considerations. As a result, the effect considered in the present paper does not arise in John’s model. Brick, Frierman, and Kim (1998) show that firms whose investment projects have a lower variance may signal quality by borrowing less and distributing a dividend. Signaling is credible because there is a problem of risk shifting between the firm and the government that arises because of the tax deductibility of interest payments. In contrast, in my model risk shifting arises because of a conflict of interest between the creditors and the shareholders (although the extension of the baseline model with bailouts in Section 6.5 has a similar flavor).

Risk Shifting. Extensive corporate finance literature analyzes the potential conflicts of interest between the creditors and the shareholders. The conflict of interest has been pointed out and studied by, among many others, Fama and Miller (1972, pp. 179–180), Black and Scholes (1973, p. 651), Jensen and Meckling (1976), Myers (1977), and Stiglitz and Weiss (1981). Akerlof and Romer (1993) explain how government debt guarantees in combination with limited liability can lead to looting, an extreme form of shifting risks to the taxpayer. The insight that high franchise values reduce the incentives to engage in risk shifting goes back to at least Marcus (1984). Keeley (1990) presents a related model and provides empirical evidence on banking firms consistent with the hypothesis that higher franchise values reduce risk shifting. However, the literature on risk shifting typically abstracts from the informational role of corporate financial policy.

Bank Dividend Payments. Finally, the paper contributes to a smaller but rapidly growing literature on dividend payments by banks. On the empirical side, key papers include Khorana and Perlman (2010), Acharya, Gujral, Kulkarni, and Shin (2012), Floyd, Li, and Skinner (2015), Hirtle (2014), and Cziraki, Laux, and Loranth (2016). On the theory side, Acharya, Le, and Shin (2016a) analyze bank dividend payments in a setting in which undercapitalized banks exert a negative externality on other financial institutions. Acharya, Le, and Shin show that unregulated banks may pay dividends that are larger than what would be optimal for the banking sector as a whole. My paper complements the work of Acharya, Le, and Shin by analyzing the informational role of dividends in a setting with scope for potential risk shifting behavior. Guntay, Jacewitz, and Pogach (2016) analyze dividend payments of banks in a setting in which a regulator can restrict dividend payments if the bank is not sufficiently well capitalized. Guntay, Jacewitz, and Pogach show
that regulatory interference may lead to excessive dividends as banks try to signal that they have passed regulatory scrutiny. My paper, instead, focuses on the incentives of firms to signal quality in the presence of risk shifting incentives.

2 Signaling Quality By Acting Prudently: Example

I begin by providing a simple example of how acting prudently can act as a credible signal of unobserved quality.

Consider the situation depicted in Figure 1. There are two periods \( (t = 0, 1) \), two players—a firm and a credit market—and no discounting. At time zero, the firm has \( x \) units of cash and an asset-in-place that yields a random return \( \tilde{y} \) at time one. The long-term asset yields \( y > 0 \) with probability \( \theta \) and zero otherwise. The quality of the long-term asset, \( \theta \), is private information to the firm. In particular, the credit market cannot directly observe \( \theta \). Holding cash is costly: one unit of cash left in the firm at time zero depreciates to \( 1 - \phi \) units of cash at time one. As I discuss below, the deadweight cost of holding cash is a simple modeling device used to capture some benefit of paying dividends, such as clientele effects or reducing agency frictions, but the key result does not hinge on holding cash being costly.

The firm needs to raise \( b \) units of short-term debt to refinance maturing debt which, to simplify algebra, is also equal to \( b \). For tractability, \( b \) is taken to be exogenous. The firm raises funds by borrowing from creditors. The creditors provide funding as long as they break even in expected terms. Through competition among themselves, the creditors exactly break even in equilibrium.

Before raising credit, the firm announces a dividend \( d \in [0, x] \). The dividend is paid out to the shareholders after the firm secures new funding and refinances its maturing debt. The dividend is chosen to maximize the true value of equity, that is, the value of equity for known \( \theta \). The prior distribution of \( \theta \) is summarized by a cumulative distribution function (CDF) \( G(\theta) \) with the support of asset quality given by a closed bounded interval \([\theta, \bar{\theta}]\).

The probability of default is positive for all firms: \( b > x(1 - \phi) \) and \( \bar{\theta} < 1 \). To simplify the exposition, I also assume that the expected return from the long-term asset of all firms is sufficiently high: \( \theta y > b \). This assumption ensures that, in expectation, all firms can repay \( b \) to the creditors at time one even if the firms pay out all of their cash as a dividend.

Let \( B \) denote the promised repayment to the creditors (face value of debt). We
Figure 1: Graphical depiction of the example. The firm has \( x \) units of cash and an asset-in-place. The asset-in-place yields \( y \) with probability \( \theta \) and zero otherwise. Asset quality \( \theta \) is private information to the firm. At time zero, the firm raises \( b \) dollars via short-term debt to repay maturing debt and distributes a dividend \( d \) to its shareholders. The amount of cash inside the firm at time one is denoted by \( x_1 \), i.e., \( x_1 = (x - d)(1 - \varphi) \), where \( \varphi \in (0, 1) \) is the cost of holding cash. Finally, \( B \) is the promised repayment to creditors (face value of debt).

see that the true value of equity is equal to

\[
\Pi_E(d, B; \theta) = d + \theta[(x - d)(1 - \varphi) + y - B],
\]

while the true value of debt is given by

\[
\Pi_D(d, B; \theta) = \theta B + (1 - \theta)(x - d)(1 - \varphi).
\]

In the present example, leaving cash inside the firm leads to a deadweight loss \( \varphi(x - d) \) but yields no benefit. Under symmetric information, therefore, all firms pay out all of their cash as a dividend.

Now consider what happens with asymmetric information about asset quality. To start, suppose that all firms pay a pooling dividend \( d = x \). To ensure that paying the pooling dividend is indeed an equilibrium strategy, assume that if a firm pays \( d < x \), the credit market attributes the deviation to the firm with lowest quality assets, i.e., \( \theta \).

Since asset quality is not observable, we need \( \int \theta \Pi_D(x, B; \theta) \, dG(\theta) = b \) for creditors to break even, which yields the promised repayment in this pooling equilibrium given by \( B_p = b / \mathbb{E}[\hat{\theta}] \). Substituting the expression for \( B_p \) into the value of
equity, the equilibrium value of equity is then given by

\[ \Pi_E(x, B_p; \theta) = (x + \theta y - b) + b \left( \frac{E[\bar{\theta}] - \theta}{E[\theta]} \right). \]  

(1)

Thus, the value of equity is equal to the expected net present value (NPV) of the firm plus a mispricing term. Since the price of debt is determined by the average quality of assets-in-place, firms with assets that are better than average in effect subsidize the funding costs of firms with assets that are worse than average.

If mispricing is sufficiently strong, high-quality firms will have an incentive to use their dividend policy as a costly signal to obtain cheaper funding, and the pooling equilibrium becomes less plausible. Specifically, to sustain the pooling equilibrium, the credit market must attribute deviations \( d < x \) to low-quality firms. However, as shown below, high-quality firms are more likely to gain from deviating to a lower dividend.\(^5\)

Even if one does not take a stand on how plausible a particular equilibrium is, dividends and quality must be (weakly) negatively related in any equilibrium. To see this, it is useful to rewrite the value of equity as

\[ \Pi_E(d, B; \theta) = [d + (x - d)(1 - \varphi) + \theta y - B] + (1 - \theta)[B - (x - d)(1 - \varphi)]. \]  

(2)

This expression decomposes the value of equity into the expected NPV of the firm and a second term, \((1 - \theta)[B - (x - d)(1 - \varphi)]\), which stems from limited liability. The second term captures the fact that when the firm is bankrupt in period one, which happens with probability \(1 - \theta\), it is protected by limited liability. As a result, instead of paying \(B\) to creditors, the shareholders only lose the cash that is inside the firm, \((x - d)(1 - \varphi)\). The difference between \(B\) and \((x - d)(1 - \varphi)\) is a source of value for the shareholders. Crucially, the value of limited liability is strictly decreasing in asset quality. Firms with better assets have a lower default probability and hence the shield from losses provided by limited liability less valuable to high-quality firms.

More formally, let \(\text{MRS}(d, B; \theta) \equiv -[\partial \Pi_E/\partial d]/[\partial \Pi_E/\partial B]\) denote the marginal rate of substitution between the dividend and the promised repayment. We calcu-

\(^5\) Formally, the pooling equilibrium does not satisfy the D1 equilibrium refinement of Cho and Sobel (1990). The Intuitive Criterion of Cho and Kreps (1987) typically does not restrict equilibrium behavior in settings with more than two types, which is also the case in the present example: the pooling equilibrium with \(d = x\) is not ruled out by the Intuitive Criterion.
Figure 2: Indifference curves through a point \((d^*, B^*)\) of a low-quality firm (low \(\theta\)) and a high-quality firm (high \(\theta\)). Suppose that the high-quality firm chooses to pay \(d^*\) with an associated repayment \(B^*\) at some given repayment schedule \(B(d)\). Then, the optimal dividend-promised repayment combination for the low-quality firm must lie in the shaded region. See the text for further discussion.

late that

\[
\text{MRS}(d, B; \theta) = \frac{1}{\theta} - (1 - \varphi).
\]

The marginal rate of substitution is strictly positive and strictly decreasing in the quality of the firm’s assets.

Therefore, higher-quality firms must pay (weakly) lower dividends in any equilibrium. To see this graphically, consider Figure 2. Suppose that all firms face a promised repayment schedule \(B(d)\) and let the optimal dividend paid by the high-quality firm be denoted by \(d^*\); the associated promised repayment is given by \(B^*\). Since the value of equity is strictly increasing in the dividend and strictly decreasing in the promised repayment, all other feasible combinations of \((d, B)\) must lie to the northwest of the indifference curve of the high-quality firm. As a result, the optimal choice of \((d, B)\) by the low-quality firm must lie in the shaded region. Hence, the firm with low-quality assets pays weakly higher dividends in any equilibrium.

To exhibit a particular equilibrium in which dividends and quality are negatively related, consider the case of a separating equilibrium. In a separating equilibrium, there is a one-to-one mapping between asset quality and equilibrium dividends. Let \(\hat{\theta}(d)\) denote a differentiable function mapping the equilibrium dividend to asset quality. Solving \(\Pi_D(d, B; \theta) = b\) for \(B\), we obtain the debt pricing equation

\[
B^*(d; \theta) = b + \frac{1 - \theta}{\theta} \left[ b - (x - d)(1 - \varphi) \right].
\]

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6 The argument is well-known and can be found in, for instance, Riley (1979, Lemma 2).
Substituting the value for the repayment \( B^* (d; \hat{\theta}(d)) \) from Eq. (3) into the expression for \( \Pi_E \) in Eq. (2) and simplifying, we see that the firm manager chooses the dividend to solve

\[
\max_{d \in [0, x]} \quad d + \theta y + (x - d)(1 - \varphi) - b + \left( \frac{\hat{\theta}(d) - \theta}{\hat{\theta}(d)} \right) \left[ b - (x - d)(1 - \varphi) \right].
\]

Similarly to Eq. (1), the payoff of shareholders is given by the expected NPV of the firm plus a mispricing term.

Because the beliefs of the credit market are correct in equilibrium, we may take the first-order condition of the problem above and impose the equilibrium condition \( \theta = \hat{\theta}(d) \) to obtain an ordinary differential equation that characterizes the equilibrium:

\[
\varphi = -\left[ b - (x - d)(1 - \varphi) \right] \frac{\hat{\theta}'(d)}{\hat{\theta}(d)}.
\]

In any separating equilibrium, the lowest quality firm must pay the same dividend as it would under symmetric information, and so we have the initial condition \( \hat{\theta}(x) = \theta \).

Eq. (4) is a separable differential equation that can be solved in closed form:

\[
\hat{\theta}(d) = C_0 \left[ b - (x - d)(1 - \varphi) \right]^{\varphi / (1 - \varphi)} \quad \text{and} \quad C_0 \equiv \theta b^{\varphi / (1 - \varphi)}. \tag{5}
\]

In equilibrium, firms with higher-quality assets pay strictly lower dividends. Since the probability of default is equal to \( \theta \)—which coincides with \( \hat{\theta}(d) \) in equilibrium—the probability of default is strictly increasing in the dividend.\(^8\)

The example in this section is special in some important respects. First, the probability of default is independent of the dividend policy. Changing this would only reinforce the result, since a lower dividend would not only be good news to the market but also reduce the probability of default by leaving more cash inside the firm. Second, default is costless. When default is sufficiently costly, risk shifting incentives are mitigated, and dividends and quality may well become positively related.

The baseline model in Section 3 presents a model that incorporates both of these

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\(^7\) To ensure that the non-negativity of dividends is not violated, we need to impose the restriction that the support of asset quality is not too large: \( \theta / \hat{\theta} \leq (b / (b - x(1 - \varphi)))^{\varphi / (1 - \varphi)} \). Of course, the solution to the differential equation has not yet been shown to indeed constitute an equilibrium. Following Riley (1979), that is straightforward to do, as shown in the proof of Proposition 2 below.

\(^8\) Equilibrium payoffs are given by \( x + \theta y - b - \frac{\varphi b}{1 - \varphi}(1 - (\theta / \theta)^{(1 - \varphi) / \varphi}) \). Comparing to Eq. (1), we see that firms with high-quality assets prefer the separating equilibrium to the pooling equilibrium with \( d = x \) whenever \( E[\hat{\theta}] \) is sufficiently close to \( \hat{\theta} \).
features and, in addition, allows for richer managerial incentives. Nevertheless, the basic insight remains the same: acting prudently can be a credible signal of quality.

3 Baseline Model

I now describe the environment of the baseline model. In essence, the baseline model generalizes the example in Section 2 by making the probability of default sensitive to the dividend policy, introducing costly default and allowing for richer managerial incentives.

The basic structure of the model is similar to Acharya, Le, and Shin (2016a). The key difference from the setup of Acharya, Le and Shin is that the firm has private information. In the baseline model, asymmetric information concerns the quality of assets-in-place but, as I discuss in Section 6.2, the exact source of asymmetric information is not essential.

3.1 Setup

There are two periods \( t = 0, 1 \), two risk-neutral agents—a firm manager and a credit market—and no discounting. The credit market consists of a unit mass of identical creditors. Since there is essentially no strategic interaction between the creditors, I will often lump all creditors together and use the words “credit market” and “creditors” interchangeably.

The firm is endowed with \( x > 0 \) units of a liquid asset, called cash, and an illiquid long-term asset. The long-term asset yields a random return of \( \tilde{y} \) units of cash in period one. Holding cash is costly. Formally, the firm has access to a storage technology that can transform one unit of cash in period zero into \((1 - \varphi) \in (0, 1)\) units of cash in period one. There is no other investment technology available to the firm.

The long-term asset is subject to asymmetric information: the firm has better information about the quality of the asset than the credit market. The return \( \tilde{y} \) is distributed according to a probability density function (PDF) \( f(y; \theta) \) with a cumulative distribution function (CDF) \( F(y; \theta) \). The index \( \theta \) is called the type or asset quality of the firm, and it is observable to the firm but not the credit market. The support of asset qualities is given by a closed bounded interval denoted by \( \Theta = [\underline{\theta}, \bar{\theta}] \). The support of \( \tilde{y} \) is also a closed bounded interval \( \mathcal{Y}(\theta) = [y, \tilde{y}(\theta)] \) with \( \tilde{y}(\theta) \geq 0 \). Both \( f(y; \theta) \) and \( F(y; \theta) \) are assumed to be continuously differentiable in \( \theta \).

Let \( \lambda(y; \theta) \equiv f(y; \theta) / [1 - F(y; \theta)] \) denote the hazard rate of the distribution of \( \tilde{y} \). I make the following assumptions on the distribution of the long-term return.
Assumption 1. Type \( \theta \) orders random variables \( \{\tilde{y}(\theta)\}_{\theta \in \Theta} \) according to the strict hazard rate order: \( \lambda(y; \theta) \) is strictly decreasing in asset quality \( \theta \) for all \( y \) with \( F(y; \theta) < 1 \).

Assumption 2. The hazard rate \( \lambda(y; \theta) \) is strictly increasing in \( y \).

The hazard rate order implies first-order stochastic dominance but is weaker than the likelihood ratio order.\(^9\) As is well-known, a sufficient condition for the hazard rate to be strictly increasing in \( y \) is that \( f(y; \theta) \) be strictly log-concave in \( y \).\(^{10}\)

The creditors have a sufficiently large endowment of cash in period zero. The creditors’ ex ante belief about \( \tilde{\theta} \) is given by a PDF \( g(\theta) \) with a corresponding CDF \( G(\theta) \). To obtain sharper analytical results, I will at times assume that the long-term return is uniformly distributed on \([0, 2\tilde{\theta}]\), i.e., \( f(y; \theta) = 1/(2\theta) \mathcal{I}(0 \leq y \leq 2\theta) \) where \( \mathcal{I}(\cdot) \) denotes the indicator function. It is straightforward to verify that this specification indeed satisfies Assumptions 1 and 2.

The firm has existing debt \( b > 0 \) that matures in period zero. To refinance the maturing debt, the firm raises \( b \) units of cash in the credit market in period zero in the form of risky debt.\(^{11}\) The promised repayment (face value) of the new debt is \( B \). If the firm fails to repay its preexisting creditors in period zero, it is declared bankrupt, and the game ends. If the firm is bankrupt in period zero, the long-term asset yields zero with probability one in period one. I assume that \( b > x \), so that the creditors seize all of the firm’s cash in the event of bankruptcy at time zero.

The assets of all firms are sufficiently good that, in expectation, all firms can repay \( b \) at time one if they pay no dividends. In addition, I assume that the probability of default is always positive. Let \( \mathbb{E}[\tilde{y} | \theta] = \int_y y dF(y; \theta) \) denote the expected return from the long-term asset, conditional on the true asset quality.

Assumption 3. All firms can repay \( b \) in expectation if they pay no dividends: \( x(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] \geq b \).

Assumption 4. All debt is risky: \( x(1 - \varphi) + \tilde{y} < b \).

In period zero, the firm pays a dividend \( d \in [0, x] \) to its shareholders. The remainder, \( x - d \), is placed in the storage technology. The shareholders make no

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\(^9\) In particular, using the identity \( F(y; \theta) = 1 - \exp \left(-\int_y^\theta \lambda(x; \theta) \right) dx \), Assumption 2 implies \( F_0(y; \theta) < 0 \) for all \( y \) with \( F(y; \theta) < 1 \), i.e., strict first-order stochastic dominance. For the definition and a characterization of the more common (weak) hazard rate order, see Shaked and Shanthikumar (2007, Chapter 1.B). The hazard rate order is also known as the monotone probability ratio order (Eeckhoudt and Gollier, 1995).

\(^{10}\) Many commonly used distributions are strictly log-concave, including the normal, logistic, and—for some parameter restrictions—chi-squared, Weibull, beta, and gamma distributions (Bagnoli and Bergstrom, 2005, Table 1). Log-concavity is preserved under truncations (Bagnoli and Bergstrom, 2005, Theorem 9).

\(^{11}\) For tractability, the firm cannot choose how much to borrow. I explore a version of the model with endogenous borrowing in Section 6.6.
decisions and merely consume any dividends paid out by the firm. I assume that
the dividend $d$ is announced before the firm borrows $b$ in the credit market ("declaration date") and is paid out to the shareholders after the interaction with the credit
market ("payment date"). This timing assumption ensures that the firm does not
loot the preexisting creditors by paying out all cash as a dividend and then default-
ing when it is not able to secure new funding.

The dividend is chosen by a firm manager whose objective function is given by
a linear combination of the true values of equity and debt in the following way.
Creditors are senior to the shareholders in the event of bankruptcy. In period zero,
the shareholders receive the period zero dividend $d$. In period one, as residual
claimants, they receive any cash that is left in the firm after the creditors are repaid.
In addition, if the firm is able to repay the creditors in period one, the shareholders
receive an additional payoff equal to $V$, called the franchise value of the firm. Thus,
for a given realization $\tilde{y} = y$, the value of equity is given by

$$\pi_E(d, B, y) = d + \max\{0, (x - d)(1 - \varphi) + y - B\} + \mathbb{1}(y \geq \hat{y})V,$$

where $\hat{y}$ is the default threshold:

$$\hat{y} \equiv B - (x - d)(1 - \varphi).$$

Importantly, franchise value—which captures the continuation value of future firm
operations—cannot be transferred to creditors or used as collateral for new loans.

Similarly, the value of debt is given by

$$\pi_D(d, B, y) = \min\{(x - d)(1 - \varphi) + y, B\}.$$

Let $\Pi_E(d, B; \theta) \equiv \int_{\tilde{y}}^{\hat{y}} \pi_E(d, B, y) \, dF(y; \theta)$ denote the expected value of equity and
$\Pi_D(d, B; \theta) \equiv \int_{\tilde{y}}^{\hat{y}} \pi_D(d, B, y) \, dF(y; \theta)$ denote the expected value of debt. For a given
promised repayment schedule $B(d)$, the firm manager chooses the dividend to solve

$$\max_{d \in [0, x]} \Pi_M(d, B(d); \theta) \equiv \Pi_E(d, B(d); \theta) + \gamma \Pi_D(d, B(d); \theta).$$

Here $\gamma \in [0, 1]$ is a parameter that measures the extent to which managerial incentives are aligned with those of the creditors. When $\gamma = 0$, the manager maximizes shareholder value. When $\gamma = 1$, the manager maximizes the total value of the firm. Descriptively, $\gamma$ may be thought as capturing the extent to which inside debt is used in managerial compensation.

The creditors provide funding to the firm as long as they break even in expected
terms. Since there are many creditors, in equilibrium, the creditors must exactly break even since otherwise they would be undercut. Thus, in equilibrium, the following break-even condition must hold:

$$\int_{\bar{\theta}}^{\theta} \int_{y}^{B(\theta)} \min\{(x - d)(1 - \varphi) + y, B\} \, dF(y; \theta) \, dB(\theta; d) = b. \tag{8}$$

The expectation over types is taken under the subjective belief of the credit market about asset quality, given by the CDF $B(\theta; d)$. In equilibrium, the beliefs must be consistent with equilibrium play, as specified later.

Finally, whenever the firm defaults on its obligations, there is a social cost of default, denoted by $\Delta \geq 0$. The social cost of default captures any externalities arising from default that are not taken into account when making private decisions.

**Timeline.** Summarizing, the sequence of events is as follows:

\[ t = 0 \]
1. The firm manager announces the dividend $d$, taking the promised repayment schedule offered by the credit market, $B(d)$, as given.
2. Given the announced dividend, the promised repayment, $B$, is determined by the break-even condition. If there is no promised repayment at which the creditors break even, the firm does not obtain funding and is declared bankrupt.
3. The dividend is paid out to the shareholders.

\[ t = 1 \]
The return on the long-term asset, $y$, is realized. If the firm does not have enough cash to repay the creditors, it is declared bankrupt.

### 3.2 Key Assumptions: Discussion

I now discuss the key elements of the model in more detail.

The franchise value, $V$, can be interpreted as the loss of future cash flows arising from disorderly liquidation. In a banking context, if a regulator needs to resolve a bank quickly, some value from future operations may be lost. More broadly, one may think of $V$ as capturing various costs of default, including reputational concerns of the firm manager, which are not modelled explicitly. The social cost of default, $\Delta$, captures any externalities arising from the default that are not taken into account by the firm.

The constraints $d \geq 0$ and $d \leq x$ roughly correspond to no issuance of equity and no asset sales, respectively. Generalizing the model to include equity issuance (negative dividends) or sales of the long-term asset is not difficult but would not change the basic results.
The imperfect storage technology introduces a non-trivial role for dividends even in the absence of asymmetric information. Paying a higher dividend avoids the deadweight cost of cash, \( \varphi \), but puts the franchise value at greater risk. The assumption that \( \varphi > 0 \) can be justified in multiple ways. One way is to interpret \( \varphi \) as the opportunity cost of holding liquid assets. In this interpretation, \( \varphi \) can be thought of as the liquidity premium on liquid assets. The setting is also equivalent to a situation in which storage is costless but the shareholders have access to a superior investment opportunity than the firm. A second interpretation of \( \varphi > 0 \) is that when the firm has a lot of cash, the manager wastes a part of it on inefficient projects, as in the agency-based explanations of dividends in Easterbrook (1984) and Jensen (1986). In this interpretation, \( \varphi \) is a measure of agency frictions. Finally, \( \varphi \) may capture clientele effects or catering incentives, as in Allen, Bernardo, and Welch (2000) or Baker and Wurgler (2004).

From an analytical perspective, setting \( \gamma \), the weight that managerial compensation puts on short-term debt, to unity is a convenient way to shut down risk shifting. When \( \gamma = 1 \), the manager cares about the total firm value, and any pure transfer of wealth between the shareholders and creditors has no effect on the managerial payoff. A positive value of \( \gamma > 0 \) can be justified by the empirical observation that a significant fraction of managerial compensation, including defined benefit pensions and deferred compensation, has a debt-like payoff structure (Sundaram and Yermack, 2007). The key results of this paper do not require the manager to place positive weight on the value of debt.

3.3 Fair Value of Debt and Equity

I now provide formulas for the fair value of debt and equity. Suppose that the creditors believe that the type of the firm is \( \theta \in [\underline{\theta}, \overline{\theta}] \) with probability one. We have the following result.

**Lemma 1** (Value of Debt). *If the credit market believes the firm’s type to be \( \theta \) with probability one, then the repayment \( B \) is implicitly defined by*

\[
B = b + \int_{\hat{y}}^{\overline{y}} F(y; \theta) \, dy \text{ where } \hat{y} = B - (x - d)(1 - \varphi).
\] (9)

*Denote the solution to Eq. (9), if it exists, by \( B^*(d, \theta) \). Then:

(a) if \( b \leq (x - d)(1 - \varphi) + \overline{y} \), the unique solution is \( B^*(d, \theta) = b \);
(b) if \( b - (x - d)(1 - \varphi) \in (\underline{y}, \mathbb{E}[\overline{y}|\theta]) \), there exists a unique solution \( B^*(d, \theta) \in (b, +\infty) \);*
(c) if \( b = (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] \), all \( B^*(d, \theta) \in [(x - d)(1 - \varphi) + \tilde{y}(\theta), +\infty) \) constitute a solution;

(d) if \( b > (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] \), no solution exists.

Finally, provided that \( F(\hat{y}; \theta) < 1 \), \( \partial B^* / \partial d = F(\hat{y}; \theta)(1 - \varphi) / [1 - F(\hat{y}; \theta)] > 0 \) and \( \partial B^* / \partial \theta = \left( \int_{\tilde{y}}^{\hat{y}} F(y; \theta) \, dy \right) / [1 - F(\hat{y}; \theta)] < 0 \).

**Proof.** In the Appendix.

In what follows, I will say that a firm is **solvent** when it pays a dividend \( d \) if the debt pricing equation Eq. (9) has a solution, i.e., the creditors can break even at the firm’s dividend policy.

The debt pricing equation decomposes the promised repayment, \( B \), into its risk-free component and a credit spread. Provided that the default probability is strictly less than one, the credit spread is strictly decreasing in the perceived quality of the firm’s assets and strictly increasing in the dividend. The results imply that the true value of debt, as a function of the dividend and the promised repayment, is given by

\[
\Pi_D(d, B; \theta) = B - \int_{\tilde{y}}^{\hat{y}} F(y; \theta) \, dy. \tag{10}
\]

I now turn to the value of equity.

**Lemma 2** (Value of Equity). Conditional on the firm obtaining funding, the value of equity is given by

\[
\Pi_E(d, B; \theta) = d + (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] + [1 - F(\hat{y}; \theta)] V + \int_{\tilde{y}}^{\hat{y}} F(y; \theta) \, dy - B, \tag{11}
\]

where \( \tilde{y} = B - (x - d)(1 - \varphi) \).

**Proof.** In the Appendix.

The value of equity consists of several components. First, the shareholders receive the first-period dividend \( d \). The remaining cash is stored until the next period and yields \( (x - d)(1 - \varphi) \). Then, the shareholders receive the expected value of the long-term return, \( \mathbb{E}[\tilde{y} | \theta] \), the expected franchise value \( [1 - F(\hat{y}; \theta)] V \) and must repay \( B \) to the creditors. Finally, the term \( \int_{\tilde{y}}^{\hat{y}} F(y; \theta) \, dy \) gives the value of limited liability. Since the shareholders get zero whenever the value of the long-term return is sufficiently low, the shield from negative payoffs provided by limited liability is a source of value to the shareholders.
Combining the results in Lemmas 1 and 2, the objective function of the firm manager can be written as

\[
\Pi_M(d, B; \theta) = d + (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y}|\theta] + [1 - F(\tilde{y}; \theta)]V + (1 - \gamma) \int_{\tilde{y}} F(y; \theta) \, dy - (1 - \gamma)B.
\] (12)

### 3.4 Benchmark: First-Best Dividend Policy

I now define and characterize the first-best dividend policy. The first-best dividend policy is found by maximizing total output in the economy, inclusive of the firm’s franchise value and the social cost of default, subject to the break-even constraint of the creditors. The social planner solves the following problem.

**Problem 1 (First-Best Dividend Policy).**

\[
V^*(\theta) \equiv \max_{d, B} \left\{ \left( x - d \right)(1 - \varphi) + \mathbb{E}[\tilde{y}|\theta] + \left( 1 - F(\tilde{y}; \theta) \right)(V + \Delta) \right\}
\]

s.t. \( B = b + \int_{\tilde{y}}^{\hat{y}} F(y; \theta) \, dy, \quad d \in [0, x]. \)

As before, the default threshold is given by \( \hat{y} = B - (x - d)(1 - \varphi). \) In what follows, I will call the dividend that solves Problem 1 first best or socially efficient, and refer to \( V + \Delta \) as the social franchise value of the firm.

As before, write \( B^*(d, \theta) \) for the value of \( B \) that solves the break-even constraint in Problem 1 for a given dividend \( d \) and type \( \theta \). By the solvency assumption (Assumption 3), \( B^*(d, \theta) \) is well-defined when \( d \) is sufficiently close to zero. However, Lemma 1 shows that if the firm pays a sufficiently large dividend, there may be no repayment \( B \) at which the creditors break even. Let

\[
\overline{d}(\theta) = \min \left\{ x, x + \frac{\mathbb{E}[\tilde{y}|\theta] - b}{1 - \varphi} \right\}
\] (13)

denote the highest feasible dividend. The highest feasible dividend is given by the smaller of (i) the firm’s cash holdings; and (ii) the highest value of the dividend at which the creditors can break even.

With this additional piece of notation, we can transform Problem 1 into an equivalent single-dimensional optimization problem.\(^{13}\)

---

\(^{12}\)The formulation in Problem 1 does not allow the social planner to liquidate the firm in period zero. This assumption involves no loss of generality since liquidation in period zero is never optimal given the assumptions.

\(^{13}\)From Lemma 1, if \( d = \overline{d}(\theta) \neq x \), there are multiple values of \( B \) at which the creditors break-even and one can pick any \( B \geq (x - d)(1 - \varphi) + \tilde{y}(\theta). \)
Problem 2 (First-Best Dividend Policy: Equivalent Formulation).

\[ \max_{d \in [0, \bar{d}(\theta)]} S(d; \theta) \equiv d + \left[ (x - d)(1 - \varphi) + \mathbb{E}[\hat{y}|\theta] + \left( 1 - F(\hat{y}; \theta) \right)(V + \Delta) \right], \]

where \( \hat{y} = B^*(d, \theta) - (x - d)(1 - \varphi) \).

The derivative of the objective function in Problem 2 with respect to the dividend is given by

\[ S_d(d; \theta) = \varphi - (1 - \varphi)\lambda(\hat{y}; \theta)(V + \Delta) \tag{14} \]

where again \( \lambda(y; \theta) = f(y; \theta) / [1 - F(y; \theta)] \) is the hazard rate. Intuitively, the benefit of paying a higher dividend is that it avoids the deadweight loss that arises from holding cash. On the other hand, a higher dividend increases the probability of losing the franchise value and incurring the social cost of default in the event of bankruptcy. Eq. (14) explains why the hazard rate plays a key role in the analysis of the model.

By Assumption 1, the hazard rate \( \lambda(y; \theta) \) is strictly decreasing in \( \theta \). As a result, the socially efficient solution has firms with higher-quality assets paying (weakly) greater dividends. By Assumption 2, the hazard rate \( \lambda(y; \theta) \) is strictly increasing in \( y \), and therefore the objective in Problem 2 is concave. Note that if \( V > 0 \), it can never be the case that the firm pays \( d = \bar{d}(\theta) \neq x \): if that were the case, \( \lambda(\hat{y}; \theta) \) would be infinite, and thus \( S_d(\bar{d}(\theta); \theta) < 0 \), a contradiction.\(^{14}\)

These basic observations allow us to fully characterize the first-best dividend policy. Write

\[ \mathbb{V}(\theta) = \frac{\varphi}{(1 - \varphi)\lambda(\hat{y}_H(\theta); \theta)} \]
\[ \bar{\mathbb{V}}(\theta) = \frac{\varphi}{(1 - \varphi)\lambda(\hat{y}_L(\theta); \theta)} \tag{15} \]

where

\[ \hat{y}_L(\theta) = B^*(0, \theta) - x(1 - \varphi) \quad \text{and} \quad \hat{y}_H(\theta) = B^*(x, \theta), \tag{16} \]

provided that \( B^*(x, \theta) \) exists.\(^{15}\) We have the following result.

**Proposition 1** (First-Best Dividend Policy). If \( V + \Delta > 0 \), then:

---

\(^{14}\)The fact that the hazard rate is infinite at \( y = \bar{y}(\theta) \) follows from the assumption that that \( f(y; \theta) \) is continuous in \( y \) on \( \mathcal{Y}(\theta) \), and therefore bounded, and, of course, \( 1 - F(y; \theta) \to 0 \) as \( y \to \bar{y}(\theta) \).

\(^{15}\)If \( B^*(x, \theta) \) does not exist (i.e., the firm is insolvent at \( d = x \)), the optimal dividend \( d^*_FB(\theta) < \bar{d}(\theta) \), as discussed above. Formally, to ensure that Proposition 1 covers all cases, set \( \mathbb{V}(\theta) = +\infty \) when \( B^*(x, \theta) \) does not exist.
(a) if \( V + \Delta \geq \overline{V}(\theta) \), \( d_{FB}^*(\theta) = 0 \);
(b) if \( V + \Delta \leq \underline{V}(\theta) \), \( d_{FB}^*(\theta) = x \);
(c) if \( V + \Delta \in (\underline{V}(\theta), \overline{V}(\theta)) \), \( d_{FB}^*(\theta) \) solves the first-order condition \( S_d(d; \theta) = 0 \).

If \( V = \Delta = 0 \) and \( \varphi > 0 \), \( d_{FB}^*(\theta) = \overline{d}(\theta) \), and if \( V = \Delta = \varphi = 0 \), the optimal dividend is indeterminate: \( d_{FB}^*(\theta) \in [0, \overline{d}(\theta)] \). In all cases, the optimal dividend \( d_{FB}^*(\theta) \) is weakly increasing in \( \theta \) and \( \varphi \) and weakly decreasing in \( V \) and \( \Delta \).

If holding cash is sufficiently costly (\( \varphi \) is sufficiently close to one) or the social franchise value, \( V + \Delta \), is sufficiently small, the optimal strategy is to pay out all cash as a dividend. Vice versa, when storage is sufficiently efficient (\( \varphi \) is sufficiently close to zero) or the social franchise value is sufficiently high, the optimal dividend is zero. The optimal dividend is increasing in asset quality. Intuitively, Assumption 2 ensures that increasing the dividend increases the probability of default by less for higher-quality firms. Therefore, the marginal cost of increasing the dividend is smaller for higher-quality firms.\(^{16}\) The marginal benefit of dividends (avoiding the costly storage of cash), in contrast, is independent of asset quality. As a result, higher types optimally pay greater dividends.

Provided that not all \( \varphi, V \) and \( \Delta \) are zero, the first-best dividend policy is uniquely determined. This feature makes the model different from the basic setting of Miller and Modigliani (1961, Section 1) in which only the present value of dividends is determinate.\(^{17}\) The reason is that the expected franchise value, expected social cost of default and the cost of holding cash are all affected by the dividend choice. Hence, dividend policy affects firm value. In contrast, when \( \varphi, V \) and \( \Delta \) are all zero, the timing of dividend payments is immaterial, and only the present value of dividends, \( x + \mathbb{E}[\tilde{y} | \theta] \), is determinate. The trade-off between losing the franchise value and the inefficiency of holding cash is economically similar to the classic trade-off models of capital structure, as surveyed, for example, in Myers (2003, Section 3).

### 3.5 Benchmark: Symmetric Information

As a second benchmark, suppose that asset quality \( \theta \) is directly observable to the credit market. The firm manager chooses the dividend to maximize her objective in Eq. (12). Substituting the expression for \( B^*(d, \theta) \) from the debt pricing equation

\(^{16}\) First-order stochastic dominance is not enough to obtain monotonicity with respect to asset quality: even if high-quality firms have a lower probability of default for a given dividend, increasing the dividend may nevertheless increase the probability of default by more for high-quality firms.

\(^{17}\) See also DeAngelo and DeAngelo (2006) for a discussion of the Modigliani-Miller dividend policy irrelevance result and additional references.
In what follows, I will write \( d^*_S(\theta) \) to denote the optimal dividend under symmetric information, i.e., the value of \( d \) that maximizes the objective function above.

Clearly, if \( \Delta = 0 \), then apart from the constant \((1 - \gamma)b\) which does not affect the choice of the dividend, the objective of the manager coincides with the objective in the first-best problem (Problem 1). Thus, the optimal dividend is the same in both problems. Importantly, even when the interests of the manager and the creditors are not perfectly aligned \((\gamma < 1)\), there are no inefficiencies that arise because of the conflict of interest. Indeed, the value of \( \gamma \) is irrelevant for the optimal dividend policy. Even if \( \gamma = 0 \) so that the firm manager only directly cares about shareholder value, the manager, in effect, maximizes total firm value as any effect on the creditors is fully internalized via changes in funding costs. As we will see, symmetric information is key for this result to obtain.

When \( \Delta > 0 \), the two objective functions are no longer identical and the privately optimal dividend choice will feature dividends that are too high from a social perspective. Specifically, from Proposition 1, we know that the first-best dividend is decreasing in the franchise value \( V \). As a result, when \( \Delta > 0 \), the privately optimal level of dividends is socially excessive.\(^{18}\)

### 4 Dividend Policy Under Asymmetric Information

I now turn to the analysis of the baseline model when the quality of the firm’s long-term asset is unknown to its creditors.

I begin by formally defining the equilibrium studied in this paper.

**Definition 1.** An equilibrium in pure strategies is a triple \( \langle d^*(\theta), B^*(d), B^*(\theta; d) \rangle \) that consists of (i) dividend policy \( d^*(\theta) \); (ii) repayment schedule \( B^*(d) \); and (iii) beliefs \( B^*(\theta; d) \), such that the following conditions hold:

1. **Optimal dividends:** for all \( \theta \in \Theta \), \( d^*(\theta) \) maximizes \( \Pi_M(d, B^*(d); \theta) \) (i.e., solves Eq. (7)), taking the repayment schedule \( B^*(d) \) as given.

2. **Creditors break even:** for all \( d, B^*(d) \) is the smallest repayment at which the creditors break even (i.e., Eq. (8) holds), given the beliefs \( B^*(\theta; d) \), or the firm obtains no funds.

\(^{18}\) Efficiency can fully be restored by imposing an appropriate linear tax on dividends. The optimal tax, however, requires the government to observe the quality of the long-term asset.
3. Bayesian consistency: if \( d = d^*(\theta) \) for some \( \theta \), equilibrium beliefs \( B^*(\theta;d) \) are determined by Bayes’ Rule; if not, beliefs \( B^*(\theta;d) \) constitute a CDF of \( \theta \) but are otherwise unrestricted.

The equilibrium definition used in this paper is, in essence, Perfect Bayesian Equilibrium, while the interaction between the firm and the credit market is a signaling game (for a recent overview of signaling games, see Sobel, 2012). The informed party—the firm—moves first by announcing its dividend. The uninformed party—the credit market—moves second and is required to best-respond to the firm’s action in the sense that the creditors must exactly break even. The credit market must satisfy the break-even constraint with respect to some set of beliefs. Beliefs are pinned down by Bayes’ Rule for dividends that are observed in equilibrium. If some dividend is never observed in equilibrium, then beliefs are not determined by Bayes’ Rule but must nevertheless be fully specified. The requirement to fully specify beliefs rules out certain non-credible threats (see, e.g., Gibbons, 1992, Chapter 4). As is common in the literature, I do not explicitly model competition between the creditors, apart from requiring the break-even constraint to hold with equality. It is well-known that there may be no Nash equilibrium in a model in which a finite number of creditors compete with each other by offering different repayment schedules (see Riley, 1979, Theorem 3).

In much of the paper, I focus on separating equilibria. An equilibrium is said to be separating if the function \( d^*(\theta) \) in Definition 1 that maps types to equilibrium dividends is one-to-one.

### 4.1 Spence–Mirrlees Single-Crossing Condition

In signaling games with a continuum of types, the Spence–Mirrlees condition plays a crucial role for the existence of separating equilibria as well as the monotonicity of the signals with respect to the underlying types. That is also the case in the present model.

**Definition 2.** The payoff function \( \Pi_M(d, B; \theta) \) is said to satisfy the (strict) Spence–Mirrlees condition if the marginal rate of substitution (MRS) between the dividend and the promised repayment

\[
\text{MRS}(d, B; \theta) \equiv -\frac{\partial \Pi_M}{\partial d} / \frac{\partial \Pi_M}{\partial B}
\]

is (strictly) monotone in \( \theta \).

For concreteness, suppose that that the managerial payoff is strictly increasing in the dividend. Consider a candidate equilibrium in which higher-quality firms pay
lower dividends. For this configuration to constitute an equilibrium, it must be the case that a lower-quality firm does not find it profitable to mimic a higher-quality firm by decreasing its dividend. The Spence–Mirrlees condition ensures this by requiring firms with better assets to be more willing to trade-off a lower dividend for a lower repayment.

As is well-known, the Spence–Mirrlees condition implies that the optimal dividend is a monotone function of asset quality for any given repayment schedule $B(d)$.

**Lemma 3.** Fix a repayment schedule $B(d)$. Suppose that the marginal rate of substitution $\text{MRS}(d, B; \theta)$ is weakly increasing (decreasing) in type $\theta$. Then, the optimal dividend $d^*(\theta) \equiv \arg \max_d \Pi_M(d, B(d); \theta)$ is weakly increasing (decreasing) in $\theta$.

**Proof.** See, for example, Riley (1979, Lemma 2) or Athey, Milgrom, and Shannon (1998, Theorem 3.2). □

When the Spence–Mirrlees condition holds, therefore, in any equilibrium (separating or not), dividends are monotone in asset quality. If that were not the case, the conjectured equilibrium would contradict the optimality of dividend choice (part 1 of Definition 1). This way, the Spence–Mirrlees condition imposes a clear structure on how equilibrium dividend policies can vary with asset quality.

In the present model, the Spence–Mirrlees condition may or may not hold, and even when the condition holds, the marginal rate of substitution may be either increasing or decreasing in the quality of the long-term asset. We calculate that

$$\text{MRS}(d, B; \theta) = \frac{\varphi + (1 - \varphi)[(1 - \gamma)F(\hat{y}; \theta) - f(\hat{y}; \theta)V]}{(1 - \gamma) - [(1 - \gamma)F(\hat{y}; \theta) - f(\hat{y}; \theta)V]}.$$  \hfill (18)

The numerator gives the marginal benefit of increasing the dividend for fixed funding costs (i.e., a fixed promised repayment $B$). Paying a higher dividend avoids the deadweight cost of holding liquidity, $\varphi$, but increases the probability of losing the franchise value $V$ by $(1 - \varphi)f(\hat{y}; \theta)$. In addition, since the funding costs are fixed, a higher dividend transfers wealth from the creditors to the shareholders. The firm is bankrupt with probability $F(\hat{y}; \theta)$, and for one dollar paid out in dividends, the creditors get $1 - \varphi$ dollars less in bankruptcy. All in all, a higher dividend increases the managerial payoff by $(1 - \gamma)(1 - \varphi)F(\hat{y}; \theta)$. As long as the interests of the manager and the creditors are not perfectly aligned ($\gamma \neq 1$), the manager has an incentive to engage in risk shifting by increasing the dividend and diverting resources from the creditors in bankruptcy states.

While the denominator in Eq. (18) is always positive, the numerator can be negative or positive depending on the franchise value $V$. As a result, the marginal rate
of substitution between dividends and repayment can be either negative or positive. Looking at Eq. (18), we see that $\theta$ enters the marginal rate of substitution only through the term in the square brackets. Thus, we have that

$$\text{sign}\left\{ \frac{\partial \text{MRS}}{\partial \theta} \right\} = \text{sign}\left\{ \frac{\partial^2 \Pi_M}{\partial d \partial \theta} \right\} = \text{sign}\left\{ F_\theta(\hat{y}; \theta)(1 - \gamma) - f_\theta(\hat{y}; \theta)V \right\}. \quad (19)$$

The rightmost expression consists of two terms. The first term, $F_\theta(1 - \gamma)$, which is always weakly negative, stems from the risk shifting effect described above: firms with higher-quality assets are less likely to default in period one. Any cash that is not paid out as a dividend is seized by the creditors in the event of default. As a result, paying a low dividend is more costly for firms with lower-quality assets.

The second term, $f_\theta(\hat{y}; \theta)V$, arises from the fact that paying a higher dividend increases the probability of losing the franchise value. The partial derivative $f_\theta$ shows how the increase in the default probability varies with asset quality. This term can be either positive or negative, depending on the specification of asset returns, as the strict hazard rate order clearly does not imply that $f(y; \theta)$ is monotone in $\theta$. For the special case of uniformly distributed returns, however, $f_\theta < 0$, and so the probability of default increases by less for firms with higher-quality assets.

To sum up, depending on the parameter values, the sum of the risk shifting and franchise value effects can be either positive or negative, and therefore the marginal rate of substitution may be either increasing or decreasing in asset quality. In the important special case when $V = 0$, the marginal rate of substitution is positive and strictly decreasing in asset quality. As a result, when $V = 0$, asset quality and equilibrium dividends are negatively related in any equilibrium.

### 4.2 Characterizing Separating Equilibria

I now turn to the analysis of separating equilibria. Separating equilibria are economically interesting because, as in Section 2, when the Spence–Mirrlees condition holds, (partially) pooling equilibria usually require implausible off-equilibrium beliefs to be sustained. In particular, such equilibria are typically ruled out by the D1 equilibrium refinement of Cho and Kreps (1987).$^{19}$

Let $\hat{\theta}(d)$ be a differentiable bijective function mapping the equilibrium dividend to the firm’s type, and, as before, denote the solution to the break-even constraint

$^{19}$In some cases, the D1 equilibrium refinement of Cho and Sobel (1990) is consistent with the firms pooling at either $d = 0$ or $d = x$. In such situations, high-quality firms would like to deviate to $d < 0$ or $d > x$ but doing so is not feasible. See Footnote 22 for more discussion.
in Eq. (9) by $B^*(d, \theta)$. The firm manager solves the following problem.

**Problem 3** (Optimal Dividend in a Separating Equilibrium).

$$\max_{d \in [0, x]} \Pi_M(d, B^*(d, \hat{\theta}(d)); \theta)$$

At an interior solution, the optimal dividend must satisfy

$$\frac{\partial \Pi_M}{\partial d} + \frac{\partial \Pi_M}{\partial B} \left( \frac{\partial B^*}{\partial d} + \frac{\partial B^*}{\partial \hat{\theta}'(d)} \right) = 0. \quad (20)$$

Provided that $\hat{y} < \bar{y}(\theta)$, the first-order condition can be rewritten as

$$\text{MRS}(d, B^*(d, \hat{\theta}(d)); \theta) = \frac{\partial B^*}{\partial d} + \frac{\partial B^*}{\partial \hat{\theta}'(d)}. \quad (21)$$

In equilibrium, beliefs of the credit market must be correct, i.e., $\theta = \hat{\theta}(d)$. Imposing this equilibrium condition, using the expressions given in Lemma 1 and Eq. (18) and simplifying, we obtain an ordinary differential equation that characterizes $\hat{\theta}(d)$:

$$\varphi - (1 - \varphi) \frac{f(\hat{y}; \hat{\theta}(d))}{1 - F(\hat{y}; \hat{\theta}(d))} V = \hat{\theta}'(d) \int_{\hat{y}}^{\bar{y}} F_\theta(y; \hat{\theta}(d)) \, dy \times \left[ (1 - \gamma) + \frac{f(\hat{y}; \hat{\theta}(d))}{1 - F(\hat{y}; \hat{\theta}(d))} V \right], \quad (22)$$

where $\hat{y} = B^*(d, \hat{\theta}(d)) - (x - d)(1 - \varphi)$. In any separating equilibrium, the firm with the lowest quality assets must pay the same dividend as it would with symmetric information, see, for instance, Mailath and von Thadden (2013, p. 1844). Thus, we have the initial condition

$$\hat{\theta}(d^*_{SI}(\theta)) = \theta. \quad (23)$$

From Eq. (17), the left-hand side of the differential equation above is just the derivative of the managerial objective function under symmetric information, evaluated at $\theta = \hat{\theta}(d)$. At the same time, the term multiplying $\hat{\theta}'(d)$ is weakly negative and

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20By the results in Mailath and von Thadden (2013), there exists no non-differentiable separating equilibrium. Assumption 1 of Mailath and von Thadden is satisfied as long as not both $\varphi$ and $V$ are zero. If $V = 0$, Theorem 3 of Mailath and von Thadden implies that there is no non-differentiable separating equilibrium. When $V > 0$, Assumption 2 of Mailath and von Thadden is automatically satisfied as an implication of Assumption 2 of the present paper. The requirement that $V_{\omega}(\omega, \omega, x) \neq 0$ (in the notation of Mailath and von Thadden) is satisfied as long as $f_\theta(\hat{y}; \theta)(1 - \gamma) - f_\theta(\hat{y}; \theta)V \neq 0$ for all $\theta \in \Theta$ and $d \in [0, x]$ where $\hat{y} = B^*(d, \theta) - (x - d)(1 - \varphi)$. If the latter condition holds, Theorem 2-a of Mailath and von Thadden shows that any separating equilibrium dividend schedule $d^*(\theta)$ is differentiable in $\theta$ whenever $\theta \neq \theta$. 
strictly so provided that not both $1 - \gamma$ and $V$ are zero. As a result, if $\hat{\theta}(d)$ is increasing, firms must pay dividends that are higher than under symmetric information for the equation to hold, and dividends must be lower than under symmetric information if $\hat{\theta}(d)$ is decreasing.

This observation allows one to show that the qualitative behavior of $\hat{\theta}(d)$ is determined by the properties of the optimal dividend policy under symmetric information.\textsuperscript{21}

Lemma 4. The solution to the differential equation in Eq. (22), $\hat{\theta}(d)$, is:

- strictly increasing in $d$ if $d^*_\text{SI}(\theta) = 0$;
- strictly decreasing for $d < d^*_\text{SI}(\theta)$ and strictly increasing for $d > d^*_\text{SI}(\theta)$ with an interior minimum at $d = d^*_\text{SI}(\theta)$ if $d^*_\text{SI}(\theta) \in (0, \bar{d}(\theta))$;
- strictly decreasing in $d$ if $d^*_\text{SI}(\theta) = \bar{d}(\theta)$.

Proof. In the Appendix. \hfill $\Box$

The differential equation in Eq. (22) was constructed by taking the first-order condition of Problem 3 and imposing the equilibrium condition $\theta = \hat{\theta}(d)$. Clearly, the solution to the differential equation may not induce a separating equilibrium. First, the second-order condition for individual maximization may not be satisfied, or we may have characterized a local instead of a global maximum. In addition, the solution may violate feasibility of the dividend choice (i.e., the fact that $d$ must lie in $[0, x]$) or wander off to a region in which the creditors cannot break even at any promised repayment. When the Spence–Mirrlees condition holds, however, the first-order approach is sufficient to ensure global optimality, and the solution to the differential equation can be shown to constitute an equilibrium under appropriate conditions (Riley, 1979).

4.3 High Dividends Are Bad News When Franchise Value Is Low

I now consider a special case of the model with $V = 0$ and $\gamma \neq 1$. In this situation, the general differential equation in Eq. (22) specializes to

$$\varphi = (1 - \gamma) \hat{\theta}'(d) \int_{\hat{y}}^{\bar{y}} F_\theta(y; \hat{\theta}(d)) \, dy \quad \text{with} \quad \hat{\theta}(\bar{d}(\theta)) = \theta.$$  \hspace{1cm} (24)

The lowest type pays the highest feasible dividend $\bar{d}(\theta)$, defined earlier in Eq. (13): since the franchise value is zero, leaving cash inside the firm has no benefit. There-\textsuperscript{21}This result is standard and arises in other signaling models, including, in particular, Riley (1979, pp. 337–8).
fore, all firms pay out the maximum feasible dividend under symmetric information.

Since the integrand is strictly negative by Assumption 1 and, in addition, \( \gamma \neq 1 \), \( \hat{\theta}'(d) \) has to be strictly negative for Eq. (24) to hold: higher types must pay lower dividends in equilibrium.

The following proposition, whose proof is a rather straightforward adaptation of Riley (1979, Theorem 1), shows that the solution to Eq. (24), \( \hat{\theta}(d) \), constitutes an equilibrium and characterizes its properties.

Proposition 2. Suppose that the franchise value \( V \) is equal to zero, \((\theta - \hat{\theta})\) is sufficiently small, and for all \( \theta > \hat{\theta} \) with \( \bar{y}(\theta) = \bar{y}(\hat{\theta}) \), \( \lim_{\hat{y} \to \bar{y}(\hat{\theta})} \frac{f(\hat{y}; \theta)}{f(\hat{y}; \hat{\theta})} > 1 \). Consider the following out-of-equilibrium beliefs:

- for \( d < \hat{\theta}^{-1}(\hat{\theta}) \), the credit market attributes the deviation to \( \theta = \hat{\theta} \);
- for \( d > \bar{d}(\hat{\theta}) \), the credit market attributes the deviation to \( \theta = \hat{\theta} \).

Then, \( \hat{\theta}(d) \) induces a separating equilibrium. In addition, \( \hat{\theta}(d) \) and \( B^*(d) \) are strictly decreasing while the probability of default \( F(\bar{y}; \hat{\theta}(d)) \) is strictly increasing in \( d \).

Proof. In the Appendix. \( \square \)

In equilibrium, firms with higher-quality assets pay lower dividends. The result stands in sharp contrast to standard models of signaling via dividends in which dividends are an unambiguously positive signal. The probability of default is strictly increasing in the dividend, while the promised repayment is strictly decreasing in the dividend. The separating equilibrium satisfies the D1 equilibrium refinement of Cho and Sobel (1990).\(^{22}\) The condition on the upper boundary of the supports, \( \bar{y}(\hat{\theta}) \), is (vacuously) satisfied if \( y'(\hat{\theta}) > 0 \). As the proof of Proposition 2 makes clear, the assumption is only used to show that deviating to \( d = \bar{d}(\hat{\theta}) \) is not profitable in the admittedly extreme case in which the lowest quality firm pays the maximum feasible dividend and defaults with probability one in period one.

\(^{22}\) Similarly to Cho and Sobel (1990), there may be a pooling equilibrium that satisfies the D1 equilibrium refinement in which all firms pay \( d = 0 \). In this equilibrium, all deviations to \( d > 0 \) are attributed to the lowest quality firm. This configuration can be an equilibrium if and only if the lowest quality firm does not find it optimal to deviate to \( \bar{d}(\hat{\theta}) \) which is the case if and only if

\[
\bar{d}(\hat{\theta}) - (1 - \gamma) \int_0^{\hat{\theta}} \int_{\hat{y}}^{\bar{y}} \{ F(y; \hat{\theta}) - F(y; \theta) \} \, dy \, dG(\theta) \leq 0,
\]

where \( \bar{y} = B^*_p - x(1 - \varphi) \) and \( B^*_p \) is the promised repayment in the pooling equilibrium. Clearly, the inequality above is violated whenever \( (\hat{\theta} - \hat{\theta}) \) is sufficiently small. Thus, whenever \( (\hat{\theta} - \hat{\theta}) \) is sufficiently small, the separating equilibrium above exists and, moreover, it is the unique outcome that satisfies the D1 equilibrium refinement of Cho and Sobel (1990).
For illustration, Figure 3 plots the separating equilibrium for a numerical example with uniformly distributed returns. To facilitate comparison with benchmark allocations, I plot \( d^*(\theta) \), the inverse of \( \hat{\theta}(d) \), which maps asset quality to the equilibrium dividend. The equilibrium dividends are decreasing in asset quality. As a consequence, the promised repayment as well as the probability of default are increasing in the dividend. Comparing the separating equilibrium to the optimal dividend policy under symmetric information, we observe that the probability of default and the promised repayment are both lower under asymmetric information; since the costs of asymmetric information are eventually borne out by the firm, the managerial payoff is also lower. The figure also illustrates the need for the support of asset quality to be small enough: if \( \theta \) is sufficiently high, the solution to the differential equation has \( d^*(\theta) < 0 \).

The figure also shows the socially optimal dividend policy. The socially optimal dividend is increasing in asset quality, as it must be by Proposition 1. While low-quality firms pay socially excessive dividends, for these particular parameter values, dividends paid by high-quality firms under asymmetric information are too low from a social point of view. That is the case even though under symmetric information, privately optimal dividends are always excessive. If the distribution of asset qualities places most of the probability mass at low-quality firms, then the separating equilibrium under asymmetric information welfare-dominates the equilibrium with symmetric information.

Importantly, the result in Proposition 2 holds not only in the knife-edge case when \( V \) is exactly zero but whenever the franchise value is small enough.

**Corollary 1.** Suppose that \( \gamma \neq 1 \) and \((\overline{\theta} - \theta)\) is sufficiently small, and if \( E[\tilde{y}|\theta] \leq b, F_\theta(\tilde{y}(\theta);\theta) < 0 \). (Note that if \( E[\tilde{y}|\theta] > b, F_\theta(\tilde{y}(\theta);\theta) \) may be zero.)\(^{23} \) Then, the solution to the general differential equation (Eq. (22)), \( \hat{\theta}(d) \), induces a separating equilibrium with \( \hat{\theta}'(d) < 0 \) whenever the franchise value is small enough.

**Proof.** In the Appendix. \( \square \)

I finish with a few remarks on the role played by the deadweight cost of holding cash, \( \varphi \). From Eq. (24), a positive cost of holding cash is clearly a necessary condition for the existence of a separating equilibrium when \( V = 0 \). Note, however, that the marginal rate of substitution is strictly decreasing in asset quality even when \( \varphi = 0 \). Thus, \( \varphi > 0 \) is not driving the negative relationship between equilibrium dividends.

\(^{23}\) The assumption that \( F_\theta(\tilde{y}(\theta);\theta) < 0 \) if \( E[\tilde{y}|\theta] \leq b \) is used to bound the expression determining the monotonicity of the marginal rate of substitution with respect to \( \theta \). A sufficient condition for \( F_\theta(\tilde{y}(\theta);\theta) < 0 \) to be true is that \( \tilde{y}'(\theta) > 0 \). If the lowest quality type firm is solvent when it pays \( d = x \), then \( F_\theta(\tilde{y}(\theta);\theta) \) may be zero (as is the case when \( \tilde{y}(\theta) \) is independent of \( \theta \)).
Figure 3: Numerical example with uniformly distributed returns ($y \sim \text{Unif}(0, 2\theta)$). Top left to right: dividend policy and promised repayment. Bottom left to right: default probability and managerial payoff. All variables are plotted as a function of asset quality $\theta$. Parameter values are $\theta = 2$, $b = 1$, $x = 0.25$, $\gamma = V = 0$, and $\Delta = 0.17$.

dividends and asset quality. To understand why $\varphi > 0$ is necessary for the existence of a separating equilibrium, suppose that $\varphi = V = 0$ and consider a candidate equilibrium schedule $\hat{\theta}(d)$. If a type $\theta$ decreases its proposed equilibrium dividend $d^*(\theta)$ to $d^*(\theta - \varepsilon)$ for some small $\varepsilon > 0$, there are two effects at play. First, since there is now more cash inside the firm, the default threshold is lower. The lower default threshold both decreases the value the firm receives from limited liability and decreases the promised repayment. However, since we have started from a conjectured equilibrium, these two effects cancel each other out as the creditors correctly price in the lower default probability. Second, the creditors update their beliefs about the quality of the firm. This latter effect is unambiguously positive. As a result, deviating to $d^*(\theta - \varepsilon)$ is profitable, and $\hat{\theta}(d)$ cannot be an equilibrium.

In effect, no separating equilibrium can exist when $\varphi = 0$ because creditors must be fully compensated for the firm’s credit risk. If, for example, the firm has some outstanding long-term debt or there is a positive probability of a bailout, separation might be possible even when $\varphi = V = 0$, as shown in Sections 6.4 and 6.5.24

The issue is not specific to this model. For example, Flannery (1986) analyzes the signaling role of debt maturity. In Section 2 of his paper, he shows that in the absence of transaction costs, the choice of debt maturity cannot be used to separate different types. To obtain separation, Flannery introduces an exogenous debt issuance cost, which serves a similar role to $\varphi$ in this paper.
4.4 Separating Equilibria With Uniform Returns

To obtain a sharper characterization of separating equilibria, I now assume that the returns from the long-term asset are uniformly distributed: \( \tilde{y} \sim \text{Unif}[0, 2\theta] \).

Lemma 1 shows that

\[
B^*(d, \theta) = (x - d)(1 - \varphi) + 2\theta - 2\sqrt{\theta + (x - d)(1 - \varphi) - b}.
\] (25)

Using Proposition 1, we further find that the optimal dividend policy under symmetric information is given by

\[
d_{SI}^*(\theta) = \begin{cases} 
0 & \text{if } V > \overline{V}(\theta) \\
\left[\frac{\theta + x(1 - \varphi) - b}{1 - \varphi}\right] - \left(\frac{1 - \varphi}{4\theta}\right) \left(\frac{V}{\varphi}\right)^2 & \text{if } V \in [\overline{V}(\theta), \overline{V}(\theta)] \\
x & \text{if } V < \overline{V}(\theta)
\end{cases}
\] (26)

where the thresholds \( \overline{V}(\theta) \) and \( \overline{V}(\theta) \) are strictly increasing in \( \theta \) and equal to

\[
\overline{V}(\theta) = \frac{2\varphi}{1 - \varphi} \sqrt{\theta(\theta - b)} \quad \text{and} \quad \overline{V}(\theta) = \frac{2\varphi}{1 - \varphi} \sqrt{\theta(\theta + x(1 - \varphi) - b)}.
\]

From Eq. (19), the sign of \( \frac{\partial \text{MRS}(d, B; \theta)}{\partial \theta} \) is the same as the sign of

\[
V - (1 - \gamma) \tilde{y} = V - (1 - \gamma)[B - (x - \varphi)].
\]

In particular, \( V - (1 - \gamma) \tilde{y} \) is independent of \( \theta \) and depends on \( d \) and \( B \) only through the default threshold \( \tilde{y} \).

I now provide sufficient conditions for the marginal rate of substitution to be strictly monotone in asset quality by bounding \( \tilde{y} \). First, observe that \( B^*(d, \theta) \) is strictly increasing in \( d \) and strictly decreasing in \( \theta \). As a result, we have the following bounds:

\[
V - (1 - \gamma) \tilde{y} \leq V - (1 - \gamma)[B^*(0, \overline{\theta}) - x(1 - \varphi)]
\]

\[
V - (1 - \gamma) \tilde{y} \geq V - (1 - \gamma)B^*(\overline{d}(\theta), \theta)
\]

Thus, provided that

\[
V < (1 - \gamma)[B^*(0, \overline{\theta}) - x(1 - \varphi)] \equiv \tilde{V}_L(\gamma),
\]
the marginal rate of substitution is strictly decreasing in asset quality. Similarly, if
\[ V > (1 - \gamma)B^* (\bar{d}(\theta), \theta) \equiv \hat{V}_H(\gamma), \]
then the marginal rate of substitution is strictly decreasing in asset quality. Clearly, \( \hat{V}_H(\gamma) \geq \hat{V}_L(\gamma) \) for all \( \gamma \in [0, 1] \) and the inequality is strict if \( \gamma \neq 1 \).

We can use these observations to prove the following result.

**Proposition 3.** Suppose that \( (\bar{\theta} - \theta) \) is small enough. Then:

1. if the franchise value \( V \) is high enough, there exists a separating equilibrium in which equilibrium dividends are strictly increasing in asset quality;
2. if the franchise value \( V \) is small enough and \( \gamma \neq 1 \), there is a separating equilibrium in which equilibrium dividends are strictly decreasing in asset quality;
3. there exists an open set of parameter values for \( (\gamma, V) \) for which there exists no separating equilibrium.

**Proof.** In the Appendix. \( \square \)

Proposition 3 is illustrated in Figure 4. In the figure, I plot the bounds \( \hat{V}_L(\gamma) \) and \( \hat{V}_H(\gamma) \) as a function of \( \gamma \), the weight placed on the creditors in the managerial objective function. In addition, I plot the threshold franchise values \( V(\theta) \) and \( \bar{V}(\theta) \). As we have just seen, when the franchise value is greater than \( \hat{V}_H(\gamma) \), the marginal rate of substitution is strictly increasing in asset quality, and therefore in any equilibrium, higher types must pay greater dividends. The situation is reversed...
when \( V < \hat{V}_L(\gamma) \) and the marginal rate of substitution is strictly decreasing. By Proposition 4, \( \hat{\theta}(d) \) is strictly increasing when \( V \geq V(\theta) \), strictly decreasing when \( V \leq V(\theta) \), and is first strictly decreasing and then strictly increasing otherwise. As a result, for example, when \( V < \min\{\hat{V}_L(\gamma), V(\theta)\} \), we obtain a prudent equilibrium in which higher-quality firms pay lower dividends. If \( V(\theta) < V < \hat{V}_L(\gamma) \), then \( \hat{\theta}(d) \) has both a strictly increasing as well as a strictly decreasing part. The strictly decreasing part (i.e., for \( d \leq d_{FB}(\theta) \)) induces an equilibrium. By the same logic, when \( V \) is sufficiently high, we obtain a classical equilibrium in which dividends and quality are positively related. When \( V \) is intermediate, by changing the weight placed on the creditors in the managerial objective function, \( \gamma \), we move from a prudent to a classical equilibrium. In the extreme case with \( \gamma = 1 \), any separating equilibrium must be classical.

For some parameter values, there may be no separating equilibrium, and all equilibria must be partially pooling. In panel (a), we see that when \( V > 0 \) and sufficiently small, while \( \gamma \) is sufficiently close to one, there is a parameter region in which the marginal rate of substitution is strictly increasing in asset quality but \( \hat{\theta}(d) \) is strictly decreasing. This case clearly cannot be an equilibrium. When the cost of holding liquidity, \( \phi \), is sufficiently small, there is another region in the parameter space for which there is no separating equilibrium, as illustrated panel (b). The second case arises when both \( \phi \) and \( \gamma \) are sufficiently close to zero.

5 Policy Implications

In this section, I consider the normative implications of the model. I first point out that signaling may increase welfare. Then, I consider policy interventions in the form of dividend taxes and restrictions.

First, it is possible for the separating equilibrium under asymmetric information to be welfare-superior to the equilibrium under symmetric information. This result stands in contrast to many signaling models in which the signaling activity is socially wasteful.\(^{25}\) For example, suppose that the franchise value is zero, but the social cost of default is sufficiently high so that \( d_{FB}(\theta) = 0 \) for all \( \theta \). In this situation, the equilibrium with symmetric information has all firms paying out the highest feasible dividend: \( d_{SI}(\theta) = \bar{d}(\theta) \). In contrast, in the separating equilibrium under asymmetric information, all firms except the lowest quality one pay a dividend that is strictly lower than under symmetric information. As a result, social welfare is strictly greater in the separating equilibrium under asymmetric information.

\(^{25}\) The idea that signaling may be socially useful is, of course, not new; see Spence (1974, Section 2.C), Bhattacharya and Ritter (1983) or, more recently, Leppämäki and Mustonen (2009).
tion. Intuitively, paying a low dividend leads to a positive externality by reducing the probability of incurring the social cost of default. Although leaving cash inside the firm is privately costly, it is beneficial from a social perspective.

To better understand efficiency properties of the different equilibria, suppose that the government levies a linear tax $\tau$ on dividends. Tax proceeds are given back to the firms in a lump-sum transfer $\rho$. The government’s budget constraint is given by

$$\rho(\tau) = \tau \int_\theta d^*(\theta, \tau) \, dG(\theta),$$

where $d^*(\theta, \tau)$ denotes the equilibrium dividend paid by type $\theta$ when the tax rate is $\tau$. The debt pricing equation (Eq. (9)) remains almost unchanged except that the default threshold is now given by

$$\hat{y} = B - [x + \rho(\tau) - d(1 + \tau)](1 - \phi).$$

To emphasize the fact that the promised repayment now depends on the dividend tax, I will write $B^*(d, \theta, \tau)$ to denote the solution to Eq. (9) when dividends are taxed.

I assume that $d^*(\theta, \tau)$ is continuous and one-to-one in $\theta$ and continuously differentiable in $\tau$; asset quality $\theta$ may or may not be directly observable to the creditors. As in Section 3.4, the government maximizes total output in the economy, inclusive of the franchise value and the social cost of default:

$$\max_{\tau} W(\tau) = \int_\theta \left\{ d^*(\theta, \tau) + [x - d^*(\theta, \tau)](1 - \phi) + \mathbb{E}[\hat{y}|\theta] ight. \left. + (1 - F(\hat{y}^*(\theta, \tau); \theta))(V + \Delta) \right\} \, dG(\theta),$$

where the equilibrium default threshold is denoted by $\hat{y}^*(\theta, \tau) = B^*(d^*(\theta, \tau), \theta, \tau) - [x + \rho(\tau) - d^*(\theta, \tau)(1 + \tau)](1 - \phi)$. By the government budget constraint, the objective coincides (up to a constant) with the aggregate managerial payoff in equilibrium, i.e., $\int_\theta \Pi_M(d^*(\theta, \tau), B^*(d^*(\theta, \tau), \theta, \tau); \theta) \, dG(\theta)$.

We calculate that

$$W'(\tau) = \int_\theta \left\{ \phi \frac{\partial d^*}{\partial \tau} - f(\hat{y}^*(\theta, \tau); \theta)(V + \Delta) \frac{\partial \hat{y}^*}{\partial \tau} \right\} \, dG(\theta),$$

where

$$\frac{\partial \hat{y}^*}{\partial \tau} = \frac{1 - \phi}{1 - F(\hat{y}^*(\theta, \tau); \theta)} \left[ d^*(\theta, \tau) - \rho'(\tau) + (1 + \tau) \frac{\partial d^*}{\partial \tau} \right].$$
Evaluating the expressions at $\tau = 0$, we obtain that

$$W'(0) = \int_{\theta}^{\theta} \left\{ \frac{\partial d^*}{\partial \tau} \left[ \varphi - (1 - \varphi) \lambda(\hat{y}^*(\theta, \tau); \theta) V \right] \right\} \ dG(\theta)$$

output distortion

$$- \Delta(1 - \varphi) \int_{\theta}^{\theta} \frac{\partial d^*}{\partial \tau} \lambda(\hat{y}^*(\theta, \tau); \theta) \ dG(\theta)$$

social cost of default

$$- (1 - \varphi)(V + \Delta) \int_{\theta}^{\theta} \lambda(\hat{y}^*(\theta, \tau); \theta) [d^*(\theta, \tau) - d_A(\tau)] \ dG(\theta),$$

redistribution

where $d_A(\tau) \equiv \int_{\theta}^{\theta} d^*(\theta, \tau) \ dG(\theta)$ denotes aggregate dividends.\(^{26}\)

The expression for the welfare effect of a small tax consists of three terms. The first term arises because of a distortion in private output. With symmetric information, the term is zero when all dividends are interior, i.e., when $d^*_{SI}(\theta) \in (0, x)$ for all $\theta$, as the expression inside the square brackets is just the first-order condition for the optimal dividend under symmetric information. However, the symmetric information dividend policy is not incentive compatible.\(^{27}\) To achieve separation under asymmetric information, therefore, firms must deviate from the symmetric information dividend policy. The output distortion term may be negative or positive depending on whether firms pay dividends that are lower or higher than under symmetric information.

The second term captures the fact that the firms do not take the social cost of default into account when setting their dividend policy. The term, of course, arises even under symmetric information.

Finally, the third term captures the fact that the dividend tax has redistributive consequences. The dividend tax redistributes resources from those firms whose dividends are higher than average to those firms that pay dividends that are lower than average. This transfer, in turn, impacts the firms’ probabilities of default. As with the output distortion term, the redistributive effect is zero under symmetric information since optimality requires the hazard rates across firms to be equalized at an interior solution (see Eq. (14) and set $\Delta = 0$). With asymmetric information, hazard rates are not equalized, and the redistributive term can be negative or positive.

\(^{26}\)The formula above is the Greenwald and Stiglitz (1986) formula for the present environment. Greenwald and Stiglitz (1986, Section II-C) analyze efficiency properties of a signaling economy that shares some similar features with the present model.

\(^{27}\)Specifically, suppose $d^*_{SI}(\theta) \in (0, x)$ for all $\theta$ and that $\hat{\theta}(d)$ in the differential equation in Eq. (22) is given by the inverse of the symmetric information dividend policy, $(d^*_{SI})^{-1}(d)$. Then, the left-hand side of Eq. (22) is equal to zero, while the right-hand side is strictly negative, a contradiction.
depending on whether higher-quality firms pay higher or lower dividends. Intu-
itively, even though the first-period payoff is linear in the dividend, the $F(\hat{y}; \theta) V$ term introduces a non-linearity and thereby a non-zero redistributive effect. The redistributive effect of dividend taxes also makes the effect of the tax on the equilibrium dividend ambiguous, and hence it is possible to have $\partial d^\ast / \partial \tau > 0$ for some types $\theta$.

To further understand the redistributive effect, suppose that higher-quality firms pay strictly lower dividends, so that $\hat{y}^\ast (\theta, \tau)$ and $d^\ast (\theta, \tau)$ are both strictly decreasing in $\theta$. Let $\theta_A$ denote the type for which $d^\ast (\theta_A, \tau) \equiv d_A (\tau)$. Then:

• for $\theta \leq \theta_A$, $d^\ast (\theta, \tau) \geq d^\ast (\theta_A, \tau)$ and $\lambda (\hat{y}^\ast (\theta, \tau); \theta) \geq \lambda (\hat{y}^\ast (\theta_A, \tau); \theta);
• for $\theta \geq \theta_A$, $d^\ast (\theta, \tau) \leq d^\ast (\theta_A, \tau)$ and $\lambda (\hat{y}^\ast (\theta, \tau); \theta) \leq \lambda (\hat{y}^\ast (\theta_A, \tau); \theta).

In both cases, we have that $\lambda (\hat{y}^\ast (\theta, \tau); \theta) [d^\ast (\theta, \tau) - d_A (\tau)] \geq \lambda (\hat{y}^\ast (\theta_A, \tau); \theta) [d^\ast (\theta_A, \tau) - d_A (\tau)] = 0$, where the equality comes from the definition of $\theta_A$. Multiplying both sides of the inequality by $g(\theta)$ and integrating, we see that the redistributive term is positive. Thus, when dividends are decreasing in asset quality, the redistributive effect makes taxing dividends less desirable.

Economically, the redistributive term highlights a weakness of dividend taxes. Mechanically, dividend taxes hurt firms that pay high dividend and help firms that pay low dividends. If firms paying high dividends are low-quality firms, then a dividend tax increases the default probability for firms whose default probabilities are already high. Dividend restrictions, in contrast, do not suffer from this flaw. On the other hand, if dividends are socially useful in avoiding the costly storage of cash, severe dividend restrictions may reduce welfare.

Instead of dividend tax, suppose now that the government imposes a quantity restriction, requiring $d \leq (1 - \alpha) x$ with $\alpha \in [0,1]$ a government policy variable. Denote the equilibrium dividend by $d^\ast (\theta, \alpha)$. In this case, the default threshold is defined by $\hat{y}^\ast (\theta, \alpha) = B^\ast (d^\ast (\theta, \alpha), \theta, \alpha) - [x - d^\ast (\theta, \alpha)](1 - \varphi)$. Therefore,

$$\frac{\partial \hat{y}^\ast}{\partial \alpha} = \frac{1 - \varphi}{1 - F(\hat{y}^\ast (\theta, \alpha); \theta)} \frac{\partial d^\ast}{\partial \alpha},$$

and so the only change with respect to Eq. (28) is that there is no redistributive effect. It is possible to have $W'(0) < 0$ in which case a local dividend restriction is not optimal.

As a final policy intervention, consider a full dividend ban: all firms are re-
stricted to set \( d^* = 0 \). The net welfare effect of such a ban is equal to

\[
\varphi \int_{\tilde{y}}^{\gamma} d^*(\theta) \, d G(\theta) - (V + \Delta) \int_{\tilde{y}}^{\gamma} [F(\tilde{y}^*; \theta) - F(\tilde{y}_0^*; \theta)] \, d G(\theta),
\]

where \( d^*(\theta) \) is the equilibrium dividend policy before the ban with an associated default threshold \( \tilde{y}^* \), and \( \tilde{y}_0^* \) is the default threshold under the dividend ban. It is clear that the welfare effect is in general ambiguous. On the one hand, the dividend ban decreases the probability of default and thereby the chance of losing \( V + \Delta \). At the same time, dividends are socially useful in reducing the deadweight cost of storage. Under a complete dividend ban, firms must incur the full deadweight cost of holding cash, as captured by the first term.

### 6 Extensions

I now consider a number of extensions of the baseline model to investigate the robustness of the main result.

#### 6.1 Signaling to Shareholders

Suppose that instead of signaling to the short-term creditors to obtain lower funding costs, the firm manager signals quality to uninformed shareholders to boost the firm’s stock price, as in the classic models of Bhattacharya (1979) and Miller and Rock (1985).

The firm has \( b \) units of debt that it must repay to its creditors in period one. The promised repayment \( b \) is now exogenous and satisfies \( b > (1 + \varphi) x + \tilde{y} \). The managerial objective function is given by a linear combination of the true value of equity, \( \Pi_E \), and the (uninformed) cum-dividend stock price, \( P \):

\[
\Pi_M(d, P; \theta) = \Pi_E(d; \theta) + \gamma P, \quad \gamma > 0. \tag{28}
\]

As in Eq. (11), the fair value of equity is

\[
\Pi_E(d; \theta) = d + (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] + [1 - F(\tilde{y}; \theta)]V + \int_{\tilde{y}}^{\gamma} F(y; \theta) \, d y - b,
\]

The fact that the manager cares about the short-run stock price may be justified by assuming that current shareholders face liquidity needs, as in John and Williams (1985). Alternatively, we may simply take the structure of managerial compensation as given, as in, for example, Ross (1977). There are well-known theoretical problems that arise from taking managerial compensation as given (Dybvig and Zender, 1991). See Guttman et al. (2010, Section 5.2) for more discussion on the structure of the objective function.
with the default threshold \( \hat{y} = b - (x - d)(1 - \varphi) \).

The stock price is equal to the expected value of equity, given the beliefs of the stock market:

\[
P = \int_{\hat{y}} B^\infty \Pi_E(d; \theta) \, dB(\theta; d). \tag{29}
\]

As previously, let \( \text{MRS}(d, P; \theta) \) denote the marginal rate of substitution between the dividend and the stock price. We see that

\[
\frac{\partial \text{MRS}}{\partial \theta} = -\frac{1}{\gamma} \frac{\partial^2 \Pi_E}{\partial d \partial \theta} = - \left( \frac{1 - \varphi}{\gamma} \right) \left[ F_\theta(\hat{y}; \theta) - f_\theta(\hat{y}; \theta) V \right].
\]

Comparing the expression above to Eq. (19), we see that the signaling incentives remain unchanged. Of course, in the baseline model, high-quality firms are rewarded with a lower repayment \( B \), whereas in the current setting high-quality firms are rewarded with a higher stock price \( P \), and thus we have the minus sign in front of the expression above.

Let us now analyze the case of a separating equilibrium. Let \( \hat{\theta}(d) \) denote the equilibrium mapping between the dividend and asset quality. Similarly, let \( P(d) \) denote the equilibrium stock price as a function of the dividend. Since \( \tilde{y} \) is a non-negative random variable, \( \mathbb{E}[\tilde{y} | \theta] = \int_{\hat{y}}^\infty [1 - F(y; \theta)] \, dy \), and we can rewrite the value of equity as

\[
\Pi_E(d; \theta) = d + \int_{\hat{y}} F(\theta(y); \theta) \, dy + [1 - F(\hat{y}; \theta)] V. \tag{30}
\]

The first-order condition for the optimal dividend is given by

\[
\varphi - (1 - \varphi) f(\hat{y}; \theta) V + (1 - \varphi) F(\hat{y}; \theta) + \gamma P'(d) = 0,
\]

and we further calculate that

\[
P'(d) = \varphi - (1 - \varphi) f(\hat{y}; \hat{\theta}(d)) V + (1 - \varphi) F(\hat{y}; \hat{\theta}(d))
- \hat{\theta}'(d) \left[ \int_{\hat{y}} \left[ F_\theta(y; \hat{\theta}(d)) \right] \, dy + F_\theta(\hat{y}; \hat{\theta}(d)) V \right]. \tag{31}
\]

All in all, imposing the equilibrium condition that \( \theta = \hat{\theta}(d) \) shows that the differ-
The differential equation characterizing $\hat{\theta}(d)$ is given by

$$(1 + \gamma) \left[ \varphi - (1 - \varphi) f(\hat{y}; \hat{\theta}(d)) V + (1 - \varphi) F(\hat{y}; \hat{\theta}(d)) \right] = \gamma \hat{\theta}'(d) \left[ \int_{\hat{y}}^{g(\hat{\theta}(d))} F_\theta(y; \hat{\theta}(d)) \, dy + F_\theta(\hat{y}; \hat{\theta}(d)) V \right].$$

When the franchise value is zero and holding cash is costless, i.e., $V = \varphi = 0$, the differential equation above specializes to

$$F(\hat{y}; \hat{\theta}(d)) = \frac{\gamma \hat{\theta}'(d)}{1 + \gamma} \int_{\hat{y}}^{g(\hat{\theta}(d))} F_\theta(y; \hat{\theta}(d)) \, dy.$$  \hspace{1cm} (33)$$

When $V = 0$, all firms choose $d = x$ under symmetric information to maximize the shield from losses provided by limited liability, and hence we have the initial condition $\hat{\theta}(x) = \theta$.

Since $F_\theta < 0$, $\hat{\theta}(d)$ must be strictly decreasing for the differential equation in Eq. (33) to hold. Thus, the result from the baseline model that acting prudently can signal quality does not hinge on the assumption that the manager signals strength to the creditors. Importantly, a separating equilibrium can exist even when holding cash is costless, that is, $\varphi = 0$.

Substituting the expression for $\hat{\theta}'(d)$ from Eq. (32) to Eq. (31) we obtain that, in equilibrium,

$$P'(d) = -\frac{1}{\gamma} \left[ \varphi - (1 - \varphi) f(\hat{y}; \hat{\theta}(d)) V + (1 - \varphi) F(\hat{y}; \hat{\theta}(d)) \right].$$

Comparing the expression above with Eq. (32), we see that when $\hat{\theta}(d)$ is strictly decreasing, $P'(d) < 0$, and a dividend increase leads to a negative stock market reaction. Similarly, when $\hat{\theta}(d)$ is strictly increasing (i.e., when $V$ is sufficiently high), a dividend increase leads to a positive stock market reaction.

### 6.2 Sources of Asymmetric Information

I now ask whether the main results are sensitive to the assumption that the firm has private information about the quality of its long-term asset. In this section, I analyze asymmetric information regarding the firm’s (i) cash holdings; and (ii) franchise value.

First, let $\theta \geq 0$ denote the amount of cash held by the firm. The amount of cash is private information to the firm and cannot be observed by the creditors. The
The managerial objective function is now

\[ \Pi_M(d, B; \theta) = d + (\theta - d)(1 - \varphi) + \mathbb{E}[\hat{y}] + \left\{ 1 - F[\hat{y}(\theta)] \right\} V \\
+ (1 - \gamma) \int_{\hat{y}}^{B} f(y) \, dy - (1 - \gamma)B, \]  

(34)

where \( \hat{y}(\theta) = B - (\theta - d)(1 - \varphi) \). I denote the default threshold by \( \hat{y}(\theta) \) to make it explicit that the default threshold now depends on information that is privately known by the firm. Since there is no asymmetric information about the long-term asset, the distribution \( F(y) \) is the same for all firms.

The expression for the marginal rate of substitution remains essentially unchanged with

\[ \text{MRS}(d, B; \theta) = \frac{\varphi + (1 - \varphi) \left\{ (1 - \gamma)F[\hat{y}(\theta)] - f[\hat{y}(\theta)]V \right\}}{(1 - \gamma) - \left\{ (1 - \gamma)F[\hat{y}(\theta)] - f[\hat{y}(\theta)]V \right\}}. \]

(35)

and

\[ \text{sign} \left\{ \frac{\partial \text{MRS}}{\partial \theta} \right\} = \text{sign}\left\{ - (1 - \gamma)f[\hat{y}(\theta)] + f'[\hat{y}(\theta)]V \right\}. \]

As before, the expression determining the monotonicity of the marginal rate of substitution consists of two terms, the first one stemming from risk shifting and the second one from the franchise value effect. If the franchise value is zero, the marginal rate of substitution is strictly decreasing in the amount of cash the firm has. Therefore, the key result remains unchanged if asymmetric information concerns the firm’s cash holdings instead of the quality of the long-term asset.

Now suppose that all firms have the same amount of cash, \( x \), but different, privately observed franchise values. Denote the franchise value by \( \theta \geq 0 \). The managerial objective remains almost identical to Eq. (34) except that the default threshold is now \( \hat{y} = B - (x - d)(1 - \varphi) \) independently of the firm’s private information, as in the baseline model, and \( V \) is replaced by \( \theta \). As a result, \( \partial \text{MRS}/\partial \theta < 0 \), provided that not both \( \gamma = 1 \) and \( \varphi = 0 \). Thus, when there is asymmetric information about the firms’ franchise values, equilibrium dividends and firm quality are always negatively related. However, this result is not driven by risk shifting incentives: neither the probability of default nor the change in the default probability for a given change in the dividend depend on the franchise value. The negative relationship is simply a manifestation of the fact that firms with higher franchise values have more to lose in the event of a default.
6.3 Signaling With Investment Risk

I now consider a setting in which, instead of paying dividends, firms can engage in risk shifting by changing the riskiness of their investment.

To simplify the exposition, suppose that as in Section 6.1, firms have existing debt and signal quality to the shareholders in order to influence the stock price. At the end of this section, I discuss what changes if firms signal quality to the short-term creditors, as in the baseline model.

As before, all firms have an asset-in-place that yields a random return $\tilde{y}$ in period one. The quality of the asset is again denoted by $\theta$ and is private information to the firm, with $\theta$ ordering the distributions of $\tilde{y}$ in the strict hazard rate order. Firms have no cash and do not distribute any dividends. Instead, each firm owns an asset whose return at time one, denoted by $\tilde{z}$, is given by $\tilde{z} = \tilde{\epsilon} \mathbb{E}[\tilde{z}]$. The disturbance $\tilde{\epsilon}$ is independent of $\tilde{y}$ and uniformly distributed on $[1-\sigma, 1+\sigma]$. The firm manager can choose the risk of investment $\sigma \in [0, 1]$; the choice of riskiness is publicly observed. To simplify, set the franchise value to zero.

I assume that $\mathbb{E}[\tilde{z}]$ is sufficiently small so that $2 \mathbb{E}[\tilde{z}] < b$. Economically, the assumption ensures that even when the manager chooses the highest feasible level of investment risk, the highest possible return from the $z$-asset is not sufficient to repay $b$. The managerial objective is given by

$$\Pi_M(\sigma, P; \theta) = \int_{1-\sigma}^{1+\sigma} \int_{\tilde{y}(\epsilon)}^{\tilde{y}(\epsilon)} \{ (y + \epsilon \mathbb{E}[\tilde{z}] - b) \, d \mathbb{P}(y; \theta) \} \, \frac{1}{2\sigma} \, d\epsilon + \gamma P,$$

where $\tilde{y}(\epsilon) = b - \epsilon \mathbb{E}[\tilde{z}]$ is the default threshold for a given realization $\epsilon$. Performing calculations similar to those in Section 3.3, we find that

$$\Pi_M(\sigma, P; \theta) = \mathbb{E}[\tilde{z}] + \mathbb{E}[\tilde{y}|\theta] + \frac{1}{2\sigma} \int_{1-\sigma}^{1+\sigma} \int_{\tilde{y}}^{\tilde{y}(\epsilon)} F(y; \theta) \, d\mathbb{P}d\epsilon - b + \gamma P.$$

By assumption, $\sigma$ orders the distributions of $\tilde{z}$ in the sense of second-order stochastic dominance. Since the inner integral above, $\int_{\tilde{y}}^{\tilde{y}(\epsilon)} F(y; \theta) \, d\mathbb{P}$, is strictly convex in $\epsilon$, the managerial payoff is strictly increasing in the riskiness of investment. If $\int_{\tilde{y}}^{\tilde{y}(\epsilon)} F(y; \theta) \, d\mathbb{P}$ is strictly concave in $\epsilon$, higher-quality firms will choose lower investment risk in any equilibrium. A sufficient condition for this to be the case is that $f_\theta(y; \theta)$ be strictly negative.

In particular, if the long-term asset return is uniformly distributed on $[0, 2\theta]$, we calculate that

$$\Pi_M(\sigma, P; \theta) = \mathbb{E}[\tilde{z}] + \mathbb{E}[\tilde{y}|\theta] + \frac{3(b - \mathbb{E}[\tilde{z}])^2 + \sigma^2 \mathbb{E}[\tilde{z}]^2}{12\theta} - b + \gamma P.$$

40
and therefore
\[
\frac{\partial^2 \Pi_M}{\partial \sigma \partial \theta} = -\sigma \frac{\mathbb{E}[\tilde{z}]^2}{6\theta^4} \leq 0.
\]
As a result, firms with higher-quality assets choose (weakly) lower investment risk in any equilibrium. We could now proceed as in Section 6.1 and derive the differential equation characterizing the separating equilibrium; I omit the details.

When firms signal quality to the short-term creditors, the cross-derivative \( \frac{\partial^2 \Pi_M}{\partial \sigma \partial \theta} \) retains the same sign, of course, but the creditors now need to break even. In the current setting, there is no element that plays the role of \( \varphi \), costly storage of cash, in the baseline model. Without such an element, a separating equilibrium does not exist. This problem does not arise if there is a positive risk-return trade-off between \( \sigma \) and the expected return from the \( z \)-asset. Formally, suppose that the expected return from the \( z \)-asset is given by \( m(\sigma) \) with \( m'(\sigma) > 0 \). With a positive risk-return trade-off, firms can separate from each other by choosing different levels of investment risk. Another possibility is to have some debt that is not repriced or a positive bailout probability, along the lines of Sections 6.4 and 6.5.

### 6.4 Outstanding Long-Term Debt

In the baseline model, the firm needs to refinance \( b \) units of short-term debt but it has no other outstanding liabilities. In this section, I consider a setting in which the firm, in addition to the maturing short-term debt, also has some outstanding long-term debt.

Suppose that the firm has \( b_L \) units of outstanding long-term debt that must be repaid at the final date. The long-term debt may also be interpreted as short-term debt that is automatically rolled over and not repriced. The latter interpretation may be particularly interesting in the context of banking as insured retail deposits often have these characteristics.

At time zero, the firm also needs to raise \( b_S \) units of short-term debt at face value \( B_S \), as in the baseline model. For simplicity, assume that the short-term creditors are junior in bankruptcy which is natural if long-term bondholders want to guard against potential risk shifting. When \( b_S = 0 \), we recover the model analyzed by Acharya, Le, and Shin (2016a) as a special case. Similarly to Assumption 4 in the baseline model, I assume that the long-term debt holders (and thereby short-term debt holders) always face a positive probability of default: \( x(1 - \varphi) + y < b_L \).

With these changes, the break-even condition for the short-term creditors (when
they believe the firm has assets of quality \( \theta \) with probability one) becomes
\[
\int_{\hat{y}_1}^{\hat{y}_2} \{ (x - d)(1 - \varphi) + y - b_L \} \, d \, F(y; \theta) + \int_{\hat{y}_2(d)}^{\hat{y}} B_S \, d \, F(y; \theta) = b,
\]
with the two default thresholds defined by
\[
\hat{y}_1 = b_L - (x - d)(1 - \varphi), \quad \hat{y}_2 = b_L + B_S - (x - d)(1 - \varphi).
\]
The firm is declared bankrupt whenever \( y < \hat{y}_2 \). When \( y \in [\hat{y}_1, \hat{y}_2] \), the long-term creditors receive their promised payment \( b_L \) but the short-term creditors get only a fraction of \( B_S \) back. When \( y < \hat{y}_1 \), the long-term creditors recover a fraction of \( b_L \) while the short-term creditors get nothing.

Some algebra shows that the pricing equation for debt is now
\[
B_S = b_S + \int_{\hat{y}_1}^{\hat{y}_2} F(y; \theta) \, d \, y.
\]
Making use of Lemma 2 (just let \( B \equiv b_L + B_S \)), we see that
\[
\Pi_E(d, B_S; \theta) = d + (x - d)(1 - \varphi) + E[\tilde{y} | \theta] + [1 - F(\hat{y}_2; \theta)] V + \int_{\hat{y}_1}^{\hat{y}_2} F(y; \theta) \, d \, y - (b_L + B_S),
\]
while the payoffs of the short-term and long-term creditors are given by
\[
\Pi_{D,S}(d, B_S; \theta) = B_S - \int_{\hat{y}_1}^{\hat{y}_2} F(y; \theta) \, d \, y \\
\Pi_{D,L}(d; \theta) = b_L - \int_{\hat{y}}^{\hat{y}_1} F(y; \theta) \, d \, y
\]
As before, the firm manager maximizes a linear combination of the values of equity, short- and long-term debt:
\[
\Pi_M(d, B_S; \theta) = \Pi_E(d, B_S; \theta) + \gamma \Pi_{D,S}(d, B_S; \theta) + \gamma \Pi_{D,L}(d; \theta),
\]
where the weight \( \gamma \in [0,1] \) is, for simplicity, taken to be the same for both short- and long-term debt.
Substitute the expressions for the payoffs to obtain that
\[
\Pi_M(d, B_S; \theta) = d + (x - d)(1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] + [1 - F(\tilde{y}_2; \theta)]V + (1 - \gamma) \int_{\tilde{y}_2}^{\tilde{y}_1} F(y; \theta) \, dy - (1 - \gamma)(b_L + B_S).
\]

Comparing the expression above to the managerial objective function in the baseline model in Eq. (12), we see that they are almost identical, except that the relevant default threshold for the firm is now \(\tilde{y}_2\) and the firm has \(b_L + B_S\) units of debt. The expression for the marginal rate of substitution, therefore, remains almost the same and
\[
\text{sign} \left\{ \frac{\partial MRS}{\partial \theta} \right\} = \text{sign} \left\{ F_{\theta}(\tilde{y}_2; \theta)(1 - \gamma) - f_{\theta}(\tilde{y}_2; \theta)V \right\}.
\]

We observe that the signaling incentives remain qualitatively unchanged by the introduction of existing debt.

Two qualitative features of the equilibrium do change, however. First, because the existing debt is not repriced, the equilibrium dividend policy is inefficient even with symmetric information and no social costs of default. Second, a positive cost of holding cash is no longer necessary to induce separation. To see this, suppose that \(\varphi = V = 0\). Deriving the differential equation for the separating equilibrium, we find
\[
F(\tilde{y}_1; \tilde{\theta}(d)) = \tilde{\theta}'(d) \int_{\tilde{y}_1}^{\tilde{y}_2} F_{\theta}(y; \tilde{\theta}(d)) \, dy
\]
with the initial condition \(\tilde{\theta}(d(\theta)) = \theta\). Thus, even when \(\varphi = V = 0\), firms with high-quality assets can separate by paying strictly lower dividends.

### 6.5 Government Bailouts

Consider the baseline model with the following change. If the firm is not able to repay its creditors at time one, the government bails out the creditors with probability \(\beta \in (0, 1)\). That is, if the firm does not have enough resources to repay \(B\), the government makes the creditors whole by supplying \((x - d)(1 - \varphi) + y - B\) to the creditors with probability \(\beta\). For simplicity, I assume that in the event of default, the government fully wipes out the shareholders. As a result, the shareholders never directly receive any funds from the government.\(^{29}\)

\(^{29}\) Of course, in equilibrium the possibility of a bailout is priced in by the creditors. Since the creditors are perfectly competitive, the full value of the subsidy is appropriated by the firm.
With bailouts, the debt pricing equation in Eq. (9) becomes

\[
B = b + (1 - \beta) \int_{y}^{\hat{y}} F(y; \theta) \, dy,
\]
where the default threshold remains \( \hat{y} = B - (x - d)(1 - \varphi) \). Since the expression for the value of equity, \( \Pi_E \), is unchanged by the possibility of a bailout, we see that the managerial payoff function is now

\[
\Pi_M(d, B; \theta) = d + (x - d)(1 - \varphi) + \mathbb{E}[\hat{y}|\theta] + [1 - F(\hat{y}; \theta)]V \\
+ [1 - \gamma(1 - \beta)] \int_{y}^{\hat{y}} F(y; \theta) \, dy - (1 - \gamma)B.
\]

We calculate that the sign of \( \partial \text{MRS}/\partial \theta \) coincides with

\[
\text{sign} \left\{ \frac{\partial^2 \Pi_M}{\partial d \partial \theta} \right\} = \text{sign} \left\{ F_\theta(\hat{y}; \theta)[1 - \gamma(1 - \beta)] - f_\theta(\hat{y}; \theta)V \right\}.
\]

The expression above is decreasing in the probability of a bailout. This way, a higher bailout probability makes it more likely that high dividends are paid by low-quality firms. Importantly, the first term in the expression above is negative even if \( \gamma = 1 \), i.e., when the interests of the creditors and the firm manager are perfectly aligned.

As in Section 6.4, the equilibrium dividend policy is inefficient even with symmetric information and no social costs of default. Similarly, with bailouts, one can obtain a separating equilibrium even when holding cash is costless. To see this, suppose that \( V = \varphi = 0 \). The expression for the differential equation becomes

\[
\beta F(\hat{y}; \hat{\theta}(d)) = \hat{\theta}'(d)(1 - \beta) \int_{y}^{\hat{y}} F_\theta(y; \hat{\theta}(d)) \, dy \left[ (1 - \gamma) - F(\hat{y}; \hat{\theta}(d))(1 - \gamma(1 - \beta)) \right],
\]

with the initial condition \( \hat{\theta}(d) = \theta \). Provided that the term in the square brackets is positive, a sufficient condition for which is \( F(B^*(\hat{d}(\theta), \theta); \theta) < (1 - \gamma)/(1 - \gamma(1 - \beta)) \), the solution \( \hat{\theta}(d) \) is strictly decreasing.

### 6.6 Endogenous Borrowing

One drawback of the baseline model is that the amount of debt raised in the credit market is exogenously fixed at \( b \). Exogenous borrowing levels seem especially difficult to justify given that holding cash inside the firm is costly. One would expect firms to use some cash to pay down existing debt and thereby reduce the amount of new borrowing.
While the insights of the baseline model generalize naturally to some settings in which the issue may be less important (e.g., when the firm has outstanding debt and signals quality to the shareholders, as in Section 6.1), I now make a few first steps in investigating the model when borrowing is endogenous. I first show that when borrowing is endogenous, there exists a separating equilibrium in which dividends are zero for all firms, and firms separate by varying the amount borrowed. Then, I show that if low-quality firms pay a positive dividend under symmetric information, there exist cases in which dividends are negatively related to asset quality in a Pareto efficient separating equilibrium. For simplicity, \( \gamma \) is set to zero throughout this section.

Letting \( u \equiv (d, b) \), the managerial objective function is given by

\[
\Pi_M(u, B; \theta) = d + [x + (b - b_0) - d](1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] + [1 - F(\tilde{y}; \theta)]V \\
+ \int_{\tilde{y}} F(y; \theta) \, dy - B,
\]

(36)

where the default threshold is \( \tilde{y} = B - [x + (b - b_0) - d](1 - \varphi) \). Here \( b_0 \) denotes the amount of existing debt that the firm must repay at time zero. The firm cannot be a lender in the credit market or have negative cash positions: \( b \geq 0 \) and \( x + (b - b_0) - d \geq 0 \). Finally, the firm does not have enough cash to repay the existing debt without raising additional funds: \( b_0 > x \). The expression for the debt pricing equation in Eq. (9) remains unchanged (except, of course, for the different default threshold).

First, consider the optimal financial policy under symmetric information. With symmetric information, the firm manager chooses \( u \) to maximize

\[
\Pi_M(u, B^*(u, \theta); \theta) = d + [x + (b - b_0) - d](1 - \varphi) + \mathbb{E}[\tilde{y} | \theta] + [1 - F(\tilde{y}; \theta)]V - b
\]

subject to the constraints that \( d \geq 0 \), \( b \geq 0 \) and \( x + (b - b_0) - d \geq 0 \). We calculate that

\[
\frac{\partial \Pi_M(u, B^*(u, \theta); \theta)}{\partial b} = -\varphi - \varphi \frac{f(\tilde{y}; \theta)}{1 - F(\tilde{y}; \theta)} V < 0,
\]

which implies that \( x + (b - b_0) - d \geq 0 \) must be binding at the optimum. Substituting \( b \) out from the objective function and differentiating with respect to \( d \), we find that the objective is strictly decreasing in the dividend along \( b = b_0 + d - x \). As a result, the optimal dividend under symmetric information is \( d^*_{\text{SI}}(\theta) = 0 \) and \( b^*_{\text{SI}}(\theta) = b_0 - x \).

Let us now see what happens under asymmetric information. I first exhibit a
particular separating equilibrium in which all signaling is done via changes in the amount borrowed. Consider the marginal rate of substitution between the amount borrowed, \( b \), and the promised repayment, \( B \). From Eq. (36), the objective function \( \Pi_M \) depends on \((b, B)\) only through \( b(1 - \phi) - B \). The marginal rate of substitution between \( b \) and \( B \), therefore, is given by \( 1 - \phi \) and is independent of asset quality \( \theta \). Thus, for a fixed dividend, the indifference curves are parallel lines in the \((b, B)\) space with slopes equal to \( 1 - \phi \).

Suppose that all firms pay no dividends. In any separating equilibrium, the lowest quality firm must choose the same \((d, b)\) as under symmetric information, and we have \( u^*(\theta) = (0, b_0 - x) \). By the discussion in the preceding paragraph, all equilibrium choices \( u^*(\theta) \equiv (d^*(\theta), b^*(\theta)) \) must lie on an indifference curve passing through \( u^*(\theta) = (0, b_0 - x) \). As a result, the equilibrium default thresholds \( \tilde{y}^*(\theta) \equiv B^*(u^*(\theta), \theta) - [x + (b^*(\theta) - b_0)](1 - \phi) \) are independent of \( \theta \) and, in fact, equal to \( B^*(u^*(\theta), \theta) \). From the debt pricing equation in Eq. (9),

\[
\tilde{y}^*(\theta) = b - [x + (b - b_0)](1 - \phi) + \int_y \tilde{y}^*(\theta) F(y; \theta) \, d\,y.
\]

Substituting \( \tilde{y}^*(\theta) = B^*(u^*(\theta), \theta) \) and solving for \( b \), we obtain

\[
b^*(\theta) = \frac{1}{\phi} \left( B^*(u^*(\theta), \theta) + x - b_0 - \int_y B^*(u^*(\theta), \theta) F(y; \theta) \, d\,y \right).
\]

Since \( b^*(\theta) \) is strictly increasing in \( \theta \) and \( b^*(\theta) = b_0 - x > 0 \), \( b^*(\theta) > 0 \) for all \( \theta \). Moreover, since \( d^*(\theta) = 0 \), the cash positions are also weakly positive for all types, and hence \( u^*(\theta) \) is feasible. By construction, all combinations of \( b \) and repayment \( B \) lie on the same indifference curve, and all firms are indifferent between their equilibrium choice \( u^*(\theta) \) and any other \( u^*(\theta') \) for \( \theta' \neq \theta \). Assuming that deviations to \( u^* \)'s not observed in equilibrium are attributed to the lowest quality firm ensures that there are no profitable out-of-equilibrium deviations.\(^{30}\)

In this separating equilibrium, high-quality firms separate by borrowing more in the credit market. The credit market rewards the high-quality firms firms by charging a lower gross interest rate \( B/b \). The equilibrium, however, is quite peculiar in that all firms are indifferent between all debt levels observed in equilibrium and have no strict incentive to not deviate. Moreover, the fact that higher-quality firms borrow more in equilibrium is not because higher-quality firms find it cheaper to leave more cash inside the firm: the marginal rate of substitution between \( b \) and

\(^{30}\)We need to ensure that the firms are solvent when they pay \( b^*(\theta) \), so that there exists a feasible repayment \( B \). This is satisfied, for example, if \( \mathbb{E}[\tilde{y}^*|\theta] > b^*(\theta) \) for all \( \theta \).
B is independent of \( \theta \). In fact, the equilibrium borrowing levels are almost completely determined by the debt pricing equation alone. On the other hand, in some numerical experiments I find that this equilibrium is the Pareto efficient separating equilibrium when firms can use both \( d \) and \( b \) to signal their quality. I have not been able to either prove that this result is always true or find a counterexample.

The difficulty in constructing separating equilibria in which dividends and quality are negatively related when borrowing is endogenous arises from the fact that firms pay zero dividends under symmetric information. If firms pay a positive dividend under symmetric information because of, say, outstanding long-term debt (as in Section 6.4) or government bailouts (as in Section 6.5), it is possible to construct equilibria that are qualitatively similar to those analyzed in the baseline setting.

To show this, suppose that there is a positive probability of a government bailout, \( \beta \in (0, 1) \). For simplicity, I consider the two-point long-term asset return distribution used in Section 2. Otherwise, the model remains unchanged.

Following Ambarish, John, and Williams (1987) and Williams (1988) I focus on a Pareto efficient separating equilibrium \( \{u^* (\theta)\}_{\theta \in \Theta} \) which solves

**Problem 4** (Pareto-Efficient Separating Equilibrium).

\[
\max_{\{u(\theta)\}_{\theta \in \Theta}} \int_0^\beta \Pi_M(u(\theta), B^*(u(\theta), \theta); \theta) \, dG(\theta)
\]

s.t. \( \{u(\theta)\}_{\theta \in \Theta} \) constitutes an equilibrium.

In the calculations, I assume that the credit market attributes all out-of-equilibrium deviations to the lowest quality firm \( \theta \). The solution method is described in Appendix B.

Figure 5 plots the results. Panel (a) shows the solution to Problem 4 for a particular set of parameter values. To maximize the implicit subsidy received from the government, the lowest quality firm borrows the highest feasible amount in the credit market, repays \( b_0 \) to the existing creditors and pays out all remaining cash as a dividend. Then, higher-quality firms both pay a lower dividend and borrow less. Nevertheless, firms leave no cash inside the firm, as can be seen from the fact that \( d^*(\theta) \) and \( b^*(\theta) \) are parallel to each other. At some point, the equilibrium dividend reaches zero. From then on, firms separate by borrowing different amounts while paying the same zero dividend, exactly as in the separating equilibrium derived above.

In panel (b), I plot the equilibrium that maximizes the objective in Problem 4 under the additional constraint that \( b^*(\theta) \) be constant for all types \( \theta \). The key point of this exercise is to show that one can obtain equilibria that are qualitatively similar to those analyzed in Sections 2 and 4 even when the amount borrowed is endoge-
The grid step in the discretization of the type space is 0.005 in panel (a) and 0.001 in panel (b).

To make sure that the problem has a solution, I restrict $\Theta$, the support of asset qualities. We see that the lowest quality firm again chooses the same financial policy as under symmetric information. All other firms choose the same level of debt but pay a strictly lower dividend. In contrast to the previous example, high-quality firms have positive cash holdings in this equilibrium.\(^{31}\)

### 7 Empirical Predictions

The baseline model and its extensions in Section 6 have a number of empirically testable predictions.

First, the baseline model predicts that when the franchise value is sufficiently low, a dividend increase should positively affect proxies of the expected default probability, such as credit default swap spreads. This prediction follows directly from Corollary 1. When the franchise value is sufficiently high so that higher-quality firms pay higher dividends, the effect of a dividend increase is ambiguous. For assets of fixed quality, a higher dividend unambiguously increases the default probability. However, when the franchise value is sufficiently high, higher dividends are paid by higher-quality firms, which have a lower probability of default for a given divided.

Second, the baseline model predicts that when the franchise value is sufficiently low (high), a dividend increase should lead to a negative (positive) stock price re-

\(^{31}\)As mentioned above, if any firm deviates from the common equilibrium borrowing level $b^*$, the deviation is attributed to the lowest quality firm. Since the marginal rate of substitution is independent of asset quality, such off-equilibrium beliefs do not seem particularly implausible.
action. Suppose that, as in Section 6.1, the firm’s stock price, \( P(d) \), is equal to the expected value of the firm’s equity, calculated using the beliefs of the firm’s uninformed shareholders. The uninformed shareholders do not observe asset quality, and their beliefs, in equilibrium, coincide with the beliefs of the credit market. The baseline model remains otherwise unchanged.

Consider the case of a separating equilibrium characterized in Eq. (22). We have the following result.

**Proposition 4.** Suppose that the solution to the differential equation in Eq. (22), \( \hat{\theta}(d) \), induces an equilibrium. Then, if \( \hat{\theta}(d) \) is strictly decreasing, \( P'(d) < 0 \), and vice versa if \( \hat{\theta}(d) \) is strictly decreasing.

**Proof.** In the Appendix.

For non-financial corporations, Lang and Litzenberger (1989) document that average stock returns on dividend announcement days are larger for firms with Tobin’s \( Q \) that is less than one. To the extent that Tobin’s \( Q \) is a good proxy for the firm’s franchise value, this evidence is inconsistent with the predictions of the baseline model. Yoon and Starks (1995) show that the differential effect across firms with different Tobin’s \( Q \)’s disappears once one controls for the size in the dividend change, dividend yield, and firm size. However, to test the predictions of the baseline model, one would arguably want to control for leverage and a measure of distance-to-default. In the baseline model of the present paper, if \( b \) is sufficiently small so that the probability of default is zero, higher-quality firms cannot signal quality by paying a lower dividend.

For bank holding companies in the US, Cziraki, Laux, and Loranth (2016, Table 7) find that dividend cuts in 2007–8 were not associated with a statistically significant stock market reaction. While Cziraki, Laux, and Loranth find a positive announcement effect for dividend increases, the effect is smaller in 2007–8 than for the years preceding the crisis; it is not clear whether the difference is statistically significant. Cziraki, Laux, and Loranth also show that dividend cuts in 2009-12 were associated with a statistically significant negative stock market reaction. This empirical finding is inconsistent with the model. However, it is possible that dividend cuts in the later stages of the financial crisis were in large part driven by regulatory intervention rather than market forces, see, e.g., Board of Governors of the Federal Reserve System (2009). For a different time period and sample of banks, Bessler and Nohel (1996, Table 2) find that dividend cuts have a lower negative announcement effect for banks that are less well-capitalized. The evidence is somewhat mixed, however, as the authors show that the regression coefficient on capital adequacy
becomes statistically insignificant once one controls for bank size. Bessler and No-"hel do not control for a measure of bank franchise value.

The baseline model also predicts that, in the cross-section, dividends and the probability of default should be positively related. However, standard risk shifting models without asymmetric information also make the same prediction; see, for example, Acharya, Le, and Shin (2016a). The cross-sectional prediction seems consistent with the pattern of dividend payments by investment banks (Acharya, Le, and Shin, 2016a, Figure 1c). As discussed by Acharya, Le, and Shin, the divergence of dividend payments between investment and commercial banks may be explained by differences in their franchise values as well as levels of leverage.

Finally, in the extension in Section 6.3, higher-quality firms are predicted to undertake investments that are less risky when the franchise value is sufficiently small. Since investment risk is not directly observable, testing this prediction empirically is difficult. Nevertheless, the observation that higher-quality firms may choose to forego risky investments may provide an additional reason for why detecting risk shifting empirically is challenging.\footnote{Existing research has, of course, provided other hypotheses that can explain why risk shifting is difficult to detect. For example, various mechanisms may be used to reduce shareholder-creditor conflicts (Jensen and Meckling, 1976; Smith and Warner, 1979). Losses of future continuation values and reputational concerns reduce risk shifting incentives (Marcus, 1984; Diamond, 1989; Hirshleifer and Thakor, 1992). Hedging observable risk may reduce the incentives to load up on unobservable risk if observable and unobservable risks are positively correlated (Campbell and Kracaw, 1990). Hernández-Lagos, Povel, and Sertsios (2016) test whether continuation values and reputational effects reduce risk shifting experimentally and find strong evidence that they do.}

I end by discussing a few practical difficulties in testing the empirical predictions of this paper. First, the existence of bond covenants may eliminate or at least mask the informational effects analyzed in this paper. For example, suppose that cash $x$ consists of two parts: existing liquid assets, $x_{-1}$, and cash flow from the current period, $\text{CF}_0$, so that $x = x_{-1} + \text{CF}_0$. As before, $x$ is the same for all firms. Suppose that all firms face a covenant stating that no dividends can be paid whenever $\text{CF}_0 < 0$. In that case, the only possible equilibrium when $\text{CF}_0 < 0$ is for all firms to pay zero dividends, and high-quality firms cannot differentiate themselves by acting prudently.

In situations in which risk shifting is an important concern, the stock market reaction to a dividend increase is likely to consist of both an informational part as well as a wealth transfer component. For example, in the model with outstanding long-term debt (Section 6.4), a higher dividend is bad news about asset quality but serves to extract wealth from the long-term creditors. Thus, the stock market reaction to a dividend increase may be positive even if the shareholders revise their assessment of the firm’s quality downwards. In such cases, it may be useful to consider earning
forecasts instead of stock prices for measuring the informational effect of dividend changes more directly.

8 Conclusions

Shareholders often have opportunities to profit at the expense of their creditors. At the same time, shareholders with favorable private information have an incentive to convey it to outside parties to avoid mispricing.

While the existing literature typically analyzes the risk shifting and signaling roles of corporate financial policy separately, I have shown that there is an important interaction between them. Specifically, not engaging in risk shifting—“acting prudently”—can be a credible signal of firm quality. The basic insight is that higher-quality firms typically have a lower incentive to engage in risk shifting. As a result, high-quality firms can distinguish themselves from low-quality firms by acting prudently.

In the baseline model of this paper, firms use dividend policy to convey private information about the quality of assets-in-place to short-term creditors. The key result is that when franchise values are sufficiently low, equilibrium dividends and asset quality are negatively related. From a normative perspective, signaling quality by acting prudently may increase welfare if firms do not fully internalize the social costs of default. The key result is robust to changes in the modeling assumptions and generalizes to situations in which firms signal quality to outside shareholders, there is a positive probability of a bailout, or firms use the level of investment risk to convey information. Finally, the model is useful for interpreting aspects of bank dividend policy observed in 2007–8.

I conclude by discussing a few limitations of this paper and providing some possible directions for future projects. For tractability, I have primarily considered settings in which firms have only one signal at their disposal (with the exception of Section 6.6). This simplification is clearly at odds with reality. In the context of the baseline model, it would be desirable to understand if dividends and share repurchases have different informational roles, and how they interact with endogenous levels of borrowing. Second, the current model, being completely static, has nothing to say about the dynamic aspects of financial policy such as the well-documented dividend stickiness. Third, the fact that firms may forego risk shifting opportunities to signal quality ex post may have important ex ante consequences for the optimal capital structure. Finally, the current model has a number of empirical predictions that should be tested directly in future work.
References


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Appendix A  Proofs

Proof of Lemma 1

Let \( x_1 \equiv (x - d)(1 - \varphi) \) denote the amount of cash that the firm has in period one. From the break-even condition in Eq. (8), we have that with degenerate beliefs

\[
b = \int_{\hat{y}}^{\bar{y}} [x_1 + y] \, dF(y) + \int_{\hat{y}}^{\bar{y}} B \, dF(y) = F(\hat{y})x_1 + [1 - F(\hat{y})]B + \int_{\hat{y}}^{\bar{y}} y \, dF(y),
\]

where I have suppressed the dependence of \( F(y; \theta) \) on \( \theta \) for brevity. Integrate by parts to obtain

\[
\int_{\hat{y}}^{\bar{y}} y \, dF(y) = F(\hat{y}) \, \hat{y} - \int_{\hat{y}}^{\bar{y}} F(y) \, dy
\]

and substitute the result to the expression above to get

\[
b = F(\hat{y}) \left[ x_1 + \hat{y} - B \right] + B - \int_{\hat{y}}^{\bar{y}} F(y) \, dy,
\]

as claimed.

For the second part, consider the roots of \( g(B) \equiv \int_{\hat{y}}^{\bar{y}} \min\{x_1 + y, B\} \, dF(y) - b \). Clearly, \( g(B) \) is continuous and, moreover, strictly increasing if \( B < x_1 + \bar{y}(\theta) \) and constant if \( B \geq x_1 + \bar{y}(\theta) \). Now \( g(b) = \int_{\hat{y}}^{\bar{y}} \min\{x_1 + y - b, 0\} \, dF(y) \leq 0 \), with equality if and only if \( x_1 + y \geq b \). Also, \( g(x_1 + \bar{y}(\theta)) = x_1 + \int_{\hat{y}}^{\bar{y}} y \, dF(y) - b \).

Results stated in the Lemma follow immediately from these observations.

Proof of Lemma 2

From the definition of \( \Pi_E \) and denoting \( x_1 \equiv (x - d)(1 - \varphi) \), we calculate

\[
\Pi_E(d, B; \theta) = d + \int_{\hat{y}}^{\bar{y}} \max\{0, x_1 + y - B\} \, dF(y) + [1 - F(\hat{y})]V
\]

\[
= d + [1 - F(\hat{y})][x_1 + V - B] + \int_{\hat{y}}^{\bar{y}} y \, dF(y),
\]

where I have suppressed the dependence of \( F(y; \theta) \) on \( \theta \). Integrating by parts as in the proof of Lemma 1, we have

\[
\int_{\hat{y}}^{\bar{y}} y \, dF(y) = \int_{\hat{y}}^{\bar{y}} y \, dF(y) - \int_{\hat{y}}^{\bar{y}} y \, dF(y)
\]

\[
= \mathbb{E}[\hat{y}] - \left( F(\hat{y}) \hat{y} - \int_{\hat{y}}^{\bar{y}} F(y) \, dy \right).
\]
Thus, $\Pi_E(d, B; \theta)$ is indeed given by

$$d + x_1 + \mathbb{E}[\hat{y}] + [1 - F(\hat{y})]V - B - F(\hat{y}) [x_1 + \hat{y} - B] + \int_{\hat{y}}^y F(y) \, dy.$$  

**Proof of Lemma 4**

Abusing notation, write $S_d(d; \theta)$ to denote the derivative of the managerial objective function under symmetric information (i.e., set $\Delta = 0$ in Eq. (14)).

First, note that the solution $\hat{\theta}(d)$ cannot be constant on any interval. Suppose, for a contradiction, that $\hat{\theta}(d)$ is constant for all $d$ in some open interval $I$. That implies that $\hat{\theta}'(d) = 0$ for all $d \in I$ which is the case if and only if $S_d(d; \hat{\theta}(d)) = 0$ for all $d \in I$. However, that contradicts the fact that $\lambda(y; \theta)$ is strictly increasing in $y$.

**Case 1:** $d_{S1}^x(\theta) = 0$. By Proposition 1, $S_d(0; \theta) \leq 0$. As a result, $\hat{\theta}'(0) \geq 0$. Clearly, it cannot be the case that $S_d(z; \hat{\theta}(z)) > 0$ for some $z > 0$: if that were the case, $\hat{\theta}'(z) < 0$, and $S_d(d; \hat{\theta}(d)) > 0$ for all $d \leq z$, contradicting the initial assumption that $S_d(0, \theta) \geq 0$. Thus, $\hat{\theta}(d)$ is strictly increasing in $d$.

**Case 2:** $d_{S1}^x(\theta) = \bar{d}(\theta)$. For this to be the case it must be true that $S_d(\bar{d}(\theta); \theta) \geq 0$ and so $\hat{\theta}'(\bar{d}(\theta)) \leq 0$ (here $\hat{\theta}'(\bar{d}(\theta))$ denotes the left derivative of $\hat{\theta}(d)$ at $d = \bar{d}(\theta)$). Since $S_d(d; \theta)$ is strictly decreasing in $d$ and strictly increasing in $\theta$, we have that $S_d(d; \theta) \geq 0$ for all $(d, \theta) \in [0, \bar{d}(\theta)] \times \Theta$. Therefore, $\hat{\theta}(d)$ is strictly decreasing in $d$ for all $d < \bar{d}(\theta)$.

**Case 3:** $d_{S1}^x(\theta) \in (0, \bar{d}(\theta))$. We have that $S_d(d_{S1}^x(\theta); \theta) = 0$. By the same arguments as above, we have that $\hat{\theta}(d)$ is strictly decreasing for $d < d_{S1}^x(\theta)$ and $\hat{\theta}(d)$ is strictly increasing for $d > d_{S1}^x(\theta)$ with an interior minimum at $d = d_{S1}^x(\theta)$.

**Proof of Proposition 2**

Assume that $\hat{\theta}(0) \geq \bar{\theta}$ for now and write $d^*(\theta) \equiv \hat{\theta}^{-1}(\theta)$. By construction, part 3 of Definition 1 is satisfied and we only need to check if parts 1 and 2 hold. I first check that parts 1 and 2 are satisfied, and then show that $\hat{\theta}(0) \geq \bar{\theta}$ when the support $\Theta$ is small enough. Finally, I give the proof for the comparative statics results.

**Optimal dividends.** From Eq. (19), the sign of $\partial \text{MRS}/\partial \theta$ is the same as that of $F_\theta(\hat{y}; \theta)$. We know that $F_\theta(\bar{y}; \theta) \leq 0$ for $\bar{y} = \bar{\theta}(\theta)$ and strictly so if $\hat{y} < \bar{\theta}(\theta)$. If $\hat{y} = \bar{\theta}(\theta)$, then the marginal rate of substitution is infinite.

Writing out the arguments in Eq. (21) explicitly, the first-order condition is

$$\text{MRS}(d, B^*(d; \hat{\theta}(d)); \theta) - \left[ B^*_d(d; \hat{\theta}(d)) + B^*_0(d, \hat{\theta}(d))\hat{\theta}(d) \right] = 0,$$  

(37)

58
where $B_\theta^*(d, \theta) \equiv \partial B^*/\partial d$ and $B^{\tilde{\theta}}_\theta(d, \theta) \equiv \partial B^*/\partial \theta$.

Suppose that type $\theta$ finds it optimal to pay $d' \in (d^*(\tilde{\theta}), d^*(\theta))$ with $\theta' \equiv \tilde{\theta}(d')$ and $\theta' \neq \theta$:

$$MRS(d', B^*(d', \tilde{\theta}(d')); \theta) - \left[ B_\theta^*(d', \tilde{\theta}(d')) + B^{\tilde{\theta}}_\theta(d', \tilde{\theta}(d')) \tilde{\theta}'(d') \right] = 0.$$  

This leads to an immediate contradiction since if $\theta' > \theta$

$$MRS(d', B^*(d', \tilde{\theta}(d')); \theta) - \left[ B_\theta^*(d', \tilde{\theta}(d')) + B^{\tilde{\theta}}_\theta(d', \tilde{\theta}(d')) \tilde{\theta}'(d') \right] >$$

$$MRS(d', B^*(d', \tilde{\theta}(d')); \theta') - \left[ B_\theta^*(d', \tilde{\theta}(d')) + B^{\tilde{\theta}}_\theta(d', \tilde{\theta}(d')) \tilde{\theta}'(d') \right] = 0,$$

and the sign of the inequality is reversed if $\theta' < \theta$. Now suppose that type $\theta < \tilde{\theta}$ finds it optimal to pay $d' = d^*(\tilde{\theta})$. For the deviation to be optimal, we must have

$$MRS(d^*(\tilde{\theta}), B^*(d^*(\tilde{\theta}), \tilde{\theta}); \theta) - \left[ B_\theta^*(d^*(\tilde{\theta}), \tilde{\theta}) + B^{\tilde{\theta}}_\theta(d^*(\tilde{\theta}), \tilde{\theta}) \tilde{\theta}'(d^*(\tilde{\theta})) \right] \leq 0,$$

with equality if $d^*(\tilde{\theta}) > 0$. However, this again leads to a contradiction since $\theta < \tilde{\theta}$, and hence we have

$$MRS(d^*(\tilde{\theta}), B^*(d^*(\tilde{\theta}), \tilde{\theta}); \theta) - \left[ B_\theta^*(d^*(\tilde{\theta}), \tilde{\theta}) + B^{\tilde{\theta}}_\theta(d^*(\tilde{\theta}), \tilde{\theta}) \tilde{\theta}'(d^*(\tilde{\theta})) \right] >$$

$$MRS(d^*(\tilde{\theta}), B^*(d^*(\tilde{\theta}), \tilde{\theta}); \theta') - \left[ B_\theta^*(d^*(\tilde{\theta}), \tilde{\theta}) + B^{\tilde{\theta}}_\theta(d^*(\tilde{\theta}), \tilde{\theta}) \tilde{\theta}'(d^*(\tilde{\theta})) \right] = 0.$$

Finally, consider a deviation to $d = d^*(\tilde{\theta})$ for some $\theta > \tilde{\theta}$. If $d^*(\tilde{\theta}) = x + (\mathbb{E}[y | \theta] - b)/(1 - \varphi)$, we can apply the same argument as for a deviation to $d^*(\tilde{\theta})$ (with the directions of the two inequalities above reversed). If $d^*(\tilde{\theta}) = x + (\mathbb{E}[y | \theta] - b)/(1 - \varphi) \leq x$, then, writing out Eq. (37) in full, for the deviation to be optimal we need

$$\varphi + (1 - \gamma) \frac{\partial \tilde{y}}{\partial d} \left[ F(\tilde{y}(\theta); \theta) - F(y(\theta); \theta) \right]$$

$$- (1 - \gamma) \tilde{\theta}'(d^*(\tilde{\theta})) \int_y \tilde{y}(\theta) F(y; \theta) \, dy \geq 0,$$

where $\tilde{y} = B^*(d, \tilde{\theta}(d)) - (x - d)(1 - \varphi)$ and $\partial \tilde{y} / \partial d$ is evaluated at $d = d^*(\tilde{\theta})$. Now

$$\lim_{d \uparrow d^*(\tilde{\theta})} \frac{\partial \tilde{y}}{\partial d} = +\infty,$$

and so the equation cannot hold if $F(\tilde{y}(\theta); \theta) < F(y(\theta); \theta)$. If $F(\tilde{y}(\theta); \theta) = F(y(\theta); \theta)$, we calculate using L'Hôpital's rule that

$$\lim_{\tilde{y} \to \tilde{y}(\theta)} \frac{F(\tilde{y}; \theta) - F(\tilde{y}; \theta)}{1 - F(\tilde{y}; \theta)} = 1 - \lim_{\tilde{y} \to \tilde{y}(\theta)} \frac{F(\tilde{y}; \theta)}{F(\tilde{y}; \theta)} = 1 - \psi,$$
and therefore the limit of the left-hand side of Eq. (38) as \( d \uparrow d^*(\theta) \) is given by

\[
LHS = \varphi + (1 - \gamma)(1 - \psi)(1 - \varphi) - (1 - \gamma)\psi\theta'(d^*(\theta)) \int_y^{\theta(\hat{\theta})} F_\theta(y; \theta) \, dy
\]

\[
= [1 - \gamma(1 - \varphi)](1 - \psi) < 0,
\]

where I have used the differential equation in Eq. (24) to obtain the second equation, and the hypothesis for the inequality.

It remains to be shown that a deviation to an off-equilibrium \( d \not\in [d^*(\hat{\theta}), d^*(\theta)] \) is not profitable. First, consider whether the highest type \( \hat{\theta} \) would like to deviate to \( d' < d^*(\hat{\theta}) \). Since \( V = 0 \), \( \Pi_M(d, B^*(d; \hat{\theta}); \hat{\theta}) \) is strictly increasing in \( d \). Thus, the highest type \( \hat{\theta} \) strictly prefers \( d^*(\hat{\theta}) \) to any \( d' < d^*(\hat{\theta}) \). By Lemma 3, all lower types would also not deviate to \( d < d^*(\hat{\theta}) \). Second, consider a deviation \( d' > d^*(\theta) \). If \( d^*(\theta) = x \), trivially, no such \( d' \) exists, so assume \( d^*(\theta) < x \). Given the equilibrium beliefs, the break-even condition cannot be satisfied for \( d' > d^*(\theta) \). Therefore, the firm secures no funding and obtains a payoff of zero. By Assumption 3, the equilibrium payoff \( \Pi^*_M(\theta) \) satisfies \( \Pi^*_M(\theta) \geq \Pi_M(\hat{\theta}(\theta), B^*(\hat{\theta}(\theta), \theta); \theta) = x + (\mathbb{E}_x[\hat{y}] - b)/(1 - \varphi) \geq 0 \), and hence no type \( \theta \) has an incentive to deviate.

**Creditors break even.** It is sufficient to check that all firms are solvent at the equilibrium dividend policy \( d^*(\theta) \), i.e., for all \( \theta \), there exists some value of \( B \) that solves Eq. (9) when \( d = d^*(\theta) \). Since \( d^*(\theta) = \hat{\theta}(\theta) \), the lowest type is solvent under the equilibrium dividend policy by construction. A fortiori, all other types are also solvent since they both pay a strictly lower dividend and have a weakly lower default probability for a given dividend.

**Continuity.** To complete the proof that \( \hat{\theta}(d) \) induces a separating equilibrium, note that since \( \hat{\theta}(x) = \theta \), \( x > 0 \), and \( \hat{\theta}(d) \) is continuous and independent of \( \theta \), \( \hat{\theta}(0) \geq \theta \) is satisfied whenever \( \theta \) is sufficiently close to \( \theta \).

**Comparative statics.** It is immediate that \( \hat{\theta}'(d) < 0 \) from Eq. (24). Since \( B^*(d; \theta) \) is strictly increasing in \( d \) and strictly decreasing in \( \theta \) by Lemma 1, we have that \( B^*(d; \hat{\theta}(d)) \) is strictly increasing in \( d \). Finally,

\[
\frac{\partial F(\hat{y}; \hat{\theta}(d))}{\partial d} = f(\hat{y}; \hat{\theta}(d)) \left( \frac{\partial B^*}{\partial d} + \frac{\partial B^*}{\partial \theta} \hat{\theta}'(d) + (1 - \varphi) \right) + F_\theta(\hat{y}; \hat{\theta}(d))\hat{\theta}'(d) > 0,
\]

as claimed.
Proof of Corollary 1

From Proposition 1 and Lemma 4, \( \hat{\theta}(d) \) has a strictly decreasing part whenever

\[
V < \frac{\varphi}{(1 - \varphi) \lambda(\hat{y}_L(\hat{\theta}), \hat{\theta})} \equiv V_1.
\]

Suppose that \( V \in (0, V_1) \). Let \( d_{SI,0}^*(\theta) \) denote the dividend paid by type \( \theta \) under symmetric information when \( V = 0 \). If the strictly decreasing part of \( \hat{\theta}(d) \) induces an equilibrium, then all equilibrium default thresholds \( \hat{y} \) must lie in

\[
\hat{Y}^* \equiv [B^*(0, \theta) - x(1 - \varphi), B^*(d_{SI,0}^*(\theta), \theta) - (x - d_{SI,0}^*(\theta))(1 - \varphi)].
\]

From Eq. (19), the sign of \( \partial \text{MRS}/\partial \theta \) is the same as the sign of

\[
F_\theta(\hat{y}; \theta)(1 - \gamma) - f_\theta(\hat{y}; \theta)V.
\]

Since \( f_\theta \) is continuous on \( \hat{Y}(\theta) \times \Theta \), there exists some \( K > 0 \) such that \( |f_\theta| \leq K \) for all \( (y, \theta) \in \hat{Y}^* \times \Theta \). If \( \mathbb{E}[\hat{y} | \theta] > b \), then clearly \( F_\theta(\hat{y}; \theta) < 0 \) for all \( (\hat{y}, \theta) \in \hat{Y}^* \times \Theta \). When \( \mathbb{E}[\hat{y} | \theta] \leq b \), then \( F_\theta(\hat{y}(\theta); \theta) < 0 \) by the hypothesis. All in all, by continuity of \( F_\theta \), we have that there exists some \( M > 0 \) such that \( F_\theta \leq -M \) for all \( (y, \theta) \in \hat{Y}^* \times \Theta \).

Thus, we have

\[
F_\theta(\hat{y}; \theta)(1 - \gamma) - f_\theta(\hat{y}; \theta)V \leq -M(1 - \gamma) + KV < 0,
\]

whenever \( V < M(1 - \gamma)/K \equiv V_2 \) for all \( (y, \theta) \in \hat{Y}^* \times \Theta \).

For \( V < \min\{V_1, V_2\} \), we have that both \( \text{MRS}(d, B; \theta) \) is strictly decreasing in \( \theta \) for the relevant values of \( \hat{y} \), and \( \hat{\theta}(d) \) has a strictly decreasing part.

We can now apply the same arguments as in the proof of Proposition 2 to show that \( \hat{\theta}(d) \) induces an equilibrium whenever \( (\hat{\theta} - \theta) \) is small enough.

Proof of Proposition 3

Part 1. Suppose that \( V > \max\{\hat{V}_H(\gamma), \hat{V}(\theta)\} \). Then, the marginal rate of substitution is strictly increasing in \( \theta \) and \( \hat{\theta}(d) \) is strictly increasing. We can use the same arguments as in the proof of Proposition 2 to show that \( \hat{\theta}(d) \) induces an equilibrium (with inequalities in the opposite direction because MRS is strictly increasing instead of strictly decreasing).

Part 2. Suppose that \( V < \min\{\hat{V}_L(\gamma), \hat{V}(\theta)\} \). Then, the marginal rate of substitution is strictly decreasing in \( \theta \) and \( \hat{\theta}(d) \) is strictly decreasing and we can proceed exactly as in Proposition 2.
Part 3. Pick any sufficient small $V > 0$ and $\gamma$ sufficiently close to one such that $\hat{V}_H(\gamma) < V(\theta)$. In this case, the marginal rate of substitution is strictly increasing in $\theta$ but $\tilde{\theta}(d)$ is strictly decreasing. By Lemma 3, $\tilde{\theta}(d)$ cannot induce an equilibrium.

Proof of Proposition 4

Using the expression for $\Pi_E$ in Eq. (30) with $\hat{y} = B^*(d, \tilde{\theta}(d)) - (x - d)(1 - \varphi)$, we calculate that

$$P'(d) = \varphi - \tilde{\theta}'(d) \int_{y}^{\hat{y}} F_{\theta}(y; \tilde{\theta}(d)) \, dy \left[ 1 + \lambda(\hat{y}; \tilde{\theta}(d))V \right] - (1 - \varphi)\lambda(\hat{y}; \tilde{\theta}(d))V$$

$$- \tilde{\theta}'(d) \left[ \int_{y}^{\hat{y}} F_{\theta}(y; \tilde{\theta}(d)) \, dy + F_{\theta}(\hat{y}; \tilde{\theta}(d)) \right].$$

If $\tilde{\theta}(d)$ is strictly decreasing, we have that

$$P'(d) \leq \varphi - \tilde{\theta}'(d) \int_{y}^{\hat{y}} F_{\theta}(y; \tilde{\theta}(d)) \, dy \left[ (1 - \gamma) + \lambda(\hat{y}; \tilde{\theta}(d))V \right] - (1 - \varphi)\lambda(\hat{y}; \tilde{\theta}(d))V$$

$$- \tilde{\theta}'(d) \left[ \int_{y}^{\hat{y}} F_{\theta}(y; \tilde{\theta}(d)) \, dy + F_{\theta}(\hat{y}; \tilde{\theta}(d)) \right]$$

$$= -\tilde{\theta}'(d) \left[ \int_{y}^{\hat{y}} F_{\theta}(y; \tilde{\theta}(d)) \, dy + F_{\theta}(\hat{y}; \tilde{\theta}(d)) \right] < 0,$$

where the equality comes from Eq. (22). If $\tilde{\theta}(d)$ is strictly increasing, the two inequalities above are reversed, and we obtain that $P'(d) > 0$. 

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Appendix B  Solving the Model With Endogenous Debt

I now describe the solution procedure for the model with endogenous borrowing, government bailouts and a two-point distribution for the long-term asset. With \( \gamma = 0 \), the objective function of the manager is given by

\[
\Pi_M(u, B; \theta) = d + \theta[(x + b - b_0 - d)(1 - \varphi) + y - B],
\]

which is linear in \( b, d \), and \( B \). The debt pricing equation becomes

\[
b = \theta B + (1 - \theta)[\beta B + (1 - \beta)(x + b - b_0 - d)(1 - \varphi)].
\]

Solving the equation for \( B \) yields \( B^*(u, \theta) \) which is also linear in \( b \) and \( d \).

I assume that all out-of-equilibrium deviations are attributed to the lowest quality firm. Consider the firm’s optimal choice of \( u \) when it is thought to have quality \( \theta \) by the creditors. Substituting the expression for \( B^* \) into the managerial objective function and differentiating, we find that, provided \( \varphi > 0, \partial \Pi_M(u, B^*(u, \theta); \theta) / \partial b < 0 \), and so the optimal solution has the constraint \( x + b - b_0 - d \geq 0 \) binding. Substituting out \( b \) and differentiating, we obtain that

\[
\frac{\partial \Pi_M([d, b_0 + d - x), B^*((d, b_0 + d - x), \theta); \theta]}{\partial d} \geq 0 \Leftrightarrow \theta \leq \sigma + (1 - \sigma)\theta.
\]

Thus, types with \( \theta < \sigma + (1 - \sigma)\theta \) pay out the highest dividend at which the creditors can break even, which is given by the solution to \( B^*((d, b_0 + d - x), \theta) = y \). Types with \( \theta > \sigma + (1 - \sigma)\theta \) pay zero dividends, while type \( \theta = \sigma + (1 - \sigma)\theta \) is indifferent between all feasible dividend policies. Denote the payoff resulting from the optimal deviation by \( \Pi_M^{\text{dev}}(\theta) \).

To solve the problem, I discretize the type space to obtain a finite number of types \( \theta_i, i = 1, \ldots, N \). For simplicity, I use a uniform grid with a constant step size \( \theta_i - \theta_{i-1} \). The probability of type \( \theta_i \) is given by \( p_i \equiv G(\theta_i) - G(\theta_{i-1}) \) with \( G(\theta_0) \equiv 0 \). Then, Problem 4 is approximated by the following linear programming problem:

\[
\begin{align*}
\max_{\{u_i\}_{i=1}^N} & \sum_{i=1}^N p_i \Pi_M(u_i, B^*(u_i, \theta_i); \theta_i) \ d G(\theta) \\
\text{s.t.} & \quad \Pi_M(u_i, B^*(u_i, \theta_i); \theta_i) \geq \Pi_M(u_j, B^*(u_j, \theta_j); \theta_i) \quad \text{for all } i, j \\
& \quad \Pi_M(u_i, B^*(u_i, \theta_i); \theta_i) \geq \Pi_M^{\text{dev}}(\theta_i) \quad \text{for all } i \\
& \quad B^*(u_i, \theta_i); \theta_i) \leq y, d_i \geq 0, b_i \geq 0, x + b_i - b_0 - d_i \geq 0 \quad \text{for all } i.
\end{align*}
\]