The Long and The Short of Corporate Debt Maturity: Endogenous Term-Structure with CDS*

R. Matthew Darst†  Md. Ehraz Refayet‡

August 28, 2015

Abstract

This paper investigates the effects that credit default swaps (CDS) have on the term-structure of corporate bonds when firms have long term production technologies. I find that when the demand for CDS is limited to the size of the underlying bond market, average debt maturity increases which is consistent with recent empirical work. In this case CDS lower borrowing costs, the benefits of which become consolidated at the time of issuance when the firm issues long term debt rather than a sequence of short term bonds, which exposes the firm to state verification. Thus, equilibrium firm liabilities and assets are more aligned and CDS help reduce the friction in debt markets which induces “maturity mismatch.” Additionally, naked CDS where the demand for the derivative is not limited to the size of the bond market, may shorten average debt maturity and actually create maturity mis-match. In this case CDS raise borrowing costs which also become more consolidated at the time of issuance when firms issue long term debt rather than short term bonds. This incentivizes the firm to issue risky short term debt and mis-align debt liabilities and assets. Furthermore, naked CDS may destroy equilibrium due to a dynamic inconsistency problem that arises whenever firms issue short term debt.

Keywords: Term structure, Credit Derivatives, Investment, Cost of Capital, Non-existence

JEL Codes: G11, G12, G32, E22

*We are immensely grateful to Ana Fostel for numerous discussions and guidance that significantly shaped and improved the quality of this paper. We would also like to especially thank committee members, Jay Shambaugh and Pam Labadie for valuable thoughts and suggestions. I would also like to thank Chao Wei and Senay Agca for helpful comments. Thank you to seminar participants at the Board of Governors of the Federal Reserve. Lastly, thank you to attendees of the George Washington University Macro and International Finance Seminar. All errors are my own. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Board of Governors or anyone in the Federal Reserve System

†Federal Reserve Board of Governors: matt.darst@frb.gov

‡Department of Treasury, OCC: Ehraz.Refayet@occ.treas.gov
1 Introduction

Credit derivatives, in particular credit default swaps (CDS), have in recent years garnered scrupulous attention from a wide audience including but not limited to academics, central bankers, financial market practitioners, and policy makers. One can easily peruse news articles and read opinions about the influence CDS had in the financial crisis beget by the collapse of investment banking firm Lehman Brothers in the summer of 2007 and the subsequent bailout of insurance giant AIG. More recently, policy makers in the European Union have banned the use of particular types of CDS all together (Financial Times, October, 19, 2011). Understanding the roll that credit derivatives play in financial markets remains a critical goal to ensure appropriate policies are being utilized when regulating financial markets.

Current research on credit derivatives focuses mainly on how derivative securities affect underlying asset prices, corporate borrowing costs, default likelihood, and rollover risk (see the literature review below for specific references.) Firms take into account the effects derivative securities have on their borrowing costs and access to capital when deciding how much debt to issue. None of the current CDS literature considers, however, the fact that when deciding how much debt to issue, firms also choose among a set of different bonds maturities. Firms typically issue debt through various maturities. Thus, the natural question to ask is how does the presence of CDS affect the term-structure of firm debt?

I show that the impact of CDS on the term structure of firm debt depends on the type of derivative security issued. For example, if all CDS purchasers must own the underlying bond (i.e. covered CDS), CDS tend to lengthen the average maturity of firm debt. This result is consistent with recent empirical work by Saretto and Tookes (2013) who show that firms with CDS actively trading on their debt tend to be able to borrow at longer maturities compared to otherwise similar firms with no CDS trading. Conversely, CDS may actually shorten the average maturity of corporate debt when CDS purchasers do not have to own the underlying bond (i.e. naked CDS). The mechanism that causes corporate term structure to lengthen in the presence of covered CDS is the same as the mechanism that causes maturity to shorten with naked CDS. Both the positive and negative impact that CDS have on borrowing costs are most concentrated around long term bond issuances. Thus, when CDS lower borrowing costs, the impact on long term bonds is more pronounced, causing the firm to issue more long term debt. Likewise, borrowing cost increases beget by CDS introduction are also most pronounced with long term debt, which induces the firm to shift a portion of debt issuance into short term bonds.

A CDS is a bi-lateral contract between a protection seller and a protection buyer. The protection buyer purchases CDS from the protection seller. The CDS contract

\footnotesize{\bibitem{CBSD} "The bet that blew up wall street," CBSD, 60 Minutes, October 26, 2008
\bibitem{Soros} Soros, George, “One Way to Stop Bear Raids,” (Wall Street Journal, March 24, 2009)
pays the protection buyer nothing if the underlying bond payments are fully honored, and the CDS seller retains the payment received from the buyer for the issuing the contract. The CDS contract pays the buyer the difference between the face value of the bond and its current market value for a pre-specified credit event.

How does the introduction of this bi-lateral contract in conjunction with ownership of the underlying asset affect the term structure of corporate debt? To answer this question, this paper uses a dynamic three-period general equilibrium model where a firm needs to issue debt (bonds) to raise capital for a long term investment project. Bonds are issued to investors with heterogeneous beliefs over the likelihood that bonds repay in full. The firm chooses among bonds with two different debt maturities; a sequence of one period short term bonds that must be rolled over at an intermediate time in order to produce, and two-period long term bonds. The fundamentals of the economy are governed by a technology shock in the final period that is common knowledge to all agents. Equilibrium is characterized by the mix of bond maturities that trade for given technology shock realizations. The firm issues the mix of bond maturities that minimizes expected-weighted borrowing costs. CDS are then introduced depending on the mix of underlying bond maturities that exist in equilibrium.

The introduction of CDS has several interesting results. Covered CDS tend to lengthen average debt maturity when firms jointly issue both long and short term bonds. The reason is that covered CDS on long term bonds lower borrowing costs for the duration of real investment projects because all borrowing costs are paid at the time of issuance. Debt financing via short term bonds allows investors to observe news announcements and adjust borrowing costs accordingly. Borrowing costs on short term bonds are lower with covered CDS as well, but only after bad news announcements, which tend to raise borrowing costs. Additionally, the amount of short term debt the firm needs to rollover falls as more capital is raised through long term bonds, which tends to further reduce the expected borrowing cost on short term debt. Thus, even though there are CDS traded on both short term and long term debt, the firm actually substitutes away from short term debt in both the mix of bonds it issues and in the level of short term debt it issues in favor of long term debt. Consequently, covered CDS help align the maturity of the firms’ liabilities with its assets and reduces any potential maturity mis-match.

Naked CDS tend to shorten average equilibrium bond maturity. Naked CDS raise borrowing cost because capital is reallocated away from debt markets and into the derivative security market. The benefit of issuing long term debt – that the firm can insulate itself from higher borrowing costs on short term bonds if intermediate news is bad – is eroded when naked CDS are introduced. The increase in borrowing costs are realized at the time of issuance and must be paid over the life of the investment project when the firm issues long term bonds. Short term bonds allow investors the possibility to receive good news and reduce their borrowing costs in subsequent
periods. The firm lowers its expected-weighted borrowing costs by substituting a portion of its debt away from long term bonds into short term bonds. Thus, the average debt maturity shortens and a potential maturity mis-match problem arrises.

Second, covered CDS have no impact on the average length of debt when investment is financed through only one maturity structure. This is because covered CDS implicitly increases the demand for the underlying asset when only one debt maturity trades in equilibrium. The increase in demand for the underlying bond prohibits bonds with alternative maturity lengths from materializing as an equilibrium source of financing.

Third, naked CDS destroy equilibrium whenever the firm raises any positive portion of its capital through short term bonds. The possibility for CDS to trade exists whenever a debt instrument carries credit risk, otherwise CDS are redundant assets. Naked CDS raise the borrowing cost for risky short term debt to the point where it is not profitable for the firm to rollover short term bonds in order to continue production. However, CDS are derivative instruments whose value is derived from an underlying asset. If the supply of the asset dries up, the derivative security cannot exist, and as soon as CDS do not exist the firm chooses to rollover over its short term debt. Thus, equilibrium ceases to exist due to a dynamic inconsistency problem in the presence of derivatives that has not yet been identified in the literature.

The baseline model without credit derivatives is related to several different strands of literature. The first of which is the optimal debt maturity structure work in corporate finance. Meyers (1977) seminal paper provides a rationale for why firms should try to match the maturity of existing debt obligations to correspond with the decision to exercise a real investment option. The reason is that existing debt lowers the future value of the firm. As long as debt payments are due before the decision to exercise investment, all positive net present value opportunities will be exercised. The model presented in this paper is concerned with how CDS affect investment financed via new debt issuance, in which case the maturity structure of the debt issuance is not always perfectly aligned with the asset’s payout. Barnea, Haugen, Senbet (1980) argue that complex financial instruments including debt contracts with different maturity structures arise to resolve an agency problem between shareholders and creditors. Different maturities arise in equilibrium in my model in response to how the market prices capital based on investor expectations.

A related strand of corporate finance literature examines corporate debt structure as an optimal response to asymmetric information problems over firm quality (Flannery (1986), Kale and Noe (1990), and Diamond (1991)). Flannery (1986) shows that absent transaction costs, good and bad firm types pool only in short term debt. Equilibrium with multiple debt maturities is possible with transaction costs.

Kale and Noe (1990) extend Flannery (1986) using sequential game theory and formally show that absent transaction costs, only a short term pooling equilibrium
survives the equilibrium selection. They then show that when the value of good firm cash flows are positively correlated over time, one can obtain the same separating equilibrium as Flannery (1986) where the good type issues short term debt and the bad type issues long term debt.

Diamond (1991) relates the choice of debt maturity to the risk of forced liquidation. He shows that high rated firms prefer short term funding absent liquidation because it lowers firm expected financing cost. Conditions are then given to describe an optimal mix of both short and long term debt, which is ultimately based on the probability a good firm is downgraded.

One of the main differences between my model and the optimal debt maturity literature is that investment is endogenous in this paper, where the aforementioned work takes investment to be exogenously fixed.

The second closely related strand of literature examines the economic impact of CDS on a variety of firm financing and capital market outcomes. The model proposed here is an extension of Darst and Refayet (2014a) who show that CDS create borrowing cost spillovers, as well as induce a trade off between borrowing costs and default risk with heterogeneous firms. I consider only a representative firm, but model investment as an optimal capital allocation problem across assets of different maturities. This allows me to analyze how CDS affect the term structure allocation problem in addition to investment and borrowing costs. Refayet (2014) shows that under asymmetric information, asset debt capacity is endogenous to the introduction of different types of CDS. He also shows that CDS can completely flip the incentive compatibility of contracts under asymmetric information. Fostel and Geanakoplos (2012a) and Che and Sethi (2014) show that CDS affect asset prices by altering the allocation of capital away from bond markets and into CDS markets. Duffee and Zhou (2001) suggest that CDS help banks overcome the “lemons” problem in the loan sale market. Bolton and Oehmke (2011) and Gi (2014) argue that CDS strengthen the ex ante commitment of firms to repay their debt obligations. The former also show that CDS give rise to empty creditors and can force firms into inefficient liquidation. Parlour and Winton (2013) show that CDS could lead to insufficient monitoring and raise default risk.

Thirdly, the implications that CDS have on the existence of equilibrium are related to Polemarchakis and Ku (1990), who show a robust non-existence example in a general equilibrium model with financial derivatives. Fostel and Geanakoplos (2014) show that credit derivatives can robustly destroy funding in a model with production despite collateralized borrowing. Che and Sethi (2010) encounter the non-existence problem Fostel and Geanokoplos (2014) prove and avoid it by assuming a retail investors will purchase bonds. The assumption that the demand for

---

4 All borrowers prefer the maturity choice good firm types choose because selecting a different maturity reveals a lower quality firm, and funding would cease for those firms.
corporate debt is always met is not sufficient to ensure equilibrium in my model due to the dynamic structure of debt financing. As I show, the fixed point problem identified in Fostel and Geanokoplos (2014) is extremely robust in a dynamic debt financing setting when firms issue short-term debt. Duffee and Zhou (2001) show that credit derivatives can cause the market for secondary bank loan sales to break down.

Fourth, the model’s results relate to literature on firm capital structure and managerial beliefs. Jung and Subramanian (2014) show in a dynamic setting that heterogeneous beliefs between managers and investors leads to more optimistic managers issuing less long-term debt. Their model relies on an agency problem whereby more optimistic managers assign more weight to their private benefit relative to the market’s valuation of their equity stake. More optimistic managers issue less long-term debt so as to avoid loosing their private benefit due to potential bankruptcy. The firm manager in my model issues a smaller portion of long-term debt as the likelihood of full debt repayment increases because the manager does not need to insulate the firm from news announcements. Landier and Thesmar (2008) and Graham et. al (2013) show that, controlling for firm risk factors and other determinants of short-term leverage, higher managerial optimism leads to more short-term debt issuance, which is consistent with my model’s equilibrium capital structure outcome for good new outcomes as well.

Lastly, the model is the first to offer an in-depth explanation of the empirical results on CDS and debt-maturity found by Saretto and Tookes (2013). The authors show that the introduction of credit derivatives allows firms with active CDS trading on their debt to issue bonds on longer debt-maturities relative to otherwise similar firms with no active CDS contracts trading. Oehmke and Zawadowski (2014) have an interpretation of this result based on liquidity needs and differences in corporate bond and CDS trading costs. The purpose of their model, however, is not to explain how CDS alter corporate equilibrium term-structure.

The organization of the paper is as follows: section 2 introduces the model without CDS and characterizes equilibrium by bonds of different debt maturities. Section 3 introduces CDS in which investor must also own the underlying bond (covered CDS economies) and analyzes the impact of the firm’s average debt maturity. Section 4 allows investors to purchase CDS without the restriction that they must also own the underlying bond. Section 5 provides concluding thoughts. Appendix B contains characterizations of equilibrium that are not considered in the main body of Sections 2, 3, and 4.
2 Non CDS Economy

2.1 Model

2.1.1 Time and Uncertainty

The model is a three-period general equilibrium model with time $t = \{0, 1, 2\}$. Uncertainty is represented by a tree of state events $s \in S$ with root $s_0$, intermediate states $s \in S$ that take values $\{U, D\}$, and a set of terminal nodes denoted $S_T = \{UU, UD, DU, UU\} \subset S$. Thus, the complete state-tree has elements set $S = \{U, D, UD, DU, DD\}$. Furthermore, each state $s \neq s_0$ as a unique predecessor denoted $s^\star$. The timing of intermediate state realization $U$ is denoted $t(U) = 1$ and terminal state $UU$ as $t(UU) = 2$. I take a state realization $U$ to be up or a “good” state and $D$ to be down or a “bad” state.

The stochastic structure of the model follows Fostel and Geanokoplos (2008 & 2012b) where bad news increases the volatility of debt repayment in the following way: The economy receives a technology shock at time 2, $A_s, s \in S_T$. The value of the technology shock received at time 2 is conditional on the information revealed at time 1. We assume that good news at time 1, $s = U$, always results in a good time 2 technology shock, whose value is normalize to 1: $A_s = 1, s \neq DD$. Bad news at time 1, $s = D$, however, raises the volatility of the technology shock at time 2. Specifically, the technology shock is good at terminal node $s = DU$, and bad at terminal node $s = DD$, where the technology shock $A_{DD} < 1$. The technology process assumed here is meant to keep the model as simple as possible while retaining enough uncertainty to study the impact of credit derivatives on the term structure of firm debt. Figure 1 depicts the economy’s state tree.

2.1.2 Debt Contracts

There is a single durable consumption good available in the economy at time 0, which is the numeraire and treated as cash. There are two bonds types, each with different maturity. There are bonds that mature after one period called short term bonds and bonds that mature after two periods called long term bonds. The distinction between short and long term bonds is whether or not debt payments come due before cash flows are realized, not specific calendar dates. We assume all bonds are zero coupon bonds. Let the quantity of bonds issued at any state and time be $q_s, s \in S/s_T$. Long term bonds can only be issued at $t = 0$ and are denoted $q_0^L$. Short term bonds may be issued at time $t = 0, 1$. Short term bonds issue at time 0 are given by $q_0^S$ and short term bonds issued at time 1 are given by $q_1^S, s = U, D$.

2.1.3 Agents

*Firm*
Figure 1: Economy State Tree

Technology Shock: $A_s, s \in S_T$

$s_0$

$t = 0$

$\gamma$

$s = U$

$(1 - \gamma)$

$A_{UV} = 1$

$t = 1$

$\gamma$

$s = D$

$(1 - \gamma)$

$A_{UD} = 1$

$t = 2$

$(1 - \gamma)$

$A_{DU} = 1$

$A_{DD} < 1$
There is a single firm owned and operated by a manager with access to a long term decreasing returns to scale production technology that produces output at time 2. The production function is denoted by \( f(I_0) = A_s I_0^\alpha \), \( \alpha < 1 \), \( s \in S_T \), where \( I_0 \) is the amount of capital the manager puts into production. I assume the firm has no internal cash flow at time 0 or 1. Thus, the manager must issue bonds at time 0 \( q_0^s \) and/or \( q_0^s \) to begin production. The manager must choose among two maturity structures to finance his project: 1) issue long term bonds that mature at time 2, and/or 2) issue a sequence of one-period, short term bonds. Because the firm does not have internal cash flow at time 1, new short term bonds \( q_s^s \), \( s = U, D \) must be issued at time 1 to repay time 0 short term debt. I assume capital raised at time 1 can only go towards repaying short term debt. Lastly, I assume the firm defaults on short term bonds issued at time 0 if it does not procure financing at time 1.

I restrict the analysis to debt issuance and do not allow for equity issuance. This allows me to focus the analysis on the impact of CDS to the term-structure of debt issuance. Incorporating equity is a natural extension to the model.

**Investors**

There exists at time 0 a continuum of uniformly distributed investors with unit mass, \( h_0 \in H_0 \sim U[0,1] \), each of which is endowed with a unit of the durable consumption good \( e^{h_0} \). The uniform distribution allows one to rank investors according to the likelihood, \( h_0 \), each places on the subsequent state being good. Investors are risk-neutral, expected utility maximizers over the consumption good at time 2, and do not discount the future. Each investor also has access to a storage technology that allows the durable consumption good to be risklessly stored across all time periods. Letting \( H_s \), \( s \in S_T \) denote the product of all \( h \) along the path from 0 to \( s_T \), the von-Neumann-Morgenstern preferences for time 0 investors are given by:

\[
U^{h_0}(x_{UU}, x_{UD}, x_{DU}, x_{DD}) = \sum_{s \in S_T} H_s x_s
\]

The set of investors at time 0 have a one-time opportunity to buy bonds. This assumption is to simplify the analysis; otherwise, one would have to keep track of different portfolio values across time periods.

Furthermore, I assume that an additional continuum of investors arrive at time 1. These investors are also endowed with one unit of the durable consumption good \( e^{h_1} \) that they can risklessly store until time 2. Time 1 investors are also risk-neutral expected utility maximizers for the consumption good at time 2 and do not discount the future. They can either hold cash or buy short term bonds issued at time 1 \( q_s^s \), \( s = U, D \). Time 1 investors will not be able to purchase long term bonds because all long term bonds are issued at time 0. Lastly, the two continua of investors are completely segmented from one another, meaning there is no communication between them and no opportunity to trade with one another.\(^5\) The von-Neumann-

\(^5\)While this assumption focuses the analysis only on primary issuance debt, the impact of CDS on the secondary corporate bond market is an important topic in its own right.
Morgenstern preferences for time 1 investors are given by:

$$U^{h_1}(x_{UU}, x_{UD}, x_{DU}, x_{DD}) = \sum_{s \in \mathcal{S}_T} h_s x_s$$ (2)

I assume two sets of investors at different time periods in order to keep the model as simple and tractable as possible. An infinitely lived set of investors requires keeping tract of their portfolio values through time. Short term debt “repayment” at time 1 becomes an accounting exercise. We believe this is the simplest way to drive home the key points of the tradeoff between choosing to issue short term versus long term debt. Moreover, the paper’s ultimate focus is on the effects of CDS on the primarily issuance portion of the corporate bond market, not the effects on the secondary bond market.

Neither the assumption of a uniform distribution of beliefs nor risk-neutrality are essential to drive home the key points of the paper; they are done for tractability. The qualitative results will hold as long as the distribution of investor beliefs is continuous and monotonic in $h_t, t = 0, 1$. Additionally, equilibrium can be preserved with risk-averse preferences, common beliefs and state contingent endowments.

### 2.1.4 Debt Repayment

The firm can issue either a sequence of short term bonds $q^s_s, s \in S/S_t$ or long term bonds $q^\ell_0$. The repayment functions of each debt type are naturally different. I describe the repayment function of short term bonds first followed long term bonds.

*Short-term Debt*
Short term bonds are one-period zero-coupon bonds, with face value normalized to 1. Bonds issued at time 0 are repaid using capital raised at time 1. The key feature of short term debt in the model is that the initial debt issuance matures before the investment project produces cash flow. Thus, the maturity structure of short term liabilities do not necessarily match the cash flow structure of the productive asset. Let \((1 - \rho)\) be the portion of capital raised at time 0 issuing short term bonds (\(\rho\) thus is the portion of capital raised via long term debt).

Time 0 creditors who hold short-term debt have the control rights to force liquidation of assets if they are not fully repaid at time 1 (Diamond 1992 and Berglöf and von Thadden 1994). We assume the liquidation value of the firm is the “outside” value to creditors whereas the value of production seen through to time 2 is the “inside” value to the manager. This assumption can be justified via many different avenues. Liquidation could be costly to investors because of costly state verification or due to incurring legal fees to settle debts in bankruptcy. Alternatively, creditors may be ill suited to operate the firm’s production process, thus they are only able to capture a portion of the initial value of the investment.

Let the liquidation value of the firm be a portion of the initial capital put into production, \(\delta I_0\) where \(\delta < 1\). Hence, time 0 debt is collateralized by the outside recovery value to time 0 creditors in the event of liquidation. This collateral serves as enforcement to repay debt obligations at time 1.

Assume for simplicity that the firm concurrently defaults on both short and long term bonds in the same states at time 2, and concurrently repays both short and long term bonds in the same states at time 2. This implies no creditors hold either senior or subordinated debt.

Given time 0 bond prices \(p_0^s\) the firm issues \(q_0^s\) one-period bonds. The firm owes \(q_0^s\) upon maturity at time 1. Let \(d_s (q_0^s), s = U, D\) describe bond repayment for short term bonds issued at time 0 and deliver repayment at time 1. Time 0 short term bonds repay in full at \(s = U\) so that \(d_U (q_0^s) = 1\). Time 0 short term bonds repay in full at \(s = D\) when \(I_0^a (1 - \rho^a) \geq q_D^s\), i.e. when the additional output generated by producing with short term debt capital exceeds the cost of rolling over short term debt. Hence the delivery will be \(d_s (q_0^s) = 1.6\) Time 0 short term bonds default when \(I_0^a (1 - \rho^a) < q_D^s\), and the bonds repay \(d_D (q_0^s) = \frac{\delta (1 - \rho) I_0}{q_0^s}\) when the firm liquidates the portion of its assets raised via short term bonds. The outside value of the liquidated assets that repays short term creditors is proportional to \(\delta\) which is the outside value to creditors when taking over the firm’s assets. Given short term bond pricing at time 1 denoted \(p_s^s, s = U, D\) the firm issues \(q_s^s\) one-period bonds to re-pay the debt issued at time 0. Short term bonds issued at time 1 repay \(d_s (q_s^s) = 1, s = UU, UD\) when intermediate news is good. Furthermore, \(d_DU (q_D) = 1\) so that any good news results

---

6This implies that absent liquidation, all time 0 short term bonds are risk free. This is the same assumption used in Flannery (1986).
in full short term bond repayment. Conversely, \(d_{DD}(q_0^F) = \min\left[1, \frac{A_{DD}(1-\rho)I_0}{q_0^F}\right]\) when news is bad for two consecutive periods. The sequence of short term bond payouts is depicted in Figure 2.

**Long-term Debt**

Long term zero-coupon bonds are issued at time 0 maturing at time 2. Given market prices for long term debt \(p_0^F\), the firm issues \(q_0^F\) long term bonds whose face value we normalize to one. Long term debt issued at time 0 is collateralized by future, contingent time 2 output. The lenders have the right to seize as much of the collateral up to the value of the promise, but no more. The collateral, just as with short term financing, serves as a repayment mechanism at time 2. Long term bonds repay in full whenever the path to the terminal state contains a period of good news. Thus, \(d_s(q_0^F) = 1, s \neq DD\). Long term debt repays \(d_s(q_0^F) = \min\left[1, \frac{A_s(p_0^F)\rho}{q_0^F}\right]\), \(s = DD\) if no assets are liquidated to repay short term creditors. Lastly, long term bonds repay \(d_s(q_0^F) = \min\left[1, \frac{A_s(p_0^F)\rho}{q_0^F}\right]\), \(s = DD\) when the firm liquidates the portion of its capital raised via short term bonds due at time 1. Figure 3 depicts the long-term bond payout.

### 2.1.5 Firm Maximization Problem

The firm chooses an initial investment amount \(I_0\) and debt maturity structure \(\rho\) at time 0 to maximize expected profits denoted \(E_0[\pi] = \Pi\). Recall that \(\rho\) denotes the portion of investment capital that is raised by issuing long term bonds \(q_0^F\). The firm weighs the relative borrowing costs in expectations between the two bond maturities
against its marginal product of capital when determining the term structure of its debt. Lastly, let $\gamma$ denote the probability the firm places on good news arriving in the following period. Formally, the firm maximizes the following problem\footnote{The probability the firm believes that $s_2 = UU$ is given by $\gamma^2$, $s_2 = UD$ is given by $\gamma (1 - \gamma)$, $s_2 = DU$ is given by $(1 - \gamma) \gamma$, and finally $s_2 = DD$ by $(1 - \gamma)^2$.}

$$
\max_{I_0, \rho} \Pi = \left\{ \sum_{s \in S_T} \gamma_s \max \left[ A_s I_0^\alpha - q_s d_s \left( q_0^s \right) - q_s d_s \left( q_s^* \right), A_s (\rho I_0)^\alpha - q_s d_s \left( q_0^s \right) \right] \right\}
$$

s.t.

$$
I_0 = p_0^U q_0^U + p_0^D q_0^D
$$

$$
p_s^U q_s^U = q_0^U, s = U, D
$$

$$
0 \leq \rho = \frac{p_0^U q_0^U}{I_0} \leq 1
$$

where $\gamma$ denotes the product of all $\gamma$ along the path from 0 to $s \in S_T$. At time 1, in either state $s = \{U, D\}$, the firm must decide whether it is beneficial to roll-over the short term component $q_0^s$ of its debt portfolio. It can either fully repay time 0 short term bond holders by raising $p_0^s q_0^s$, $s = U, D$ at time 1, for which it pays $q_s d_s \left( q_s^* \right)$, $s \in S_T$ on maturity at time 2, or it can liquidate the portion of its assets raised via short term debt at time 0. The firm ultimately owes $q_s d_s \left( q_0^s \right) + q_s d_s \left( q_s^* \right)$ at time 2 when all short term debt is re-financed. When short term assets are liquidated at time 1, the firm continues production using only the capital raised via long term debt at time 0 given by $\rho I_0$, thus owing $q_s d_s \left( q_0^s \right)$ on maturity.

The maximization problem is subject to the following constraints: The amount of capital the firm can use for production has to be raised by issuing bonds at time 0. Conditional on rolling over short term debt at time 1, the firm issues new short term debt in the amount $p_0^s q_0^s$, $s = U, D$ to fully repay its time 0 debt obligations $q_0^s$. Note that when the firm can credibly commit to rolling over short term debt, all time 0 short term financing will be risk free.\footnote{Flannery (1986) and Diamond (1991) use the same risk-free assumption on short term debt.} Lastly, the portion of the firm’s investment that is raised through long term bonds $\rho$ is naturally bound between zero and one.

### 2.1.6 Investor Maximization Problem

We can now characterize each investors’ budget set. We begin with investors at time 0 and then move on to time 1 investors.

Given time 0 bond prices $(p_0^U, p_0^D)$, each time 0 investor $h_0 \in H_0$ chooses cash holdings $\{x_0^{h_0}\}$, bond holdings $\{q_0^{U h_0}, q_0^{D h_0}\}$ and final period consumption decisions $\{x_s^{h_0}\}$ to maximize utility given by (1) subject to the budget set defined by:

$$
B^{h_0} (p_0^U, p_0^D) = \left\{ \left( x_0, q_0^{U h_0}, q_0^{U h_0}, x_s \right)_{h_0 \in H_0} \in R_+ \times R_+ \times R_+ : \right\}
$$

$$
x_0^{h_0} + p_0^U q_0^{U h_0} + p_0^D q_0^{D h_0} = e^{h_0},
$$

$$
x_s^{h_0} = x_0^{h_0} + d_s \left( q_0^s \right) + d_s \left( q_s^* \right), s \in S_T \right\}.
$$
Each investor uses their cash to purchase either type of bond at time 0. All cash that is not used to purchase bonds is carried forward for consumption. All final period consumption comes from initial bond purchases and cash holdings.

Given short term bond prices \( p^*, s = U, D \) at time 1, each time 1 investor \( h_s \in H_1 \) chooses cash holdings \( \{ x_{s h_1} \} \), bond holdings \( \{ q_{s h_1} \} \), and final period consumption decisions \( \{ x_s^{h_o} \}, s \in S_T \) to maximize utility given by (2) subject to the following budget set:

\[
B^{h_1} (p^*_s) = \left\{ (x_{s*}, q^*_s, x_s)_{h_1 \in H_1} \in R_+ \times R_+ \times R_+ : \\
x_{s*}^{h_1} + p^*_s q^*_s = e^{h_1} , \\
x_{s}^{h_1} = x_{s*}^{h_1} + d_s (q^*_s), s \in S_T \right\}.
\]

As with time 0 investors, each time 1 investor uses their cash to purchase bonds or stores it for final period consumption. Total final period consumption is the sum of cash stored at time 1 and bond deliveries at time 2.

### 2.1.7 Equilibrium

An equilibrium in the Non-CDS Economy is a collection of bond prices, firm investment decision, investor cash holdings, bond holdings, and final consumption decisions

\[
\left( (p^0_f, p^0_s, p^s_s), I_s \right) = \left( (x_0, q_0^0, q_0^s, x_s)_{h_0 \in H_0} \right) \left( (x_1, q_1^*, x_s)_{h_1 \in H_1} \right) \in (R_+ \times R_+ \times R_+) \times (R_+ \times R_+ \times R_+ \times R_+) \times (R_+ \times R_+ \times R_+)
\]

such that the following are satisfied:

1. \( \int_0^1 x_{s}^{h_0} dh_0 + \int_0^1 q_{s}^{h_0} dh_0 + \int_0^1 q_{s}^{h_0} dh_0 = \int_0^1 e^{h_0} dh_0 \)
2. \( \int_0^1 x_{s*}^{h_1} dh_1 + \int_0^1 q_{s*}^{h_1} dh_1 = \int_0^1 e^{h_1} dh_1 \)
3. \( f_s (I_0) = \pi_s + q_s d_s (q^0_s) + q_{s*} d_s (q^*_s) \)
4. \( I_0 = p^f_0 \int_0^1 q_{s}^{h_0} dh_0 + p^s_0 \int_0^1 q_{s}^{h_0} dh_0 \)
5. \( p^s_0 \int_0^1 q_{s}^{h_1} dh_1 = q^s_0 \)
6. \( \pi (I_0, \rho) \geq \pi (I_0, \rho), \forall I_0 \geq 0 \text{ and } 0 \leq \rho \leq 1 \)
7. \( \left( x_0, q_0^0, q_0^s, x_s^{h_0} \right) \in B^{h_0} (p^f_0, p^0_s, p^s_s) \Rightarrow U^{h_0} (x) \leq U^{h_0} (x^{h_0}), \forall h_0 \)
8. \( \left( x_{s*}^{h_1}, q_{s*}^{h_1}, x_{s}^{h_1} \right) \in B^{h_1} (p^*_s) \Rightarrow U^{h_1} (x) \leq U^{h_1} (x^{h_1}), \forall h_1 \)

\( s \in S_T \).
Conditions (1) and (2) state that all investor cash endowment at time 0 and time 1 respectively, is used for consumption or bond purchases. The time 2 goods market clearing condition states that all firm output goes toward repaying bond debt and firm profits as shown by condition (3). Condition (4) states that all of the capital raised by issuing both short and long term bonds at time 0 is used as input in final goods production. Condition (5) says all capital raised at time 1 is used to fully repay time 0 short term creditors. Condition (6) states that the firm chooses investment to maximize profits, while conditions (7) and (8) state that investors at time 0 and 1 choose portfolios of bonds and cash holdings to maximize their respective utilities within their respective budget sets.

To find an equilibrium in this model we first guess a particular regime and check to see if it is optimal. This entails determining whether or not the firm’s capital structure consists of debt raised through both long and short term bonds, only long term bonds, or only short term bonds. Furthermore, if there is any short term debt issued in equilibrium it must be determined whether it is optimal to rollover at time 1 or to liquidate short term assets. It is important to completely characterize equilibrium for the entire parameter space because it ultimately determines what type of CDS may exist in the following sections. CDS are derivative securities whose value is derived from an underlying asset. This implies that CDS cannot exist on an asset that does not exist. Thus, the introduction of CDS on a particular type of asset cannot, in equilibrium, destroy the underlying asset’s existence. The mapping to an economy with CDS depends crucially on the types of assets available in the baseline economy from which CDS ultimately derive their value. The system of equations used to solve for each of the possibilities depends crucially on the value of \( \rho \) - the portion of capital raised by issuing long term bonds.

Equilibrium is ultimately characterized by three different funding regimes:

1. The firm issues a portfolio of short and long term bonds and rolls-over short term debt at time 1
2. The firm issues only short term debt, which it rolls-over at time 1
3. The firm issues only long term debt.

These regimes are shown in Figure 4. We consider each of the above cases in the following Results section.

2.2 Results

The natural question at this point is: What is the trade-off facing the firm when deciding whether to issue long term or short term bonds? Raising debt by issuing long term bonds allows the firm to insulate itself from bad news that may arise at time 1. The benefit of insulating its production from bad news comes at the cost of higher borrowing costs at time 0. Short term bonds allows the firm to potentially
borrow at lower or even risk-free interest rates. This is because good news at time 1 always results in a good technology shock in the final period, which assures time 1 investors of full repayment at time 2. Hence short term borrowing is risk free at time 1. On the other hand, short term borrowing at time 1 entails higher borrowing costs than long term borrowing if bad news arises because the probability any investor places on a bad technology shock at time 2 is higher when \( s = D \) than at \( s_0 \).\(^9\)

The firm’s optimal equilibrium capital structure minimizes the expected marginal per-unit borrowing cost or default premium the firm must pay to issue bonds. Consider first the default premium when only long term bonds are issued. The firm defaults when \( s = DD \) and repays in full for \( s \neq DD \). Thus, the firm only forms expectations over states governed by default \((1 - \gamma)^2\) and non-default \((1 - (1 - \gamma)^2)\). Since the firm walks away with no proceeds when it defaults, its marginal investment decision is irrespective of its expectation. Thus, the expected default premium on long term bonds is simply its borrowing cost \((1 - p_0)\).

The expected default premium on short term bonds is the result of the temporal capital allocation problem, which issuing exclusively long term debt avoids. The expected borrowing cost of short term debt issued at time 1 is \((1 - \gamma)(1 - p_D^c)\).\(^{10}\) Thus all short term debt issued at time 0 considers the fact that with probability \((1 - \gamma)\) bad news arrives at time 1 and borrowing costs will rise from the risk free levels at which short debt is issued at time 0.

\(^9\)The reason is that the probability any investor places on the subsequent state being bad is given by \(0 \leq (1 - h^m) \leq 1\). At time 0, the probability an investor places on two consecutive bad states is therefore \((1 - h^m)^2 < (1 - h^m)\).

\(^{10}\)The default premium on short term bonds issued at time 0 is zero because all debt is rolled over and risk free.
The final default premium the firm considers in that which it expects to pay on a portfolio of long and short term debt bonds. Simply put, the expected default premium is the weighted sum of the two individual default premia: \( \rho (1 - \gamma_p^0) + (1 - \rho)(1 - \gamma)(1 - \gamma_p^D) \). Finally, note of course that bond pricing changes when the firm issues both long and short term bonds jointly versus when it issues only long term or only short term bonds.

**Portfolio**

The firm always prefers to issue a portfolio of long and short term bonds when it can credibly commit to full rollover. A portfolio of the two debt maturities allows the firm to issue debt to investors not only at time 0, but also at time 1 as shown in Figure 5. Raising capital by issuing both long and short term bonds allows the firm to take advantage of relatively low interest rates on long term debt, thus reducing the amount of short term debt issued at time 0 that needs to be roll over at time 1 subject to potentially higher interest rates. The only way to receive funding at time 1 is to issue and roll over time 0 short term bonds. The firm cannot raise capital from these investors if it issues only long term bonds or does not rollover its time 0 short term bonds.

Equilibrium for which the firm issues a portfolio of long and short term bonds is characterized by the following parameters: \( \alpha = 0.8, \delta = 0.7, \gamma = 0.5, A_{DD} = 0.5 \). The results do not depend on this particular parameterization per se. There is some leeway in choosing parameters for each of the particular term structure regimes. A particular point is chosen to illustrate the comparative static results on the firm’s optimal term structure \( \rho \) in subsequent sections as CDS are introduced.

In equilibrium, as a result of linear utilities and the continuity of utility in \( h \), and the connectedness of the set of agents \( H = (0, 1) \), at initial node \( s_0 \) there will be a *marginal buyer*, \( h_0 \). Every agent \( h > h_0 \) will buy long term bonds and every agent \( h < h_0 \) will hold a portfolio of cash and short term bonds because short term bonds are risk free with full rollover at \( s = \{U, D\} \). Additionally, since there is rollover in equilibrium, at \( s = D \) there will be *marginal buyer* \( h_D \). All investors \( h > h_D \) will purchase short term bonds at time 1, and all agents \( h < h_D \) will remain in cash. This regime is shown in Figure 5.

Table 1 shows the results for the baseline economy without CDS when the firm issues both short and long term debt. Note that the firm raises roughly 65 percent of its capital through long term bonds. Though we are ultimately interested in how CDS affect this capital allocation, the fact that the firm issues short term debt absent any asymmetric information problem is something the existing literature is lacking.

---

11The firm commits to roll over short term debt at time 1. Hence time 0 short term borrowing is risk free.

12All investors \( h < h_0 \) hold 13.42 percent of their portfolio in short term bonds and the remainder in cash.
Table 1: Portfolio Results

<table>
<thead>
<tr>
<th>$(A_{DD}, \gamma) = (0.5, 0.5)$</th>
<th>$p_0^s$</th>
<th>$p_D^s$</th>
<th>$p_0^l$</th>
<th>$q_0^s$</th>
<th>$q_D^s$</th>
<th>$I_0$</th>
<th>$\rho$</th>
<th>$E[\pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.577</td>
<td>0.9855</td>
<td>0.1078</td>
<td>0.1997</td>
<td>0.3046</td>
<td>0.6460</td>
<td>0.0579</td>
</tr>
</tbody>
</table>

Market incompleteness alone in this model generates the simultaneous existence of short and long term debt contracts.

Note that a portfolio of both short term and long term bond maturities is endogenously issued in the general equilibrium setting with perfect information. This is differs from the partial-equilibrium optimal risky debt work of Flannery (1986), Kale and Noe (1990), and Diamond (1991). In these models, asymmetric information about firm types leads to separating equilibria in which the firm types separate by issuing either short term or long term debt. In this model, both the firm and investors have the same information regarding the value of the aggregate technology shock and firm productivity. Issuing bonds to investors across time lowers the expected-weighted average cost of capital because it spreads financing risk across investors who are most willing to pay the highest prices.

The benefit of this modeling strategy is that one can study the impact of CDS on the firm’s average maturity in a very simple manner. An alternative model with asymmetric information and multiple firms such as those cited above may allow for the introduction of CDS, but solving for equilibrium becomes much more cumbersome. The last thing to note is the firm cannot credibly commit to full roll over at time 1 for the entire parameter space. Figure 4 shows the parameters for which a portfolio of both debt maturities are issued versus only long term and only short term bonds.

The reason the firm does not credibly commit to full rollover is easiest seen by examining the rollover decision rule given by $I_0^s (1 - \rho^s) \geq q_D^s$. Rewriting this as $I_0^s (1 - \rho^s) \geq \frac{q_0}{p_D}$ we see that the additional output generated by short term capital that is rollover over at time 1, $I_0^s (1 - \rho^s)$, must be at least as large as the debt obligations owed to time 0 creditors in states for which borrowing costs at time 1 increase, $\frac{q_0}{p_D}$. Honoring this debt obligation is most difficult as $p_D^s$ decreases. The lower is the technology shock $A_{DD}$ the lower is $p_D^s$ because small technology shocks reduce the amount investors can recover in the event of bond default. The resulting equilibrium is one in which the firm must issue either exclusively short term bonds or long term bonds when agents expect sufficiently low technology shocks. These two equilibrium possibilities are discuss in the respective subheadings below.

The remainder of the section describes how to solve for the equilibrium in the Non-CDS Economy when the firm issues a portfolio of bonds. Readers not interested in the technical details may move on to the Long and Short term portions of the Results section.
The ten endogenous variables are \( (p_0^\delta, p_0^\ell, p_D^\delta, q_0^\delta, q_0^\ell, I_0, \rho, h_0, h_D) \). The system of equations is:

\[
p_0^\delta = 1 \\
1 = \frac{1 - (1 - h_0)^2 + (1 - h_0)^2d_{DD}(q_0^\ell)}{p_0^\ell} \\
1 = \frac{h_D + (1 - h_D)d_{DD}(q_D^\delta)}{p_D^\delta} \\
I_0 = p_0^\ell q_0^\ell + p_0^\delta q_0^\delta \\
\frac{\gamma}{p_0^\delta} + \frac{\gamma(1 - \gamma)}{p_D^\delta} = 1 - (1 - \gamma)^2 \\
\alpha I_0^{-1} \left[1 - (1 - \gamma)^2\right] = \rho \left[\frac{1 - (1 - \gamma)^2}{p_0^\ell} + \frac{(1 - \rho)}{p_0^\delta} \left[\gamma + \gamma(1 - \gamma)\right]\right] \\
\rho = \frac{p_0^\ell q_0^\ell}{I_0} \\
q_0^\delta = p_s^\ell q_s^\ell \\
1 - h_0 = p_0^\ell q_0^\ell \\
1 - h_D = p_D^\delta q_D^\delta
\]

The first three equations are bond pricing equations. Equation (4) shows that short term bonds issued at time 0 are risk free because all short term debt is rolled over at time 1. Equation (5) states that long term bonds are priced based on the time 0 marginal investor’s expectations because he is indifferent between buying the bond
and holding a cash equivalent asset. Similarly, equation (6) states that time 1 short term bonds are priced based on the time 1 marginal investor’s expectations because cash is the only other alternative asset.

Equation (7) says that the amount of capital the firm raises in the bond market is equal to the investment it puts into its production technology. Equations (8) and (9) are the first order conditions w.r.t. the portfolio allocation \( \rho \) and investment level \( I_0 \), respectively. A necessary condition for the firm to issue a portfolio of both long and short term bonds is that on the margin the expected cost of issuing either type of bond must be the same. The left hand side of (9) is the expected marginal product of capital irrespective of whether or not it is issued via long term or short term bonds. The right hand side is the expected-weighted marginal cost of capital. Equation (10) sets \( \rho \) equal to the portion of the firm’s investment that is raised via long term debt. Equation (11) shows that the firm will issue as many short term bonds at time 1 as it takes to fully repay its time 0 short term creditors. Equations (12) and (13) are, respectively, the long term and time 1 short term bond market clearing conditions.

**Short Term Funding**

Equilibrium is characterized by only short term debt at times 0 and 1 when the firm is relatively optimistic about good news at time 1. This result is intuitive because the firm does not need to issue long term bonds at higher interest rates to insulate itself from news arriving at time 1 when it expects the news to be good. In fact, it is better off structuring its debt maturity to take advantage of its expectations by issuing only short term debt.

The firm is likely to issue at least some short term debt for a given level of optimism as the value of the technology shock increases. The reason is that investors price time 1 short term bonds on increasingly better terms as \( A_{DD} \) increases so that the firm can borrow short term for two consecutive periods at low interest rates.

The following parameters are chosen to characterize equilibrium when the firm issues only short term bonds. \( \alpha = 0.8, \delta = 0.7, \gamma = 0.8, A_{DD} = 0.2 \). The qualitative results do not depend on this particular parameterization.

In equilibrium, as a result of linear utilities and the continuity of utility in \( h \), and the connectedness of the set of agents \( H = (0,1) \), at state \( s_0 \) all investors \( h \) will hold a portfolio of cash and short term bonds because short term bonds are risk free with complete rollover at \( s = \{U,D\} \). Additionally, since there is rollover in equilibrium, at \( s = D \) there will be marginal buyer \( h_D \). All investors \( h > h_D \) will purchase short term bonds at time 1, and all agents \( h < h_D \) will remain in cash. This regime is shown in Figure 6.

\[13\] All investors hold 27.46 percent of their portfolio in short term bonds and the remainder in cash.
Table (2) shows the model’s results when the firm only issues short term bonds \( i.e. \rho = 0 \). Appendix B A.1 shows the system of equations used to solve for the five endogenous variables \((p_0^0, p_D^0, q_0, q_D, I_0)\).

Lastly, the firm cannot honor time 0 short term bond payments in full for all parameters, in which case creditors will liquidate assets and no final output is produced. Thus, for parameters regions in which the firm cannot honor its debt obligations, the only funding option available to the firm will be through long term bonds.

**Long Term Funding**

The previous section highlighted the fact that the firm will not issue short term bonds for low values of \( A_{DD} \) and will not want to issue short term bonds when it believes time 1 news will be bad–low \( γ \). For these parameter regions the firm issues long term debt. Long term debt allows the firm to insulate itself from borrowing at even higher interest rates when time 1 news is bad \( s = D \). The firm is more likely to issue exclusively short term debt as it becomes more optimistic because it does not view \( s = D \) as a likely event.

The following parameters are chosen to characterize equilibrium when the only funding available is through long term bonds. \( \alpha = 0.8, \delta = 0.7, 5, \gamma = 0.5, A_{DD} = \)
In equilibrium, as a result of linear utilities and the continuity of utility in $h$, and the connectedness of the set of agents $H = (0, 1)$, at state $s_0$ there will be a marginal buyer $h_0$. All agents $h > h_0$ will purchase long term bonds, and all agents $h < h_0$ will hold cash. This regime is shown in Figure 7.

Table 3 shows the model’s results when only long term bonds are issued. Appendix B A.2 shows the system of equations used to solve for the four endogenous variables $(p_0^f, q_0^f, h_0, I_0)$.
3 Covered CDS Economy

In this section credit default swaps (CDS) are incorporated to the baseline model to analyze their impact on debt maturity. Whether CDS incentivize more long versus short term debt depends on how the credit derivative affects the different bonds’ relative default premia. Recent research suggest that whether or not CDS purchasers must own the underlying bond (covered versus naked CDS) ultimately determines how borrowing costs and default premia change (see Fostel and Geanakoplos (2012a), Darst and Refayet (2014) and (Che and Sethi (2014)). As a result, a distinction in the literature is drawn between covered and naked CDS. This paper follows the same distinction and analyzes the effects of covered CDS versus naked CDS separately.

A CDS is a financial contract in which a CDS seller compensates the CDS buyer for losses to the value of an underlying asset for a pre-specified credit event or default. The underlying assets in this economy are firm bonds. CDS contracts will compensate the contract holder the difference between a bond’s face value at maturity and its market value at the time of the credit event. Thus, CDS allow investors to hedge against idiosyncratic default risk.\(^{14}\) I follow Bolton and Oehmke (2011) and assume that only default constitutes a credit event that triggers CDS payment. Debt renegotiation is increasingly excluded as a trigger for CDS payment (see ISDA 2009).

Consider first covered CDS, where CDS buyers are required to also hold the underlying asset i.e. the firm’s bond for whom the CDS is written. Let \( \phi_0^{\ell} \) be the price of a CDS contract issued at time 0 on bond \( q_0^{\ell} \) where the superscript \( \ell \) denotes long term debt and the superscript \( \varsigma \) denotes short term debt as in the Non CDS Economy. Let \( v_{h0}^{\ell} \) be the number of CDS contracts investor \( h \) can issue at time

\(^{14}\)CDS do not allow investors to insure away aggregate risk.
holdings defined by: own collateral that he must post in order to sell a CDS, where a CDS proceeds from which to consume. 1 defaults therefore will equal the total payment the seller is required to make if the bond defaults \(1 - d_s (q^\ell_s)\), \(s \in S/s_0\).

Take, for example, a CDS issued on a long term bond whose payout to the CDS seller and CDS buyer are depicted in Figure 8. At time 0 the CDS seller receives payment \(\phi_0^\ell\) from the CDS buyer. The seller believes with probability \(1 - (1 - h_0)^2\) that the firm will not default and the CDS will not payout. The CDS seller believes with probability \((1 - h_0)^2\) that there will two consecutive periods of bad news, in which case he is expected to pay the difference between the face value of the bond and its recovery value at time 2, \(1 - d_s (q^\ell_0), s \in S_T\), leaving the seller with no proceeds from which to consume.

Notice that selling a CDS is equivalent to buying an Arrow-Up security as it permits state-contingent consumption. The price of an Arrow-Up security \(1 - d_s (q^\ell_0) - \phi_0^\ell\) is the portion of a CDS seller’s own collateral, \(\frac{1}{\phi^{\ell h_0}}\) that he must post in order to sell a CDS.\(^{15}\)

### 3.1 Investor Maximization Problem

We can now characterize each investors budget set. Given time 0 bond and CDS prices \((p^\ell_0, p^\xi_0, \phi^\ell_0, \phi^\xi_0)\), each time 0 investor \(h_0 \in H_0\) chooses cash holdings \(\{x^{\ell h_0}_0\}\), bond holdings \(\{q^\ell_0, q^\xi_0\}\), CDS positions \(\{v^{\ell h_0}, v^{\xi h_0}\}\) and final period consumption decisions \(\{x^{\ell h_0}_S\}\), \(s \in S_T\) to maximize utility given by (1) subject to the budget set defined by:

\[
B^{\ell h_0} (p^\ell_0, p^\xi_0, \phi^\ell_0, \phi^\xi_0) = \left\{ (x_0^\ell, q^\ell_0, q^\xi_0, v^\ell_0, v^\xi_0, x_s)_{h_0 \in H_0} \in R_+ \times R_+ \times R_+ \times R \times R \times R_+ \right\}:
\]

\[
x_0^\ell + p^\ell_0 q^\ell_0 + p^\xi_0 q^\xi_0 + \phi^\ell_0 v^\ell_0 + \phi^\xi_0 v^\xi_0 = e^{\ell h_0}
\]

\[
x_s^{\ell h_0} = x_0^\ell + d_s (q^\ell_0) + d_s^* (q^\xi_0) - \phi^\ell_0 v^{\ell h_0} - \phi^\xi_0 v^{\xi h_0}
\]

\[
\max \left[ 0, v^{\ell h_0}_0 \right] \leq q^\ell_0
\]

\[
\max \left[ 0, v^{\xi h_0}_0 \right] \leq q^{\xi h_0}_0, s \in S_T\right\}.
\]

\(^{15}\)The price of a time 1 Arrow-Up security \(1 - d_s (q^\ell_* - q^\xi_*)\), \(s \in S_T\) is the portion of a CDS seller’s own collateral that he must post in order to sell a CDS, where \(\phi^\ell_*\) is the price of a CDS on a time 1 bond \(q^\ell_*\).
Investors use their cash endowment to purchase bonds and CDS, as collateral to issue CDS, or for direct consumption. Note, that there is no sign restriction on \( u_0^{\ell, \xi} \). Selling CDS implies that \( u_0^{\ell, \xi} < 0 \), while \( u_0^{\ell, \xi} > 0 \) corresponds to purchasing CDS. In all terminal states investors consume from their cash holdings and the payouts from their portfolio of bonds and CDS positions. The maximum number of CDS contracts an investor can purchase with either bond maturity as the underlying reference entity cannot be greater than the number of bonds the investor owns because CDS buyers are required to hold the underlying asset. Lastly, as in the Non-CDS Economy, we assume there is no short selling of bonds, given by the restriction \((q_0^d, q_0^s) \in (R_+ \times R_+)\).

Given time 1 short term bond and CDS prices \((p_s^e, \phi_s^e)\), \(s = U, D\) each time 1 investor \(h_s \in H_1\), \(s = U, D\) chooses cash holdings \(\{x_{s}^{h_1}\}\), bond holdings \(\{q_s^{h_1}\}\), CDS positions \(\{v_s^{h_1}\}\), and final period consumption decisions \(\{x_{s}^{h_0}\}, s \in S_T\) to maximize utility given by (2) subject to the following budget set:

\[
B^{h_1}(p_s^{e, \star}, \phi_s^{e, \star}) = \left\{ (x_{s}^{\star, \xi}, q_s^{\xi, \star}, x_s)_{h_1 \in H_1} \in R_+ \times R_+ \times R \times R_+ : \\
\begin{align*}
&x_s^{h_1} + p_s^{e, \star} q_s^{\xi, h_1} + \phi_s^{e, \star} v_s^{\xi, h_1} = \epsilon_s^{\star}, \\
&x_s^{h_1} = x_s^{h_1} - d_s (q_s^{\xi, \star} v_s^{\xi, h_1} + (1 - d_s (q_s^{\xi, \star})) v_s^{\xi, h_1} \\
&\max \left[ 0, q_s^{\xi, h_1} \right] \leq q_s^{\xi, h_1}, s \in S_T \right\}.
\]

As with period 0 investors, investors use their cash endowment to purchase bonds and CDS, as collateral to issue CDS, or for direct consumption. In all terminal states investors consume from their cash holdings and the payouts from bond and CDS portfolios. Selling CDS implies that \(v_s^{\xi, h_1} < 0\), while buying a CDS implies \(v_s^{\xi, h_1} > 0\). Lastly, the number of CDS contracts owned cannot exceed the number of bonds owned because all CDS must be covered by the underlying bond.

### 3.2 Equilibrium

An equilibrium in the Covered CDS Economy is a collection of bond prices, CDS prices, firm investment decision, investor cash holdings, bond holdings, CDS positions and final period consumption decisions

\[
\left( (p_0^{e, \xi}, p_0^s), (\phi_0^{e, \xi}, \phi_0^s, \phi_s^e), I_0, (x_0, q_0^d, q_0^s, u_0^e, v_0^e, x_0)_{h_0 \in H_0} (x_s^{\star, \xi}, q_s^{\xi, \star}, v_s^{\xi, \star}, x_s)_{h_1 \in H_1} \right) \\
\in (R_+ \times R_+ \times R_+) \times (R_+ \times R_+ \times R_+) \times R_+ \times (R_+ \times R_+ \times R_+ \times R \times R_+) \times \\
(R_+ \times R_+ \times R \times R_+)
\]
such that conditions (1) – (8) from the Non CDS Economy hold as well as the following:

9. \( \int_0^1 v_{0}^{\ell,ho} dh_0 = 0 \)

10. \( \int_0^1 v_{0}^{s,ho} dh_0 = 0 \)

11. \( \int_0^1 v_{s}^{s,h_1} dh_1 = 0 \)

Equilibriums conditions (1) - (8) are analogous to the Non CDS Economy. Conditions (9) – (11) state that the CDS markets at time 0 for both bond maturities and time 1 are, respectively, in zero net supply.

I make use of the following lemma to characterize equilibrium in the covered CDS economy.

**Lemma 1** If \( 0 < d_s (q_0^\ell) < 1 \) and \( 0 < d_s (q_s^\star) < 1 \), then no bonds for which CDS are sold will be purchased unprotected.

**Proof.** See Appendix B. ■

The intuition behind Lemma 1 is that any investor optimistic enough to buy a bond without a CDS will be better off selling CDS on that bond. Additionally, if bond delivery is equal to zero, then CDS and bonds pay the same amount in both states, making CDS redundant assets. Finally, if the bond repays in full, then bonds are risk free and no CDS will trade in equilibrium.

### 3.3 Results

Equilibrium in the Covered CDS Economy must be based on the bond maturity mappings implied by the Non CDS Economy. CDS are derivative assets whose value depends on the existence of the underlying bond. Thus, CDS cannot be introduced on short term bonds for parameter regions in the Non CDS Economy where equilibrium is characterized by only long term bonds. Similarly, CDS cannot exist on long term bonds for parameter regions in the Non CDS Economy for which equilibrium is characterized by only short term bonds. Therefore, equilibrium with CDS on both long term debt and short term debt is only possible when there exists an interior solution for \( \rho \) – the portion of firm funding that is financed through long term debt – that characterizes equilibrium in the Non CDS Economy.\(^{16}\) This does not mean, however, that short (long) term bonds cannot exist as a funding possibility once CDS

\(^{16}\)Note that CDS will not trade on time 0 short term bonds in this model because all short term financing in the Non-CDS Economy is rolled over at time 1. This implies that time 0 short term debt is risk free and CDS will be redundant.
are introduced on long (short) term bonds. This means that it is possible for CDS to change the average maturity structure for all Non CDS Economy regimes.\textsuperscript{17}

In equilibrium, as a result of linear utilities and the continuity of utility in \( h \), and the connectedness of the set of agents \( H = [0, 1] \), at state \( s_0 \) there will be a marginal buyer, \( h_0 \). Every agent \( h > h_0 \) sells CDS on long term bonds, and every agent \( h < h_0 \) holds a portfolio of covered long term bond positions, short term bonds, and cash.\textsuperscript{18} Additionally, since there is rollover in equilibrium, at \( s = D \) there will be marginal buyer, \( h_D \). All investors \( h > h_D \) sell CDS on short term bonds at time 1, and all agents \( h < h_D \) hold a portfolio of covered CDS positions and cash.\textsuperscript{19} This regime is shown in Figure 9.

Table 4 shows the intra-regime results when covered CDS are introduced for the same parameterizations used on the Non-CDS Economy. We see that Covered CDS induce the firm to issue a greater percentage of long term bonds, even when covered CDS are traded on both short and long term bonds. In other words, covered CDS lengthen the average debt maturity of the firm’s bond liabilities, just as documented empirically by Saretto and Tookes (2013). Covered CDS improve the borrowing cost terms on which the firm borrows across both debt maturities. The reason bond prices rise in covered CDS economies, as highlighted in Darst and Refayet (2014) is that CDS and bonds are complimentary assets. This implies that investors who wish to sell CDS implicitly increase the demand for bonds because the number of CDS than can be issued is tied to the number of bonds given by the restriction \[ \max \left[ 0, v_{s}^{c,h} \right] \leq q_{s}^{c,h}, s \in S_T \]. The subsequent increase in long term bond prices allows the firm to substitute long term debt for short term debt at time 0. This has the effect of reducing the supply of short term debt the firm needs issue at time 1 to honor its initial short term debt obligations. Hence, the average length of the firm’s debt composition lengthens.

Note also that not only does the firm proportionally issue more long term debt than short term debt, it actually reduces the overall amount short term debt issued even though there are CDS on short term bonds at time 1 as well. The change in short term bond pricing coming from increased investor demand for bonds in and of itself is not sufficient for the firm to maintain its optimal expected-weighted cost of capital at pre-CDS levels. The firm then re-optimizes its capital structure by reducing the amount of capital raised via short term bonds so that time 1 short term bond prices rise sufficiently to maintain its optimal expected weighted cost of capital.

\textsuperscript{17}I present in the main body of the paper only detailed equilibrium results for the Covered CDS Economy for which a portfolio of both long and short term debt \( 0 < \rho < 1 \) are issued in the Non CDS Economy. The Covered CDS Economy equilibrium characterizations that correspond to only long term or short term bonds \( \rho = 0, 1 \) can be found in Appendix B A.3.

\textsuperscript{18}All investors \( h < h_0 \) hold 7.72 percent of their portfolio in short term bonds, 27.93 percent in long term covered CDS positions, and the remainder in cash.

\textsuperscript{19}All investors \( h < h_1 \) hold 7.25 percent of their portfolio in short term bonds and the rest in cash.
In sum, covered CDS induce the firm to better match its liabilities with the timing of its asset payouts and reduce the incidence of maturity mis-match.

The complimentarily of the CDS and the underlying bond underscores why the firm’s term structure is unaltered for parameter regions in which only one debt maturity trades in the Non CDS Economy. The implicit increased demand for the underlying asset prohibits funding through alternative maturity structures from materializing. This is why the parameter regions for which only long term (short term) bonds are issued continue to be characterized by only long term (short term) bonds when CDS are introduced. Figure 10 shows the parameters space that characterizes each of the possible funding regimes.

Lastly, consistent with other general equilibrium models of investment with CDS, the reduction in borrowing cost causes the firm the increase investment demand and leads to more profitable production.

The remainder of the section describes how to solve for equilibrium in the Covered CDS Economy when the firm issues both long and short term debt maturities.\(^{20}\)

\(^{20}\)The system of equations used to solve for only long term or short term debt can be found in Appendix B A.3
Table 4: Covered Results

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>( p_0^* )</th>
<th>( p_D^* )</th>
<th>( q_0^* )</th>
<th>( q_D^* )</th>
<th>( I_0 )</th>
<th>( \rho )</th>
<th>( E[\pi] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covered CDS</td>
<td>1</td>
<td>.9901</td>
<td>.9967</td>
<td>.0700</td>
<td>.0707</td>
<td>.2531</td>
<td>.3223</td>
</tr>
<tr>
<td>Non-CDS</td>
<td>1</td>
<td>.9577</td>
<td>.9855</td>
<td>.1078</td>
<td>.1126</td>
<td>.1997</td>
<td>.3046</td>
</tr>
</tbody>
</table>

Figure 10: Covered CDS Economy

Readers not interested in the technical details may move on to the *Naked CDS Economy* section.

The twelve endogenous variables are \(( p_0^*, p_D^*, q_0^*, q_D^*, I_0, \rho, h_0, h_D, \phi_0^*, \phi_D^* )\). The
system of equations is

\[ p_0^\phi = 1 \]

\[ 1 = \frac{(1-(1-h_0)^2)(1-d_{DD}(q_0^\phi))}{1-\phi_0^\phi-d_{DD}(q_0^\phi)} \quad (14) \]

\[ 1 = \frac{h_D(1-d_{DD}(q_D^\phi))}{1-\phi_D^\phi-d_{DD}(q_D^\phi)} \quad (15) \]

\[ I_0 = p_0^{\phi_L}q_0^L + p_0^{\phi_S}q_0^S \quad (16) \]

\[ \frac{\gamma}{p_0^\phi} + \frac{\gamma(1-\gamma)}{p_D^\phi} = \frac{1-(1-\gamma)^2}{p_0^\phi} \quad (17) \]

\[ \alpha I_0^{\phi_L-1} \left[ 1 - (1-\gamma)^2 \right] = \frac{\rho(1-(1-\gamma)^2)}{p_0^\phi} + \frac{(1-\rho)}{p_0^\phi} \left[ \gamma + \frac{\gamma(1-\gamma)}{p_D^\phi} \right] \quad (18) \]

Equations (14)-(16) are the bond and CDS pricing equations. Time 0 short term debt is risk free because of rollover at time 1. Long term CDS prices are determined by marginal investor \( h_0 \), who is indifferent between writing a long term CDS and holding cash. The return to writing a long (short) term CDS is the total expected collateral retained in the upstate over the portion of the seller’s own collateral required to issue the contract. Equations (17)-(21) are the same as in the Non-CDS Economy. Equations (22) and (23) are, respectively, the time 0 long term and time 1 short term CDS market clearing conditions. They state the that optimists’ demand for issuing CDS is equal to the supply of bonds the firm issues since any purchased CDS must be accompanied by the underlying bond. Lastly, equations (24) and (25) are the long term and short term no arbitrage conditions which state CDS prices and underlying bond prices must sum to 1.

4 Naked CDS Economy

This section extends the model by allowing investors to hold naked CDS positions: investors are not required to hold the underlying asset to purchase a CDS.

A naked CDS buyer receives the difference between the face value of the bond and recovery value in the event of default, \( 1-d_s \left( q_s^{\phi_s} \right) \), \( s \neq s_0 \). Naked CDS sellers receive nothing if the bond repays in full. CDS buyers must make an upfront payment in the amount \( \phi_s^{\phi_s} \), \( s \neq s_0 \) in order to purchase a contract. Notice that purchasing a naked
CDS effectively buys the Arrow-Down security. The naked CDS payout structure is given in Figure 11.

As in the *Covered CDS Economy*, all CDS contracts are fully collateralized. The CDS seller must post enough collateral to make the required payment if the firm defaults. The total collateral posted per CDS contract is equal to $1 - d_s(q^c_{s^*})$, $s \neq s_0$. A portion of the total collateral amount comes from the price a CDS seller receives for selling the contract $\phi^c_{s^*}$, $s \neq s_0$, and the remainder of the collateral amount comes from his own endowment $\frac{1}{v_{s^*}}$, $s \in S/S_T$. Moreover, the implications of Lemma 1 still hold so that no agent purchases a bond unprotected for which CDS protection is sold.

### 4.1 Investor Maximization Problem

Given time 0 bond and CDS prices $(p^c_0, p^\ell_0, q^c_0, q^\ell_0)$, each time 0 investor $h_0 \in H_0$ chooses cash holdings $\{x_{h_0}^0\}$, bond holdings $\{q_{h_0}^c, q_{h_0}^\ell\}$, CDS positions $\{v_{h_0}^c, v_{h_0}^\ell\}$ and final period consumption decisions $\{x_{s_0}^h\}$, $s \in S_T$ to maximize utility given by (1) subject to the budget set defined by:

$$B^{h_0}(p_0^c, p_0^\ell, q_0^c, q_0^\ell) = \left\{ (x_{0_0}^c, q_{0_0}^c, q_{0_0}^\ell, v_{0_0}^c, v_{0_0}^\ell, x_s)_{h_0 \in H_0} \in R_+ \times R_+ \times R_+ \times R \times R_+ : \right.$$

$$x_{s_0}^h = x_0 + d_s(q_0^c) + d_s^*(q_0^\ell) - \phi^c_{0_0} v_{0_0}^c - \phi_{0_0}^\ell v_{0_0}^\ell, s \in S_T \bigg\}.$$  

Investor budget sets are the same as those described in the *Covered CDS Economy* except that investors can now buy CDS without holding the underlying asset, hence the restriction $\max \left[ 0, v_{c_0}^{c_0} \right] \leq q_{c_0}^{c_0}$ is no longer imposed. As a result the number of
CDS contracts investors may purchase is not restricted to the number of bonds they own.

Given time 1 short term bond and CDS prices \((p_s^c, \phi_s^c)\), \(s = U, D\) each investor \(h_1 \in H_1\) chooses cash holdings \(\{x_{h_1}^1\}\), bond holdings \(\{q_{s,h_1}^c\}\), CDS positions \(\{\nu_{s,h_1}^c\}\), and final period consumption decisions \(\{x_{h_0}^s\}\), \(s \in S_T\) to maximize utility given by (2) subject to the following budget set:

\[
B_{h_1}^s (p_s^c, \phi_s^c) = \left\{ (x_{s,h_1}^c, q_{s,h_1}^c, \nu_{s,h_1}^c, x_s)_{h_1 \in H_1} \in R_+ \times R_+ \times R \times R_+ : \right. \\
x_{s,h_1}^c + p_s^c q_{s,h_1}^c + \phi_s^c \nu_{s,h_1}^c = c_{h_1}^c, \\
x_{s}^h = x_{s}^{h_1} - \phi_s^c \nu_{s}^{c,h_1} + d_s (q_s^c) q_{s}^c + (1 - d_s (q_s^c)) \nu_{s}^{c,h_1}, \ s \in S_T \right\}.
\]

The time 1 budget set in the *Naked CDS Economy* is also the same as the *Covered CDS Economy* except there is no restriction that ties the maximum number of CDS contracts bought to the number of bonds held.

**Equilibrium Existence and Institutional Investor**

Fostel and Geanakoplos (2014) identify an existence problem in production economies where collateral equilibrium may breakdown in the presence of naked CDS. Darst and Refayet (2014) and, in an earlier working paper version, Che & Sethi (2010) assume that a retail investor will always demand bonds so that equilibrium does not break down. I follow this assumption to compute equilibrium in the current model.

Assume institutional investors arrive at times 0 and 1 who are required to hold no risk on their balance sheet. Furthermore, assume that the investors are endowed with enough capital to satisfy all potential firm funding needs and CDS premiums in equilibrium. One may, for example, take the investors to be mutual fund managers. For simplicity, assume each institutional investor may only purchase assets that are introduced into the economy concurrent with his arrival. For example, the retail investor arriving at time 0 can only purchase CDS and bonds issued at time 0.\(^{21}\)

Lastly, the presence of the institutional investor would not affect equilibrium in the *Non-CDS* and *Covered-CDS Economies*. Limiting the risk exposure of the investor will prevent him from purchasing bonds without any CDS protection. Thus he will remain in cash in an economy without CDS. Moreover, in the covered CDS economy, agents who did not sell CDS held a portfolio of cash and covered CDS positions. The institutional investor could just as easily replace the agents’ covered CDS positions, leaving them only holding cash and equilibrium unperturbed.

\(^{21}\)A formal presentation of the Retail Investor Problem is presented in Appendix B B.2
4.2 Equilibrium

An equilibrium in the *Naked CDS Economy* is a collection of bond prices, CDS prices, firm investment decision, individual and retail investor cash holdings, bond holdings, CDS positions and final consumption decisions

$$\left( (p_0^0, p_0^0, p_{s^*}^0), (\phi_0^0, \phi_0^0, \phi_{s^*}^0), I_0, (x_0, q_0^0, q_0^0, v_0^0, v_0^0, x_s) \right)_{h_0 \in H_0, M_0} (x_s^*, q_s^*, v_s^*, x_s)_{h_1 \in H_1, M_1}$$

$$\in (R_+ \times R_+ \times R_+) \times (R_+ \times R_+ \times R_+) \times R_+ \times (R_+ \times R_+ \times R_+ \times R \times R_+ \times R_+)$$

such that the following are satisfied:

1. $$\int_0^1 x_0^h dh_0 + x_0^{M_0} + p_0^h q_0^0 + \phi_0^h \phi_0^0 = \int_0^1 e^{h_0} dh_0 + e^{M_0}$$
2. $$\int_0^1 x_s^h dh_1 + x_s^{M_1} + p_s^h q_s^M_1 = \int_0^1 e^{h_1} dh_1 + e^{M_1}$$
3. $$f_s (I_0) = \pi_s + s \cdot d_s (q_s^0) + q_s^* d_s (q_s^*)$$
4. $$I_0 = p_0^h q_0^0 + \phi_0^h \phi_0^0$$
5. $$p_s^h q_s^M_1 = q_s^0$$
6. $$\pi (I_0, \rho) \geq \pi (I_0, \rho), \forall \hat{I}_0 \geq 0 \text{ and } 0 \leq \hat{\rho} \leq 1$$
7. $$\left( x_0^h, q_0^h, q_0^h, x_s^0 \right) \in B^0 \left( p_0^h, p_0^h, p_{s^*}^h \right) \Rightarrow U^{h_0} (x) \leq U^{h_0} (x^h), \forall h_0$$
8. $$\left( x_s^h, q_s^h, x_s^h \right) \in B^1 \left( p_s^h \right) \Rightarrow U^{h_1} (x) \leq U^{h_1} (x^h), \forall h_1$$
9. $$\left( x_s, c_0^M_0 \right) \in B^0 \left( p_0^h, p_0^h, \phi_0^h \phi_0^0 \right) \Rightarrow U^{M_0} (x) \leq U^{M_0} \left( x_s, c_0^M_0 \right)$$
10. $$\left( x_s, c_s^M_1 \right) \in B^1 \left( p_s^h, \phi_{s^*}^h \right) \Rightarrow U^{M_1} (x) \leq U^{M_1} \left( x_s, c_s^M_1 \right)$$
11. $$\int_0^1 v_0^h dh_0 + v_0^M_0 = 0$$
12. $$\int_0^1 v_0^h dh_0 + v_0^{M_0} = 0$$
13. $$\int_0^1 v_s^h dh_1 + v_s^{M_1} = 0$$

$$s \in S_T$$

Conditions (1) & (2) state that all endowments in period 0 and 1 have one of three potential uses: (a) held as collateral to issue CDS, (b) held as cash by the *retail investor* for consumption, or (c) used to buy bonds and CDS. Condition (3) says the goods market clears such that total firm output is consumed by firm managers in the form of profits and used to repay bond holders. Condition (4) states that all of the capital raised at time 0 goes towards producing output, while condition (5) says all capital raised at time 1 is used to fully repay time 0 short term creditors. Condition (6) states that the firm chooses investment to maximize profits while
Conditions (7) – (10) say that all investors choose utility maximizing portfolios given their budget sets. Finally, conditions (11) – (13) state that the CDS markets in either period are in zero net supply.

Results

As with the Covered CDS Economy, equilibrium in the Naked CDS Economy must be mapped to the Non CDS Economy. Figure 12 shows the different maturity structures that characterize equilibrium for the parameter space when naked CDS are introduced. Equilibrium with naked CDS is characterized by only two different maturity structures: 1) Portfolio of both bond maturities with short term liquidation at time 1, and 2) only long term debt. What this means is that Naked CDS robustly destroy equilibrium when any short term debt is issued at time 0, a point to which I return later in the section.

Term Structure

Consider first equilibrium characterized by CDS only on long term debt with short term liquidation at time 1. To analyze this equilibrium, consider the following parameters: $\alpha = 0.8$, $\gamma = 0.5$, $A_{DD} = 0.1$, $\delta = 0.7$. In equilibrium, as a result of linear utilities and the continuity of utility in $h$, and the connectedness of the set of agents $H = (0, 1)$, at state $s_0$ there will be two marginal buyers, $h^1_0$ and $h^2_0$. Every agent $h > h^1_0$ will sell CDS on long term bonds, every agent $h^2_0 < h < h^1_0$ will buy short term bonds, and every agent $h < h^2_0$ will buy naked CDS on long term bonds. This regime is shown in Figure 13.

The grey region in Figure 12 is the parameter space that is characterized by short term debt that is in default when $s = D$ and long term debt with naked CDS. There
are no CDS on short term debt issued at time 0 because only long term bonds are issued when CDS do not exist. It is the introduction of naked CDS on long term bonds that cause short term debt at time 0 to become a debt financing option. Unlike the complementarily of covered CDS and the underlying bond, naked CDS and the underlying bond are substitute assets (Darst and Refayet (2014), Fostel and Geanakoplos (2014) – henceforth denoted FG). Naked CDS induce investors to move their capital into derivative markets and out of the underlying bond market. This reduces the demand for bonds and reduces bond prices. The firm responds to lower long term bond prices by substituting a portion of the capital it raises from long term debt into short term debt as shown by the reduction in $\rho$ from 1 to .9173 in Table 5. Naked CDS thus induce some “maturity mis-match” between the payments due on firm liabilities and revenues generated from investment when there otherwise would be no maturity mis-match absent the trading of the derivative.

Maturity mis-match arises in equilibrium and long term bonds cease to be the exclusive funding instrument when agents view the technology shock to be very poor for a simple reason. Naked CDS raise long term bond prices to the point where the benefit to the firm of insulating itself from bad intermediate news is “partially” eroded for certain values of $A_{DD}$. It is only “partially” eroded because the firm still opts to issue mostly long term debt. Instead of issuing all long term bonds at high interest rates, the firm issues risky short term debt that is not rollover over if $s = D$. The firm liquidates assets to repay short term creditors and continues producing using the capital it raised from long term bonds. The firm rolls over the short term debt if $s = U$ at risk free interest rates. Thus, when borrowing costs on long term debt are sufficiently high, the firm finds it optimal not to insulate itself from intermediate
news announcements, rather it uses the news announcements to its advantage to rollover short term bonds risk free in good states, and default on short term debt in bad states.

Additionally, it is precisely the technology shock outcomes where short term borrowing is most costly at time 1 that the firm chooses to issue short term debt *i.e.* very low $A_{DD}$, on which it eventually defaults. This result is similar to the rollover problem identified by Che and Sethi (2014). They argue that conditional on an equilibrium with default, short term debt is not rolled over when investors believe that revenues will be sufficiently low. Specifically, debt financiers’ pessimism about the firm’s ability to repay debts becomes self-fulfilling when naked CDS can be traded. The rollover problem in this model is distinct because it is actually profit maximizing for the firm and there are no CDS on short term debt. The firm can always choose to issue all long term bonds and make positive expected profits even in the presence of naked CDS, which is does for certain parameters (see the white region in Figure 12). The naked CDS on long term bonds induces the firm to issue risky short term debt when the difference between good and bad technology shock realizations is most pronounced. In that sense, asset liquidation is similar to Che and Sethi when expectations about technology outcomes are most pessimistic. The difference in the two results is that naked CDS lead to an *issuance freeze* whereby the firm opts to liquidate assets and not rollover short term debt.

*Existence*

Now consider the black region in Figure 12 labeled No Equilibrium. Naked CDS destroy equilibrium for these parameters consistent with FG (2014). Che and Sethi (2010), Darst and Refayet (2014), and Refayet (2014) circumvent this the non-existence issue by assuming a institutional investor always demands corporate bonds as long as the firm demands investment. For this reason, the assumption of a retail investor always standing by to buy bonds is sufficient to restore equilibrium with naked CDS in other general equilibrium production economy models.

In this model, however, institutional investors are not sufficient to restore equilibrium because the demand for assets is not what collapses when CDS are introduced. The firm raises a portion of its funding through short term debt that is rolled over in full at $s = D$ for the entire black parameter space in Figure 12 in the *Non CDS Economy*. This means that CDS can be written on long term bonds at time 0 and short term bonds at time 1. Introducing naked CDS on the short term bond at time 1 increases the default premium such that it is no longer profitable to raise funds through short term debt at $s = D$. But, because CDS are derivative instruments whose value is derived from an underlying bond, if the underlying bond does not exist, CDS cannot exist. As shown in the *Non CDS Economy*, equilibrium with short term rollover always exists for this parameter space. Hence, even assuming a retail investor’s with sufficient capital to purchase bonds and CDS does not restore equilibrium because the firm does not issue bonds! Thus the *dynamic inconsistency*
problem due to short term debt is present for this region as well. In fact, whenever the firm’s assets and liabilities are not perfectly timed i.e. if any debt is issued that matures before the investment generates revenue, naked CDS destroy equilibrium. The assumption of ever present demand for bonds in a static setting is not sufficient to restore equilibrium in a dynamic setting. Thus, the non-existence result of FG is much more robust in dynamic economies than in static economies.

The remainder of the section describes the set of equations used to solve for equilibrium in the *Naked CDS Economy*. Readers not interested in the technical details should proceed to the paper’s concluding remarks. The ten endogenous variables that solve for equilibrium characterized by both long and short term debt with naked CDS only on long term bonds are \((p_0^\ell, p_0^s, \phi_0^\ell, q_0^\ell, q_0^s, v_0^\ell, I_0, h_0^1, h_0^2, \rho)\). The set of equations which solve the system is:

\[
\left(1 - (1 - h_0^1)^2\right) \left(1 - d_{DD}(q_0^\ell)\right) = \frac{h_0^1 + (1 - h_0^1) d_{DD}(q_0^\ell)}{p_0^\ell} \tag{26}
\]

\[
\frac{h_0^2 + (1 - h_0^2) d_{DD}(q_0^s)}{p_0^s} = \frac{(1 - h_0^2)^2 \left(1 - d_{DD}(q_0^s)\right)}{\phi_0^s} \tag{27}
\]

\[
I_0 = \frac{p_0^\ell q_0^\ell + p_0^s q_0^s}{p_0^s} \tag{28}
\]

\[
\alpha (\rho I_0)^{\alpha - 1} \left[\gamma (1 - \gamma)\right] = \frac{\gamma (1 - \gamma) + \gamma - \gamma}{p_0^\ell} \tag{29}
\]

\[
\alpha I_0^{\alpha - 1} (\gamma + \gamma (1 - \gamma) \rho^\alpha) = \frac{[\gamma + \gamma (1 - \gamma)] \rho + \gamma (1 - \rho)}{p_0^\ell} \tag{30}
\]

\[
\rho = \frac{p_0^\ell q_0^\ell}{I_0} \tag{31}
\]

\[
\frac{(1 - h_0^1)}{1 - \phi_0^\ell - d_{DD}(q_0^\ell)} = q_0^\ell + \left(\frac{h_0^2}{\phi_0^\ell}\right) \tag{32}
\]

\[
\frac{h_0^2 - h_0^1}{p_0^s} = q_0^s \tag{33}
\]

\[
p_0^\ell + \phi_0^\ell = 1 \tag{34}
\]

Equations (26) and (27) are the asset pricing equations. The former says that, at time 0, the marginal buyer \(h_0^1\) is indifferent to selling a naked CDS on the long term bond and buying the short term bond. The latter says that at time 0, marginal buyer \(h_0^2\) is indifferent between buying the short term bond and buying a naked CDS against the long term bond. Equation (28) is the firm funding condition as in previous economies. Equations (29) and (30) are the firm first order conditions w.r.t. the portfolio allocation \(\rho\) and investment \(I_0\), respectively. Equation (31) defines \(\rho\) to be the portion of capital the firm raises by issuing long term bonds as in previous economies.

\[\text{Equation (32) defines the collateral condition for long term bonds.}\]

\[\text{Equation (33) defines the collateral condition for short term bonds.}\]

\[\text{Equation (34) is the budget constraint.}\]

\[\text{The system of equations used to solve for equilibrium in the *Naked CDS Economy* characterized by only long term debt can be found in Appendix B A.5.}\]
Table 5: Naked Results

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$p_0$</th>
<th>$p'_0$</th>
<th>$q_0$</th>
<th>$q'_0$</th>
<th>$I_0$</th>
<th>$\rho$</th>
<th>$E[\pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naked CDS</td>
<td>.6640</td>
<td>.6602</td>
<td>.0053</td>
<td>.0588</td>
<td>.0423</td>
<td>.9173</td>
<td>.0117</td>
</tr>
<tr>
<td>Covered CDS</td>
<td>–</td>
<td>.9551</td>
<td>–</td>
<td>.2727</td>
<td>.2605</td>
<td>1</td>
<td>.0511</td>
</tr>
<tr>
<td>Non-CDS</td>
<td>–</td>
<td>.9460</td>
<td>–</td>
<td>.2625</td>
<td>.2483</td>
<td>1</td>
<td>.0492</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long</th>
<th>$(A_{DD}, \gamma)=(.2, .5)$</th>
<th>$p_0$</th>
<th>$p'_0$</th>
<th>$q_0$</th>
<th>$q'_0$</th>
<th>$I_0$</th>
<th>$\rho$</th>
<th>$E[\pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naked CDS</td>
<td>–</td>
<td>.7074</td>
<td>–</td>
<td>.0821</td>
<td>.0581</td>
<td>1</td>
<td>.0154</td>
<td></td>
</tr>
<tr>
<td>Covered CDS</td>
<td>–</td>
<td>.9680</td>
<td>–</td>
<td>.2877</td>
<td>.2785</td>
<td>1</td>
<td>.0539</td>
<td></td>
</tr>
<tr>
<td>Non-CDS</td>
<td>–</td>
<td>.9512</td>
<td>–</td>
<td>.2682</td>
<td>.2551</td>
<td>1</td>
<td>.0503</td>
<td></td>
</tr>
</tbody>
</table>

Economies. Equations (32) and (33) are respectively the time 0 market clearing conditions for long term naked CDS and short term bonds. Lastly, (34) is the no arbitrage condition stating that owing the long term bond with a CDS is equivalent to holding cash.

5 Conclusion

This paper finds that covered CDS tend to lengthen the debt maturity of corporate bonds, consistent with recent empirical evidence. Covered CDS raise bond prices. The benefits of lower bond prices on long term bonds can be realized all at the date of issuance. The benefits of lower borrowing costs on short term bonds only materialize in states where bad news raises the expected likelihood of default. However, the presence of naked CDS may induce firms to issue risky short term debt when it otherwise would issue exclusively long term debt. This raises the possibility of maturity mis-match in the real sector where a portion of real long term investment is financed through short term bonds. Finally, naked CDS cause a debt issuance freeze when any portion of investment is finance through short term debt, even if there are always investors standing buy willing to buy bonds.
References


A Appendix

A.1 Baseline: Short Term Funding

The following equations are used to solve the baseline model when the firm only issues short term debt ($\rho = 0$).

\[
\begin{align*}
    p_{st} & = 1 \\
    1 & = \frac{h_{1,D} + (1 - h_{1,D}) R_{st}}{p_{st1,D}} \\
    I & = p_{st} q_{st} \\
    \alpha I^{\alpha-1} [\gamma + \gamma (1 - \gamma)] & = \frac{\gamma}{p_{st}} + \frac{\gamma(1 - \gamma)}{p_{st} p_{st1,D}} \\
    q_{st} & = \frac{p_{st1,D} q_{st1,D}}{p_{st1,D}} \\
    1 - h_{1,D} & = q_{st1,D}
\end{align*}
\]

All time 0 short term debt is risk free and priced accordingly because the firm commits to rollover at time 1. The marginal buyer at time 1 who buys short term bonds is indifferent between buying short term bonds and cash. Thus, in equilibrium, the returns of the two assets must be equivalent according to the marginal investor’s beliefs. All investment that the firm raises will come through issuing short term debt. The optimal amount of capital the firm raises equates the expected marginal product of issuing short term bonds against the expected marginal cost of repayment. The marginal cost of issuing more short term debt at time 0 is reflected by the fact that
Table 6: Covered Short Term - Results

<table>
<thead>
<tr>
<th>$(A^{DD}, \gamma) = (0.2, 0.8)$</th>
<th>$p_{st}$</th>
<th>$p_{st1,D}$</th>
<th>$q_{st}$</th>
<th>$q_{st1,D}$</th>
<th>$I$</th>
<th>$\rho$</th>
<th>$E[\pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covered CDS</td>
<td>1</td>
<td>0.8375</td>
<td>0.2795</td>
<td>0.3337</td>
<td>0.2795</td>
<td>0</td>
<td>0.0692</td>
</tr>
<tr>
<td>Non-CDS</td>
<td>1</td>
<td>0.8223</td>
<td>0.2746</td>
<td>0.3339</td>
<td>0.2746</td>
<td>0</td>
<td>0.0683</td>
</tr>
</tbody>
</table>

it will be rolled over at time 1 where the firm must pay interest according to time 1 interest rates. All time 0 short term debt will be paid in full by issuing more short term debt at time 1. Lastly, investor demand for time 1 short term bonds is equal to the supply of bonds the firm issues at time 1, which are used to repay time 0 short term creditors.

A.2 Baseline: Long Term Funding

The following equations are used to solve the baseline model when the firm only issues long term debt ($\rho = 1$).

\[
1 = \frac{1 - (1 - h_0)^2 + (1 - h_0)q_{lt}}{p_{lt}}
\]

\[
I = \frac{p_{lt}q_{lt}}{p_{lt}}
\]

\[
\alpha I^\alpha - 1 = \frac{1}{p_{lt}}
\]

\[
\frac{1 - h_0}{p_{lt}} = q_{lt}
\]

The long term bonds are priced by the marginal investor who is indifferent between holding bonds and cash. The marginal buyer’s evaluation of long term bond returns in equilibrium will be equal to cash. All capital raised by the firm for production will come through the long term bond market. The optimal amount of capital the firm raises will equal the marginal product of issuing an additional bond to the marginal cost of repayment. Lastly, investor demand for long term bonds at time 0 will equal the supply of bonds the firm issues.
A.3 Covered: Short Term Funding

The following equations are used to solve the model when the firm only issues short term debt ($\rho = 0$) in a Covered CDS Economy.

$$p_{st} = 1$$
$$1 = \frac{h_{1,D} + (1 - h_{1,D}) R_{st}^{*}}{p_{st1,D}}$$

$$I = p_{st}q_{st}$$
$$\alpha I^{-1}[\gamma + (1 - \gamma) (1 - \gamma) (1 - \gamma)] = \gamma + (1 - \gamma)$$
$$q_{st} = p_{st1,D}q_{st1,D}$$

$$q_{st1,D} = \frac{1 - h_{1,D}}{1 - R_{st}^{*} - \phi_{st1,D}}$$

The short term funding economy is analogous to the model in the baseline economy, with the exception of the bond market clearing condition at time 1 since the demand for bonds is implicitly determined by the CDS market. The CDS market clearing condition simply states that the investor demand for (covered) CDS is equal to the supply of CDS.
Figure 15: Covered - Long Term

\[ h = 1 \]
\[ h_0 = .7934 \]
\[ t = 0 \]

LT CDS Sellers

Table 7: Covered Long Results

<table>
<thead>
<tr>
<th>((A_{DD}, \gamma)=(.2, .5))</th>
<th>(p_0)</th>
<th>(q'_0)</th>
<th>(I_0)</th>
<th>(\rho)</th>
<th>(E[\pi])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naked CDS</td>
<td>.7074</td>
<td>.0821</td>
<td>.0581</td>
<td>1</td>
<td>.0154</td>
</tr>
<tr>
<td>Covered CDS</td>
<td>.9680</td>
<td>.2877</td>
<td>.2785</td>
<td>1</td>
<td>.0539</td>
</tr>
<tr>
<td>Non-CDS</td>
<td>.9512</td>
<td>.2682</td>
<td>.2551</td>
<td>1</td>
<td>.0503</td>
</tr>
</tbody>
</table>
A.4 Covered: Long Term Funding

The following equations are used to solve the baseline model when the firm only issues long term debt ($\rho = 1$).

\[
1 = \frac{1 - (1 - h_0)^2 + (1 - h_0) d_{DD} \left(q_0^\ell\right)}{p_0^\ell}
\]
\[
I = p_0^\ell q_0^\ell
\]
\[
\alpha I_0^{-1} = \frac{1}{p_0^\ell}
\]
\[
q_0^\ell = \frac{1 - h_0}{1 - \phi_0^\ell - d_{DD} \left(q_0^\ell\right)}
\]

The long term funding Covered CDS Economy is analogous to long term funding in the Non CDS Economy, with the exception of the time 0 bond market clearing condition, because the demand for bonds is implicitly determined by the CDS market. The CDS market clearing condition simply states that demand for (covered) CDS—which is equivalent to the number of bonds issued—is equal to the supply of CDS issued by CDS sellers.

A.5 Naked: Long term funding

Consider the second possible equilibrium characterized by long term with CDS. In equilibrium, as a result of linear utilities and the continuity of utility in $h$, and the connectedness of the set of agents $H = (0, 1)$, at state $s = 0$ there will be marginal buyer $h_1$. Every agent $h > h_1$ will sell CDS on long term bonds, every agent $h < h_1$ will buy naked CDS on long term bonds. This regime is also shown in Figure 13. The same parameters used to solve for long term funding in both the baseline and covered CDS economies are used to solve for long term funding in the naked CDS economy as well. The five endogenous variables in this economy are $(p_0^\ell, q_0^\ell, \phi_0^\ell, I_0, h_1)$. The system of equations used to solve for these variables is:

\[
\frac{(1 - (1 - h_1)^2) + (1 - h_1)^2 d_{DD} \left(q_0^\ell\right)}{p_0^\ell} = 1
\]  \hspace{1cm} (35)
\[
\frac{1 - h_1}{1 - \phi_0^\ell - d_{DD} \left(q_0^\ell\right)} = q_0^\ell + \left(\frac{h_1}{\phi_0^\ell}\right)
\]  \hspace{1cm} (36)
\[
\alpha I_0^{-1} = \frac{1}{p_0^\ell}
\]  \hspace{1cm} (37)
\[
I_0 = p_0^\ell q_0^\ell
\]  \hspace{1cm} (38)
\[
\phi_0^\ell + p_0^\ell = 1
\]  \hspace{1cm} (39)

Equation (35) says that the marginal CDS seller prices long term bonds such that his expected return on a long term bond is equivalent to cash. Equation (36) is CDS market clearing which says that supply of naked CDS issued by CDS sellers is equal...
to the demand for CDS coming from both covered CDS held by the retail investor and naked CDS from CDS buyers. Equation (37) equates the firm’s marginal product of investment through long term bonds with the marginal cost of issuing an additional bond. (38) is the firm’s funding which says that all capital raised from debt goes into production. Lastly, (39) is the non arbitrage condition which says that holding a long term bond with the CDS is equivalent to holding cash.

\[ B \text{ Appendix B} \]

\[ \text{B.1 Proof of Lemma 3} \]

The proof follows directly from Lemma 1 from Darst & Refayet (2014). If an investor at either time 0 or time 1 would buy a bond unprotected, then there must be an investor who is indifferent to selling a CDS on that bond and purchasing that bond. Lemma 1 shows that if that is the case, then the investor selling the CDS is also indifferent to holding a risk free asset. Thus there is no investor who believes that the bond returns more in expectation than the risk free asset, hence the bond goes un-purchased.

\[ \text{B.2 Formalization of Retail Investor} \]

In this Appendix I present one possible way to formalize the retail investor introduced in Section 4.1.

Let the utility of the retail investors in each period be identical and given by:

\[ U^{M}(x_{s}, c_{s^*}) = x_{s} + \sum_{s^* \in S} c_{s^*} \epsilon_{s^*}, \quad \epsilon_{s^*} > 0, \quad s \in S_{T} \]

where \( x_{s}, s \in S_{T} \) is time 2 consumption and \( c_{s^*}, s^* \in S \) is a covered CDS package made up of a bond and corresponding CDS: \( c_{s^*} \equiv q_{s^*}^{\ell, \xi} + v_{s^*}^{\ell, \xi}, s^* \in S \).\(^{23}\) Furthermore, let \( U^{M_{0}}(c_{s^*}) > 0 \) so that the investors always prefer to invest when the opportunity exists. We can now characterize the retail investors’ budget sets. First consider the retail investor at time 0:

\[ B^{M_{0}}(p^{0, \xi}, \phi^{0}) = \left\{ \left( x^{M_{0}}_{0}, q^{\ell, M_{0}}, q^{\xi, M_{0}}, v^{\ell, M_{0}}, v^{\xi, M_{0}}, x^{M_{0}}_{s} \right) \in R_{+} \times R_{+} \times R_{+} \times R_{+} \times R_{+} \right\} : \]

\[ x^{M_{0}}_{0} + (p^{0} + \phi^{0}) c^{\ell, M_{0}}_{0} + (p^{0} + \phi^{0}) c^{\xi, M_{0}}_{0} = e^{M_{0}}_{0}, \]

\[ x^{M_{0}}_{s} = x^{M_{0}}_{s} + c^{\ell, M_{1}}_{s} + c^{\xi, M_{0}}_{s}, \quad s \in S_{T} \].

\[ ^{23}\text{The micro-foundations of the investor’s preferences are not explicitly modeled, though it is easy to do so. For example, one could assume the investor is a fund manager who invests in covered CDS for which he receives a fee given by } \epsilon. \]
Time 0 investor uses his endowment to either purchase bonds with covered CDS or stores it for consumption. He consumes the same amount in all terminal states since covered CDS positions have identical payouts in all states e.g. 1. Thus, final consumption is the sum of cash carried forward and the number of covered CDS packages purchased at time 0. Similarly time 1 retail investor’s budget set is

\[
B^{M_1} (p_s^*, \phi_s^*) = \left\{ \left( x_0^{M_0}, q_s^{M_1}, u_s^{M_1}, x_s^{M_1} \right) \in R_+ \times R_+ \times R_+ \times R_+ \right\}:
\]

\[
x_0^{M_1} + (p_s^* + \phi_s^*) c_{s}^{M_1} = e_s^{M_1}, \\
x_s^{M_1} = x_0^{M_1} + c_{s}^{M_1}, s \in S_T.
\]

Time 1 retail investor uses his endowment to either purchase bonds with covered CDS or stores it for consumption. He consumes the same amount in all states since covered CDS positions have identical payouts in all states e.g. 1. Thus, final consumption is the sum of cash carried forward and the number of short term covered CDS packages purchased at time 1.