Learning about Debt Crises*

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[VERY PRELIMINARY]

Abstract
The European debt crisis of 2008-2014 was marked with the spikes in government bond yields of unprecedented magnitude among developed countries in the modern post-war history. These panics occurred with a significant delay with respect to the initial negative output shocks, even though the governments did not undertake the fiscal adjustments necessary to prevent a subsequent increase in default risk. I show that these observations are at odds with the predictions of existing sovereign debt models and propose a new theory that features incomplete (symmetric and asymmetric) information about the economy’s growth prospects. The lag arises as a consequence of the markets’ learning process, while the government optimally postpones debt reduction in order not to send a negative signal about the state of the economy.

Keywords: Sovereign debt, Bayesian learning, signaling

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1 Introduction

The European debt crisis of 2008-2014 was an unprecedented period of time that put into question the conjecture shared by many policymakers and researchers about the developed countries’ resilience to sovereign default. From the perspective of time it is important to understand not only the factors that lead to such high levels of indebtedness of the peripheral Eurozone countries, but also what drove their optimal policy when the economies were first hit by negative output shocks. Furthermore, any modern theory of sovereign debt that aims at explaining the debt crisis in Europe should also be able to replicate the evolution of interest rates on government bonds over time.

In this paper, I review the chronology of the debt crisis in Europe with an emphasis on its early stage. I start by noticing that the European economies entered the 2008 financial crisis following a decade of low volatility and stable growth. This stands in contrast with the experience of many previous sovereign defaulters whose emerging economies usually exhibit considerable volatility around a long-run trend. In this context, I make two crucial observations about the timing of events and the behavior of governments across the GIIPS countries during the recent crisis. First, in spite of the high debt stocks held by the European governments and relatively large output shocks that first hit their economies in 2008, the corresponding interest rates on government bonds increased with a significant lag, reaching up two years following the beginning of the recession. As I show using the model developed in this paper, such behavior of the capital markets is puzzling from the perspective of existing theories of sovereign debt, which tend to predict an instant reaction of interest rates to negative output shocks.

Second, the governments of GIIPS countries did not undertake any adjustment to their debt policy following the outset of the financial crisis. On the contrary, we observe that indebtedness went significantly up across the peripheral European countries in between the beginning of the recession in 2008 and the actual debt crisis in 2010-2012. Once again, such behavior contrasts sharply with the predictions of the majority of existing sovereign debt theories. In those models, a government whose economy experiences bad output shocks should be desperate to reduce debt levels in order to stave off the possibility of defaulting and to secure higher bond prices. Consequently, to understand the European debt crisis we must crucially be able to explain the factors that drove the debt policy of the governments.
In line with the title of this paper, I propose a new theory of sovereign debt based on a learning process exhibited by the capital markets. I conjecture that international lenders initially perceived the bad output shocks in 2008 as a merely temporary downturn, one of the many short-lived recessions observed in the post-war European history. They were only able to learn slowly over time, by observing the subsequent unfavourable releases of the national accounts statistics, that the initial recession would turn into the largest and most persistent downturn in many decades. When the lenders finally realized that a prompt recovery was nowhere on the horizon, they started requiring much higher compensation for the risk related to holding the government bonds, resulting in the observed sharp spikes in interest rates around 2011-2012.

Apart from the fact that lenders could learn independently about the economies’ growth prospects by observing the incoming statistics, they could also learn by analyzing the governments’ actions. In today’s highly globalized financial markets, many of the bondholders reside in remote parts of the world and often have much inferior understanding of the nature of output process in European countries. Moreover, many lenders actually choose to neglect their own knowledge of the fundamentals and adopt passive investment strategies such as index-tracking. Such lenders will observe the governments’ policy with particular attention, trying to infer the private information about future default probabilities from the actual debt levels. Considering this channel, a government whose economy experiences negative income shocks must be careful with conducting fiscal policy adjustments, as they might send a negative signal to the markets about the state of the economy. In other words, policymakers in the indebted countries who already realize that the currently experienced downturn is likely to be a very persistent one have strong incentive to continue behaving like a good-economy government which expects the shocks to go away soon. In such a pooling equilibrium, a bad-economy government keeps its debt high to mimic the good one, lenders continue to learn about the output process slowly, and eventually respond with dramatic spikes in the interest rates on government bonds.

In order to investigate the proposed conjectures, I build a quantitative model of sovereign debt that features several novel elements. First, I introduce a regime-switching output process to better capture the distinct feature of the European economies that contrasts with those of the emerging markets studied by previous models. Second, I introduce partial information about the growth regime in which the economy is currently operating and show the delaying effect it can have on the response of financial markets. Finally, I assume that the government has private information about the growth regime and engages in a signaling
game with international lenders. The model proposed in this paper predicts that an optimal government’s policy in response to a sudden regime switch induces a pooling equilibrium, in which a hidden bad type government continues to issue excessive debt in order to appear more resilient to the markets. In the calibrated version of the model I show that this theory is capable of explaining the delayed structural adjustments observed across the European governments following the outset of the financial crisis.

1.1 Literature review

This work is closely related to the sovereign debt literature, in particular the one building on the seminal work of Eaton and Gersovitz [1981] and, more recently, Cole and Kehoe [1996], [2000], Aguiar and Gopinath [2007] and Arellano [2008]. The models presented in these papers set the foundations for our understanding of the dynamics of sovereign default risk and the mechanics of equilibrium defaults actually observed in the world.

The paper also relates to the significant literature on learning about the economic conditions in macroeconomics. In particular, Boz, Daude and Durdu [2011] take the model of Aguiar and Gopinath [2007] and assume that market participants are unable to distinguish between the incoming permanent and transitory shocks. In a calibrated model, they show how a learning process can explain some of the observed differences between developed and emerging market economies.

Finally, the main model in this paper is related to a growing branch of literature that embeds a signaling mechanism in the unsecured debt framework. Chari and Kehoe [2003] develop a model of investor herd behavior in which a government uses debt repayment in difficult times as a way to show off its good type. They show that an outside bailout may be undesirable because it deprives the resilient sovereigns of an opportunity to separate themselves from the weak ones. Similarly, Alfaro and Kanczuk [2005] build a quantitative model with adverse selection to show that some countries may choose to delay default until the times are bad enough to make it look “excusable”. D’Erasmo [2011] adds private information and government reputation to a quantitative sovereign debt model and shows that it is capable of producing much higher debt levels than many previous papers in the field. Chatterjee et al. [2008] and Athreya et al. [2012] both analyze the role of borrower signaling in the unsecured private credit markets. The former develop a theoretical mechanism in which debtors signal their low-risk status by avoiding a default, while the latter explore quantitatively the effects of improvements in
lenders’ information over time on the stock of consumer debt and personal bankruptcy rates.

The remainder of the paper is structured as follows. Section 2 presents a discussion of the chronology of the European debt crisis and points out its distinctive features that constitute a challenge for the sovereign debt literature. Section 3 introduces the main model in this paper and discusses various assumptions regarding the information structure. Section 4 applies the calibrated model in the context of the European debt crisis and highlights the features that help us understand the patterns observed in the data. Section 5 concludes.
2 Background

In this section, I document the peculiar features of the recent European debt crisis. I explain how, in particular, the timing of events and the evolution of the government bond yields contrasts with the basic predictions of current models of sovereign debt. I show it formally in the next section by calibrating a benchmark full-information case of my model.

2.1 European debt crisis

Prior to 2008, the peripheral economies of the Eurozone enjoyed a decade of relative prosperity and stable growth, fuelled by the European integration, rising trade and benefits of the common currency. As a result of such long period of stable growth, the capital markets have become somewhat accustomed to the idea that sovereign debt crises are predominantly a problem of highly volatile emerging economies. By contrast, the securities issued by governments of the developed countries in the western world seemed risk-free, in spite of the extraordinarily high debt stocks and slowing growth in these economies. The general tendency to put excessive faith in Europe’s ability to repay their debts continued even after the severe recession began in the second half of 2008. Figure 1 depicts the times series of real GDP for four of the troubled European economies, together with the spreads on long-term government bond yields. As can be noticed, markets did not express significant concern about the European governments’ ability to repay for a long time following the initial slump in output. Instead, the bond spreads only exhibited small “wiggles” when the financial crisis first began, and only since then went on a gradual increase, leading eventually to the dramatic spikes observed in 2011-2012.

The idea that financial markets have remained relatively calm for a long time after the crisis actually started off can also be seen by examining in detail the case of Portuguese sovereign rating downgrades. Figure 2 presents the plot of Portugal’s real GDP and its linear trend, together with the indicators for the quarters in which the Portuguese sovereign rating was officially downgraded by the two leading agencies, S&P and Moody’s. Even though the economy fell permanently below the trend as early as in the second quarter of 2008, markets still kept perceiving Portuguese bonds as a relatively risk-free investment.

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1In this paper, I focus mostly on the cases of Greece, Ireland, Italy and Portugal. The fifth member of the colloquial GIIPS group, Spain, is left out for the purpose of clear exposition. Nevertheless, the empirical patterns discussed in this section can be observed for the Spanish case as well.

2The bond spread is defined as the difference between the annualized interest rate on the given ten-year government bonds and the interest rate on the German long-term bonds, assumed to be a risk-free asset.

3Similar patterns can also be uncovered for the remaining GIIPS economies.
Note: The GDP series are in constant 2010 prices, and the logged values are normalized such that the third quarter of 2008 equals 100 (beginning of the financial crisis). The bond spreads are expressed in percentage points.

Figure 1: Real GDP and the spread on government bonds for the troubled European economies: 2000-2014

until two years later. As a result, only in 2010 and 2011 do we observe a sequence of Portugal’s sovereign rating downgrades which indicates that the market sentiment about the government’s debt-repayment probability had deteriorated significantly.

The evidence presented above suggests that learning about the economies’ growth and debt repayment prospects was an important factor in the evolution of the recent debt crisis in Europe. The small reactions of bond spreads to the initial negative growth shocks seem to suggest that markets were either viewing the shocks as temporary, or expecting a prompt adjustment of the government’s fiscal policies. Judging from the perspective of time, this reaction of the financial markets seems significantly delayed. Not surprisingly, as I formally show in the next section, the existing theories of sovereign debt fail to predict
such pattern of the bond spreads. If the markets are endowed with perfect foresight in terms of the magnitude and duration of the looming recession, then given the actual magnitude of debt stocks and growth shocks, the spreads should have increased with no lag at all.

Another interesting aspect of the crisis in the peripheral economies arises from examining the government debt policy over the time period 2000-2014. Figure 3 contrast the evolution of real GDP and its trend over time consolidated government debt of the four economies of interest. Two observations stand out in the graph. First, prior to 2008 the four economies exhibited differing patterns of government debt accumulation. While Ireland managed to reduce the debt stock over time, Italy kept it roughly constant, while Portugal and Greece increased their debt steadily throughout the boom years. Second, and more importantly for the present paper, following the negative shocks that hit all countries in 2008 the governments uniformly responded by increasing the rate of debt accumulation over time. The actual slowdown of the debt accumulation was only observed after 2012 and mostly enforced by the international financial institutions (IMF and the European Commission), or a default event for the case of Greece. As I also show on the benchmark full-information version of the model developed in the subsequent section, such behavior of the government is strongly
at odds with the predictions of current sovereign debt models. In a world where default is costly, a government close to the default edge will be desperate to reduce the debts in response to bad shocks in order to lower the probability of default and to secure high bond prices.

Figure 3: Real GDP and the consolidated government debt of the troubled European economies: 2000-2014

*Note: The GDP series are in constant 2010 prices, and the logged values are normalized such that the third quarter of 2008 equals 100 (beginning of the financial crisis). The debt series represent the consolidated general government debt in constant 2010 prices, and its logged values are normalized such that the first quarter of 2000 equals 100.*
3 The model

In this section I present a new model of sovereign debt that features an enhanced specification of the output process and partial information about its realization. I consider several variants of the model, which differ in terms of the assumed information structure, and show how they contribute to our understanding of the European debt crisis.

3.1 Economic environment

Consider a small open economy with a representative consumer and a benevolent sovereign government that borrows internationally from a large number of perfectly competitive lenders. There is no production or labor. Instead, the economy faces a stochastic stream of endowment realizations. Markets are incomplete and the only asset available for trading is the one-period non-contingent bond.

**Endowment process** Suppose the country’s endowment follows an autoregressive regime-switching process. I assume that there are two possible regimes, High and Low, and each of them is possibly characterized by its own mean, persistence and the variance of shocks. Specifically, the evolution of output, detrended with a deterministic long-run mean growth rate, is given by

\[ y_t = \mu_j (1 - \rho_j) + \rho_j y_{t-1} + \sigma_{\varepsilon,j} \varepsilon_t \]  

where \( \varepsilon_t \sim \mathcal{N}(0, 1) \) is an i.i.d. random shock and \( \{\mu_j, \rho_j, \sigma_{\varepsilon,j}\}_{j=L,H} \) are parameters of the two regimes. Regimes change according to a Markov process with the transition probability matrix given by

\[ \Pi = \begin{bmatrix} \pi_L & 1 - \pi_L \\ 1 - \pi_H & \pi_H \end{bmatrix} \]  

The specification of dichotomous endowment regimes given by formula (1) is non-standard in the sovereign debt literature. It is motivated however by the growth pattern of the European economies in the recent decade. This point is illustrated in Figures (), which show the real GDP and its linear trend of ... [ADD GRAPH]

**Preferences** Time is discrete and a representative household has preferences given by the expected utility of the form:

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \]  

\[ \beta = \begin{cases} \pi_L & 1 - \pi_L \\ 1 - \pi_H & \pi_H \end{cases} \]
where I assume the function \( u(\cdot) \) is strictly increasing, concave and twice continuously differentiable. The discount factor is given by \( \beta \in (0, 1) \).

**Government** In each period, the government chooses a consumption rule and the level of asset holding in order to maximize the household’s lifetime utility. The only asset available is the one-period zero-coupon bond. The government may save at an international risk free rate. If it decides to borrow, however, the government is not committed to repay the debt next period. Consequently, the bond is priced endogenously by risk-neutral lenders to account for the possibility of default. As it is commonly assumed in the sovereign debt literature, the government who refuses to honor its obligations faces an exogenous cost of default and is further excluded from borrowing in the financial markets, with a small probability of being readmitted in every subsequent period.

**Market clearing** There is no storage technology and, under the aforementioned assumptions on the utility function, implies that the endowment is fully divided between current consumption and net borrowing. This market clearing condition is given by

\[
c_t + q_t b_{t+1} - b_t = y_t
\]

(4)

where \( q_t \) is the price of the bond \( b_{t+1} \) (to be repaid next period), and the negative value of \( b_t \) means that the government is a borrower.

### 3.2 Information structure

The two state variables mentioned so far, current bond holdings \( (b) \) and income \( (y) \), are typical in sovereign debt literature. In addition, the present model features another exogenous state, \( z \in \{z_L, z_H\} \), which represents the regime (low or high) in which the economy is currently operating. While all market participants are always knowledgeable about the recent endowment realization, the model in this paper will be differentiated by three possible assumptions regarding the information about the current growth regime.

The benchmark model features Full Information (FI hereafter) and is therefore similar in nature to the model of Arellano [2008]. The only difference boils down to a richer stochastic process for endowment, as specified in equation (1). As can be seen in Figure ??, the assumption of separate growth regimes appears particularly appropriate for the case of European economies, which tend to experience long streaks of persistent over- and under-performance relative to a long run trend.
The second version of the model considered features Partial Symmetric Information (PSI). Under this assumption, neither the government nor the lenders have knowledge of the current growth regime. Instead, they share a common belief $p$, defined as the probability assigned to being in the High regime, formally $s = \text{Prob}(z = z_H)$. Intuitively, this variable can be thought of as a market sentiment about the economy’s growth prospects. As it is described in more detail in the subsequent section, market participants use the incoming output data every period to Bayesian-update their belief before any actions take place. This assumption allows us to capture the uncertainty around the persistence of negative shocks that first hit the European countries following the 2008 financial crisis.

Finally, the third version of the present model assumes Partial Asymmetric Information (PAI), such that the current realization of $z$ is private information of the government. International lenders still form their own belief $p$, while government may either of High or Low type, depending on the economy’s growth regime. Under this specification, the lenders gain an additional channel of learning about the hidden state, namely by observing the government’s actions. Consequently, the next period debt allocation becomes a productive signal sent by the government.

### 3.3 Timeline

Every period, the timing of events in the PSI/PAI models is as follows:

1. The new regime $z \in \{z_L, z_H\}$ is drawn, with the probability distribution as specified in (2).

2. The new realization of endowment $y$ is drawn, according to the newly updated regime $z$ and conditional on its level from last period.

3. International lenders observe the new level $y$ and mechanically form a new prior belief $p$ about the growth regime, conditional on the previous and current endowment realizations, as well as last period’s posterior belief $\tilde{p}$.

4. Default and redemption decisions take place:
   - The government that has recently defaulted on its debt draws a random number to determine whether it can be readmitted to the financial markets.
   - The government that has recently been current on its debt decides whether to repay or default this period.
5. Equilibrium allocations take place:

- If the government defaults, it is excluded from financial markets this period and simply consumes its endowment, subject to a default penalty.

- If the government repays, it chooses the new allocation of bonds $b'$, while the lenders update their posterior belief $\tilde{p}$ and post the bond price $q(b', y, \tilde{p})$.

The treatment of the lenders’ belief is a novel aspect of the present model and requires a further comment. Notice that in each period there are up to two stages of updating the lenders’ belief. One of them, common to both PSI and PAI models, results from the mechanical application of Bayes’ formula and always occurs at the beginning of the period, as soon as the new endowment realization has arrived. In that stage, the last period $\tilde{p}$ is taken as input and the new state variable $p$ is returned. The other stage, a distinct feature of the PAI model, is an outcome of the signaling interaction between the lenders and the government, and is determined simultaneously with equilibrium allocations. Here, the state variable $p$ is mapped to a new posterior belief $\tilde{p}$. Consequently, both variables $p$ and $\tilde{p}$ constitute either a prior or a posterior belief, depending on the updating stage considered. For notational coherence, and given the chronological order of events within a period, throughout the paper I will refer to the state variable $p$ as the prior, and the endogenous variable $\tilde{p}$ as the posterior. Notice that in the PSI model we always have that $p = \tilde{p}$ and the only true posterior is $p'$, the belief that arises from Bayesian-updating of the future income realization. To avoid confusion however, I will still refer to variables $p$ and $p'$ as the current-period prior and the next-period prior, respectively, even though no signaling interaction takes place in that model.

3.4 Recursive formulation

In the following section I formalize the economic environment by stating the problems faced by market participants in the recursive form. Define the vector of aggregate state variables that are common knowledge as $s = (b, y, p)$. In order to keep the distinction between the PSI and PAI models tractable, I treat the regime state $z$ as a separate argument; naturally it becomes irrelevant in the version with symmetric information.

**Government** The government that is current on its debt obligations has the general value function given by

$$v^0(s; z) = \max_{d \in \{0, 1\}} \left\{ (1 - d)v^r(s; z) + dv^d(y, p; z) \right\}$$

(5)
A sovereign who decides to default \((d = 1)\) is excluded from international credit markets and has the probability \(\theta\) of being readmitted every subsequent period. The associated default value is given by

\[
v^d(y, p; z) = u(h(y)) + \beta \sum_{z' \in \{z_L, z_H\}} \pi(z'|z) \int_Y f_{z'}(y', y) \left[ \theta v^0(0, y', p'; z') + (1 - \theta) v^d(y', p', z') \right] dy'
\]

subject to the law of motion for the lenders’ belief

\[
p'(y, y', \tilde{p}) = \frac{f_{z_H}(y', y) \times [\tilde{p} \pi(z_H|z_H) + (1 - \tilde{p}) \pi(z_H|z_L)]}{\sum_{z' = z_L,z_H} f_{z'}(y', y) \times [\tilde{p} \pi(z'|z_H) + (1 - \tilde{p}) \pi(z'|z_L)]}
\]

\[
\tilde{p} = \psi^d(s)
\]

In equation (6), \(h(\cdot)\) is an arbitrary function that represents the cost of defaulting; \(f_{z'}(y'|y)\) denotes the probability density of transitioning from state \(y\) to state \(y'\) given that tomorrow’s regime is \(z'\). Furthermore, \(\psi^d\) is a function that describes the posterior belief of the lenders, which in turn depends on today’s state and potentially allows for a strategic default. The next period belief \(p'\), described in equation (7), depends on the current and future income realization, as well as the current period posterior belief \(\tilde{p}\). It results from applying Bayes’ rule and taking into account a potential regime switch at the beginning of next period, in accordance to the transition matrix given by (2).

The value of the government associated with repayment is given by

\[
v^r(s; z) = \max_{c,b'} \left\{ u(c) + \beta \sum_{z' \in \{z_L, z_H\}} \pi(z'|z) \int_Y f_{z'}(y', y) v^0(s'; z') dy' \right\}
\]

subject to the budget constraint and the law of motion for the lenders’ belief

\[
c + q(b', y, \tilde{p})b' = y + b
\]

\[
p'(y, y', \tilde{p}) = \frac{f_{z_H}(y', y) \times [\tilde{p} \pi(z_H|z_H) + (1 - \tilde{p}) \pi(z_H|z_L)]}{\sum_{z' = z_L,z_H} f_{z'}(y', y) \times [\tilde{p} \pi(z'|z_H) + (1 - \tilde{p}) \pi(z'|z_L)]}
\]

\[
\tilde{p} = \psi^r(b'; s)
\]

In the PAI model, the lenders’ posterior belief \(\tilde{p}\) not only depends on the current state, but also on the next period bond allocation selected by the government. Notice that in
this model, the bond price $q$ varies not only with the level of future borrowing and the current income (as it typically does in most models of sovereign debt), but also with the lenders’ posterior belief. This is due to the fact that lenders attempt to read the signal embedded in the bond allocation and map it to one of the government types. Note that, by contrast, in the PSI model both posterior functions $\psi^d$ and $\psi^r$ are always equal to the prior $p$.

**Lenders** Every period the lenders only observe $(b, y)$ and share a market belief $p$. Although they do not see the current growth regime $z$, they know its distribution and independently update their belief about it, as described by the law of motion in formulas (7) and (11). Notice that the denominator in those equations is always greater than zero and the resulting next period belief $p'$ is strictly interior on the interval $(0, 1)$.

In the PAI model, the government’s debt choice conveys a signal about the economy’s type and international lenders therefore gain another channel of updating their belief. Formally, this occurs through the application of Bayes’ rule and is given by

$$\psi^r(b'; s) = \frac{\text{Prob}(b' | z = z_H) \times p}{\text{Prob}(b' | z = z_H) \times p + \text{Prob}(b' | z = z_L) \times (1 - p)}$$

(13)

Notice that formula (13) may return one of the three possible values for posterior belief, $\{0, 1, s\}$. In the case of 0 or 1, all uncertainty that period is resolved and we have a separating equilibrium - essentially, the lenders are able to associate the observed bond allocation with a government type exactly. However, the equilibrium might also be pooling in that both types of government settle on the same debt amount and the lenders learn nothing from observing that allocation. In such case, the posterior belief $\tilde{p}$ is simply equal to the prior.

In solving the PAI model, it is essential to specify how the lenders view the term $\text{Prob}(b' | z = z_i), \forall i \in \{H, L\}$, a probability that the observed allocation is coming from government type $i$. In particular, a typical problem arises in defining the off-equilibrium beliefs for all those cases where the denominator of formula (13) is equal to zero (i.e. no government type optimally chooses the observed allocation in equilibrium). A definition of Bayesian equilibrium allows for such beliefs to be arbitrary. However, according to the classic argument by Rothschild and Stiglitz (1976) in regard to the pricing of insurance contracts, an equilibrium might fail to exist at all in such case. Intuitively, under certain conditions we can construct an infinite sequence of profitable deviations in which the bad (high risk) type mimics the good (low risk) type, competitive insurers respond with a higher price (in order to avoid a loss), and in turn the bad type resorts back to a separating contract. For this reason,
the insurers have preference for cream-skimming, i.e. offering the contract at terms that only the good type will find attractive. In this spirit, Wilson (1977) introduces the notion of anticipatory equilibrium, in which all deviations that will eventually become unprofitable once the initial contracts are themselves withdrawn, are ruled out. Under that assumption, the pooling contract may actually survive against separation in equilibrium, because it needs to be such that it does not become unprofitable when both types decide to deviate to it.

Throughout this paper, I follow Wilson in modeling the lenders’ behavior in the spirit of anticipatory equilibrium. Intuitively, a government is only offered a high-type separating bond price if at the allocation \( b' \) considered the low type has no incentive to deviate from its own low-type separating contract. If a high type government chooses \( b' \) such that a low type would find it optimal to mimic that behavior and get \( \psi^r(b';s) = 1 \) as a result, the lenders respond with a pooling contract, i.e. \( \psi^r(b';s) = p \).

To make this point formally, suppose that the lenders need to update their posterior belief based on the observed bond allocation \( b' \) in state \( s \). With a slight abuse of notation, define the following sets

\[
B_H(s) = \{ b' : v^r_H(s, b', \tilde{p} = \cdot) > v^0_H(s, b_H^*, \tilde{p} = 0) \} \supseteq \{ b' : v^r_H(s, b', \tilde{p} = 1) > v^0_H(s, b_H^*, \tilde{p} = 0) \} \]

\[
B_L(s) = \{ b' : v^r_H(s, b', \tilde{p} = \cdot) \leq v^0_H(s, b_H^*, \tilde{p} = 0) \} \supseteq \{ b' : v^r_H(s, b', \tilde{p} = 1) \leq v^0_H(s, b_H^*, \tilde{p} = 0) \} \]

\[
B_{HL}(s) = \{ b' : v^r_H(s, b', \tilde{p} = \cdot) > v^0_H(s, b_H^*, \tilde{p} = 1) \} \supseteq \{ b' : v^r_H(s, b', \tilde{p} = 1) > v^0_H(s, b_H^*, \tilde{p} = 0) \} \]

where \( v^r_i(s, b', \tilde{p} = \cdot) \) is type \( i \)'s value of repaying the debt and selecting \( b' \) as the next period bond allocation under a price implied by the posterior belief \( \tilde{p} \). Similarly, \( v^0_i(s, b_i^*, \tilde{p} = \cdot) \) is type \( i \)'s value of repaying the debt and choosing its optimal bond allocation \( b_i^* \) under a price implied by the posterior belief \( \tilde{p} \). Hence, \( B_i(s) \) denotes the set of all possible deviations from optimal policy that only the government of type \( i \) finds optimal in order to get the maximum price, while \( B_{HL}(s) \) denotes the set of all deviations that attract both types, if the best possible price were to be offered. With this notation in place, in an anticipatory equilibrium the lenders will update their posterior in response to allocation \( b' \) as follows

\[
\psi^r(b';s) = \begin{cases} 
1, & \text{if } b' \in B_H(s) \\
 p, & \text{if } b' \in B_{HL}(s) \& b' = b_H^* \\
0, & \text{if } b' \in B_L(s) 
\end{cases} \quad (14)
\]

Formula (14) implies that the lenders recognize a high-type government only as long as
it chooses the next period debt level that a low type would never find optimal. In contrast, the resulting equilibrium is pooling if and only if the selected debt level lies in the interval that potentially attracts both types and is equal to the high type’s optimal allocation in the PSI model. Naturally, it remains to mention that the debt amounts which solely attract the low type government result in a separating equilibrium.

It is worth pointing out that under this strategy of the lenders it does not matter how the off-equilibrium beliefs are specified. The reason is that they belong to the complement of the set $B_H(s) \cup B_L(s) \cup B_{HL}(s)$ and therefore will never be optimally chosen by either government type, regardless of what the resulting bond price is.

The last issue in discussing the lenders’ strategy involves specification of their posterior belief in response to an observed default $\psi^d(s)$. Technically, in certain cases a low type government might find it optimal to strategically default on its debt to influence the lenders’ belief. However, taking a full account of such sophisticated strategies would complicate the model and increase the computational burden significantly. However, given the commonly assumed low values of parameter $\theta$ (probability of being readmitted to financial markets after default - typically around 0.1), it seems to matter little how we model the lenders’ reaction to an observed default. This is because during the expected 10 periods of staying in exclusion from financial markets, the lenders’ independent belief will have diverged away from the one updated on impact. To keep the model simple, I assume that $\psi^d(s) = p$, i.e. the lenders do not interpret a default as any sort of signal and rather stick to their prior.

Having the lenders strategy in place, for both PSI and PAI models I follow Arellano [2008] by classifying the possible endowment realizations into default and repayment sets. The set of $y$’s for which default is the optimal choice is defined as

$$D(s; z) = \{ y \in \mathcal{Y} : v^d(y, p; z) > v^r(s; z) \}$$

(15)

The complement of $D$, the repayment set, is then defined as

$$R(s; z) = \{ y \in \mathcal{Y} : v^d(y, p; z) \leq v^r(s; z) \}$$

(16)
Then, the government’s default policy can be characterized as

\[ d(b, \ell, y) = \begin{cases} 
1, & \text{if } y \in \mathcal{D} \\
0, & \text{if } y \in \mathcal{R}
\end{cases} \] (17)

Consequently, using the default set we can characterize the next period expected default probability as

\[
\delta(b', y, \tilde{p}) = \tilde{p} \sum_{z'} \pi(z'|z_H) \int_{\mathcal{D}(s';z')} f_{z'}(y', y) dy' + (1 - \tilde{p}) \sum_{z'} \pi(z'|z_L) \int_{\mathcal{D}(s';z')} f_{z'}(y', y) dy'
\] (18)

where \( s' = (b', y', p'(y, y', \tilde{p})) \). As it is common in the quantitative models of sovereign debt, lenders are assumed to be risk-neutral and the resulting equilibrium bond price is such that they make zero profits in expectation.

\[
q(b', y, \tilde{p}) = \frac{1 - \delta(b', y, \tilde{p})}{1 + r^*}
\] (19)

A novel aspect of the PAI model that is worth emphasizing is the dependence of bond prices on the level of borrowing in both the next period (as in the standard debt models) as well as the current one. This occurs through the posterior \( \tilde{p} \) which in turn is a function of the current debt level, as specified in equation (12). Consequently, even if we hold the typical arguments \( b' \) and \( y \) fixed, the bond price will be different for various levels of the current debt \( b \), depending on whether the government decides to separate or pool.

Concluding this section, definition 1 introduces the equilibrium concept that I will use in the present paper.

**Definition 1** A stationary Perfect Bayesian Equilibrium for this economy consists of the government’s policy functions \( c(s; z) \), \( b'(s; z) \), \( d(s; z) \), the posterior belief functions \( \psi^r(b'; s) \) and \( \psi^d(s) \), and the bond price schedule \( q((b', y, \psi^r(b'; s)) \) such that:

1. The policy functions \( c, b', d \) solve the government’s problem.
2. Bond prices \( q \) are such that the lenders make expected zero profit (subject to their imperfect beliefs).
3. The posterior beliefs \( \psi^r, \psi^d \) are updated according to Bayes’ rule, whenever possible, and are arbitrary elsewhere.
3.5  Quantitative analysis

3.5.1 Data

In this section I bring the model to the data by matching it to the Italian economy. In what follows, quarterly data on the National Accounts and the government debt are obtained from the Eurostat (1995Q1-2014Q4), while the data on bond prices are from Bloomberg. The spread on Italian bonds is computed at quarterly frequency as the difference between the interest paid on Italian 10-year government bonds and the German 10-year bonds.

3.5.2 Functional forms and calibration

I follow the general trends in the literature to select specific functional forms. The household’s utility is a CRRA function of the form \( u(c) = c^{1-\sigma} \). The default penalty is specified as \( h(y) = -d_0 y + d_1 y^2 \) following Chatterjee and Eyigungor (2012). Table 1 presents a summary of the parameter values used to compute the model’s equilibrium.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Meaning</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>Risk aversion</td>
<td>Literature</td>
</tr>
<tr>
<td>( r^* )</td>
<td>0.01</td>
<td>Risk-free rate</td>
<td>Literature</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>0.659</td>
<td>Default penalty parameter</td>
<td>Calibration</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>0.716</td>
<td>Default penalty parameter</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.101</td>
<td>Re-entry probability</td>
<td>Calibration</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.979</td>
<td>Discount factor</td>
<td>Calibration</td>
</tr>
</tbody>
</table>

Note: Three parameters are jointly calibrated to match the following moments: St. dev. of trade balance to GDP, ratio of debt service to GDP, correlation of consumption and GDP, and the annualized default probability.

The relative risk aversion coefficient is set to 2, which is a standard value in the business cycle models. Following the recent sovereign debt literature, I set the risk-free rate equal to 1%. The remaining four parameters of the model \( (d_0, d_1, \beta) \) are calibrated jointly to match the real moments in the data: the standard deviation of trade balance, the ratio of debt service to GDP, the correlation coefficient of consumption and GDP, and the annualized probability of default.

To calibrate the dichotomous endowment process, I use the following reasoning. The High regime in my model is interpreted as “normal” times, while the Low regime represents
an extremely rare “depression” mode. Apart from the second world war, which can be viewed as a purely exogenous event from the point of view of pure economic mechanisms, the countries in Europe had not suffered a recession this severe since the Great Depression. This gives us roughly 70 years (280 quarters) or High regime and translates into the expected probability of switching from High to Low of just 0.003. Similarly, because the Great Depression lasted around 10 years (40 quarters), we obtain the likelihood of staying in the Low regime of 0.025. Having pinned down these two key probabilities, we obtain the entire transition matrix defined in (2).

I consequently proceed by estimating the output process using Italian quarterly GDP data with Maximum Likelihood, under the assumption that the persistence and volatility parameters, $\rho$ and $\eta$, are equal across the regimes. While this assumption may not necessarily be true, we do not have long enough times series to distinguish reliably between them. This assumption is important however for the single crossing property to hold in the PAI version of the model. The unconditional mean of the High regime is normalized to 0. Thus, the only free parameter left to be pinned down is the unconditional mean of the Low regime, $\mu_L$. I discipline this parameter by matching the economy’s learning process during the European debt crisis with the evolution of the sovereign rating.

Table 2 summarizes the obtained results. As can be expected, the estimated AR(1) process using data from European economies exhibits high persistence and low volatility compared to most studies on emerging economies. The unconditional mean of the Low regime, estimated at $-0.15$, is such that the learning process in the model economy matches the one inferred from the dynamics of the sovereign ratings.

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4The specification of these exact numbers is not crucial, because I eventually discipline the output process estimation by matching the learning process with sovereign rating data. Instead, what matters is that the Low regime is generally viewed as a very rare and extreme event.

5Generally, it seems intuitive that the High regime should be preferred to the Low regime. However, depending on calibration, there may be states of nature in which this preference is reversed. For example, imagine that $\rho_H > \rho_{HL}$ and $\eta_H < \eta_L$. If the income today is low enough then the high persistence and low volatility of the High regime implies that the economy will suffer for a number of periods, before slowly rebounding back to the high unconditional mean. In such case, being in the Low regime might be more desirable as it gives the government a higher chance of getting better income shock already next period. Consequently, the single crossing property is broken.

---
### Table 2: Parameters of the endowment process

<table>
<thead>
<tr>
<th>Regime</th>
<th>Mean $\mu$</th>
<th>Persistence $\rho$</th>
<th>St. dev. $\eta$</th>
<th>Transition Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>$-0.15$</td>
<td>0.965</td>
<td>0.008</td>
<td>0.975 0.025</td>
</tr>
<tr>
<td>High</td>
<td>$0.00$</td>
<td>0.965</td>
<td>0.008</td>
<td>0.003 0.997</td>
</tr>
</tbody>
</table>

#### 3.5.3 Characterization of the equilibrium

In the following section I characterize some of the key properties of the equilibrium computed in the two versions of the model, PSI and PAI. Figure 4 illustrates the default set in the two models, as function of the current debt (x-axis) and endowment (y-axis). As can be noticed, the set of states in which government prefers to default expands with the drop in market belief, which is in line with the basic intuition about the mechanics of the model (lower belief leads to a worse price of bonds and therefore less incentive to repay).

![Default sets for different beliefs](image)

Figure 4: Default set for different prior beliefs in the PSI model

Figure 5 presents the policy functions for next period debt, plotted at different values of the market belief. It is clear that the higher the belief, the more debt is desired by an impatient government, due to higher expectation of the economy’s future performance.

Figure 6 plots the resulting bond prices as functions of the next period debt allocation, for various levels of prior belief. It can easily be noticed that when the market sentiment about
the growth regime is high, the government is offered attractive bond prices that depreciate leniently as the level of indebtedness increases. However, as the markets adjust their belief downwards, the bond prices not only fall, but also the entire schedule becomes much steeper, punishing the government for excessive debt at much lower levels of $b$.

Turning to the PAI version of the model, Figure 7 presents the default sets for both types of government and depending on different levels of the prior belief. As can be noticed in Figure 7(a), the Low type defaults in the smallest number of states for the highest possible belief $p = 0.99$, similarly as it is for the PSI case. When the market belief falls however, this default set expands only marginally due to the fact that the Low type now has an option to mimic the high type and avoid defaulting on its debt. By contrast, as depicted in Figure 7(b), the High type’s default set does not vary with market beliefs, due to the fact that this type of government always has an option to force separation and reveal itself, as an alternative to defaulting.

Figures 8 through 10 contrast the debt policy of the two types depending on today’s level of income, for the high, medium and low market belief, respectively. The general interpretation of the optimal policy is similar across the figures. For a low enough
Figure 6: Bond price as function of next period debt for different levels of prior belief endowment, both types prefer to default and their next period debt allocation consequently is equal to zero. As the level of income rises, the High type finds it optimal to repay the debt and subsequently enjoys an interval of mostly undistorted debt policy. At some point however, the low government type also chooses to repay its debt and engages in a pooling equilibrium, by mimicking the High type’s optimal policy as in the PSI model. As a result, the two lines merge on the graph. Notice also that at the endpoints of the pooling interval the High type makes and “escape”, by increasing the borrowing level to prevent the low type from mimicking its behavior. Eventually, for large enough endowments, the two types decide to separate and settle their debt policy on their individually optimal levels.

As can be noticed by eyeballing Figures 8 through 10, the prior market belief matters for the level at which the policy function in a pooling equilibrium is established. Figure 8 shows that for a very high market belief it is the Low type who bears the entire burden of mimicking the High type. In that case, the former has potentially a lot to gain (improve the market confidence from $p = 0.00$ to $p = 0.99$), while the latter has almost nothing to lose (one percentage point). As the market belief drops to $p = 0.53$, as shown in Figure 9, the pooling equilibrium policy function settles in between the individual optimal levels of the two types, as both of them have now something significant to win and lose, respectively. Figure 10 shows that for an extremely low prior belief, the Low type does not bother to
mimic (because the possible gain is very small) and pursues its own individually optimal policy instead. The High type on the other hand, unable to reveal its type over the interval of pooling, adjusts its debt policy downwards due to the very low price that international lenders are willing to offer. This type of pooling equilibrium is an interesting result of the anticipatory equilibrium concept assumed in this paper - even though the two types choose separate policy functions, the High one is not offered a good separating price in order to prevent the Low one from mimicking and rendering the contract unprofitable.
Figure 8: Policy functions for the two types and a high market belief in the PAI model

Figure 9: Policy functions for the two types and a medium market belief in the PAI model
4 Simulating the European debt crisis

In this section, I use the calibrated models to analyze the timing and pattern of events during the debt crisis in Italy. I start with the benchmark, full-information version of the model and gradually add the assumptions about the information structure discussed in the previous section. This allows me to point out the elements of the model that make it possible to explain the features of the European debt crisis.

4.1 Full Information case

I start by feeding in the actual detrended Italian GDP observations into the benchmark Full Information version of the model. The upper panel of Figure 11 presents the simulated evolution of the debt level and the bond spread in that world. The economy is started with zero debt service and gradually accumulates it throughout the 2000s, up to the level of around 25%. At the same time, the bond spread is equal to zero in all periods prior to 2008, implying a zero probability of default during that time. When the crisis first hits in the third quarter of 2008, the bond spread jumps up, while the government reduces its debt. With lower level of indebtedness, the subsequent negative output shocks only induce much smaller spikes in the spread, while the government continues to reduce its debt level. As
can be noticed then, the Full Information version of the model delivers predictions opposite to what we observe in the data for the European debt crisis. The key assumption of the analysis is that the regime switches from High to Low in the third quarter of 2008, and both the government and the lenders are aware of this. This information structure is confirmed in the lower panel of Figure 11 which shows that the market belief is equal to 1 before and 0 after that period.

![Figure 11: Simulated debt crisis in the FI model](image)

4.2 Partial Symmetric Information case

Now I conduct the simulation under the assumption that the market and the government share a common belief about the growth regime in which the economy is operating. The upper panel of Figure 12 presents the evolution of the debt level and the bond spread in the PSI model. The economy is once again started with zero debt service and gradually accumulates it throughout the 2000s, up to the level of around 25%. The bond spread is once again equal to zero in all periods prior to 2008. However, in this world when the crisis begins and the regime switches to Low, the market belief only drops slightly, which
can be viewed in the lower panel of Figure 12. As a result, the bond spread only increases slightly, while the government reduces its debt by a smaller amount as compared with the FI case. Subsequently however, when the bad income shocks hit the economy again at the beginning of 2010, the markets finally start to realize that this prolonged downturn must in fact be driven by the underlying regime switch, and reduce their belief correspondingly. As a result, the bond spread goes on a sharp increase, much higher in magnitude that initially during late 2008. This exercise shows therefore that market learning is a mechanism capable of delivering the delay in the outset of a debt crisis, as witness for peripheral European economies during 2008-2014.

![Bond spread and debt level](image.png)

![Market belief about the government type](image.png)

Figure 12: Simulated debt crisis in the PSI model
4.3 Partial Asymmetric Information case

[TO BE ADDED]

References


