Liquidity Fluctuations in Lemon Markets and Policy Interventions

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[Preliminary and Incomplete]

Abstract

This paper studies endogenous liquidity fluctuations in dynamic lemon markets. Agents trade heterogeneous qualities of an asset over the counter and bargain under asymmetry of information. Future liquidity begets current liquidity as current prices increase with the resale value of a lemon. Conversely, current liquidity crowds out future liquidity through an endogenous composition effect in the pool of sellers. When this latter effect dominates, stationary cycles emerge where price and volume oscillate. From a normative point of view, unstable markets call for specific policy interventions to restore liquidity. I show that when buyers can post prices before matching, liquidity and welfare may actually decrease. These results provide new insights on the role of competition and information for market efficiency.

Keywords: Adverse Selection, Equilibrium Cycles, Market Structure.

JEL Codes: D47, D82, G01.

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1 Introduction

The recent 2007 crisis started with a widespread shortage of liquidity in the financial system. Difficulties to sell or finance securities on secondary markets triggered a collapse in the issuance of many assets, impacting credit and ultimately the real economy. The severity of the bust prompted a two stage policy response with regulators acting both as market participants and market designers. During the first phase, government institutions intervened massively to prop up trading and liquidity in asset markets\textsuperscript{1}. In a second ongoing phase, policy makers have pushed structural reforms to redesign various segments of these markets\textsuperscript{2}. Many assets indeed trade Over The Counter (OTC) where the lack of competition and transparency may cause illiquidity and unstability. This paper presents a theory of trading based on asymmetry of information to explain these endogenous liquidity dynamics. I show that asset purchase programs to isolate bad assets may help restore liquidity. However, in the presence of adverse selection, introducing more competition and information in the market can sharply decrease liquidity and welfare.

In the model, heterogeneous agents have different private valuations for a long-term asset, generating gains from trade. These preferences switch over time so that buyers may ultimately need to re-sell an asset previously acquired. In order to trade, agents match bilaterally in an OTC market where buyers make a Take It Or Leave It offer. In a meeting, sellers privately know the quality of the asset they hold as well as their valuation. Hence trade may fail to occur because of adverse selection as in Akerlof (1970). A buyer indeed proposes a pooling price only when information rents to low quality sellers are not too high. Otherwise, he targets only those low quality sellers. We say that a market is liquid when all types of sellers can trade despite these information frictions. In a dynamic environment, liquidity may fluctuate in non-trivial ways because buyers’ decisions exhibit both complementarity and substitutability across time. First, high future resale prices raise buyers’ willingness to pay for an asset of unknown quality. In other words, with

\begin{itemize}
\item \textsuperscript{1}Central Banks like the Fed provided credit lines to financial institutions (e.g. Term Auction Facility) or extended their open market operation with large scale asset purchase programs
\item \textsuperscript{2}In Europe, The Market in Financial Instruments Directive (MIFID II, Regulation 600/2014 of the European Parliament) requires “all standardised derivatives to be traded on organised and transparent venues”. Title VII of the Dodd Frank in the US contains similar provisions for Over the Counter Derivatives.
\end{itemize}
the resale effect, future liquidity begets present liquidity. Second, buyers willingness
to pay depends on the quality of the pool of sellers. Among owners of high quality
assets, only those with low valuation wish to sell. Hence, if markets were liquid in
the past, these assets are precisely held by high valuation agents who are the natural
owners. The supply of low quality assets however remains constant since both low
and high valuation owners want to flip their lemon. The composition effect thus
implies that present liquidity crowds out future liquidity.

I solve for stationary equilibria of the model and show that this interaction yields
novel predictions. When the composition effect dominates the resale effect, that is
for low discount factors and persistent preferences, (endogenous) equilibrium cycles
emerge. The economy then fluctuates between periods of high trading volume at
high price and periods of low trading volume at depressed prices, in the absence of
any aggregate shock. At the peak of the cycle, the high average quality among assets
for sale leads buyers to propose a pooling price. This lowers the future quality of
the pool because of the composition effect. During the trough of the cycle, demand
and price stay low for fear of increased adverse selection. Selling pressure from high
quality asset owners accumulates until the quality in the pool of sellers reaches a
new peak. To the best of my knowledge, this paper is the first to describe these
dynamics in a model of adverse selection.

My analysis offers novel insights in line with observed market price movements
both in normal and crisis times. Duffie (2010) show that sharp price decrease and
slow recovery often follow the arrival of new information. In my model, at the peak
of a cycle, there is good news about the quality of assets for sale as many high
quality owners wish to sell. The model also sheds light on asset price bubbles, a
phenomenon commonly associated to financial crises. During the trough of the cycle,
buyers are willing to pay a premium for the lemon since they rationally anticipate
to resell it at a higher price. The high pooling price at the peak supports agents
conjecture. After a peak period, the bubble “bursts” as prices fall sharply. The low
quality asset thus resembles a hot potato that agents try to pass to the next investor
in the chain for a profit. Importantly in my model these fluctuations arise in the
absence of aggregate shocks.

Equilibrium cycles emerge when the share of high quality assets in the economy
is intermediate. In this region, a steady state equilibrium also exists in which buyers
randomize between high and low prices. Intuitively, when the average share of high quality assets is too low to sustain full trade, there must be some degree of illiquidity in the market. In a steady state, illiquidity is distributed evenly across periods whereas it fluctuates in a cycle. I show that these differences matter for welfare and study asset purchase programs aimed at restoring full trade. A utilitarian planner seeks to take lemons out of the market in order to increase the average quality in the pool of sellers. An asset purchase programs thus trades off the one-time cost of purchasing lemons at inflated prices with the permanent increase of liquidity in the market. I show that for the same parameters, a full-trade intervention may increase welfare only if the economy is in a cycle but not in a steady state. In the trough of a cycle, the very low level of trade prompts a government intervention. The steady state only exhibits partial illiquidity that never justifies paying the cost of the intervention. Hence, my analysis suggests that unstability and not only illiquidity matter for the design of such policy interventions.

Concerns that the very structure of OTC markets would cause illiquidity and unstability fueled proposals to move trading towards organized venues. Section 5 introduces a slightly different trading infrastructure - called Exchange - to study current market design reforms. Buyers may now post and commit to terms of trade prior to meeting a counterparty rather than making offers after a meeting. Price posting generates competition and provides sellers with pre-trade information, in line with the stated objectives of the current set of reforms\(^3\). In the OTC market, monopsonist buyers offer either a pooling price to all sellers or a low price to low quality sellers as in Samuelson (1984). In the Exchange, within period competition prevents pooling allocations as buyers may cream-skim high quality sellers. Price posting with exclusivity thus implements the least cost separating allocation thanks to rationing at high prices\(^4\). With two-dimensions of private information however - private valuation and asset quality -, high quality assets do not trade in equilib-

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3The comparison is also helpful from a positive point of view as many financial markets feature price posting. This is the very concept underlying a public limit order book in an Exchange where all offers are simultaneously available to an investor. In some OTC markets, dealers may also advertise quotes.

4Note that exclusivity is crucial to generate separation in competitive search models. Although not formalized in a search environment, Wilson (1980) provides a useful comparison on this issue. In assuming exclusivity, I follow most of the recent competitive search literature building on Gale (1996)
rium, generating an extreme form of illiquidity. The crucial insight for the result is that the presence of high valuation lemon owners generates a no gap at the bottom condition. Whereas low valuation lemon owners may sacrifice high prices for high probability of trade, high valuation lemon owners may only gain by mimicking high quality sellers. In a dynamic setting, high valuation agents precisely end up holding lemons to realize gains from trade with low valuation agents.

The Exchange is competitive but illiquid as high quality assets do not trade. As a result, an OTC equilibrium typically dominates the Exchange equilibrium both in the ex-ante and interim welfare sense. The first result is not surprising given the nature of equilibrium: pooling against separating. The conditions for interim improvement however are usually much stronger with only one dimension of private information. These results hint at the dual role of competition in an environment with adverse selection. Local monopsony power reduces demand but also shields buyers from cream-skimming deviations. In a sense, opaqueness of the asset traded and the trading structure are complements. Indeed, when there is asymmetry of information about the asset value, a transparent exchange is not necessarily desirable.

Relation to the literature

My paper contributes primarily to the growing literature on dynamic markets with asymmetric information. Many recent contributions (e.g. Deneckere and Liang, 2006, Camargo and Lester, 2014, Moreno and Wooders, 2013, Fuchs and Skrzypacz, 2015) have shown that, as a screening device, trading delay is tantamount to rationing in a static environment. As low types are more eager to sell, time on the market signals asset quality. This argument may no longer hold when agents can re-trade the asset in secondary markets. Guerrieri and Shimer (2014) and Chang (2014), building on the pioneering work of Gale (1996) still obtain static separation through rationing at different prices in a competitive market with exclusivity. Section 5 uses the same competitive search market structure as these two papers but shows that high quality assets do no trade. High valuations owners of lemons, who can not sell when private valuations are public knowledge, indeed generate a worse form of adverse selection. The paper closest to mine is by Chiu and Koeppel (2014)

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5Kurlat (2013) is a notable exception where sellers may visit several markets simultaneously.
6Guerrieri et al. (2010) and Chang (2014), the latter with two dimensions of private information,
who use a similar OTC framework where agents trade two heterogeneous asset qualities. There also, monopsonist buyers propose either a low price or a pooling price and do not screen sellers. However, they only study steady states while I characterize all stationary equilibria with different positive and normative properties. In addition, I compare these results with those obtained in a different market structure with price posting.

There is a vast literature dealing with government interventions in illiquid markets. In the presence of asymmetry of information, a planner who faces the same constraints as private agents acts as a mechanism designer. In a static model, Philippou and Skreta (2012) and Tirole (2012) show that government interventions raise the endogenous outside option to participate in the market, in turn raising the cost of removing lemons. This effect is present in my model but the gains from intervention are permanent in a dynamic environment. Chiu and Koepppl (2014) consider the same type of open market purchase to analyze gradual policies and announcement effects. As a complement, I show that cyclical fluctuations may warrant policy interventions that are not desirable when the market is illiquid but stable. My analysis thus emphasizes the role of unstability on top of illiquidity for the desirability of these policies.

Section 5 compares price posting with directed search - labeled Exchange - to the trading protocol of the OTC market: random search with ex-post bargaining. I show that while the first trading institution increases competition and information, it may generate lower trading volume and welfare. In such a common value environment, Hörner and Vieille (2009) and Fuchs and Skrzypacz (2015) show that information about past trades can lower welfare as high types try to signal by rejecting offers. Using the Burdett and Judd (1983) framework, Lester et al. (2015) find that an intermediate degree of competition maximizes welfare. In these papers as in mine, the variation in competition or information is exogenous and does not result from the optimal choice of agents. Many papers have studied the coexistence of platforms with similar trading protocols, mostly in a private value environment. With homog...
geneous investors, competition between similar platforms typically leads to liquidity concentrating in one venue (see Pagano, 1989). Conversely, several trading posts may open to accommodate the need of heterogeneous investors. In Pagnotta and Philippon (2015) or Lester et al. (2015), platforms compete in price and execution speed. In my model, this trade-off is central within the Exchange as heterogeneous sellers must choose between low price or low execution probability. However, my model remains silent about the endogenous choice between different trading protocols: Exchange and OTC. In Michelacci and Suarez (2006) or Bolton et al. (2014) this choice is relevant because trading takes place under complete information in the OTC market, a rather strong assumption.

Although the mechanism I emphasize is new, a significant literature has shown that markets may be unstable in the absence of aggregate shocks. In a survey of endogenous business cycle models, Boldrin and Woodford (1990) stress that exotic price dynamics typically require a low discount factor or some market friction, as in my model. More recently, the monetary literature (see Rocheteau and Wright (2013) among others) highlighted the importance of cycles or chaotic dynamics in economies where assets are used as a mean of exchange. In these models, self-confirming expectations generate these dynamics. In my model, cycles result from endogenous composition dynamics - a backward looking effect and equilibrium multiplicity is not as severe.

The rest of the paper is organized as follows. Section 2 introduces the model. I discuss the main dynamic effects and solve for stationary equilibria in Section 3 including cycles. Section 4 discusses policy interventions aimed at restoring liquidity in the market. Section 5 analyzes a market structure change by allowing agents to post prices before meetings and finally, Section 6 concludes.

2 The Model

2.1 Environment

Time is discrete and runs forever \( t = 0, 1, \ldots, \infty \). The economy is populated by a continuum of agents with discount factor \( \delta < 1 \). There is a fixed supply \( S \) of infinitely lived assets in the economy. Agents consume a non-storable numeraire good \( c \) and
dividends or fruits \(d\) from the assets. Agents may be of two types \(i = 1, 2\) with the following instantaneous preferences over a numeraire/fruit consumption bundle:

\[
u^i(c, d) = c + \tau^i d
\]

where \(1 = \tau^1 < \tau^2 = \tau\). Differences in valuation for the dividends generate gains from trading the tree. Types are persistent but may switch from one period to the next with probability \(\gamma \in [0, 1/2]\). This Markov Process is identical and independently distributed across agents. The type of an agent is private information.

Agents are endowed with \(e\) units of the numeraire good every period. There are two varieties of the asset in the economy with low and high dividends. A fraction \(q\) (resp. \(1 - q\)) of the assets yield \(d_H\) units of fruits (resp. \(d_L < d_H\)) at the end of every period. In the following, asset \(a \in \{L, H\}\) refers to the asset paying \(d_a \in \{d_L, d_H\}\). The quality \(a\) is private information to the current holder of the asset. As it is standard in the literature, agents may hold either one or zero unit of the asset. Endowment is large in the following sense:

\[
e \geq \frac{\tau d_H}{1 - \delta}
\]

(A1)

There is free entry of non-owners who must pay a flow cost \(\kappa > 0\) to participate in the market.

For conciseness, I label \((\tau^i, a)\) an agent with asset holding \(a \in \{0, L, H\}\) and type \(\tau^i \in \{\tau^1, \tau^2\}\). By convention \(a = 0\) means that the agent holds no asset. Let us denote \(v_a^i(t)\) the value function of an agent \((\tau^i, a)\) in period \(t\), net of the present discounted value of the endowment flow. For notation ease, I also introduce the expected value for each agent \((\tau^i, a)\).

\[
\bar{v}_a^i(t) := v_a^i(t) + \gamma (\bar{v}_0^i(t + 1) - v_a^i(t))
\]

At the end of period \(t - 1\), \(\bar{v}_a^i(t)\) is the expected utility in period \(t\) for type \((\tau^i, a)\), given that he might switch type with probability \(\gamma\). Let us now define the reservation value for an asset owner as follows:

\[
r_a^i(t) := \tau^i d_a + \delta (\bar{v}_a^i(t + 1) - \bar{v}_0^i(t + 1)), \quad i = 1, 2, \quad a = L, H
\]

(1)
The reservation value is the utility an agent derives from holding on to his asset in period $t$. As we will see, $r^i_a(t)$ is thus the minimum price for which the asset owner will sell his asset. The price must compensate the owner for the current dividend $\tau^i d_a$ and the future net value of owning the asset $\bar{v}^i_a(t+1) - \bar{v}^i_0(t+1)$. Finally, observe that $r^i_a(t)$ is also the net value a potential $(\tau^i, 0)$ buyer attaches to the asset in period $t$. For future reference, we also define the autarky valuation $v^i_{a, aut}$ for an agent holding asset $a$

$$v^i_{a, aut} = \frac{(1-\delta)\tau^i + \delta\gamma(\tau + 1)}{(1-\delta)(1-\delta(1-2\gamma))}d_a$$

This is the utility agent with type $\tau^i$ derives from holding his asset forever.

Let $\mu^i_a(t)$ be the mass $(\tau^i, a)$ agents at the beginning of period $t$. These quantities must verify the following aggregate consistency requirements:

$$\mu^1_H(t) + \mu^2_H(t) = Sq$$
$$\mu^1_L(t) + \mu^2_L(t) = S(1-q)$$

In this lemon market, adverse selection is severe, that is

$$\tau d_L < d_H$$

(LC)

Intuitively, our lemon condition (LC) implies that the value of a $L$ asset to high valuation 2 agent is lower than that of a $H$ asset to a lower valuation 1 agent.

### 2.2 The OTC market

Trading is decentralized and agents must search for counterparties. Buyers pay the flow cost $\kappa$ to match with sellers while sellers may search for free\(^8\). Matches are bilateral and buyers make offers that sellers can accept or reject.

Matching\(^8\)

\(^8\)This asymmetric structure is justified because buyers make TIOLI offers. Since high quality sellers may never obtain more than their reservation value, they would not search with a positive cost. The positive search cost also serves a technical purpose. For low valuation non-owners, that is type $(\tau^1, 0)$, it breaks the tie between searching and not searching in favor of the latter option.
All asset owners (resp. non-owners) are potential sellers (resp. buyers). Let us denote $\mu^B(t)$ (resp. $\mu^S(t)$) the mass of active buyers (resp. sellers). Since sellers do not incur a search cost, all sellers search in the OTC market. For buyers, participating in the market results from an optimal entry decision in a way specified below. The probability for a seller to meet a buyer in period $t$ is

$$\lambda^S(t) = \min\left\{ \frac{\mu^B(t)}{\mu^S(t)}, 1 \right\}$$

Symmetrically, a buyer meets a seller with probability

$$\lambda^B(t) = \min\left\{ \frac{\mu^S(t)}{\mu^B(t)}, 1 \right\}$$

The matching technology exhibits efficient rationing$^9$. Agents on the short-side of the market finds a counterparty for sure. The probability for a buyer (resp. for a seller) to meet a given type of seller (resp. buyer) is proportional to the fraction of that type among the population of sellers (resp. buyers). All meetings are anonymous.

**Bargaining**

In a meeting, the buyer makes a Take It Or Leave It (TIOLI) offer to the seller. An offer is a menu of contracts formed by a price and a probability to trade. However, following Samuelson (1984), we know that uninformed buyers ultimately offer a single price to sellers$^{10}$. For buyer $(\tau^k, 0)$ where $k = 1, 2$, a strategy in period $t$ is thus a distribution of price offers $\Pi^k(t,.) : \mathbb{R}_+ \mapsto [0, 1]$ with density $\pi^k(t,.)$ and support $\Gamma^k(t,.)$. A seller $(\tau^i, a)$ may only accept or refuse an offer. His strategy in period $t$ can be represented by an acceptance function $\alpha_a^i(t,.) : \mathbb{R}_+ \mapsto [0, 1]$. The strategies used by buyers and sellers in the bargaining game must form a sub-game perfect equilibrium.

$^9$Observe that matching is not purely random as there cannot be a buyer-buyer or a seller-seller meeting. I choose efficient rationing for its tractability. The model may still accommodate a matching technology of the form $\alpha \min\{\mu^B(t), \mu^S(t)\}$ where $\alpha \in (0, 1]$ would capture the severity of search frictions.

$^{10}$The result that uninformed buyers do not wish to screen sellers rests on the linearity of the buyer’s problem in trading probabilities.
Sellers’ Problem

In a bilateral match, the acceptance rule takes a simple form whereby a seller accepts any offer above his reservation value. Indeed, when offered $p$, the seller solves:

$$\max_{\alpha^i_a(t,p)} \alpha^i_a(t,p)[p + \delta \bar{v}_0(t+1)] + (1 - \alpha^i_a(t,p))[\tau^i d_a + \delta \bar{v}_a(t+1)]$$

If he accepts, a seller enjoys the price $p$ and the future utility from being a non-owner $\bar{v}_0(t)$. If he refuses the offer, he consumes the dividend and obtains next period expected utility from the asset. We can rewrite this problem as follows:

$$\max_{\alpha^i_a(t,p)} \alpha^i_a(t,p)[p - r^i_a(t)] + \tau^i d_a + \delta \bar{v}_a(t+1) \quad (4)$$

With this formulation, Lemma 1 immediately follows.

**Lemma 1.** Optimal Acceptance rules verify:

$$\forall t, \quad \alpha^i_a(t,p) = 1_{\{p \geq r^i_a(t)\}} \quad (5)$$

The term between bracket in (4) is equal to $p - r^i_a(t)$. Since buyers never meet again with probability 1, the result is then immediate invoking sub-game perfection in the bargaining game. The seller may not use non-credible out of equilibrium threats to extract high offers from the seller. A seller $(\tau^i, a)$ acceptance rule depends on the period $t$ only through the reservation value $r^i_a(t)$. Lemma 1 has an immediate corollary.

**Corollary** For $k = 1, 2$, $\Gamma^k \subset \{r^i_a(t)\}_{a=L,H}^{i=1,2}$

**Proof** Suppose a buyer makes offer $p$ such that there exists $(\tau^i, a)$ with $p > r^i_a(t)$. Then, for $\epsilon < p - r^i_a(t)$ the buyer is strictly better off making offer $\hat{p} = p - \epsilon$ since he attracts the same pool of sellers and increases his profit per unit traded. In addition, with a positive search cost, a buyer never proposes a loosing price as he would otherwise make a net loss. \(<\)
Let now $\Pi(t,\cdot)$ be the distribution of offers received in period $t$ by sellers$^{11}$. The value function of a $(\tau^i, a)$ seller verifies:

$$v^i_a(t) = \tau^i d_a + \delta \bar{v}^i_a(t + 1) + \lambda^S(t) \sum_{r_{a'}^i(t) \geq r^i_a(t)} \pi(t, r_{a'}^i(t)) \left[ r^i_{a'}(t) - r^i_a(t) \right]$$  \hspace{1cm} (6)

where we used the optimal acceptance rule defined in (5). The next Lemma helps us to characterize the non-owners’ problem.

**Lemma 2.** For all $t$,

$$r^1_L(t) \leq r^2_L(t) < r^1_H(t) \leq r^2_H(t)$$  \hspace{1cm} (7)

Observe that assumption (LC) completes a static order on bi-dimensional types whereby $(\tau^i, a) \succ (\tau^{i'}, a')$ if $\tau^i d_a \geq \tau^{i'} d_{a'}$. Lemma 2 states that the model dynamics do not modify this order when appropriately defined on (endogenous) reservation values$^{12}$. For this proof, we use the zero profit condition for a buyer, anticipating Part 1 of equilibrium definition 1. Intuitively, a higher type can always mimic a lower type so that he cannot get a lower utility. Observe in particular, that in any equilibrium, type $(\tau^2, L)$ has a lower reservation value than type $(\tau^1, H)$

** Buyers Problem**

A non-owner $(\tau^k, 0)$ for $k = 1, 2$ must choose whether to pay the search cost $\kappa$ taking as given the meeting probability $\lambda^B(t)$. Conditional on a meeting, he chooses the offer distribution $\Pi^k(t,\cdot)$ optimally given the acceptance rules defined in (5). We can write a non-owner net gain from trading as the solution in $\pi^k(t,\cdot)$ of

$$g^k_0(t) = -\kappa + \lambda^B(t) \left[ \max_{\{\pi^k(t,\cdot)\}} \sum_{(i,a)} \pi^k(t, r^i_a(t)) \sum_{r_{a'}^i(t) \leq r^i_a(t)} \frac{\mu^{a'}}{\mu^B(t)} (r^i_{a'}(t) - r^i_a(t)) \right]$$  \hspace{1cm} (8)

$^{11}$This distribution is a mixture of the distributions $\Pi^2(t,\cdot)$ and $\Pi^1(t,\cdot)$ weighted by probability to meet each type of buyer. With a positive search cost however, low-valuation agents never enter the market to buy, as we show later.

$^{12}$This condition may not hold in Guerrieri and Shimer (2014) where agents trade because of different discount factors. Patient agents with $L$ assets may have a higher reservation value than impatient agents with $H$ assets. In their set-up however, it is not entirely clear what a static ranking between types would be.
When making offer $r^i_a(t)$, a buyer attracts all the sellers with reservation values $r^j_a(t) \leq r^i_a(t)$ and weighs them accordingly. Hence, a buyer faces adverse selection because he cannot tailor his offer to each seller’s type. The value function of a non-owner $(\tau^k, 0)$ thus verifies.

$$v^k_0(t) = \max \{0, g^k_0(t)\} + \delta v^k_0(t + 1)$$

(9)

It is clear that a non-owner $(\tau^k, 0)$ finds it optimal to search if and only the net gain from trading $g^k_0(t)$ is non-negative. Lemma 2 has two important consequences. First, type $(\tau^1, 0)$ never search as they have negative gains from trade with any seller type. This appears clearly in (8). Hence only type $(\tau^2, 0)$ non-owners may enter the market as buyers and sellers only receive offers from these agents. In the following, we then drop the superscript for the type on the variables describing buyers’ strategies. Building on Lemma 2, we can further reduce the set of possible offers from buyers.

**Lemma 3.** $\Gamma \subset \{r^A_a(t)\}_{a=L,H}$, the distribution of offers has a two points support.

**Proof** $\triangleright$ We know from Lemma 2 that $\Gamma \subset \{r^i_a(t)\}_{i=1,2}^{a=L,H}$. Then observe that offer $r^2_a(t)$ is weakly dominated by $r^1_a(t)$ as the former yields at best a zero gross profit to $(\tau^2, 0)$ buyers. It is strictly dominated if there is a positive mass of every seller’s type, which we show to be the case in equilibrium.$\triangleleft$

Lemma 2 and 3 imply that $H$ asset owners may never obtain more than their reservation value, that is

$$v^i_H(t) = r^i_H(t) = v^i_{H,aut}$$

(10)

where $v^i_{H,aut}$ is the autarky valuation defined in (2). In the remainder of the paper, I drop the time argument in the value and reservation functions of $H$ asset owners. This result is a version of the Diamond paradox in search models. Repeated trading opportunities do not increase competition when buyers have all the bargaining power. In the remainder of the paper, I use the notation $r^i_H$ to denote both the value (function) and the reservation value of type $(\tau^i, H)$ Using Lemma 3, with a slight abuse of notation, we define $\pi(t) := \pi(t, r^i_H)$ to be the probability that buyers make a high offer $r^i_H$ in period $t$. We can then rewrite the buyer’s optimal offer problem
in a more condensed way:

\[
\pi(t) \in \arg \max_{\hat{\pi}(t)} \left\{ \hat{\pi}(t) \left[ \mu^1_H(t)(r^2_H - r^1_H) + (\mu^2_L(t) + \mu^1_L(t))(r^2_L(t) - r^1_L) \right] + (1 - \hat{\pi}(t))\mu^1_L(t)(r^2_L(t) - r^1_L) \right\}
\]

Equation (11) captures the rent-efficiency trade-off faced by uninformed buyers. Offer \( r^1_H \) (probability \( \hat{\pi}(t) \)) attracts all potential sellers but leaves information rents \( r^1_H - r^i_L(t) \) to low types. With a low offer \( r^1_L(t) \), buyers do not overpay the \( L \) asset but forgo gains from trade on the \( H \) asset.

**Law of Motions**

Finally, we need to characterize the law of motions for each type \((\tau^i, a)_{a=L,H}\). Using equations (3), we only need to keep track of low valuation \( \tau^1 \) agents. We have:

\[
\mu^1_L(t + 1) = \left[ 1 - \gamma - \lambda^S(t)(1 - 2\gamma) \right] \mu^1_L(t) + \gamma \mu^2_L(t) \\
\mu^1_H(t + 1) = \left[ 1 - \gamma - \lambda^S(t)(1 - 2\gamma)\pi(t) \right] \mu^1_H(t) + \gamma \mu^2_H(t)
\]

Let us explain in detail equation (12). In period \( t + 1 \) low valuation lemon owners might come from three different sources. First, some \((\tau^1, L)\) traders do not find a counterparty and do not switch type. This happens with probability

\[(1 - \gamma)(1 - \lambda^S(t))\]

Indeed, conditional on finding a buyer, these agents always trade. For those \((\tau^1, L)\) agents who traded, the new type \( \tau^2 \) owners may switch to type 1 in period \( t + 1 \). This represents a mass of

\[\gamma \lambda^S(t) \mu^1_L(t)\]

Third, they may be former \((2, L)\) traders who switched type. This happens with probability \( \gamma \). Whether owners in this latter group traded or not is irrelevant since assets would stay in the hands of type \( \tau^2 \) traders. Combining these three different cases, we obtain equation (12). The law of motion (13) for \((\tau^1, H)\) owners differs
from (12) to the extent that these sellers receive sub-optimal offers with probability $1 - \pi(t)$. We can now introduce the definition of an OTC equilibrium.

**Definition 1 (OTC Equilibrium).** For a given initial distribution $\{\mu^i_a\}_{a=L,H}^{i=1,2}$, an equilibrium of the OTC market is given for any $t$ by value functions $\{v^i_a(t)\}_{a=L,H}^{i=1,2}$, distribution of asset owners $\{\mu^i_a(t)\}_{a=L,H}^{i=1,2}$, a mass of buyers $\mu^B(t)$ and price offers $\pi(t)$ such that:

1. Offer $\pi(t)$ solves buyers problem (11) (optimality) $\mu^B(t)$ solves the zero-profit condition $v^B_k(t) = 0$ for $k = 1, 2$ (free entry).
2. Distribution $\{\mu^i_a\}_{a=L,H}^{i=1,2}$ verifies equations (3) and law of motion (12)-(13)
3. Value functions $\{v^i_a(t)\}_t$ verify equation (6).

Observe that we implicitly built sellers optimal acceptance rule $\alpha^i_a(t,.)$ into the equilibrium definition through Lemma 1. Ultimately an equilibrium of this economy can conveniently be described by a sequence $\{\pi(t)\}_{t=1,\ldots,\infty}$, the probability of a pooling offer $r^H_1$. I consider $\pi(t)$ as the measure of liquidity in period $t$.

### 3 Stationary Equilibria

In this section, I solve for (long-run) stationary equilibria of the model and show that endogenous liquidity fluctuations arise in the absence of aggregate shocks. The existence of cycles depends on the interaction between two central dynamic forces at play in the model: the resale effect and the composition effect. I explicit these mechanisms in the paragraph below. From equation (11), the solution to the buyers’ problem writes:

$$\pi(t) = \begin{cases} 
0 & \text{if } G(t) < 0 \\
\pi \in [0, 1] & \text{if } G(t) = 0 \\
1 & \text{if } G(t) > 0
\end{cases}$$

where

$$G(t) := \mu^1_H(t)r^2_H - (\mu^1_H(t) + S(1-q))r^1_H + \mu^2_L(t)r^2_L(t) + \mu^1_L(t)r^1_L(t)$$

15
In the expression of $G(t)$, we may classify endogenous variables into two different groups. The reservation values for lemons $\{r^i_L(t)\}_{i=1,2}$ are forward-looking increase with future trading opportunities $\{\pi(t+l)\}_{l=1,\infty}$. Indeed, high future liquidity facilitates the resale of lemons. I simply call this dynamic complementary in traders’ decision the \textit{resale effect}: higher future trading prices increase current buyers’ marginal willingness to buy an asset. Masses of traders $\{\mu^i_a(t)\}_{i=1,2, a=H, L}$ however are backward looking variables that depend on past trading decisions. In particular, the mass of $(\tau^1, H)$ types evolves according to law of motion (13):

$$
\mu^1_H(t) = \left[1 - \gamma - \lambda^S(t - 1)(1 - 2\gamma)(1 - \pi(t - 1))\right] \mu^1_H(t - 1) + \gamma \mu^2_H(t - 1)
$$

$$
= \gamma S + (1 - 2\gamma)(1 - \lambda^S(t - 1)\pi(t - 1)) \mu^1_H(t - 1)
$$

where I used equation (3) to replace $\mu^2_H(t)$. Hence, a liquid market ($\pi(t - 1)$ high) decreases the current share of $H$ quality assets for sale $\mu^1_H(t)$ and ultimately the probability $\pi(t)$ of a high offer. We call \textit{composition effect} this substitutability in traders decision across time, which is the source of liquidity fluctuations. We now introduce the definition for stationary equilibria.

\textbf{Definition 2 (Stationary Equilibrium).} \textit{A long-run equilibrium of the model is stationary if $\exists T \in \mathbb{N}_+$ such that:}

$$
\forall t, \quad \pi(t + T) = \pi(t)
$$

\textit{The period of the equilibrium is the lowest $T$ such that condition (14) holds. When $T \geq 2$, the equilibrium is a cycle}

Observe that steady states belong to this class of equilibria as stationary equilibria of period $T = 1$. I first characterize those in Section 3.1 and later consider higher order cycles for $T \geq 2$ in Section 3.2

\subsection*{3.1 Steady State Equilibria}

As I showed, a steady state equilibrium can be fully characterized by the value of the endogenous variable $\pi$, the (constant) probability of a high offer.
Proposition 1. In any equilibrium \( \lambda^S = 1 \), that is sellers meet a buyer for sure. There exists two values \((q, \bar{q}) \in [0, 1]^2\) such that:

i) When \( q \leq \frac{1}{2} \) a steady state equilibrium with \( \pi = 0 \) exists.

ii) When \( q \geq \frac{1}{2} \) a steady state equilibrium with \( \pi = 1 \) exists.

iii) For \( q \in (\min\{\frac{1}{2}, \bar{q}\}, \max\{q, \bar{q}\}) \), there exists an equilibrium in mixed strategy with \( \pi(q) \in (0, 1) \).

These are the only possible steady state equilibria.

In equilibrium, enough buyers enter so that no seller is left unmatched. Proposition 1 states that there always exists a steady state equilibrium and that several equilibria coexists when \( q > \frac{1}{2} \). The expressions for the thresholds \((q, \bar{q})\) are the following:

\[
q = \frac{2a}{(1 - \delta)(\tau - 1)d_H + 2a}
\]

\[
\bar{q} = \frac{a - b}{\gamma(\tau - 1)d_H + a - b}
\]

where

\[
a = (1 - \delta)(d_H - \tau d_L) + \gamma \delta (\tau + 1)(d_H - d_L) + \gamma (1 - \delta)(\tau - 1)d_L
\]

\[
b = \delta \gamma (1 - 2\gamma)(\tau - 1)d_L
\]

Hence there are multiple steady state equilibria \(^{13}\) when

\[
\frac{2\gamma}{1 - \delta} > \frac{a - b}{a}
\]

Multiplicity stems from the dynamic strategic complementarities in buyers’ decision to pool sellers. Indeed, buyers take into account the endogenous re-sale price for the asset when choosing their offer. A quick examination of condition (17) above shows that equilibrium multiplicity is more likely when \( \delta \) is high. This is intuitive since

\(^{13}\)When the \( L \) asset is a pure lemon \((d_L = 0)\), equation (17) simplifies to \( 2\gamma > 1 - \delta \), a condition similar to that obtained by Chiu and Koeppl (2014) in a continuous time environment.
the strength of this resale effect depends on the weight attached to future payoffs. Let us now write the value function \( \{ v_i^j \}^i=1,2 \) for a lemon holder.

\[
v_1^L = \pi \frac{1}{1 - (1 - \pi)\delta} v_1^H + (1 - \pi) \left[ 1 - (1 - \pi)\delta + \delta(1 - \pi)\gamma(\tau + 1) \right] d_L \tag{18}
\]

\[
v_2^L = \pi \frac{1}{1 - (1 - \pi)\delta} v_1^H + (1 - \pi) \left[ 1 - (1 - \pi)\delta \right] \tau + \delta(1 - \pi)\gamma(\tau + 1) \right] d_L \tag{19}
\]

Agents’ valuation for a lemon is a weighted average of the pooling price and the holding value of a lemon. Both \( v_1^L \) and \( v_2^L \) increase in \( \pi \) as lemon holders extract larger information rents. Let us now write down the equilibrium mass of traders as a function of \( \pi \). We have

\[
\mu_1^L = \gamma S(1 - q)
\]

\[
\mu_1^H = \frac{\gamma}{\pi + 2\gamma(1 - \pi)} S q
\]

\[
\mu_2^L = (1 - \gamma) S(1 - q)
\]

\[
\mu_2^H = \frac{(1 - \gamma)\pi + (1 - \pi)\gamma}{\pi + 2\gamma(1 - \pi)} S q \tag{20}
\]

As an illustration of the composition effect, the share of \((\tau^L, H)\) agents depend negatively on \( \pi \). Indeed, high market liquidity reduces the supply \( \mu_1^H \) of \( H \) assets to the market.

In the following, I assume that (17) does not hold, that is there is a unique steady state for any value of \( q \). Equilibrium entry of non-owners \( \mu^B(t) \) is pinned down by the zero profit condition. We can write the free-entry condition directly as a function of \( q \).

\[
-\kappa + \frac{1}{\mu^B(q)} \left[ \mu_1^L \left( r_2^L(q) - r_1^L(q) \right) + \pi^*(q) G(\pi^*(q)) \right] = 0
\]

We obtain

\[
\mu^B(q) = \frac{1}{\kappa} \begin{cases} 
\frac{(\tau - 1)d_L \gamma S(1 - q)}{1 - \delta(1 - 2\gamma)} & \text{if } q \leq \bar q \\
\frac{(\tau - 1)d_L \gamma S(1 - q)}{1 - \delta(1 - \pi^*(q)(1 - 2\gamma))} & \text{if } q \in [\bar q, \bar q] \\
(\tau - 1)d_L \gamma S(1 - q) + G(q) & \text{if } q \geq \bar q 
\end{cases} \tag{21}
\]
Endogenous gains from trades drive equilibrium entry for non-owners. When $q \leq \bar{q}$, only $L$ assets are traded. Hence buyers may extract all the surplus from trading with $(\tau^1, L)$ sellers. When $q \in [q, \bar{q}]$, buyers must concede information rents to $(\tau^1, L)$ sellers, driving down profit and equilibrium entry. In this region, global competition drives down individual profit although buyers act as local monopsonists. Indeed, a buyer could obtain a higher profit if other traders would make low price offers only. Finally, when $q \geq \bar{q}$, the net gain from trading the $H$ asset $G(q)$ becomes positive, driving up profit and equilibrium entry.

3.2 Deterministic Liquidity Cycles

We now study cycles, that are stationary equilibria with period $T$ equal to or higher than 2. For simplicity, we focus on deterministic cycles\(^{14}\) where $\pi(t) \in \{0, 1\}$ for any $t$. Lemma 4 provides some guidance as to the nature of cycles we should expect.

**Lemma 4.** In a deterministic cycle, if $\pi(t) = 1$, then $\pi(t + 1) = 0$.

Lemma 4 states that there cannot be two consecutive periods where buyers offer a pooling price in a cycle. If it were the case, the proof shows that buyers would prefer pooling in every period preceding $t$, a consequence of the resale effect. For instance, a sequence of period 3 with $(\pi(t), \pi(t + 1), \pi(t + 2)) = (1, 1, 0)$ cannot be an equilibrium while $(1, 0, 0)$ is a possible candidate. This result stems from the asymmetry in the composition effect dynamics. When only $(\tau^1, L)$ types trade (that is $\pi(t) = 0$), the selling pressure from $H$ quality owners slowly increases because the (exogenous) type switching process is persistent. Offering a pooling price however (that is $\pi(t) = 0$), decreases endogenously the mass of $(\tau^1, H)$ agents, and the quality decline in the pool of sellers is much steeper. Building on Lemma 4, I first characterize equilibrium cycles and give conditions for existence in Proposition 2. As before, I guess and verify that $\lambda^S(t) = 1$ in equilibrium, that is sellers meet a buyer with probability 1. Let us now denote without ambiguity by 0 the periods $t, t + T, t + 2T$ where $\pi(t) = 1$. Labels 1, 2, ..., $T - 1$ are for the intermediate periods. In the following, the additional subscript $T$ captures the dependence of endogenous

\(^{14}\)Given that the model may have stochastic steady states, it can also generate stochastic cycles. Deterministic cycles do not entail any loss of generality as to the forces at play in the model.
variables on the cycle period when relevant. Using $\lambda_S(t) = 1$, together with equation (12) yields:

$$\mu_1^L(t) = \mu_1^L = \gamma(1-q)S$$
$$\mu_2^L(t) = \mu_2^L = (1-\gamma)(1-q)S$$

Let us now turn to type $(\tau^1, H)$ traders. Since $\pi(0) = 1$, and $\pi(t) = 0$ for $t = 1, \ldots, T - 1$, we have

$$\mu_{H,T}^1(t) = \gamma Sq + 1_{\{t \neq 1\}}(1 - 2\gamma)\mu_{H,T}^1(t - 1)$$

The mass of $(\tau^1, H)$ agents $\mu_{H,T}^1(t)$ is a sufficient statistic for the quality in the pool of sellers in period $t$. The first terms is the mass of $(\tau^1, H)$ sellers after period 0. The second terms captures the accumulation of the selling pressure during the trough.

The last variables of interest are the reservation values for a lemon $\{r_{L,T}^i(t)\}_{i=1,2}$. Observe that lemon owners sell at pooling price $r_1^H$ in period 0. We thus have

$$r_{L,T}^i(t) = \begin{cases} 
\tau^id_L + \delta r_1^H & \text{if } t = T - 1 \\
\tau^id_L + \delta \left[ (1-\gamma)r_{L,T}^i(t + 1) + \gamma r_{L,T}^{-i}(t + 1) \right] & \text{if } t = 0, \ldots, T - 2
\end{cases} \quad (22)$$

Straightforward computations in the proof of Proposition 2 show that:

$$r_{L,T}^1(t) := \left[ \frac{1 - \delta^{T-t}}{1 - \delta}(\tau + 1) - \frac{1 - (\delta(1-2\gamma))^{T-t}}{1 - \delta(1-2\gamma)}(\tau - 1) \right] \frac{d_L}{2} + \delta^{T-t}r_1^H \quad (23)$$
$$r_{L,T}^2(t) := \left[ \frac{1 - \delta^{T-t}}{1 - \delta}(\tau + 1) + \frac{1 - (\delta(1-2\gamma))^{T-t}}{1 - \delta(1-2\gamma)}(\tau - 1) \right] \frac{d_L}{2} + \delta^{T-t}r_1^H$$

In particular, the reservation value $r_{L,T}^1(t)$ of $(\tau^1, L)$ sellers increases during a through as the peak period looms. Since $r_{L,T}^1(t)$ is also the price buyers will offer in period $t \in \{1, \ldots, T - 1\}$, the cycle exhibits non-trivial price dynamics. We can now state the main Proposition of the paper.

**Proposition 2.** Let $T \geq 2$. An equilibrium cycle with period $T$ exists if and only if

$$\frac{1 - (1-2\gamma)^T}{1-(1-2\gamma)^{T-1}} \geq r_1^H - r_{L,T}^2(0) + \gamma (r_{L,T}^2(0) - r_{L,T}^1(0)) \left( \frac{1}{1-\delta}r_1^H - \tau d_L + \gamma (\tau - 1)d_L \right)$$

(E_T)
and
\[ q_T \leq q \leq \bar{q}_T \]
where the thresholds \((q_T, \bar{q}_T)\) are defined in the appendix.

The proof of Proposition 2 and the following Corollary are in the Appendix.

**Corollary to Proposition 2:**
There exists \( T \in \mathbb{N}_+ \) such that \((E_T)\) does not hold for \( T > \bar{T} \). In addition \( \bar{q}_T \) and \( q_T \) are decreasing in \( T \) with \( \bar{q}_2 = \bar{q} \) and \( \lim_{T \to \infty} q_T = q \).

Proposition 2 and its corollary show that the model can generate equilibrium cycles with period up to a finite number \( \bar{T} \). Cycles may not have arbitrarily long periods because of the resale effect. Indeed, in the last period \( T - 1 \), resales prices \( \{r_{i, T}(T - 1)\}_{i=1,2} \) are high but buyers should find it optimal to propose a low price, that is \( \pi(T - 1) = 0 \). The opposite holds in period 0 as they expect many periods with low liquidity but must still be willing to offer a high price. This behavior is optimal only if the composition effect is strong enough so that the quality in the pool of sellers increases significantly between \( T - 1 \) and 0. However, as the cycle period \( T \) goes up, improvement in the pool quality between these two periods decreases as measured by the difference:

\[
\mu_{H,T}^1(0) - \mu_{H,T}^1(T - 1) = \frac{1}{2} \left[ (1 - 2\gamma)^{T - 1} - (1 - 2\gamma)^T \right]
\]

Unlike the composition effect, the resale effect does not depend on the length of the cycle but merely on the discounting between two consecutive periods. Hence, short frequency cycles are more likely.

Proposition 2 thus shows that multiple stationary equilibria can coexist with different period together with a steady state. Intuitively, when \( q \) takes an intermediate value, there must be some degree of illiquidity. If buyers were to propose a high price \( r_{H}^1 \) in every period, the average quality in the pool of sellers would be too low and buyers would make a loss. Thanks to the composition effect, some illiquidity in period \( t \), that is \( \pi(t) < 1 \) increases the quality in the pool in period \( t + 1 \). In a steady state, illiquidity spreads out evenly across periods as sellers face a constant
probability to receive a high price. In a cycle, illiquidity fluctuates with the average quality in the pool of sellers.

Figure 1: A 3 Period Cycle ($\delta = 0.35, d_L = 1, d_H = 4, \tau = 2, \gamma = 0.01$)

Prices
In an equilibrium cycle $\pi(t) = 1_{t=0}$. We can thus write the asset price in period $t$ as follows:

$$p(t) = \begin{cases} r^1_H & \text{if } t = 0 \\ r^1_L(t) & \text{otherwise} \end{cases}$$

(24)

Figure 1 illustrate these dynamics in a 3 period cycle. As the peak $t = 0$ gets close, the value of a $L$ asset goes up. During the trough (periods $t = 1, 2$), the price increases to reflect $(\tau^1, L)$ sellers’ outside option of waiting for better offer in $t = 3$. Buyers’ willingness to propose a high price for a lemon rests on the ability to resell it in the future. In periods $t = 1, \ldots, T-1$, the $L$ assets trades above the full information price since

$$r^1_L(t) > \frac{1 - \delta + \delta \gamma (\tau + 1)}{(1 - \delta)[1 - \delta(1 - 2\gamma)]} d_L = v^1_{L, aut}$$

while only $L$ assets are traded. Although buyers do not face any uncertainty about the quality of the asset they acquire, this premium ultimately reflects future infor-
mation rents conceded at the peak of the cycle. As in the infamous “hot potato” story, agents know they purchase bad assets at inflated prices and hope to resell them at an even higher price in the future.

**Equilibrium Entry**

In a cycle, entry patterns of non-owners exhibit interesting dynamics. Using the zero profit condition for buyers, we obtain:

\[ \mu_B(t) = \frac{\mu_L^1 \left( r_L^2(t) - r_L^1(t) \right) + \pi(t)G(t)}{\kappa} \]  

(25)

The first term in the denominator captures the profit extracted from type \((\tau^1, L)\) sellers. These profits decrease from period 0 onward because \((\tau^1, L)\) sellers’ outside option improves as the peak of the cycle looms. The second terms captures the additional gains a buyer may receive from a pooling offer \(r_H^1\). At the peak of the cycle, both these terms are maximized and so is equilibrium entry \(\mu_B(0)\). Hence \(\mu_B(t)\) decreases during the trough of a cycle before a surge at the peak. Trade volumes during the trough remain constant and equal to \(\mu_L^1 = \gamma S q\), the mass of \((\tau^1, L)\) sellers.

4 Welfare and Policy

In order to restore liquidity in dysfunctional markets, central banks like the Fed took an active role as buyers of last resort during the financial crisis. In this section, I analyze the desirability of an asset purchase program whereby a benevolent government buys and removes lemons out of the market. In particular, we want to know whether cyclical economies warrant different policy interventions. For simplicity, we suppose that only 2 period cycles may emerge in equilibrium, that is \((E_T)\) holds only for \(T = 2\) and \(q \in [\tilde{q}_2, \bar{q}_2]\).

The government is a large agent with deep pockets\(^{15}\) and fully persistent type \(\tau^1\). This assumption captures the government lower usage for the assets than agents

\(^{15}\)This assumption is rather innocuous as illiquidity does not stem from a shortage of funds or “cash in the market” pricing in this model. See Faria-e-Castro et al. (2015) for an analysis of the availability of funds on the design of government interventions with adverse selection.
in the market. Figure 2 describes the modified timeline of a period with government intervention. A government intervention is a pair \( (P^g(t), S^g(t)) \in \mathbb{R}_+^2 \) whereby the government stands ready to buy \( S^g(t) \) assets at unit price \( P^g(t) \) at the beginning of period \( t \). The government does not resell the assets he purchases. We let \( o^i_a(t) \in [0,1] \) denotes the (possibly random) decision variable for type \((\tau^i, a)\) where \( o^i_a(t) = 1 \) means opting in and \( o^i_a(t) = 0 \) opting out. The benevolent government maximizes the sum of the agents and his own utility from holding the asset he purchases. The objective function of the government thus writes:

\[
\max_{(P^g(t), S^g(t))} W(t) = \sum_{i,a} (1 - o^i_a(t)) \mu^i_a(t) v^i_a(t) + \sum_{i,a} o^i_a(t) \mu^i_a(t) \frac{d_a}{1 - \delta}
\]

s. to

\[
S^g(t) = \sum_{i,a} o^i_a(t) \mu^i_a(t)
\]

\[
o^i_a(t) = \begin{cases} 
0 & \text{if } P^g(t) < v^i_a(t) \\
1 & \text{if } P^g(t) > v^i_a(t) 
\end{cases}
\]

where \( v^i_a(t) \) is the market utility of type \((\tau^i, a)\) in the continuation equilibrium post intervention. The quantity \( \mu^i_a(t) \) is the mass of type \((\tau^i, a)\) coming into period \( t \)

---

\[16\] Exit strategies are a current concern of policy makers. For instance, Central Banks may think about re-selling previously acquired assets in the market. Exit strategies are relevant provided that market conditions eventually improve other times, which is not the case in my environment. The issue is certainly interesting and I leave it for future research.
(hence before intervention). The first constraint that the intervention must be fully subscribed is without loss of generality since the government may always adjust $S^g(t)$. The last equality is the participation constraint. As a difference with standard mechanism design, the outside option $v^i(t)$ for agent $(\tau^i, a)$ typically depends on the intervention. Indeed, the market may become more liquid after the intervention, raising the opportunity cost of opting in. With transferable utility, the welfare criterion is the sum of the market utility for agents opting out and the government utility from holding the assets of participating agents. Building in agents participation decisions, we can rewrite (26) as follows:

$$W(t) = \sum_{i,a} \mu^i(t)v^i_a(t) - \sum_{i,a} o^i_a(t)\mu^i_a(t) \left( v^i_a(t) - \frac{d_a}{1 - \delta} \right)$$  \hspace{1cm} (27)

An intervention thus trades-off the benefits from increased liquidity in the market with the losses incurred by the government when purchasing assets. Observe that it is optimal to attract only $L$ assets. Purchasing $H$ assets would reduce market liquidity and increase misallocation costs.

4.1 Full Trade Intervention

The general solution to Problem (26) is difficult to characterize. Indeed, the continuation equilibrium after an intervention typically exhibits non-stationary and potentially cyclical dynamics. Instead, we focus on an intervention that restores full trade in the continuation equilibrium and analyze whether it is desirable\textsuperscript{17}.

Lemma 5 (Full Trade Intervention). The following intervention at $t_{inv}$ restores full trade:

$$P^g_f = r^1_H, \quad S^g_f(q) = \frac{S(q - \bar{q})}{\bar{q}}$$  \hspace{1cm} (28)

that is $\pi(t) = 1$ for $t \geq t_{inv}$ is an equilibrium.

Provided that the intervention only attracts lemons, the value for $S^g_f$ implies that

\textsuperscript{17}Intuitively, if some degree of intervention is desirable, a government should try to restore full trade after the intervention. I cannot prove this conjecture however because problem (26) is non-linear in the size of the intervention.
the post-intervention share of $H$ assets is $\bar{q}$. This is the threshold above which the unique stationary equilibrium exhibits full trade. In the proof, we show by backward induction that full trade can be sustained for any $t \geq t_{\text{inv}}$. Lemon holders outside option $v^*_L(t)$ is then equal to $r^*_H$ the market price offered by buyers post intervention. Observe that $H$ asset holders may also be willing to participate in the program at this price. This indeterminacy disappears if we introduce arbitrary small frictions in the matching process. I formalize this argument in Appendix B. Indeed, we would then obtain $v^*_L(t) < r^*_H$ and the government could attract only $L$ asset holders. I now refer to $(P^g_f, S^g_f)$ defined in (28) as the full trade intervention. The next proposition shows that a full trade intervention is indeed welfare enhancing.

**Proposition 3.** Let $d_L = 0$ and $q \in [q_2, \bar{q}_2]$. The full trade intervention $(P^g_f, S^g_f)$ increases welfare whether the economy is in a cycle of period 2 or in a steady state.

With an intervention, the government internalizes the permanent liquidity gains from taking lemons out of the market. Private traders however always seek to resell lemons once they acquired it. The decomposition in equation (27) helps to understand the cost benefit analysis. The government must buy $S^g_f(q)$ lemons at price $P^g_f = r^*_H$ and keep them out of the market. However, the increase in liquidity benefits every holder of a $L$ asset in the economy who have a total mass of $S(1 - q)$, not only agents remaining in the market. Indeed, agents participating in the program enjoy the high intervention price. Proposition 3 states these benefits overcome the cost born by the government. Observe that when $d_L = 0$, our assumption about the government type is without loss of generality as no type would value holding a lemon. Interestingly, the proof shows that the size of welfare gains depends on the nature of the equilibrium absent intervention. If the economy is in a cycle, the net welfare gain from a full trade intervention is

$$W_f - W_0 = S(1 - q)r^*_H(1 - \delta) - S^g_f(q)r^*_H$$

Absent intervention, $L$ asset owners would enjoy price $r^*_H$ only next period which makes the gains proportional to $(1 - \delta)$. The same expression when the economy is

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18In this economy, $H$ asset owners do not benefit from the intervention since monopsony pricing leaves them at their reservation utility. We show in the next Section that this is also the case if buyers compete in prices.
in a steady state yields:

\[ W_f - W_\emptyset = S(1 - q)r_H^1 \left( 1 - \frac{\pi(q)}{1 - \delta(1 - \pi(q))} \right) - S_f^q(q)r_H^1 \]

In this case, the intervention raises the probability of a high offer from \( \pi(q) \) every period to 1. Since \( \pi(q) \to 1 \) as \( q \to \bar{q} \), the per-trader benefits vanish in that case. This is not the case in a cycle as the pre-trader benefit is independent of \( q \).

### 4.2 Costly Government Funds

We now argue that this difference matters when public funds are costly. Suppose indeed that the government must incur an extra proportional cost \( \theta \geq 0 \) to raise funds. A transfer from the government to asset owners now has a negative net value of \( \theta \) per unit transferred. The cost of intervention (28) thus raises from \( S_f^q(q)r_H^1 \) to \((1 + \theta)S_f^q(q)r_H^1\). We can now state the Corollary to Proposition 3.

**Corollary to Proposition 3**

Let \( d_L = 0, q \in [\bar{q}, \tilde{q}] \), and \( \theta \) be the extra cost of funds for the government. If \( \theta \geq \hat{\theta} \), the full trade intervention decreases welfare if the economy is in steady state but increases welfare in a cyclical economy for \( q \in [\hat{q}(\theta), \bar{q}] \) where

\[ \hat{\theta} := \frac{2\gamma}{1 - \delta - 2\gamma}, \quad \hat{q}(\theta) = \frac{\theta + \delta}{1 + \theta - \bar{q}(1 - \delta)\bar{q}} \]

We stressed earlier that benefits from a full-trade intervention differ across no-intervention equilibria. When \( q \) is inferior but close to \( \bar{q} \), illiquidity is very mild in a steady state since \( \pi(q) \to \pi(\bar{q}) = 1 \). Hence, the government does not wish to intervene if the cost of funds \( \theta \) is too high. In a cycle however, illiquidity is severe in a trough as \( \pi = 0 \). Hence the benefits from intervention may still overcome the costs. The corollary thus shows that an asset purchase program is all the more desirable when the economy exhibits cycles. Hence, the nature of equilibria absent intervention matters for the design of government policies although stability itself is not an objective of the government. However, fluctuations entail a high degree of illiquidity in the trough of a cycle which motivates the intervention.
5 Liquidity and Market Structure

As part of the current policy response to the financial crisis, regulators pointed at the very structure of OTC markets as a source of illiquidity and unstability\textsuperscript{19}. Ongoing reforms notably mandate central clearing of OTC instruments such as derivatives and swaps with a stated objective to increase transparency and competition.

In the OTC market, there are indeed two important frictions. First, meetings between buyers and sellers are random and resources are wasted on useless meetings. Second, ex-post bargaining generates local monopsony power and decreases trading volume since buyers reduce demand to extract rent. This section introduces a slight variation of the model where buyers can post and commit to prices before meeting a counterparty. This alternative trading platform generates more competition and provides pre-trade information, two desirable features according to the recent regulatory proposals. Surprisingly, higher competition and trade information might significantly lower volume and welfare in the presence of dynamic adverse selection.

5.1 Market Structure

In this environment, buyers post prices\textsuperscript{20} before meetings $p \in \mathbb{R}^+$ and sellers direct their search towards corresponding sub-markets.

Matching

Formally, an exchange is a continuum of markets $p \in \mathbb{R}^+$ where $p$ is a price for the asset and $\theta(t, p)$ the buyer to seller ratio in that market. Buyers and sellers choose the market they want to trade in, taking $\theta(t, p)$ as given. The probability for a seller to meet a buyer in market $p$ is

$$\lambda^S(t, p) = \min \{\theta(t, p), 1\}$$

\textsuperscript{19}On its website, the IMF referring to OTC markets explains that “some types of market arrangements can very quickly become disorderly, dysfunctional, or otherwise unstable”.

\textsuperscript{20}There is no loss in generality in assuming that buyers post prices and not contracts since rationing plays the same role as probabilities of trade.
Symmetrically, a buyer meets a seller with probability

$$\lambda^B(t, p) = \min \{\theta(t, p)^{-1}, 1\}$$

Hence, for a seller, $\theta(t, p)$ measures rationing in market $p$. Importantly, sellers cannot sell their asset in two different markets $p$ and $\hat{p} \neq p$ in the same period. Hence, an attempt to sell at a price $p$ is a commitment not to try and sell at a different price $p'$. This commitment may act as a signal of quality if sellers expect rationing at high prices\(^{21}\). As in the OTC market, agents have several opportunities to trade over time: exclusivity only restricts intra-period trades.

### Sellers Problem

The problem faced by asset owner $(\tau^i, a)$ can thus be written:

$$v^i_a(t) = \max_{p \in \mathbb{R}^+} v^i_a(t, p)$$  \hspace{1cm} (29)

where

$$v^i_a(t, p) = \lambda^S(t, p)p + (1 - \lambda^S(t, p))r^i_a(t)$$

is the utility derived in market $p$. Asset owners may always choose a very high price where $\theta(t, p) = 0$ if they do not want to trade.

### Buyers Problem

In every market $p \in \mathbb{R}_+$, let $\gamma^a_i(t, p) \in [0, 1]$ denote buyers’ belief about the share of type $(\tau^i, a)$ in market $p$. Given these beliefs, the net gain from trading in market $p$ writes:

$$v^2_0(t, p) = -\kappa + \lambda^B(t, p) \left[ (\gamma^2_H(t, p) + \gamma^1_H(t, p)) r^2_H(t) + (\gamma^2_L(t, p) + \gamma^1_L(t, p)) r^2_L(t) - p \right] + \delta v^2_0(t+1, p)$$  \hspace{1cm} (30)

We let $\mu^B(t, p)$ be the measure of buyers in market $p$ with measure $\mathcal{P}(t)$. A buyer

\(^{21}\)Models of competitive adverse selection such as Gale (1996), (Guerrieri and Shimer, 2014), Chang (2014) also impose this exclusivity assumption, which leads to separation. Kurlat (2013) allows for non-exclusivity in a static model where heterogeneously informed buyers may screen through selection rules. For an analysis of non-exclusivity with a strategic equilibrium concept, see Biais et al. (2000) and Attar et al. (2011).
ultimately cares about the quality of the asset $a$ not the type $\tau^i$ of the seller.

**Law of Motion**
The law of motion for types $(\tau^1, a)$ writes:

$$\mu^1_a(t + 1) = \left[ 1 - \gamma - (1 - 2\gamma) \int \gamma^1_a(t, p) \lambda^B(t, p) \mu^B(t, p) dp \right] + \gamma \mu^2_a(t)$$  \hspace{1cm} (31)

The expression is similar to the one derived for OTC markets except that agents might visit different markets $p$ with different trading probabilities

**Beliefs**

On markets where trade takes place, beliefs $\{\gamma^i_a(t, p)\}_{i=1}^{1,2, a=L,H}$ about seller’s type are pinned down by the physical proportion of sellers choosing this market. A complete description of the exchange requires buyers to hold beliefs about inactive markets as well. Without a refinement on beliefs, many pessimistic equilibria could be sustained if buyers expect the $L$ asset to be supplied in inactive markets. We follow Gale (1996) to specify out of equilibrium beliefs. Point 1 of the definition formalizes this refinement and adapts Cho and Kreps (1987) intuitive criterion to a competitive environment. On inactive markets, buyers expect to see the type of sellers who finds it more profitable to deviate to that market. This refinement is standard in the competitive search literature and we refer to the papers mentioned above for a more extensive discussion.

**Definition 3** (Exchange Equilibrium). For an initial distribution $\{\mu^i_a(0)\}_{a=L,H}^{i=1,2}$, an Equilibrium of the Exchange is given for any $t$ by value functions $\{v^i_a(t)\}_{a=L,H}^{i=1,2}$, distributions $\{\mu^i_a(t)\}_{a=L,H}^{i=1,2}$ a measure $\mu^B(p, t)$ with support $\mathcal{P}(t)$, a rationing function $\theta(t, p) : \mathbb{R}^+ \mapsto \mathbb{R}^+ \cup \{\infty\}$ and belief function $\gamma(t, p) : \mathbb{R}_+ \mapsto \Delta^4$ such that

---

22 The formula above seems convoluted but economizes on notation as we do not need to introduce measure of sellers $\mu^i_a(t, p)$ in the market. We would have

$$\lambda^S(t, p) \mu^i_a(t, p) = \lambda^B(t, p) \mu^B(t, p) \gamma^i_a(t, p)$$

23 In the OTC market, there is only one trading place to visit or one “active market”. Hence physical proportion of traders pin down beliefs, as in the open markets $\mathcal{P}$ here.
1. Buyers optimality and free-entry. For all $p \in \mathbb{R}^+$, $v^i_0(t, p) \leq 0$ with equality if $p \in \mathcal{P}(t)$.

2. Sellers optimality. For all $p \in \mathbb{R}^+$, $i = 1, 2$ and $a \in \{L, H\}$, $v^i_a(t) \geq v^i_0(t, p)$ with equality if $\theta(t, p) \leq \infty$ and $\gamma^i_a(t, p) > 0$.

3. Market Clearing. For $i = 1, 2$ and $a = L, H$

$$\int_{\mathcal{P}(t)} \frac{\gamma^i_a(t, p)}{\theta(t, p)} \mu^B(t, p) dp \leq \mu^i_a(t)$$

with equality if $v^i_a(t) > r^i_a(t)$.

4. Law of motion : $\{\mu^i_a(t)\}_{i=1,2, a=L,H}$ verify (31) and (3).

Except for the dynamic part in Point 4, the definition follows closely Guerrieri et al. (2010). Point 1 translates free entry into buyers’ zero profit condition. Active markets $\mathcal{P}(t)$ are those where buyers actually search. Point 2 formalizes the requirement that sellers search in the market(s) which maximizes their utility. In addition, on markets where $\theta(t, p) < \infty$, buyers should expect to see sellers who are indifferent between that market and their optimal choice. This is formally the refinement we discussed above. Point 3 ensures that supply on active markets is consistent with buyers beliefs. Observe that, some buyers might find it optimal not to trade ($v^i_a(t) = r^i_a(t)$) in which case they should supply their asset on inactive markets where $\theta(t, p) = 0$. Finally, point 4 says that the mass of owners obtains from the optimal trading decision in the previous period.

5.2 Equilibrium

The following Lemma states that reservation values are ranked in the same way as on the OTC market.

Lemma 6. For all $t$,

$$r^1_L(t) \leq r^2_L(t) < r^1_H(t) \leq r^2_H(t)$$

(32)

I omit the proof since it follows the same argument as that of Lemma 2 and does not rely on the price formation process. Again, agents with a more valuable
asset in the static sense cannot be worse off as they can always mimic lower types. While Lemma 6 is easy to prove, it is crucial for Proposition 4. With exclusivity, the competitive search equilibrium is typically separating\textsuperscript{24}. Sellers with a higher type in the sense of Lemma 6 would accept (more) rationing to trade at higher prices and signal their quality. With two dimensions of private information however, we show that rationing for H quality assets is so severe that they do not trade. Define first

\begin{align*}
  r^1_L & = \frac{[\delta \tau + (1 - \delta)]d_L - \delta[1 - \gamma - \delta(1 - 2\gamma)]\kappa}{1 - \delta} \\
  r^2_L & = \frac{\tau d_L - \delta \gamma \kappa}{1 - \delta}
\end{align*}

(33) (34)

**Proposition 4.** There exists a unique stationary exchange equilibrium where \( \mathcal{P}(t) = \{p_L\} \) and \( p_L = r^2_L - \kappa \). Only \((\tau^1, L)\) owners sell in equilibrium. Value functions are given by

\[
\begin{cases}
  v^1_L(t) = p_L \\
  v^2_L(t) = r^2_L
\end{cases}
\quad
\begin{cases}
  v^1_H(t) = r^1_H \\
  v^2_H(t) = r^2_H
\end{cases}
\]

The Rationing function \( \theta \) satisfies

\[
\theta(t, p) = \begin{cases}
  0 & \text{if } p < p_L \\
  \frac{p_L - r^1_L}{p - r^2_L} & \text{if } p \in [p_L, r^2_L] \\
  0 & \text{if } p > r^2_L
\end{cases}
\]

The belief function is

\[
(\gamma^1_L, \gamma^1_H, \gamma^2_L, \gamma^2_H)(t, p) = \begin{cases}
  (1, 0, 0, 0) & \text{if } p \in [p_L, r^2_L] \\
  (0, 0, 1, 0) & \text{if } p > r^2_L
\end{cases}
\]

\textsuperscript{24}Chang (2014) obtains pooling with 2 dimensions of private information because these 2 dimensions can be collapsed to one payoff relevant statistic for sellers. In my model, there is a strict ranking between reservation values which blocks pooling.
The masses of traders are

\[
\begin{align*}
\mu_1^L &= \gamma S(1 - q) \\
\mu_2^L &= (1 - \gamma)S(1 - q)
\end{align*}
\]

while equilibrium entry is \( \mu^B = \mu_1^L \)

Proposition 4 shows that high quality asset owners may never sell their asset\(^{25}\). These gains from trade are never realized because buyers expect to see \( L \) assets at any price. Let us indeed consider a market \( \hat{p} \in (r_1^L, r_2^L - \kappa) \). At this price \( \hat{p} \), \((\tau^1, H)\) owners accept to sell while \( \tau^2 \) non-owners accept to buy quality \( H \). However, from the belief function, since \( \hat{p} > r_2^L \), buyers only expect to find \( L \) assets. Given these beliefs, buyers would make a loss and do not post price \( \hat{p} \), that is \( \theta(t, \hat{p}) = 0 \). This conjectures is not overly pessimistic as it satisfies the refinement. Indeed,
\( v_2^L(t, \hat{p}) = v_2^L(t) = r_2^L \). However, since \((\tau^1, H)\) sellers are willing to sell at \( \hat{p} \), buyers could actually attract both \((\tau^2, L)\) and \((\tau^1, H)\) sellers at price \( \hat{p} \). The crucial part in the proof of Proposition 3 is that there cannot be pooling of \((\tau^2, L)\) and \((\tau^1, H)\) owners. The argument resembles cream-skimming deviation in strategic models. If pooling were to take place, there would be a market \( p' > \hat{p} \) with \( \theta(t, p') < \theta(t, \hat{p}) \) that would strictly benefit buyers and \((\tau^1, H)\) sellers. This logic stops only when \( \theta(p') = 0 \). Sellers’ ability to pick between different prices paves the way for this negative signaling spiral\(^{26}\).

As the discussion above suggests, the argument holds only with two dimensions of private information: asset quality and private valuation. When private valuations are common knowledge, only low valuations owners may sell. Then, \((\tau^1, L)\) and \((\tau^1, H)\) may trade in two different markets \( p_L \) and \( p_H \) if

\[
\theta(p_L)(p_L - r_1^L) \geq \theta(p_H)(p_H - r_1^L)
\]

that is the incentive constraint of \((\tau^1, L)\) types is satisfied. Hence, with one dimen-

---

\(^{25}\)The result is stronger since a liquidity freeze occurs whenever \((\tau^2, L)\) agents are in the market and not only in a stationary equilibrium. In addition \( \mu_2^L > 0 \) with probability 1 after the first period since \( \tau^2 \) agents acquire the asset.

\(^{26}\)A benevolent government could increase liquidity and welfare by taxing high price markets to block this spiral. I formalize this intuition in Appendix C
sion of private information, the Exchange equilibrium would implement the least cost separating allocation. However, when agents trade, high valuation agents naturally become the owners of lemons, generating this extreme form of adverse selection\(^{27}\). The reader familiar with models of adverse selection will observe that the presence of high valuation agents among sellers endogenously generates a no gap at the bottom condition.

In the OTC market as well, bi-dimensional private information reduces liquidity since buyers are less willing to propose a high price. However, when the equilibrium is pooling, the presence of \((\tau^2, L)\) sellers change the degree and not the nature of the adverse selection problem. There, the effect is not as dramatic though as it merely lowers the quality in the pool of sellers. Introducing competition with price posting comes at the expense of liquidity for the \(H\) quality asset. In OTC meetings indeed, bilateral monopoly protects buyers from cream-skimming deviations. I derive welfare implications of these different liquidity patterns in the next Section.

### 5.3 Welfare

We are interested in comparing welfare between the OTC market and the Exchange, \(W_{OTC}\) and \(W_E\) respectively. In the interest of space, I focus on the steady state equilibrium of the OTC market. I index the endogenous variables related to the Exchange equilibrium by \(E\) when relevant. In the Exchange, the stationary equilibrium is unique. Using the results from Proposition 4, we obtain

\[
W_E = \gamma S(1 - q)(r_{L,E}^2 - \kappa) + (1 - \gamma)S(1 - q)r_{L,E}^2 + \frac{1}{2}Sqr_{H}^1 + \frac{1}{2}Sqr_{H}^2
\]

In the Exchange, only \((\tau^1, L)\) sellers trade at price \(p_L = r_{L}^2 - \kappa\) while all other owners enjoy their reservation utility. Market shutdown in \(H\) assets generates important mis-allocation costs since half of these assets are held by low valuation \(\tau^1\) agents. In the OTC equilibrium, we can write the steady state welfare \(W_{OTC}\) as a function

---

\(^{27}\) Previous papers identified two cases where such freeze occurs. First, when non-owners are scarce, trading concentrates in the low quality good because \((\tau^1, L)\) are the most eager to sell. Second, market freeze occurs when the low quality asset is worthless, that is \(d_L = 0\). Proposition 4 requires none of these assumptions although the result ultimately relies on a similar no gap at the bottom condition.
of $\pi \in [0,1]$.

$$W_{OTC} = \gamma S(1 - q)v_L^1(\pi) + (1 - \gamma)S(1 - q)v_L^2(\pi) + \mu_H^1(\pi)r_H^1 + \mu_H^2(\pi)r_H^2$$

As $\pi$ increases, $L$ asset owners obtain more information rents and the allocation of $H$ assets improves. Since $\pi$ depends monotonically on $q$, the Proposition naturally follows:

**Proposition 5.** There exists $q_W$ in $[q, \bar{q}]$ such that welfare is higher in the OTC steady state equilibrium than in the Exchange equilibrium, that is

$$W_{OTC} \geq W_E \quad \text{if} \quad q \geq q_W$$

**Proof** $\triangleright$ Using the monotonicity of $W_{OTC}$ in $q$, it is enough to prove that the inequality holds strictly in $q = 1$. Observe first that $v_L^1(1) = r_H^1 > r_L^2,E$. Then we can conclude using $\mu_H^1(1) = \gamma Sq < 1/2 Sq$ and $r_H^2 > r_H^1$. $\triangleleft$

When $q$ is low, only lemons are traded both in the Exchange and in the OTC market ($\pi = 0$). However, price posting in the Exchange brings sellers and buyers together more efficiently. Indeed, total search costs go down with directed search which increases the overall gains from trade. More generally, without asymmetry of information, the Exchange would always deliver higher welfare than the OTC market. As $q$ increases, monopsonist buyers increase their offered price in the OTC market to attract ($\tau_1, H$) types. Buyers thus pool $H$ and $L$ asset sellers increasing the information rents for the later and improving the asset allocation. In the Exchange, $H$ assets never trade for any value of $q$ because the equilibrium is separating. Adverse selection always matters in the Exchange and illiquidity persists even for high values of $q$.

When $q$ is large, our results suggest that low valuation asset owners would not favor a reform to organize the OTC market as an Exchange. In any model of adverse selection, low types indeed prefer the pooling allocation over the separating outcome. In the pooling allocation of the OTC market, they benefit from an implicit transfer by the $H$ asset owners. These agents would not push for a reform either. Indeed, with the extreme form of illiquidity, they do not benefit from competition in
the Exchange. They enjoy the same utility in the OTC markets where monopsonist buyers offer their reservation value. Hence in this context, both types of agents weakly benefit from an OTC market structure. To put it otherwise, the Ex-ante Welfare criterion is aligned with Pareto efficiency.\(^{28}\)

6 Conclusion

This paper presented a theory of endogenous liquidity fluctuations based on asymmetry of information and re-trade in secondary markets. Liquidity increases with the average quality of the pool of sellers and the gains from re-selling a lemon. In a dynamic model, both these components are endogenous and may fluctuate over time. This economy features equilibrium cycles, where prices and trading volume vary in the absence of aggregate shocks. In a cycle, buyers rationally purchase lemons above their value, hoping to resell them in the future. The future increase in the quality of asset for sale supports dynamically this bubble component.

The economy is inefficient as traders do not internalize there impact on market liquidity. I study asset purchase programs by a benevolent government aimed at restoring full liquidity. The program trades off the permanent increase in market liquidity with a better pool of sellers with the cost of removing lemons out of the market. I show that such programs deliver higher welfare gains when the economy is in a cycle rather than in steady state, highlighting the cost of instability.

In the last part of the paper, I consider a variation of the model where buyers post prices before meeting. This comparison is motivated by the recent set of market reforms to increase competition and transparency in financial markets. Price posting indeed fosters competition and increases pre-trade information since sellers may direct their search. However, these forces may significantly reduce welfare in the presence of adverse selection. While price posting suppresses buyers’ monopsonist rents, directed search makes cream-skimming deviations possible. With bi-dimensional private information, high quality assets never trade. Hence, the role of

\(^{28}\)This result highlights the role of two dimensions of private information where the high type seller always (weakly) prefer the pooling allocation over the least cost separating one. This result would be weaker with one dimension of private information as the ex-ante welfare gain would not always yield a Pareto improvement.
competition and information on market liquidity ultimately appears ambiguous.

This analysis pleaves many interesting questions open for the analysis of markets with adverse selection. I considered one-time interventions whereby the government holds the assets he purchased forever. In practice, policy makers consider selling back these assets once market start functioning normally. The optimal design and timing of these exit strategies is a timely issue worth investigating. At a more theoretical level, my analysis presents an interesting, yet not fully satisfying argument on the role of competition and information in markets with adverse selection. In particular, it seems important to bring random search with ex-post bargaining and directed search with price posting within a single unified framework. Steps in this direction would allow to better endogenize the choice of trading institutions.

References


Appendices

A Proofs

A.1 Lemma 2

We want to show that for any \((\tau^i, \tau^j, a, a')\) such that \(\tau^i d_a \geq \tau^j d_{a'}\), we have \(r^i_a(t) \geq r^j_a(t)\). Using the buyers’ zero profit condition (Part 1 of Definition 1), we have \(r^i_a(t) = \tau^i d_a + \delta \bar{v}^i_a(t)\). Hence, we establish the following sufficient condition for the result: \(\bar{v}^i_a(t) \geq \bar{v}^j_a(t)\).

Observe first that the asset value can grow unbounded since every agent has at most \(e\) units of consumption good to spend on the asset. In particular, the following transversality condition holds for \(i = 1, 2\) and \(a = L, H\):

\[
\lim_{T \to \infty} \delta^T v^i_a(T) = 0 \tag{35}
\]

By optimality, agent \((i, a)\) obtains a higher utility than if he follows from \(t\) onwards the acceptance rule of agent \((j, a')\) for \(j \in \{i, -i\}\). That is

\[
v^i_a(t) \geq r^i_a(t) + \lambda^S(t) \sum_{p \in \Gamma(t)} \pi(t, p) \alpha^j_{a'}(t, p)[p - r^i_a(t)]
\]
where \( \alpha_{a'}^j(t, p) \) is the acceptance rule of type \((j, a')\). Hence

\[
v_i^a(t) - v_i^{a'}(t) \geq r_i^a(t) - r_i^{a'}(t) - \lambda^S(t) \sum_{p \in \Gamma(t)} \pi(t, p) \alpha_{a'}^j(t, p) [r_H^i(t) - r_H^{a'}(t)]
\]

Using the expression for \( r_i^a(t) \) and denoting

\[
f_j^{a'}(t) = \lambda^S(t) \sum_{p \in \Gamma(t)} \pi(t, p) \alpha_{a'}^j(t, p) \leq 1
\]

we obtain

\[
v_i^a(t) - v_i^{a'}(t) \geq (1 - f_j^{a'}(t))(\tau_i^a - \tau_j^{a'}) + \delta(1 - f_j^{a'}(t))(\bar{v}_i^a(t) - \bar{v}_j^{a'}(t))
\]

\[
= (1 - f_j^{a'}(t))(\tau_i^a - \tau_j^{a'}) + \delta(1 - \gamma)(1 - f_j^{a'}(t))(v_i^a(t + 1) - v_i^{a'}(t + 1))
\]

\[
+ \delta \gamma(1 - f_j^{a'}(t))(v_i^a(t + 1) - v_i^{a'}(t + 1))
\]

Hence we can rewrite the following expression as

\[
\begin{bmatrix}
v_2^L(t) - v_1^L(t) \\
v_2^H(t) - v_1^H(t) \\
v_2^H(t) - v_1^H(t)
\end{bmatrix} \geq
\begin{bmatrix}
(\tau - 1)d_L & 0 & 0 \\
d_H - \tau d_L & 0 & 0 \\
(\tau - 1)d_H & 0 & 0
\end{bmatrix} + \delta M(t)
\begin{bmatrix}
v_2^L(t + 1) - v_1^L(t + 1) \\
v_2^H(t + 1) - v_1^H(t + 1) \\
v_2^H(t + 1) - v_1^H(t + 1)
\end{bmatrix}
\]

(36)

where

\[
M(t) =
\begin{bmatrix}
(1 - f_L^1(t))(1 - 2\gamma) & 0 & 0 \\
\gamma(1 - f_L^2(t)) & 1 - f_L^2(t) & \gamma(1 - f_L^2(t)) \\
0 & 0 & (1 - f_H^2(t))(1 - 2\gamma)
\end{bmatrix}
\]

Iterating on expression (36) and using condition (35), we obtain that

\[
\tau^i d_a \geq \tau^j d_{a'} \implies v_i^a(t) \geq v_i^{a'}(t)
\]

It is then straightforward to show that the result extends to reservation values.
A.2 Proof of Proposition 1

To prove Proposition 1, we proceed as follows. First, we write all endogenous variables as a function of $\pi$ under the conjecture that $\lambda^S = 1$. Then we solve for a fixed point equation in $\pi$. Finally, we check that our partial equilibrium conjecture holds.

Step 1

Given that $\lambda^S = 1$, the steady state masses of traders solve the following equations:

\[
\mu^1_L = \gamma \mu^1_L + \gamma \mu^2_L = \gamma Sq
\]
\[
\mu^1_H(\pi) = [1 - \gamma - (1 - 2\gamma)\pi] \mu^1_H(\pi) + \gamma \mu^2_H(\pi)
\]
\[
= \gamma Sq + (1 - 2\gamma)(1 - \pi) \mu^1_H(\pi)
\]

We obtain solution (20) using the equations above and the consistency requirements (3), that is:

\[
\mu^1_L = \gamma S(1 - q)
\]
\[
\mu^1_H(\pi) = \frac{\gamma}{\pi + 2\gamma(1 - \pi)} Sq
\]
\[
\mu^2_L = (1 - \gamma) S(1 - q)
\]
\[
\mu^2_H(\pi) = \frac{(1 - \gamma)\pi + (1 - \pi)\gamma}{\pi + 2\gamma(1 - \pi)} Sq
\]

As we showed in the text, $H$ quality traders always obtain their reservation value. Hence $(v^1_H, v^2_H)$ solve the following system:

\[
v^1_H = d_H + \delta [(1 - \gamma)v^1_H + \gamma v^2_H]
\]
\[
v^2_H = \tau d_H + \delta [(1 - \gamma)v^2_H + \gamma v^1_H]
\]

We obtain:

\[
v^2_H + v^1_H = (\tau + 1)d_H + \delta (v^1_H + v^2_H)
\]
\[
v^2_H - v^1_H = (\tau - 1)d_H + \delta (1 - 2\gamma)(v^2_H - v^1_H)
\]
Finally we have,

\[
v_1^H = r_1^H = \frac{1 - \delta + \delta \gamma (\tau + 1)}{(1 - \delta)(1 - \delta(1 - 2\gamma))} \cdot d_H
\]

\[
v_2^H = r_2^H = \frac{(1 - \delta)\tau + \delta \gamma (\tau + 1)}{(1 - \delta)(1 - \delta(1 - 2\gamma))} \cdot d_H
\]

We turn now to \(L\) asset owners for which \((v_1^L, v_2^L)\) solve

\[
v_1^L = d_L + \pi(r_1^H - d_L - \delta \bar{v}_1^L) + \delta \bar{v}_1^L
\]

\[
v_2^L = \tau d_L + \pi(r_1^H - \tau d_L - \delta \bar{v}_2^L) + \delta \bar{v}_2^L
\]

We get

\[
v_2^L + v_1^L = (1 - \pi)(\tau + 1)d_L + 2\pi r_1^H + \delta(1 - \pi)(v_2^L + v_1^L)
\]

\[
v_2^L - v_1^L = (1 - \pi)(\tau - 1)d_L + \delta(1 - \pi)(1 - 2\gamma)(v_2^L - v_1^L)
\]

From which we obtain

\[
v_1^L(\pi) = \pi \cdot \frac{1}{1 - (1 - \pi)\delta} \cdot r_1^H + (1 - \pi) \cdot \frac{1 - (1 - \pi)\delta + \delta(1 - \pi)\gamma(\tau + 1)}{1 - \delta(1 - \pi)(1 - 2\gamma)} \cdot d_L
\]

\[
v_2^L(\pi) = \pi \cdot \frac{1}{1 - (1 - \pi)\delta} \cdot r_1^H + (1 - \pi) \cdot \frac{1 - (1 - \pi)\delta + \delta(1 - \pi)\gamma(\tau + 1)}{1 - \delta(1 - \pi)(1 - 2\gamma)} \cdot d_L
\]

and \(r_1^L(\pi) = \tau d_L + \delta \bar{v}_1^L(\pi)\). We have thus written all the endogenous variables as functions of \(\pi\).

Step 2

Let us now turn to the problem of the buyer. Define the correspondence

\[
\pi \to \arg \max_{\hat{\pi}} \left\{ \hat{\pi} \left[ \mu_1^H(\pi)(r_1^H - r_1^H) + S(1 - q)(r_2^L(\pi) - r_1^H) \right] \\
+ (1 - \hat{\pi})\mu_1^L(r_2^L(\pi) - r_1^L(\pi)) \right\}
\]
An equilibrium is a fixed point \( \pi^* \) of the correspondence defined above. Given that maximization problem is linear in \( \hat{\pi} \), we have

\[
\pi^* = \begin{cases} 
0 & \text{if } G(0) \leq 0 \\
\in (0, 1) & \text{if } G(\pi^*) = 0 \\
1 & \text{if } G(1) \geq 0 
\end{cases} 
\] (37)

where

\[
G(\pi) = \mu_H^1(\pi)(r_H^2 - r_H^1) - S(1 - q)(r_H^1 - r_H^2(\pi)) - \mu_L^1(r_L^2(\pi) - r_L^1(\pi))
\]

is the term in factor of \( \hat{\pi} \) in the expression of \( F \). Using equation (37), we can characterize too cutoffs \((q, \bar{q})\) for the existence of the pure strategy equilibria \( \pi^* = 0 \) and \( \pi^* = 1 \). Plugging the expressions obtained above, we have

\[
G(0) = \frac{q}{2} \frac{\tau - 1}{1 - \delta(1 - 2\gamma)} d_H - (1 - q) \frac{(1 - \delta)(d_H - vd_L) + \delta\gamma(v + 1)(d_H - d_L)}{(1 - \delta)(1 - \delta(1 - 2\gamma))} \\
- \gamma(1 - q) \frac{\tau - 1}{1 - \delta(1 - 2\gamma)} d_L
\]

and thus \( G(0) \leq 0 \) if and only if

\[
q(1 - \delta)(\tau - 1)d_H - 2(1 - q)(1 - \delta)(d_H - \tau d_L) - 2(1 - q)\delta\gamma(\tau + 1)(d_H - d_L) \\
- 2\gamma(1 - q)(1 - \delta)(\tau - 1)d_L \leq 0
\]

We thus obtain the threshold \( q \) introduced in the main text:

\[
q = \frac{2a}{(1 - \delta)(\tau - 1)d_H + 2a}
\]

where

\[
a = (1 - \delta)(d_H - \tau d_L) + \gamma\delta(\tau + 1)(d_H - d_L) + \gamma(1 - \delta)(\tau - 1)d_L
\]
Similarly, we have

\[ G(1) = \gamma q \frac{\tau - 1}{1 - \delta(1 - 2\gamma)} d_H - (1-q) \left[ \frac{1 - \delta + \delta\gamma(\tau + 1)}{1 - \delta(1 - 2\gamma)} d_H - \tau d_L \right] - \gamma(1-q)(\tau-1)d_L \]

and thus \( G(1) \geq 0 \) if and only if \( B(q) \geq 0 \) where

\[ B(q) = \gamma q(\tau - 1)d_H - (1 - q)(1 - \delta)d_H - (1 - q)\delta\gamma(\tau + 1)d_H + (1 - q)(1 - \delta(1 - 2\gamma))\tau d_L \\
- \gamma(1-q)(1-\delta(1-2\gamma))(\tau-1)d_L \\
= \gamma q(\tau - 1)d_H - (1 - q)(1 - \delta)(d_H - \tau d_L) - (1 - q)\delta\gamma(\tau + 1)(d_H - d_L) \\
+ 2\gamma\delta(1-q)\tau d_L - (1 - q)\delta\gamma(\tau + 1)d_L - \gamma(1-q)(1-\delta(1-2\gamma))(\tau-1)d_L \\
= \gamma q(\tau - 1)d_H - (1 - q)(1 - \delta)(d_H - \tau d_L) - (1 - q)\delta\gamma(\tau + 1)(d_H - d_L) \\
- \gamma(1-q)(1-\delta)(\tau-1)d_L + \delta\gamma(1-q) \left[ 2\tau - (\tau + 1) - 2\gamma(\tau-1) \right] d_L \]

We obtain

\[ \bar{q} = \frac{a - b}{\gamma(\tau - 1)d_H + a - b} \]

where

\[ b = \delta\gamma(1-2\gamma)(\tau-1)d_L \]

When \( q \in [\min\{\bar{q}, \bar{q}\}, \max\{\bar{q}, \bar{q}\}] \), the mixed equilibrium \( \pi^* \) solves \( G(\pi^*) = 0 \).

Step 3

We must now find conditions on \( \kappa \) such that our partial equilibrium conjecture holds, that is \( \lambda^S = 1 \) in any equilibrium. Let \( q \) be given and \( \pi^*(q) \) an equilibrium at \( q \). Remember that the buyers’ zero profit condition implicitly defines \( \mu^B(\pi^*(q)) \) as

\[ -\kappa + \frac{1}{\mu^B(\pi^*(q))} \left[ \mu^1_L(\pi^*(q)) - \pi^*(q)G(\pi^*(q)) \right] = 0 \]

We must then check that

\[ \min_{q,\pi^*(q)} \mu^B(\pi^*(q)) \geq S \]

From our previous analysis, it is clear that the expression between brackets is min-
imal either in $q$ or $\bar{q}$. Hence ultimately, we must verify that

$$\min\left\{\frac{1-q}{1-\delta(1-2\gamma)}, 1-\bar{q}\right\} \gamma(\tau - 1) d_L \geq \kappa$$

This inequality holds for $\kappa$ small enough.

### A.3 Proof of Lemma 2

The proof is by contradiction. Let $t_0$ be such that $\pi(t_0) = 1$. Suppose then that $\pi(t_0 + 1) = 1$. We want to establish that $\pi(t_0 - 1) = 1$. Then by backwards induction, it means that for all $t$, $\pi(t) = 1$, a contradiction with having a cycle of period $T \geq 2$.

We have seen that buyer’s optimality condition writes

$$\pi(t) = \begin{cases} 
0 & \text{if } G(t) < 0 \\
\pi \in [0, 1] & \text{if } G(t) = 0 \\
1 & \text{if } G(t) > 0 
\end{cases}$$

where

$$G(t) := \mu_1^1(t)(r_H^2 - r_H^1) - S(1-q)r_H^1 + \mu_2^2(t)r_L^2(t) + \mu_L^1(t)r_L^1(t)$$

Hence $\pi(t_0 + 1) = 1$ means that $G(t_0 + 1) > 0$. We want to show that $G(t_0 - 1) > 0$. Given our partial equilibrium conjecture that $\lambda^S(t) = 1$ for all $t$, we have that $\mu_L^1(t) = \mu_L^1 = \gamma Sq$ and $\mu_L^2(t) = \mu_L^2 = (1 - \gamma)Sq$ for all $t$. Remember also that

$$\mu_H^1(t) = \gamma Sq + (1 - 2\gamma)(1 - \pi(t - 1)) \mu_H^1(t - 1)$$

where we also used that $\lambda^S = 1$. Hence $\mu_H^1(t_0 + 1) = \gamma Sq \leq \mu_H^1(t_0 - 1)$ Next, observe that

$$r_L^1(t_0 - 1) = \tau d_L + \delta \bar{v}_L^1 = \tau d_L + \delta r_H^1$$

Indeed, a $L$ asset owner in $t_0 - 1$ will be able to sell his asset at price $r_H^1$ since $\pi(t_0) = 1$. This is the highest possible value for $r_L^1(t)$ in equilibrium. Hence,
\[ r_i^L(t_0 - 1) \geq r_i^L(t_0 + 1) \text{ for } i = 1, 2. \] This shows that \( G(t_0 - 1) \geq G(t_0 + 1) > 0 \) so that \( \pi(t_0 - 1) = 1 \). Hence, by contradiction, it must be that \( \pi(t_0 + 1) = 0. \)

### A.4 Proof of Proposition 2

In the following, we characterize a \( T \) period cycle and then derive conditions for it to be an equilibrium. Let us first write down the endogenous variables in the conjectured equilibrium. Using the results in the main text, we have

\[ \mu^1_H(t) = \frac{1 - (1 - 2\gamma)^t}{2} Sq, \quad t = 1, \ldots, T \quad (38) \]

Reservation values \( \{r_i^L(t)\}_{i=1,2} \) verify the following equations for \( t = 0, \ldots, T - 2 \)

\[ r_1^L(t) = d_L + \delta [(1 - \gamma)r_1^L(t + 1) + \gamma r_2^L(t + 1)] \]
\[ r_2^L(t) = \tau d_L + \delta [(1 - \gamma)r_2^L(t + 1) + \gamma r_1^L(t + 1)] \]

while

\[ r_i^L(T - 1) = \tau^i d_L + \delta r_i^H \]

By summing and subtracting, we obtain

\[ r_1^L(t) + r_2^L(t) = (\tau + 1)d_L + \delta [r_1^L(t + 1) + r_2^L(t + 1)] \]
\[ r_2^L(t) - r_1^L(t) = (\tau - 1)d_L + \delta (1 - 2\gamma) [r_2^L(t + 1) - r_1^L(t + 1)] \]

Hence we obtain for \( t = 0, \ldots, T - 1 \)

\[ r_1^L(t) + r_2^L(t) = \frac{1 - \delta^{T-t}}{1 - \delta} (\tau + 1)d_L + 2\delta^{T-t}r_1^H \]
\[ r_2^L(t) - r_1^L(t) = \frac{1 - [\delta(1 - 2\gamma)]^{T-t}}{1 - \delta(1 - 2\gamma)} (\tau - 1)d_L \]
from which we get for \( t = 0, \ldots, T - 1 \)

\[
\begin{align*}
  r_{L,T}^1(t) &= \left[ \frac{1 - \delta^{T-t}}{1 - \delta} + \frac{1 - (\delta(1 - 2\gamma))^{T-t}}{1 - \delta(1 - 2\gamma)} \right] \frac{dL}{2} + \delta^{T-t} r_H^1 \\
  r_{L,T}^2(t) &= \left[ \frac{1 - \delta^{T-t}}{1 - \delta} + \frac{1 - (\delta(1 - 2\gamma))^{T-t}}{1 - \delta(1 - 2\gamma)} \right] \frac{dL}{2} + \delta^{T-t} r_H^1
\end{align*}
\]

A quick argument by backward induction shows that

\[
r_L^i(t) \leq r_L^i(t + 1), \quad t = 0, \ldots, T - 2
\]  \hspace{1cm} (39)

Now remember that in any period \( t \), buyers’ optimality condition gives us

\[
\pi(t) = \begin{cases} 
0 & \text{if } G(t) < 0 \\
\pi \in [0, 1] & \text{if } G(t) = 0 \\
1 & \text{if } G(t) > 0
\end{cases}
\]

where

\[
G(t) := \mu_H^1(t)(r_H^2 - r_H^1) - S(1 - q)r_H^1 + \mu_L^2 r_L^2(t) + \mu_L^1 r_L^1(t)
\]

Using equations (38)-(39) together, we have \( G(t) \leq G(t + 1) \), for \( t = 1, \ldots, T - 2 \). Hence to verify that \( \pi(t) = 1(t=0) \) is an equilibrium, it is enough to check that \( G(0) \geq 0 \) and \( G(T - 1) \leq 0 \). These conditions are respectively equivalent to

\[
\begin{align*}
q &\geq q_T := \frac{2(r_H^1 - r_H^2(0)) + 2\gamma(r_L^2(0) - r_L^1(0))}{1 - (1 - 2\gamma)^T} + 2(r_H^1 - r_H^2(0)) + 2\gamma(r_L^2(0) - r_L^1(0)) \\
q &\leq \tilde{q}_T := \frac{2(r_H^1 - r_H^2(T - 1)) + 2\gamma(r_L^2(T - 1) - r_L^1(T - 1))}{1 - (1 - 2\gamma)^{T-1}} + 2(r_H^1 - r_H^2(T - 1)) + 2\gamma(r_L^2(T - 1) - r_L^1(T - 1))
\end{align*}
\]  \hspace{1cm} (40) \hspace{1cm} (41)

Hence the \( T \) periods cycle exists if and only if \( q_T \leq \tilde{q}_T \) that is

\[
\frac{1 - (1 - 2\gamma)^T}{1 - (1 - 2\gamma)^{T-1}} \geq \frac{r_H^1 - r_H^2(0) + \gamma(r_L^2(0) - r_L^1(0))}{r_H^1 - r_H^2(T - 1) + \gamma(r_L^2(T - 1) - r_L^1(T - 1))}
\]
Using the expressions derived above, we obtain
\[
\frac{1 - (1 - 2\gamma)T}{1 - (1 - 2\gamma)^{T-1}} \geq \frac{r_H^1 - r_L^2(0) + \gamma(r_H^2(0) - r_L^1(0))}{(1 - \delta)r_H^1 - \tau d_L + \gamma(\tau - 1)d_L} \quad (E_T)
\]
The LHS decreases with \(T\). On the RHS, the denominator does not depend on \(T\) while the numerator is equal to \(r_H^1 - (1 - \gamma)r_L^2(0) - r_L^1(0)\) and increases in \(T\). This proves that for \(T' \geq T\)
\[(E_T) \Rightarrow (E_{T'})\]

Observe also that the LHS of \((E_T)\) tends towards 1 while the RHS is strictly superior and bounded away from 1. Hence by continuity, there exists \(\bar{T}\) such that no equilibrium cycle exists for \(T > \bar{T}\).

It appears from equation (41) that \(\bar{q}_T\) only depends on \(T\) through the first term of the denominator \(1 - (1 - 2\gamma)^{T-1}\). It is then immediate that the sequence \(\{\bar{q}_T\}\) is decreasing in \(T\). Observe also that \(\bar{q}_2 = \bar{q}\).

Consider now the sequence \(\{q_T\}\). Clearly, we have \(q_{T+1} \leq q_T\) if and only if
\[
\frac{1 - (1 - 2\gamma)^{T+1}}{1 - (1 - 2\gamma)^T} \geq \frac{r_H^1 - r_L^2(T+1)(0) + \gamma[r_L^2(T+1)(0) - r_L^1(T+1)(0)]}{r_H^1 - r_L^2(T)(0) + \gamma[r_L^2(T)(0) - r_L^1(T)(0)]}
\]
Using existence condition \((E_{T+1})\) the inequality above holds if
\[
r_H^1 - r_L^2(T - 1) + \gamma[r_L^2(T - 1) - r_L^1(T - 1)] \leq r_H^1 - r_L^2(T)(0) + \gamma[r_L^2(T)(0) - r_L^1(T)(0)]
\]
which holds true given the monotonicity of \(r_L^2(T)(t)\) in \(t\).

We are left to show that our partial equilibrium conjecture holds, that is \(\lambda^S(t) = 1\), that is \(\mu^B(t) \geq 2\) for \(t = 0, \ldots, T - 1\) in a cycle with period \(T\). Again, \(\mu^B(t)\) is defined implicitly by the buyers’ zero profit condition.
\[
g_\delta^2 = -\kappa + \frac{1}{\mu^B(t)} \left[\mu_L^1(r_L^2(t) - r_L^1(t)) + \pi(t)G(t)\right] = 0 \quad (42)
\]
Hence \(\mu^B(t)\) is minimal in period \(T - 1\) when we have \(\mu^B(T - 1) = \gamma S(1 - q)(\tau - \ldots)\).
1) $d_L/\kappa$. Since, $\bar{q}_T \leq \bar{q}$, it is enough that 

$$\kappa \leq \gamma (1 - \bar{q}) (\tau - 1) d_L$$

### A.5 Proof of Lemma 3

We want to show that the intervention in (28) restores full trade. Suppose first that the intervention only attracts lemon holders. Then the steady state share of $H$ assets is 

$$q_{ss} = \frac{Sq}{Sq + S(1-q)S_{int}(q)} = \bar{q}$$

We know from Proposition 1 and 2 that the only stationary equilibrium when $q_{ss} \geq \bar{q}$ exhibits full trade, that is $\pi_{ss} = 1$.

Suppose that $\pi(t) = 1$ in $t = t_{inv}$. Then in period $t + 1$, equilibrium masses are given by

$$\mu_1^H(t+1) = \gamma Sq, \quad \mu_1^L(t+1) = \gamma \left[ S(1-q) - S_{int}(q) \right], \quad \mu_2^L(t+1) = (1-\gamma) \left[ S(1-q) - S_{int}(q) \right]$$

Since these are also the steady state values, by backward induction, $\pi(t) = 1$ for all $t \geq t_{inv} + 1$. We are left to show that buyers indeed find it optimal to propose $\pi(t) = 1$ for $t = t_{inv}$. Since $(\tau_1, L)$ and $(\tau_2, L)$ sellers are indifferent about opting in or out of the intervention, let us assume that participation is proportional to their share of the population. Hence, equilibrium masses of traders after intervention are also equal to the steady state levels so that $\pi(t_{inv}) = 1$ is an equilibrium. At price $P_{int} = r^1_H$, $L$ sellers are indeed willing to participate since they would receive the same utility from staying in the market.

### A.6 Proof of Proposition 4

To establish that the intervention increases (resp. decreases) welfare it is enough to prove that $W_f \geq W_\emptyset$ (resp. $W_f \leq W_\emptyset$) where $W_\emptyset$ is welfare in the absence of any

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29As we already noted, all owners but $(\tau_2, H)$ types are actually indifferent about participating. Our selection can be motivated by introducing arbitrarily small search frictions as in Appendix B. With $\alpha < 1$ but close to 1, the government can design a program that uniquely attracts this selection of traders. In the case $d_L = 0$ this selection is irrelevant among $L$ asset owners.
intervention.
i) Cycle of Period 2
The increase in welfare is equal to
\[ W_f(q) - W_\emptyset(q) = S(1 - q)(1 - \delta)r_H^1 - S_f(q)r_H^1 \]
\[ = \left[ (1 - \delta)(1 - q) - \left( 1 - \frac{q}{\bar{q}} \right) \right] S r_H^1 \]
This expression is strictly increasing in \( q \). We are thus left to show that it is positive for \( q = q_2 \). From Proposition 2 with \( d_L = 0 \), we have
\[ q_2 = \frac{2(1 - \delta) + 2\delta\gamma(\tau + 1)}{\frac{2\gamma(1 - \gamma)}{1 + \delta}(\tau - 1) + 2(1 - \delta) + 2\delta\gamma(\tau + 1)} \]
Hence we obtain
\[ W_f(q_2) - W_\emptyset(q_2) = \frac{\gamma S r_H^1(\tau - 1)}{\frac{2\gamma(1 - \gamma)}{1 + \delta}(\tau - 1) + 2(1 - \delta) + 2\delta\gamma(\tau + 1)} \left[ \frac{2(1 - \delta)(1 - \gamma)}{1 + \delta} - \frac{2(1 - \gamma)}{1 + \delta} + 1 \right] \]
The terms between brackets is increasing in \( \gamma \) and positive when \( \gamma = 0 \). Hence we have proven that the full-trade intervention increases welfare when \( q \in [q_2, \bar{q}_2] \)

ii) Steady state.
When \( q \in [q_2, \bar{q}_2] \), using the characterization in the proof of Proposition 1 with \( d_L = 0 \), the steady state probability of a high pooling price \( r_H^1 \) is
\[ \pi(q) = \frac{(1 - \delta)(1 - \bar{q})q - 2\gamma\bar{q}(1 - q)}{(1 - 2\gamma)(1 - q)\bar{q} - \delta q(1 - \bar{q})} \]
The net welfare increase from a full-trade intervention is equal to
\[ W_f(q) - W_\emptyset(q) = S r_H^1 \left[ (1 - q) \left( 1 - \frac{\pi(q)}{1 - \delta(1 - \pi(q))} \right) - \left( 1 - \frac{q}{\bar{q}} \right) \right] \]
Straightforward computations show that
\[ W_f(q) - W_\emptyset(q) = \frac{2\gamma S r_H^1(\bar{q} - q)}{(1 - 2\gamma - \delta)\bar{q}} \geq 0 \]
A.7 Proof of Corollary 2

The welfare gain from intervention now incorporates the extra cost of funds $\theta$ for the government.

i) Steady State

Building on the Proof of Proposition 4, the welfare gain from an intervention now writes

$$W_f(q, \theta) - W_\emptyset(q, \theta) = Sr_H^1 \left[ (1-q) \left( 1 - \frac{\pi(q)}{1 - \delta(1 - \pi(q))} \right) - (1 + \theta) \left( 1 - \frac{\bar{q}}{\bar{q}} \right) \right]$$

which can be simplified to

$$W_f(q, \theta) - W_\emptyset(q, \theta) = \frac{Sr_H^1(\bar{q} - q)}{q(1 - 2\gamma - \delta)} \left[ 1 - \delta - (1 + \theta)(1 - \delta - 2\gamma) \right]$$

This is negative for all $q$ when

$$\theta \geq \hat{\theta} := \frac{2\gamma}{1 - \delta - 2\gamma}$$

ii) Cycle

We are left to show that the welfare gains from a full-trade intervention can be positive in a cycle when $\theta \geq \hat{\theta}$. We have:

$$W_f(q, \theta) - W_\emptyset(q, \theta) = \left[ (1 - \delta)(1 - q) - (1 + \theta) \left( 1 - \frac{\bar{q}}{\bar{q}} \right) \right] Sr_H^1$$

This expression is positive for

$$q \geq \hat{q}(\theta) := \frac{\theta + \delta}{1 + \theta - q(1 - \delta)} \bar{q} \leq \bar{q} = \bar{q}_2$$

A.8 Proof of Proposition 5

The proof is in several steps. First, we argue that there are $(\tau^2, L)$ sellers in equilibrium from $t = 1$ onwards. Second, we show that there cannot be a market where $\theta(t, p) > 0$ and $\gamma_H(t, p) > 0$. Finally, we show that the allocation in Proposition 4 is the only equilibrium possible.
Step 1: \( \mu^2_L(t) > 0 \) for \( t \geq 1 \)

Suppose first that \( \mu^2_L(t) > 0 \) for some \( t \geq 1 \). Then \( \mu^2_L(t+1) > 0 \). Indeed, during period \( t \), \( L \) assets initially held by \( \tau^2 \) agents stay with (possibly different) \( \tau^2 \) agents who are the only potential buyers. Hence accounting for the type-switching probability \( \mu^2_L(t+1) \geq (1-\gamma)\mu^2_L(t) > 0 \). We are left to show that \( \mu^2_L(1) > 0 \). If \( \mu^2_L(0) > 0 \) our previous argument concludes the proof. If not, \( \mu^2_L(1) \geq \min\{\gamma,1-\gamma\}\mu^2_L(0) > 0 \) The first case is when no \((\tau^1,L)\) agent trades in \( t = 0 \). The second case is when they all trade.

Step 2

Let \( t \geq 1 \). The argument is by contradiction. Observe first that

\[
\max \mathcal{P}(t) < r^2_H(t)
\]

Indeed, the maximum price buyers will pay for an asset is \( r^2_H(t) - \kappa \), that is the value of a \( H \) asset minus the search cost. Hence, \((\tau^2,H)\) will never sell their asset. Define now

\[
\mathcal{P}_H(t) = \{ p \in \mathcal{P}(t) \mid \gamma^3_H(t,p) > 0 \}
\]

The set \( \mathcal{P}_H(t) \) is the set of active markets where \( H \) assets are for sale. We want to show that \( \mathcal{P}_H(t) = \emptyset \). By \((\tau^1,H)\) sellers optimality condition, we have \( \min \mathcal{P}_H(t) \geq r^1_H(t) \). Let \( \bar{p}_H(t) = \max \mathcal{P}_H(t) = \max \mathcal{P}(t) \).

Suppose first that \( \gamma^3_H(t,p) = 1 \). It must be that \((\tau^2,L)\) sellers are at most indifferent about trading at that price. Let thus be

\[
\bar{p}^2_L(t) = \max \{ p \in \mathcal{P}(t) \mid \gamma^2(t,p) > 0 \} < \bar{p}_H(t)
\]

Market \( \bar{p}^2_L(t) \) is the maximum price at which agents \((\tau^2,L)\) trade. For \( \bar{p}^2_L(t) \) to be optimal, it must be that:

\[
\lambda^S(t,\bar{p}^2_L(t))(\bar{p}^2_L(t) - r^2_L(t)) \geq \lambda^S(t,\bar{p}_H(t))(\bar{p}_H - r^2_L(t))
\]

\footnote{They choose a market \( p > r^2_H(t) \) where \( \theta(t,p) = 0 \)}
In particular, we must have $\bar{p}_L^2(t) > r_L^2(t)$. Since $r_L^2(t) - \kappa$ is the maximum price a $L$ asset can command, it must be that $\gamma_H^1(t, p') > 0$. By seller's optimality, agents $(\tau^1, H)$ must weakly prefer market $p_L^2(t)$ to any market $p' > \bar{p}_L^2(t)$, that is

$$\lambda^S(t, \bar{p}_L^2(t))(\bar{p}_L^2(t) - r_H^1(t)) \geq \lambda^S(t, \bar{p}_H)(p' - r_H^1(t))$$

This is only possible if $\lambda^S(t, p') < \lambda^S(t, \bar{p}_L^2(t)) \leq 1$. Then since $r_L^2(t) < r_H^1(t)$, agents $(\tau^2, L)$ strictly prefer market $\bar{p}_L^2(t)$ over $p'$. Hence, using Part 2 of the Equilibrium definition, $\gamma_H^1(t, p') = 1$. Let us now write buyers profit in market $\bar{p}_L^2(t)$ and $p' > \bar{p}_L^2(t)$

$$v_0^2(t, \bar{p}_L^2(t)) = -\kappa + \lambda^B(t, \bar{p}_L^2(t))[(\gamma_H^1(t) + \gamma_L^2(t, p))r_L^2(t) + \gamma_H^1(t, p)r_H^2(t) - \bar{p}_L^2(t)]$$

$$v_0^2(t, p') = -\kappa + r_H^2(t) - p'$$

Hence,

$$\lim_{p' \to \bar{p}_L^2(t)} v_0^2(t, p') > v_0^2(t, \bar{p}_L^2(t))$$

which is incompatible with buyers' optimality condition.

But if we suppose now that $\gamma_H^1(t, \bar{p}_H) < 1$, the same argument applies. Hence, we have shown that $P_H(t) = \emptyset$.

**Step 3**

We want to show that there exists a unique price $p_L(t)$ at which there is trade. As we observed earlier, the maximum price a $L$ asset can command is $r_L^1(t) - \kappa$. Hence $(\tau^2, L)$ sellers find it optimal not to supply their asset.

Let us show first that $P(t)$ is a singleton. Suppose otherwise : there exists $(p, p') \in P(t)$ such that $p' > p$. Then type $(\tau^1, L)$ sellers must be indifferent between $p$ and $p'$

$$\lambda^S(t, p)[p - r_L^1(t)] = \lambda^S(t, p')[p' - r_L^1(t)]$$

so that $\lambda^S(t, p') < \lambda^S(t, p) \leq 1$. Suppose that $\theta(t, p) \leq 1$. Then $\lambda^B(t, p) = \lambda^B(t, p') = 1$ which is incompatible with buyers optimality. If $\theta(t, p) > 1$. Let then $\epsilon > 0$. Rationing at $p + \epsilon$ is pinned down by $(\tau^1, L)$ sellers indifference condition so that $\lambda^S(t, p + \epsilon) < 1$ and then $\lambda^B(t, p + \epsilon) = 1$ which is again incompatible.
with buyers’ optimality. Finally \( p_L(t) = r^2_L(t) - \kappa \) results from buyers zero profit condition.

The belief and rationing functions of Proposition 4 immediately follow.

**B Search Frictions and Government Intervention**

In this model, search frictions are rather mild as a seller is certain to find a buyer in equilibrium. Search costs thus boils down to resources wasted in useless meetings (for instance between a \((\tau^2, H) \) seller and a buyer). We now suppose that an asset owner may fail to find a match even when buyers outnumber sellers. The probability for a seller to find a buyer and vice versa are now respectively:

\[
\lambda^S(t) = \alpha \min \left\{ \mu^B(t), 1 \right\}, \quad \lambda^B(t) = \alpha \min \left\{ \mu^S(t), 1 \right\}
\]

where \( \alpha \in (0, 1] \). The model of the main text has \( \alpha = 1 \).

All our results on equilibrium existence hold qualitatively for \( \alpha \) in a neighborhood of 1. First, there always exists a steady state equilibrium. Second, for any \( T \in \mathbb{N}_+ \setminus \{1\} \), there exists thresholds \((q_T(\alpha), \bar{q}_T(\alpha)) \in (0, 1)^2 \) such that a \( T \) period cycle exists if \( q_\alpha(\alpha) \leq \bar{q}_T(\alpha) \) and

\[
\lim_{\alpha \to 1} q_T(\alpha) = q_T, \quad \lim_{\alpha \to 1} \bar{q}_T(\alpha) = \bar{q}_T
\]

where for each equality, the right hand side is the threshold defined in the main text. Intuitively, all relevant endogenous variables are continuous in \( \alpha \). Hence, a limit of an equilibrium when \( \alpha \to 1 \) is an equilibrium of the main text. A formal proof is available upon request.

We want to show that when \( \alpha \) is close to 1, a government can implement a full trade intervention and selects uniquely \( L \) asset owners. Following Lemma 3, define a full-trade intervention \((P^g_T(\alpha), S^g_T(\alpha, q)) \in \mathbb{R}^2_+ \) as an intervention such that \( \pi(t) = 1 \) for any period \( t \) following the intervention. As in the main text, the reservation value of a type \((\tau^1, H) \) seller is equal to \( r^1_H(t) \). We are left to show that after intervention \((P^g_T(\alpha), S^g_T(\alpha, q), v^1_L(\alpha, t) < r^1_H \) for \( i = 1, 2 \). Then any intervention price \( P^g_T(\alpha) \in (v^2_L(\alpha, t), r^1_H) \) would attract \( L \) asset owners only. To see this, observe
that the value function of a $L$ asset holder post intervention solves:

$$v^L_i(t) = \alpha \min \left\{ \frac{\mu^B(t)}{\mu^S(t)}, 1 \right\} r^1_H + \left( 1 - \alpha \min \left\{ \frac{\mu^B(t)}{\mu^S(t)}, 1 \right\} \right) r^1_L(\alpha, t)$$

We know\(^{31}\) from Lemma 2 that $r^i_L(\alpha, t) < r^1_H$. The result that $v^i_L(t, \alpha) < r^1_H$ naturally follows since $v^i_L(\alpha, t)$ is a convex combination of $r^1_H$ and $r^i_L(t, \alpha)$.

\section{C Government Intervention with Price Posting}

In this section, we analyze government intervention in the market structure of Section 5. Under laissez-faire, the equilibrium is separating and independent from the share of $H$ assets in the market. This observation suggests that asset purchase programs would not increase liquidity as in the OTC market. Indeed, unless the government removes all $L$ assets, the post-intervention equilibrium would be exactly as in Proposition 4. Instead, we argue that a tax schedule levied on appropriate sub-markets can increase liquidity.

\underline{Definition} : Tax Schedule

A tax schedule is a mapping $\sigma : \mathbb{R}^+ \to \mathbb{R}^+$ such that buyers must pay $p(1 + \sigma(p))$ to purchase an asset in market $p$ while sellers obtain $p$.

Our discussion in the main text helps understand why such a tax schedule may be beneficial. Absent intervention, $(\tau^1, H)$ can always defeat a pooling allocation with $(\tau^2, L)$ by deviating towards high price and high rationing markets. As we have shown in Proposition 4, this signaling spiral may only stop if agents $(\tau^1, H)$ trade with probability zero. By setting prohibitive tax rates on these markets, the government can block this negative spiral.

For a given tax schedule, the definition of equilibrium is exactly as in the main text with the appropriate modification to buyers’ problem (30). We use the same utilitarian welfare criterion as in Section 4 whereby the government maximizes a weighted average of agents’ utility and his own utility derived from the tax proceeds.

\(^{31}\)Strictly speaking, Lemma 2 establishes the result for $\alpha = 1$ but it extends naturally for any $\alpha \in (0, 1]$.\)
The welfare criterion thus writes:

\[ W(t) = \sum_{i,a} \mu_{ia}^i(t)v_{ia}^i(t) + \int_{p \in \mathbb{R}^+} p\sigma(p)\mu^B(t,p)dp \]

By setting a tax schedule \( \sigma(,.) \), the government can ultimately pick open markets where sellers trade under the zero profit condition for buyers and incentive constraints. We formalize this observation with the following auxiliary problem where the planners pick prices and trading probabilities. We drop the dependence on \( t \) since the problem is stationary.

**Definition : Auxiliary Problem**

Let \( \mathcal{P}_G = \{p_1^L, p_2^L, p_1^H, p_2^H\} \) be a set of prices and \( \lambda^S : \mathcal{P}_G \mapsto [0, 1] \) a trading probability function. The auxiliary problem for the government is

\[
\max_{\mathcal{P}_G, \lambda^S(.)} \sum_{i,a} \mu_{ia}^i v_{ia}^i \\
\text{subject to} \quad v_{ia}^i = v_{ia}^i(p_a^i) \geq v_{ia}^i(p), \quad \text{for } p \in \mathcal{P}_G \\
v_{ia}^2(p) = 0, \quad \text{for } p \in \mathcal{P}_G \\
\mu_1^L = \frac{\gamma S(1 - q)}{1 - (1 - \gamma)(1 - \lambda^S(p_1^L))} \\
\mu_2^L = \frac{\gamma S q}{1 - (1 - \gamma)(1 - \lambda^S(p_1^H))} \\
\mu_1^H = \frac{\gamma S q}{1 - (1 - \gamma)(1 - \lambda^S(p_1^H))}
\]

In the auxiliary problem, the government picks prices and probability of trade for each type of seller respecting incentive constraints. Buyers must make zero profit. Their beliefs are in accordance with the sellers optimal choice of a market in \( \mathcal{P}_G \) proposed by the government. Finally, the last equations give the mass of each type in steady state given the optimal choice of the government.

**Proposition 5 : Solution to the auxiliary problem**

The government sets \( \theta(p_2^H) = 0 \). There exists thresholds \( q_G, \bar{q}_G \) such that

i) If \( q \leq \bar{q}_G \), \( \theta(p_1^H) = \theta(p_2^L) = 0 \).

ii) If \( q \in [q_G, \bar{q}_G] \), \( p_1^L < p_1^H = p_2^L \) and \( \theta(p_1^H) < 1 \) while \( \theta(p_1^L) = 1 \).
iii) If $q \geq \bar{q}_G$, $p_{1L}^L = p_{2L}^L = p_{1H}^L$ and $\theta(p_{1H}^L) = 1$

The government can implement this solution to the auxiliary problem with tax schedule $\sigma(p) = r_{2H}^1 - r_{1H}^1$ for $p \geq p_{1H}^1$

A proof and the full characterization of the allocation and prices is available in an Online Appendix. Intuitively, the government seeks to maximize the trading probability for every type of seller. When $q \geq \bar{q}_G$, there are enough good quality assets so that type $(\tau^1, H)$ are willing to trade at the full pooling price $p_{1H}^L$. When $q$ decreases, full pooling is not sustainable. Hence the optimal allocation separates $(\tau^1, L)$ types on the one hand and pools $(\tau^1, H)$ and $(\tau^2, L)$ types on the other hand. These sellers trade with probability less than 1 not to attract $(\tau^1, L)$ types. Finally, when $q \leq \bar{q}_G$, the average quality in the pool made of $(\tau^1, H)$ and $(\tau^2, L)$ sellers is too low and $(\tau^1, H)$ sellers would refuse the partial pooling price. However, as in the laissez-faire equilibrium, $(\tau^2, L)$ sellers will necessarily trade in the same market than the $(\tau^1, H)$ types if any. Hence, trading of the $H$ quality asset is not possible.

Since type $(\tau^1, H)$ do not trade in the laissez-faire equilibrium, any improvement according to the utilitarian criterion brought about by government intervention is also a Pareto improvement.