Transmission of Monetary Policy with Heterogeneity in Household Portfolios*

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Abstract

This paper assesses the importance of heterogeneity in household portfolios for the transmission of monetary policy in a New Keynesian business cycle model with incomplete markets and portfolio choice under liquidity constraints. In this model, the consumption response to changes in interest rates depends on the joint distribution of labor income, liquid and illiquid assets. The presence of liquidity-constrained households weakens the direct effect of changes in the real interest rate on consumption, but at the same time makes consumption more responsive to equilibrium changes in labor income. The redistributive consequences, including debt deflation, amplify the consumption response, whereas they dampen the investment response. Market incompleteness has important implications for the conduct of monetary policy as it relies to a larger extent on indirect equilibrium effects in comparison to economies with a representative household.

Keywords: Monetary Policy, Heterogeneous Agents, General Equilibrium

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1 Introduction

A household’s portfolio generally consists of non-tradable and tradable assets. The most important non-tradable asset is human capital. It is the primary source of income for most households and at the same time subject to substantial idiosyncratic shocks. The presence of such shocks gives rise to both precautionary savings and cross-sectional differences in holdings of tradable assets when markets are incomplete. Importantly, tradable assets vary in their degree of liquidity. In fact, a large fraction of households in the United States holds low levels of liquid assets relative to their income, although most households exhibit considerable positive net worth. This has important implications for the transmission of monetary policy, because consumption of low-liquidity households is less sensitive to interest rates but responds more strongly to current income. While the monetary authority has direct control over the interest rate, changes in income only arise indirectly out of second-round general equilibrium effects.

This paper assesses quantitatively the implications of heterogeneity in household portfolios for the transmission of monetary policy and the relative importance of the direct and indirect transmission channels. Toward this end, I build a New Keynesian dynamic stochastic general equilibrium (DSGE) model with asset-market incompleteness, idiosyncratic income risk, and sticky prices. The novel feature of the model is to allow for portfolio choice between liquid and illiquid assets in a business-cycle framework. The illiquid asset is real capital. It can only be traded with a certain probability each period but pays a higher return than the liquid asset, which comprises nominal government and household debt and can be traded without frictions. These characteristics make sure that the model endogenously generates the distribution of portfolio shares and marginal propensities to consume across households as documented for the United States. In particular, illiquidity of the real asset leads to “wealthy hand-to-mouth” households that explain the high aggregate propensity to consume out of current income (cf. Kaplan and Violante, 2014).

1See Kaplan et al. (2014) for a documentation of this fact for the U.S. and other industrialized countries.
2See the empirical literature on the consumption response to transfers; e.g. Johnson et al. (2006), Parker et al. (2013), or Misra and Surico (2014).
My main finding is that, when markets are incomplete, the direct response to changes in the interest rate makes up only 35% of the consumption response to monetary shocks, while indirect effects account for the remaining 65%. Indirect effects mainly work through equilibrium changes in labor income (about 85%), which represents the most important income source for the majority of households. The revaluation of nominal claims, including debt deflation, adds another indirect channel of monetary policy, which roughly accounts for 15% of the total response.

The importance of indirect effects contrasts sharply with the standard New Keynesian model that builds on a representative household. In the latter, the direct effect explains close to all of the consumption response. The indirect effect is quantitatively unimportant, because it works exclusively through changes in life-time income that monetary shocks hardly affect. With complete markets, savings adjust to undo any temporary mismatch between income and consumption. In an economy with incomplete markets, by contrast, borrowing constraints and precautionary motives are important. They make savings and, thus, consumption less sensitive to interest rate changes. This renders the direct effect of monetary policy less potent. At the same time, indirect effects become stronger because current income is a binding constraint for households at or close to the borrowing constraint. What is more, revaluation of nominal assets impacts on the tightness of borrowing constraints as inflation changes the real value of debt. This amplifies the effect of borrowing constraints through Fisher (1933) debt deflation.

The indirect effect through changes in income is, therefore, the key determinant of the consumption response to monetary shocks in an economy with incomplete markets. This reversal of the importance of direct and indirect effects explains how monetary policy may have sizable effects on aggregate consumption while interest rate elasticities at the household level are low.3

While the consumption response to monetary shocks works through different channels and is also stronger in total, a monetary shock moves output to a similar extent in the representative and heterogeneous agent version of the model. Investment falls by 25% less in the incomplete-markets setting and, thus, cancels out the stronger consumption response. The reason for this smaller reaction of investment

3See for example the handbook chapter on monetary policy by Christiano et al. (1999) for evidence on the aggregate consequences of monetary policy shocks.
relates to the fact that monetary policy has non-trivial redistributive consequences that interact with heterogeneity in household portfolios. In line with the empirical findings by Coibion et al. (2012), a monetary tightening increases inequality and makes households at the top of the wealth distribution, who primarily hold real assets, richer. Thereby they stabilize investment demand after a contractionary monetary policy shock.

With these results, my paper contributes to the recently evolving literature that incorporates market incompleteness and idiosyncratic uncertainty into New Keynesian models. As such it builds on the New Keynesian literature with its focus on nominal rigidities. This literature has proven successful in replicating the impulse responses to monetary policy shocks as identified from time-series data (cf. Christiano et al., 2005). What my paper and other recent contributions add to this literature is the attempt to endogenize heterogeneity in wealth. In this class of models, the response of consumption and savings depends on the distribution of wealth, which evolves in response to aggregate shocks.

Relative to this literature, my paper is the first to analyze monetary policy in a business cycle framework with portfolio choice. My work is most closely related to Kaplan et al. (2015), which originated in parallel. They also decompose the effects of monetary policy into direct and indirect effects but focus on the consumption response to a one-time unexpected monetary shock. My model, in contrast, is calibrated to match business cycle statistics and, thus, adds to their analysis by considering the response of consumption, investment, and output in unison.

This paper also contributes to the assessment of debt deflation as transmission mechanism of monetary policy. My analysis shows that redistribution through inflation is of secondary importance relative to the effect of monetary policy on aggregate income in models with sticky prices.

The remainder of the paper is organized as follows. Section 2 presents the model, and Section 3 discusses the solution method. Section 4 explains the calibration of the model. Section 5 presents the quantitative results. Section 6 concludes.

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5Exogenous heterogeneity is well-established in New Keynesian models. See for example Iacoviello (2005) and Gali et al. (2007).

6See Doepke et al. (2015) for an analysis of this channel in a flexible-price model.
2 Model

The model economy consists of households, firms, and a government/monetary authority. Households consume, supply labor, obtain profit income, accumulate physical capital, and trade in the bonds market. Firms combine capital and labor services to produce goods. The government issues bonds, raises taxes, and purchases goods, while the monetary authority sets the nominal interest rate. Let me describe each agent in turn.

Households face idiosyncratic income risk, but insurance markets are incomplete. They self-insure by trading nominal bonds and illiquid physical capital. I model this illiquidity as infrequent participation in the capital market. Every period a fraction of households is randomly selected to trade physical capital. Households are either workers or entrepreneurs with a certain probability. Worker-households supply labor on a perfectly competitive market and are subject to idiosyncratic shocks to their labor productivity. Entrepreneur-households do not supply labor, but instead receive an equal share of economy-wide profits.

There are three types of firms. Perfectly competitive intermediate-goods producers hire capital and labor from households and sell the homogeneous intermediate good at marginal costs. Monopolistically competitive resellers then differentiate the intermediate good and set prices above marginal costs. They may, however, only adjust their prices with some positive probability each period as in Calvo (1983). As a result, demand determines output in the short-run, because a fraction of firms has to satisfy demand at given prices. The differentiated goods

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7This setup builds on previous joint work with Christian Bayer, Lien Pham-Dao, and Volker Tjaden. Where there is overlay the exposition closely follows Bayer et al. (2015). We choose to exclude trading as a choice and hence use a simplified framework relative to Kaplan et al. (2015) for numerical tractability. Random participation keeps the households’ value function concave, thus makes first-order conditions sufficient, and therefore allows us to use a variant of the endogenous grid method as algorithm for our numerical calculations. See Bayer et al. (2015) for proofs.

8According to the Congressional Budget Office, the top 1% of the income distribution receives about 30% of their income from financial income, a much larger share than any other segment of the population.

9For reasons of tractability, I abstract from tradable shares in monopoly profits and instead introduce an exogenous employment state that receives all profits.

10Fixed types of workers and entrepreneurs (or capitalists) without stochastic transitions can be found in Walsh (2014) or Broer et al. (2015), while Romei (2014) uses stochastic transitions.
are finally bundled again by perfectly competitive final-good producers to final goods used for consumption and investment.

Monetary policy follows a Taylor (1993)-type rule that determines the nominal interest rate at which the government and households may borrow and lend to each other. It thereby affects the real rate of interest because of sticky prices. The balance sheet of the central bank is not modeled explicitly. The government collects proportional income taxes to finance its interest expenses and government purchases. The latter follow a simple rule to stabilize debt.

The model economy is subject to aggregate shocks as in Krusell and Smith (1998). The shocks affect total factor productivity of intermediate-goods production and the Taylor-rule. I next describe the model in more detail.

2.1 Households

There is a continuum of ex-ante identical households of measure one indexed by \( i \in [0, 1] \). Households are infinitely lived, have time-separable preferences with time-discount factor \( \beta \), and derive felicity from consumption \( c_{it} \) and leisure. Households can be entrepreneurs \( (s_{it} = 0) \) or workers \( (s_{it} = 1) \). Transition between both types is exogenous and stochastic, but the fraction of households that are entrepreneurs at any given time \( t = 0, 1, 2, ... \) is constant.

Workers supply labor. Their labor income \( w_t h_{it} n_{it} \) is composed of the wage rate, \( w_t \), hours worked, \( n_{it} \), and idiosyncratic labor productivity, \( h_{it} \), which evolves according to the following first-order autoregressive process:

\[
\log h_{it} = \rho_h \log h_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_h).
\] (1)

Entrepreneurs have zero productivity on the labor market, but instead receive an equal share of the economy’s total profits \( \Pi_t \). They pay the same tax rate as workers, \( 1 - \tau \).

Households have GHH preferences (cf. Greenwood et al., 1988) and maximize the discounted sum of felicity:

\[
V = E_0 \max_{\{c_{it}, n_{it}\}} \sum_{t=0}^{\infty} \beta^{t} u (x_{it}),
\] (2)
where \( x_{it} = c_{it} - h_{it} G(n_{it}) \) is household \( i \)'s composite demand for the physical consumption good \( c_{it} \) and leisure.

The disutility of work, \( h_{it} G(n_{it}) \), determines a workers’ labor supply given the aggregate wage rate through the first-order condition:

\[
h_{it} G'(n_{it}) = \tau w_t h_{it}. \tag{3}
\]

Under the above assumption, a workers’ labor decision does not respond to idiosyncratic productivity \( h_{it} \), but only to the net aggregate wage \( \tau w_t \). Thus I can drop the household-specific index \( i \), and set \( n_{it} = N_t \).

The Frisch elasticity of aggregate labor supply is constant with \( \gamma \) being the inverse elasticity:

\[
G(N_t) = \frac{1}{1 + \gamma} N_t^{1 + \gamma}, \quad \gamma > 0.
\]

Exploiting the first-order condition on labor supply, the disutility of working can be expressed in terms of the net wage rate:

\[
h_{it} G(N_t) = h_{it} \frac{N_t^{1 + \gamma}}{1 + \gamma} = \frac{h_{it} G'(N_t) N_t}{1 + \gamma} = \frac{\tau w_t h_{it} N_t}{1 + \gamma}.
\]

In this way the demand for \( x_{it} \) can be rewritten as:

\[
x_{it} = c_{it} - h_{it} G(N_t) = c_{it} - \frac{\tau w_t h_{it} N_t}{1 + \gamma}.
\]

Total labor input supplied is given by:

\[
\tilde{N}_t = N_t \int s_{it} h_{it} di.
\]

Asset markets are incomplete. Households may only self-insure in nominal bonds, \( \tilde{b}_{it} \), and in capital, \( k_{it} \). Holdings of capital have to be non-negative, but households may issue nominal bonds up to an exogeneously specified limit \(-b \in (-\infty, 0]\). Moreover, trading capital is subject to a friction.

This trading friction only allows a randomly selected fraction of households, \( \nu \), to participate in the market for capital each period. All other households obtain dividends, but may only adjust their holdings of nominal bonds. For those
households participating in the capital market, the budget constraint reads\footnote{The household problem can be expressed in terms of composite good $x_{it}$ by making use of $c_{it} = x_{it} + \frac{\tau w_i h_{it} N_t}{1 + \gamma}$.}:

$$
c_{it} + b_{it+1} + q_t k_{it+1} = \frac{R_{t-1}^B}{\pi_t} b_{it} + (q_t + r_t) k_{it} + \tau \left[ s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right],
$$

$$
k_{it+1} \geq 0, b_{it+1} \geq -b
$$

where $b_{it}$ is the real value of nominal bond holdings, $k_{it}$ are capital holdings, $q_t$ is the price of capital, $r_t$ is the rental rate or “dividend”, $R_{t-1}^B$ is the gross nominal return on bonds, and $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate. I denote real bond holdings of household $i$ at the end of period $t$ by $b_{it+1} := \frac{b_{it+1}}{P_t}$.

For those households that cannot trade in the market for capital the budget constraint simplifies to:

$$
c_{it} + b_{it+1} = \frac{R_{t-1}^B}{\pi_t} b_{it} + r_t k_{it} + \tau \left[ s_{it} w_t h_{it} N_t + (1 - s_{it}) \Pi_t \right],
$$

$$
b_{it+1} \geq -b.
\tag{5}
$$

Note that I assume that the depreciation of capital is replaced through maintenance such that the dividend, $r_t$, is the net return on capital.

A household’s optimal consumption-savings decision is a non-linear function of that household’s asset portfolio \{\(b_{it}, k_{it}\)\} and employment type \{\(s_{it}, h_{it}\)\}. Accordingly, the price level $P_t$ and aggregate real bonds $B_{t+1} = \frac{B_{t+1}}{P_t}$ are functions of the joint distribution $\Theta_t$ over idiosyncratic states (\(b_t, k_t, h_t, s_t\)). This makes the distribution $\Theta_t$ a state variable of the households’ planning problem. The distribution $\Theta_t$ fluctuates in response to aggregate monetary and total factor productivity shocks. Let $\Omega$ stand in for aggregate shocks.

With this setup, two Bellman equations characterize the dynamic planning problem of a household; $V_a$ in case the household can adjust its capital holdings.
and $V_n$ otherwise:

$$V_a(b, k, h, s; \Theta, \Omega) = \max_{k', b'_a} u[c(b, b'_a, k, k', h, s)] + \beta [\nu EV^n(b'_a, k', h', s', \Theta', \Omega')$$

$$+ (1 - \nu) EV^n(b'_a, k', h', s', \Theta', \Omega')]$$

$$V_n(b, k, h, s; \Theta, \Omega) = \max_{b'_n} u[c(b, b'_n, k, k, h, s)] + \beta [\nu EV^n(b'_a, k, h', s', \Theta', \Omega')$$

$$+ (1 - \nu) EV^n(b'_a, k, h', s', \Theta', \Omega')] \quad (6)$$

In line with this notation, I define the optimal consumption policies for the adjustment and non-adjustment cases as $c^*_a$ and $c^*_n$, the nominal bond holding policies as $b^*_a$ and $b^*_n$, and the capital investment policy as $k^*$. See Appendix A for the first order conditions.

### 2.2 Intermediate Good Producer

Intermediate goods are produced with a constant returns to scale production function:

$$Y_t = Z_t \tilde{N}_t^\alpha K_t^{(1-\alpha)},$$

where $Z_t$ is total factor productivity (TFP). It follows a first-order autoregressive process:

$$\log Z_t = \rho \log Z_{t-1} + \epsilon^Z_t, \quad \epsilon^Z_t \sim N(0, \sigma_Z). \quad (7)$$

Let $MC_t$ be the relative price at which the intermediate good is sold to resellers. The intermediate-good producer maximizes profits,

$$MC_t Y_t = MC_t Z_t \tilde{N}_t^\alpha K_t^{(1-\alpha)} - w_t \tilde{N}_t - (r_t + \delta) K_t,$$

but it operates in perfectly competitive markets, such that the real wage and the user costs of capital are given by the marginal products of labor and capital:

$$w_t = \alpha MC_t Z_t \left( K_t / \tilde{N}_t \right)^{1-\alpha}, \quad (8)$$

$$r_t + \delta = (1 - \alpha) MC_t Z_t \left( \tilde{N}_t / K_t \right)^\alpha. \quad (9)$$
2.3 Resellers

Resellers differentiate the intermediate good and set prices. They are risk neutral and have the same discount factor as households. For tractability reasons, I assume that resellers obtain an arbitrarily small share of profits and do neither participate in the bond nor capital market. This assumption separates the resellers’ price setting problem from the households’ saving problem.

By setting prices of final goods, resellers maximize expected discounted future profits:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \Pi_{jt}. \tag{10}
\]

Resellers buy the intermediate good at a price equaling the nominal marginal costs, \(MC_t P_t\), where \(MC_t\) are the real marginal costs at which the intermediate good is traded due to perfect competition, and then differentiate them without the need of additional input factors. The goods that resellers produce come in varieties uniformly distributed on the unit interval and each indexed by \(j \in [0, 1]\). Resellers are monopolistic competitors, and hence charge a markup over their marginal costs. They are, however, subject to a Calvo (1983) price setting friction, and can only update their prices with probability \(\theta\). They maximize the expected value of future discounted profits by setting today’s price, \(p_{jt}\), taking into account the price setting friction:

\[
\max_{\{p_{jt}\}} \sum_{s=0}^{\infty} (\theta \beta)^s E \Pi_{jt,t+s} = \sum_{s=0}^{\infty} (\theta \beta)^s E y_{jt,t+s} (p_{jt} - MC_{t+s} P_{t+s}) \tag{11}
\]

\[
s.t.: y_{jt,t+s} = \left( \frac{p_{jt}}{P_{t+s}} \right)^{-\eta} y_{t+s},
\]

where \(\Pi_{jt,t+s}\) are profits and \(y_{jt,t+s}\) is the production level in \(t + s\) of a firm \(j\) that set prices in \(t\).

I obtain the following first-order condition with respect to \(p_{jt}\):

\[
\sum_{s=0}^{\infty} (\theta \beta)^s E y_{jt,t+s} \left( \frac{p^*_t}{P_{t-1}} - \frac{\eta}{\mu} MC_{t+s} \frac{P_{t+s}}{P_{t-1}} \right) = 0, \tag{12}
\]
where $\mu$ is the static optimal markup.

Recall that resellers are risk neutral, and that they do not interact with households in any intertemporal trades. Therefore, I can solve the resellers’ planning problem locally by log-linearizing around the zero-inflation steady state, without having to know the solution of the households’ problem. This yields the New Keynesian Phillips curve, see e.g. Galí (2008):

$$\log \pi_t = \beta E_t (\log \pi_{t+1}) + \kappa (\log MC_t + \mu),$$

where

$$\kappa = \frac{(1 - \theta)(1 - \beta \theta)}{\theta}.$$

Besides differentiating intermediate goods, I assume that resellers also obtain rents from adjusting the aggregate capital stock. The cost of adjusting the stock of capital is $\frac{\phi}{2} \left( \frac{\Delta K_{t+1}}{K_t} \right)^2 K_t$. Hence, resellers will adjust the stock of capital until the following first-order condition holds:

$$q_t = 1 + \phi \frac{\Delta K_{t+1}}{K_t}. \quad (14)$$

### 2.4 Final Good Producer

Perfectly competitive final good producers use differentiated goods as input taking input and sell price as given. Final goods are used for consumption and investment. The problem of the representative final good producer is as follows:

$$\max_{Y_t, y_{jt} \in [0, 1]} \quad P_t Y_t - \int_0^1 p_{jt} y_{jt} dj$$

s.t.

$$Y_t = \left( \int_0^1 y_{jt}^{\frac{n-1}{\eta}} dj \right)^{\frac{\eta}{n-\eta}},$$

where $y_{jt}$ is the demanded quantity of differentiated good $j$ as input. From the zero-profit condition, the price of the final good is given by

$$P_t = \left( \int_0^1 p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}.$$
2.5 Central Bank and Government

Monetary policy sets the gross nominal interest rate, $R^B_t$, according to a Taylor (1993)-type rule that reacts to inflation deviations from target and exhibits interest rate smoothing:

$$\frac{R^B_t}{R^B_{t-1}} = \left( \frac{R^B_{t-1}}{R^B_{t-1}} \right)^{\rho_R} \left( \frac{1 + \pi_t}{1 + \pi} \right)^{\theta_\pi} \epsilon^D_t, \tag{16}$$

where $\log \epsilon^D_t \sim N(0, \sigma_D)$ are monetary policy shocks. All else equal, the central bank raises the nominal rate above its steady-state value $R^B$ whenever inflation exceeds its target value. It does so by more than one-to-one to guarantee a non-explosive price path ($\theta_\pi > 1$). The parameter $\rho_R$ captures “intrinsic policy inertia”, a feature supported by empirical evidence, see Nakamura and Steinsson (2013).

The fiscal authority decides on government purchases, $G_t$, raises tax revenues, $T_t$, and issues nominal bonds. Let $B_{t+1}$ denote their time $t$ real value. The government budget constraint reads:

$$B_{t+1} = \frac{R^B_{t+1}}{1 + \pi_t} B_t + G_t - T_t, \tag{17}$$

where real tax revenues are given by:

$$T_t = (1 - \tau) \left[ (N_t W_t \int s_i h_i \Theta_t(b, k, h, s)) + \Pi_t \right]. \tag{18}$$

I assume that government purchases stabilize the debt level,

$$G_t = \gamma_1 - \gamma_2 (B_t - B), \tag{19}$$

with $B$ equal to steady state debt, while the tax parameter $\tau$ remains constant. Adjustment via government purchases is the baseline formulation because changing taxes would directly redistribute across households. This also applies to lump-sum taxes in this environment. Government purchases, in contrast, do not have any direct distributional consequences.
2.6 Bonds, Capital, Goods, and Labor Market Clearing

The labor market clears at the competitive wage given in (8); so does the market for capital services if (9) holds. The nominal bonds market clears, whenever the following equation holds:

\[ B_{t+1} = \int \left[ \nu b^*_a(b, k, h, s; q, \pi) + (1 - \nu) b^*_n(b, k, h, s; q, \pi) \right] \Theta_t(b, k, h, s) db dk dh ds. \]  

(20)

Last, the market for capital has to clear:

\[ q_t = 1 + \phi K_{t+1} - K_t = 1 + \nu \phi \frac{K^*_t - K_t}{K_t} \]  

(21)

\[ K^*_t := \int k^*(b, k, h, s; q_t, \pi_t) \Theta_t(b, k, h, s) db dk dh ds \]

\[ K_{t+1} = K_t + \nu (K^*_t - K_t), \]

where the first equation stems from competition in the production of capital goods, the second equation defines the aggregate supply of funds from households trading capital, and the third equation defines the law of motion of aggregate capital. The goods market then clears due to Walras’ law, whenever both, bonds and capital markets, clear.

2.7 Recursive Equilibrium

A recursive equilibrium in this model is a set of policy functions \( \{c^*_a, c^*_n, b^*_a, b^*_n, k^*\} \), value functions \( \{V_a, V_n\} \), pricing functions \( \{r, R_B, w, \pi, q\} \), aggregate bonds, capital, and labor supply functions \( \{B, K, N\} \), distributions \( \Theta_t \) over individual asset holdings, types, and productivity, and a perceived law of motion \( \Gamma \), such that

1. Given \( V_a, V_n, \Gamma, \) prices, and distributions, the policy functions \( \{c^*_a, c^*_n, b^*_a, b^*_n, k^*\} \) solve the households’ planning problem, and given prices, distributions, and the policy functions \( \{c^*_a, c^*_n, b^*_a, b^*_n, k^*\} \), the value functions \( \{V_a, V_n\} \) are a solution to the Bellman equations (6).

2. The labor, the final-goods, the bonds, the capital, and the intermediate-good markets clear, i.e. (8), (13), (20), and (21) hold.
3. The actual law of motion and the perceived law of motion $\Gamma$ coincide, i.e. $\Theta' = \Gamma(\Theta, \Omega')$.

3 Numerical Implementation

The dynamic program (6) and hence the recursive equilibrium is not computable, because it involves the infinite dimensional object $\Theta_t$.

3.1 Krusell-Smith Equilibrium

To turn this problem into a computable one, I assume that households predict future prices only on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998). Specifically, I make the assumption that households condition their expectations on last period’s aggregate real bond holdings, $B_t$, last period’s nominal interest rate, $R_{t-1}^B$, and the aggregate stock of capital, $K_t$. If asset-demand functions, $b^*_{n,n}$ and $k^*$, are sufficiently close to linear in human capital, $h$, types, $s$, and in non-human wealth, $b$ and $k$, at the mass of $\Theta_t$, I can expect approximate aggregation to hold. For my exercise, the three aggregate states – $B_t$, $R_{t-1}^B$, $K_t$ – are sufficient to describe the evolution of the aggregate economy conditional on the aggregate shocks $\Omega$.

While the law of motion for $Z_t$ is pinned down by (7), households use the following log-linear forecasting rules for current inflation and the price of capital:

$$
\log \pi_t = \beta_1^\pi(\Omega) + \beta_2^\pi(\Omega) \hat{B}_t + \beta_3^\pi(\Omega) \hat{K}_t + \beta_4^\pi(\Omega) \hat{R}_{t-1}^B, \tag{22}
$$

$$
\log q_t = \beta_1^q(\Omega) + \beta_2^q(\Omega) \hat{B}_t + \beta_3^q(\Omega) \hat{K}_t + \beta_4^q(\Omega) \hat{R}_{t-1}^B, \tag{23}
$$

where $\hat{()}$ refers to log-differences from the steady state value of each variable and $\Omega$ indicates the dependence on aggregate shocks. The law of motion for aggregate real bonds, $B_t$, then follows from the government budget constraint (17). The Taylor-rule (16) determines the motion of the nominal interest rate, $R_t^B$. The law of motion for $K_t$ results from (21).

Technically, finding the equilibrium is similar to Krusell and Smith (1997), as I need to find market clearing prices within each period. Concretely, this means
the posited rules, (22) and (23), are used to solve for households’ policy functions. Having solved for the policy functions conditional on the forecasting rules, I then simulate \( n \) independent sequences of economies for \( t = \{1, \ldots, T\} \) periods, keeping track of the actual distribution \( \Theta_t \). In each simulation the sequence of distributions starts from the stationary distribution implied by the model without TFP and monetary policy shocks. I then calculate in each period \( t \) the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rule (22) and (23) from period \( t + 1 \) onward. Having determined the market clearing prices, I obtain the next period’s distribution \( \Theta_{t+1} \). In doing so, I obtain \( n \) sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. I next re-estimate the parameters of (22) and (23) from the simulated data and update the parameters accordingly. By using \( n = 20 \) and \( T = 1250 \), it is possible to make use of parallel computing resources and obtain 20,000 equilibrium observations. Subsequently, I recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules (22) and (23) approximate the aggregate behavior of the economy well. The minimal within sample \( R^2 \) is above 99.9%. Out-of-sample performance as defined by Den Haan (2010) is also good. See Appendix B.

### 3.2 Solving the household planning problem

In solving for the households’ policy functions I apply an endogenous grid point method as originally developed in Carroll (2006) and extended by Hintermaier and Koeniger (2010), iterating over the first-order conditions. I approximate the idiosyncratic productivity/employment state process by a discrete Markov chain with 4 states, using the method proposed by Tauchen (1986). \(^{12} \)

\(^{12} \)I solve the household policies for 40 points on the grid for bonds and 40 points on the grid for capital. For aggregate bonds, aggregate capital, and last period’s nominal interest rate I use a grid of 3 points each, while for TFP I use 7 points and 3 for the \( \text{iid} \) monetary shock. \(^{13} \)

\(^{13} \)Details on the algorithm can be found in Bayer et al. (2015).
4 Calibration

I calibrate the model to the U.S. economy over the time period 1984Q1 to 2008Q3 as my focus lies on conventional monetary policy. One period in the model is a quarter. Table 1 summarizes the calibration. In detail, I choose the parameter values as follows.

4.1 Households

I assume that the felicity function is of constant-relative-risk-aversion form: \[ u(x) = \frac{1}{1-\xi} x^{1-\xi} \], where \( \xi = 2 \), a standard value. The time-discount factor, \( \beta = 0.985 \), and the capital market participation frequency, \( \nu = 0.075 \), are jointly calibrated to match the ratio of capital and government bonds to output. I equate capital to all capital goods relative to nominal GDP. The annual capital-to-output ratio is therefore 290%. This implies an annual real return on capital of about 4%. I equate government bonds to the outstanding government debt held by private domestic agents, which implies an annual bonds-to-output ratio of 31%.

I set the borrowing limit in bonds, \( b \), such that 20% of households have negative net worth as in the Survey of Consumer Finances (2007). This implies a relatively tight borrowing limit that equals the average quarterly income.

I calibrate the stochastic process for the employment state to capture the distribution of wealth in the U.S. economy. In particular, I determine the share of entrepreneur-households to match a Gini coefficient of 0.82. For simplicity, I assume that the probability of becoming an entrepreneur is the same for workers independent of their labor productivity and that, once they become a worker again, they draw their labor productivity from a uniform distribution. I set the quarterly standard deviation of persistent shocks to idiosyncratic labor productivity to 0.08. The quarterly autocorrelation is 0.987 – a standard value in the literature. This implies for the baseline calibration that on average 1% of households are entrepreneurs.

---

14The participation frequency of 7.5% is higher than in the optimal participation framework of Kaplan and Violante (2014). They find a participation frequency of 4.5% for working households given a fixed-adjustment cost of $500.
### Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.985</td>
<td>Discount factor</td>
<td>$K/Y = 290%$ (annual)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>7.5%</td>
<td>Participation frequency</td>
<td>$B/Y = 23%$ (annual)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2</td>
<td>Coefficient of rel. risk av.</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
<td>Inv. Frisch elasticity</td>
<td>Standard value</td>
</tr>
<tr>
<td><strong>Intermediate Goods</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>72%</td>
<td>Share of labor</td>
<td>Income share of labor of 66%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.35%</td>
<td>Depreciation rate</td>
<td>NIPA: Fixed assets &amp; durables</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>0.9</td>
<td>Persistence of TFP shock</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>0.05</td>
<td>STD of TFP shock</td>
<td>STD($Y$)=1</td>
</tr>
<tr>
<td><strong>Final Goods</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.08</td>
<td>Price stickiness</td>
<td>Avg. price duration of 4 quarters</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.06</td>
<td>Markup</td>
<td>6% markup (standard value)</td>
</tr>
<tr>
<td><strong>Capital Goods</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>10</td>
<td>Capital adjustment costs</td>
<td>STD($I$)/STD($Y$)=3.5</td>
</tr>
<tr>
<td><strong>Fiscal Policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 - \tau$</td>
<td>0.3</td>
<td>Tax rate</td>
<td>Budget balance</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.05</td>
<td>$G$ in steady state</td>
<td>$G/Y = 20%$</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.1</td>
<td>$G$ reaction function</td>
<td>Small value</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi$</td>
<td>1.005</td>
<td>Inflation</td>
<td>2% p.a.</td>
</tr>
<tr>
<td>$R^B$</td>
<td>1.0136</td>
<td>Nominal interest rate</td>
<td>5.5% p.a.</td>
</tr>
<tr>
<td>$\theta_\pi$</td>
<td>1.5</td>
<td>Reaction to inflation</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\rho_{RB}$</td>
<td>0.95</td>
<td>Interest rate smoothing</td>
<td>Nakamura and Steinsson (2013)</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>18e-3</td>
<td>STD of monetary shock</td>
<td>Christiano et al. (1999)</td>
</tr>
<tr>
<td><strong>Income Process</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>0.987</td>
<td>Persistence of productivity</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>0.08</td>
<td>STD of innovations</td>
<td>Standard value</td>
</tr>
</tbody>
</table>
4.2 Intermediate, Final, and Capital Goods Producers

The labor and capital share including profits (2/3 and 1/3) align with long-run U.S. averages. The persistence of the TFP shock is set to $\rho_Z = 0.9$. The standard deviation of the TFP shock, $\sigma_Z = 0.005$, is calibrated to make the model match the standard deviation of H-P-filtered U.S. output.

To calibrate the parameters of the resellers’ problem, I use standard values for markup and price stickiness that are widely employed in the New Keynesian literature (c.f. Christiano et al., 1999). The Phillips curve parameter $\kappa$ implies an average price duration of 4 quarters, assuming flexible capital at the firm level. The steady state marginal costs, $exp(-\mu) = 0.95$, imply a markup of 6%. I calibrate the adjustment cost of capital, $\phi = 10$, to match the relative investment volatility in the United States.

4.3 Central Bank and Government

I target an average annual inflation rate of 2% according to the Federal Reserve System’s inflation objective. I set the real return on bonds to 3.5% in line with the average federal funds rate in the U.S in real terms from 1984 to 2008.\textsuperscript{15} Hence, the nominal return is $R^B = 1.0136$ quarterly. Nakamura and Steinsson (2013) provide an estimate for the parameter governing interest rate smoothing, $\rho_{R^B} = 0.95$, while the central bank’s reaction to inflation deviations from target is standard, $\theta_\pi = 1.5$. The standard deviation of the monetary policy shock, $\sigma_D$, is 71 basis points annually (c.f. Christiano et al. 1999).

The government levies a proportional tax on labor income and profits to finance government purchases and interest expense on debt. I adjust $1 - \tau = 0.3$ to close the budget constraint given the interest expense and a government-spending-to-GDP ratio of 20% in steady state. Government purchases, in turn, react to debt deviations from steady state such that the debt level remains bounded. Specifically, I choose $\gamma_2 = 0.1$, which ensures that the reaction of government spending builds up very slowly and, thus, interference with the aggregate consequences of monetary shocks is minimized.

\textsuperscript{15}I obtain real returns by subtracting the GDP deflator from the Effective Federal Funds Rate. Both time series are retrieved from the FRED database, Federal Reserve Bank of St. Louis.
4.4 Model Fit

Table 2 reports the business cycle statistics implied by the model. The volatility of output and investment are calibrated to U.S. data, while the remaining statistics and variables are not targeted. The most striking fact is the low volatility of government spending. This is the result of the passive nature of government spending in the model as it only moderately reacts to stabilize debt and does not feature any shocks. The volatility of government spending is deliberately low to keep interference with monetary shocks minimal.

<table>
<thead>
<tr>
<th></th>
<th>Model STD</th>
<th>Model CORR</th>
<th>Model AC(1)</th>
<th>Data STD</th>
<th>Data CORR</th>
<th>Data AC(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.02</td>
<td>1.00</td>
<td>0.74</td>
<td>0.97</td>
<td>1.00</td>
<td>0.72</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.77</td>
<td>0.99</td>
<td>0.74</td>
<td>0.85</td>
<td>0.88</td>
<td>0.68</td>
</tr>
<tr>
<td>Investment</td>
<td>3.50</td>
<td>0.94</td>
<td>0.71</td>
<td>4.42</td>
<td>0.87</td>
<td>0.79</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>0.43</td>
<td>0.15</td>
<td>0.96</td>
<td>1.22</td>
<td>-0.08</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Notes: Standard deviation, correlation with GDP, and autocorrelation after log-HP(1600)-filtering. Standard deviation is multiplied by 100.

Figure 1 (a) shows the liquidity of portfolios across the wealth distribution. I measure this as the total amount of liquid assets a household might withdraw (up to the borrowing constraint) relative to total wealth. The poorest households hold almost all their wealth in liquid assets. As households become richer the share of liquid assets in their portfolios falls. As the wealth-to-income ratio increases, the optimal share of liquid assets falls because the marginal value of liquidity declines for given income and households rather invest in the high-return illiquid asset. The declining share of liquid assets in household portfolios generated by the model approximately matches U.S. data from the Survey of Consumer Finances (2007).

The model also performs well in matching U.S. wealth inequality. Figure 1 (b) compares the Lorenz curve of wealth implied by the model to U.S. data. The U.S. Gini coefficient of 0.82 is matched by construction, but the model also generates realistic shares in total wealth across all percentiles of the wealth distribution.
5 Results

This section discusses the transmission of monetary policy shocks to the aggregate economy and, in particular, the transmission channels. I first discuss the theoretical channels through which monetary policy affects aggregates in this model and then compare the aggregate effects in the economy with heterogeneity in household portfolios to the same economy with a representative household. I elaborate on heterogeneity in the savings response across the wealth distribution to highlight the importance of heterogeneity in household portfolios for aggregate outcomes. The section concludes with an assessment of redistribution through inflation as a potential transmission channel of monetary policy. 

(a): Maximum withdrawal of liquid assets relative to total wealth. U.S.: Liquid assets include all financial assets except for stocks (incl. mutual funds that primarily hold stocks) minus unsecured credit. Illiquid assets include all non-financial assets plus stocks minus secured credit. I assume the same borrowing limit as in the model ($20,000) and exclude all households with more unsecured credit. 
(b): Wealth Lorenz curve in the model (dashed line) against Lorenz curve of wealth defined as financial plus nonfinancial assets minus debt for the U.S. (solid line).
5.1 Transmission Channels of Monetary Policy

Key for understanding the transmission of monetary policy in any DSGE model is the household consumption-savings decision. The decision problem of households in an incomplete-markets setting differs from that of a representative household in that borrowing constraints apply. This gives rise to differences in optimal decisions as households take the existence of borrowing constraints into account or might actually be at the constraint. The response of consumption and savings to monetary shocks hence differs between an economy with and without complete markets. The effect of monetary policy on household decisions, in turn, can be split into direct and indirect effects along the lines of Table 3.

Table 3: Monetary policy transmission mechanism in the model

<table>
<thead>
<tr>
<th>Decision</th>
<th>Variable</th>
<th>Determined by</th>
<th>Relevant prices</th>
<th>Effect is</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sequence of Euler equations</td>
<td>${R^B_t/R_t-1/\pi_t}$</td>
<td></td>
<td>direct</td>
</tr>
<tr>
<td>intertemporal consumption-savings</td>
<td>${X_t}_{t=0…\infty}$</td>
<td>life-time budget</td>
<td>${w_t, r_t, \pi_t}$</td>
<td>indirect</td>
</tr>
<tr>
<td></td>
<td>borrowing constraints</td>
<td>${w_t, r_t, \pi_t, q_t}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>intratemporal labor-leisure</td>
<td>${N_t}$</td>
<td>marginal dis-utility of work</td>
<td>${w_t}$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table breaks the household problem down into inter- and intratemporal decisions. The gray shaded block represents the effects of monetary policy through general equilibrium changes in prices, i.e. the indirect effects. Borrowing constraints (in bold) only bind in the incomplete markets version of the model.

Consider a contractionary monetary policy shock. All else equal, an increase in the nominal interest rate increases the real return on nominal assets and, thus, the intertemporal relative price of composite consumption of leisure and goods,
$X_t$, today vs. tomorrow. I refer to the interest rate channel as the direct effect of monetary policy. Since prices are sticky, the decrease in consumption is not completely offset by lower prices, and output falls. Lower output, in turn, decreases income and consumption, which again reduces income and so forth. I refer to the equilibrium changes in income and prices as the indirect effects of monetary policy.

In the complete markets economy, these indirect effects matter for composite consumption only in so far as they change life-time income, because the consumption path is determined by a sequence of Euler equations and a single life-time budget constraint. The consumption of final goods, $C_t$, and labor supply, $N_t$, follows then through the intratemporal consumption-leisure trade-off that solely depends on the wage rate. With incomplete markets, however, current income becomes an important determinant of composite consumption and, thus, consumption of final goods because of borrowing constraints.

The indirect effects of monetary policy, therefore, work through the (life-time) budget constraint and the complementarity of consumption and hours worked inherent in GHH preferences in this model. This paper is about the effect of borrowing constraints on household decisions through the budget constraint channel. For this purpose, GHH preferences and the specific form of the disutility of labor adopted are helpful. They rule out wealth effects on labor supply and more generally make labor supply independent of all idiosyncratic states. As a result, labor supply only depends on the aggregate wage rate in both versions of the model and, thus, does not contribute to differences in the response to monetary shocks.

The complementarity of leisure and hours worked does matter for the total response, of course, as this is an important factor in the determination of consumption of final goods. It is hence the difference in the response of composite consumption between both economies that identifies the effect of borrowing constraints. The quantitative assessment of the differences between both economies comes next.

Recall that the household problem can be expressed in terms of composite consumption $X_t$ with GHH preferences: $x_{it} = c_{it} - \tau \frac{w_t h_t N_t}{1 + \gamma}$. It is therefore the intertemporal allocation of composite consumption that matters for the household in this model.

More leisure time decreases the marginal utility of consumption with GHH preferences such that, all else equal, consumption falls with labor supply $N_t$. 
5.2 Aggregate Effects of a Monetary Policy Shock

In the following, I consider the effect of a monetary surprise that, all else equal, would increase the nominal interest rate by one-standard deviation, i.e. 18 basis points (quarterly), in period 0. Figure 2 compares the responses of the economy with and without heterogeneity in household portfolios.

What stands out immediately is that the aggregate responses of both economies are very similar. In particular the responses of employment and wages are nearly identical due to the specification of the preferences. The initial drop in output is 0.54 percent in the full model and 0.48 percent in the representative household version of the model. The composition of the output drop, however, is quite different. The fall in consumption is steeper and more persistent in the economy with heterogeneous households, while the reverse is true for investment.\footnote{Government spending does not respond in period 0. The responses differ at the maximum by 0.02 percentage points and, thus, are not of importance for differences in the output responses.} Consumption falls by 0.13 percentage points more and the total consumption loss over 4 years is 0.31 percentage points higher with incomplete markets. Looking at composite consumption $X_t$, which leaves out the effect of GHH preferences, makes this more evident.

To assess the importance of the direct effect on consumption in both economies, I compare the response of composite consumption to interest rate changes keeping all other prices at steady state values. With complete markets, changing the path of the real interest rate to the path in Figure 2 lowers composite consumption by 0.065 percent.\footnote{I feed the path of the real interest rate into the Euler equation and budget constraint of the representative household without changing any other prices to determine the partial consumption response.} This number reduces to 0.056 percent with incomplete markets.\footnote{The path of the real interest rate differs between both economies. Assuming the same path as in the complete markets benchmark shows that consumption falls by 25\% less with market-incompleteness.}

Comparing these numbers to the equilibrium response of composite consumption in Figure 2 identifies the indirect effect through income. Composite consumption falls almost three times more in the economy with incomplete markets relative to the direct response, whereas it barely changes with complete markets. What matters for composite consumption of the representative household is life-time in-
**Notes:** Impulse responses to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points. Solid line: the model with heterogeneous households. Dashed line: same calibration with a representative household. Dots: solid-dashed. The y-axis shows either percent or basis points deviations from the no-shock path. The x-axis shows time since the shock in quarters. $\text{LP} = \left( q_{t+1} + r_{t+1} \right) / q_t - R_{t}^{B} / \pi_{t+1}$

$$X_t = \int \left( c_{it} - h_{it} \frac{\nu_i 1 + \gamma}{1 + \gamma} \right) dt$$
come as savings adjust to undo any effect of temporary income losses on consumption. Therefore, the indirect effect through the life-time budget constraint is of minor importance. Current income, however, responds strongly and so does composite consumption with market-incompleteness because of borrowing constraints. The indirect effect through tighter borrowing constraints, therefore, more than outweighs the muted direct effect of interest rate changes on consumption in the full model.

Quantitatively, the indirect effect explains 65% of the drop in composite consumption with market-incompleteness, while the direct effect through interest rate changes accounts for only 35%. This difference becomes substantially higher when the indirect effect through GHH preferences is included. Looking at consumption of final goods, indirect effects make up for more than 90% of the total response. The GHH effect, however, is also present in the complete-markets setting. It also accounts for 87.5% of the response in consumption of final goods there. This is driven by the adopted preference specification, of course, and vanishes with additively separable preferences in consumption and leisure. With such preferences, the response of composite consumption applies, which is completely determined by the direct effect with complete markets.

The stronger reaction of consumption to monetary shocks is not reflected in output because investment falls by 25% less with incomplete versus complete markets. The smaller reaction of investment is a consequence of the redistributive effects of monetary policy in the full model. A tightening of monetary conditions increases inequality because it redistributes from borrowers to lenders and from households that earn wage-income to those that earn profit-income. Both channels transfer from the bottom to the top of the wealth distribution and hence increase inequality. Wealthy households hold relatively more high-return real assets in their portfolios, recall Figure 1 and, thus, stabilize investment demand as they get richer through redistribution.

This points to the importance of heterogeneity in portfolios for aggregate outcomes. The redistributive consequences of monetary policy are discussed next.

22These findings mirror recent empirical evidence by Coibion et al. [2012]
5.3 Importance of Heterogeneity for the Transmission

With incomplete markets, the transmission of monetary policy also works through redistributive effects. Household portfolios in the model differ in net nominal positions, real asset holdings, and human capital. This section quantifies the relative importance of gains and losses on these three dimensions for the transmission. Let me discuss the channels in turn. A higher real rate of interest benefits bondholders at the expense of debtors. Both lenders and borrowers, however, lose on their real asset holdings as asset prices and dividends fall. Labor income declines as well, while income from profit increases.

Table 4: Exposure to monetary shocks by wealth holdings

<table>
<thead>
<tr>
<th>By wealth percentiles</th>
<th>Income gains/losses</th>
<th>Capital gains/losses on real assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta(R^B_{t-1}/\pi_t)$</td>
<td>$\Delta r_t$</td>
</tr>
<tr>
<td>0-10</td>
<td>-0.23</td>
<td>-0.00</td>
</tr>
<tr>
<td>10-20</td>
<td>-0.10</td>
<td>-0.01</td>
</tr>
<tr>
<td>20-30</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>30-40</td>
<td>0.02</td>
<td>-0.05</td>
</tr>
<tr>
<td>40-50</td>
<td>0.04</td>
<td>-0.08</td>
</tr>
<tr>
<td>50-60</td>
<td>0.06</td>
<td>-0.11</td>
</tr>
<tr>
<td>60-70</td>
<td>0.08</td>
<td>-0.14</td>
</tr>
<tr>
<td>70-80</td>
<td>0.10</td>
<td>-0.20</td>
</tr>
<tr>
<td>80-90</td>
<td>0.15</td>
<td>-0.52</td>
</tr>
<tr>
<td>90-100</td>
<td>0.29</td>
<td>-1.27</td>
</tr>
</tbody>
</table>

Notes: Gains and losses in percent of within group consumption in period 0 to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points. Results are expressed in terms of steady-state consumption of each decile and averaged by using frequency weights from the steady-state wealth distribution.

Table 4 summarizes the gains and losses on each of the three portfolio dimensions across the wealth distribution relative to average consumption of each wealth bracket. The sizable fall in labor income represents the single largest loss for the bottom 80% of the wealth distribution. Households in the top quintile of
the wealth distribution, in contrast, enjoy higher returns on their human capital on average, because an over-proportionate share are entrepreneurs. As such, they receive profit income, which increases. The top quintile incurs the highest losses on the real asset position. However, most of it is due to lower asset prices that are not completely realized. In addition, those households are also the largest bondholders and, thus, gain on that account from higher real returns. All in all, the top 10% of households in terms of net worth stands to gain from a monetary tightening as long as they do not realize more than 8% of their capital losses. This is clearly the case as Figure 3 shows.

5.3.1 Investment Response

Figure 3 provides an overview of the portfolio adjustments to the monetary shock across the wealth distribution. The charts in the first row show the change in bond and capital holdings, whereas the second row depicts the contribution of each decile of the wealth distribution to the total change.

Two results stand out. First, the response of the top 20% in terms of net worth explains more than 50% of the change in aggregate savings in bonds and capital. Second, the response of capital declines in wealth holdings. The wealthiest households liquidate only a very small fraction of their capital holdings, while capital holdings by the poorest decile falls 5 times more than aggregate capital. Wealthy households do not only adjust their capital holdings little, but they also account for the majority of the aggregate capital response. As a consequence, higher inequality stabilizes investment as it broadens the difference in the capital response between the top and the bottom. It is therefore the redistributive consequences of monetary policy that explain the significantly weaker response of investment in the incomplete markets economy relative to the complete markets setting, in which no redistribution occurs by definition.

In fact, the investment response would be even more muted if the liquidity premium, i.e. the return on capital relative to the return on bonds, remained at its steady-state value. The relative return on bonds, however, increases to absorb the additional supply of government bonds created by the shortfall in tax revenues. In equilibrium, the liquidity premium falls 3 by basis points (cf. Figure 2). This
makes wealthy household invest more in bonds as they would otherwise do.

Figure 3: Portfolio adjustments to a monetary shock

Notes: Change in savings in bonds and capital to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points in period 0. The first row shows the average savings response of all households in a given decile of the wealth distribution. The second row shows how much each decile contributes to the aggregate change in bond and capital holdings. The policies are averaged using frequency weights from the steady-state wealth distribution.

5.3.2 Consumption Response

The redistributive consequences of monetary policy do not only matter for the investment response, but also for the consumption response. Debt deflation is a prominent channel, going back at least to [Fisher (1933)](https://example.com), that has potentially strong effects on consumption as indebted households are closer to the borrowing
constraint. In the following, I assess the importance of this channel for consumption across households and aggregate outcomes.

Figure 4: Consumption response to a monetary shock with nominal vs. real debt

(a) Consumption $c_{it}$
(b) Composite Consumption $x_{it}$

Notes: Average consumption response of all households in a given quintile of the wealth distribution to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points, in period 0. The left columns correspond to the full model with nominal debt and right ones to the same model with real debt. The policies are averaged using frequency weights from the steady-state wealth distribution.

Heterogeneity in portfolios and, thus, in the exposure to monetary shocks generates sizable heterogeneity in household consumption. The left bars in Figure 4 plot the change in consumption by wealth holdings in the baseline economy with nominal debt. The effect on consumption of final goods includes both the effect of lower income and less hours worked through the complementarity of consumption and leisure. Consumption by households in the first quintile declines by 2 times more than consumption by the top quintile in terms of net worth. This difference becomes more pronounced by considering consumption of the composite good $X_t$ as it leaves out the GHH effect that applies to all households. The ratio of consumption between top and bottom quintile of the wealth distribution increases from 2 to 3 in this case.

Households in the first quintile of the wealth distribution suffer not only from
substantially lower earnings, but also from a higher real rate on nominal debt (c.f. Table 4). The effect of the latter goes only through surprise inflation in period 0. By assumption, the monetary shock affects the nominal interest rate tomorrow but not today.

This timing assumption allows to shut down the initial redistribution through differences in net nominal positions by considering an economy with real debt (inflation-indexed bonds). The right bars in Figure 4 show the response of consumption in the same economy but with real debt. Indebted households gain the most, but also lenders gain because the indirect effects of monetary policy become weaker. Considering consumption of composite goods identifies the importance of indirect effects through the budget constraint. Clearly, this effect is the dominant one for the first quintile of the wealth distribution, while richer households gain little as they are further away from the borrowing constraint.

Figure 5: Marginal propensity to consume

\[ Notes: \Delta c_{it}/\Delta Y_t \text{ with } \Delta Y_t = 0.01Y^{SS} \text{ across capital and bond holdings expressed relative to quarterly income. Human capital is integrated out using the steady-state joint-distribution.} \]

All in all, redistribution through inflation explains about 15% of the total output loss in Figure 2. Redistribution through inflation amplifies the indirect effects of monetary policy because it is strongly correlated with marginal propensities
to consume (MPC). Figure 5 shows the MPCs across capital and bond holdings. Clearly, consumption becomes more sensitive to current income the less liquid bonds a household holds. Lower than expected inflation therefore redistributes from households with high MPCs to households with low MPCs. This depresses aggregate consumption and, thus, output.

6 Conclusion

Heterogeneity in household portfolios has important implications for the transmission of monetary policy. This paper quantifies the consumption and savings response to monetary shocks in a New Keynesian business cycle model with incomplete markets and assets with different degrees of liquidity. When markets are incomplete, the direct effect of changes in interest rates explains less than half of the consumption response to monetary policy shocks. The response of consumption is primarily driven by indirect equilibrium changes in income that strongly affect consumption by liquidity-constrained households. The equilibrium effects, in turn, mainly work through changes in labor income. Redistribution through the revaluation of nominal claims, including Fisher (1933) debt deflation, reinforces the effect on consumption. At the same time, the redistributive consequences of monetary policy imply a muted investment response. The share of real assets in household portfolios increases in household wealth such that second-round changes in inequality affect the investment response.

This is in stark contrast to the transmission mechanism in standard New Keynesian models that build on a representative household. When borrowing constraints do not apply, temporary changes in income are not of importance and monetary shocks do hardly affect life-time income. Consequently, consumption responds solely to the changes in interest rates. Savings, in contrast, react strongly and undo any temporary mismatch between income and consumption.

The reversal of the importance of direct and indirect effects in the transmission mechanism has important implications for the conduct of monetary policy. When markets are incomplete, the power of interest rates to affect aggregate economic activity relies to a large extent on equilibrium effects on labor income. The response of labor income, in turn, depends on a functioning labor market. In particular,
how much does demand for labor respond to changes in aggregate demand? Labor market frictions or financial frictions on the side of firms might, therefore, impede the transmission of monetary policy. Provided the transmission works, mistakes in the setting of the interest rate still imply larger consumption volatility. Therefore, welfare costs of monetary policy shocks might be substantially higher than previously thought. Moreover, the weakening of the interest rate channel questions the existing results on optimal monetary policy rules. It is thus important to reassess the optimality of the properties of the Taylor-rule in a New Keynesian model with incomplete markets.

References


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A First Order Conditions

Denote the optimal policies for consumption, bond holdings, and capital holdings as $x_i^*, b_i^*, k^*, i \in \{a, n\}$ respectively. Let $z$ be a vector of potential aggregate states.

The first-order conditions for an inner solution in the (no-)adjustment case read:

$$k^* : \frac{\partial u(x^*_a)}{\partial x} q = \beta E \left[ \nu \frac{\partial V_a(b^*_a, k^*; z')}{\partial k} + (1 - \nu) \frac{\partial V_n(b^*_n, k^*; z')}{\partial k} \right]$$  \hspace{1cm} (24)

$$b^*_a : \frac{\partial u(x^*_a)}{\partial x} = \beta E \left[ \nu \frac{\partial V_a(b^*_a, k^*; z')}{\partial b} + (1 - \nu) \frac{\partial V_n(b^*_n, k^*; z')}{\partial b} \right]$$  \hspace{1cm} (25)

$$b^*_n : \frac{\partial u(x^*_n)}{\partial x} = \beta E \left[ \nu \frac{\partial V_n(b^*_n, k^*; z')}{\partial b} + (1 - \nu) \frac{\partial V_n(b^*_n, k^*; z')}{\partial b} \right]$$  \hspace{1cm} (26)

Note the subtle difference between (25) and (26), which lies in the different capital stocks $k'$ vs. $k$ in the right-hand side expressions.

Differentiating the value functions with respect to $k$ and $b$, I obtain the following:

$$\frac{\partial V_a(b, k; z)}{\partial k} = \frac{\partial u[x^*_a(b, k; z)]}{\partial x} (q(z) + r(z))$$  \hspace{1cm} (27)

$$\frac{\partial V_a(b, k; z)}{\partial b} = \frac{\partial u[x^*_a(b, k; z)]}{\partial x} \frac{R^B(z)}{\pi(z)}$$  \hspace{1cm} (28)

$$\frac{\partial V_n(b, k; z)}{\partial b} = \frac{\partial u[x^*_n(b, k; z)]}{\partial x} \frac{R^B(z)}{\pi(z)}$$  \hspace{1cm} (29)

$$\frac{\partial V_n(b, k; z)}{\partial k} = r(z) \frac{\partial u[x^*_n(b, k; z)]}{\partial x}$$  \hspace{1cm} (30)

$$+ \beta E \left[ \nu \frac{\partial V_a(b^*_n, b, k; z)}{\partial k} + (1 - \nu) \frac{\partial V_n(b^*_n, b, k; z)}{\partial k} \right]$$

$$= r(z) \frac{\partial u[x^*_n(b, k; z)]}{\partial x} + \beta \nu E \frac{\partial u[x^*_n(b, k; z)]}{\partial x} (q(z') + r(z'))$$

The marginal value of capital in the case of non-adjustment is defined recursively.

Substituting the second set of equations into the first set of equations I obtain
the following Euler equations (in slightly shortened notation):

$$\frac{\partial u[x^*(b, k; z)]}{\partial x} q(z) = \beta E \left[ \nu \frac{\partial u[x^*(b^*, k^*; z')]}{\partial x} [q(z') + r(z')] + (1 - \nu) \frac{\partial V^n(b^*, k^*; z')}{\partial k^*} \right]$$

(31)

$$\frac{\partial u[x^*(b, k; z)]}{\partial x} = \beta E \frac{R^B(z')}{\pi(z')} \left[ \nu \frac{\partial u[x^*(b^*, k^*; z')]}{\partial x} + (1 - \nu) \frac{\partial u[x^*_n(b^*_n, k^*; z')]}{\partial x} \right]$$

(32)

$$\frac{\partial u[x^*_n(b, k; z)]}{\partial x} = \beta E \frac{R^B(z')}{\pi(z')} \left[ \nu \frac{\partial u[x^*_n(b^*_n, k^*; z')]}{\partial x} + (1 - \nu) \frac{\partial u[x^*_n(b^*_n, k^*; z')]}{\partial x} \right]$$

(33)

B Quality of the Numerical Solution

The equilibrium forecasting rules are obtained by regressing them in each iteration of the algorithm on 20,000 observations. I generate the observations by simulating the model in parallel on 20 machines, letting each economy run for 1250 periods and discarding the first 250 periods. The $R^2$ is generally above 99.9%.

Following Den Haan (2010), I also test the out-of-sample performance of the forecasting rules. For this I initialize the model and the forecasting rules at steady state values, feed in the same shock sequence, but otherwise let them run independently. Figure 6 plots time series of the prices $q$ and $\pi$ taken from the simulation of the model and the forecasting rules. The equilibrium forecasting rules track the evolution of the underlying model without any tendency of divergence. Table 5 summarizes the mean and maximum difference between the series generated by the model and the forecasting rules. The mean error for all four time series is less than 0.005%. The maximum errors are small, too.
Figure 6: Out-of-sample forecast performance of forecasting rules

Notes: Out-of-sample comparison between forecasting rules and model zoomed in at $t = \{1000, ..., 1.500\}$ for visibility; see [Den Haan (2010)].
Table 5: Forecasting Errors

<table>
<thead>
<tr>
<th></th>
<th>Price of Capital $q_t$</th>
<th>Inflation $\pi_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error</td>
<td>0.004%</td>
<td>0.002%</td>
</tr>
<tr>
<td>Max Error</td>
<td>0.016%</td>
<td>0.007%</td>
</tr>
</tbody>
</table>

*Notes:* Percentage differences in out-of-sample forecasts between forecasting rules and model; see Den Haan (2010).

C Distributional Consequences: Gini Indexes

Figure 7 displays the Gini indexes for total wealth, income, and consumption. Inequality in income and consumption instantaneously react to the expansionary monetary policy shock, whereas wealth inequality slowly falls. The initial decrease in the Gini index for income is about 5 times larger than the decrease in the Gini index for consumption. This points to substantial consumption smoothing. The dynamics of income inequality follow the response of inflation, which quickly returns to its steady state value and with it profits as well. The decline in consumption inequality, by contrast, is more persistent because of a prolonged time of lower wealth inequality.
Notes: Impulse responses of Gini indexes of wealth, income, and consumption to an 18 basis points monetary policy shock, $e^D$. The y-axis shows basis point changes (an increase of “100” implies an increase in the Gini index from, say, 0.81 to 0.82).

D  Recalibration of Investment Volatility

Figure 8 shows the impulse responses to a one-standard deviation monetary tightening for the economies with incomplete and complete markets. In this section, I have recalibrated the capital adjustment costs in the latter economy to make both versions match the same business cycle statistics.
Figure 8: Response with recalibrated business cycle statistics

Output $Y_t$, Consumption $C_t$, Investment $I_t$, Gov. Spend. $G_t$

Capital $K_t$, Bonds $B_t$, Labor $N_t$, Share at Constraint

Inflation $\pi_t$, Nom. interest $R_t^B$, Wages $W_t$, Profits $\Pi_t$

Price of Capital $q_t$, Dividend $r_t$, Liquidity Premium* Composite $X_t^{**}$

Notes: Impulse responses to a one-standard deviation monetary policy shock, $\epsilon^D = 18$ basis points. Solid line: the model with heterogeneous households. Dashed line: same economy with a representative household and recalibrated capital adjustment costs. The y-axis shows either percent or basis points deviations from the no-shock path. The x-axis shows time since the shock in quarters. *$LP = (q_{t+1} + r_{t+1})/q_t - R_t^B/\pi_{t+1}$ **$X_t = \int (c_{it} - h_{it} \frac{\pi_{t+1}}{1+\gamma})di$