Competing Under Financial Constraints*

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Abstract

This paper presents novel results in the literature of credit rationing by analysing the strategy of a supplier extending trade credit to a financially constrained retailer. A supplier setting linear wholesale prices may lower them to minimize the agency costs generated by his customer’s competitive pressure. Thus, the retailer gains competitive advantage due to the need of honouring her own debt. Furthermore, the retailer’s competitors make lower profits and final consumers face lower retail prices. Moreover, it is shown that optimal contracts can be implemented by two-part tariffs. There, agency problems rationalize lump-sum transfers from suppliers to retailers. (JEL G32, L11, D43)

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1 Introduction

Firms’ limited access to credit is a widespread phenomenon even in countries with developed financial systems. Among other effects, the lack of credit reduces companies’ investment capacity, thereby shaping both production chains and the market structure. Introducing elements of industrial organization in a model of credit rationing, we study the commercial relation between a financially constrained retailer and its supplier. We show that when the supplier is not able to extract all its customer’s surplus, it may internalize agency costs by reducing input prices. As a result, a credit constrained retailer can be at a competitive advantage due to the need to honour her debt. Further, we study the optimal contract for the supplier and show that it can be implemented with a two-part tariff.

We consider a vertical relation where a supplier (he) extends trade credit to a competitive retailer (she). This occurs when the supplier allows his customer to delay payment for goods already delivered. Hence, it can be seen as a financial transaction where the supplier becomes a lender and the retailer acts as a borrower. Extensive evidence shows that this is an important source of firms’ finance (see, for example, Raghuram G. Rajan and Luigi Zingales, 1995; Mariassunta Giannetti, 2003). We assume that the retailer’s investment in the downstream market is not contractible. Hence, limited liability creates an incentive to divert the product and obtain private benefits rather than honouring the debt. As a consequence, the supplier needs to reduce the volume of credit extended in order to make his customer incentive compatible. This leads to financial constraints, which arise when a borrower cannot obtain the loan that she wants even though she is willing to pay it back (Jean Tirole, 2006).

Importantly, in our setting the line of credit is a combination of quantity of input and input price that the supplier defines to maximize profits. Therefore, financial constraints not only affect the quantity of input the retailer can purchase, but also her marginal cost. The retailer’s incentives to divert depend on the profitability of the product market. Thus, high wholesale prices lower the benefits of investing fairly and provide incentives to divert the
borrowed funds and not pay back. Competitive pressure in the downstream market has an equivalent effect. However, retailer’s competitors are also affected by this transaction, since it determines the volume of output that our retailer is able to launch in the market. Hence, our approach captures the effects that financial constraints and product market competition have on each other.

Our main result shows that an increase of competition downstream can make it optimal for the supplier to lower input prices while maintaining the amount of input lent. As a consequence, the retailer makes higher profits than in a situation with no financial constraints. In particular, by reducing retailer’s marginal costs, i.e. its input price, the supplier induces his customer to increase purchases and thereby hit the financial constraint. This informs other firms in the downstream market about their competitor’s plan before they simultaneously decide their own investment. Thus, the supplier uses the financial constraint as a commitment device that provides the retailer with a leader advantage. The mechanism has further effects in the retail market, where competitors reduce their output and become less profitable than if their rival was self-financed. Nonetheless, the overall effect is positive for final consumers. In particular, total output increases and so retail prices are lower due to the presence of credit constraints.

The result arises under the standard assumption that the supplier is restricted to set linear input prices. This assumption has been used in the literature to approximate situations where a manufacturer cannot to extract the entire surplus from his retailer, and thus where the retailer earns positive profits. Hence, our vertical relation is characterized by double marginalization. The supplier sets the input price and the retailer decides how much to invest in the downstream market, where she competes in quantities with multiple incumbents. The double marginalization is complete when the number of competitors is small. However, as competitive pressure increases, the incentive compatibility constraint (ICC) becomes binding and the supplier is forced to reduce the line of credit through either prices and/or quantities. Finally, when competition is strong, the supplier does not extend any credit and so the
retailer cannot enter the market.

The supplier’s market power is a key element to the mechanism developed in this study. Low competition in the upstream market allows the supplier to set a wholesale price above marginal costs. As a consequence, he can optimally combine the reduction of input price and quantity of input lent when he is forced to shrink the volume of credit extended. In our model, the setting is simplified by assuming that he is a monopolist. With the same aim, it is also assumed that the retailer is a penniless entrepreneur, so she needs full funding to enter the downstream market, which also features some incumbent firms. Another important characteristic is that our firms compete à la Cournot. This implies that higher output leads to bigger profits and that the best response of the retailer is linearly decreasing in input price. As a result, it might be optimal for the supplier to reduce the value of credit extended through lowering the price rather than the volume of input lent. At the end of the paper, we discuss the robustness of our results to alternative assumptions regarding the market structure and the nature of product market competition.

We also characterize the optimal contract, defined as that maximizing the supplier’s profits subject to the ICC. In this setting the supplier is not restricted to any contractual form. Hence, it is equivalent to first-degree price discrimination, where the supplier can extract the whole surplus of his customer. Nonetheless, in our model the retailer still makes a positive profit, which is granted by the supplier in order to satisfy the ICC. We show that an optimal contract provides the supplier with the profits of a Stackelberg leader operating in the downstream market. In particular, the wholesale price is used to generate a retailer’s demand that equals the output of a first-mover. However, the effective marginal cost of this leader incorporates the agency cost carried by the transaction with the retailer.

An optimal contract can be implemented with a lump-sum tariff in exchange of a specific quantity of input. This resembles quantity-forcing contracts, which are not widely extended in the economy. However, we show that the optimal contract can also be implemented with a two-part tariff, a more popular form in industries such as supermarket and food
retailing. Moreover, we characterize a two-part tariff by which the retailer is never financially constrained. As we later discuss, this result opens a door for the analysis of the link between price discrimination and the identification of credit constraints. Finally, and contrary to common beliefs, we show that for high levels of competitive pressure the supplier transfers a positive lump sum to his customer. This negative fixed fee is subsidized with a relatively high unit price.

The results have implications for the study of interactions between financial constraints and product market competition. In our model the supplier is also the financier. Nonetheless, it is easy to separate the two activities and obtain similar results under standard assumptions. Hence, we establish a link between credit rationing and pricing policies and show that firms’ financial constraints can also affect their marginal costs. Furthermore, we provide a new mechanism through which companies exploit their limited liability. Particularly, information frictions constraint the line of credit extended to a retailer. In response to that, the supplier might reduce input price and make his client more competitive in order to minimize agency costs. Finally, we also note that the identification of credit constraints can be related to the ability of the supplier to extract the retailer’s surplus. More specifically, we show that whereas constraints are binding under linear pricing, with two-part tariff the retailer’s demand might always be satisfied.

1.1 Related Literature

The forces interacting in our model are similar to those where financial contracts serve as strategic commitments affecting the outcome of the market game. This idea is explored by Chaim Fershtman and Kenneth Judd (1987), who study the incentive contracts that principals (owners) will choose for their agents (managers) in an oligopolistic context. Most notably, James Brander and Tracy Lewis (1986) attribute such a strategic property to debt financing. They show that a firm may want to choose its financial structure so as to commit to being aggressive and thus become more profitable. In our model similar results arise via
different mechanisms. The retailer’s commitment is induced by its supplier and achieved through its own financial constraint. Hence, we model a vertical relation characterized by agency problems. A supplier (principal) can financially constraint a retailer (agent) by lowering wholesale prices and thereby increasing input demand. This gives to the retailer a first-mover advantage and allows the supplier to extract higher rents. However, this is only optimal when the ICC is tight. Otherwise, the supplier prefers a standard double marginalization that leaves the retailer competing à la Cournot.

This paper is relevant for the research linking corporate financing and product market competition. There is an extensive debate in the literature about the effects of leverage on firms’ ability to compete in the product market, e.g. James Brander and Tracy Lewis (1986) versus Patrick Bolton and David S. Scharfstein (1990).\(^1\) We show that credit constraints influence the supplier’s pricing strategy and therefore, the competitiveness of the borrower. Our outcome reconciles different results in the literature. Particularly, financial constraints can allow a company to be at a competitive advantage due to the need to honour its own debt. However, if competition downstream is severe, the same financial constraints might be harmful and even prevent the company from entering the market. The results are also significant for the literature in credit rationing. Contrary to the common wisdom (e.g. Bengt Holmstrom and Jean Tirole, 1997), we show that credit constraints can lead to higher levels of investment.

Moreover, we extend the already large literature in trade credit. The paper builds on Mike Burkart and Tore Ellingsen (2004), in which the nature of the product determines the ability of the supplier to extend trade credit. There, the more liquid is the input in alternative markets, the more incentives a retailer has to divert it, and thus the tighter is the financial constraint.\(^2\) To this source of moral hazard, we add competition in the downstream market, which affects the profitability of honouring the debt. Hence, our approach is closely related to

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\(^1\) See Giacinta Cestone (1999) for a survey of corporate finance and product market competition.

\(^2\) See also Vicente Cuñat (2007) and Mariassunta Giannetti, Mike Burkart and Tore Ellingsen (2011) for other relevant studies explaining differences in the extension of trade credit through product characteristics.
a strand of literature analysing the impact of product market competition on in-kind finance. There, several studies reach different conclusions about how supplier’s competitive pressure affects the extension of trade credit (see, for instance, Mitchell Petersen and Raghuram Rajan, 1997; John McMillan and Christopher Woodruff, 1999; Raymond Fisman and Mayank Raturi, 2004; Daniela Fabbri and Leora Klapper, 2014).

Notably, we differ from previous work in that we analyse the effect of competition in the borrower’s market. By giving all the bargaining power to the supplier, our model describes the optimal combination of prices and quantities subject to the ICC of his customer. This links our study those papers analysing the relation between input prices and the extension of trade credit. Janet K. Smith (1987) and Michael J. Brennan, Vojislav Maksimovic and Josef Zechner (1988) show that price discrimination can be used to reveal the creditworthiness of customers. More recently, Arup Daripa and Jeffrey Nilsen (2011) propose a new rationale for inter-firm credit, which acts as a subsidy for inventory holding costs against lost sales. In this context, their paper evaluates input price adjustment as an alternative to the extension of credit. However, as noted in Mariassunta Giannetti, Mike Burkart and Tore Ellingsen (2011), it is still not clear whether a supplier would reduce input prices to credit-constrained customers. We propose an answer to that question by studying the interaction between the financial and the commercial relation of two vertically related companies.

The remainder of the paper is organized as follows. Sections 2 and 3 set up the model and solve it. Section 4 presents the results when the supplier is forced to set linear prices. Section 5 characterizes the optimal contract and shows that it can be implemented with a two-part tariff. Finally, Section 6 discusses the scope of our results and Section 7 concludes.

2 The Model

- Vertical Relation. Consider a risk-neutral penniless retailer (she) that wishes to enter a market by borrowing inputs from a supplier (he) and pay it back at a later date.
Both the non-wealth and the absence of financial institutions - she only obtains in-kind finance, are assumptions made for the sake of simplicity. Nonetheless, equivalent results can be obtained by relaxing these assumptions. The supplier faces a constant cost \( c_u \) per unit of input produced and has all the bargaining power. Furthermore, we assume that there is no production, so each unit of input borrowed by the retailer can be transformed into a unit of output in the retailing market.

The retailer borrows an amount of input \( I \leq L \) and incurs a debt \( T(I) \). Hence, \( T(\cdot) \) stands for the input tariff set by the supplier. Moreover, \( L \) represents the credit line offered by the supplier and hence a limit to the resources that the retailer can borrow. As will become clear, the retailer is financially constrained when, for a given tariff \( T(\cdot) \), her optimal demand is higher than \( L \).

- **Markets.** The supplier is a monopolist for the retailer. Even though our results hold when the supplier only has some market power, this assumption simplifies significantly the model. Furthermore, numerous studies argue that low competitive pressure for the supplier encourages the extension of trade credit.\(^3\)

Downstream, the retailer competes à la Cournot in a market characterized by a standard inverse demand function \( P(Q) = M - Q \), where \( M \) stands for the size of the market and \( Q \) represents the total quantity of homogeneous output that it is sold in it. There are \( N \) firms that we call *incumbents*. Firm \( i \in \{0, 1, 2, \ldots, N\} \) produces \( q_i \) and each firm has the same unit cost \( c \). We allow for \( c \neq c_u \) so we can study how the relative efficiency of the supplier affects our results. However, these are robust to both \( c_u \leq c \) and \( c < c_u \). Modelling identical incumbents simplifies our setup and let us to exploit the vertical relation as a function of the competitive pressure in an intuitive way. For the same reason, we study symmetric equilibria, so all incumbents react to the retailer’s entrance in the same way.

\(^3\)This argument is extended in Section 6.
• **Moral Hazard.** In the spirit of Mike Burkart and Tore Ellingsen (2004), we assume that the volume of input borrowed by the retailer is observable to the supplier, but not to third parties. Furthermore, sales revenues are verifiable but neither input purchase nor investment decisions are contractible. A moral hazard problem arises because the retailer can divert resources which were destined for sale into private use and so obtain private benefits. As a consequence, the debt is honoured only to the extent of market revenues and the retailer enjoys these only after honouring all repayment obligations. Let $q_e \in [0, I]$ be the retailer’s output in the downstream market. When she diverts $I - q_e$ each unit of input diverted generates a private payoff of $\beta > 0$. Hence, the retailer’s utility is

$$\pi_e = \max \left\{ \left( M - \sum_{i=1}^{N} q_i - q_e \right) q_e - T(I), 0 \right\} + \beta (I - q_e)$$

(1)

The retailer is financially constrained when, conditional on honouring the debt, the optimal $I$ exceeds the amount of input she can borrow, $L$.

**Assumption 1:** $c_u > \beta$

The assumption says that alternative uses of input do not generate enough revenue to cover its cost. Otherwise, the supplier could sell directly in that market.

• **Timing.** The sequence of events is as follows:

$t = 0$ The supplier offers a credit line of up to $L$ units of input with a price scheme $T(\cdot)$. Importantly, the tariff $T(\cdot)$ is observed by incumbents. Hence, retailer and incumbents know each others’ marginal costs.$^4$

$t = 1$ The retailer borrows $I \leq L$ and incurs a debt $T(I)$. Furthermore, the supplier incurs a cost $c_u I$. This transaction is not observable to third parties.

$^4$Alternatively, one could assume that $L$ is observable to incumbents but not $T(\cdot)$. 
$t = 2$ Retailer and incumbents simultaneously launch $q_i$ and $q_e \leq I$ in the retail market whereas $I - q_e$ is diverted. Finally, consumers purchase and the debt $T(I)$ is honoured to the extent of market revenues.

3 Subgame Perfect Equilibria

We solve for a subgame perfect equilibrium using backwards induction.

3.1 $t = 2$ : Diversion and Investment, $q_e(I)$

The retailer invests $q_e(I)$ and competes with the incumbents in the downstream market. The profit function for any incumbent $i$ is $\pi_i = (M - \sum_{-i} q_{-i} - q_i - q_e) q_i - cq_i$. The best response to $\sum_{-i} q_{-i}$ and $q_e$ is

$$R_i(q_{-i}, q_e) = \frac{M - \sum_{-i} q_{-i} - q_e - c}{2}$$

This allows us to define a representative reaction that crucially depends on both the retailer’s output and the competitive pressure. The best response of any incumbent is

$$R_i(q_{-i}, q_e) = \frac{M - q_e - c}{2}, \forall i$$

(2)

Hence, we can model competition as a two-player game between the retailer and N aggregated incumbents. Our retailer’s objective is given in equation (1). Note that an important feature is that she never invests so little to pay only a fraction of the debt, $T(I)$. The reason is that market revenues are contractible. Hence, if the debt was honoured partially, the invested resources would not generate any utility - the value of the maximand would still be zero. In contrast, if the same resources are diverted, the retailer obtains a payoff of $\beta > 0$ per unit.

We denote the outcome of diverting with the superscript $\beta$, so $q_e^\beta = 0$ and $\pi_e^\beta = \beta I$. On the other hand, $q$ indexes for the strategy consisting in honouring the debt. In this case, the
The retailer wants to invest in the market the amount that maximizes profits in (1). Her best response is therefore \( q_e(q_i) = (M - Nq_i - \beta) / 2 \). If she has more resources, she assigns the remaining part to alternative uses that generate revenue of \( \beta \) per unit. Note that we do not call this diversion because the retailer is honouring her debt. In contrast, when she does not have enough resources to invest optimally, the concavity of profits implies that she invests all the product she has. Therefore, the retailer’s best response when she honours her debt is

\[
R_e^q(q_i) = \begin{cases} 
I & \text{if } I \leq \frac{M-Nq_i-\beta}{2} \\
\frac{M-Nq_i-\beta}{2} & \text{otherwise}
\end{cases}
\] (3)

and the corresponding profits

\[
\pi^q_e(q_i) = \begin{cases} 
(M - Nq_i - I) I - T(I) & \text{if } I \leq \frac{M-Nq_i-\beta}{2} \\
\frac{1}{4} [(M - Nq_i)^2 + 2\beta (2I - M + Nq_i) + \beta^2] - T(I) & \text{otherwise}
\end{cases}
\] (4)

Figure 1 plots the reaction functions of both our retailer \( q_e = R_e(q_i) \), and a representative incumbent \( q_i = R_i(q_e) \). Their interactions are marked with black dots and represent the Nash Equilibria of this subgame. We show three panels in which \( I \) is progressively increased (from left to right) to illustrate the effect of the volume of input borrowed on the Nash Equilibria. It is possible to note that the incumbent’s reaction is a standard best response function of a firm competing in a Cournot market. Instead, the reaction of the retailer may have up to three different regions: (i) it is flat and positive when the retailer invests all the input, \( R_e(q_i) = I \); (ii) it slopes down when only a fraction of the input is invested, \( R_e(q_i) = (M - Nq_i - \beta) / 2 \); and (iii) it is zero when diverting is a dominant strategy, \( R_e(q_i) = 0 \).

**FIGURE 1**

The Figure shows that the retailer’s reaction shifts up and to the left as we move from the first to the last panel. Hence, the region where input is diverted increases with the volume of input held by the retailer. When she has a small quantity of product, investing it all is
optimal. As $I$ increases, different equilibria may appear. Notably, there exist situations in which both investment and diversion can be supported as Nash Equilibrium. Finally, when $I$ is sufficiently big, it is strictly dominant for the retailer to divert and not honour the debt. Although the last case is not represented, it is easy to imagine how, in the third panel, the equilibrium with investment disappears as $I$ increases.

In sum, the more inputs a retailer has, the stronger are her incentives to divert it. The reason is that market revenues are concave in quantity, whereas those from diversion are linear. As a consequence, the supplier will need to set a constraint $L$ on the amount of input extended in credit. Otherwise, the retailer would borrow and divert unbounded levels of resources. Finally, two equilibria can coexist. In this case, we focus on the equilibrium in which the retailer repays, as otherwise the supplier will not extend credit, rendering the equilibrium uninteresting.

3.2 $t = 1$ : Borrowing, $I(L, T(\cdot))$

Before investing or diverting, the retailer must choose the amount of input she wants to borrow given the supplier’s offer $\{L, T(\cdot)\}$. As argued above, diversion is an all-or-nothing decision. Hence, we can split our analysis between situations where the retailer invests the input and situations where she diverts it. When the retailer plans to honour the debt, she wishes to borrow

$$R_e(q_i) = \frac{M - Nq_i - T'(I)}{2} \quad (5)$$

To avoid an uninteresting analysis of off-equilibrium paths, we argue that the supplier will always set a contract that prevents his customer from investing in alternative markets. The intuition is as follows. Note that the supplier will always offer a contract that makes the retailer incentive compatible because otherwise he would make loses. Hence, the retailer’s profits must be, at least, $\beta I$. A direct consequence is that the supplier cannot extract any surplus from his customer for the input invested in alternative markets, which generates $\beta$ per
unit. Therefore, since the supplier incurs a unit cost of $c_u$ for the product lent, Assumption 1 implies that each unit of input invested in alternative markets reduces his profits. To prevent this, the supplier reduces the input price and caps the amount of input extended in credit, $L$.

Summarizing, if the retailer plans to honour her debt, she borrows $I = \min \left\{ L, \frac{M-Nq_e-T'(I)}{2} \right\}$. Instead, if she plans to divert, she exhausts the credit extended by the supplier ($I = L$).

### 3.3 $t = 0$: Supplier’s Offer, $\{L, T(\cdot)\}$

At $t = 0$ the supplier anticipates both our retailer’s and incumbents’ behavior. His offer consists of tariff a $T(\cdot)$ and a maximum amount of input $L$ that he is willing to lend. The supplier makes losses whenever his customer diverts. Therefore, he maximizes profits subject to $\pi_e^\beta \leq \pi_e^q$, which is the retailer’s ICC. For this, his policy combines both the price and the amount of input he is willing to extend in credit. Hence, the supplier’s problem reads

$$\max_{\{T(I), L\}} \pi_u = T(I) - c_u I \quad \text{s.t.} \quad \begin{cases} I \leq L \\ \pi_e^\beta \leq \pi_e^q \\ I \in \arg \max_I \{\pi_e^q\} \end{cases}$$

**Corollary 1.** The supplier’s offer $\{L, T(I)\}$ is such that either the retailer honours her debt fully or there is no trade, i.e. $L = 0$.

Note that the less profitable is the downstream market with respect to the gains from diverting, the higher is the incentive to divert for the retailer. Hence, both downstream competition and input liquidity increase agency costs. When either $N$ or/and $\beta$ increase, the supplier needs to lower the value of the credit extended to the retailer in order to make her incentive compatible. This may lead to a financial constraint, i.e. the retailer can’t borrow as much input as she wishes even when she plans to honour her debt. Recall that with no constraint, the retailer would borrow infinitely many resources and divert them.
Our goal is to analyse the interaction between competitive pressure in the downstream market and the terms of the contract offered by the supplier to our retailer. As argued, an increase in the number of incumbents raises agency costs. In response to that, the supplier combines the reduction of both the price $T(\cdot)$ and the amount of input the entrepreneur can borrow $L$ in order to shrink the line of credit he extends. As agency costs increase, the contract offered by the supplier converges to $\{L, T(I)\} = \{0, c_u\}$. At this point, there is no room for surplus conditional on the ICC. Hence, the penniless retailer does not get any credit and so she cannot enter the market.

**Proposition 1.** If $c_u + \beta > c$, the retailer cannot enter the market for $N \geq N = \frac{M-I-c_u}{c_u+\beta-c}$.

Proof: See Appendix.

**Assumption 2.** $c_u + \beta > c$

As a result, either the supplier is not too efficient relative to the incumbents and/or revenues in alternative markets are sufficiently high.

We make this assumption to focus on the interesting insights generated by the model. We want to consider the case where, due to the financial constraint, the retailer’s entrance might not be feasible. Importantly, this holds when the supplier and the incumbents are equally efficient ($c = c_u$).

4 **Linear Prices**

In this section the supplier is restrained to set a constant price per unit of input, so $T(I) = p_u I$. Upstream and downstream firms often do not trade intermediate goods through a simple linear price mechanism. Instead, their relationship involves more general terms for payments as well as vertical restraints.\(^5\) Nonetheless, the literature often uses linear prices to approximate situations where a manufacturer is not able to extract the entire surplus from

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\(^5\)See Patrick Rey and Jean Tirole (1986) and Patrick Rey and Thibaud Vergé (2005) for a classification and a review of pricing strategies and vertical restraints in input transaction contracts.
a retailer and thus, where the retailer earns positive profits. This could occur, for example, due to further information asymmetries such as those regarding the reservation prices of the buyer. As a result, there is a double marginalization by which both firms, the supplier and the retailer, make positive profits.

Our model enriches the traditional framework in different ways. First, note that the value of the transaction, i.e. the volume of credit extended, is limited by exogenous factors. In particular, it depends on the profitability of the retailing market with respect to input diversion. Hence, the line of credit shrinks when either input liquidity ($\beta$) or/and competitive pressure downstream ($N$) increase. Contrary, credit constraints are set strategically by the supplier. Recall that the retailer is constrained when she cannot borrow as much input as she wants even though she is willing to pay back. Furthermore, her optimal investment is a function of her marginal cost, namely the input price. Therefore, by setting $\{L, p_w\}$ the supplier decides whether to make his customer financially constrained. Intuitively, note that if the supplier set a sufficiently large $p_w$ the retailer would never be constrained because her optimal investment would be zero.

The supplier’s ability to financially constrain his customer is key. Incumbents observe the offer $p_w$ before competing in the retailing market, so they can infer whether our retailer is constrained. When the retailer’s best response is lower than $L$, she is not constrained and all firms simultaneously decide quantities. Hence, the game is equivalent to a standard Cournot. Instead, if $R^i_e(q_e) \geq L$ incumbents know that their competitor will exhaust all credit ($q_e = L$). Hence, the financial constraint, which is induced through the wholesale price, acts a commitment device that gives to the retailer a first-mover advantage. We show that when the competitive pressure downstream is sufficiently high, the supplier uses this mechanism to maximize the surplus he can extract from his customer. Furthermore, this is also beneficial for the retailer, which makes higher profits than in a situation with no agency problems, e.g. $\beta = 0$.

We present our findings by describing the amount of input borrowed (and invested) as
a function of the number of incumbents. The following proposition characterizes how the retailer’s output is affected by competitive pressure downstream when the supplier sets linear input prices.

**Proposition 2.** For a given set of market characteristics \((M, c, c_u, \beta)\), pure strategy Nash Equilibrium in the downstream market determines credit constraints and output as follows:

(i) If \(N \leq N_1\) the retailer is not constrained and a standard Cournot is played with \(N + 1\) companies;

(ii) If \(N \in (N_1, N_2]\) the retailer is constrained, but active and \(q_e'(N) = 0\);

(iii) If \(N \in (N_2, \bar{N})\) the retailer is constrained, but active and, \(q_e'(N) < 0\);

(iv) If \(N \geq \bar{N}\) the retailer cannot enter the market, \(L = 0\), and incumbents play a standard Cournot with \(N\) companies.

Proof. See Appendix.

The first and the last parts of the proposition show the extreme cases. With few incumbents \((N \leq N_1)\), the downstream market is relatively profitable. Then, the supplier’s surplus is maximized by not constraining his customer and inducing her to play a simultaneous game downstream. As a result, the outcome in the retailing market is that of a Cournot game and the vertical relation is characterized by a standard double marginalization. In contrast, when the number of competitors is sufficiently large \((N \geq \bar{N})\), the supplier is not able to offer any contract satisfying the ICC. Hence, there is no trade and so the retailer cannot enter the market. Intuitively, note that the supplier’s problem consists in reducing both \(L\) and \(p_w\) to make his client incentive compatible. Nonetheless, this policy is limited by \(L = 0\) and \(p_w = c_u\), which is the equilibrium for \(N \geq \bar{N}\).

For intermediate levels of competition, the supplier sets a \(p_w\) so that the retailer is financially constrained. Hence, he uses the financial constraint as a commitment device to grant his customer with a leader advantage. More specifically, when \(N \in (N_1, N_2)\), the
supplier responds to an increase in the number of incumbents by sharply reducing $p_w$ so that $L$ remains constant. Therefore, the acute reduction in price allows the supplier to keep his customer’s demand stable on $L$ despite the increasing competition in the retailing market. In contrast, for $N \in (N_2, \overline{N})$, he reduces the sensitiveness of the wholesale price to the competitive pressure downstream. As a consequence, $L$ is also reduced when the number of incumbents increases. In this region, the retailer’s output equals that of a Stackelberg leader with a unit cost of $c_u + \beta$, which is linearly decreasing in $N$. As it is further explained in the next section, this output maximizes the supplier’s surplus under any type of contractual form.

Figure 2 compares the outcome of this game (solid) with the benchmark case, where there are no financial constraints (dashed). The left panel presents equilibrium quantities as a function of the competitive pressure $N$ whereas the right panel plots the retailer’s profits. Changes in the slope of the solid line identify the thresholds $N_1$, $N_2$ and $\overline{N}$. As it is possible to see, there is a significant region where the retailer makes higher profits due to her own financial constraints – where the solid line is above the dashed line.

FIGURE 2

The contract described above has further effects on other agents. The higher volume of output of the financially constrained retailer changes the market outcome similarly to James Brander and Tracy Lewis (1986). The induced aggressiveness reduces the optimal investment of incumbent firms, which become less profitable than if their rival was self-financed. Moreover, the same mechanism increases final consumers’ surplus. In particular, in the region where credit rationing makes the retailer more aggressive, the total quantity of output in the downstream market is higher than in a setting with no agency problems. In other words, the overall reduction of output of incumbent companies is smaller than the entrant’s induced aggressiveness. As a consequence, consumer prices are lower due to the financial constraints of a retailer.
The left panel in Figure 3 shows how the profits of the seller decline smoothly as the number of incumbents in the downstream market increases. Thus, even though the optimal price-quantity policy may have positive externalities for the retailer, it is designed to minimize the supplier’s agency cost of an increment of competition in the downstream market. Furthermore, the right panel plots input price as a function of the number of incumbents. As expected, it converges to the marginal cost $c_u$, where the supplier makes zero profits. Changes in the first derivative are a consequence of the adjustments in the optimal policy. We draw the discontinuities in the first derivative with dashed lines, which correspond to the values $N_1$, $N_2$ and $N$. Notably, one can observe that the slope is higher in the region $N \in (N_1, N_2)$ and that $p_w = c_u$ when $N = N$.

**FIGURE 3**

We conclude highlighting the possibility that financial constraints can allow a retailer to gain first-mover advantage in the product market. This occurs for intermediate levels of competitive pressure, where the supplier can extend credit and the reduction in prices required to financially constrain the retailer is not too strong. Furthermore, incumbent companies in the retail market lower their profits and consumers face lower prices. Next, we evaluate the implications of our assumptions by not restricting the supplier to any specific contractual form. Furthermore, in Section 6 we discuss the results and some potential applications.

## 5 Optimal Contracts

In the preceding section the supplier had to set a linear wholesale price. We argued that this is a proxy for situations in which the seller cannot extract all rents from his customer. We now study the case where the supplier can set an optimal contract, i.e. there is no restriction in the contractual form. Hence, the supplier is a monopolist with full information about the retailer and so able to extract all her surplus. Although this is rarely the case, we aim to
study the implications of our assumptions in a setting that approximates situations where suppliers are better informed about the market conditions of their customers.

The supplier holds all bargaining power. Hence, the optimal contract maximizes his profits with no restriction in the contractual form. Consequently there is no double marginalization, implying that without the presence of diversion opportunities ($\beta = 0$) the retailer would make zero profits. However, $\beta > 0$ and so our supplier optimizes subject to the ICC. As a result, due to the non-contractibility of investment, he has to offer an agency cost of $\beta$ per unit of input extended in credit. Hence, the retailer obtains information rents and so she makes positive profits. In particular, she is granted with $\beta$ per each unit of input that borrows.

Optimality can be obtained through different contractual forms. However, they all of them must reach the same outcome. Thus, it is useful to characterize the sufficient conditions for a contract to be optimal. In our setting, these are given by a well-known market structure: the Cournot-Stackelberg.

**Proposition 3:** A contract is optimal if and only if supplier’s profits are those of a Stackelberg leader in the downstream market with a unit cost of $c_u + \beta$ and retailer’s profits are those that make her just incentive compatible, i.e. $\pi_e = \beta I$.

Proof. See Appendix

Proposition 3 provides an elegant framework to study the optimality of different contracts. The intuition behind this result is as follows. Note that the best the supplier could do would be to directly participate in the retailing market. However, since this is not possible, he needs to use the retailer, which carries an agency cost of $\beta$ per unit of output. This makes an optimal contract with the retailer equivalent to operating in the downstream market with a marginal cost $c_u + \beta$. Furthermore, incumbents observe the wholesale price offered by the supplier before competing downstream. Therefore, the supplier can induce his customer to launch the quantity of output of a firm with a first-mover advantage. As a result, the outcome equivalent to directly participating in the retailing market is that of a Stackelberg
leader.

In contrast to where the supplier is restricted to set a linear wholesale price, the non-contractibility of the transaction unambiguously leads to an increase of retail prices when optimal contracts can be implemented. The agency cost reduces the optimal quantity of a Stackelberg leader. In equilibrium, the decrease in output of our production chain is not fully compensated by competitors in the downstream market and so the overall quantity of output is reduced. Therefore, while the presence of financial constraints can be beneficial for consumers when suppliers cannot fully extract their customer’s surplus, their welfare is always decreased if suppliers can implement optimal contracts.

A simple form of an optimal contract can be implemented by a lump sum \( T(I) = T \) that the retailer pays in exchange of a specific amount of input set by the supplier. Let \( q^S_i \) and \( q^S \) denote respectively the outputs of a Stackelberg leader with a unit cost \( c_u + \beta \) and a representative follower. Then, the next proposition follows:

**Proposition 4.** An optimal contract can be implemented with an offer consisting of a unique amount of input \( L = q^S_i \) and a tariff \( T = (M - Nq^S_i - q^S_i) q^S_i - \beta q^S_i \).

**Proof.** See Appendix.

This pricing scheme resembles a quantity-fixing contract.\(^6\) In fact, in our setting the retailer accepts any offer with a positive amount of input because it ensures positive profits – at least \( \beta \) per unit of product. Therefore, the supplier can offer the quantity of a Stackelberg leader and set the tariff that makes his customer just incentive compatible. Nevertheless, despite the simplicity of these contracts, their use is not extended in the real economy. Among the possible explanations, there are the precise information required by the supplier or the numerous allegations of anticompetitive exclusion by other retailers.\(^7\)

Having characterized an optimal contract, in the next subsection we show that it can also be implemented with a two-part tariff. Contrary to quantity-fixing contracts, this price

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\(^6\)See Rey and Vergé (2005) for a detailed description of quantity-fixing contracts.

\(^7\)See Marius Schwartz and Daniel Vincent (2008) and references therein.
scheme is commonly used in input transactions. Thus, we aim to provide a more plausible application of our results.

5.1 Two-Part Tariffs

Two-part tariffs are often used in wholesale markets, where manufacturers charge a lump sum to a retailer for the right to carry their product plus a constant charge per unit ordered by the retailer, i.e. \( T(I) = F + p_w I \). As with the linear price, the unit price \( p_w \) becomes the marginal cost of the retailer. However, here a fixed fee \( F \) is used by the supplier to extract his customer’s surplus.\(^8\)

One can approach the quantity-fixing contract as a restricted case of a two-part tariff where the unit price is set equal to zero. Therefore, from Proposition 4, it is straightforward to see that an optimal contract can also be implemented by a two-part tariff. Moreover, because the retailer can make use of the quantity constraint \( L \), there is a continuum of combinations \( \{F, p_w\} \), i.e. of two-part tariffs, that meet the sufficient conditions in Proposition 3. Next, we characterize a contract that establishes an interesting link with the financial constraints. In particular, we show that there is a pair \( \{F, p_w\} \) with which the retailer always fulfils her demand. In other words, she would only wish to borrow more input if she had the intention to strategically default on her debt.

**Proposition 5.** The optimal contract can be implemented with a two-part tariff such that the retailer is never financially constrained, so she fulfils her demand when she plans to honour the debt. The credit line is \( L = q_i^S \) and the tariff satisfies:

\[
p_w = \frac{(\beta + c_u)(N + 1)(N + 2) - N(M - cN)}{2(N + 1)}; \quad F = \left( M - Nq_i^S - q_i^S \right) q_i^S - \beta q_i^S - p_w q_i^S.
\]

Proof. See Appendix

This contract induces the retailer to demand the input of Stackelberg leader with marginal

\(^8\)See Massimo Motta (2004), for an overview on two-part tariffs
cost \( c_u + \beta \) through the unit price \( p_w \). Furthermore, the fixed fee \( F \) is used to extract the retailer’s rents while satisfying the ICC. As a consequence, conditional on investing, the retailer fulfils her demand and thereby is not financially constrained. Figure 4 plots both the unit price and the fixed fee as a function of the number of competitors in the retailing market.

\[ \text{FIGURE 4} \]

Contrary to common wisdom, the optimal fixed fee can be negative at the expense of a relatively high unit price. In general, the supplier’s response to downstream competition is to make his customer more aggressive by setting unit prices below marginal costs. Then, the fixed fee accomplishes an additional function consisting of subsidizing the low per-unit price (Hayes, 1987). However, in our setting agency costs have an opposing effect. For high levels of competitive pressure, a negative fixed fee is subsidized by a relatively high unit price. In Figure 4, it can be appreciated that this occurs in the region where \( N \in (N_2, N) \). There, the retail price \( P \) is below the effective marginal cost of the supplier, \( \beta + c_u \).\(^9\) Yet, he can make profits by compensating the retailer with a lump sum. Notably, this region vanishes when \( \beta = 0 \).

We finish by noticing two features of the two-part tariff described in Proposition 5 that will be further discussed in the next section. First, a negative fixed fee consists of a transfer from the supplier to the retailer. Such transfer needs to be made ex-post. Otherwise, it would provide additional incentives for the retailer to divert and so she would not be incentive compatible. Thus, an implicit assumption is that, different from the retailer, the supplier is able to commit to honouring his obligations. Finally, a financial constraint might not exist despite of information frictions when the supplier can fully discriminate in prices. As a consequence, the presence of agency problems becomes noticeable only through input prices and firms’ profits.

\(^9\text{When } N = N_2 \text{ then } F = 0 \text{ and } P = p_w + \beta.\)
6 Discussion and Scope

Our results apply to industries where suppliers have positive markups. Otherwise, a supplier would not be able to reduce the wholesale price in response to an increase of competitive pressure in his customer’s product market. Furthermore, an important strand of inter-firm credit literature argues that low competition is also one of the main elements that enables companies to extend trade credit. Few alternative suppliers (Mitchell Petersen and Raghuram G. Rajan 1995), long withstanding commercial relations (Vicente Cuñat 2007) or limited opportunities to divert customer-tailored inputs (Mike Burkart and Tore Ellingsen 2004; and Mariassunta Giannetti, Mike Burkart and Tore Ellingsen 2011) are shown to encourage in-kind finance. Hence, we expect a positive correlation between the presence of trade credit and the forces represented in our model. Regarding the retailing market, our model represents situations where firms compete simultaneously, which leads to a first-mover advantage. Hence, similar results could be obtained in a market with Hotelling competition. There, distance between products plays a similar role to the number of incumbents in our model.

A natural question is whether our results can extend to more standard financial transactions. In fact, it is possible to think of the supplier as a financial institution setting an interest rate instead of an input price. Then, the input price accounts for the cost of raising funds for this institution. According to Mike Burkart and Tore Ellingsen (2004), conventional cash loans can be represented in our setting by a fully liquid input, i.e. $\beta = 1$. The results of our model are robust to this assumption and so make it reasonable to think of such an extension. However, financial markets are often highly competitive. Hence, the applicability in this context seems limited.

Alternatively, we propose a connection between input suppliers and the banking sector, which are often seen as two completely separate industries. In our model the supplier is also the financier. However, it is easy to separate both activities and obtain equivalent results under standard assumptions. In that case, the retailer’s access to credit has an effect on the
supplier’s pricing policy and therefore, on how competitive is the retailer in the downstream market. Hence, we think that our results should be taken into account by institutions working on the alleviation of credit constraints. In particular, we showed that benefits from relaxing financial constraints can be absorbed by suppliers, who may raise prices instead of extending more input in credit.\(^{10}\)

By comparing our results under linear pricing and optimal contracts, we argued that credit rationing may be related to the capacity of the supplier to extract his customer’s surplus. Accordingly, we think that future empirical work in this direction can contribute to the understanding of both the motives and the consequences of financial constraints. For this, several elements can be used to identify whether a commercial relation is closer to either setting in the model. For instance, using the classification of Mariassunta Giannetti, Mike Burkart and Tore Ellingsen (2011), we expect higher price discrimination (and thus less financial constraints) in markets of customer-tailored products than in markets of generic products.

Our results have further implications for studies evaluating credit rationing. The common view is that financial constraints lead to underinvestment of borrowers (e.g. Bengt Holmstrom and Jean Tirole 1997). Instead, we show that a financially constrained retailer may produce higher levels of output than if it had full access to credit. The same mechanism can also explain the allocation of debt across group-affiliated companies. Relatedly, Magda Bianco and Giovanna Nicodano (2006) suggest that these corporations may rely on the limited liability of their subsidiaries to raise debt that is subsequently allocated to the holding company. Similarly, our results imply that holding companies could use the limited liability of their subsidiaries to obtain inputs at a better price.

Finally, the model also shows that lump-sum transfers from suppliers to their customers can be optimal when investment decisions are not contractible. Notably, the seller can use a fixed payment to subsidize high unit prices that otherwise would prevent its customer

\(^{10}\)Similar arguments are provided, for instance, in Timothy Besley (1994).
from being incentive compatible. There are several real life examples of contracts involving lump-sum payments from suppliers to their retailers. This is the case, for instance, of volume rebates or slotting fees. To our knowledge, we present a new mechanism that rationalizes this sort of contracts and may complement other existing theories.\footnote{An extended rationale for rebates is that they are an instrument to elicit information from retailers and to mitigate demand uncertainty (see, for instance, Terry A. Taylor and Wenqiang Xiao, 2009, and references therein). Regarding slotting fees, empirical evidence suggests that are a risk-sharing device between suppliers and retailers (Volker Nocke and John Thanassoulis 2014, and references therein).} Hence, we suggest that empirical work in this direction could find causality between retailers’ incentives to strategically default in their debt and the use of these type of contracts.

\section{Concluding Remarks}

This paper aims to contribute to the understanding of how financial constraints shape the contracts between vertically related firms. To this end, we introduce elements of industrial organization in a model of in-kind finance. More specifically, we characterize the trade credit contract offered by a supplier to a retailer with no funds when the transaction is not contractible. The supplier has market power and this allows him to define a price-quantity policy that minimizes his agency costs. Moreover, the retailer competes in quantities in a market with multiple self-financed companies. Competition downstream affects the retailer’s incentives to divert and in turn the conditions at which she obtains the input, namely the contract offered by the supplier.

Our main result shows that a retailer can gain competitive advantage due to her own financial constraint. Furthermore, this effect can be exacerbated by competitive pressure in the retailer’s market, which worsens the agency problems. The effect also has an impact on the other market participants. In particular, retailer’s competitors in the downstream market make lower profits than if their rival had full access to credit. Nevertheless, the overall quantity of output in the downstream market is higher. This lowers the retailing price and as a consequence, final consumers are better off due to the credit rationing of a retailer. This
result provides a new mechanism through which firms can benefit from limited liability and furthermore, it shows that financial constraints can lead to higher levels of output.

The previous finding arises when we restrict the supplier to set a linear wholesale price, which represents situations where he is not able to extract the entire surplus of his customer. By relaxing this assumption, we characterize an optimal contract for the supplier and show that it can be implemented with a two-part tariff. The presence of information frictions rationalizes a transfer from the supplier to the retailer and thus contributes to the understanding of contracts with this characteristic such as rebates. Moreover, the demand of the retailer may always be fulfilled when she wants to honour her debt. Hence, we open a door to further analysis about the relation between suppliers’ ability to discriminate in prices and the identification of financial constraints faced by their customers.
Appendix

Figure 1

Compare $\pi^\beta_e = \beta I$ with the profit function in (4), where $T(I) = p_w I$. With the sake of clarity, denote $\pi^q_1$ the profits in (4) when $I \leq \frac{M-Nq_i-\beta}{2}$ and $\pi^q_2$ the profits otherwise.

Moreover, denote $q^{q1}_i$ the value of $q_i$ such that $\pi^\beta_e \leq \pi^q_1$ if $q_i \leq q^{q1}_i$. Analogously, let $q^{q2}_i$ be the value for which $\pi^\beta_e \leq \pi^q_2$ when $q_i \leq q^{q2}_i$. Finally, for $q^{I}_i$ it holds that $\pi^q_2 \leq \pi^q_1$ when $q_i \leq q^{I}_i$. The corresponding expressions are:

$$q^{q1}_i = \frac{M - I - p_w - \beta}{N}, q^{q2}_i = \frac{M - \beta - 2\sqrt{Ip_w}}{N} \quad \text{and} \quad q^{I}_i = \frac{M - 2I - \beta}{N}$$

When $I \leq p_w$, it holds that $q^{I}_i > q^{q1}_i$. Then $R_e(q_i) = I$ for $q_i \leq q^{q1}_i$ whereas $R_e(q_i) = 0$ otherwise. If instead $I > p_w$, then $q^{I}_i < q^{q1}_i$. In this case $R_e(q_i) = I$ for $q_i \leq q^{I}_i$; $R_e(q_i) = \frac{M-Nq_i-\beta}{2}$ if $q_i \in \left[q^{I}_i, q^{q2}_i \right]$ and $R_e(q_i) = 0$ for $q_i > q^{q2}_i$. As $I$ increases, $q^{q1}_i$ and $q^{q2}_i$ subsequently become negative. Hence, the best responses corresponding to $q_i$ below these values disappear. Figure 5 plots the three profit functions with the corresponding thresholds as $I$ increases (from left to right).

**FIGURE 5**

Proposition 1

There can’t be trade if the total surplus is less than $\beta I$ because otherwise it is not possible to satisfy the ICC. Hence, it must be the case that $\pi_u = T(I) - c_u I \geq 0$ and $\pi_e = (M - Nq_i - I)I - T(I) \geq \beta I$ where $q_i$ is the best response in (2). Suppose the supplier has zero profits, i.e. $T(I) = c_u I$. Then it needs to hold that

$$\left[ M - N \left( \frac{M - I - c}{N + 1} \right) - I \right] I - c_u I \geq \beta I \rightarrow I \leq M - (\beta + c_u)(N + 1) + cN$$
Since $I \geq 0$, the condition can’t be satisfied for $N \geq N \equiv \frac{M-\beta-cu}{cu+\beta-c}$.

**Proposition 2**

**Diversion and investment:** $q_e(I)$. The ICC is satisfied when $\pi_e^\beta \leq \pi_e^q$ where $\pi_e^\beta = \beta I$. Hence, we want to define $\pi_e^q$ as a function of $I$.

Given $I$, it is optimal for the retailer $q_e(q_i) = \frac{M-Nq_i-\beta}{2}$. This, and incumbents’ response in (2), lead to the equilibrium

$$q_e^I = \frac{M - \beta(N + 1) + cN}{N + 2}; \quad q_i^I = \frac{M - 2c + \beta}{N + 2} \quad (7)$$

If $I < q_e^I$ then $q_e = I$ and $\pi_e^q = (M - Nq_i - I)I - pwI$ where $q_i = R_i(I)$ in equation (2). Contrary, when $I \geq q_e^I$ the market outcome is (7) and the retailer invests $I - q_e^I$ in alternative activities. Plugging $q_i^I$ into the second expression in (4) we obtain

$$\pi_e^q = \begin{cases} \frac{[M-I-pw(N+1)+cN]}{N+1} - F & \text{if } I \leq q_e^I \\ \frac{1}{(N+2)^2} \left\{ \begin{array}{l} M^2 + N [Nc^2 - Ipw(N + 4)] \\ -4Ipw + 2MNc + \beta^2(N + 1)^2 \\ + \beta [I(N + 2)^2 - 2(N + 1)(M + cN)] \end{array} \right\} - F & \text{otherwise} \end{cases} \quad (8)$$

where $F = 0$.

We now derive the retailer’s strategy in terms of $I$. Let $\pi_e^{q1}$ and $\pi_e^{q2}$ denote the first and the second expressions in (8) respectively. Denote $I^{q1}$ the value of $I$ such that $\pi_e^{\beta} \leq \pi_e^{q1}$ when $I \leq I^{q1}$. Analogously, $I^{q2}$ satisfies $\pi_e^{\beta} \leq \pi_e^{q2}$ for $I \leq I^{q2}$.\[^{12}\]

$$I^{q1} = M - (\beta + pw)(N + 1) + cN \quad \text{and} \quad I^{q2} = \frac{[M - \beta(N + 1) + cN]^2}{pw(N + 2)^2} \quad (9)$$

Compare these thresholds with quantities in (7). When $I^{q1} \leq q_e^I$, then $q_e = I$ for

\[^{12}\]Contrary to the case where $F > 0$, i.e. two-part tariff, here $\pi_e^{\beta} = \pi_e^q = 0$ when $I = 0$. Hence, $\pi_e^{\beta}$ and $\pi_e^q$ only meet once ($I^{q1}$) for $I > 0$.  

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I \leq I^{q_{13}} \text{ and } q_e = 0 \text{ otherwise. Instead, when } q^{T}_e < I^{q_{13}} \text{ there are three potential outcomes: } 

i) \text{ if } I < q^{T}_e \text{ then } q_e = I; \ ii) \text{ when } I \in [q^{T}_e, I^{q_{23}}] \text{ then } q_e = q^{T}_e \leq I \text{ and } iii) \text{ if } I > I^{q_{23}} \text{ then } q_e = 0.

\textbf{Borrowing: } I(L, p_w). \text{ With a linear price, it must be that } p_w \geq c_u. \text{ Hence, by Assumption } 1, p_w > \beta. \text{ Therefore, if the retailer is incentive compatible she only wants to invest in the downstream market, i.e. } q_e = I. \text{ Thus, only } I^{q_{13}} \text{ in (9) is relevant for the analysis.}

If she plans to invest, her best response is that in (5) where } T'(I) = p_w. \text{ This reaction and that in (2) lead to a standard Cournot outcome }

\begin{align*}
q^*_e &= \frac{M - p_w (N + 1) + N c}{N + 2}; q^*_i = \frac{M - 2c + p_w}{N + 2} 
\end{align*}

and retailer’s profits

\begin{align*}
\pi^*_e = \pi^{q1}(q^*_e, q^*_i) = \frac{[M - p_w(N + 1) + c N]^2}{(N + 2)^2} - F, 
\end{align*}

where } F = 0. \text{ The profit in (11) is the highest the retailer can obtain by investing. This makes convenient to split the analysis into two cases: } q^*_e > I^{q_{13}} \text{ and } q^*_e \leq I^{q_{13}}. \text{ Now define } p'_w = \frac{M - \beta(N + 2) + c N}{N + 1} \text{ be the price such that } q^*_e \leq I^{q_{13}} \text{ for } p_w \leq p'_w \text{ and } q^*_e > I^{q_{13}} \text{ otherwise. }

\text{When } q^*_e \leq I^{q_{13}} \text{ there are three regions of interest: } \ i) \text{ if } L \leq q^*_e, \text{ then } I = q_e = L; \ ii) \text{ for } L \in \left(q^*_e, \frac{\pi^*_e}{\beta}\right], \text{ then } I = q^*_e < L \text{ and } iii) \text{ when } L > \frac{\pi^*_e}{\beta} \text{ then } I = L \text{ but } q_e = 0. \text{ Note that } \frac{\pi^*_e}{\beta} \text{ is the amount of input that by being diverted generates the maximum profit that can be achieved by investing. Hence, the retailer will always divert if } L > \frac{\pi^*_e}{\beta}. \text{ On the other hand, if } q^*_e > I^{q_{13}}, \text{ the entrepreneur always exhausts all credit, } I = L. \text{ Moreover, she invests if } L \leq I^{q_{13}} \text{ and diverts otherwise.}

\textbf{Offer } \{L, p_w\}. \text{ The supplier has profits } \pi_u = (p_w - c_a)q_e \text{ if the retailer is incentive compatible. For this, he sets } L = \frac{\pi^*_e}{\beta} \text{ when } p_w \leq p'_w \text{ and } L = I^{q_{13}} \text{ when } p_w > p'_w. \text{ However, } p_w \text{ is also determined endogenously. Hence, the supplier sets the } p_w \text{ that maximizes profits}
subject to the ICC, which is the policy for $L$. The problem reads:

$$\arg \max_{p_w} \pi_u = (p_w - c_u)I \quad \text{s.t.} \quad \begin{cases} I = q_e^s & p_w \leq p_w' \\ I = I^{q_1\beta} & p_w > p_w' \end{cases}$$ (12)

If $p_w \leq p_w'$ the constraint $I = q_e^s$ applies. Then, a standard Cournot is played and the solution to (12) is:

$$p_w^s = \frac{M + c_u(N + 1) + cN}{2(N + 1)}.$$

Plugging this into both (10) and $\pi_w^s/\beta$, we obtain

$$q_e^s = \frac{M - c_u(N + 1) + cN}{2(N + 2)}, \quad q_i^s = \frac{M - 2c + \frac{M + c_u(N + 1) + cN}{2(N + 1)}}{N + 2}, \quad L^s = \frac{[M - c_u(N + 1) + cN]^2}{4\beta(N + 2)^2}.$$

This is the solution for $p_w^s \leq p_w'$, or equivalently $q_e^s \leq L^s$, which is satisfied when $N \leq N_1 = \frac{M - c_u - 4\beta}{c_u - c + 2\beta}$. 

If $p_w > p_w'$ the financial restraint is binding, so we index the corresponding outcome with $r$. Plugging $I^{q_1\beta}$ into (12) and solving the problem leads to:

$$p_w^r = \frac{M + (N + 1)(c_u - \beta) + Nc}{2(N + 1)}.$$

In this case the retailer exhaust all the credit, so $I^{q_1\beta} = q_e^r = L^r$. Plugging $p_w^r$ into $I^{q_1\beta}$ in (9), we obtain

$$q_e^r = L^r = \frac{M - (N + 1)(c_u + \beta) + Nc}{2}, \quad q_i^r = \frac{M + (N + 1)(c_u + \beta) - (N + 2)c}{2(N + 1)}.$$

This outcome is valid for $p_w^r \geq p_w'$, which holds when $N \geq N_2 = \frac{M - c_u - 3\beta}{c_u - c + \beta}$. Moreover, one can check that $L^r \leq 0$ if $N \geq \overline{N} = \frac{M - c_u - 3\beta}{c_u - c + \beta}$. Therefore, the restrained outcome is an

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13 Note that when $p_w < p_w'$, then $q_e^r < I^{q_1\beta} < \frac{\pi^s_L(q_e^r, q_i^r)}{\beta}$ and for $p_w = p_w'$ it holds that $q_e^r = I^{q_1\beta} = \frac{\pi^s_L(q_e^r, q_i^r)}{\beta}$.

14 When $N = \overline{N}$ the optimal price reaches marginal cost, $p_w^r = c_u$. Then, the lender is not able to reduce $p_w$ and so relax the financial constraint anymore.
equilibrium when $N \in (N_2, N)$ and no credit is extended if $N \geq N$.

Finally, for $N \in (N_1, N_2)$ the lender sets $p_w = p_w'$. As a result, $q^c = q^r = \beta$ and 
$q^e = q^r = \frac{M-c-\beta}{N+1}$.

**Proposition 3**

With no frictions, the supplier makes the profits of directly participating in the downstream market. Instead, moral hazard imposes an agency cost of $\beta I$. Hence, if the supplier could operate in the downstream market but had to afford the cost of satisfying the ICC, he would produce $I \in \arg\max \{(M - Nq_i - I)I - (c_u + \beta)I\}$. However, in our model incumbents observe $T(I)$ before competing and so they can infer $L$. Hence, a fair comparison implies that the supplier can commit to a certain quantity, so is a Stackelberg leader.

On the opposite direction the argument is almost trivial. It follows from the fact that in no case the presence of the retailer can increase supplier’s profits with respect to the situation where he is a Stackelberg leader with unit cost $c_u + \beta$.

**Stackelberg game.** Denote a Stackelberg leader with the subscript $l$ and assume his unit cost is $c_u + \beta$. The equilibrium quantities are

$$q^S_l = \frac{M - (c_u + \beta)(N + 1) + cN}{2}; q^S_i = \frac{M - c(N + 2) + (c_u + \beta)(N + 1)}{2(N + 1)}$$

which lead to a profit for the leader

$$\pi^S_l = \frac{[M - \beta - c_u - N(\beta + c_u - c)]^2}{4(N + 1)}$$

**Proposition 4**

**Diversion and Investment** $q_e(I)$. The analysis is analogous to that in Proposition 2 with $F > 0$. If $I < q^S_i$ then $q_e = I$ and obtains $\pi^S_e = (M - Nq_i - I)I - F - p_wI$ where $q_i = R_i(I)$
in (2). Contrary, when \( I \geq q_e^7 \) the market outcome is that in (7) and the retailer assigns \( I - q_e^7 \) to alternative activities. Profits are represented in (8).

Let \( \pi_e^{q_1} \) and \( \pi_e^{q_2} \) denote the first and the second expressions in (8) respectively. Denote \( I_1^{q_{1\beta}} \) and \( I_2^{q_{1\beta}} \) the values such that \( \pi_e^{\beta} \leq \pi_e^{q_1} \) when \( I \in [I_1^{q_{1\beta}}, I_2^{q_{1\beta}}] \). Note there is a new threshold with respect to Proposition 2. This is due to the fact that when \( F > 0 \) the retailer is not incentive compatible for small \( I \). Furthermore, \( I_{q_2^{\beta}} \) satisfies \( \pi_e^{\beta} \leq \pi_e^{q_2} \) for \( I \leq I_{q_2^{\beta}} \); and \( I_{q_1^{q_2}} \) satisfies \( \pi_e^{q_2} \leq \pi_e^{q_1} \) if \( I \leq I_{q_1^{q_2}} \).

\[
\begin{align*}
I_1^{q_{1\beta}} &= \frac{1}{2} \left[ M - (\beta + p_w)(N + 1) + cN - \sqrt{\Gamma} \right]; \\
I_2^{q_{1\beta}} &= \frac{1}{2} \left[ M - (\beta + p_w)(N + 1) + cN + \sqrt{\Gamma} \right]; \\
I_{q_2^{\beta}} &= \frac{\beta^2(N + 1)^2 - F(N + 2)^2 - 2\beta(N + 1)(M + cN) + (M + cN)^2}{p_w(N + 2)^2}; \\
I_{q_1^{q_2}} &= \frac{(N + 1)[M - \beta(N + 1) + cN]}{N + 2},
\end{align*}
\]

where \( \Gamma = [M - \beta - p_w - (\beta - c + p_w)N]^2 - 4F(N + 1) \).

The retailer is incentive compatible when \( I \in [I_1^{q_{1\beta}}, \max\{I_2^{q_{1\beta}}, I_{q_2^{\beta}}\}] \). If \( I_2^{q_{1\beta}} < I_{q_2^{\beta}} \), she sets \( q_e = I \) for \( I \in [I_1^{q_{1\beta}}, I_{q_1^{q_2}}] \), invests \( q_e^7 \) for \( I \in (I_{q_1^{q_2}}, I_{q_2^{\beta}}] \) and misbehaves when \( I > I_{q_2^{\beta}} \). Instead, when \( I_{q_2^{\beta}} < I_2^{q_{1\beta}} \) there is no situation in which the entrepreneur wants to invest partially, so \( I_{q_2^{\beta}} \) is not relevant. Hence, \( q_e = I \) when \( I \leq I_2^{q_{1\beta}} \) and \( q_e = 0 \) otherwise.

**Borrowing:** \( I(L, p_w) \). As argued, the contract will be such that the retailer never invests partially. Thus, \( q_e \in \{0, I\} \) and so only \( I_1^{q_{1\beta}} \) and \( I_2^{q_{1\beta}} \) are in (13). If the response in (5) is feasible, the outcome in (10) and profits in (11) are obtained. This makes convenient to split our analysis in two cases depending on the relation between \( \pi_e^s \) and \( \pi_e^{\beta} \) when \( I = q_e^s \). Let us use \( F \) (rather than \( p_w \)) to characterize the two situations. Define \( F' \) and \( F'' \) so that: when \( F \leq F' \) then \( q_e^s \leq I_2^{q_{1\beta}} \); if \( F \in (F', F'') \) then \( q_e^s > I_2^{q_{1\beta}} \); for \( F = F'' \) then \( I_1^{q_{1\beta}} = I_2^{q_{1\beta}} \); and if \( F > F'' \) neither \( I_1^{q_{1\beta}} \) nor \( I_2^{q_{1\beta}} \) are definite. In the latter case, \( \pi_e^{\beta} > \pi_e^{q_1} \) for any \( I \) so the
retailer never plans to honour her debt.

\[
F' = \frac{[M - p_w - (p_w - 2) N] [M - p_w - 2\beta - (p_w + \beta - 2) N]}{(N + 2)^2}; \tag{14}
\]

\[
F'' = \frac{[M - p_w - \beta - (p_w + \beta - 2) N]^2}{4(N + 1)}
\]

Finally we describe the retailer’s strategy as a function of \(\{L, (F, p_w)\}\). When \(F > F''\) then \(I = L\) and \(q_e = 0\). If \(F \in (F', F'')\) again \(I = L\), but \(q_e = I\) when \(L \in [I^q_1, I^q_2]\) and \(q_e = 0\) otherwise. Last, if \(F < F'\) there are four regions of interest: \(i)\) for \(L < I^q_1\) then \(I = L\) and \(q_e = 0\); \(ii)\) if \(L \in [I^q_1, q^*_e]\) then \(I = q_e = L\); \(iii)\) when \(L \in [q^*_e, \pi^*_e/\beta]\) then \(q^*_e = I \leq L\); and \(iv)\) for \(L > \pi^*_e/\beta\) then \(I = L\) and \(q_e = 0\).

**Offer:** \(\{L, F\}\). Impose \(p_w = 0\) and solve the supplier’s problem by constructing a fixed fee that makes the retailer just incentive compatible. The ICC holds when \(\pi^3_e \leq \pi^q_e\) or equivalently, when \(\beta I \leq (M - Nq_i - q_e) q_e + \beta (I - q_e) - F\). Hence, the supplier maximizes profits by setting

\[
F = (M - Nq_i - q_e) q_e - \beta q_e \tag{15}
\]

The fee is designed so that \(\pi^3_e = \beta I\). This implies \(i)\) that the retailer maximizes profits with \(I = L\) and \(ii)\) that she is indifferent between \(q_e = I\) and \(q_e = 0\). Hence, \(q_e = I = L\). By Proposition 3, the contract is optimal obtained for \(L = q^S_i\)

**Proposition 5**

**Diversion and Investment and Borrowing:** As in Proposition 4

**Offer:** \(\{L, (F, p_w)\}\). With the reasoning used in (15), note that now

\[
F = (M - Nq_i - q_e) q_e - \beta q_e - p_w I
\]
If the retailer is not constrained, the outcome is that in (10). Hence, the supplier solves

$$\max_{p_w} \pi_u = F(q_e^S, q_i^S) + p_w q_e^S - c_u q_e^S$$

which leads to

$$p_w = \frac{(\beta + c_u)(N + 1)(N + 2) - N(M - cN)}{2(N + 1)}$$

This $p_w$ satisfies $q_e^S = q_i^S$ and $q_i^S = q_i^S$. Moreover, $F$ is such that $\pi_e^S = \beta I$ and $\pi_u = \pi_u^S$. 

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Figures and Tables

Figure 1: The retailer’s reaction may have up to three different regions that lead to distinct type of equilibria. Here \( I \) is progressively increased from left to right. See appendix for the technical details of the figure.

Figure 2: Output and profits of the retailer under moral hazard (solid) versus perfect contractibility (dashed).
Figure 3: Supplier profits and input price as a function of competitive pressure

Figure 4: \( \{F, p_w\} \) offered by the supplier as a function of the competitive pressure in entrepreneur’s market. \( M = 10; \beta = 0.8; c = 2; c_u = 2.5 \)

Figure 5: \( M = 15; \beta = .8; p_w = 2; c = 2; N = 1 \). From left to right: \( I = 1.5; I = 6; I = 8 \).
References


