Financing Innovation Under Asymmetric Information: signalling through internal financing

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Abstract

This paper considers the financing of innovation in the presence of information asymmetry between innovators and investors. Innovators are privately informed of high or low prior belief in the risky project and choose to exert costly effort to experiment before investment. As innovators themselves normally cover the learning cost, the choice of internal funding spent on the experimentation could be adopted to signal their confidence in the project. The more the internal funding allocated, the faster the learning, so the posterior belief in the project being good will be increased if no bad news arrives during experimentation. In the first-best case, high-type innovators tend to allocate less internal financing based on the higher prior belief compared with the low-type ones. When borrowing from external financiers, revenue shares contingent on success or failure and internal financing will be included in the financial contract. Under imperfect information, multiple pooling equilibria exist where cross-subsidisation from the high to the low is commonly involved. While under the unique least-costly separating equilibrium, the high-type innovators are willing to invest even more than the low-type first-best internal funding in order to repel others from mimicking, and the low-type remain at their first-best internal financing, in which case financier could distinguish between different types of innovators via the signal of internal funding.

JEL classification: D82,D86,G32,O32

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1 Introduction

Nowadays, under-investment in innovation has become a widely recognized problem, especially for those small and medium-sized innovative firms (Hall and Lerner, 2010). Early theoretical studies have suggested that severe asymmetric information between innovators and financiers may result in under-provision of innovation investment. Moral hazard problem exists in the sense that the financiers can barely keep inventions as secrets, which makes themselves reluctant of undertaking initial investment (Nelson, 1959; Arrow, 1962). On the other hand, financiers could not observe the quality of innovation project as much as innovators do, which gives rise to the adverse selection issue (Brealey et al., 1977; Myers and Majluf, 1984). However, compared with external financing sources (venture capitalists or business angel), internal funding is a more convenient and effortless way of financing, which particularly works well at the initial learning stage of innovation. In fact, internal financing plays a central role in R&D spending among small high-tech firms in US and UK, and the development of innovation is primarily based on the availability of internal financing (Spence, 1979; Himmelberg and Petersen, 1994; Bougheas, 2004). Although the importance of internal financing has been corroborated empirically, there exists few theoretical work exploring the crucial role of internal financing in innovation, especially under asymmetric information. Hence, this paper tends to provide insight into firms’ optimal decision of internal and external financing, and theoretically proves that internal financing could be adopted as a signalling tool in innovation financing so as to alleviate the adverse selection problem.

To begin with, the paper addresses the existence of adverse selection problem within innovation process. In practice, innovation often goes through trial and error learning phase before investment where innovators usually use the internal funding to learn how good the project is. Assume innovators are initially endowed with an identical innovative project with either high or low prior belief on success, but the payoff of which in the event of success or failure is the same\(^1\). Whereas, prior beliefs are unobservable to the financiers. In this case, adverse selection problem arises where low-type innovators may pretend of owning a high-type project and the uninformed financiers can hardly distinguish. Without any signalling scheme, it may even result in a ‘lemons’ market.

\(^1\)In this paper, we sometimes refer to innovators as high-type or low-type according to their endowed beliefs in the project, which means there is no heterogeneity in innovators' learning ability or marginal cost of learning.
problem where no financier is willing to fund the project and high-type innovators are driven out of the market (Akerlof, 1970). To address this issue, we claim that not only does the internal funding support the learning cost but also reveals innovator’s prior belief to the financiers.

To see this, we first look into the first-best internal financing of high and low types. The model adopts the exponential-bandit game in accordance with the uncertainty nature of innovation where internal funding is allocated in order to explore the unknown state of world (Bergemann and Hege, 2005; Manso, 2011; Horner and Samuelson, 2013; Bouvard, 2014, etc.). In the good state, risky project succeeds and generates a higher revenue; in the bad state, it fails and receives a lower revenue. Experimentation would randomly generate bad signals which indicate the state is bad following the setup of bad news bandit model (Keller et al., 2005; Keller and Rady, 2010). The arrival of bad news follows an exponential distribution with intensity $\lambda$, which depends on the amount of internal funding provided, such that more internal funding accelerates learning and provides a more accurate prediction of the bad state. According to the Baye’s rule, given a fixed period of learning, posterior belief on good state tends to increase in internal funding when no bad news arrives. Therefore, under perfect information, the first-best internal financing is determined by the profit-maximisation problem of innovators. As a result, for the project with relative lower prior belief in the good state, a larger amount of internal funding will be chosen; given a higher prior belief, innovators use less of internal funding. Intuitively, projects with higher prior of success require less learning input as they would easily succeed, so innovators are prone to divert funding to other inventions, however, innovation with lower prior requires more experiment and attention in order to avoid future failure. The above intuition appears to be in line with the general result of real-option signaling model where innovators hold the option of investment timing, such that innovators with higher prior belief are apt to provide less learning and invest earlier (Grenadier and Wang, 2005; Bouvard, 2014; Bobtcheff and Levy, 2014). Instead, internal financing is treated as the optimal decisions of innovators in this paper, which conveys private information on the quality of the innovative project. Despite a larger internal funding is allocated by the low-prior project holders, by the time of investment, they still hold a lower expectation on the state being good than those with a higher prior. Indeed, for those cash-constrained innovative companies, it’s fairly risky to allocate too much internal funding on the project with a relative low prior of success at early stage.
In this set-up, we then explore external financial contracting between informed innovators and less-informed financiers. Innovative projects, irrespective of prior beliefs, require the same amount of investment, which could not be financed internally by assumption. Innovators are expected to share a certain proportion of revenue with the financier in return of external investment. Hence, revenue shares conditioning on success and failure of the innovation will be included ex-ante in the financial contract. Specifically, innovators with higher prior belief will keep higher revenue share themselves given higher expected possibility of success compared with low-type ones. Thus, equilibrium contracts are composed of revenue shares and internal funding, which jointly convey the information on the quality of the project. We also find relevant contractual terms appear in real-world financial contracts within the venture capital and start-up firms. As documented in Kaplan and Strömberg (2003), cash flow rights contingent on the accomplishment of milestones are widely used in incentivizing entrepreneurs under asymmetric information, which functions the same as revenue shares in our model. Internal funding is normally implicitly regarded as a criteria at which financiers may judge how much confidence innovators have in this project.

However, information asymmetry may generate distortions in the sense that first-best contracts are no longer attainable for both types. For the low-type innovators will deviate by choosing the high-type contract by offering less of the shares and internal financing to the financier. To resolve this problem, the high-type would endeavor to separate from the low ones by signalling. One of our key proposition states that larger amount of internal financing will be adopted by the high-type in this case as compared with high-type full-information optimum as well as low-type's first-best. In fact, increasing internal funding would upward adjust project’s expected probability of success if no bad news is observed during experimentation, which in return building up financier’s expectation of investing this innovative project. Thus, the expansion of high-type internal financing compensates financier’s loss on the low-type contract, which avoids the market-breakdown. More importantly, internal financing functions as a separation tool by the high-type which repels the low-type from mimicking. Regarding high and low innovators’ preferences over internal funding and revenue shares, the single-crossing property is proved to be true. It indicates that it costs the low-type more internal funding in exchange of an additional increase in revenue shares in comparison with the high ones. Details of this property will be shown in the analysis later. Consequently, mimicking high-type contract
by exerting more internal funding is no longer profitable for the low-type as the low-
type incentive compatibility constraint is relaxed. Proposition 4 formally describes this
separating equilibrium which manages to distinguish agent’s type and make the financier
break-even on average. This paper also covers other pooling equilibria, which involves
cross-subsidisation from the high to the low, when either one of the contractual tools
is available. However, the least-costly separating equilibrium is proven to be interim
efficient and thus weakly Pareto dominates other equilibria in the sense that both types
of agents would perform their type-equivalent contract and the low-type become weakly
better-off.

Supportive evidence regarding innovator’s distinctive choices over internal funding
could be found in various empirical studies on the pharmaceutical industry where R&D
plays a pivotal role. As indicated in Danzon et al. (2005) and Lerner et al. (2003),
those more experienced and larger pharmaceutical companies are more likely to develop
drugs relying on an internal alliance with other biotech companies, and their drugs get
approved more often by the administration in comparison with those produced by a
single and inexperienced company. Intuitively, forming up such alliance may convey a
positive signal to outside investors. Predictions of our model also match the study by
Guedj and Scharfstein (2004) which shows the correlation between initial learning and
success rate. For those young and cash-constrained drug firms, they only own several
drug candidates although some of which are not promising. However, the managers of
these drug companies behaved more aggressive in putting forward as many candidates
as possible from Phase I to Phase II with a less extensive experiment conducted, it turns
out that most of which failed in Phase III, compared with mature ones. Within two
years Phase I trial, 61.4% of candidates were promoted by young companies compared
to 45.3% promotion rate among mature ones.

Financing innovation under adverse selection has been examined in several recent pa-
ters that are closely related to ours (Grenadier and Wang, 2005; Bouvard, 2014; Bobtcheff
and Levy, 2014). In their models, the duration of experimenting (investment timing) is
an active contractual variable and served as a signaling tool. In Bouvard (2014), when
there is information asymmetry on innovator’s prior beliefs, high-type innovators would
extend investment timing in order to make their contracts no longer attractive to low-
type ones, which forms up a separating equilibrium. Bobtcheff and Levy (2014) model
innovators with different learning speed. As optimal investment date is non-monotonic in
learning speed, it may result in either under or over investment under imperfect information. Although the papers as mentioned earlier have pointed out that internal funding alleviates the information distortion in adverse selection, none of them have regarded it as a signaling tool nor related it with learning. Thus, our paper fills the gap in the innovation-financing literature by introducing internal financing as a contractible and signaling tool by heterogeneous innovators.

This article belongs to a broader literature that combines experimentation and agency problem. The seminal papers by Bergemann and Hege (1998, 2005) have examined agency conflict under imperfect information and its correlation with funding releasing speed. Under arm-length financing when agent’s actions are observable, funding releasing rate decreases since the posterior belief on good state goes down as long as no good news arrives over time. However, when actions are unobservable under relationship financing mode, moral hazard problem arises. In this case, the financier tends to downgrade his posterior belief on good state after a deviation is detected, which leads to a shorter financing horizon. In two-period experimentation model by Drugov and Macchiavello (2014), investment cost is revealed after first period’s experimentation. More recently, the mechanism designed to alleviate moral hazard in financing innovation has received much attention (Manso, 2011; Yu et al., 2012; Sannikov, 2014; Horner and Samuelson, 2013). While our paper is more related to another strand of innovation literature concentrating on the adverse selection conflict between informed innovators and less-informed investors, Gomes et al. (2013) have examined screening contracts which help the monopolistic investor to distinguish private information of the innovators. Free-riding and communication between a group of innovators have been studied under innovation competition set-up (Halac et al., 2016a; Heidhues et al., 2015; Akcigit and Liu, 2015). Compared with the above-mentioned papers, the distinctive feature of our paper is the inverting roles of innovator and financier. As we assume there are a lot of financiers in the market, the innovator, the owner of innovative project, captures the entire bargaining power and proposes the financing contracts. Thus, we draw attention on the equilibrium behaviour of innovators with heterogeneous prior beliefs on the future success probabilities.

As motivation and related literature are illustrated above, this paper proceeds as follows. Section 2 introduces model set-up and socially first-best internal financing. Section 3 explores the equilibrium contracts under perfect and imperfect information respectively, including the analysis of pooling equilibria and least-costly separating equilibrium. Sec-
tion 4 concludes.

2 Model

2.1 Set-up

To match the real-world innovation investment, we model it regarding two-stage funding allocation, referring to Figure 1. At the start-up testing stage, innovators use internal funding to learn the quality of the project. As allocated prior to learning, it’s fixed and irreversible even if some bad signals may arrive during the experiment process afterwards. Let $\lambda$ denote the internal financing, which represents the entire cost of learning. It involves not only financial cost but also non-financial inputs, such as managerial skill, observable effort, etc. Thus, internal funding may not be sufficient enough to cover further investment, which amounts to $I$ and $I > \lambda$. At the investing stage, the financier lends the funding $I$ to the innovator according to the pre-negotiated contract. The details of the contract will be discussed below. The result of the investment, either success or failure, would be realised immediately upon investing. In the good state, the project succeeds and generates revenue $\bar{R}$. However, it fails in the bad state and receives 0. Whenever the expected revenue of the project is greater than the investment cost, it’s worthwhile to initiate the experiment. Let $T$ be the exogenously fixed investment timing. Imagine in practice, given a certain period of experimentation, the project without any negative news will normally receive more attention and deserve further investment.

Innovators are privately assigned with a prior belief $p_0^h$ of the project being good, which could either be high or low, such that $p_0^h > p_0^l$. While within the distribution, it is commonly known that there are $q$ proportion of high-type innovators and $(1-q)$ of low-type ones. We also impose the condition to make both types of innovators have incentives to experiment: assume the expected revenue of the project based on prior beliefs is higher than total investment, i.e., $p_0^h \bar{R} > p_0^l \bar{R} \geq I$.

Uncertainties exist in the innovation in the sense that whether the project will succeed or not is unknown until investment, but learning will provide additional information about the project via the arrival of bad news. Here we adopt the exponential distribution to model the arrival of a bad signal during experimentation, following Keller et al. (2005) and Keller and Rady (2010). Thus, learning would randomly generate bad news
with intensity $\lambda$ if the state is bad following an exponential distribution, which is public observable. As soon as bad news arrives, posterior belief on the good state will jump to 0, innovators and financier will abandon the project immediately. However, if no bad news comes, based on the Baye’s rule, one would upward adjust their beliefs on the good state.

We use $U$ and $V$ describe the expected payoff of the innovator and financier respectively. Both risk-neutral parties discount future with rate $r$ and have full commitment power.

![Figure 1: Timing of events](image)

### 2.2 First-best internal financing

This section considers the optimal allocations of internal funding by the social planner who has perfect information on the project’s prior. At time 0, he chooses internal funding $\lambda^\theta$ given the realisation of type-$\theta$ prior belief in order to maximise the expected surplus of the project, denoted as $\Pi(\lambda^\theta)$:

$$
\max_{\lambda^\theta} \Pi(\lambda^\theta) = e^{-rT} \left[ p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T} \right] \left( p_T^\theta R - I \right) - \lambda^\theta.
$$

(1)

In continuous time setting, $e^{-rT}$ is the time discounting and $T$ is the exogenous investment timing known to the public. $[p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T}]$ is the probability of getting no bad news before $T$: in the good state, no bad news will come with certain; in the bad state, bad news comes with probability $(1 - e^{-\lambda^\theta T})$, thus, $(1 - p_0^\theta)e^{-\lambda^\theta T}$ is the conditional probability of not receiving any bad news when the state is bad. The third term indicates the expected net return from investing at time $T$. $p_T^\theta$ is the updated posterior belief on good state at time $T$: $p_T^\theta = \frac{p_0^\theta}{p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T}}$ following the Baye’s rule if no bad news arrives; otherwise, $p_T^\theta = 0$. The last term $\lambda^\theta$ represents the entire experimentation cost for $\theta$-type project.
However, there is a trade-off of choosing internal funding $\lambda^\theta$. On the one hand, a larger amount of internal funding would accelerate one’s learning speed, in the event of no bad news before $T$, one would be more confident in the good state as a higher posterior belief will be perceived. On the other hand, internal investment is allocated before learning and irreversible, which involves many risks. Thus, internal funding allocation varies across different types of projects.

As Equation (1) could be rearranged by substituting posterior belief $p_t$ expression:

$$\max_{\lambda^\theta} \Pi(\lambda^\theta) = e^{-rT} \{ p_0^\theta R - [p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T}]I \} - \lambda^\theta \quad (2)$$

Then first-best $\bar{\lambda}^\theta$ for $\theta$-type project could be solved as following:

$$\bar{\lambda}^\theta = -r + \frac{1}{T} \ln T(1 - p_0^\theta)I \quad (3)$$

Specifically, for $p_0^h > p_0^l$, $\bar{\lambda}^h < \bar{\lambda}^l$. Intuitively, for a high-type innovation, less internal funding is needed to learn the state of nature given the higher prior belief of success; however, for a low-type project, larger initial funding is required in order to discover whether it’s worth investing.

3 The External Financial Contracting

This section focuses on the financial contracting issue within innovators and financiers in the perfect competitive capital market. Assume innovators have limited cash at hand, which could only cover the learning expenditure instead of making the final investment. Thus, innovators have to seek external funding to finance the investment. This paper adopts the ex-ante contracting setting where informed innovators contract with less-informed financiers before exerting their internal funding, which firstly proposed as the informed-principal problem by Maskin and Tirole (1992). Note that the informed-principal setting inverts the timing of typical signalling game, such as Spence (1973) where workers acquire educational certificates so as to send signals to uninformed employers before signing labour contracts. One can show that this educational signalling game has different classes of Perfect Bayesian equilibria, including multiple pooling and separating equilibria, however, some of which are inefficient and fail to fulfill Cho-Kreps
intuitive criterion (Cho and Kreps, 1987, Laffont and Martimort, 2009). This paper adopts ex-ante contracting setup which overcomes the inefficiency of multiple equilibria and renders opportunity to select the possible least-costly separating equilibrium.

Let the contract proposed by $\theta$-type agent include internal funding and revenue share, denoted as $C^\theta = \{\lambda^\theta, \alpha^\theta\}$ where $\alpha^\theta$ denotes the revenue share of the financier in the event of success, $\theta = \{h, l\}$. As there are only two types of innovators, we could concentrate on two possible incentive compatible contracts, i.e., $C^h$ and $C^l$, corresponding to the revelation principle.

In the environment of imperfect information, the innovator, regardless of his type, is assumed to propose a menu of two contracts to the financier. After belief updating, the financier decides whether to accept the offer or not. If accepted, the innovators chooses one of the contract to execute. Otherwise, proposing to another financier. According to Tirole (2006), delivering a menu of two contracts to the uninformed party is to ‘get rid of their bad expectations’. As in our case the low-type would gain extra by mimicking the high-type contract under information asymmetry, and the financier makes a loss as a result. Thus, the financier is reluctant of taking high-type contract solely as it might be from a low-type agent. However, such bad expectation would be eliminated if high-type agent includes a low-type incentive-compatible contract into the proposal, which makes the financier breaks even on average. After contracting, innovators choose one of the contracts to perform and initiate experimentation.

### 3.1 Perfect information equilibrium contracts

In this section, financiers have complete information of the prior belief on the project being good when contracting with the innovator. However, learning is still necessary to explore whether the project indeed is good or bad. From the previous section, we know that $\tilde{\lambda}^h$ and $\tilde{\lambda}^l$ are the first-best internal financing, which are supposed to be achievable under full information contracting. In this setup, financiers would sign the contract if the expected net return is no less than the outside option 0. Thus, $\alpha^\theta$ is proposed to make the financier at least break-even under $\theta$-type financial contract. The expected payoff of the financier and innovator at period 0 are denoted as $V^\theta(C^\theta)$ and $U^\theta(C^\theta)$ respectively.
as following:

\[ V^\theta(C^\theta) = e^{-rT}[p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T}] (\alpha^\theta p_T^\theta \bar{R} - I) = e^{-rT}[p_0^\theta \alpha^\theta \bar{R} - [p_0^\theta + (1 - p_0^\theta)e^{-\lambda^\theta T}]I] \]

\[ U^\theta(C^\theta) = e^{-rT}(1 - \alpha^\theta) p_0^\theta \bar{R} - \lambda^\theta. \]

Note that \( U^\theta \) is monotonically decreasing in \( \lambda^\theta \). As internal funding increases, it builds up one’s posterior belief in the project being good in the event of no bad news. However, at period 0, innovator’s expected payoff depends on the ex ante success probability rather than the updated posterior belief. Therefore, internal funding solely adds extra cost to innovator’s total expected payoff.

Here we write down the optimisation programme of \( \theta \)-type innovators:

\[
\max_{C^\theta} U^\theta(C^\theta), \quad \text{subject to } V^\theta(C^\theta) \geq 0
\]

Under full-information, financier breaks even, thus \( V^\theta(C^\theta) = 0 \). Substitute this condition back into the maximisation problem, we could determined the following:

\[ \lambda^\theta = -r + \frac{1}{T} \ln T(1 - p_0^\theta)I \quad (4) \]

\[ \alpha^\theta = \frac{I}{p_T^\theta \bar{R}} = \frac{I}{\frac{p_0^\theta}{p_0^\theta + e^{-rT}T} \bar{R}} = \frac{I}{\bar{R}} + \frac{1}{p_0^\theta \bar{R} e^{-rT}T}, \quad (5) \]

For \( p_0^h > p_0^l \), \( \lambda^h < \lambda^l \) and \( \alpha^h < \alpha^l \) hold. Thus, the high-type innovators allocate less internal funding and keep larger fraction of revenue themselves compared to low ones.

As single-crossing property is satisfied, i.e., \( \frac{MU^l_1}{MU^l_{1-a}} > \frac{MU^h_1}{MU^h_{1-a}} \), we find that it costs a low-type innovator more internal funding to achieve an additional increase in revenue share than a high-type maintaining expected payoffs at the same level. Therefore, though larger optimal internal funding is provided by low-type ones, they still claim less revenue share. Moreover, provided optimal internal funding, posterior beliefs in good state at time \( T \) could be determined respectively, that is, \( p_T^h = \frac{\frac{p_0^h}{p_0^l + e^{-rT}T}}{\frac{p_0^l}{p_0^l + e^{-rT}T}} > p_T^l = \frac{\frac{p_0^l}{p_0^l + e^{-rT}T}}{\frac{p_0^l}{p_0^l + e^{-rT}T}} \). Thus, at time of investing, high-type projects still hold a higher posterior belief in succeeding than low-type ones, and it costs less internal funding, which altogether results out in a higher expected returns.

Above all, the full-information equilibrium contracts for high and low types of inno-
Innovators are $\bar{C}^h = \{\bar{\lambda}^h, \bar{\alpha}^h\}$ and $\bar{C}^l = \{\bar{\lambda}^l, \bar{\alpha}^l\}$ under which innovators capture the entire surplus. As shown in Figure 2, optimal allocations of $\lambda$ and $\alpha$ are the tangent points of innovators’ indifference curves and financiers’ zero-expected-payoff curves.

3.2 Equilibrium contracts under imperfect information

When financiers have imperfect information of the type of the project, the low-type innovators have strong incentives to undertake the high-type optimal contract, shown as an inward shifting of the low-type indifference curve in Figure 3, which makes full-information equilibrium contracts no longer stable. This section attempts to develop perfect Bayesian Equilibria under imperfect information and pays extra attention on the Pareto-dominant one, or the least-cost separating equilibrium.

Firstly, we aim at defining an equilibrium with the minimum requirement to be stable and incentive compatible under which both types of innovators will choose their type equivalent contracts and get funded by the financier. Thus, we shall follow Maskin and Tirole (1992) and characterise the Rothschild-Stiglitz-Wilson (RSW) allocation for both types (Rothschild and Stiglitz, 1976; Wilson, 1977), also known as the low-information intensity equilibrium. Later on, we check if the RSW allocation is interim efficient with respect to financier’s prior beliefs, if so, a Pareto-dominant equilibrium could be deter-
Definition 1 $(\hat{\lambda}^h, \hat{\alpha}^h)$ is an RSW-h allocation of the high-type agent if it maximises the total expected payoff of the high-type subject to a set of incentive compatible allocations for both types of innovators and non-negativity expected profit constraints of the financier regardless of his beliefs.

The following the maximisation programme of the high-type leads to the RSW-h outcomes:

Programme A (high-type)

$$\max_{C^h, C^l} U^h(C^h)$$

subject to

$$U^h(C^h) \geq U^l(C^l)$$
$$U^l(C^l) \geq U^l(C^h)$$
$$V^h(C^h) \geq 0$$
$$V^l(C^l) \geq 0,$$

where the first two incentive compatibility constraints are set in order to make sure innovators in the equilibrium choose their type-equivalent contracts, and the next two constraints are to guarantee the financier is type-by-type profitable. There is a symmetric maximization programme for the low-type agent, let $(\hat{\lambda}^l, \hat{\alpha}^l)$ be the RSW-l allocation of the low-type innovators. As RSW equilibrium could be achieved regardless of financier’s prior beliefs in agent’s type under imperfect information, the agent could guarantee himself a RSW payoff by proposing $(\hat{\lambda}^\theta, \hat{\alpha}^\theta)$ under no circumstances, and such RSW payoff can be regarded as one’s reservation utility.

Lemma 2 The low-type agent could receive no more than his full-information equilibrium payoff in the RSW-l allocation, that is, $U^l(\hat{\lambda}^l, \hat{\alpha}^l) \leq U^l(\bar{\lambda}^l, \bar{\alpha}^l)$.

Proof. For $(\hat{\alpha}^l, \bar{\lambda}^l)$ solve the first-best maximisation problem of low-type with the individual rationality constraint of the financier being binding, thus, $V^l(\bar{\lambda}^l, \hat{\alpha}^l) = 0$. In the Programme A of the low-type, $V^l(\bar{\lambda}^l, \hat{\alpha}^l) \geq 0$ should be fulfilled, which means $V^l(\bar{\lambda}^l, \hat{\alpha}^l) \geq V^l(\bar{\lambda}^l, \bar{\alpha}^l)$. Thus, $U^l(\bar{\lambda}^l, \hat{\alpha}^l) \leq U^l(\bar{\lambda}^l, \bar{\alpha}^l)$ holds as the agent claims the remaining surplus of the project. ■
Intuitively, the low-type innovators maximise profit given that the individual rationality constraint of the financier binds under full information. That is, for \( \tilde{C}^l \) to be the first-best contract, the financier makes zero profit: \( V^l(\tilde{C}^l) = 0 \). Under Programme A, financier makes a non-negative profit on both types of innovators irrespective of his expectation. Thus, the agent could not keep more than \( U^l(\tilde{C}^l) \) without violating the non-negativity profit condition of the financier. In addition, the low-type can always achieve at least the first-best payoff by revealing their type and obtain the investment.

However, as the financier is assigned with a prior belief in agent’s type in the model, we should build on the above RSW equilibrium and relax the type-by-type non-negativity constraints of the financier. With probability of \( q \), the financier takes the agent as the high-type; with probability of \( (1 - q) \), the agent is regarded as the low-type. Given this belief, the financier would accept the financial contract if it makes himself break-even on average. In this case, we could form up another efficient equilibrium that is interim efficient relative to the prior belief \( \{q, 1 - q\} \) and incentive compatible for both types of agents. The term interim refers to the stage where innovators propose the financial contract to the financier with private information on their types. Moreover, interim efficient equilibrium weakly Pareto dominates the RSW equilibrium, which is the key results of Maskin and Tirole (1992). In intuition, as the individual rationality constraints of the financier are relaxed, the interim efficient equilibrium allows the financier make a loss on either types of the innovators, which leads to a Pareto improvement without hurting the other type’s benefit. Define it formally as follows:

**Definition 3** The set of contracts \( \{\tilde{C}^h, \tilde{C}^l\} \) is interim efficient if it optimises high and low innovators’ expected payoff respectively, is incentive compatible and leaves the financier non-negative profit in expectation.

Following Lemma 3, the low-type could at most receive their first-best payoff in the RSW equilibrium. As the interim efficient equilibrium weakly Pareto dominates the RSW equilibrium, we could expect that the payoff of the low-type innovators in the interim efficient equilibrium weakly dominate their RSW equilibrium payoff, which is at most first-best payoff. Thus, we could generate another constraint in the programme describing the above relationship. The following the maximisation programme leads to the interim efficient outcomes:
Programme B (high-type)

\[
\begin{align*}
\max_{C^h, C^l} & \quad U^h(C^h) \\
\text{subject to} & \quad U^h(C^h) \geq U^h(C^l), \quad (IC^h) \\
& \quad U^l(C^l) \geq U^l(C^h), \quad (IC^l) \\
& \quad qV^h(C^h) + (1 - q)V^l(C^l) \geq 0, \quad (IR) \\
& \quad U^l(C^l) \geq U^l(\bar{C}^l), \quad (IE^l)
\end{align*}
\]

where \(IC^h\) and \(IC^l\) are the incentive compatibility constraints of high and low types of innovators respectively, \(IR\) is the individual rationality constraint of the financier, which ensures he gets non-negative profit in expectation, and \(IE^l\) constraint indicates that the low-type agent would receive at least their first-best expected profit in any equilibrium.

### 3.2.1 Pooling Equilibria

As mentioned before, full-information equilibrium is not robust under information asymmetry on agent’s type since the low-type agent would pretend to be high-type ones. Such deviation of the low-type would leave the financier negative profit, and without any signaling scheme, the high-type innovator will be mistaken as a low-type, which would further result in financier’s reluctance of investing. The key criterion of a pooling equilibrium is to make sure there is no profitable deviation for either type of the innovators and guarantee the financier on average break-even. To achieve this, the high-type may need to compromise on their revenue in order to subsidise financier’s welfare losses on the low-type.

Revenue shares and internal funding are proposed in our paper to be signaling tools of the innovators. We will show next that if only one of them is available, the same signal will be chosen by both types of agents where the financier learns nothing about their type, which consists of a pooling equilibrium.

**Proposition 1** Assume the revenue share to be exogenously fixed, high and low types of innovators choose the same amount of internal funding, i.e., \(\lambda^h = \lambda^l = \lambda\) as a pooling equilibrium.

**Proof.** See appendix. \(\blacksquare\)
Proposition 2 Assume the internal funding to be exogenously fixed, high and low types of innovators choose the same revenue share, i.e., $\alpha^h = \alpha^l = \alpha$ as a pooling equilibrium.

Proof. See appendix. ■

As shown in Figure 4 and 5, $V(q)$ is financier’s zero-profit indifference curve as a linear combination of his zero-profit indifference curves under high and low types of contracts, denoted as $V^h = V^l = 0$. Notice that both pooling equilibria involve a cross-subsidy from the high to the low. In the first case, with exogenously fixed $\alpha$, high and low types pool at same internal funding level, $\lambda$. It turns out that the high-type tend to undertake more internal financing than needed under full-information, i.e., $\lambda > \bar{\lambda}^h$, as shown in Figure 4, which results in a positive gain of the financier. However, the low-type would choose a lower amount of internal financing than their full-information equilibrium, i.e., $\lambda < \bar{\lambda}^l$, which induces a loss of the financier. In this case, as either type of the innovators could get any better without jeopardising the other party’s welfare, it can be regarded as a stable equilibrium outcome. Moreover, as the allocation $\{\alpha, \lambda\}$ is on the $V(q) = 0$ indifference curve, the financier is willing to accept the financial contract and exert external investment.

Similarly, as stated in Proposition 2, with internal funding to be exogenous, the only
signaling tool available is the revenue share. While the high-type are willing to give more revenue shares to the investor in order to compensate her payoff-loss from the low-type. On the other hand, the best reaction of the low-types is to adopt this $\alpha$-revenue share, which generates a positive gain without being distinguished from the high-type.

As the above two pooling equilibria are achieved with either of the signalling tool available, it’s worth exploring the existence of any other pooling equilibrium with two signalling dimensions.

**Proposition 3** There exists no pooling equilibrium when both revenue shares and internal funding are available under imperfect information according to Programme B.

**Proof.** See appendix. ■

### 3.2.2 The Least-costly Separating Equilibrium

As there is no pooling outcomes, we should solve the programme fully and try to get any separation equilibrium.

**Proposition 4** There exists a least-costly separating equilibrium: $\{\lambda^l*, \alpha^l*\}$, $\{\lambda^h*, \alpha^h*\}$ such that:

- Regarding internal funding, low-type innovators remain at their first-best optimum, whereas, high-type innovators invest even more than low-type first-best internal funding such that $\lambda^l* = \bar{\lambda}^l$ and $\lambda^h* > \bar{\lambda}^l > \bar{\lambda}^h$;

- On revenue share, the low-type allocate less than full-information revenue shares to the financier, thus follows the order: $\alpha^h* < \alpha^l* \leq \bar{\alpha}^l$.

**Proof.** See appendix. ■

In practise, the separation equilibrium works like the following: the innovator, either high or low type, proposes a menu of $\{\lambda^l*, \alpha^l*\}$ and $\{\lambda^h*, \alpha^h*\}$ contracts to the financier. Upon observing these two contracts, the financier still stays uninformed of the type of the innovator but will accept the offer as it makes him break-even. Then the innovator will execute one of the contracts according to her type as there is no profitable deviation for either party. However, in the case when the informed innovator proposes only one contract of her type, the financier will update his belief in agent’s type accordingly. The low-type would have incentive to deviate as the high-type contract is most likely to be
accepted by the financier. Thus, to avoid further distortion, a set of two contracts should be proposed ex ante.

The above proposition states that the low-type innovators remain at their full-information equilibrium internal financing, whereas, the high-type invest a bit more than their first-best internal financing. Firstly, in terms of the internal financing allocation, \( \{ \lambda^h, \bar{\lambda}^l \} \) allows the high-type being fully separated from the low-type. By overinvesting on learning, the high-type refrains the low-type from mimicking under imperfect information. Indeed, by increasing internal financing from \( \bar{\lambda}^h \) to \( \lambda^h \), the incentive compatibility constraint of the low-type agents has been relaxed. Thus, for the low-type agents, they prefer choosing their full-information contracts to the high-type contract, which ensures as much as a RSW optimum payoff.

Moreover, as \( \lambda^h > \bar{\lambda}^l \), the internal funding of the high-type increases even further and turns to be greater than low-type’s first-best. By definition of separating equilibrium, all innovators would prefer their type-equivalent contracts. The separation would not be possible if the high-type chose less than low-type first-best internal funding, which gives rise to a possible deviation by the high-type. This could be verified intuitively as the single-crossing property holds in this case. Let \( MU^\theta_\alpha \) and \( MU^\theta_\lambda \) denote the marginal utility of revenue share and internal funding given a certain utility level for the \( \theta \)-type agent. We could calculate the marginal rate of substitution of revenue for internal funding as a ratio of \( MU^\theta_\alpha \) over \( MU^\theta_\lambda \) respectively for high-type and low-type. Taking the difference between the ratio of high-type and low-type, we find the following relationship:

\[
\frac{MU^h_\alpha}{MU^h_\lambda} - \frac{MU^l_\alpha}{MU^l_\lambda} > 0
\]

which indicates that the high-type are willing to sacrifice more internal funding in exchange of a further decrease of revenue shares compared with the low-type. In other words, revenue share is more valuable to the high-type and internal funding is more important to the low-type. As a result, more internal funding than the low-type first-best amount is allocated by the high-type in order to preserving a larger proportion of revenue share for themselves in this equilibrium.

Secondly, we analysis the welfare under this separating equilibrium. Under the financial contract \( \{ \bar{\lambda}^l, \alpha^l^* \} \), as \( \alpha^l^* \leq \bar{\alpha}^l \) the low-type would achieve at least their full-information payoff and have no incentive to deviate. \( \bar{\lambda}^l \) amount of internal funding
guarantees the low-type a sufficient learning and generate the optimum expected payoff, which means they can not get any better by adjusting the spending. Whereas, reducing the revenue share from $\bar{\alpha}^l$ in the RSW equilibrium to $\alpha^r$ makes the low-type weakly better off. Thus, this interim efficient equilibrium weakly pareto dominates the above-mentioned RSW equilibrium where the low-type at most receive their first-best payoff. In addition, it manages to make the separation between two types of the agents. Although the financier is not perfectly informed ex ante, proposing this set of two contracts ensures him a participation, he would also believe that each type of innovators will act according to their types after contracting.

Figure 6: Separating equilibrium

In Figure 6, an outward shift of high-type’s indifference curve $U^h$ generates a tangent at the point $(\lambda^{h*}, \alpha^{h*})$ to financier’s zero-average-profit curve (the Blue line), which represents a cross-subsidisation from the high to the low and a fulfilling of the interim efficiency of the IR constraint in Programme B. As a response, the low-type could make themselves better off by inward shifting indifference curve until it crosses high-type’s new allocation. Actually, any point on that dotted new indifference curve makes the low-type receive as much payoff as if taking the high-type contract. As a set of red indifference curves of the financier $V^l$ are vertical transformations of each other, $\bar{\lambda}^l$ remains to be
the optimum internal funding. Whereas, the inward shifting results in a revenue share decrease from $\alpha^l$ to $\alpha^l^*$, which, in all, illustrates that $(\lambda^l, \alpha^l^*)$ is low-type’s best attainable under separation.

In the following, we deliver some implications and thoughts from this main result. To begin with, the above separating equilibrium can be related with Spence’s (1973) labor market signaling model where heterogeneous employees decide how much costly education to acquire under adverse selection. They also propose a separating equilibrium under which the high-type agents are willing to get an education just for signaling purpose and the low-type agents get no education. In this case, education severs as a signal to less-informed employers as it’s more costly for the low-type to acquire same education compared with the high-type. Similarly, as in our model, high-type innovators would otherwise undertake less internal funding than the low-type given higher expected probability of success. Directly driven by the purpose of separating, the high-type are prone to spend even more internal funding than the low-type, which comprises of a pareto dominating separating equilibrium.

Proposition 4 also echoes with the well-known pecking order theory according to which firms prefer internal funding over external resources and debt over equity at the external financing stage in the presence of information frictions (Myers and Majluf, 1984; Myers, 1984). This theory has also be confirmed by some empirical studies as surveyed in Frank and Goyal (2011). Apparently, our model predicts that innovators regardless of their types would allocate at least first-best level of internal financing to support learning under imperfect information. Although external financing happens endogenously between financier and innovator without any cost, if borrowing is costly we expect that the high-type innovators would adopt more of the internal funding and borrow less proportion of the external funding in comparison with the low-type ones. As innovator’s investing decision is contingent on the expected probability of success, it follows that innovators with high-type project tend to provide more internal funding based on their higher probability of success. Additionally, by doing so, the high-type could save extra borrowing cost and less compensation is needed for the less-informed outside investor. Thus, implication could be found in our model that heterogeneous agents reveal a slightly different preference over internal and external financing.
4 Conclusion

This paper presents an exponential bandit model to show heterogeneous innovation firms’ equilibrium options of internal and external financing. Given that prior belief in the project being good is private information, separating equilibrium contract indicates that the high-type could be distinguished from the low-type by committing to a higher level of internal financing. Otherwise, heterogeneous innovators will be pooled at same revenue share and internal financing level where the high-type is strictly worse-off due to cross-subsidisation to the low-type. The model manages to capture several features of real-world innovation: costly learning is required before investment; the outcome of an innovation project is uncertain; the more in depth the innovator learns, the higher posterior belief on success conditional on no bad news. Several important predictions could be made from our model: innovators with high-type project usually put more effort in the initial learning stage and are more likely to succeed; financiers could judge a project’s success likelihood by looking into the innovator’s internal financing on experimentation; internal financing is more preferred to the high-type especially when external borrowing is more costly, or information asymmetry problem is severer compared with the low-type.

This article can be extended in the following ways: first, let the innovation firms become less cash-constrained where internal funding can cover part of the investment. Attention could be focused on heterogeneous firms’ optimal internal and external financing allocation under information frictions. Second, initial learning stage of innovation could be bought under the contest environment where only the most promising project could be funded. It is similar as Halac et al. (2016b) where the principal plays a role of attributing prizes and providing incentives to contestants via partly disclosing information. However, innovators hold private information on their prior beliefs in our contest setting.
A  Appendix

A.1  Proof of Proposition 1 and 2

According to the assumption, $\alpha^h = \alpha^l = \alpha$. As $U(\lambda)$ is monotonically decreasing in $\lambda$, therefore, both incentive compatibility constraints indicate that $\lambda^h \leq \lambda^l$ and $\lambda^l \leq \lambda^h$. Thus, $\lambda^h = \lambda^l = \lambda$ in this case.

As the individual rationality constraint of the financier binds when innovators receive the optimal payoff, we choose the smallest $\lambda$ that makes $IR$ binds as following:

$$qe^{-rT}\{p^h_0\alpha R - [p^h_0 + (1 - p^h_0)e^{-\lambda T}]I\} + (1 - q)e^{-rT}\{p^l_0\alpha R - [p^l_0 + (1 - p^l_0)e^{-\lambda T}]I\} = 0$$

$$\implies \bar{\lambda} = \frac{1}{\bar{T}} \ln \frac{I[q(1-p^h_0) + (1-q)(1-p^l_0)]}{(\alpha R - I)[qp^h_0 + (1-q)p^l_0]}$$

Since the full-information internal financing of the low-type under fixed revenue share is

$$\bar{\lambda}^l = \frac{1}{\bar{T}} \ln \frac{I(1-p^l_0)}{(\alpha R - I)p^l_0}$$

Thus, $\bar{\lambda} \leq \bar{\lambda}^l$ for $q \in [0, 1]$ and $IE^l$ holds. Thus Proposition 1 is proved.

Following the same reasoning. For $\lambda^h = \lambda^l = \lambda$, two IC constraints imply that $\alpha^h = \alpha^l = \alpha$. Then according to the IR constraint’s binding condition, one could show that $\bar{\alpha} \leq \bar{\alpha}^l$, then the $IE^l$ constraint holds as well.

A.2  Proof of Proposition 3

Prove by contradiction. Suppose that there exists a pooling equilibrium where high and low types of innovators propose the same financial contract, i.e $C^h = C^l$. In the following we first solve the Programme $B$ given $C^h = C^l$ to see the most efficient outcome in the high-type’s best interest. Then we attempt to analysis the symmetric Programme $B$ of the low-type when $C^h = C^l$. If the outcomes of these two programmes coincide, there is a pooling equilibrium. If not, there is always profitable deviation for either type, which results in no pooling result under imperfect information.

In Programme $B$, when $C^h = C^l$, $IC^h$ and $IC^l$ hold with equality and could be ignored. We first ignore $IE^l$ and check whether it will be satisfied later. Write down the
reduced version of Programme B as the following:

\[
\max_{\lambda, \alpha^h} U(\alpha^h, \lambda^h) = e^{-rT}(1 - \alpha^h)p_0^hR - \lambda^h
\]

subject to \[ V(q) = qV^h(\lambda^h, \alpha^h) + (1 - q)V^l(\lambda^h, \alpha^h) \geq 0 \] (IR)

As the optimum allocation is where indifference curve of the high-type tangent with financier’s zero-profit curve where \( MRS_{\alpha^h, \lambda^h}^h(\bar{U}) = MRS_{\alpha^l, \lambda^l}(V(q)) \) for given utility level \( \bar{U} \) and 0, we calculate in the following way:

\[
MRS_{\alpha^h, \lambda^h}^h(\bar{U}) = \frac{\partial U}{\partial \alpha^h} \frac{\partial U}{\partial \lambda^h} = e^{-rT}p_0^hR
\]

\[
MRS_{\alpha^l, \lambda^l}(V(q)) = \frac{\partial V}{\partial \alpha^l} \frac{\partial V}{\partial \lambda^l} = \frac{qp_0^hR + (1 - q)p_0^lR}{[q(1 - p_0^h) + (1 - q)(1 - p_0^l)]TTe^{-\lambda^hT}}
\]

then equalise two \( MRS \) finding the tangent point:

\[
\lambda^h = -r + \frac{1}{Tlnp_0^h} \frac{TIp_0^h[q(1 - p_0^h) + (1 - q)(1 - p_0^l)]}{qp_0^h + (1 - q)p_0^l},
\]

\[
\alpha^h = \frac{I}{R} + \frac{1}{p_0^hR}e^{-rT} = \bar{\alpha}^h.
\]

According to the same logic, we solve the low-type version of Programme B and get the result as following:

\[
\lambda^l = -r + \frac{1}{Tlnp_0^l} \frac{TIp_0^l[q(1 - p_0^l) + (1 - q)(1 - p_0^l)]}{qp_0^l + (1 - q)p_0^l},
\]

\[
\alpha^l = \frac{I}{R} + \frac{1}{p_0^lR}e^{-rT} = \bar{\alpha}^l.
\]

Contradictions appear where \( \lambda^h \neq \lambda^l \) and \( \alpha^h \neq \alpha^l \). Moreover, whether \( IE^l \) will hold or not depends on the \( q \) parameter value, so it’s not satisfied under no circumstances.
A.3 Proof of Proposition 4

Solving the Programme B (high-type) to prove the above proposition:

\[
\max_{\lambda^h, \alpha^h} \quad U^h(\alpha^h, \lambda^h) = e^{-rT}(1 - \alpha^h)p_0^h \bar{R} - \lambda^h \\
\text{subject to} \quad U^h(C^h) \geq U^h(C^l), \quad (IC^h) \\
U^l(C^l) \geq U^l(C^h), \quad (IC^l) \\
qV^h(C^h) + (1 - q)V^l(C^l) \geq 0 \quad (IR) \\
U^l(C^l) \geq U^l(\bar{C}^l) \quad (IE^l)
\]

Firstly, as IR constraint binds at the optimum, we could obtain the following condition:

\[
qV^h(\alpha^h, \lambda^h) + (1 - q)V^l(\lambda^l, \alpha^l) = 0 \quad (7)
\]

As the expected payoffs of high-type innovators could be expressed as the difference between expected surplus of the high-type project and expected payoff of the financier, so \(U^h(\alpha^h, \lambda^h)\) can be rewritten as the following:

\[
U^h(\alpha^h, \lambda^h) = e^{-rT}(1 - \alpha^h)p_0^h \bar{R} - \lambda^h \\
= e^{-rT}\{p_0^h \bar{R} - [p_0^h + (1 - p_0^h)e^{-\lambda^h T}]I\} - \lambda^h - V^h(\alpha^h, \lambda^h)
\]

Substitute (7) into \(U^h(\alpha^h, \lambda^h)\) and cancel variable \(\alpha^h\):

\[
U^h(\lambda^h) = e^{-rT}\{p_0^h \bar{R} - [p_0^h + (1 - p_0^h)e^{-\lambda^h T}]I\} - \lambda^h + \frac{1 - q}{q}V^l(\lambda^l, \alpha^l)
\]

We ignore \(IC^h\) at this stage as it will be shown satisfied ex post.

Lemma 4 In Programme B, if \(IE^l\) constraint is binding, \(IC^l\) constraint must be binding.

Proof. Under unconstrained maximization problem, \(\bar{\lambda}^l\) and \(\bar{\alpha}^h\) are the first-best solution of high-type agents. If \(IE\) constraint is binding under this constrained optimisation problem, thus for \(\{\lambda^l, \alpha^l\}\):

\[
U^l(\lambda^l, \alpha^l) = U^l(\bar{\lambda}^l, \bar{\alpha}^l) < U^l(\lambda^h, \alpha^h) \quad \text{holds},
\]
which implies that $IC^l$ must be binding under constrained optimisation problem. In other words, $\{\bar{\lambda}^h, \bar{\alpha}^h\}$ are no longer achievable as a second-best solution for the high-type agents.

As $IE^l$ constraint binds is the sufficient condition that leads $IC^l$ to be binding according to Lemma 4, so Programme $B$ becomes:

$$\max_{\lambda^h} U^h(\lambda^h) = e^{-rT}\{p_0^h \bar{R} - [p_0^h + (1 - p_0^h)e^{-\lambda^h T}]I\} - \lambda^h + \frac{1 - q}{q} V^l(\lambda^l, \alpha^l)$$

subject to $U^l(\lambda^h) \geq U^l(\bar{\lambda}^l, \bar{\alpha}^l)$ \hspace{1cm} (IE$^{l''}$)

The key to this high-type maximisation problem is defining a $\lambda^h$ makes $IE^l$ binding. So we rewrite the left-hand-side of $IE^l$ as the following:

$$U^l(\lambda^h) = \frac{p_0^l}{p_0^h}[e^{-rT} p_0^h (1 - \alpha^h) \bar{R}] - \lambda^h$$

$$= \frac{p_0^l}{p_0^h} (U^h(\lambda^h) + \lambda^h) - \lambda^h$$

$$= \frac{p_0^l}{p_0^h} U^h(\lambda^h) + (\frac{p_0^l}{p_0^h} - 1)\lambda^h \hspace{1cm} (8)$$

The objective function $U^h(\lambda^h)$ is concave in $\lambda^h$ since $U^{h''}(\lambda^h) < 0$, which guarantees a unique $\bar{\lambda}^h$ that maximises $U^h$. Moreover, for any $U^{h'}(\lambda^h) < 0$, we could induce that $\lambda^h < \bar{\lambda}^h$.

Then we study the property of $U^l(\lambda^h)$ in order to identify the $\lambda^h$ which makes $IE^{l''}$ binding. $U^l(\lambda^h)$ shows the same concavity property in $\lambda^h$. As $\frac{dU^l}{d\lambda^h} = \frac{p_0^l}{p_0^h} U^{h'}(\lambda^h) + (\frac{p_0^l}{p_0^h} - 1)$, and for $\bar{\lambda}^h$ to be the first-best, $U^{h'}(\bar{\lambda}^h) = 0$ holds. Thus, we find out that: $U^l(\lambda^h)$ decreases at the $\bar{\lambda}^h$-neighborhood as $\frac{dU^l}{d\lambda^h} < 0$. Moreover, since $U^l(\bar{\lambda}^h) > U^l(\bar{\lambda}^l)$ and $U^{l'}(\bar{\lambda}^h) < 0$, there exists a $\lambda^h > \bar{\lambda}^h$ that satisfying $U^l(\lambda^h) = U^l(\bar{\lambda}^l)$.

We then turn our attention to the $IC^h$ constraint: $U^h(C^h) \geq U^h(C^l)$. Rearrange the right-hand-side $U^h(C^l)$ as following:

$$U^h(C^l) = \frac{p_0^h}{p_0^l} U^l(C^l) + (\frac{p_0^h}{p_0^l} - 1)\lambda^l \hspace{1cm} (9)$$

Substituting the condition that $IC^l$ binds in the equilibrium, i.e., $U^l(C^l) = U^l(C^h)$, into
the above $U^h(C^l)$:

$$U^h(C^l) = \frac{p_0^h}{P_0^l} U^l(C^h) + \left(\frac{p_0^h}{P_0^l} - 1\right) \lambda^l$$

$$= e^{-rT} p_0^h (1 - \alpha^h) \bar{R} - \lambda^h + (\lambda^h - \lambda^L)(1 - \frac{p_0^h}{P_0^l})$$

$$= U^h(C^h) + (\lambda^h - \lambda^L)(1 - \frac{p_0^h}{P_0^l})$$

For $IC^h$ to be hold: $U^h(C^h) \geq U^h(C^l) = U^h(C^h) + (\lambda^h - \lambda^L)(1 - \frac{p_0^h}{P_0^l})$, as $(1 - \frac{p_0^h}{P_0^l}) < 0$, thus we induce that $\lambda^h > \lambda^L$ should be satisfied.
References


