European Capital Flows.
Sovereign Debt, Intermediation and Trade
Preliminary draft. Link to latest version

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Abstract

The experience of the European Monetary Union was characterized by two distinct phases. An initial one witnessed growing capital flow imbalances between core and periphery countries. Capital flows from the rest of the world were also intermediated by core countries, financing deficits in the periphery. In a second phase, a severe sovereign debt crisis took place, with spillovers and recessions in multiple countries. What is the relationship between the two phases? Can features of the EMU help explain the observation of imbalances, intermediation of capital flows and crisis spillovers? I argue that subsidies on cross-border holdings of euro-denominated assets have contributed to all three phenomena, by inducing a relaxation of borrowing costs and cross-border financial linkages. The interaction between trade and financial linkages amplifies, in turn, the severity of a debt crisis. I outline the theoretical mechanism behind these results by means of a simple, two-period model, where countries in an economic union trade financial assets with the rest of the world. In an infinite horizon, heterogeneous countries model, I show that the introduction of a subsidy on asset holdings generates predictions for net and gross asset flows that closely resemble the EMU experience. In addition, the model generates crisis contagion across borrowers, with heterogeneous amplification depending on countries’ exposure to trade with union partners.

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1 Introduction

The experience of the European Monetary Union was characterized by two very distinct phases. In an initial period, lasting approximately from the inception of the union itself until the global financial crisis, a phenomenon of widening current account imbalances was observed. In addition, gross capital flows also rose, with surplus countries in the euro area issuing gross liabilities to the rest of the world to fund lending to deficit ones. In the aftermath of the global financial crisis, a second phase emerged, characterized by the occurrence of a sovereign debt crisis, a collapse in net flows, and recessions in multiple countries. What is the relationship between these two phases? Can institutional features of the European Monetary Union help explain the observed pattern of current account imbalances, intermediation of capital flows and crisis? I document the presence of regulatory and policy frictions in the European Monetary Union, subsidizing holdings of risky assets issued within the union itself. I analyze the effect of such frictions on net and gross capital flows and welfare in a model with heterogeneous countries, trading assets among each other and the rest of the world.

It is a well known fact that, in the initial phase of the European Monetary Union, a pattern of widening current account deficits and surpluses was observed in the euro area. In the so-called group of core countries, comprising Austria, Belgium, France, Germany and the Netherlands, the total current account surplus approached, at its peak, 3% of total GDP in the monetary union. An aggregate deficit of similar magnitude was observed in the group of debtor countries composed by Greece, Ireland, Italy, Portugal and Spain.

In addition to net flows, gross capital flows also increased substantially in this period. The sum of gross external asset and liabilities of the euro area, a common measure of financial integration, rose to 350% of GDP in 2007, a figure almost twice as large as the one observed in the United States, as shown by Waysand et al. (2010). In core countries in particular, both gross assets and liabilities increased substantially in magnitude, with a distinctive pattern emerging in terms of geographical location of counterparties. Against the rest of the euro area, core countries held bilateral net foreign assets amounting to
15% of total area GDP in 2007, as Chen et al. (2013) document. At the same time, the bilateral net foreign asset position of this countries vis-à-vis the rest of the world was negative, with liabilities amounting to almost 10% of euro area GDP. Gross liabilities of the so-called periphery were mostly held, instead, by core countries in the euro area. The heterogeneity in counterparties on the two sides of core countries’ external balance sheet can be interpreted as evidence of intermediation of capital flows from the rest of the world, which were ultimately directed towards countries in the core euro area periphery.

The presence of large capital flows has typically been cause of concern, especially in presence of rigidities hindering the adjustment from a phase of high deficits and surpluses, as discussed by Blanchard (2007). The European sovereign debt crisis is a prime example of a painful adjustment phase. Debtor economies suffered from severe recessions between 2010 and 2013, which compounded the previous contraction in correspondence of the global financial crisis. As well as in the periphery, economic performance was muted in the core of the euro area, too. Since 2011, the recovery in core countries slowed down significantly, with GDP growing by 1% only in core countries in the three years to 2013. In comparison, in the United States, GDP grew by 7% in the same period.

This paper analyzes the relationship between net and gross capital imbalances in the euro area and the subsequent crisis, focusing on the key role played by distortions subsidizing holdings of risky assets issued within the euro area. Several institutional features of the European Monetary Union have subsidized ownership by union residents of assets issued by other members of the euro area. Regulation on capital requirements in the European Union allowed financial institutions not to hold bank capital against euro-denominated government debt of euro area countries, thanks to the presence of a zero risk weight. Lack of discrimination by nationality of sovereign debt collateral in transactions with the European Central Bank provided an additional incentive for institutions accessing ECB facilities in holding risky government debt.

I introduce a simple model of international capital flows between two economies in an economic union and the rest of the world. In the model, holders of sovereign debt issued in the union are offered a subsidy in the form of a bailout promise in the event
of default by borrowers. The model serves to outline three main effects of a subsidy on asset holdings, and it can be used to assess its implications on welfare. First, the presence of a subsidy induces intermediation of capital flows. Agents enjoying a subsidy leverage by borrowing from the rest of the world, to arbitrage high returns they perceive as risk-free. Second, it induces widening of current account balances: Higher supply of funds reduces borrowing costs for debtors, allowing for larger current account deficits. As saving becomes more attractive due to the subsidy, current account surpluses increase as well. Finally, in the event of default, the interaction between financial linkages and trade integration in an economic union exacerbates the severity of the crisis. The crisis transmits to saving countries via a reduction in the value of their assets, as their balance sheet expanded with purchases of risky sovereign debt. For debtors, the recession is made more severe by the fall in consumption in intermediating countries, as these coincide with their major export destinations.

The introduction of a subsidy on holdings of risky assets can be welfare-improving for the union as a whole. In a second-best world, the presence of limited commitment frictions that make sovereign debt risky prevents borrowers from allocating consumption evenly across states of the world. By distorting the problem of lenders in the union, a bailout promise induces a relaxation in debtors’ borrowing costs, transferring resources across countries in the union. If the borrowing constraint faced by debtors is sufficiently severe, the gains from redistribution of resources across countries can be shown to outweigh the cost of distorting lenders’ problem, even when a full bailout promise is introduced.

In a dynamic general equilibrium model with heterogeneous countries, I show that the introduction of a bailout promise induces capital flows dynamics that closely resemble the experience of the European Monetary Union. Starting from an allocation with no bailout promises, I consider the effects of the introduction of this distortion, letting the model economy transition towards its new stationary equilibrium. Two main patterns emerge. First, economies with relatively high debt in the wealth distribution react to the relaxation in borrowing costs by running larger current account deficits, letting their net foreign asset position deteriorate as they converge towards the new stationary allocation.
Second, wealthier economies choose to further accumulate foreign assets, in order to reap benefits of the subsidy on asset holdings. In addition, they issue liabilities to the rest of the world, intermediating these funds to deficit countries by purchasing gross assets.

The dynamic model also serves to analyze the transmission of a debt crisis in an economic union. The reduction in the value of savers’ gross assets following a crisis causes an increase in borrowing costs for all debtors. Following the increase in interest rates, debtor economies deleverage, reducing consumption to limit their exposure to financial markets. The spillover effects of the crisis are heterogeneous across union members, depending on their exposure to trade with the rest of the union. As demand is depressed for debtors and savers alike, economies with substantial trade linkages to other union members suffer from a reduction in export demand. A fall in the terms of trade occurs as countries are attempting to run trade surpluses, making the adjustment process more difficult for debtors in a crisis. This mechanism highlights how the interaction between financial and trade linkages can generate perverse feedback effects in crisis times, for economies less diversified in terms of destinations of exports.

The next subsection reviews the contribution of this paper to the existing literature. Section 2 presents evidence on net and gross capital flows in the euro area, on the presence of distortions that affected trade in euro-denominated debt and on heterogeneous exposure to intra-union trade for European countries. In section 3, I analyze the implications of a subsidy on asset holdings on current account imbalances, intermediation and crisis spillovers in a two-period model. Section 4 presents the effects of the introduction of a bailout promise in a dynamic model with heterogeneous countries. Section 5 concludes.

1.1 Literature review

This paper contributes to a large literature on international capital flows. A growing body of research has analyzed in particular the recent European experience, motivated by the stark developments observed in terms of current account balances, gross financial positions as well as by the sovereign debt crisis.

Chen et al. (2013) present clear evidence on the pattern of intermediation of capi-
tal flows via core countries in the euro area. These authors also suggest that implicit bailout promises and preferential regulatory treatment on euro-denominated assets may have contributed substantially to generate this pattern. This paper follows from that suggestion, analyzing the role of policy distortions for net and gross capital flows in the context of a model, further looking at their implications for welfare in the economic union and in the event of a debt crisis.

The empirical observation that motivates this work, on current account imbalances and growing gross asset and liability positions, has been documented by the seminal contribution of Lane and Milesi-Ferretti (2007). In addition, Waysand et al. (2010) document bilateral gross positions for European economies, providing a detailed description of the financing of current account imbalances. Lane (2013) also provides rich empirical evidence on net and gross flows of capital in the European Monetary Union.

The presence of institutional distortions affecting holdings of EMU-issued assets for residents of the union itself has also been widely documented. Buiter and Sibert (2005) focus on aspects of collateral eligibility rules in transactions with the European Central Bank that subsidized risky sovereign debt issued by euro area governments. Coeurdacier and Martin (2009) and Kalemli-Ozcan et al. (2010) highlight the role of the removal of currency risk implied by the introduction of the euro. The role of such institutional features in affecting capital flows, especially via the banking sector, has also been analyzed by Acharya and Steffen (2013), focusing on behavior by European banks between 2007 and 2012, at the peak of the sovereign debt crisis. Hale and Obstfeld (2014) detail how in the years following the inception of the monetary union, the role of banks in the euro area core was crucial in determining current account imbalances and intermediation of capital flows, also attributing the cause of such behavior to the institutional distortions present in the framework of the European Monetary Union.

From a modelling perspective, this paper builds on the approach developed by Clarida (1990). There, a continuum of countries trade claims on financial markets, giving rise to a stationary wealth distribution. I extend that framework by considering trade in financial assets with the rest of the world and the presence of a bailout promise on risky
assets. Fornaro (2014) studies the implications of a debt deleveraging episode in a similar framework, in presence of nominal rigidities. The assumption of heterogeneous transport costs affecting trade in different goods was originally introduced in this context by Sachs (1982).

This paper shares with Mendoza (2010), Bianchi (2011) and Benigno et al. (2013) the assumption that households fail to internalize a key externality when interacting with international financial markets. These authors highlight the role of pecuniary externalities due to feedback effects between external borrowing and the relative price of output. In the framework here introduced, households that are offered a bailout promise do not internalize default risk associated with their purchases of risky assets.

The effects of a debt crisis in an economic union have also been analyzed thoroughly in the literature. The emphasis of this paper is on understanding what factors give rise to financial linkages among union members, as well as understanding their implications on the magnitude of net flows and their interaction with trade linkages, especially in the event of a crisis. Guerrieri et al. (2013) study the transmission of a debt crisis in the periphery through the banking sector of core countries, while Arellano and Bai (2014) study transmission of debt crises across debtors borrowing from a common lender. Broner et al. (2014) introduce a model where sovereign debt crises have real effects by crowding out investment. In addition, they introduce an economic union whose residents enjoy preferential treatment in the event of default, similarly to the model here presented. Benigno and Romei (2014) analyze a debt deleveraging episode causing substantial fluctuations in relative prices and the terms of trade, giving rise to heterogeneous welfare implications across countries, a feature common to the crisis event here considered. An empirical analysis of the recent Eurozone sovereign debt crisis is provided, among others, by Lane (2012) and Neri (2013).
Figure 1: Current account balances of core and periphery countries in the euro area, as fraction of total euro area GDP. Core includes Austria, Belgium, France, Germany and Netherlands. Periphery includes Greece, Ireland, Italy, Portugal and Spain.

2 Empirical evidence

2.1 Current account imbalances in the Euro area

The observation that current account dynamics of economies in the Euro area displayed significant imbalances in the years preceding the global financial crisis has been widely documented. Figure 1 displays aggregate current account balances for the two subsets of so-called “core” and ”periphery” members of the monetary union, expressed as ratio to euro area GDP. The figure clearly shows the widening gap between countries in these groups in terms of current account balance. The current account surplus in core countries rose above 2% of euro area GDP in 2007 from a balanced position in 2000, while in the periphery the current account deficit doubled in the same period, also reaching 2% of euro area GDP in 2007.

The significance of the observed pattern for current account balances with respect to

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1See, e.g., Lane (2013) and papers cited in section 1.1.
2Core includes Austria, Belgium, France, Germany and Netherlands. Periphery includes Greece, Ireland, Italy, Portugal and Spain.
individual economies can be appreciated by disaggregating these data at country level, as reported in Figure 2. We observe that, among core countries, the rise in current account surplus is particularly large in Germany, where it reaches 7% of GDP in 2007, with Austria and the Netherlands displaying a similar pattern. The surplus diminishes in magnitude in France and Belgium, turning into a deficit towards the end of the sample. Among periphery countries, we observe increasing current account deficits in all countries but Portugal, which started from a large deficit position at the beginning of the sample. In 2008, the current account deficit was above 5% of GDP in Ireland and Spain, and above 10% in Portugal and Greece.

2.2 Gross capital flows and intermediation

The euro area is characterized by the presence of sizable gross asset and liability positions among member economies, as well as by a pattern of intermediation of capital flows from the rest of the world towards debtors in the Union.

The largest fraction of total assets held by core economies in the euro area is constituted by claims issued by partners in the European Union, as documented by Waysand
et al. (2010). As the mirror image, liabilities of periphery countries were mostly held by savers within the European Union. The bilateral net foreign asset position of selected core and periphery euro area economics vis-à-vis all financial partners and with respect to EU partners, only is displayed in figure 3. Bilateral net foreign asset positions are shown for 2002 and 2008, as a fraction of GDP in individual economies. In core euro area countries, Germany, Belgium and France, growth in bilateral net foreign asset positions against European partners was larger than overall growth observed in this period, as shown in the upper panel of this figure. In France, in particular, while the total net foreign asset position turned negative in this period, the country accumulated positive claims against the rest of the European Union. As France was accumulating gross assets with respect to European partners, then, it was accumulating a larger volume of gross liabilities with respect to extra-EU lenders. A similar pattern of intermediation, to a lower extent, was also taking place in Belgium, while Germany increased its net savings with respect to all economies.

The increase in total net foreign liabilities observed in debtor countries can instead be almost completely explained by an increase in their net liabilities towards European counterparties, as shown in the bottom panel of Figure 3. In Italy, Greece, Spain and Portugal, in particular, the bilateral net foreign asset position with respect to extra-EU partners was relatively stable while these countries were accumulating debt, largely against other economies in the European Union.

This evidence shows how capital flows from the rest of the world directed to debtor countries in the periphery of the euro area were intermediated by their European partners. The next subsection presents evidence on features of regulation and policy in the European Monetary Union that contributed to the observed pattern of intermediation of international capital flows.
Figure 3: Bilateral net foreign asset positions of selected European economies. Light colored bars represent net foreign asset positions against all counterparties (World), as a fraction of GDP of individual economies. Dark colored bars represent bilateral net foreign asset positions with respect to partners in the European Union, only. Data are presented for years 2002 and 2008. An increase in the bilateral net foreign asset position with respect to European counterparties matched by a smaller, or negative, increase in the overall position implies that a country is simultaneously accumulating net assets against EU partners and net liabilities against economies in the rest of the world. Source: Waysand et al. (2010).

2.3 Distortions on asset holdings in the European Monetary Union

Several aspects of policy and regulation in the European Monetary Union are likely to have contributed to the pattern of intermediation of capital flows observed in the euro area. Financial institutions operating in the European Union were allowed by regulators not to set aside capital against holdings of sovereign debt issued by governments of member states. Specifically, according to the Capital Requirement Regulation of the European Commission, when measuring risk-weighted assets for regulatory purposes,
financial institutions were allowed to assign a “zero-risk weight” to government debt of EU member states denominated in own currency. Such a provision may have provided financial institutions in the European Union with a comparative advantage in holding government debt issued within the Union, either respecting to banks in the rest of the world that were subject to less lenient regulation.

Banking systems in the euro area were, indeed, highly exposed to debt issued within the EU. Evidence on the average exposure of individual banking systems to sovereign and total debt of large euro area borrowers is presented in Table 1. These data summarize results of a 2011 stress test on EU banks, as published by the European Banking Authority and OECD (2011a). The small size of the Greek economy implied that total exposure to Greek debt of French and German banks was limited to 27.2% and 15.7%, respectively, of regulatory Tier I capital. Exposure was significantly higher when considering exposure of these banking systems to Belgian, Italian and Spanish debt, rising to 312% and 203% and highlighting their vulnerability to adverse fluctuations in the value of debt issued by these countries.

Exposure to periphery debt was highly heterogeneous across individual financial institutions. The extreme example of the Franco-Belgian banking group Dexia is documented in Acharya and Steffen (2013). The size of the sovereign bond portfolio of this institution amounted to “almost three times of its book equity”, and it was largely composed of Italian and Greek government debt. In the second half of 2011, as sovereign bond prices fell and this bank found it harder to access sources of funding, Dexia received a bailout from the governments of Belgium, France and Luxembourg.

The possibility that bailout expectations may have led financial institutions in the euro area to perceive assets issued by different economies within the monetary union as more substitutable than investors in the rest of the world did is also suggested by Chen et al. (2013). Moreover, these authors highlight how the possibility to use euro-denominated debt as collateral with the European Central Bank made this type of asset

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3 Regulation (EU) No 575/2013, Part Three, Title II, Chapter 2, Section 2, Article 114.4 contains the most recent version of relevant legislation.
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<td>Portugal</td>
<td>8.5</td>
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Table 1: Average exposure, by nationality of banks, in percentage of core, Tier I capital, to Greek and Belgian, Italian and Spanish debt. Source: OECD (2011a) and European Banking Authority, 2011 EU-wide stress test.

relatively more attractive for those financial intermediaries accessing financing by this institution. Buiter and Sibert (2005) also point out how regulation on collateral eligibility for ECB transactions weakened fiscal discipline in the euro area, by implicitly subsidizing purchases of risky government debt. Coeurdacier and Martin (2009) highlight how the removal of exchange rate risk implied by the monetary union constituted a reduction in financial transaction costs, further contributing to additional lending by core economies to euro area-periphery ones.

In the model presented in this paper, I introduce institutional distortions giving preferential treatment to residents of an economic union when purchasing risky issued assets issued in the union itself as a simple bailout promise. While this modeling choice does not allow to separately evaluate the contribution of the different aspects of regulation highlighted above, it is sufficient to characterize in a simple way subsidies and distortions leading agents in an economic union to intermediate capital flows directed towards fellow member economies.
2.4 Trade integration and specialization

Trade with partners in the European Union accounted in 2005 for approximately two thirds of total trade by EU-28 economies. Geographical proximity, similarity in tastes and explicit trade liberalization make it an unsurprising fact that European economies largely trade with each other, highlighting the high degree of goods market integration characterizing the Union.

Economies in the European Union, however, differ significantly in terms of relative specialization in those industries whose output is more extensively traded internationally. Manufacturing accounted in 2005 for 23% of Gross Value Added (GVA) in Germany, while representing only 10% and 16% of GVA in Greece and Spain, respectively. Peripheral economies, on the other hand, are relatively specialized in industries characterized by a lower degree of tradability of output. In the same year, construction accounted for only 4% of GVA in Germany, rising to 6% and 12% in Greece and Spain.

In order to document the asymmetric pattern of specialization of European economies, accounting for heterogeneity in tradability of output produced by different industrial sectors, I construct a country-level measure of average tradability of output. I define tradability of individual industrial sectors as the ratio, for each sector, of total exports and imports attributed to that sector in a sample of advanced economies, to total Gross Value Added (GVA) of that sector. Formally, tradability of sector $s$ is defined as

$$\text{TDTY}_s = \frac{\sum_{i \in I} \text{EXP}_{s,i} + \text{IMP}_{s,i}}{\sum_{i \in I} \text{GVA}_{s,i}}$$

where EXP$_{s,i}$ and IMP$_{s,i}$ represent total exports and imports attributed to sector $s$ in country $i$. $I$ represents the set of all countries in my sample. Hence, $\sum_{i \in I} \text{GVA}_{s,i}$ represents total GVA of sector $s$ in the entire sample.

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[5]In the System of National Accounts, GVA “is defined as the value of output less the value of intermediate consumption and is a measure of the contribution to GDP made by an individual producer, industry or sector.”.

[6]Data are drawn from the OECD Structural Analysis (STAN) database.

[7]A complete description of the data used to construct this measure is presented in Appendix ??.
At country level, average tradability of output is given by the average of tradability in all industrial sectors, weighted by the shares of GVA represented by each sector in the economy of each country. Formally, this is given by

\[ TDTY_i = \sum_{s \in S} TDTY_s \frac{GVA_{s,i}}{GVA_i} \]

where \( S \) represents the set of all sectors that compose the economy and \( GVA_i = \sum_{s \in S} GVA_{s,i} \) represents total GVA in economy \( i \).

Figure 4 presents values of average tradability of output for EU economies. Consistently with the observation that tradable sectors such as manufacturing are under-represented in these economies, Greece, Spain and Portugal feature low average tradability of output. On the other hand, average tradability is high in Germany, especially in comparison to other very advanced economies. This variable, in fact, is closer in Germany to the level encountered in recent accession countries such as Hungary and Slovenia than in the United Kingdom or France. Low values for average tradability of output observed in Southern European economies are due to relative specialization in these countries towards those sectors characterized by lower sectoral tradability. Detailed evidence on EU economies’ sectoral specialization is presented in Appendix ??.

In the context of the model presented in the following section, I will study how specialization in industries characterized by lower tradability determines an amplification of the adverse effects of a debt crisis, when debt is owed to partners in an economic union. The ability of a country to generate trade surpluses through exports, in fact, is hampered by weak economic conditions in union partner economies if industrial specialization determines a high degree of exposure to export demand from these countries.

3 A simple model of default, intermediation and trade

Consider a two-period world economy. Time is discrete and indexed by \( t \in \{1, 2\} \). The world economy is inhabited by two small open economies, \( H \) and \( F \), and by the rest of the world. This measure is analogous to that of tradedness in Betts and Kehoe (2001).
Figure 4: Average tradability of output in the European Union. This measure is computed as the weighted average of sector-level tradability of output, with weights given in each country by the shares of total GVA corresponding to each sector. Tradable of output at the sector level is given by the ratio of total exports and imports attributed to that sector in all countries in my sample, to total GVA of that sector. Data source: OECD STAN and TiVA databases, year 2005.

the world, ROW. The ROW is characterized as a large, risk-neutral agent. All countries receive endowments of two types of consumption good, good A and good B. All goods are tradable among all countries, subject to transport costs that are heterogeneous across country pairs and good types. H is subject to a limited commitment friction, and it cannot promise to always repay external debt issued in the initial period. Households in F enjoy a subsidy when purchasing H-issued assets, in the form of a bailout promise in the event of default by H.I will use the expression “economic union” when referring to the set of countries H and F.

9For simplicity, F is not subject to limited commitment frictions. This assumption, which is not essential, will be relaxed in the infinite horizon model presented in section 4.1.
Country H  

H is inhabited by a continuum of identical households and by a benevolent fiscal authority. In each period, the representative household derives utility from consumption of goods A and B, according to the aggregator:

\[ c_t = \frac{(c_{A,t})^a (c_{B,t})^{1-a}}{a^a (1-a)^{1-a}}. \]  

(1)

The parameter \( a \) governs the utility weight of consumption good A, \( c_{A,t} \). \( c_{B,t} \) denotes consumption by the representative household in H of good B and \( c_t \) denotes its aggregate consumption.

Lifetime utility of the representative household is given by:

\[ u(c_1) + \beta E[u(c_2)], \]

(2)

where \( 0 < \beta < 1 \) is the subjective discount factor and \( E[\cdot] \) is the mathematical expectation operator. The period utility function, \( u(\cdot) \), is increasing and concave. I will impose that \( u(\cdot) \) takes a logarithmic form: \( u(c) = \log(c) \).

H receives a constant stream of of endowments of good B: \( y_{B,1} = y_{B,2} = y_B \). The stream of good A received by H is stochastic and skewed towards the terminal period:

\[ y_{A,1} = y_A - \epsilon \]

where \( y_{A,1} \) denotes initial-period endowment of good A and \( \epsilon > 0 \) represents the amount by which it falls short of a long-run value given by \( y_A \). In the terminal period, H faces income risk: it only receives the amount \( y_A \) with probability \( \pi \) and, with probability \( 1 - \pi \), a lower amount \( y_{A,L} < y_A \):

\[ y_{A,2} = \begin{cases} y_A & \text{w.p. } \pi \\ y_{A,L} & \text{w.p. } 1 - \pi. \end{cases} \]

Households do not have access to international financial markets. The fiscal authority in H, however, can issue debt in the initial period subject to a limited commitment friction. In the terminal period, it can choose to default on its debt, suffering in this event a default cost \( \zeta \). The default cost takes the form of a reduction in the amount of
endowment of good $A$. I assume that default costs are increasing in the realization of the endowment process.\footnote{This is a common assumption in the endogenous sovereign default literature. Mendoza and Yue (2012) study how frictions in markets for imported inputs give rise to endogenous default costs that are increasing in TFP.} For simplicity, default costs are nil in the event of a low output realization and positive otherwise:

$$\zeta = \begin{cases} 0 & \text{if } y_{A,2} = y_{A,L} \\ \hat{\zeta} & \text{otherwise.} \end{cases}$$

The fiscal authority in $H$ maximizes welfare of the representative household, \footnote{Given intratemporal preferences the price index of the consumption bundle is given in each period by $p = (p_B)^{1-a}$.} by issuing debt in the initial period and choosing between default and repayment in the terminal period. Resources raised on international financial markets are rebated in lump-sum fashion to the representative household. Formally, the problem is given by:

$$V_H (\epsilon) = \max_{b_H, c_1, c_2, D \in \{0, 1\}} u (c_1) + \beta E[u (c_2)]$$

subject to:

$$p_1 c_1 + q_H b_H = y_A - \epsilon + p_{B,1} y_B,$$

$$p_2 c_2 = y_{A,2} + p_{B,2} y_B - [D \zeta - (1 - D) b_H],$$

All quantities are denominated in units of the numeraire good $A$. The relative price of good $B$ to good $A$ is denoted by $p_{B,t}$. $p_t$ is the price index of the consumption bundle $c_t$. The amount of debt issued by $H$ in the initial period is given by $-b_H$, while $q_H$ denotes the price of a unit of debt. $D$ is an indicator taking value of unity in the event of default. The fiscal authority takes good prices as given while it internalizes the effect of its choice for debt issued on the probability of terminal period default and on the debt price $q_H$.

Proceeding backwards from the terminal period, it is optimal for the fiscal authority to repay debt when $|b_H| \leq \zeta$. When debt is positive, default occurs in the event of a low output realization, which is accompanied by absence of default costs. Default would occur with certainty if debt were to be higher than the upper bound of default costs,
The following proposition establishes optimality, from the point of view of the fiscal authority in \( H \), of issuing debt carrying positive but not certain default probability:

**Proposition 1.** \( H \)'s optimal choice for \( b_H \) lies in the interval \( (0, \zeta) \) for a sufficiently low realization of the initial-period A-good endowment, \( y_A - \epsilon \), given values for \( y_{A,L}, \pi, \) and \( \beta \).

**Proof.** See appendix A.1

Given this choice, as default in the terminal period only occurs in the event of a low output realization, the probability of terminal-period default is given by \( 1 - \pi \). The solution to the problem of the fiscal authority can be summarized by the optimality condition:

\[
q_H \frac{u'(c_1)}{p_1} = \beta \pi \frac{u'(c_{2,R})}{p_{2,R}} \tag{4}
\]

where \( p_{2,R} \) and \( c_{2,R} \) denote the price index and consumption by \( H \) in the terminal-period state of the world where repayment is optimal. The marginal benefit of issuing a marginal unit of debt is equal at an optimum to its marginal cost. The cost of repaying debt in the terminal period is only borne by the representative household in the state of the world where it is optimal to do so. Finally, demand for the two individual goods is given in each period by:

\[
c_A = apc \quad \text{and} \quad c_B = (1 - a) \frac{p}{p_B} c. \tag{5}
\]

**Country F** As is country \( H \), \( F \) is inhabited by a continuum of identical households. Preferences of households in \( F \) are identical to those of households in \( H \), as defined in (1) and (2). Variables pertaining to country \( F \) are indexed by an asterisk, so that \( c^*_t, c^*_{A,t}, \) and \( c^*_{B,t} \) indicate aggregate consumption and consumption of the two types of good by the representative household in country \( F \) in period \( t \).

Households in \( F \) can trade assets on international financial markets in the initial period. In particular, households in \( F \) can issue risk-free debt to \( ROW \) and purchase debt issued by \( H \). \( F \)-households are promised by their domestic fiscal authority that they will be bailed out of losses incurred on assets issued by \( H \) in the event of its default.
The extent of partial bailout offered to households in $F$ is captured by the parameter $\xi \in [0, 1]$. Individual households have full information on the state variables determining the solution to the problem in $H$, and they correctly understand what is the optimal default policy by $H$. Due to the presence of the bailout promise the return on an asset issued by $H$ in the event of default, as perceived by a resident of $F$, is equal to $\xi$. Absent a bailout promise, the return on debt issued by $H$ in the event of default would be zero. In the event of default by $H$, bailout transfers received by individual households in $F$ are financed by a lump-sum tax $T$ on all households in the country. Consumption of the representative household in the terminal period then fully bears the negative wealth shock implied by default of $H$, even as each individual household perceives that it will be bailed out of possible losses on its assets.

The problem solved by the representative household in $F$ in the initial period is the following:

$$V_F(\epsilon) = \max_{c_1^*, c_2^*, b_F, b_{HF} \geq 0} u(c_1^*) + \beta E[u(c_2^*)]$$

s.t. \[ p_1 c_1^* + q_F b_F + q_H b_{HF} = y_A^* + p_{R,1} y_B^* , \]
\[ p_2 c_2^* = y_A^* + p_{R,2} y_B^* + b_F + [D(\xi b_{HF} - T) + (1 - D) b_{HF}] . \]

The amount of $H$-issued debt purchased by the representative household in $F$ is denoted by $b_{HF}$. Liabilities issued to $ROW$ at price $q_F$ are denoted by $-b_F$. The representative household in $F$ acts as a price taker with respect to good prices. Debt prices $q_H$ and $q_F$ are also taken as given, as is the value of the lump-sum tax $T$. In equilibrium, tax receipts finance bailout payments in the default state of the world, $T = \xi b_{HF}$.

Restricting again attention to cases where $H$ issues risky debt and gross assets purchased by $F$ are positive, $b_H \in \left(0, \hat{\zeta}\right)$ and $b_{HF} > 0$, the solution to the problem in $F$ can

\[ \text{restrict the bailout promise } \xi \text{ to take a value of zero if } |b_H| \geq \hat{\zeta}. \text{ Absent this restriction, a positive price for } b_H \text{ would be bid by households in } F \text{ even when default would occur with certainty. The problem faced by the fiscal authority in } H \text{ would not have a finite solution, as the choice of debt issued by } H \text{ would be unbounded.} \]
be summarized by the following system of first-order conditions:

\[
\begin{align*}
q_F u'(c^*_1) &= \beta E \left[ \frac{u'(c^*_2)}{p_2} \right] \\
q_H &= q_F - \beta (1 - \pi)(1 - \xi) \frac{u'(c^*_2)}{u'(c^*_1)} \frac{p_2}{p_1}.
\end{align*}
\]  

(7)

The subscript \(D\) denotes the terminal-period state of the world where default by \(H\) is optimal and, again, \(R\) denotes the the one where repayment is preferred. The first equation is standard and it simply describes the optimal intertemporal allocation of consumption by \(F\). The second equation characterizes the price \(q_H\) at which households in \(F\) are willing to purchase a positive amount of \(H\)-issued assets being purchased by households in \(F\). \(F\)-household need to be compensated for default risk, given by the probability of default times the loss suffered on each unit of asset: \((1 - \pi)(1 - \xi)\). In addition, households need to be compensated for volatility in terminal-period consumption that default risk induces, as captured by the term \(\frac{u'(c^*_2)}{u'(c^*_1)} \frac{p_2}{p_1} D\). Note that if a full bailout promise is offered, \(\xi = 1\), the price that \(F\)-households are willing to bid for \(H\)-issued assets equals the one of risk-free liabilities, \(q_F\). This is the same price they would bid on truly risk-free assets, if \(\pi = 1\).

Demand functions for individual goods are determined as in \(H\):

\[
c^*_A = apc^* \text{ and } c^*_B = (1 - a) \frac{p}{p_B} c^*.
\]  

(8)

**ROW** The representative household inhabiting \(ROW\) has risk-neutral preferences over consumption. Its intratemporal preferences are identical to those in \(H\) and \(F\), as is its discount factor, \(\beta\). The economy in \(ROW\) is large when compared to \(H\) and \(F\). In particular, endowments of either good in \(H\) and \(F\) are of negligible size when compared to those in \(ROW\). As in country \(F\), the stream of endowments received by \(ROW\) is deterministic and constant: \(y_{A,1}^{ROW} = y_{A,2}^{ROW} = y_A^{ROW}, y_{B,1}^{ROW} = y_{B,2}^{ROW} = y_B^{ROW}\). Bond holdings by \(ROW\) of risk-free assets are given by \(b_{ROW}\). The price of a risk-free asset issued by \(ROW\) is given by \(q_{ROW}\). Bond holdings by \(ROW\) of \(H\)-issued assets are given by \(b_{H,ROW}\)\(^{13}\))

\(^{13}\)The optimization problem solved by \(ROW\) is described in detail in Appendix A.2.
Transport costs  Good $A$ can be costlessly traded across all country pairs. International trade of good $B$ is subject to an iceberg transport cost $\tau$ that only applies to trade between either $H$ or $F$ and the ROW: For each unit of good $B$ that is shipped by an exporter, only $1 - \tau$ units are received by the destination country. Absence of transport costs affecting trade between $H$ and $F$ captures strong goods market integration among members of an economic union. Heterogeneous transport costs affecting goods $A$ and $B$ serve instead as a proxy for heterogeneous tradability of output of different industrial sectors. In the model, countries that are relatively abundant in good $B$ describe economies that are characterized by low average tradability of output. Relative abundance of the two types of good in $H$ and $F$ is captured by the ratios $\frac{y_A}{y_B}$ and $\frac{y_A^*}{y_B^*}$.

The relative price of good $B$ to good $A$ prevailing in $H$ and $F$ can be interpreted as the terms of trade of the country that is relatively abundant in good $B$. Due to the presence of transport costs, good prices in ROW may differ from those prevailing in the union formed by $H$ and $F$. The relative price of good $B$ to good $A$ in ROW is denoted by $p_{B,ROW}$.

Equilibrium  I will now proceed to define a competitive equilibrium in the world economy.

**Definition 1.** An equilibrium is defined as a vector of quantities $b_H$, $b_{HF}$, $b_F$, $b_{ROW}$, $b_{H,ROW}$, $c_s$, $c_{A,s}$, $c_{B,s}$, $c_{s}^*$, $c_{A,s}^*$, $c_{B,s}^*$, $c_{ROW}$, $c_{A,s}^{ROW}$, $c_{B,s}^{ROW}$, prices $q_H$, $q_F$, $q_{ROW}$, $p_{B,s}$, $p_s$, $p_{B,s}^{ROW}$, $p_s^{ROW}$ and default indicator $D$ such that, given values for the bailout promise $\xi$, transport costs $\tau$ and output realization $\epsilon$, in all time periods and states of the world, as indexed by $s \in \{1; 2, R; 2, D\}$:

- $b_H$ and $\{c_s\}_s$ solve the intertemporal allocation problem in $H$, (3), given $q_H$ and $\{p_s\}_s$.
- $D$ solves optimal default policy by $H$, as defined by (3).
- $b_F$, $b_{HF}$ and $\{c_s^*\}_s$ solve the intertemporal allocation problem in $F$, (3), given $q_F$, $q_H$ and $\{p_s\}_s$. 

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• \( \{c_{A,s}, c_{A,s}^*, c_{B,s}, c_{B,s}^*\} \), solve demand for individual goods by \( H \) and \( F \), (5) and (8), given \( \{c_s, c_s^*\} \) and prices \( \{p_{B,s}, p_s\} \).

• \( b_{ROW}, b_{H,ROW} \) and \( \{c_{s,ROW}\} \), solve the intertemporal allocation problem in \( ROW \) given \( q_{ROW}, q_H \) and \( \{p_{ROW}^s\} \). \( \{c_{A,s}^*, c_{B,s}^*\} \) satisfies \( ROW \)'s demand for individual goods, given \( \{c_s^\text{ROW}\} \) and prices \( \{p_{ROW}^s, p_s^\text{ROW}\} \).

• All markets for assets clear:
  \[
  0 = b_H + b_{HF} + b_{H,ROW} \\
  0 = b_F + b_{ROW}
\]

• The world market for good \( B \) clears, net of resources lost to transport costs between \( H,F \) and \( ROW \):
  \[
  y_B + y_B^* + y_B^{ROW} = c_{B,s} + c_{B,s}^* + c_{B,s}^{ROW} + \tau |y_B^{ROW} - c_{B,s}^{ROW}|
\]

• The market for good \( A \) clears by Walras’ law.
  \[
  c_{A,s} + c_{A,s}^* + c_{A,s}^{ROW} = y_A + y_A^* + y_A^{ROW}
\]

**World prices**  In equilibrium, world prices follow from the solution of \( ROW \)'s problem, given the small size of \( H \) and \( F \) compared to the world economy. The world price of a risk-free bond is given by \( q_{ROW} = \beta \), by risk-neutrality of \( ROW \).\footnote{Since assets are denominated in units of numeraire good \( A \), the first order condition in \( ROW \) implies \( q_{ROW} = \beta \frac{p_{ROW}^1}{p_{ROW}^2} \). Given the constant endowment streams, \( p_{ROW}^1 = p_{ROW}^2 \), so that \( q_{ROW} = \beta \) holds.} Bonds issued by \( F \) are risk-free, so that their price is also given by \( q_F = q_{ROW} = \beta \).

If \( ROW \) holds a positive amount of bond issued by \( H \), the price of this asset must compensate \( ROW \) for default risk. Hence, \( b_{H,ROW} > 0 \) implies
  \[
  q_H = \beta E [1 - D].
\]
In particular, if \( b_H \in (0, \zeta) \), the probability of repayment is given by \( \pi \) and \( q_H = \beta \pi \).

\( ROW \) is not willing to buy assets issued by \( H \) if \( q_H > \beta E [1 - D] \). In this event, \( F \)
would be the marginal and sole buyer of assets issued by $H$: $b_{H,\text{ROW}} = 0$ and, by market clearing, $b_{HF} = b_H$.

World good prices are also determined in $\text{ROW}$. When $H$ or $F$ export good $B$ to $\text{ROW}$, arbitrage and transport costs imply $p_B = (1 - \tau) p_{B,\text{ROW}}^{\text{ROW}}$. Symmetrically, when $\text{ROW}$ exports good $B$ to either $H$ or $F$, $p_B = \frac{1}{(1 - \tau)} p_{B,\text{ROW}}^{\text{ROW}}$. Finally, when $\tau > 0$, a no-trade equilibrium arises, where price differentials are too small, compared to transport costs, for intratemporal trade in $B$ to arise. Indeed, when $p_B \in \left( (1 - \tau) p_{B,\text{ROW}}^{\text{ROW}}, \frac{1}{(1 - \tau)} p_{B,\text{ROW}}^{\text{ROW}} \right)$, no trade in good $B$ takes place between $H$ or $F$ and $\text{ROW}$. When trade in good $B$ between $H,F$ and $\text{ROW}$ does not take place, the markets for good $B$ in $H$ and $F$ and in $\text{ROW}$ have to individually clear:

$$y_B + y_B^* = c_{B,s} + c_{B,A}^* \quad \text{and} \quad y_B^{\text{ROW}} = c_{B,s}^{\text{ROW}}.$$  

Demand for good $B$ by $H$ and $F$ is then satisfied entirely by supply within these countries:

$$c_B + c_B^* = y_B + y_B^*. \quad (9)$$

Within this interval, the relative price of good $B$ in $H$ and $F$ is determined by market clearing within the economic union. From intratemporal preferences, this relative price is given by:

$$p_B = \frac{1 - a}{a} \frac{c_A + c_A^*}{c_B + c_B^*} = \frac{1 - a}{a} \frac{c_A + c_A^*}{y_B + y_B^*}. \quad (10)$$

### 3.1 Results

**Intermediation of capital flows**  What drives intermediation of international capital flows in this framework? When is it optimal for an economy like $F$ to be the main holder of risky debt issued by its partner in the economic union? Risk aversion and the presence of a bailout promise play a crucial role in answering these questions.

Absent a bailout promise, risk-averse households in $F$ are not willing to hold the entire amount of debt issued by $H$. From $F$’s Euler equation, $(7)$, imposing $q_F = \beta$, the price

\[ \text{From } (5) \text{ and } (5), \text{ it can be seen that } c_A + c_A^* = \frac{a}{1 - a} \frac{c_B + c_B^*}{p_B}. \]
of $H$-assets bid by $F$-households is given by

$$q_{H,F} = \beta \left[ 1 - (1 - \pi) \frac{u'(c^*_2)/p_{2,D}}{u'(c^*_1)/p_1} \right],$$

which is below the price bid by $ROW$, $\beta \pi$, if

$$\frac{u'(c^*_2)/p_{2,D}}{u'(c^*_1)/p_1} > 1.$$

$F$ would suffer from low consumption in the state of the world where $H$ defaults, and it has to be compensated for such risk, as captured by the ratio of marginal utilities across initial and terminal periods. Hence, $F$ is not willing to bid a price higher than that offered by $ROW$, $q_{H,ROW} = \beta \pi$, for assets issued by $H$.\(^{16}\)

When a full bailout promise is offered, $H$-issued assets are perceived as risk-free by $F$-households. Accordingly, the price they bid for this asset is the same as the one at which their risk-free liabilities are issued:

$$q_{H,F} = q_F = \beta.$$

To the extent that these assets are not truly risk-free ($\pi < 1$), the price bid by $H$ is not sufficiently high to compensate $ROW$ for default risk, $\beta > \beta \pi$, and this agent will not buy any positive amount of $H$-issued debt. The entire debt issued by $H$ is then held in equilibrium by $F$.

Gross asset purchases by $F$ are partly financed with own resources, as well as by intermediating funds borrowed from $ROW$. The fraction of gross asset purchases that $F$ finances by issuing gross liabilities lies in the interval:\(^{17}\)

$$\frac{b_F}{b_H} \in \left[ \frac{\beta}{1 + \beta}, 1 \right] \text{ for } \pi \in [0, 1]. \quad (11)$$

---

\(^{16}\)In an equilibrium where $\frac{p_{2,D}}{p_1}$ is sufficiently high, it is optimal for $F$ to hold $H$-issued debt even in absence of the bailout promise, in order to transfer resources across states of the world and exploit differences in good prices. This equilibrium would not arise for sufficiently small transport costs and I ignore it for simplicity.

\(^{17}\)This result follows from the solution to the system of optimality conditions in \(^{17}\). I consider here for simplicity a setup where $\tau = 0$, so that good prices are constant across states of the world as determined in $ROW$, and normalized to unity. I define total endowment in $F$ as $y^* = y^*_A + y^*_B$.\(^{25}\)
If repayment by $H$ is certain, $F$ entirely finances its purchase of $H$ debt by borrowing from $ROW$, $b_F = b_H$, and the allocation is identical to the one where $H$-debt is held directly by $ROW$. In the opposite extreme where default is certain, $\pi = 0$, gross liabilities issued by $F$ are smaller than gross assets, $b_F = \frac{\beta}{1+\beta} b_H$, as households transfer resources to the terminal period, when they will certainly face low consumption due to default by $H$. The riskier is debt issued by $H$, the larger the fraction of gross asset purchases that is financed by $F$ with own resources, as lower expected consumption in the terminal period induces higher net saving. For values of $\pi \in (0, 1)$, the optimal amount of gross liabilities issued by $F$ is presented graphically in Figure 5. The fraction of assets that is financed by intermediation falls as the relative size of gross assets to income rises. Intuitively, larger amounts of gross assets imply higher consumption risk in the terminal period when $\pi \in (0, 1)$ and saving provides a means to insure against such risk.

**Figure 5:** Issuance of gross liabilities by $F$ as function of repayment probability by $H$ and size of gross assets. The blue, solid line represents the fraction of gross assets purchased by $F$ that is financed by borrowing from $ROW$, for a low amount of gross assets relative to total income ($b_H/y^* = .25$). The red, dashed line represents the same variable for a higher ratio of gross assets to income, $b_H/y^* = 1$.

**Debt crisis in an economic union** Motivated by the recent Eurozone experience, we can employ the model here presented to study spillover and amplification effects of
a debt crisis in an economic union. To this purpose, I analyze consumption by \( H \) and \( F \) in the terminal-period crisis state, where \( H \) faces a low output realization, \( y_A = y_{A,L} \), and it does not repay its debt. By comparing the equilibrium allocation with bailout promise to the one with no bailout, we are able to analyze how this particular friction affects consumption in crisis times in the economic union.

In presence of a bailout promise, it is easy to see how the crisis spreads to lending countries. The bailout promise induces \( F \) to take on exposure to default risk by \( H \), issuing debt to finance asset purchases. Wealth in \( F \) is lower in the intermediation scenario as gross assets give no return in the crisis state and gross liabilities to ROW are repaid. Formally, consumption by \( F \) in the terminal-period crisis state is given by:

\[
c^{*,2}_{D} = \left( y_A^* + p_B y_B^* + b_F \right) \frac{1}{p}
\]

As gross liabilities \( b_F \) are larger in the bailout promise allocation, crisis-times consumption in \( F \) is lower.

In an economic union, the adverse effect of the crisis on the intermediating economy \( F \) can potentially feed back into the crisis country itself, as the two are linked by trade integration as well as by financial exposures. In the crisis state, \( H \) is reduced to financial autarky, but it is able to trade in goods \( A \) and \( B \) according to relative abundance of the two in its endowment basket. Consumption by \( H \) in the crisis then crucially depends on the relative price of the good it exports. When good \( B \) is not traded with ROW, its price is function of its relative abundance in the union, in terms of good \( A \). In the crisis state, this relative price is given by:

\[
p_B = \frac{1 - a \frac{c_{A,2,D} + c_{A,2,D}^*}{c_{B,2,D} + c_{B,2,D}^*}}{a \frac{y_{A,L} + y_A^* + b_F}{y_B + y_B^*}}
\]

Higher liabilities issued by \( F \) in the intermediation scenario, as captured by a higher absolute values of \( b_F \), imply that a lower amount of good \( A \) is available in the union, causing a fall in the relative price of good \( B \). Lower wealth in the union can have a detrimental or beneficial effect on the crisis country, \( H \), depending on its status as net exporter or net importer of the good subject to transport costs, \( B \). Consider for simplicity
the two extreme cases of full specialization, \( Y_{A,L} = 0, Y_B > 0 \), and \( Y_{A,L} > 0, Y_B = 0 \). In the first case, \( H \) is a net exporter of good \( B \), and it suffers from the fall in value of its export good. Denoting by \( I \) and \( NI \) the intermediation and non-intermediation scenarios, it can be shown that the relative fall in \( H \)-consumption across the two scenarios is proportional to the relative change in the terms of trade:

\[
\frac{c_I}{c_{NI}} = \left( \frac{p_{B,I}}{p_{B,NI}} \right)^a
\]

Conversely, if it were fully specialized in production of \( A \), consumption in \( H \) would rise in response to a fall in \( p_B \):

\[
\frac{c_I}{c_{NI}} = \left( \frac{p_{B,I}}{p_{B,NI}} \right)^{a-1},
\]

as \( a - 1 < 0 \). Finally, when relative abundance of the two goods in \( B \) equals relative abundance in the union, the fall in \( p_B \) has no effect on consumption, as no intra-temporal trade occurs. Formally, \( c_I = c_{NI} \) if

\[
\frac{y_B}{y_B + y_B^*} = \frac{y_A}{y_{A,L} + y_A^* + b_{F,NI}}
\]

with \( c_I < c_{NI} \) for values of \( \frac{y_B}{y_B + y_B^*} \) above this threshold and vice versa. The higher the share of good \( B \) in the endowment basket of \( H \), the more severe will be the crisis in this country, as creditors to whom debt is not repaid coincide with the destination of exports. If the crisis country were instead a net exporter of good \( A \), it would benefit from cheaper imports of good \( B \), while being able to export to \( ROW \) which is unaffected by the crisis.

Prior to the Eurozone crisis, economies like Greece, Spain and Portugal were highly exposed to trade with European partners in industries whose output is subject to relatively high trade costs, as detailed in section 2.4. The analysis above highlights how the interaction between financial and trade links amplifies the severity of a crisis, a mechanism that may have played an important role during the recent recession in the Euro area periphery. I present in section 4 a quantitative analysis of these channels, in the context of an infinite horizon framework.

\[18\] The result in (14) only holds locally, for \( \frac{p_{B,I}}{p_{B,NI}} \approx 1 \). A full derivation of results in this section is provided in Appendix A.3.
**Optimal bailout promise** The introduction of a bailout promise distorts the allocation in $F$, inducing excessive risk taking and leverage. In the economy subject to the limited commitment friction, $H$, the same distortion determines a higher price of debt issued, making borrowing cheaper. In a second-best world, it need not be the case that the introduction of an additional distortion diminishes welfare. In this framework, there exist indeed equilibrium allocations where aggregate welfare in the economic union with a bailout promise is higher than that attained absent such a distortion. Intuitively, when the borrowing constraint is sufficiently severe, due for example to scarce initial-period resources in $H$, the benefits of redistribution would outweigh the costs of distortion, making a bailout promise desirable.

Consider welfare in the union as the sum of welfare in the two countries, as function of the initial period income shock in $H$, $\epsilon$:

$$V(\epsilon) = V_H(\epsilon) + V_F(\epsilon)$$

We are interested in studying the change in aggregate welfare when comparing an allocation with no bailout promise, $\xi = 0$, to another one with bailout, e.g. $\xi = 1$. It is possible to analytically compare welfare in the union across these two different allocations by focusing on the limiting case where $\pi$, the probability of repayment by $H$, tends to zero. Absent a bailout promise, neither $H$ nor $F$ would engage in financial trade, for very different reasons. The flat endowment path in $F$ ensures that this economy achieves in financial autarky the optimal allocation of consumption across period and states, so that no borrowing or lending takes place:

$$c_1^* = c_2^* = y^*$$

Given scarcity of initial period resources, $H$ would instead prefer to borrow. It cannot do so if default is certain, however, as its debt is issued at price $q_H = \beta \pi = 0$. The higher

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19 A classic reference is Lipsey and Lancaster (1956).

20 As in the previous discussion on intermediation, I am considering here an allocation with no transport costs and constant good prices determined by $ROW$. As prices are constant, I am also ignoring endowments of the two different goods and focusing simply on total endowments $y$ and $y^*$. 


is \( \epsilon \), then, the higher is marginal utility of initial-period consumption in \( H \) and the more skewed will be the path for consumption in this economy:

\[
c_1 = y - \epsilon, c_{2,D} = y_L
\]

When a bailout promise is offered, default by \( H \) still occurs with certainty, as \( \pi = 0 \), but \( F \) is now willing to lend, financing asset purchases with own resources and by borrowing from \textit{ROW}. Lending simply amounts to a free transfer of resources to \( H \), so that wealth in \( F \) falls as consequence of the bailout distortion. \( H \) receives resources in the initial period, when they are the most needed, and its welfare rises in this allocation.\(^{21}\) Importantly, the welfare increase in \( H \) is increasing in the initial period shock, by concavity of the utility function. Hence, as welfare in \( F \) is unaffected by \( \epsilon \) in either the full or no bailout promise allocations, the following result can be established:

**Proposition 2.** For a sufficiently high value of the initial period shock \( \epsilon \), an allocation with bailout promise, \( \xi = 1 \) delivers the same welfare as the allocation where no bailout is promised, \( \xi = 0 \). The bailout promise allocation is preferable to the no-bailout one for values of \( \epsilon \) above that threshold.

*Proof.* See appendix A.4 \( \square \)

Intuitively, the bailout distortion allows resources to be transferred across members of the union. This transfer, however, is only desirable if resources in the destination country are sufficiently scarce. This result can help to shed light on the rationale for the introduction of similar distortions in the Euro area. At the cost of generating critical financial linkages, implicit subsidies on cross-border asset holdings may have contributed to a relaxation on borrowing constraints faced by peripheral countries. The relaxation of such constraints could have been seen as desirable, in turn, if relative incomes were expected to converge across countries.

\(^{21}\)The amount of borrowing by \( H \) is bounded above by the amount of resources available in the repayment state, as except in the limit \( \pi > 0 \) holds, and marginal utility of consumption would otherwise be unbounded if \( H \) would have to repay debt.
4 Net and gross foreign assets with a continuum of countries

In the previous section, I outlined some theoretical channels through which policy distortions, in the form of subsidies on holdings of specific assets, affect capital flows, gross financial positions and crisis spillovers in an economic union. I will introduce here an infinite-horizon, heterogeneous countries general equilibrium model to study the dynamic interaction between the policy distortion and net and gross capital flows among economies in a union. In addition, we are able to analyze in general equilibrium the interplay between current account and trade within the union, as countries are specialized in production of heterogeneous goods. The model also allows for a richer characterization of a debt crisis, delivering predictions on the effects of the crisis on borrowing costs for debtors in the union, as well as on relative good prices and terms of trade.

I will first introduce the general environment of the model, where a continuum of small open economies trade financial assets with the rest of the world, describing the stationary allocation in this framework. I will then introduce a bailout promise inducing cross-border holdings of assets within the union, presenting again the stationary allocation of this economy. I will interpret the transition of the model economy towards the stationary equilibrium with bailout promise as a stylized description of the episode of integration among heterogeneous economies that was observed following the inception of the European monetary union. I will then analyze simulations for net and gross foreign asset positions arising from the model, to assess the extent to which the introduction of policy distortions can explain the observed pattern in European economies. Finally, I will analyze the predictions of the models in the event of a debt crisis, modeled as a fall in the value of gross assets, describing the heterogeneous behavior in this setting of economies that differ in terms of exposure to trade with the rest of the union.
4.1 Infinite horizon model

Environment The world economy is inhabited by a unit-measure continuum of small open economies, $I$, and by the rest of the world, $ROW$. Countries in the economic union are indexed by $i \in [0,1]$. Each country is inhabited in turn by a continuum of identical households. There are two types of goods, $A$ and $B$. I retain the assumption on heterogeneous transport costs made in the previous section: Good $A$ is freely tradable across all country pairs, while trade in good $B$ between $ROW$ and countries in $I$ is subject to an iceberg transport cost $\tau$. Each economy in $I$ receives a stream of endowments of either type of good. $F = [0, \kappa]$ denotes the subset of economies in $I$ that receive endowments of good $A$, while economies in the complementary set $H = (\kappa, 1]$ receive endowments of good $B$. Good $A$ is the numeraire in the world economy. Time is discrete and indexed by $t$. Each economy receives a stream of endowments $\{y_{i,t}\}_{t=0}^\infty$ which is determined according to a stochastic Markov process. The process is i.i.d. across countries and there is no aggregate uncertainty in the world economy.

Households in all economies $i$ enjoy consumption of both types of good. In each period, they derive utility from a consumption basket defined as the CES aggregator:

$$c_i = \left[ a^{\vartheta} c_{A,i}^{\vartheta-1} + (1-a)^{\vartheta} c_{B,i}^{\vartheta-1} \right]^{\vartheta/(\vartheta-1)}$$

(15)

where $\vartheta$ represents the elasticity of substitution between the two types of consumption good and $a$ is the utility weight of good of type $A$. Preferences of the representative household over consumption paths are indexed according to:

$$U_i = E_0 \sum_{t=0}^\infty \beta^t u(c_{i,t}),$$

(16)

where $E_0$ is the mathematical expectation operator conditional on information available at time zero. The period utility function takes the CRRA form $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where the parameter $\gamma$ captures the degree of relative risk aversion.

The $ROW$ is represented by a large, risk-neutral agent, enjoying consumption of both goods. $ROW$ is relatively more patient than households in the economic union. Its
stochastic discount factor, $\beta^{ROW} > \beta$ pins down the price of a risk-free asset in the world economy, $q^{ROW} = \beta^{ROW}$. The relative price of good $B$ in $ROW$ is given by $p_B^{ROW}$.

**Risky sovereign debt** Countries in the economic union issue risky debt on international financial markets. External debt is subject to a simple non-repayment friction: For a given amount of debt $b'_i$ issued by $i$, lenders will fail to receive debt repayments with probability $1 - \pi$. The exogenous repayment probability $\pi$ is function of current-period output $y_i$ and of the amount of debt issued. The function $\pi(y_i, b'_i)$ is continuously differentiable, increasing in $y_i$ and decreasing in $b'_i$. All countries can buy risk-free assets issued by $ROW$: $\pi = 1$ if $b' > 0$. All agents have full information on non-repayment probabilities. Lenders in $ROW$ behave competitively, and they are willing to buy debt issued by $i$ as long as the price of this asset compensates them for non-repayment risk. The price at which $i$ can issue debt to $ROW$ is then given by the function

$$q(y_i, b_i) = q^{ROW} \pi(y_i, b'_i).$$

(17)

The debt price function inherits properties of $\pi$. In particular, the price of debt issued by $i$ is decreasing in its quantity. Borrowers always repay debt in full, even though a fraction of debt repayments are not received by lenders. This assumption captures in a simple way sovereign risk faced by creditors on international financial markets. In addition, as the probability of non-repayment is priced by lenders, borrowers have to take into account how the quantity of debt they issue affects its price. These features will be crucial to our analysis on the direction of international capital flows and intermediation in an economic union.

---

22 As in other models of external debt, e.g. Schmitt-Grohe and Uribe (2003), this assumption ensures that $\beta/q^{ROW} < 1$, ruling out divergence of external positions in presence of a borrowing limit. In addition, these authors review properties of alternative stationarity-inducing methods in similar frameworks.

23 The simplifying assumption that debt is always repaid in full allows us to study the pattern of intermediation and current account imbalances while ignoring optimal default choice by borrowers. In particular, I disregard strategic considerations by borrowers that would arise when issuing defaultable debt whose price does not reflect the true default probability. While interesting, these issues fall beyond
**Equilibrium in no-bailout economy**  Consider the case where, in all economies in the economic union $I$, no distortions are imposed on trade in financial assets by households. In particular, no bailout is offered in the event of losses suffered on risky assets. In this scenario, all economies in $i$ directly trade financial assets with $ROW$. The budget constraint of the representative household in $i$ is given in each period by:

$$pc_i + q(y_i, b'_i) b'_i = p_i y_i + b_i.$$  \hfill (18)

where, adopting recursive notation, I remove time subscripts and indicate with a prime superscript next-period values. The relative price of the endowment good received by $i$ is given by $p_i$. This variable takes value of unity if $i \in F$ and receives an endowment of good $A$, and it is given by $p_B$ otherwise. The relative price of the consumption basket $c$ is given by $p = [a + (1 - a) p_B^{1 - \sigma}]^{\frac{1}{1 - \sigma}}$. Bond holdings by $i$ are given by $b_i$. Negative values of this variable indicate that the economy owes debt to $ROW$. Note that the amount of resources that $i$ can raise on international financial markets is bounded, as the price at which debt is traded is decreasing in the amount issued.

The problem solved by the representative household in $i$ is to maximize expected lifetime utility \hfill (16) subject to the sequence of budget constraints \hfill (18):

$$V_{k \in \{H,F\} \in \{H,F\}} (y_i, b_i) = \max_{b'_i, c_i} \{u(c_i) + \beta E [V_k (y'_i, b'_i)]\}$$

s.t. $pc_i + q(y_i, b'_i) b'_i = p_i y_i + b_i,$

where $V_{k \in \{H,F\} \in \{H,F\}}$ denotes the value function of a generic economy $i \in k$, where $k$ equals either $H$ or $F$. The solution can be summarized by the following intertemporal optimality condition,

$$\frac{u'(c_i)}{p} \left( q(y_i, b'_i) + \frac{\partial q}{\partial b'_i} b'_i \right) = \beta E \left[ \frac{u'(c'_i)}{p'} \right].$$  \hfill (20)

The reader is referred to Conesa and Kehoe (2015) for an analysis of the strategic interaction between sovereign borrowers and lenders in a model of self-fulfilling debt crisis.

As these assets are priced by a risk-neutral agent, $ROW$, and they do not offer insurance against own income risk, it is not optimal for the risk-averse households in $I$ to hold positive amounts of them.
equating the discounted marginal cost of an additional unit of debt with its marginal benefit, taking into account how a marginal increase in debt issued affects its price $q(y_i,b_i')$.

Households allocate consumption across the two goods according to standard CES demand:

$$c_{A,i} = a \left( \frac{1}{p} \right)^{-\vartheta} c_i, \quad c_{B,i} = (1-a) \left( \frac{p_B}{p} \right)^{-\vartheta} c_i.$$ (21)

The total amount of $B$-good that economies in $I$ are endowed with is given by

$$y_B = (1-\kappa) \int_{i \in I} y_i \, di.$$ Absent aggregate uncertainty $\int_{i \in I} y_i \, di = E[y_i]$, where $E[y_i]$ is the unconditional expected value of $y$. Given intra-temporal demand, (21), and defining aggregate consumption as $c = \int_{i \in I} c_i \, di$, aggregate consumption of good $B$ in $I$ is given by:

$$c_B = \int_{i \in I} c_{B,i} \, di = (1-a) \left( \frac{p_B}{p} \right)^{-\vartheta} c.$$ (22)

Due to the presence of transport costs, the relative price of $B$ prevailing in $I$, $p_B$, may differ from the one in $ROW$, $p_B^{ROW}$. In the range $p_B \in \left( (1-\tau) p_B^{ROW}, \frac{1}{1-\tau} p_B^{ROW} \right)$ no trade in good $B$ occurs between $I$ and $ROW$, as gains from trade are not sufficiently large to outweigh transport costs. Trade in good $B$ between $I$ and $ROW$ only emerges if total demand for good $B$, (22), would imply $p_B \notin \left( (1-\tau) p_B^{ROW}, \frac{1}{1-\tau} p_B^{ROW} \right)$ when $c_B = y_B$. Otherwise, the aggregate endowment of good $B$ is entirely consumed within the union, and $p_B$ is determined according to (22), with $c_B = y_B$.

I now define a competitive equilibrium in the world economy when no bailout is promised.

**Definition 2.** A recursive competitive equilibrium in the no-bailout economy is defined as a set of functions $p_B$, $c_i$, $c_{A,i}$, $c_{B,i}$, $b'_i$ for $i \in \{H,F\}$ that solve households’ optimization problems and clear the market for good $B$, as well as by a transition function $\Gamma$ for the joint distribution of $(y,b)$:

---

25 Differently than in standard Euler equations in Eaton-Gersovitz frameworks, repayment occurs here in all states of the world. Hence, the expectation operator is conditional on information available in the current period only and not on a particular realization of the following-period exogenous state.
Consumption $c_i(y,b)$ and bond holdings $b_i'(y,b)$ solve households’ maximization problem (19) for economies $i$ in either $H$ or $F$.

Consumption of individual goods $c_{A,i}(y,b), c_{B,i}(y,b)$ solves the intra-temporal allocation (21).

The relative price $p_B$ ensures market clearing for good $B$, $y_B = \int_{i \in I} (1 - a) \left( \frac{p_B}{p} \right)^{-\vartheta} c_i \, di$ if no trade in good $B$ between $I$ and ROW occurs, $y_B = c_B$, and it is determined consistently with transport costs and the price prevailing in world markets otherwise:

$$p_B = \begin{cases} (1 - \tau) p_B^* & \text{if } y_B > c_B \\ \frac{1}{(1 - \tau)} p_B^* & \text{if } y_B < c_B \end{cases} \quad (23)$$

The Markov transition probability of the exogenous shock process for $y$ and the optimal choice of bond holdings determine the transition function $\Gamma$ for the joint distribution of $(y,b)$ in $H$ and $F$.

**Equilibrium in the economy with full bailout promises** Consider now the case where a bailout promise is offered to all households in all economies in $I$, when holding risky debt issued by other economies in the union. The government in each $i$ offers a full bailout promise were repayments owed on holdings of risky sovereign debt not to be received. The bailout promise is only offered, however, if the total amount of liabilities issued by each individual borrower $j$ falls below a certain threshold $\bar{b}$: $b_j \geq \bar{b}(y_j)$. In each period, governments finance bailout payments to asset holders by setting a lump-sum tax $T_i$ that is paid by all households in $i$. Define $\tilde{b}_{i,j}$ as holdings by the representative household in $i$ of debt issued by $j$. The amount of resources that is lost by the representative household in $i$ of debt issued by $j$. The amount of resources that is lost by the representative household in $i$ on risky assets issued by other union members is given by $\int_{j \in I} \left[ 1 - \pi(y_{j,-1}, \tilde{b}_j) \right] \tilde{b}_{i,j} \, dj$. As the government fully compensates asset holders for losses incurred on risky debt, the amount of tax that is paid by the representative asset holder.

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26 As in section 3, this restriction on bailout promises is required to impose a borrowing limit and to prevent borrowers’ liability positions to grow unbounded.
household in each period is exactly equal to the aggregate amount of resources that are lost by bond holders:

\[ T_i = \int_{j \in I} \left[ 1 - \pi \left( y_{j,-1}, \tilde{b}_j \right) \right] \tilde{b}_{ij} dj. \]  

(24)

Economies in the union only differ ex-ante in terms of the type of good they are endowed with, which is given by \( A \) or \( B \) depending on whether \( i \) belongs to the set \( F \) or \( H \). Depending on their wealth and realization of the exogenous shock, economies optimally choose to behave as gross savers, buying assets issued by union members, or whether to be borrowers instead, issuing debt to other economies.

In gross saving economies, due to the full bailout promise, households perceive assets issued by all other union members as being perfect substitutes for each other, even as they differ in terms of repayment probabilities. I define \( \tilde{b}_i = \int_{j \in I} \tilde{b}_{ij} dj \) as the aggregate amount of union-issued assets that is held by economy \( i \). The price bid by union members for bonds issued by other economies in \( I \) is given by \( \tilde{q} \). As in the no-bailout setting, \( b_i \) denotes holdings by \( i \) of risk-free bonds issued by \( ROW \). This variable takes negative values when \( i \) owes gross liabilities to \( ROW \). Beginning-of-period net wealth of the representative household in a gross saving economy is defined as gross assets, net of gross liabilities owed to \( ROW \) and taxes paid to the domestic government: \( n_i = \tilde{b}_i + b_i - \theta_i \).

Note that, even though it enjoys a full bailout promise, the representative household still incurs in losses on holdings of gross assets, as bailout transfers are financed by taxation. The budget constraint of the representative household in a gross saving economy is given by:

\[ pc_i + \tilde{q}\tilde{b}_i' + q (y_i, b_i') b_i' = p_i y_i + n_i \]

describing how purchases of consumption and gross assets are financed with own resources, \( n_i \), and by issuing liabilities to \( ROW \).

The problem faced by the representative household in \( i \) includes as a state variable aggregate wealth in the economy, \( N_i \). Aggregate wealth is a state variable as it allows

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27By the law of large numbers, \( \pi (y, b') \) both indicates the probability of repayment by a given economy with \((y, b')\) as well as the total fraction of resources that are received by a lender on his holdings of assets issued by all economies with \((y, b')\).
households to forecast the next-period value of gross asset holdings in \(i\), which determines in turn next-period taxation, according to (24)\(^{28}\). I define \(\Phi (y_i, N_i)\) as the forecast for the next-period value of aggregate wealth, \(N'_i\), given the current aggregate state in \(i\), \((y_i, N_i)\). The other state variables for the household are given by its net wealth, \(n_i\), and by the realization of the endowment shock \(y_i\). The maximization problem solved by the representative household in a gross saving economy is the following:

\[
V_{k \in \{H,F\}}^{S} (y_i, N_i, n_i) = \max_{b'_i, \tilde{b}'_i > 0,} \{ u(c_i) + \beta E [V_k (y'_i, N'_i, n'_i)] \}
\]

subject to

\[
\begin{align*}
pc_i + q (y_i, b'_i) b'_i + \tilde{q} \tilde{b}'_i &= y_i + n_i \\
n'_i &= \tilde{b}'_i + b'_i - \theta'_i \\
N'_i &= \Phi (y_i, N_i)
\end{align*}
\]

(25)

Economy \(i\) is not restricted to be a gross saver in the following period, hence the continuation value of households is not given by \(V_{k}^{S} (\cdot)\) but rather by \(V_k (\cdot)\), to be defined shortly.

I define \(\mathcal{W}\) as the subset of the state space where exists a solution to the problem in (25) with positive holdings of gross assets, \(\tilde{b}'_i > 0:\ \mathcal{W} = \{(y, N, n) | \tilde{b}'_i (y_i, N_i, n_i) > 0\}\). The solution to the household problem implies a law of motion for net wealth of the representative household, \(n'_i (y_i, N_i, n_i)\), which, in general, differs from the perceived aggregate one, \(N'_i = \Phi (y_i, N_i)\). The two must coincide in equilibrium.

Under some regularity conditions, the problem can be summarized by the following system of two Euler equations, for gross liabilities and gross assets, respectively\(^{29}\):

\[
u' (c_i) \left( q (y_i, b'_i) + \frac{\partial q}{\partial b'_i} b'_i \right) = \beta E [u' (c'_i)]
\]

(26)

\[
u' (c_i) \tilde{q} = \beta E [u' (c'_i)].
\]

(27)

As gross assets and gross liabilities are equally perceived as risk-free, the portfolio of gross asset and liability positions of the representative household is set by equating the price of

\(^{28}\)The optimization problem in Bianchi (2011) exhibits similar features, in presence of a pecuniary externality. Here, the household fails to internalize the effect of its choice for gross assets on taxation. There, household choices determine relative good prices that affect in turn borrowing conditions.

\(^{29}\)We need to insure that \(E [V (\cdot)]\) is differentiable in \(n'\), and \(\frac{\partial E [V (\cdot)]}{\partial n'} = E [u' (c')]\).
gross assets $\tilde{q}$ with the amount of resources obtained by issuing a marginal unit of debt, $q(y_i, b'_i) + \frac{\partial q}{\partial b'_i} b'_i$. In equilibrium, the amount of debt issued to the ROW is pinned down by the arbitrage condition:

$$\tilde{q} = q(y_i, b'_i) + \frac{\partial q}{\partial b'_i} b'_i. \quad (28)$$

Economy $i$ is a borrower if, given the vector of state variables $(y_i, N_i, n'_i)$, there exists no solution to the maximization problem in (25) with positive holdings of gross assets, $\tilde{b}'_i > 0$. Borrowers simply issue debt to other economies in $I$ and their budget constraint is given by

$$pc_i + \tilde{q} \tilde{b}'_i = p_i y_i + n_i$$

The problem faced by the representative household in a borrowing economy is the following:

$$V_{k \in \{H,F\}} (y_i, N_i, n_i) = \max_{\tilde{b}'_i \geq 0} \left\{ u(c) + \beta \mathbb{E} \left[ V_k \left( y'_i, N'_i, \tilde{b}'_i \right) \right] \right\}$$

s.t.

$$pc_i + \tilde{q} \tilde{b}'_i = p_i y_i + n_i$$

$$\tilde{b}'_i \geq \bar{b}(y_i)$$

$$n'_i = \tilde{b}'_i$$

$$N'_i = \Phi (y_i, N_i). \quad (29)$$

As households in borrowing economies do not purchase gross assets, no taxes will be paid in the following period. Next-period net wealth simply follows from the amount of debt issued in the current period. The borrowing constraint $\tilde{b}'_i \geq \bar{b}(y_i)$ stems from the limited bailout promise that is offered to gross savers in the union, as they are only willing to bid $\tilde{q}$ on debt issued by $i$ if the amount issued is below $\bar{b}(y_i)$. The Euler equation describing the solution to borrowers’ problem is the following:

$$u'(c_i) \tilde{q} = \beta \mathbb{E} [u'(c'_i)]$$

Borrowers do not take into account how marginal debt issues affect the repayment probability $\pi$, as lenders, due to the bailout promise, do not require to be compensated for non-repayment risk associated with individual assets. For gross savers and borrowers.
alike, the intra-temporal allocation of consumption across the two goods is determined according to CES demand, (21), as in the no-bailout setting.

We can now define the value function of the representative household in a generic economy $i$, $V_{k \in \{H,F\}}(y_i, N_i, n_i)$, as the value function of a gross saving economy, $V^S_{k \in \{H,F\}}(y_i, N_i, n_i)$, in the subset of the state space $\varpi$ where a solution to the problem in (25) with positive $\tilde{b}$ exists. In the complementary subset of the state space, the economy $i$ will be a borrower and its value function is given by $V^B_{k \in \{H,F\}}(y_i, N_i, n_i)$:

$$V_{k \in \{H,F\}}(y_i, N_i, N_i) = \begin{cases} 
V^S_{k \in \{H,F\}}(y_i, N_i, N_i) & \text{if } (y, N) \in \varpi \\
V^B_{k \in \{H,F\}}(y_i, N_i, N_i) & \text{otherwise.}
\end{cases}$$

(30)

I now define an equilibrium in the economy with bailout promises.

**Definition 3.** A recursive competitive equilibrium in the economy with bailout promises is defined as a set of functions $c_i, c_{A,i}, c_{B,i}, \tilde{b}_i, b_i, \theta_i$ for $i \in \{H,F\}$, $p_B, \tilde{q}$ that solve households’ optimization problems and clear the markets for assets traded in the union and good $B$, as well as by a perceived law of motion for wealth $\Phi(y_i, N_i)$ and a transition function $\Gamma$ for the joint distribution of $(y, N)$:

- In the state space subset $\varpi$ where a solution to the problem in (25) with positive $\tilde{b}$ exists, consumption $c_i(y, N, n)$, bond holdings of union-issued assets $\tilde{b}_i(y, N, n)$ and of ROW assets $b_i(y, N, n)$ solve gross savers’ maximization problem (25) for economies $i$ in either $H$ or $F$

- In the complement subset to $\varpi$, consumption $c_i(y, N, n)$ and bond holdings of union-issued assets $\tilde{b}_i(y, N, n)$ solve borrowers’ maximization problem (29) for economies $i$ in either $H$ or $F$

- In gross saving economies, taxes $T_i$ are consistent with losses incurred on gross assets held in each economy, according to (24).

- Consumption of individual goods $c_{A,i}(y, b), c_{B,i}(y, b)$ solves intra-temporal allocation (21) for all $i$
The price $\hat{q}$ ensures market clearing for assets traded within the union:

$$\int \hat{b}_i \, di = 0$$

The relative price $p_B$ ensures market clearing for good $B$, $y_B = (1 - a) \left( \frac{p_B}{p} \right)^{-\theta} \int_{i \in I} c_i \, di$ if no trade in good $B$ between $I$ and ROW occurs, $y_B = c_B$, and it is determined consistently with transport costs and the price prevailing in world markets otherwise:

$$p_B = \begin{cases} 
(1 - \tau) p_B^* & \text{if } y_B > c_B \\
\frac{1}{(1-\tau)} p_B^* & \text{if } y_B < c_B 
\end{cases} \quad (31)$$

The aggregate law of motion for wealth perceived by the representative household in each economy, $N_i' = \Phi (y_i, N_i)$, coincides with the one implied by the optimization problems $(25)$ and $(29)$.

The Markov transition probability for the exogenous shock and the optimal choice of bond holdings determine the the transition function $\Gamma$ for the joint distribution of $(y, N)$ in $H$ and $F$.

**Calibration** The model cannot be solved analytically and I obtain its numerical solution via a global method based on the endogenous grid point algorithm. Choices for parameter values closely follow the literature on open-economy macroeconomics. Parameter values and calibration targets are reported in Table 2. In order to compare simulated series from the model with empirical evidence on the current account, the model is calibrated at quarterly frequency.

The exogenous function $\pi (y, b')$, determining the probability that sovereign debt repayments not received by lenders, equals unity for $b' \geq 0$ and it takes the following functional form otherwise:

$$\pi (y, b') = \frac{(\eta - 1) \bar{b}^\eta}{(\eta - 1) b^\eta + (-b'/y)^\eta} \quad (32)$$

where $\bar{b}$ and $\eta$ are parameters. The price of debt issued to ROW by economies in $I$ is then given by $q (y, b') = q^{ROW} [\pi (y, b')]$ which is continuous, differentiable and positive.
for all real values of $b'$. The upper bound for resources raised on international financial markets when debt is priced by \textit{ROW} is given by the solution to

$$\max_{b'} (-q(y, b') b')$$

The value for $b'$ that maximizes resources raised from borrowing is given by

$$\bar{b}(y) = \arg\max_{b'} (-q(y, b') b')$$

which, from the function in \[32\], is given by $\bar{b}(y) = y\bar{b}$. The parameters $\beta$, $\bar{b}$, and $\eta$ are set to match properties of the distribution of external positions and interest rates before the inception of the monetary union in Euro area countries. In particular, I target an average yearly real interest rate of 5% for economies in $I$ in the stationary equilibrium with no bailout promise, which captures the average benchmark real interest rate on government bonds of Euro area countries in 1997. The difference in yearly interest rates paid by economies in the richest and poorest decile of the stationary distribution equals 1.3 percentage points, which equals the difference in interest rates on Italian and German government debt in that same year. Finally, I target an average ratio of of net-foreign assets to GDP of -7%, which corresponds to the aggregate one observed in Euro Area economies at the onset of the European monetary union.

The subjective discount factor in \textit{ROW}, $q^{\text{ROW}}$ is set to match a world risk-free real interest rate of 2.5%. Due to the presence of a downward secular trend in world interest rates, this value understates the risk-free rate prevailing in the pre-EMU period, while it can describe well the low interest rate environment observed in the world economy in the 2000s.

Endowment streams are determined according to the autoregressive stochastic process:

$$\log (y_{i,t}) = (1 - \rho_y) \mu_y + \rho_y \log (y_{i,t-1}) + \epsilon_{i,t}, \quad (33)$$

where $\epsilon_{i,t}$ is a normal, zero-mean i.i.d shock with standard deviation $\sigma_\epsilon$. To normalize the unconditional mean of $y_i$ to unity, $\mu_y$, is set to equal $-\frac{1}{2} \frac{\sigma^2}{1-\rho_y^2}$. Standard deviation $\sigma_\epsilon$ and the autoregressive parameter $\rho_y$ are set to match standard deviation and autocorrelation
Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.96</td>
<td>Average interest rate in $I$</td>
</tr>
<tr>
<td>$\beta^{ROW}$</td>
<td>.99</td>
<td>World real interest rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>.55</td>
<td>Average NFA in $I$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>6</td>
<td>Spread in borrowing costs between Italy and Germany, pre-EMU</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>.85</td>
<td>Relative size of Greece, Spain, Portugal to euro area</td>
</tr>
<tr>
<td>$a$</td>
<td>$= \kappa$</td>
<td>Normalize $p_B = 1$ in autarky</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>.85</td>
<td>Corsetti, Dedola, Leduc (2008)</td>
</tr>
</tbody>
</table>

of 1.5% and 0.88 of HP-filtered GDP for the euro area. In the numerical solution, the process is approximated by a discretized process obtained via the Rouwenhorst method (Rouwenhorst, 1995, Kopecky and Suen, 2010).

Relative risk aversion of households in $I$, $\gamma$, is set to the standard value of 2. The elasticity of intratemporal substitution between $A$ and $B$ good, $\vartheta$, is set to 0.85, consistent with estimated values in Corsetti, Dedola, Leduc (2008). The relative mass of countries specialized in high-trade cost good is given by $1 - \kappa$, which I set to 0.15 to match the relative size of Greece, Portugal and Spain to the Euro Area, consistently with evidence of low average tradability of output in these countries, as presented in section 2.4. The utility weight $a$ is set to equal relative abundance of the two goods, $\kappa$.

**Steady states** The steady state allocations of the no-bailout and bailout economies differ substantially across several dimensions. This is mainly due to differences in how the price of union-issued assets is determined in the two allocations. While in the no-bailout allocation economies issue liabilities at a price that is elastic with respect to the amount issued, this is not the case in the bailout allocation, as lenders do not differentiate across borrowers due to the bailout promise.

The policy functions for current account, $n' - n$, as function of net wealth in the two
allocations are represented in Figure 7 for an economy with average realization of the endowment shock. In the no-bailout equilibrium, the current account policy function is highly elastic in wealth. Economies with low wealth have a strong incentive to save and reduce their liabilities, as doing so will reduce their borrowing costs. Economies with positive bond holdings choose to reduce their wealth, due to their impatience relative to ROW implying a relatively high price of risk-free assets. These mechanisms are muted in the allocation with bailout promise. Wealthy economies have a lower incentive to dissave, as union issued assets offer high returns that are perceived as risk-free. At lower levels of wealth, borrowing costs are no longer increasing in debt, so that economies with large liabilities have a weaker incentive to run current account surpluses. For very high levels of debt, however, due to the presence of the hard borrowing constraint implied by the limit $\bar{b}$, saving rises steeply again, to reduce the likelihood of a binding borrowing constraint occurring in the future.

The different policy functions for the current account across the two allocations have important implications for the distribution of net foreign asset positions in the economic union $I$. In the no-bailout allocation, as initial net wealth significantly affects choices for
net borrowing and lending, the level of this state variable is not persistent for individual economies. Countries in the union then tend to converge towards similar levels of wealth in the stationary allocation. Net wealth becomes more persistent in the bailout allocation, as the incentive to save is weaker for borrowers due to inelastic borrowing costs, and stronger for wealthy economies due to high returns on union issued assets. As consequence, the support of the net wealth distribution widens significantly, and economies in $I$ with positive and negative levels of net wealth both feature in the stationary distribution. Figure 7 displays the stationary distributions in the no-bailout and bailout allocations of this model.

The distribution of borrowing costs for union-member economies also differs significantly across the two steady states of this model. In the no-bailout allocation, economies characterized by different levels of wealth and income issue debt at different prices depending on their borrowing choices, through the function $q(y, b')$. In the bailout allocation, all union-issued assets are perceived as perfect substitutes by lenders in the union, and they all trade at the same price $\tilde{q}$. When comparing the two distributions, we observe then a compression of borrowing costs, as risky and less risky debtors can borrow at the
same interest rate in the allocation with bailout promise.

### 4.2 Results

I analyze in this section simulated series from the model economy under two experiments. First, in order to assess the ability of the model to describe the pattern of net and gross capital flows observed in the Euro area after the inception of the monetary union, I will analyze simulations for these variables generated when a full bailout promise is introduced in the model economy. Second, starting from the stationary equilibrium of the economy with bailout promises, I will study the effects of a debt crisis, analyzing simulated series following an unexpected fall in savers’ external wealth. I will analyze in this setting borrowing and saving choices by individual economies as well as movement in interest rates and relative good prices.

**Intermediation and the current account**  I consider here the transition of the model economy after the unexpected introduction of a full bailout promise, starting from the stationary equilibrium with no bailout. The behavior of individual economies during this transition episode differs substantially depending on their initial position in the wealth distribution. I will focus here for clarity on two extreme examples, describing first the path of net foreign assets for an economy with high debt and low income, analyzing thereafter the simulation for the same variable in a wealthier economy, with a high endowment realization. For the latter, I will describe in addition the path of gross financial positions, detailing the intermediation behavior in which savers engage.

The path of net foreign asset positions for the high debt economy is presented in Figure 8. This is an economy in $F$ at the XXX percentile of the stationary wealth distribution.

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30 Individual policy functions feature substantial non-linearities, due to the presence of the borrowing constraint and of the bailout promise. Rather than presenting results from one particular simulation for each exercise, I will show below median simulations of individual variables from a large sample of simulated paths, given a large number of draws for individual shock processes. In presence of non-linearities, this procedure ensures that simulations presented do not capture the effects of specific shock realizations.
Figure 8: Transition after introduction of bailout promise. Net foreign asset position of high debt economy (blue, solid line). The red, dashed line displays the counterfactual simulation if no bailout promise is introduced.

with a realization of the endowment shock XXX standard deviations below the long-run mean. This economy is significantly poorer than average, with consumption XXX below the stationary distribution average in the no-bailout economy. In a counterfactual simulation where the bailout promise is not introduced, due to high borrowing costs for high values of debt, this economy would run current account surpluses, reducing its external liabilities as the income shock reverts towards its mean, as shown by the red, dashed line in figure. The introduction of the bailout promise on risky assets causes a shift in the path for current account balances in this economy. The country runs a current account deficit rather than a surplus, letting its net foreign asset position deteriorate, as described by the solid, blue line. Given the low income shock realization, the economy has an incentive to borrow to finance consumption, but it was constrained in doing so by increasing borrowing costs in the no-bailout allocation. As the price of debt is no longer falling in its quantity, due to lenders not fully internalizing default risk, the borrowing constraints is relaxed, allowing economies to issue larger amounts of liabilities without incurring in high interest rates.
Figure 9: Transition after introduction of bailout promise. Net foreign asset position of economy with low debt (blue, solid line). The red, dashed line displays the counterfactual simulation if no bailout promise is introduced.

Current account dynamics of an economy with higher than average wealth in the stationary distribution are the mirror image of those for borrowers just analyzed, and they are displayed in Figure 9. I focus here on the path for net foreign assets of an economy in $F$ with income XXX standard deviations above the long-run average, with net foreign assets corresponding to the XXX percentile of the stationary wealth distribution. Again, the path for net foreign assets in the counterfactual simulation where the bailout promise is not introduced is displayed by the red, dashed line. As for the less wealth economy, standard consumption smoothing motives imply that this economy optimally would let its net foreign asset position deteriorate as the income shock process reverts towards its mean. When a bailout promise is introduced, the high return on union-issued assets that are perceived as risk-free induces this economy to save, increasing the level of its net foreign assets. While in the left tail of the wealth distribution the introduction of the bailout promise induces current account deficits, the reverse is true in the opposite tail, where wealthier economies run current account surpluses rather than running down their assets to finance consumption.
The path of gross asset and liability positions summarizes the intermediation behavior of saving economies in the union, as shown in Figure 10. The path for net foreign assets in this economy is the same one above described in Figure 9. In addition, the path for gross liabilities versus ROW is shown by the red, dotted line and gross assets held against other union members are represented by the black, dashed-dotted line. Saving economies in the union such as the one here represented extend credit to other union members over and above net savings financed by own resources, channelling towards union partners funds that are raised by borrowing from ROM and leveraging the high returns on union-issued assets that are perceived as risk-free.

**Debt crisis in the economic union**  To analyze a debt crisis in this framework, I will study here the effects of an unexpected fall in the value of union-issued assets held by bond holders in the union. I will focus in particular on the path in response to the shock of aggregate variables such as the interest rate on risky assets and the relative price of high trade-cost good. I will analyze then the heterogeneous response to the shock of
economies in the union, focusing in particular on behaviour by countries in $H$ and $F$.

The path for the aggregate value of gross assets held by economies in $I$ is presented in Figure 11. After the fall caused by the initial shock, the amount of gross assets held in the union rises towards its long-run value, as the economy transitions towards its stationary distribution.

The interest rate on debt traded within the union rises in response to the shock. As gross savers’ wealth is reduced by the deterioration in asset values, a lower price of debt ensures equilibrium in the market for union-issued assets, by inducing a reduction in the amount of liabilities issued by borrowers. As economies in the union gradually rebuild their gross asset positions, the interest rate falls again towards its stationary equilibrium value, as shown in Figure 12.

The reduction in asset values implied by the debt crisis negatively affects the relative price of the high transport-cost good, $B$. Aggregate consumption demand falls, as wealth of gross savers in the union is reduced without a corresponding fall in the amount of their liabilities. As aggregate supply of good $B$ is unchanged, the relative price $p_B$ needs to fall to clear the market, as demand for both goods is lower and $B$ can only be exported.
Figure 12: Transition after debt crisis. Relative price of high-transport cost good, \( p_B \), and interest rate on borrowers, \( \tilde{r} \). Percentage deviations from stationary allocation values.

to ROW subject to transport costs. The response of \( p_B \) to the shock is also presented in Figure XXX.

Economies in the union are differently affected by the crisis shock according to their relative specialization with respect to the different types of good, as well as their position in the wealth distribution. The path of consumption for two economies characterized by the same level of net foreign assets, corresponding to XXX, but specialized in production of different types of output is represented in Figure XXX. Consumption falls in both economies, as they reduce their external liabilities in response to the rise in the interest rate. The fall in consumption, however, is compounded for economies in \( H \) by the deterioration in the terms of trade implied by the fall in \( p_B \): As \( p_B \) falls, the value of output relative to the consumption basket falls at the same time as the interest rates, magnifying the reduction in consumption. The same effect acts in the opposite way for economies in \( F \): Being able to export their output to ROW, these countries do not suffer from the fall in consumption in \( I \). On the other hand, the fall in \( p_B \) allows them to enjoy cheaper consumption goods, dampening the negative response of consumption to the rise in the interest rate.
Figure 13: Transition after debt crisis. Aggregate consumption in $F$ economies, specialized in good of type $A$ (blue line) and in $H$, specialized in good of type $B$ (red line). Percentage deviations from stationary allocation values.

5 Concluding remarks

TO BE WRITTEN
A Appendix

A.1 Proof of Proposition 1

Consider the case where a risk-neutral agent is the marginal buyer of debt issued by $H$. Given $H$’s terminal-period default policy, the price this agent is willing to bid for $b_H$ is given by

$$q_H = \begin{cases} 
\beta & \text{if } b_H \in (-\infty, 0] \\
\pi \beta & \text{if } b_H \in (0, \hat{\zeta}] \\
0 & \text{if } b_H \in (\hat{\zeta}, \infty) 
\end{cases} \quad (34)$$

The interior solution for $b_H$ in $(0, \hat{\zeta}]$ to the problem in (3) is given by the solution to the following first-order condition:

$$\pi \beta \frac{1}{p_1} u' \left( \frac{y_A - \epsilon + p_{B,1}y_B + \pi \beta b_H}{p_1} \right) = \pi \beta \frac{1}{p_{2,R}} u' \left( \frac{y_A + p_{B,2,R}y_B - b_H}{p_{2,R}} \right).$$

where $p_{B,2,R}$ and $p_{2,R}$ denote prices in the terminal-period state of the world with high output realization, where repayment by $H$ is optimal.

Imposing logarithmic utility, the optimal amount of debt issued by $H$ is given by

$$b_H = \frac{\epsilon + y_B (p_{B,2} - p_{B,1})}{1 + \pi \beta}.$$ 

I will restrict attention to cases where $\epsilon > y_B (p_{B,1} - p_{B,2,R})$, so that debt optimally issued is positive, $b_H > 0$. The proposed interior solution for $b_H$ is indeed optimal if it is preferred by $H$ to all values of $b_H$ in $(-\infty, 0] \cup (\hat{\zeta}, \infty)$.

Consider now the interior solution with $b_H < 0$ and $q_H = \beta$. Such a solution is given by the following first-order condition:

$$\beta \frac{1}{p_1} u' \left( \frac{y_A - \epsilon + p_{B,1}y_B + \beta b_H}{p_1} \right) = \pi \beta \frac{1}{p_{2,R}} u' \left( \frac{y_A + p_{B,2,R}y_B - b_H}{p_{2,R}} \right) + (1 - \pi) \beta \frac{1}{p_{2,D}} u' \left( \frac{y_{A,L} + p_{B,2,D}y_B - b_H}{p_{2,D}} \right).$$
where $p_{B,2,D}$ and $p_{2,D}$ denote prices in the terminal-period state of the world with low output realization. Note that, for $b_H < 0$, default never occurs. Imposing logarithmic utility, the first-order condition becomes:

$$\frac{1}{y_A - \epsilon + p_{B,1}y_B + \beta b_H} = \pi \frac{1}{y_A + p_{B,2,R}y_B - b_H} + \frac{1}{(1 - \pi) y_{A,L} + p_{B,2,D}y_B - b_H}.$$  

Such a condition cannot be satisfied by a value for $b_H < 0$ whenever

$$\epsilon > y_A + p_{B,1}y_B - \left[ \pi \frac{1}{y_A + p_{B,2,R}y_B} + (1 - \pi) \frac{1}{y_{A,L} + p_{B,2,D}y_B} \right]^{-1}. \quad (35)$$

Let us now consider the corner solution $b_H = 0$. Welfare of the representative household is given in this case by:

$$V_{H,\text{corner}} = \log (y_A - \epsilon + y_Bp_{B,1}) - \log (p_1) + \pi \beta \left[ \log (y_A + y_Bp_{B,2,R}) - \log (p_{2,R}) \right] + (1 - \pi) \beta \log \left( \frac{y_{A,L} + y_Bp_{B,2,D}}{p_{2,D}} \right).$$

Compare this with welfare under the interior solution for $b_H$ in $(0, \hat{\zeta})$:

$$V_{H,\text{risky}} = - \log (p_1) - \pi \beta \log (p_{2,R}) + (1 + \pi \beta) \left[ \log \left( y_A - \frac{\epsilon}{1 + \pi \beta} + \frac{1}{1 + \pi \beta} y_Bp_{B,1} + \frac{\pi \beta}{1 + \pi \beta} y_Bp_{B,2,R} \right) \right] + (1 - \pi) \beta \log \left( \frac{y_{A,L} + y_Bp_{B,2,D}}{p_{2,D}} \right).$$

Removing identical terms, and ignoring the effect of the planner’s choice on good prices, due to price-taking behaviour, the risky choice is preferred to the corner solution if and only if

$$(1 + \pi \beta) \log \left( y_A - \frac{\epsilon}{1 + \pi \beta} + \frac{1}{1 + \pi \beta} y_Bp_{B,1} + \frac{\pi \beta}{1 + \pi \beta} y_Bp_{B,2,R} \right) > \log (y_A - \epsilon + y_Bp_{B,1}) + \pi \beta \log (y_A + y_Bp_{B,2,R})$$

noting that

$$y_A - \frac{\epsilon}{1 + \pi \beta} + \frac{1}{1 + \pi \beta} y_Bp_{B,1} + \frac{\pi \beta}{1 + \pi \beta} y_Bp_{B,2,R} = \frac{1}{1 + \pi \beta} (y_A - \epsilon + y_Bp_{B,1}) + \frac{\pi \beta}{1 + \pi \beta} (y_A + y_Bp_{B,2,R}).$$
this condition is satisfied by concavity of the logarithm function and Jensen’s inequality. It is trivial to rule out choices for \( b_H \) in \((\hat{\zeta}, \infty)\). Given \( q_H = 0 \), such choices are dominated by setting \( b_H = 0 \). While the same amount of resources is obtained in the initial period, i.e. zero, by choosing \( b_H > \hat{\zeta} \) an output cost of default is suffered with certainty in the terminal period.

Suppose an agent were willing to bid \( q_H > \pi \beta \) for \( b_H \in (0, \hat{\zeta}] \) and \( 0 \) and \( \beta \) for \( b_H > \hat{\zeta} \) and \( b_H < 0 \), respectively. A fortiori, then it will still be optimal for \( H \) to issue \( b_H \) in the \((0, \hat{\zeta}]\) interval.

The above proves that a risky borrowing choice is preferred to all other choices when

\[
\epsilon > \min \left\{ y_{BH} (p_{B,1} - p_{B,2}), y_A + p_{B,1}y_B - \frac{1}{y_A + p_{B,2}y_B} + \frac{1}{y_{A,L} + p_{B,2,2}y_B} \right\}^{-1}.
\]

This condition is function of goods prices that are determined in general equilibrium and, in general, it must be verified on a case-by-case basis. In the special case where \( p_B \) is constant across periods and states of the world, the condition simplifies to

\[
\epsilon > (1 - \pi) (y_A - y_{A,L}) \frac{y_A + p_{B}y_B}{\pi y_{A,L} + (1 - \pi) y_A + p_{B}y_B}.
\]

### A.2 Optimization problem in ROW

The \textit{ROW} is inhabited by a risk-neutral representative household enjoying utility from consumption of goods of type \( A \) and \( B \).

The problem faced by the representative household in \textit{ROW} is the following:

\[
\begin{align*}
\max_{c_1^{\text{ROW}}, c_2^{\text{ROW}}, b^{\text{ROW}}, b_{H,\text{ROW}} \geq 0} & \quad c_1^{\text{ROW}} + \beta E c_2^{\text{ROW}} \\
\text{s.t.} & \quad c_t^{\text{ROW}} = (c_{A,t}^{\text{ROW}})^a (c_{B,t}^{\text{ROW}})^{1-a}, \\
& \quad p_1^{\text{ROW}} c_1^{\text{ROW}} = y_A^{\text{ROW}} + p_{B,1} y_B^{\text{ROW}} + q_H b_H^{\text{ROW}} - q_H b_{H,\text{ROW}}, \\
& \quad p_2^{\text{ROW}} c_2^{\text{ROW}} = y_A^{\text{ROW}} + p_{B,2} y_B^{\text{ROW}} - b_{\text{ROW}} + (1 - D) b_{H,\text{ROW}},
\end{align*}
\]

where I allow relative prices and price indices in \textit{ROW}, to differ from those prevailing in \( H \) and \( F \). All other variables have their intuitive meaning, in particular, \( b_{H,\text{ROW}} \) denotes
the amount of assets issued by \( H \) and purchased by \( ROW \) and \( b_{ROW} \) denotes the amount of risk-free assets that are issued by \( ROW \).

A.3 Crisis amplification in \( H \)

Consider terminal-period consumption in \( H \) in the crisis state where it receives a low amount of good \( A \) endowment. This is given by

\[
c = \frac{y_{A,L} + p_B y_B}{p}.
\]

Denoting with subscripts \( I \) and \( NI \) allocations in the intermediation and non-intermediation scenarios, respectively, the crisis is amplified in \( H \) in presence of intermediation when

\[
\frac{c_I}{c_{NI}} = \frac{p_{NI} y_{A,L} + p_{B,I} y_B}{p_I y_{A,L} + p_{B,NI} y_B} < 1
\]

It is easy to see that this condition holds when specialization in \( B \) good in \( H \) is extreme, namely when \( y_{A,L} = 0, y_B > 0 \). In this case,

\[
\frac{c_I}{c_{NI}} = \frac{p_{NI}}{p_I} \frac{p_{B,I}}{p_{B,NI}} = \frac{p_{B,I}}{p_{B,NI}}.
\]

Default by \( H \) liabilities owed to \( F \) rather than to \( ROW \) causes lower wealth in the union in the intermediation scenario. The ensuing lower price of the high-transport cost good \( B \) is unambiguously detrimental to consumption in \( H \), when this economy displays extreme specialization in this type of good, as stated in XXX. Consider now the opposite case where \( H \) is extremely specialized in good \( A \), \( y_{A,L} > 0, y_B = 0 \). Now,

\[
\frac{c_I}{c_{NI}} = \frac{p_{NI}}{p_I} = \frac{p_{B,I}}{p_{B,NI}}^{a-1}.
\]

In this event, a fall in the relative price of good \( B \) implies an appreciation of the terms of trade and, thanks to cheaper imports of good \( B \), intermediation dampens the severity of the crisis in \( H \). Finally, consider the general case where both \( y_{A,L} > 0 \) and \( y_B > 0 \):

\[
\frac{c_I}{c_{NI}} < 1 \iff \frac{y_{A,L} + p_{B,I} y_B}{y_{A,L} + p_{B,NI} y_B} < \frac{p_I}{p_{NI}}
\]
Noting that \( \frac{p_I}{p_{NI}} = \left( \frac{p_{B,I}}{p_{B,NI}} \right)^{1-a} \), taking logs and making use of the fact that \( \log(1 + x) \approx x \), at first order, the condition above reduces to

\[
\frac{(p_{B,I} - p_{B,NI}) y_B}{y_{A,L} + p_{B,NI} y_B} < (1 - a) \frac{p_{B,I} - p_{B,NI}}{p_{B,NI}}.
\]

Imposing \( p_{B,NI} = \frac{y_{A,L} + y_A^* + b_{F,NI}}{y_B + y_B^*} \), this reduces to the condition on relative abundance of good \( B \) stated in XXX:

\[
\frac{y_B}{y_B + y_B^*} > \frac{y_{A,L}}{y_B + y_B^*} y_{A,L} + y_A^* + b_{F,NI}.
\]
A.4 Proof of Proposition [2]

Consider the allocation with $\xi = 0$, $\pi \to 0$. Consumption by $H$ in financial autarky is given by $c_1 = y - \epsilon$, $c_{2,D} = y_L$, $c_{2,R} = y$, noting that the repayment state is realized with zero probability in the limit. In the limit, welfare in $H$ is given by

$$V_{H}^{\xi=0}(\epsilon) = \log(y - \epsilon) + \beta \log(y_L).$$

Consumption by $F$ is given by $c_s^* = y^*$ in all periods and states $s$. Welfare in $F$ is

$$V_{F}^{\xi=0}(\epsilon) = (1 + \beta) \log(y^*).$$

Under the bailout allocation, borrowing by $H$ is bounded in the limit by the amount of resources available in the repayment state, $b_H = -y$. Consumption is given by $c_1 = 2y - \epsilon$, $c_{2,D} = y_L$ and $c_{2,R} \to 0$. Using the fact that $\lim_{x \to 0} x \log(x) = 0$, welfare in $H$ tends to

$$V_{H}^{\xi=1}(\epsilon) = \log(2y - \epsilon) + \beta \log(y_L).$$

As established in [11] borrows $b_F = \frac{\beta}{1+\beta} b_H = -\frac{\beta}{1+\beta} y$ to finance asset purchases when $\pi = 0$. In the limit for $\pi = 0$, consumption is equalized across the initial period and terminal period default state: $c_1^* = c_{2,D}^* = y^* - \frac{\beta}{1+\beta} y$. Welfare in $F$ in the bailout allocation is then given by

$$V_{F}^{\xi=1}(\epsilon) = (1 + \beta) \log\left(y^* - \frac{\beta}{1+\beta} y\right).$$

Note that the change in $F$ welfare induced by the bailout promise is independent of $\epsilon$:

$$V_{F}^{\xi=1}(\epsilon) - V_{F}^{\xi=0}(\epsilon) = (1 + \beta) \log\left(1 - \frac{\beta}{1+\beta} \frac{y}{y^*}\right).$$

The change in welfare in $H$ is instead increasing in $\epsilon$:

$$V_{H}^{\xi=1}(\epsilon) - V_{H}^{\xi=0}(\epsilon) = \log\left(\frac{2 - \epsilon/y}{1 - \epsilon/y}\right).$$

Hence, the change in union welfare induced by the bailout promise, $V_{\xi=1}(\epsilon) - V_{\xi=0}(\epsilon)$, is increasing in $\epsilon$. Note that this change tends to infinity for $\epsilon \to y$. For $\epsilon = 0$, $V_{\xi=1}(\epsilon) - V_{\xi=0}(\epsilon) < 0$ under the following condition for $y/y^*$ and $\beta$:

$$\left(1 - \frac{\beta}{1+\beta} \frac{y}{y^*}\right)^{1+\beta} < \frac{1}{2}.$$
By continuity of $V^{\xi=1}_H (\epsilon) - V^{\xi=0}_H (\epsilon)$ and the intermediate value theorem, it can be shown that there exist a value $\bar{\epsilon} \in (0, \infty)$ such that $V^{\xi=1}_H (\bar{\epsilon}) - V^{\xi=0}_H (\bar{\epsilon}) = 0$ and $V^{\xi=1}_H (\epsilon) - V^{\xi=0}_H (\epsilon) > 0$ for $\epsilon > \bar{\epsilon}$. 
References


