Amplification and Spillover with Financial Arbitrage, Production and Collateral Constraints

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Highlight

- general equilibrium of collateral constrained arbitrage in a production economy

- agenda:
  - price difference between identical assets
  - collateral constraints
  - financial markets and production sector

- contribution:
  - merge two strands of literature
  - spillover effects and amplification
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contribution:

- merge two strands of literature
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Motivation I – Lessons from Financial Crisis

- **1998 financial crisis**
  - Arbitrage
    - hedge funds bet on convergence of prices of similar-payoff assets
    - during crisis, prices diverged.
    - hedge funds experienced heavy losses + distress
    - force to liquidate profitable positions

- **2008 financial crisis**
  - shocks from the housing sector spill over into financial sectors and reinforce with each other

- Asset prices and liquidity:
  - prices pushed away from fundamentals
  - liquidity dried up
  - cross-sector contagion
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Motivation II

- empirical evidence about persistent price differences
  - “Siamese-twin” stocks:
    - Rosenthal and Young (1990) and Dabora and Froot (1999)
  - newly issued “on-the-run” bonds Vs older “off-the-run” bonds

- market segmentation
  - Before 2014, A shares traded in mainland China and H shares in Hongkong

- links between financial friction and macroeconomy
Main Conclusions

- with perfect foresight of market demand
  - self-recovery

- with inaccurate estimation of future market demand
  - looser collateral constraints trigger recession
    - spillover and amplification
  - tighter collateral constraints stabilize the economy
Related Literature

  - No production sector
  - The financial constraints must cover the maximum loss of the arbitrageurs.

- Brunnermeier and Sannikov (2014)
  - no financial arbitrage
Baseline Model

Figure: The structure of the economic system.
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Agents

- a continuum of competitive IM and HH
- only one perishable consumption goods
  - IM can convert consumption and capital
- IM are both arbitrageurs and entrepreneurs
  - IM invest capital and HH offer labor.

\[ y_t = F(K_{t-1}) = aK_{t-1}^\alpha L_t^\gamma \]

- separate collateral posting with capital investment
Exogeneous Shocks

- HH’s fixed-size production / natural endowment

\[ y_{i,t} = bK_H + u_{i,t-1}\theta_t, \quad i \in \{A, B\}, \quad t \in \{1, 2, \ldots \}. \]

- \( \theta_t \) follows a symmetric distribution around zero on \([-\bar{\theta}, \bar{\theta}]\).
- the shock intensities/market demand \( u_{A,t} = -u_{B,t} =: u_t \).

- opposite shocks, opposite hedging demand
Financial Assets

- identical financial assets in each market
  - dividend $\theta_t$ mimicks the shock
  - long-lived, in zero net supply
  - IM and HH’s position $x_{i,t}^{IM}$ and $y_{i,t}^{HH}$.

- prices differ across markets
  - opposite hedging demand A : $-u_{A,t} = -u_t$; B : $-u_{B,t} = u_t$.

- IM exploit arbitrage profit
IM take identical but opposite positions $x_{A,t} = -x_{B,t} = x_t$.

- collateral constraints
  - separately post capital input as collateral
  - cover HH’s maximum loss if IM default or walk away from their positions
  - total collateral limit: IM’s capital rent $\alpha F(K_t)$. 
IM’s Optimization Problems

max_{c_s^{IM}, x_i, s, K_s} \mathbb{E}\left[\sum_{s=t}^{\infty} \rho^s \log \left(c_s^{IM}\right)\right], \quad i \in \{A, B\}.

subject to

\begin{align*}
c_t^{IM} &= \sum_{i \in \{A, B\}} x_{i, t-1}^{IM} p_{i, t} - \sum_{i \in \{A, B\}} x_{i, t}^{IM} p_{i, t} + \underbrace{a(1 - \gamma)K_{t-1}^{\alpha} L^\gamma - K_t}_{\text{entrepreneur income: net production output minus wage and investment}} + \underbrace{\min \left\{ \min_{p_{i, t+1}, \theta_{t+1}} \left\{ x_{i, t}^{IM} (p_{i, t+1} + \theta_{t+1}) \right\}, 0 \right\}}_{\text{value of previous period's investment in financial asset } i} + \alpha F(K_t) \geq 0.
\end{align*}
HH’s Optimization Problems

\[
\max_{c_{i,s}^{HH}, y_{i,s}^{HH}} E \left[ \sum_{s=t}^{\infty} \beta^s \log (c_{i,s}^{HH}) \right]
\]

subject to

\[
c_{i,t}^{HH} = \left( y_{i,t-1}^{HH}(p_{i,t} + \theta_t) - y_{i,t}^{HH} p_{i,t} \right) + \frac{1}{2} a \gamma K_{t-1}^\alpha L^\gamma + (bK_H + u_{i,t-1} \theta_t)
\]

income from trading financial asset

labor Income

endowment

- Ideally, \( y_{i,t-1}^{HH} = -u_{i,t-1} \) so that households are fully protected from the endowment shock \( \theta_t \).
Competitive Equilibrium

For any initial capital endowment, an equilibrium is described by the price process $p_{i,t}$, IM’s capital investment $K_t$, financial asset positions $y_{i,t}^{HH}$ and $x_{i,t}^{IM}$, and consumption choices $c_{t}^{IM}$ and $c_{i,t}^{HH}$ for $i \in \{A, B\}$ such that

- all agents solve their optimization problems given prices;
- markets clear for financial assets, that is $y_{i,t}^{HH} + x_{i,t}^{IM} = 0$. 
Riskless Arbitrage

When shock intensity is known to agents, there exist steady states

- if $\beta \geq \frac{(2 \max\{|u_t|\} \bar{\theta})^{1-\alpha}}{\alpha^{\alpha \bar{\alpha}}}$, then $\rho F'(K^*) = 1$, $x_t = u_t$, price difference $|\psi^*|$ is 0.

- otherwise if $u_t$ is constant, then $K_t$ converges over time to a unique $K^*$, with price discrepancy $|\psi^*| > 0$ and market liquidity $x^* = \frac{\alpha F(K^*)}{2\bar{\theta} + |\zeta^*|}$.

- no arbitrage opportunity case, $\alpha \beta F'(K_n^*) = 1$
- here $\alpha' \beta F'(K_n^*) = 1$, where the production return parameter $\alpha' = \alpha + \alpha \frac{1-\beta}{\beta} \frac{|\psi^*|}{2\bar{\theta} + |\zeta^*|}$
- $K^* > K_n^*$
- nominal zero-interest debt, leveraged production
- increased marginal return of production
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Proposition

*When the shock intensity* $u_t$ *is constant, a unique competitive equilibrium exists in which the price difference* $\psi_t$, *intermediaries’ capital investment* $K_t$ *and the positions of the financial assets* $x_t^i$ *are deterministic.*
Shock Reactions–Self-recovery

In case of a sudden loss in capital input or financial income,

- immediate reaction

  price difference: $|\psi_{t+1}| > |\psi^*|$, market liquidity: $|x_{t+1}| < x^*$,
  $K_{t+1} < K^*$.

- marginal return of capital

- arbitrage profitability

- long term

  price difference: $|\psi_t| > |\psi_{t+1}| > \cdots > |\psi^*|$,  
  market liquidity: $|x_t| < |x_{t+1}| < \cdots < |x^*|$,  
  $K_t < K_{t+1} < \cdots < K^*$.
Risky Arbitrage – Looser Collateral Constraints

With uncertainty, agents might underestimate or overestimate the market demand.

- underestimate case
  - looser collateral constraints
  - overinvestment in financial markets
  - spillover and amplification
  - trigger recession & systemic risk
Spillover and Amplification Effects

- lower wage for HH
- looser collateral constraints
- dropping supply of market liquidity
- over investment in financial markets
- unexpected loss; less financial income
- IM’s overall income decreases
- less capital investment
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overestimate case

- tighter collateral constraints
- underinvestment in financial markets
- higher income for both IM and HH
- stabilize the economy by boosting production at the cost of market liquidity
Thank you!