Price Discrimination in Input Markets under Asymmetric Agency Cost

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Abstract

This paper analyzes a model of price discrimination in input markets where small firms have to buy a larger share of their inputs from other firms than large firms. I show that the model gives rise to cases where the large firm, the small firm, or none of the firms is favored. A ban on price discrimination may thus either be beneficial or detrimental for the profit of small firms. The welfare implications of a ban on price discrimination typically depend on whether small or large firms are favored. Furthermore, welfare implications in this model with asymmetric agency costs differ from welfare implications if technological costs are asymmetrically distributed for the small and the large firm.

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1 Introduction

In many markets for intermediate products, sellers with market power charge buyers different prices based on observable buyer characteristics. A prominent and easily observable characteristic of a firm is its size. Price discrimination based on firm size has been in the focus of policy makers since the beginning of legislative activity regarding price discrimination.\(^1\)

In the extant literature on price discrimination in input markets, smaller firms are usually assumed to operate with less efficient technology. In this paper, I want to provide a complementary perspective based on the following idea: When producing a product with the input purchased from an upstream firm, a smaller firm may have to rely more on further inputs purchased on other input markets than a large firm which may produce a larger share of the necessary components itself. This implies that even if a small firm uses the same technology as a larger firm, it may have to give more rents to third parties. Thus, the small firm tends to have higher private costs than a larger firm although its social cost of production may be the same.

In order to capture this idea, I analyze a model where a manufacturer sells its product to two downstream firms that operate in separated final markets. Both firms need a second input in order to produce their final good. The first firm is large and can produce the second input by itself. The second firm is small and needs to buy the second input from a subcontractor. I assume that the subcontractor and the first have private information on their cost of producing the second input. This gives rise to an information rent which adds to the second firm’s private production cost, and restricts the manufacturer from extracting the complete industry profit. Furthermore, I assume that the manufacturer cannot sign a direct contract with the subcontractor. Thus, all communication between the subcontractor and the manufacturer has to go via the second firm. This gives rise to double marginalization.

In this framework, I analyze the following two research questions: First, what kind of pricing is optimal for the manufacturer in the situation described above? Under what conditions does the manufacturer favor the small firm, the large firm or none of the two? Second, what are the implications for welfare if the manufacturer is banned from using price discrimination, relative to models where small firms have higher technological costs?

\(^1\)Inderst and Valletti (2009) cite Wright Patman, initiator of the Wright-Patman Act of 1936, saying that the act aimed to protect smaller merchants from unfair competition.
I show that depending on a simple property of the distribution function of marginal production cost of the first firm and the subcontractor, the model either predicts the small or the large firm to be favored by the manufacturer. Surprisingly, the manufacturer may find it optimal to treat the two firms equally. This is for instance the case if the firm’s private information is uniformly distributed, but holds as well if the distribution comes from the family of generalized Pareto distributions.

A ban on price discrimination may therefore benefit either the small or the large firm. However, even if the small firm is favored by the manufacturer, its advantage is never so large that it makes up the handicap caused by the information rent which has to be given to the subcontractor. This implies that a ban on price discrimination may reduce or increase the distortion on the market which is served by the smaller firm, depending on whether or not the small firm is favored under price discrimination. However, general implications of a ban on price discrimination remain, as in most of the literature, rather hard to draw. Nevertheless, I can show that in the standard case of linear demand and constant marginal cost of production, a ban on price discrimination is likely to increase welfare if the large firm is favored, whereas it tends to reduce welfare if the small firm is favored.

Furthermore, comparing my welfare results with those obtained by Herweg and Müller (2014), I show that asymmetric agency costs change the welfare implications of a ban on price discrimination relative to asymmetric technological costs.

Related literature Starting from Marshall (1920) and Robinson et al. (1933), economists have analyzed third-degree price discrimination from a positive and normative perspective. This paper is closely related to the literature on third-degree price discrimination in input markets. Herweg and Müller (2014) analyze a model where the downstream firms hold private information about their production cost. In contrast to this paper, their cost of production represents asymmetric technologies. I relate my welfare results to theirs in order to shed light on the differences that arise due to asymmetric technological costs vs. asymmetric agency cost. The literature does not agree on whether a manufacturer is more likely to favor small and weak buyers, as in DeGraba (1990) and Yoshida (2000), or large and efficient buyers, as in Inderst.

\[^2\]cf. Bergemann et al. (2015) for the potential welfare effects under third degree price discrimination
\[^3\]See Aguirre et al. (2010) for an extension of Robinson’s analysis
and Valletti (2009) and Inderst and Shaffer (2009). The idea that large firms may are more likely to produce their inputs themselves is not new to the literature on third degree price discrimination. Both Katz (1987), Inderst and Valletti (2009), and O’Brien (2014) consider the possibility of backward integration. The literature draws different welfare implications of a ban on price discrimination.\footnote{See also Arya and Mittendorf (2010) for an analysis if downstream firms may serve different markets, and Herweg and Müller (2012) for the effects of entry in the downstream market.}

Finally, this paper is related to the literature on bilateral contracts (see for instance O’Brien and Shaffer (1992) and Rey and Vergé (2004)). Closest from this literature is Dequiedt et al. (2015) who analyze optimal bilateral contracting under private information. Whereas in the literature on bilateral contracting, the principal is restricted from using a grand mechanism, the manufacturer is in my paper restricted from communicating directly with the subcontractor.

In the next section, I present the model. Section 3 defines price discrimination in the context of the model. Section 4 analyzes optimal contract if price discrimination is allowed. Section 5 considers the manufacturer’s problem when price discrimination is banned. Section 6 considers some implications for welfare. Section 7 concludes.

## 2 The model

A manufacturer produces goods for two independent markets which are served by two retailers. Retailer 1 serves market 1 and retailer 2 serves market 2. The inverse demand function on market $i$ is given by $P_i(y_i)$ where $y_i$ is the quantity which is sold on market $i$. $P_i(y_i)$ is strictly decreasing in $y_i$. If the manufacturer produces the quantities $y_i$ and $y_j$, total production costs are $C_1(y_1) + C_1(y_2)$, which is increasing in $y_1$ and $y_2$.

For $y_1$ units sold by retailer 1 on market 1, she incurs retailing costs of $c_1K_1(y_1)$, where $c_1$ is a parameter which is private information to retailer 1. $c_1$ is drawn according to the cumulative distribution function $F_1(c_1)$ from the interval $[\underline{c}, \overline{c}]$. If retailer 2 wants to sell on market 2, she needs to cooperate with a subcontractor who provides a necessary input to the retailing process. If $y_2$ units are sold on market 2, the subcontractor incurs retailing costs of $c_2K_2(y_2)$, whereas retailer 2 has no further costs. $c_2$ is a parameter which is private information to the subcontractor, it is drawn from the interval $[\underline{c}, \overline{c}]$ according to the cumulative distribution function $F_2$. 


If \( t_i \) is a transfer from retailer \( i \) to the manufacturer and \( t_s \) is a payment from retailer 2 to the subcontractor, then the manufacturer receives a profit of \( t_1 + t_2 - C_1(y_1) - C(y_2) \), retailer 1 enjoys a profit of \( P_1(y_1)y_1 - c_1K_1(y_1) - t_1 \), retailer 2’s profit is given by \( P_2(y_2)y_2 - t_s - t_2 \), and the subcontractor earns \( t_s - c_2K_2(y_2) \).

In order to focus on the asymmetry which arises from the differences in the organization of the retailing process, I assume that both markets, goods, and production technologies are otherwise perfectly symmetric, i.e. \( P_i(y) = P(y), C_i(y) = C(y), K_i(y) = K(y) \), and \( F_i(c) = F(c) \) with \( i \in \{1, 2\} \). Furthermore I assume that \( yP_1(y) \) is twice differentiable and concave in concave in \( y \), \( C(y) \) and \( K(y) \) are continuously differentiable and convex and satisfy \( C(0) = K(0) = 0 \). Finally, I assume that \( y_i \in [0, \bar{y}] \), and that \( F'(c_i) = f(c_i) \) which satisfies that \( z(c_i) \equiv c_i + F(c_i)/f(c_i) \) is twice differentiable in \( c_i \).

**The contracting game** I study the following contracting game between the manufacturer, the retailers 1 and 2, and the subcontractor.

At the beginning of the game, \( c_1 \) and \( c_2 \) are drawn. \( c_1 \) is only observed by the retailer 1 and \( c_2 \) is only observed by the subcontractor. Next, the manufacturer offers wholesale contracts to retailers 1 and 2. Each contract specifies a payment \( t_i \) from retailer \( i \) to the manufacturer, and a quantity \( y_i \), both as functions of a reports \( \hat{c}_i \) of the cost type \( c_i \) from the retailers to the manufacturer. These contracts also allows both retailers to send the message \( \hat{c}_i = \emptyset \) and not to participate in the contract. In this case, they make zero profit. The offered contracts take the form \( \{t_i(\hat{c}_i), y_i(\hat{c}_i)\} \) and are observable to both retailers and the subcontractor.

After the contract is offered and before the retailers have to decide whether to accept it, retailer 2 offers a subcontract to the subcontractor. The subcontract specifies a transfer \( t_s \) from the retailer 2 to the subcontractor and a reporting strategy \( r \) specifying the report by the retailer 2 in the wholesale contract, both as functions of the report \( \hat{c}_2 \) of the subcontractor on the cost parameter \( c_2 \). Formally a reporting strategy is a function that maps from the set of cost parameters \( [\underline{c}, \overline{c}] \) to the set of possible reports in the wholesale contract \( [\underline{c}, \overline{c}] \cup \{\emptyset\} \). The subcontract takes the form \( (t_s(\hat{c}_2), r(\hat{c}_2)) \). Importantly, I assume that the subcontract is not observable to any outside party, such that the manufacturer cannot force retailer 2 to offer a certain subcontract.

Next, the subcontractor has to decide whether to accept the subcontract. If the subcon-
tractor rejects the offer, both the subcontractor and retailer 2 receive zero profits. Finally, the reports in the wholesale contracts are made and the goods are sold on the markets.

I restrict attention to truthful Perfect Bayesian Equilibria (PBE) of this game, i.e in equilibrium, retailer 1 reports \(c_1\) truthfully to the manufacturer, the subcontractor reports \(c_2\) truthfully to retailer 2, and the subcontract prescribes a reporting strategy \(r^*(c_2) = c_2\).

**Industry optimum** The total profit of the industry is given by

\[
\sum_{i=1}^{2} y_i p_i(y_i) - c_i K(y_i) - C(y_i).
\]

For the retailing cost parameters \(c_1\) and \(c_2\), I denote by \((y^*_1(c_1), y^*_2(c_2))\) the unique maximizer of the above function.

**Public Information** If all information on cost parameters is public, the double marginalization problem can be solved using two-part tariffs and the manufacturer can achieve the optimal industry profit. For any \(c_1\) and \(c_2\), the manufacturer optimally sets piece rates of \(w_i(c_i) = C'(y^*_i(c_i))\) and a fixum of \(\Phi_i(c_i) = P(y^*_i(c_i))y^*_i(c_i) - c_i K(y^*_i(c_i)) - w_i(c_i)y^*_i(c_i)\).

Under private information, an informational variant of the double marginalization problem arises, which does not resolve even if sophisticated contracts are used.

### 3 Defining Price Discrimination

Before I turn to the analysis of optimal wholesale contracts in the cases where price discrimination is permitted and when it is not, this section provides an analysis of the subcontracts which are signed between retailer 2 and the subcontractor in equilibrium. This is a necessary step, in order to sensibly define what discrimination between the two retailers actually means in the present context. It turns out that with an optimal subcontract for retailer 2, the retailers are symmetric apart from the fact that retailer 2 has a *virtual* retailing cost function in contrast to the *real* retailing cost function of retailer 1. This allows to define when wholesale contracts are discriminating and when not.

Thus I now analyze which subcontract retailer 2 optimally offers to the subcontractor for a given wholesale contract \(\{t_2(\hat{c}_2), y_2(\hat{c}_2)\}\). I denote the subcontractor’s expected profit from
truths\(\text{telling, for a reporting rule } r,\) by

\[ \Pi_s(c_2; r) \equiv t_s(c_2) - c_2 K(y_2(r(c_2))). \]

Retailer 2 faces the following problem:

\[
\max_{r(\cdot), t_s(\cdot)} \int_{c_2}^{c_2} [y_2(r(c))] P(y_2(r(c))) - t_2(r(c)) - t_s(c_2)] \, dF(c_2)
\]

\[ \text{s.t. } (IC_s) : \Pi_s(c_2; r) \geq t_s(\hat{c}_2) - c_2 K(y_2(r(\hat{c}_2))) \quad \forall c_2, \hat{c}_2 \in [\underline{c}, \overline{c}], \]

\[ (PC_s) : \Pi_s(c_2; r) \geq 0 \quad \forall c_2 \in [\underline{c}, \overline{c}]. \]

The constraints in this problem can be characterized in the following way:

**Lemma 1.** \((IC_s)\) and \((PC_s)\) are satisfied if and only if

\[ i) \quad \Pi'_s(c_2; r) = -K(y_2(r(c_2))), \]

\[ ii) \quad y_2(r(c_2)) \text{ is weakly decreasing in } c_2, \]

\[ iii) \quad \Pi_s(\overline{c}; r) \geq 0. \]

This result can be used to express the profit of retailer 2 from a subcontract \(\{r(\cdot), t_s(\cdot)\}\), as the total profit of the distribution channel reduced by the profit that has to be given to the subcontractor.

\[
\int_\underline{c}^{\overline{c}} [y_2(r(c_2)) P(y_2(r(c_2))) - t_2(r(c_2)) - t_s(c_2)] \, dF(c_2)
\]

\[ = \int_\underline{c}^{\overline{c}} [y_2(r(c_2)) P(y_2(r(c_2))) - z(c_2) K(y_2(r(c_2))) - t_2(r(c_2))] dF(c_2) - \Pi_s(\overline{c}) \]

For any reporting rule \(r(\cdot)\) which retailer 2 may set, the optimal transfer rule \(t_s(\cdot)\) makes the participation constraint for the highest cost type of the subcontractor binding, i.e. \(\Pi_s(\overline{c}; r) = 0\). Together with condition \(i)\) in Lemma 1, this fixes the transfer rule that retailer 2 sets in a BPE as well as in attractive deviations from a truthful reporting strategy.

**Lemma 2.** For a given wholesale contract and a reporting rule \(r(\cdot)\), retailer 2 optimally chooses
the transfer

\[ t_s(c_2) = c_2 K(y_2(r(c_2))) + \int_{c_2}^{r} K(y_2(r(x)))dx. \]

I denote the resulting expected profit for retailer 2 by \( \Pi_2(r(\cdot)) \) and I define retailer 2’s profit for a given realization of \( c_2 \) by

\[ \Pi_2(c_2; r(\cdot)) \equiv y_2(r(c_2))P(y_2(r(c_2))) - z(c_2)K(y_2(r(c_2))) - t_2(r(c_2)). \]

In a truthful BPE of the game, the truthful reporting strategy \( r^* \) with \( r^*(c_2) = c_2 \) is set in the subcontract. To simplify notation, I denote \( \Pi_2 \equiv \Pi_2(r^*(\cdot)) \) and \( \Pi_2(c_2) \equiv \Pi_2(c_2; r^*(\cdot)) \) and retailer 2’s profit in a truthful BPE is

\[ \Pi_2(c_2) = y_2(c_2)P(y_2(c_2)) - z(c_2)K(y_2(c_2)) - t_2(c_2). \]

This expression can now be compared with the profit of retailer 1 from a wholesale contract \( \{y_1(\hat{c}_1), t_1(\hat{c}_1)\} \) in a truthful BPE:

\[ \Pi_1(c_1) = y_1(c_1)P(y_1(c_1)) - c_1K(y_1(c_1)) - t_1(c_1). \]

\( \Pi_1(c_1) \) and \( \Pi_2(c_2) \) are symmetric apart from the fact that retailer 1 has retailing costs of \( c_1K(y_1(c_1, y_2)) \) whereas retailer 2 has virtual retailing costs of \( z(c_2)K(y_2(c_2, c_1)) \). If retailer 2 has a cost parameter \( c_2 = c \) and retailer 1 has a cost parameter \( c_1 = z(c) \), both retailers have the same retailing cost function. If the two retailers are treated differently in this case, I say that the manufacturer discriminates between the retailers. In order to define this idea more generally, I denote by \( c^* \) the solution to \( z(c) = \bar{c}. \)

**Definition 1.** _There is no price discrimination in the input market if for all \( c \in [\bar{c}, c^*] \)

\[ y_1(z(c)) = y_2(c) \quad \text{and} \quad t_1(z(c)) = t_2(c). \]

Otherwise, there is price discrimination in the input market._

Without price discrimination, the two retailers are treated symmetrically given their retailing
cost functions. However, if \( \bar{c} < \infty \), then the (virtual) cost parameter of retailer 2 may lie outside the range of the cost parameter for retailer 1. This is the case for all \( c_2 \) with \( z(c_2) > \bar{c} \). The manufacturer may then still offer the two retailers symmetric contracts by allowing retailer 1 to make reports in the interval \([\underline{c}, z(\bar{c})]\). However, it needs to be ensured that reports in the interval \((\bar{c}, z(\bar{c})]\) are never optimal. This point is important to keep in mind when considering the definition of price discrimination.

4 Optimal wholesale contracts when price discrimination is permitted

I now turn to the analysis of optimal wholesale contracts if price discrimination is allowed. In this case, the manufacturer faces no further restrictions apart from the incentive and participation constraints of the two retailers. As the two markets are independent, the optimal design of a wholesale contract for market 1 is independent of the wholesale contract for market 2 and vice versa. I can thus analyze the two problems separately.

Optimal wholesale contract on market 1 I first characterize conditions under which a wholesale contract gives retailer 1 the incentive to report the cost parameter truthfully. If the manufacturer offers retailer 1 a wholesale contract \( \{t_1(\hat{c}_1), y_1(\hat{c}_1)\} \), retailer 1’s profit for cost parameter \( c_1 \) is given by

\[
\Pi_1(c_1) = P(y_1(c_1))y_1(c_1) - c_1K(y_1(c_1)) - t_1(c_1).
\]

It is optimal for retailer 1 to participate in the wholesale contract and to truthfully report \( c_1 \) if

\[
(IC_1) : \quad \Pi_1(c_1) \geq P(y_1(\hat{c}_1))y_1(\hat{c}_1) - c_1K(y_1(\hat{c}_1)) - t_1(\hat{c}_1) \quad \forall c_1, \hat{c}_1 \in [\underline{c}, \bar{c}]
\]

\[
(PC_1) : \quad \Pi_1(c_1) \geq 0 \quad \forall c_1 \in [\underline{c}, \bar{c}].
\]
The manufacturer’s problem of designing the wholesale contract for market 1 is to

$$\max_{y_1, t_1} \int [t_1(c_1) - C(y_1(c_1))]dF(c_1)$$

s.t. \((IC_1), (PC_1)\).

The optimal wholesale contract on market 1 can thus be found using standard techniques from principal-agent models under the following assumption.

**Assumption 1.** \(z(c)\) is strictly increasing for \(c \in [c, \bar{c}]\).

**Proposition 1.** If price discrimination is permitted and Assumption 1 is satisfied, the manufacturer sets the wholesale contract \(\{y^{PD}_1(c), t^{PD}_1(c)\}\) on market 1:

$$y^{PD}_1(c) = \arg \max_{y_1} y_1 P(y_1) - z(c_1)K(y_1) - C(y_1),$$

$$t^{PD}_1(c) = y^{PD}_1(c_1)P(y^{PD}_1(c_1)) - \int_{c_1}^{\bar{c}} K(y^{PD}_1(x))dx.$$  

**Optimal wholesale contract on market 2** In a truthful BPE of the game, the truthful reporting strategy \(r^*\) with \(r^*(c_2) = c_2\) is set by retailer 2 in the subcontract. It thus needs to hold that the truthful reporting strategy gives at least the same payoff to retailer 2 as any other reporting strategy which could be implemented. The set of implementable reporting strategies is restricted by condition \(ii)\) in Lemma 1. I denote this set by \(R:\)

$$R = \left\{ r : [c, \bar{c}] \rightarrow [c, \bar{c}] \cup \{\emptyset\} \left| y_2(r(c_2)) \right. \text{is weakly decreasing in } c_2 \right\}.$$  

The following incentive compatibility constraints needs to be satisfied in any truthful equilibrium:

$$\left( IC_2 \right): \Pi_2 \geq \Pi_2(r(\cdot)) \quad \forall r \in R.$$  

Note that this constraint contains a participation constraint, as retailer 2 may implement a reporting strategy that implies non-participation for certain reports of the subcontractor. The
manufacturer’s optimal wholesale contract for market 2 is the solution to

$$\max_{y_2(t), t_2(t)} \int [t_2(c_2) - C(y_2(c_2))]dF(c_2)$$

s.t. \((IC_2)\).

The constraint \((IC_2)\) is not a standard incentive constraint, as retailer 2 can only choose reporting strategies from the set of implementable reporting strategies \(R\). However, the set of implementable strategies is rich enough to derive a simple and standard characterization of incentive compatibility.

**Lemma 3.** \((IC_2)\) is satisfied if and only if

1. \(\Pi'_2(c_2) = -z'(c_2)K(y_2(c_2))\),
2. \(y_2(c_2)\) is weakly decreasing in \(c_2\),
3. \(\Pi_2(\overline{c}) \geq 0\).

Given this characterization, the optimal wholesale contract for market 2 can be easily derived under the following assumption.

**Assumption 2.** \(z(c) + z'(c)(z(c) - c)\) is weakly increasing for \(c \in [\underline{c}, \overline{c}]\).

**Proposition 2.** If price discrimination is permitted and Assumption 2 is satisfied, the manufacturer sets the wholesale contract \(\{y_2^{PD}(c_2), t_2^{PD}(c_2)\}\) on market 2:

$$y_2^{PD}(c_2) = \arg \max_{y_2} y_2P(y_2) - (z(c_2) + z'(c_2)(z(c_2) - c_2))K(y_2) - C(y_2),$$

$$t_2^{PD}(c_2) = y_2^{PD}(c_2)P(y_2^{PD}(c_2)) - \int_{c_2}^{\overline{c}} z'(x)K(y_2^{PD}(x))dx.$$

**Comparing wholesale contracts** I now want to compare the optimal wholesale contracts with respect to quantities and profits that the two retailers receive. This allows to answer the question which of the two retailers is favored under price discrimination, or whether the manufacturer may even find it optimal not to discriminate between the retailers. For some \(c \in [\underline{c}, \overline{c}]\), I say that retailer 2 is favored over retailer 1 if \(\Pi_2(c) > \Pi_1(z(c))\) and vice versa.

The quantities in the optimal wholesale contracts can be compared as follows.
Lemma 4. If \( z(\cdot) \) is convex, \( y_1^{PD}(z(c)) \leq y_2^{PD}(c) \) for all \( c \in [\bar{c}, c^*] \). If \( z(\cdot) \) is concave, \( y_1^{PD}(z(c)) \geq y_2^{PD}(c) \) for all \( c \in [\bar{c}, c^*] \).

Proof. If \( z(\cdot) \) is convex, \( z(z(c)) \geq z(c) + z'(c)(z(c) - c) \) which implies \( y_1^{PD}(z(c)) \leq y_2^{PD}(c) \). The reverse holds if \( z(\cdot) \) is concave. \( \square \)

In order to answer whether there is price discrimination and, if yes, which of the retailers is favored, it is useful to define the threshold \( \bar{c} \) as the smallest cost parameter for which a market would not be served even in the industry optimum:

\[
\bar{c} \equiv \inf \left\{ c \in \mathbb{R} : \arg \max_y yP(y) - cK(y) - C(y) = 0 \right\}.
\]

In order to compare profits for the two retailers, note that for \( c \in [\bar{c}, c^*] \)

\[
\Pi_1(z(c)) = \int_{z(c)}^{c} K(y_1(x))dx = \int_{z(c)}^{c} K(y_1(z^{-1}(x))))dx = \int_{c}^{c^*} K(y_1(z(\chi))))z'(\chi)d\chi,
\]

where the last equality follows from a change in variable from \( x \) to \( \chi = z^{-1}(x) \). This has to be compared to

\[
\Pi_2(c) = \int_{c}^{c^*} K(y_2(x))z'(x)dx.
\]

The are two sources of differences between the profits of the retailers. The first are the quantities \( y_1(z(c)) \) and \( y_2(c) \) for \( c \in [\bar{c}, c^*] \). The second difference arises if retailer 2 is served for cost parameters \( c > c^* \). For \( c > c^* \), retailer 2 has a virtual cost parameter \( z(c) > \bar{c} \) which exceeds all values that the cost parameter can take for retailer 1. Clearly, the second source of difference does not arise if the range of cost parameters is not bound from above.

Proposition 3. Suppose price discrimination is permitted and Assumptions 1 and 2 are satisfied.

1. If \( z(\cdot) \) is linear, there is no price discrimination if \( z(\bar{c}) \geq \bar{c} \). If \( z(\bar{c}) < \bar{c} \), retailer 2 is favored for all \( c_2 \in [\bar{c}, c^*] \).

2. If \( z(\cdot) \) is convex, retailer 2 is favored for all \( c_2 \in [\bar{c}, c^*] \).
3. If \( z(\cdot) \) is concave, retailer 1 is favored if \( c + z'(c^*)(\bar{c} - c^*) \geq \bar{c} \). If \( \bar{c} + z'(c^*)(\bar{c} - c^*) < \bar{c} \), there exists a cutoff \( c^+ \in [\bar{c}, c^*] \) such that retailer 1 is favored if \( c_2 = z(c_1) < c^+ \), and retailer 2 is favored if \( c_2 = z(c_1) \geq c^+ \).

Note that \( z(\cdot) \) is linear if \( F(c) \) is uniform on \([0,1]\), or satisfies \( F(c) = c^\alpha \) for \( \alpha > 0 \) on \([0,1]\). Furthermore it is easy to check, that \( z(\cdot) \) is also linear if \( F(c) \) comes from the family of (reverted) Generalized Pareto distributions, which includes the (reverted) exponential distribution as a special case. If \( z(\cdot) \) is linear, the quantities satisfy the condition given in Definition 1. If \( z(\bar{c}) \geq \bar{c} \), retailer 2 does not serve market 2 if the cost parameter lies above \( c^* \). Thus, retailer 2 receives for \( c_2 = c^* \) a profit of zero, the same profit which retailer 1 receives for the cost parameter \( c_1 = \bar{c} = z(c^*) \). However, if \( z(\bar{c}) < \bar{c} \), the manufacturer serves retailer 2 for values of the cost parameter above \( c^* \). This implies that retailer 2 receives a positive profit for \( c_2 = c^* \), whereas retailer 1 receives a profit of zero for \( c_2 = \bar{c} \). Since retailer 2 with cost parameter \( c_2 < c^* \) sells the same quantity as retailer 1 with parameter \( c_1 = z(c_2) \), the difference in profit is the same for all \( c_2 \leq c^* \). Thus, retailer 2 is favored.

If \( z(\cdot) \) is convex, retailer 2 with parameter \( c_2 \leq c^* \) sells more on market 2 than what retailer 1 with \( c_1 = z(c_2) \) sells on market 1. Furthermore, if market 2 is also served for \( c_2 > c^* \), retailer 2 enjoys an additional advantage over retailer 1 as it makes a positive profit for \( c_2 = c^* \), whereas retailer 1’s profit is zero for \( c_1 = \bar{c} = z(c^*) \).

If \( z(\cdot) \) is convex, retailer 1 sells more than retailer 2 for the cost parameters \( c_1 = z(c_2) \) and \( c_2 \leq c^* \). From this respect, retailer 1 is favored. However, if retailer 2 is served also for cost parameters above \( c^* \), retailer 2 enjoys the advantage of receiving a positive profit for \( c_2 = c^* \), whereas retailer 1 receives again zero profit for \( c_1 = \bar{c} \).

5 Optimal wholesale contracts when price discrimination is banned

If price discrimination is not permitted, the manufacturer’s optimal wholesale contracts need to satisfy the following additional constraint:

\[
(ND) : y_1(z(c)) = y_2(c) \quad \text{and} \quad t_1(z(c)) = t_2(c) \quad \forall c \in [\bar{c}, c^*].
\]
The manufacturer’s problem is then to

\[
\max_{y_1(\cdot),y_2(\cdot),t_1(\cdot),t_2(\cdot)} \sum_{i=1}^{2} \int [t_i(c_i) - C(y_i(c_i))]dF(c_i)
\]

\[\text{s.t. } (IC_1), (PC_1), (IC_2), (ND).\]

The constraints can be characterized as follows.

**Lemma 5.** (IC_1), (PC_1), (IC_2), and (ND) are satisfied if and only if for all \(c_1, c_2 \in [c, \bar{c}]\) and \(c \in [c, c^*]\)

\[
i) \quad \Pi'_1(c_1) = -K(y_1(c_1)), \quad \Pi'_2(c_2) = -z'(c_2)K(y_2(c_2))
\]

\[
ii) \quad y_1(c_1) \text{ is weakly decreasing in } c_1, \quad y_2(c_2) \text{ is weakly decreasing in } c_2,
\]

\[
iii) \quad \Pi_2(\bar{c}) \geq 0, \quad \Pi_1(\bar{c}) = \Pi_2(\bar{c}) + \int_{c}^{\bar{c}} z'(x)K(y_2(x))dx,
\]

\[
iv) \quad y_1(z(c)) = y_2(c).
\]

Using condition \(i)\) from the lemma, the manufacturer’s profit can be expressed as

\[
\int_{c}^{\bar{c}} \int_{c}^{\bar{c}} \sum_{i=1}^{2} \left[ y_i(c_i)P(y_i(c_i)) - C_i(y_i(c_i)) - \Pi_i(\bar{c}) \right.
\]

\[
- z(c_1)K(y_1(c_1)) - \left. (z(c_2) + z'(c_2)(z(c_2) - c_2))K(y_2(c_2)) \right] dF(c_1)dF(c_2).
\]

It is optimal to set \(\Pi_2(\bar{c})\) to zero. This allows to express the manufacturer’s profit as

\[
\int_{\underline{c}}^{\bar{c}} [y_1(z(x))P(y_1(z(x))) - z(x)K(y_1(z(x)))) - C(y_1(z(x))))] f(z(x))z'(x)dx
\]

\[
+ \int_{\underline{c}}^{\bar{c}} [y_2(z(x)) - (z(x) + z'(x)(z(x) - x))K(y_2(x)) - C(y_2(x))] f(x)dx
\]

\[
+ \int_{\underline{c}}^{\bar{c}} [y_2(z(x)) - (z(x) + z'(x)(z(x) - x + 1/f(x))))K(y_2(x)) - C(y_2(x))] f(x)dx.
\]

Including the constraint in condition \(iv)\) of the Lemma with a Lagrange-multiplier, the expression can be pointwisely maximized with respect to the quantities. The maximizers are indeed a solution to the manufacturer’s problem under the following assumption.
Assumption 3.

\( i \) \( \frac{f(z(c))z'(c)z(z(c))}{f(z(c))z'(c) + f(c)} + \frac{f(c)(z(c) + z'(c)(z(c) - c))}{f(z(c))z'(c) + f(c)} \) is weakly increasing for \( c \in [c, c^*] \),
\( ii \) \( z(c) + z'(c)(z(c) - c + 1/f(c)) \) is weakly increasing for \( c \in (c^*, \bar{c}] \).

**Proposition 4.** If price discrimination is banned and Assumption 3 is satisfied, the manufacturer sets the wholesale contracts \( \{ y_{1D}^{ND}(c_1), t_1^{ND}(c_1) \} \) and \( \{ y_{2D}^{ND}(c_2), t_2^{ND}(c_2) \} : 

For \( c \in [c, c^*] : y_{1D}^{ND}(z(c)) = y_{2D}^{ND}(c) = \arg \max_y yP(y) - \left( \frac{f(z(c))z'(c)z(z(c))}{f(z(c))z'(c) + f(c)} + \frac{f(c)(z(c) + z'(c)(z(c) - c))}{f(z(c))z'(c) + f(c)} \right) K(y) - C(y), 

For \( c \in (c^*, \bar{c}] : y_{2D}^{ND}(c) = \arg \max_y yP(y) - (z(c) + z'(c)(z(c) - c + 1/f(c)))K(y) - C(y), 

For \( c \in [\bar{c}, \bar{c}] : t_2^{ND}(c) = y_{2D}^{ND}(c)P(y_{2D}^{ND}(c)) - \int_c^{\bar{c}} z'(x)K(y_{2D}^{ND}(x))dx, 

For \( c \in [c, c^*] : t_1^{PD}(z(c)) = t_2^{PD}(c). 

A ban on price discrimination thus has the following effect on the quantities sold on the two markets:

**Proposition 5.** If Assumptions 1 to 3 are satisfied, a ban of price discrimination has the following effects on quantities:

1. If \( z(\cdot) \) is linear, the quantities on both markets remain the same for all \( c_1, c_2 \in [c, \bar{c}] \) if \( z(\bar{c}) \geq \bar{c} \). If \( z(\bar{c}) < \bar{c} \), the quantity on market 1 remains the same for all \( c_1 \in [c, \bar{c}] \). The quantity on market 2 remains the same for \( c_2 \in [c, c^*] \) and are reduced for \( c_2 \in (c^*, \bar{c}] \).

2. If \( z(\cdot) \) is convex, the quantity on market 1 increases for all \( c_1 \in [c, \bar{c}] \), whereas the quantity on market 2 decreases for all \( c_2 \in [c, \bar{c}] \).

3. If \( z(\cdot) \) is concave, the quantity on market 1 decreases for all \( c_1 \in [c, \bar{c}] \), whereas the quantity on market 2 increases for \( c_2 \leq c^* \) and decreases for \( c_2 > c^* \).
6 The welfare effects of a ban on price discrimination

In this section, I analyze the welfare effects of a ban on price discrimination. The curvature of the virtual retailing cost parameter also determines whether or not a ban on price discrimination has a positive effect on welfare. I then show that the welfare implications of a ban on price discrimination with asymmetric agency costs differ remarkably from the welfare implications when retailers differ with respect to their technological costs.

Total welfare is given by

\[ W = \sum_{i=1}^{2} \int_{0}^{y_i(c_i)} \left[ P(x)dx - c_i K(y_i(c_i)) - C(y_i(c_i)) \right] dF(c_i). \]

As shown in the previous section, a ban on price discrimination can only reduce quantities if \( z(\cdot) \) is linear. In this case, the welfare implications are therefore simple. If \( z(\cdot) \) is not linear, a ban of price discrimination has positive effects on one market and negative effects on the other market. I therefore make welfare statements for the case where demand is linear and marginal production and retailing costs are constant. The inverse demand function is then given \( P(y) = \max \{ A - y, 0 \} \), and production and retailing costs satisfy \( C(y) = By \) and \( K(y) = y \). \( B \) can thus be normalized to zero.

I first show that the welfare implications usually depend on whether retailer 1 or retailer 2 is favored by price discrimination. Comparing my results with those obtained by Herweg and Müller (2014), reveals that the welfare implication of a ban on price discrimination may be very different under asymmetric technological costs, as studied in Herweg and Müller, and asymmetric agency costs, as studied in this paper.

**Proposition 6.** Suppose Assumptions 1 to 3 are satisfied. If \( z(\cdot) \) is linear, a ban on price discrimination has no effect on welfare if \( z(\bar{e}) \geq \hat{c} \) and decreases welfare if \( z(\bar{e}) < \hat{c} \). Suppose furthermore that demand is linear and marginal costs of production and retailing are constant. If \( z(\cdot) \) is convex, then a ban on price discrimination decreases welfare for \( z'(\hat{c}) \leq 4 \). If \( z(\cdot) \) is concave and markets are served sufficiently often, a ban on price discrimination increases welfare.

If \( z(\cdot) \) is linear, the manufacturer does voluntarily not discriminate between the two retailers if \( z(\bar{e}) \geq \hat{c} \). In this case, a ban of price discrimination has clearly no effect on welfare. If
if \( z(\bar{c}) < \hat{c} \), then the manufacturer does not change the quantity which is sold on market 1 if price discrimination is banned. However, the quantity sold on market 2 is reduced if \( c_2 \in (c^*, \bar{c}] \). Thus, welfare decreases if market 2 is served for \( c_2 \in (c^*, \bar{c}] \) under price discrimination.

7 Conclusion

This paper analyzes a model of price discrimination in input markets where small firms have to buy a larger share of their inputs from other firms than large firms. I show that the model gives rise to cases where the large firm, and to such where the small firm is favored. A ban on price discrimination may thus either be beneficial or detrimental for the profit of small firms. The welfare implications of a ban on price discrimination typically depend on whether small or large firms are favored. Furthermore, welfare implications in this model with asymmetric agency costs differ from welfare implications if technological costs are asymmetrically distributed for the small and the large firm.

8 Appendix

Proof of Lemma 1

The result follows from standard arguments. \((IC_s)\) can be rewritten as

\[
\Pi_s(c_2; r) \geq \Pi_s(\hat{c}_2; r) + (\hat{c}_2 - c_2)K(y_2(r(\hat{c}_2))), \forall c_2, \hat{c}_2 \in [c, \bar{c}].
\]

(1)

Using the constraint for the type \( c_2 \) who reports \( \hat{c}_2 \) and the type \( \hat{c}_2 \) who reports \( c_2 \), it holds that

\[
(\hat{c}_2 - c_2)K(y_2(r(\hat{c}_2))) \leq \Pi_s(c_2; r) - \Pi_s(\hat{c}_2; r) \leq (\hat{c}_2 - c_2)K(y_2(r(c_2))).
\]

Condition i) follows if we consider the expression above when \( \hat{c}_2 \) converges to \( c_2 \). Condition ii) follows from the fact that the first term is smaller than the third term in the expression above. \((PC_s)\) implies trivially condition iii). To prove sufficiency, note that it follows by absolute continuity of the terms from condition i) that

\[
\Pi(c_2; r) = \Pi_s(\bar{c}; r) + \int_{c_2}^{\bar{c}} K(y_2(r(c))) dc.
\]
Thus \((PC_s)\) is implied by condition \(iii\) and (1) can be rewritten as

\[
\int_{c_2}^{\hat{c}_2} K(y_2(r(x)))dx \geq (\hat{c}_2 - c_2)K(y_2(r(\hat{c}_2))) \forall c_2, \hat{c}_2 \in [\underline{c}, \bar{c}],
\]

which is true by condition \(ii\).

Proof of Lemma 2

\(\Pi_s(\bar{c}) = 0\) implies

\[
\Pi(c_2; r) = \int_{c_2}^{\bar{c}} K(y_2(r(c)))dc = t_s(c_2) - c_2K(y_2(r(c))).
\]

Solving for \(t_s(c_2)\) gives the result.

Proof of Proposition 1

\((IC_1)\) and \((PC_1)\) can be characterized in the following standard way

Lemma 6. \((IC_1)\) and \((PC_1)\) are satisfied if and only if

\[
i) \quad \Pi'_1(c_1) = -K(y_1(c_1)),
\]

\[
ii) \quad y_1(c_1) \text{ is weakly decreasing in } c_1,
\]

\[
iii) \quad \Pi_1(\bar{c}) \geq 0.
\]

Using, condition \(i\) from this lemma, the manufacturer’s profit from interaction with retailer 1 can be rewritten as

\[
\int [t_1(c_1) - C(y_1(c_1))]dF(c_1)
\]

\[
= \int [y_1(c_1)P(y_1(c_1)) - z(c_1)K(y_1(c_1)) - C(y_1(c_1))]dF(c_1) - \Pi_1(\bar{c}).
\]

One can now maximize this expression pointwisely with respect to \(y_1(c_1)\) and \(\Pi_1(\bar{c})\). It is clearly optimal to set \(\Pi_1(\bar{c}) = 0\), i.e. to make condition \(iii\) of Lemma 6 binding. \(y_1^{PD}(c_1)\) is optimal since it decreases in \(c_1\) due to assumption of \(z(c_1)\) being increasing in \(c_1\). As \(K(y)\) is increasing in \(y\), \(y_1^{PD}(c_1)\) satisfies condition \(ii\) in Lemma 6.
Proof of Lemma 3

The sufficiency of conditions $i)$, $ii)$, and $iii)$ is shown first. $(IC_2)$ is clearly implied by the following two constraints:

$$(IC'_2) : \ \Pi_2(c_2) \geq \Pi_2(\hat{c}_2) + (z(\hat{c}_2) - z(c_2))K(y_2(\hat{c}_2)), \ \forall c_2, \hat{c}_2 \in [\underline{c}, \bar{c}],$$

$$(PC'_2) : \ \Pi_2(c_2) \geq 0, \ \forall c_2 \in [\underline{c}, \bar{c}].$$

Note that $(IC'_2)$ and $(PC'_2)$ ensure that any reporting strategy $r : [\underline{c}, \bar{c}] \to [\underline{c}, \bar{c}] \cup \{\emptyset\}$, whether in $R$ or not, is not better than the truthful reporting strategy. By standard arguments (the same as in the proof of Proposition 1), $(IC'_2)$ and $(PC'_2)$ are satisfied if and only if condition $i)$, $ii)$, and $iii)$ in the Lemma are satisfied.

It remains to show necessity of $i)$, $ii)$, and $iii)$. Define the reporting strategies $\underline{r}(\cdot)$ and $\overline{r}(\cdot)$ such that for any $c_l, c_h \in [\underline{c}, \bar{c}]$ with $c_l \leq c_h$ and $\varepsilon \geq 0$ small enough

$$\underline{r}(c) = \begin{cases} 
  c & \text{if } c \not\in [c_l, c_h] \\
  c_l & \text{if } c \in [c_l, c_l + \varepsilon) \\
  c - \varepsilon & \text{if } c \in [c_l + \varepsilon, c_h], 
\end{cases}$$

and

$$\overline{r}(c) = \begin{cases} 
  c & \text{if } c \not\in [c_l, c_h] \\
  c + \varepsilon & \text{if } c \in [c_l, c_h - \varepsilon] \\
  c_h & \text{if } c \in (c_h - \varepsilon, c_h]. 
\end{cases}$$
($IC_2$) implies $\Pi_2 \geq \Pi_2(r)$ which is equivalent to

$$
\int_{c_l}^{c_h} \Pi_2(c) dF(c) \geq \int_{c_l}^{c_l+\varepsilon} (\Pi_2(c_l) - (z(c) - z(c_l))K(y_2(c_l))) dF(c) \\
+ \int_{c_l+\varepsilon}^{c_h} (\Pi_2(c) - (z(c) - z(c - \varepsilon))K(y_2(c - \varepsilon))) dF(c)
$$

$$
\Leftrightarrow \int_{c_l}^{c_l+\varepsilon} (\Pi_2(c) - \Pi_2(c_l)) dF(c) + \int_{c_l+\varepsilon}^{c_h} (\Pi_2(c) - \Pi_2(c - \varepsilon)) dF(c) \geq \\
\int_{c_l}^{c_l+\varepsilon} (z(c) - z(c_l))K(y_2(c_l)) dF(c) - \int_{c_l+\varepsilon}^{c_h} (z(c) - z(c - \varepsilon))K(y_2(c - \varepsilon))) dF(c).
$$

Dividing by $\varepsilon$ on both sides of the inequality sign, gives for $\varepsilon \to 0$

$$
\int_{c_l}^{c_h} \Pi_2'(c) dF(c) \geq \int_{c_l}^{c_h} -z'(c)K(y_2(c)) dF(c)
$$

This argument can be repeated for $\Pi_2 \geq \Pi_2(\bar{r})$ which implies

$$
\int_{c_l}^{c_h} \Pi_2'(c) dF(c) \leq \int_{c_l}^{c_h} -z'(c)K(y_2(c)) dF(c)
$$

Thus for any $c_l, c_h \in [\underline{c}, \bar{c}]$, it follows

$$
\int_{c_l}^{c_h} \Pi_2'(c) dF(c) = \int_{c_l}^{c_h} -z'(c)K(y_2(c)) dF(c),
$$

which implies condition $i)$ in the lemma. Note that condition $ii)$ is inherited from Lemma 1 and $r^* \in R$. Thus, one can express retailer 2’s profit as

$$
\Pi_2(c_2) = \Pi_2(\bar{r}) + \int_{c_2}^{\bar{r}} K(y_2(x)) dx.
$$

Thus, $iii)$ needs to hold. Suppose towards a contradiction that $\Pi_2(\bar{r}) < 0$. By continuity of $\Pi_2(c_2)$, it would follow that there is a interval $[c', \bar{c}]$, for which non-participation is better for retailer 2. The reporting strategy $\tilde{r}(\cdot)$ with $\tilde{r}(c_2) = c_2$ for $c_2 < c'$ and $\tilde{r}(c_2) = \emptyset$ for $c_2 \in [c', \bar{c}]$ would therefore be a profitable deviation. $\tilde{r}(\cdot) \in R$ follows from $y_2(\emptyset) = 0$ and the fact that $y_2(c_2)$ is nondecreasing, which allows retailer 2 to implement non-participation at the upper end of the interval $[\underline{c}, \bar{c}]$. \hfill \Box
Proofof Proposition 2

Given Lemma 3, one can express the manufacturer’s profit as

$$\int [t_2(c_2) - C(y_2(c_2))]dF(c_2) = \int [y_2(c_2)P(y_2(c_2)) - z(c_2)K_2(c_2)) - \Pi_2(c_2)]dF(c_2) = \int [y_2(c_2)P(y_2(c_2)) - (z(c_2) + z'(c_2)(z(c_2) - c_2))K(y_2(c_2)) - C(y_2(c_2))]dF(c_2) - \Pi_2(\tau)$$

where the last equality follows from

$$\Pi_2(c_2)dF(c_2) = \Pi(\tau) + \int \int_{c_2} z'(c)K(y_2(c))dcdF(c_2) = \Pi(\tau) + \int z'(c_2)F(c_2)K(y_2(c_2))dF(c_2)$$

and $F(c_2)/f(c_2) = z(c_2) - c_2$. It is optimal to set $\Pi(\tau) = 0$. The pointwise maximizer of the remaining expression, i.e. $y_2^{PD}(c_2)$, is the solution to the problem if it weakly decreasing in $c_2$.

This is the case under Assumption 2. The transfers can be derived as

$$t_2(c_2) = y_2^{PD}(c_2)P(y_2^{PD}(c_2)) - z(c_2)K(y_2^{PD}(c_2)) - \Pi_2(c_2) = y_2^{PD}(c_2)P(y_2^{PD}(c_2)) - z(c_2)K(y_2^{PD}(c_2)) - \int_{c_2} z'(c)K(y_2(c))dc.$$

Proof of Proposition 3

For $c \in [c^*, c^+]$, I compare $\Pi_1(z(c)) = \int_{c^*}^{z(c)} K(y_1(z(x)))z'(x)dx$ with $\Pi_2(c) = \int_{c^*}^{z(c)} K(y_2(c))z'(x)dx$. If $z(\cdot)$ linear, $y_1(z(c)) = y_2(c)$ for $c \in [c^*, c^+]$. If $z(\tau) \geq c$, $y_2(c) = 0$ for all $c \in (c^*, \bar{c}]$, thus $\Pi_1(z(c)) = \Pi_2(c)$. If $z(\tau) < c$, $y_2(c) > 0$ for some $c \in (c^*, \bar{c}]$, thus $\Pi_1(z(c)) < \Pi_2(c)$. If $z(\cdot)$ convex, $y_1(z(c)) \leq y_2(c)$ for $c \in [c^*, c^+]$. Thus, $\Pi_1(z(c)) < \Pi_2(c)$. If $z(\cdot)$ concave, $y_1(z(c)) \geq y_2(c)$. If $z(c^*) + z'(c^*)(c^* - c^*) = \tau + z'(c^*)(\tau - c^*) \geq \hat{c}$, $y_2(c) = 0$ for all $c \in (c^*, \bar{c}]$, thus $\Pi_1(z(c)) \geq \Pi_2(c)$. If $\tau + z'(c^*)(\tau - c^*) < \hat{c}$, $y_2(c) > 0$ for some $c \in (c^*, \bar{c}]$. Thus, $\Pi_1(\tau) < \Pi_2(c^*)$.

As $c$ decreases $\Pi_1(z(c))$ grows more quickly than $\Pi_2(c)$. Therefore, there exists some threshold $c^* \in [c^*, c^+]$ such that $\Pi_1(z(c)) < \Pi_2(c)$ if $c \geq c^*$ and $\Pi_1(z(c)) \geq \Pi_2(c)$ if $c < c^*$. \qed
Proof of Lemma 5

\((IC_1)\), \((PC_1)\), and \((IC_2)\) are satisfied if and only if \(i, ii)\), and \(\Pi_i(\bar{c}) \geq 0\) with \(i \in \{1, 2\}\) is satisfied. \((ND)\) implies \(\Pi_1(z(c)) = \Pi_2(c)\) for all \(c \in [c, c^*]\). Under \(y_1(z(c)) = y_2(c)\) for all \(c \in [c, c^*]\), this is satisfied if and only if \(\Pi_1(\bar{c}) = \Pi_2(c^*) = \Pi_2(\bar{c}) + \int_{c}^{\bar{c}} z'(c)K(y_2(c))dc\). □

Proof of Proposition 4

Maximize the following Lagrangian to the manufacturer’s problem:

\[
\mathcal{L} = \int_{c}^{c^*} \left[ [y_1(z(x))P(y_1(z(x))) - z(x)K(y_1(z(x))) - C(y_1(z(x)))] f(z(x))z'(x)
+ [y_2(x)P(y_2(x)) - (z(x) + z'(x)(z(x) - x))K(y_2(x)) - C(y_2(x))] f(x)
- \lambda(x)(y_1(z(x)) - y_2(x)) \right] dx
+ \int_{c}^{\bar{c}} [y_2(x)P(y_2(x)) - (z(x) + z'(x)(z(x) - x + 1/f(x)))K(y_2(x)) - C(y_2(x))] f(x)dx.
\]

The pointwise maximizer of the Langrangian gives the optimal quantities for the manufacturer if condition \(ii)\) of Lemma 5 is satisfied. Due to part \(i)\) of Assumption 3, \(y_1^{ND}(c)\) is weakly decreasing for \(c \in [c, \bar{c}]\) and \(y_2^{ND}(c)\) is weakly decreasing for \(c \in [c, c^*]\). From \(ii)\) it follows that \(y_2^{ND}(c)\) is weakly decreasing for \(c \in (c^*, \bar{c}]\). \(y_2^{ND}(c)\) is also weakly decreasing at \(c = c^*\). This follows from

\[
\frac{f(z(c))z'(c)z(z(c))}{f(z(c))z'(c) + f(c)} + \frac{f(c)(z(c) + z'(c)(z(c) - c))}{f(z(c))z'(c) + f(c)} \leq z(c) + z'(c)(z(c) - c + 1/f(c))
\]

for all \(c \in [c, \bar{c}]\). In order to see this, rewrite the inequality as

\[
z(c) + \frac{F(z(c))z'(c)}{f(z(c)) + f(c)} + \frac{z'(c)F(c)}{f(z(c)) + f(c)} \leq z(c) + z'(c)\frac{1 + F(c)}{f(c)} \iff \frac{F(z(c)) + F(c)}{f(z(c)) + f(c)} \leq \frac{1 + F(c)}{f(c)}.
\]

The transfers can then be determined from the condition \(i)\) in Lemma 3. □

Proof of Proposition 5

Since \(\frac{f(z(c))z'(c)z(z(c))}{f(z(c))z'(c) + f(c)} + \frac{f(c)(z(c) + z'(c)(z(c) - c))}{f(z(c))z'(c) + f(c)}\) is bracketed by \(z(z(c))\) and \(z(c) + z'(c)(z(c) - c)\), the result follows for \(c \in [c, c^*]\) from the comparison of quantities under price discrimination in Lemma 4. Moreover, since \(z(c) + z'(c)(z(c) - c + 1/f(c)) > z(c) + z'(c)(z(c) - c)\), the quantity
on market 2 is reduced \( c \in (c^*, \bar{c}] \) by a ban of price discrimination if market 2 is served under price discrimination for \( c \in (c^*, \bar{c}] \).

\[ \square \] 

**Proof of Proposition 6**

For linear \( z(\cdot) \), the result follows immediately from the effect of a ban of price discrimination on the quantity on both markets. Under the assumption of linear demand, and constant marginal costs it follows that

\[ y_{1}^{PD}(c_1) = \left[ \frac{A - z(c_1)}{2} \right] + \quad \text{and} \quad y_{2}^{PD}(c_2) = \left[ \frac{A - z(c_2) - z'(c_2)(z(c_2) - c_2)}{2} \right] + . \]

where \([X]_+ = \max \{X, 0\}\). With a ban on price discrimination, the quantities are given by

\[ y_{2}^{ND}(c) = \left[ \frac{A - \frac{f(z(c))z'(c)}{f(z(c))z'(c) + f(c)} z(z(c)) - \frac{f(c)}{f(z(c))z'(c) + f(c)} (z(c) + z'(c)(z(c) - c)}{2} \right] + \]

and \( y_{1}^{ND}(c) = y_{2}^{ND}(z^{-1}(c)) \). The welfare effect of a ban on price discrimination is given by

\[ W^{ND} - W^{PD} = \sum_{i=1}^{2} \int_{c^*}^{\bar{c}} \left[ \int_{0}^{y_{2}^{ND}(c)} (A - x)dx - c_{1} y_{1}^{ND}(c_1) \right] dF(c_i) - \sum_{i=1}^{2} \int_{c^*}^{\bar{c}} \left[ \int_{0}^{y_{1}^{PD}(c)} (A - x)dx - c_{1} y_{1}^{PD}(c_1) \right] dF(c_i) = \]

\[ \int_{c^*}^{\bar{c}} \left[ \int_{0}^{y_{2}^{ND}(c)} (A - x)dx - y_{1}^{ND}(c) y_{1}^{ND}(c) - \int_{0}^{y_{1}^{PD}(c)} (A - x)dx + z(c) y_{1}^{PD}(z(c)) \right] f(z(c)) z'(c) dc \]

\[ + \int \left[ \int_{0}^{y_{2}^{ND}(c)} (A - x)dx - c y_{2}^{ND}(c) - \int_{0}^{y_{2}^{PD}(c)} (A - x)dx + c y_{2}^{ND}(c) \right] f(c) dc. \]

where the second equality follows from a change in variable from \( c_1 \) to \( c = z^{-1}(c_1) \). From linear demand and constant marginal costs it follows that \( y_{1}^{ND}(z(c)) + y_{2}^{ND}(c) = 2y_{2}^{ND}(c) = y_{1}^{PD}(z(c)) + y_{2}^{PD}(c) \). This can be used to prove the results. I define

\[ \alpha(c) \equiv \frac{f(z(c))z'(c)}{f(z(c))z'(c) + f(c)}. \]
Then the difference in welfare can be written as

\[
\int_{c^*}^{c} \left[ (f(z(c))z'(c) + f'(c)) \left( (A - c)(y_2^{ND}(c) - \alpha(c)y_1^{PD}(z(c)) - (1 - \alpha(c))y_2^{PD}(c) \right)
+ \frac{1}{2} (\alpha(c)y_1^{PD}(c)^2 + (1 - \alpha(c))y_1^{PD}(c)^2 - y_2^{ND}(c)^2) - (z(c) - c)\alpha(c)(y_1^{ND}(z(c)) - y_1^{PD}(z(c)) \right] dc
+ \int_{c^*}^{c} [(A - c)(y_2^{ND}(c) - y_2^{PD}(c)) - \frac{1}{2}(y_2^{ND}(c)^2 - y_2^{PD}(c)^2)]f(c)dc.
\]

Note that the third term is always negative, as \(y_2^{ND}(c) \leq y_2^{PD}(c)\) for \(c \in (c^*, \bar{c}]\). For \(c\) such that \(y_1^{PD}(z(c)) > 0\) and \(y_2^{PD}(c) > 0\), one can use that \(y_2^{ND}(c) = \alpha(c)y_1^{PD}(z(c)) + (1 - \alpha(c))y_2^{PD}(c)\) to write the first two lines below the integral as

\[
\frac{1}{2} \frac{f(z(c))z'(c)f'(c)}{f(z(c))z'(c) + f'(c)} [y_1^{PD}(z(c)) - y_2^{PD}(c)] \left[ y_1^{PD}(z(c)) - y_2^{PD}(c) + 2(z(c) - c) \right].
\]  

(2)

If \(z(\cdot)\) is convex, \([y_1^{PD}(z(c)) - y_2^{PD}(c)] < 0\). Then the expression above is negative if \([y_1^{PD}(z(c)) - y_2^{PD}(c) + 2(z(c) - c)] > 0\). Using the specific expressions for quantities \(y_1^{PD}\) and \(y_2^{PD}\), this is equivalent to

\[
\frac{z(z(c)) - z(c) - z'(c)(z(c) - c)}{2} < 4(z(c) - c).
\]

By convexity of \(z(\cdot)\), this is implied by \(z'(c) < 4\) for all \(c \in [c, \bar{c}]\). If \(z'(c) < 4\), equation (2) remains negative even for cases where some of the quantities become zero. This follows from the fact that \(y_1^{PD}(z(c))\) becomes zero for smaller values than \(y_2^{ND}(c)\). It follows that a ban of price discrimination decreases welfare if \(z(\cdot)\) convex.

If \(z(\cdot)\) is concave, \([y_1^{PD}(z(c)) - y_2^{PD}(c)] > 0\) and \([y_1^{PD}(z(c)) - y_2^{PD}(c) + 2(z(c) - c)] > 0\). Thus equation (2) is always positive for \(z(\cdot)\) concave. For a ban on price discrimination to be welfare increasing under \(z(\cdot)\) concave, we therefore need that the cases where all quantities are positive, outweigh those cases not all markets are served, and the cases where \(c > c^*\) occur not too often. \(\Box\)
References


