Financial Crises & Debt Rigidities

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Abstract

This paper develops a dynamic general equilibrium model which includes financial intermediation and endogenous financial crises. Within this framework, financial crises occur out of prolonged boom periods and are initiated by a moderate adverse shock. A boom period increases the likelihood of a financial crisis since financial intermediaries expand their balance sheets and issue new loans which are backed by relatively less collateral in good times. After a sudden bust, default ratios on outstanding loans sharply increase and the financial sector is left with an increased debt burden. Both effects pressure the financial sector’s balance sheet and lead to a creditor run. The model is extended to include price rigidities, nominal debt contracts, and monetary policy. Within this extended version, I analyze the impact of monetary policy on financial stability. Both expansionary and contractionary monetary policy can contribute to financial instability. A typical path leading to a financial crisis is characterized by a longer period of expansionary monetary policy, followed by sharp contractionary monetary policy.

JEL codes: E44, E52, G01, G21

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1 Introduction

Recent contributions to the literature on financial crises highlight two facts about financial crises. First, they occur out of (credit) boom periods as shown empirically by Schularick and Taylor (2012) and Boissay, Collard and Smets (2015) among others. Second, financial crises are the response to relatively small initial triggers. For example, the initial trigger for the Great Recession originated from the subprime mortgage credit market and the initial losses for this market were relatively small (Gorton and Guillermo, 2014). These two observations pose challenges to current macroeconomic models. Why does the likelihood of a financial crisis increase during good times, when agents have the resources to build up precautionary buffers? Why do we observe credit booms which result in financial crises? Moreover, how can a relatively ‘small’ adverse shock trigger a financial crisis which results in large changes of macroeconomic variables?

This paper develops a dynamic general equilibrium model to address these challenges. The paper focuses in particular on three aspects, which are important in this regard. First, the modeling of the financial sector’s balance sheet and the financial sector’s lending behavior aim to be in accordance with current institutional realities and empirical evidence. The crucial features of the model are the maturity mismatch of the financial sector and its procyclical lending - both elements and their interaction play key roles around financial crises. Second, the model incorporates occasional financial crises which arise endogenously and are determined by the risk-taking behavior of the agents. Third, in order to capture the nonlinear dynamics characteristic to episodes of financial distress, a nonlinear global solution is obtained. In this introduction, I describe these points in more detail by giving a brief overview of the model and the results.

Financial Intermediation. An institutional characteristic of the financial sector is its role in transforming short-term borrowing into long-term lending, which exposes the sector to a maturity mismatch. Intermediaries borrow short-term debt from households and issue long-term and defaultable debt to entrepreneurs. The model incorporates an endogenous credit risk cycle, as both the leverage and the default decision of entrepreneurs are endogenous. Regarding the default probability of loans, Jiménez et al. (2009, 2014) show empirically that after a contractionary monetary policy shock, default ratios on outstanding loans increase, while financial intermediaries issue new loans with lower default probabilities. The first result can be explained by lower firm profits due to a fall in aggregate demand and a higher real burden of debt due to lower inflation, both leading to higher default ratios on outstanding loans. The latter result is harder to explain. The price of capital is lower after a monetary tightening and firms may want to take on more debt (relative to their collateral) to take advantage of the good investment opportunities. In order to reconcile the model with the empirical results by Jiménez et al. (2009, 2014), I introduce an agency problem between intermediaries and entrepreneurs which determines the financial sector’s lending over the business cycle. Entrepreneurs may want to risk-shift and invest in projects with low expected returns, but potentially high upsides which imply a negative net present value investment for financial intermediaries. Credit is rationed in order to ensure that entrepreneurs do not choose to invest into such projects (similar to Stiglitz and Weiss, 1981). This risk-shifting problem becomes more severe in recessions as the incentives to risk-shift increase, and therefore the quantity and the loan-to-collateral ratio of newly issued loans decrease in recessions - while
the opposite holds for boom periods. Due to the risk-shifting problem, the loan-to-collateral ratio of newly issued loans is procyclical and the model is able to replicate the asymmetric response in default probabilities by Jiménez et al. (2009, 2014).

The lending decision of financial intermediaries plays an important role in the build-up of systemic risk. During a prolonged boom period, intermediaries expand their balance sheets. They take on more debt and replace old, maturing loans with new loans which are backed by relatively less collateral because incentives to risk-shift decrease in good times. The longer the boom lasts, the stronger the expansion of the financial sector’s balance sheet and the larger the fraction of loans on the financial sector’s balance sheet which has been issued under good economic conditions. However, the increased debt burden and the issuance of loans which are backed by less collateral also makes the financial sector’s balance sheet susceptible to a creditor run after a sudden, adverse change in economic conditions - a sudden bust. After a sudden bust, the value of outstanding loans falls sharply since current and future default ratios increase. If this drop in the value of loans is strong enough and the financial sector is unable to quickly reduce its debt or raise new equity, then the financial sector’s market leverage increases. Figure (1) shows that the market leverage of U.S. financial institutions behaved in such a way around the Great Recession. During the build-up period, the financial sector’s market leverage slowly increased and then spiked sharply during the crisis. The model is able to reproduce such a behavior of leverage around financial crises.\footnote{Similar empirical evidence can be found in Ang, Gorovyy and van Inwegen (2011).}

![Figure 1: Market Leverage. The graph shows the evolution of market leverage for different types of U.S. financial institutions around the Great Recession. See Appendix A.8 for a description of the data and definitions.](image-url)

**Endogenous Financial Crises.** The economy occasionally ends up in financial crises. When financial sector’s market leverage exceeds a certain threshold, creditors run on intermediaries. A creditor run results in a discontinuous drop in the amount of funding available to the financial sector which captures both depositor runs, as for example during the Great Recession.
Depression, as well as wholesale funding market runs, as recently experienced during the Great Recession. Moreover, since creditor runs only occur if the intermediary’s leverage exceeds a certain threshold, they are linked to the balance sheet of the financial intermediary. In this regard, a creditor run is not a self-confirming equilibrium in a setting with multiple equilibria (as in Diamond and Dybvig, 1983), but is instead based on aggregate information. Gorton (1988) gives empirical evidence for this approach. He shows that banking panics during the U.S. National Banking Era were systematic responses by depositors to changing perceptions of risk, based on the arrival of new information rather than random events.

Taken together, the model includes standard business cycle dynamics, the risk-taking behavior of borrowing agents over the business cycle, and endogenous financial crises. Capturing these dynamics in one model is particularly important since I am interested in how systemic risk builds up and how the economy transits from a normal business cycle period into a financial crisis. In a quantitative analysis of the model, I find that the typical build-up path leading to a financial crisis is characterized by a prolonged boom period, followed by a sudden bust which is triggered by a relatively moderate adverse shock. The boom increases the likelihood of a financial crisis due to the expansion of the financial sector’s balance sheet in good times and the described procyclical lending. Moreover, the model is able to replicate three stylized facts regarding banking crises as highlighted by Boissay, Collard and Smets (2015): financial crises are rare, break out in the midst of a credit-intensive boom, and the associated recessions are deeper than standard recessions.

One objective of the paper is to demonstrate how to introduce financial crises into a New-Keynesian dynamic general equilibrium model, commonly used at central banks. In order to arrive at such a model, several steps are taken which structure the rest of the paper. First, the main mechanism driving financial distress in the general equilibrium models is illustrated with the help of a partial equilibrium example (Section 3). Second, these ideas are embedded into a real dynamic general equilibrium model, in which the aggregate source of risk is a technology shock, and the model is quantitatively analyzed (Section 4). And third, the model is extended to include price rigidities, nominal debt contracts, and monetary policy (Section 5). In this extended version, the aggregate exogenous disturbance is a monetary policy shock and monetary policy is non-neutral for two reasons. First, goods producing firms are subject to price stickiness (see for example Nakamura and Steinsson, 2008, for micro-data evidence on price rigidities). Second, debt contracts are written in nominal terms. Changes to inflation therefore adjust the real burden of debt which has real effects on output under default and default costs (as in Gomes, Jermann and Schmid, 2014). In a quantitative analysis of the model, I show that a typical path to a financial crisis is characterized by a longer period of expansionary, followed sharp contractionary monetary policy. Thus, it is a U-shaped pattern of the policy target rate which is most likely to increase financial instability - a pattern followed by monetary authorities in the U.S. and in Japan prior to the recent financial crises in those countries. For both the real and the nominal model, I obtain empirical support for the response of the models’ main indicator of financial stability (the market leverage of the financial sector) to the models’ aggregate sources of risk (technology and monetary policy shock). The models are calibrated to replicate the empirical impulse response of leverage - an approach similar to Christiano, Eichenbaum and Evans (2005), but linking aggregate sources of risk to financial stability.
2 Related Literature

This paper builds on past contributions to the literature on financial frictions within macroeconomic settings. The seminal contributions in this field by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Carlstrom and Fuerst (1997) focus on the role of financial frictions in the amplification and persistence of shocks during business cycle episodes. This paper considers a (monetary) dynamic general equilibrium model similar to Bernanke, Gertler and Gilchrist (1999). The model differs in comparison to the mentioned contributions along two dimensions. First, financial intermediaries are modeled explicitly and occasional financial crises are introduced. Second, the model is solved nonlinearly in order to capture the nonlinear dynamics characteristic to episodes of financial distress. In this regard, the paper contributes to a recent literature which developed in response to the Great Recession. In what follows, I explain how the model relates to existing contributions within this literature.

Mendoza (2010), Bianchi (2011), and Bianchi and Mendoza (2010) study financial crises and sudden stops within small open economy (or partial equilibrium) settings. In their models, (firm-)households are subject to a leverage constraint which binds occasionally. As here, leverage determines the distance to a financial crisis. In contrast to these contributions, I consider a closed economy in a general equilibrium setting and model financial intermediaries explicitly.

Brunnermeier and Sannikov (2014), Adrian and Boyarchenko (2012), and He and Krishnamurthy (2014) build models which incorporate financial intermediation. These models are parsimonious on the modeling of the macroeconomy, have few state variables, and are written in continuous time. Brunnermeier and Sannikov (2014) show that endogenous systemic risk remains even if exogenous risk decreases because agents adjust their risk-taking behavior. The authors find that endogenous systemic risk can even increase if exogenous risk decreases, a result which is termed “volatility paradox”.

Motivated by the research of Adrian and Shin (2010) who show that the leverage of certain types of financial institutions is procyclical based on book value data, Adrian and Boyarchenko (2012) construct a model in which financial intermediaries’ leverage is procyclical since intermediaries raise their debt according to a value-at-risk constraint. Here, in contrast to Adrian and Boyarchenko (2012), financial intermediaries are unconstrained in their funding in normal times and occasionally experience a discontinuous drop in the amount of funding available to them if a creditor run occurs. In contrast to the empirical results by Adrian and Shin (2010) which are based on book value data, I show empirically that the U.S. financial sector’s market leverage is acyclical and the model’s implied cyclicity of leverage is in line with this evidence.

In He and Krishnamurthy (2014), the amount of equity relative to the market value of assets which financial intermediaries can raise is determined by their contemporaneous return on equity. In a crisis, intermediaries experience low returns on equity, restricting the available amount of equity, and increasing their market leverage. The behavior of leverage around a financial crisis in He and Krishnamurthy (2014) is similar to the one in this model. However, the mechanism is different. Here, intermediaries are unable to raise equity. In a crisis, their market leverage increases because the value of
assets falls faster than intermediaries are able to reduce their debt burden.

Further contributions to the aforementioned literature are Boissay, Collard and Smets (2015), Bocola (2015), and Martinez-Miera and Suarez (2012). The closest paper to this one is Boissay, Collard and Smets (2015). In their paper, financial crises occur out of credit booms and are initiated by relatively moderate adverse shocks - as in this one. What differentiates the paper from Boissay, Collard and Smets (2015) is the mechanism. The one in their paper works as follows. After a boom period, when productivity returns, households accumulate savings which increases the available funds in financial markets. The increased availability of funding gives banks incentives to engage in risky activities which can result in a collapse of the interbank market. In contrast to Boissay, Collard and Smets (2015), in this paper it is the interaction between leverage, procyclical lending, and the maturity mismatch of intermediaries which plays the key role for the boom-bust-cycle. Similar ideas can be found in the work of John Geanakoplos, see for example Fostel and Geanakoplos (2008). Bocola (2015) focuses on sovereign risk and solves a model with a similar number of state variables, using a comparable solution algorithm to handle the curse of dimensionality. Additionally, he shows how to estimate the model using Bayesian techniques. Martinez-Miera and Suarez (2012) study welfare effects of capital requirements within a model in which banks risk-shift due to government guarantees and endogenously expose themselves to exogenous systemic risk.

This paper contributes to this literature along two dimensions. First, I show how to model maturity transformation which gives a realistic representation of the financial sector’s balance sheet. Using a stochastic maturity of loans, I show that three aggregate state variables can account for all vintage-specific outstanding loans, their collateral, and their default ratios. Second, all of the mentioned contributions are real models. In contrast, this paper considers a monetary dynamic general equilibrium model in which the non-neutrality of monetary policy arises due to price stickiness and nominal debt contracts. To the best of my knowledge, this is the first paper which introduces financial crises into a standard New-Keynesian dynamic general equilibrium model and obtains a global solution of the model.

There is a large (empirical) literature about the impact of monetary policy on financial stability (as for example the mentioned work by Jiménez et al., 2009, 2014) and I refer the reader to a summary by Smets (2013). Recent empirical papers have found that expansionary monetary policy spurs greater risk-taking by banks, occurring in particular at lower capitalized banks (as emphasized by Jiménez et al., 2014). The latter may arise due to risk-shifting incentives which can be present under government guarantees and limited liability. Here, I do not consider government guarantees or limited liability of financial institutions and the finding that lower capitalized banks take more risk after expansionary monetary policy is therefore not confirmed.

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2 Andreasen, Ferman and Zabczyk (2013) illustrate how to introduce maturity transformation within a general equilibrium model. I follow their reasoning by assuming that long-term loans exist because firms face long-term projects. In contrast to Andreasen, Ferman and Zabczyk (2013), firms can additionally default on their long-term loan.

3 Brunnermeier and Sannikov (2012) also study the impact of monetary policy on financial stability. In contrast to this paper, their framework puts financial frictions at the center of the monetary policy transmission mechanism. The main role of monetary policy in their model is to redistribute wealth towards financially constrained agents in order to avoid deflationary spirals after adverse shocks. In contrast to their paper, monetary policy is non-neutral in this model due to price rigidities.
3 A Partial Equilibrium Example

I start out by giving a stylized partial equilibrium example of the main mechanism. The purpose of this section is to convey the central ideas of the general equilibrium models which follow.

Consider a representative financial intermediary holding zero-interest, short-term debt $B_t$ and long-term, defaultable loans with face value $L_t$ in period $t$. The loans $L_t$ are backed by collateral $K_t$. Assume that loans have an exogenous stochastic maturity. They do not mature with probability $\gamma$ and are only repaid if they mature. The price per unit of loan is given by

$$Q_t = E_t \left[ \sum_{k=1}^{\infty} \Lambda_{t,t+k}\gamma^{k-1}(1-\gamma)\Pi_{t+k} \left( \frac{L_t}{K_t}, \{\epsilon_t\}_{t=0}^{t+k} \right) \right]$$

where $\Lambda_{t,t+k}$ is some exogenous stochastic discount factor and $\Pi_{t+k}$ is the repayment per unit of face value in period $t+k$. The repayment may be smaller than one because borrowers have the option to default in which case less than the face value is recovered. $\Pi_{t+k}$ depends on the history of some exogenous aggregate source of risk $\epsilon_t$ from period 0 until $t+k$ and on the loan-to-collateral ratio $\frac{L_t}{K_t}$. Default is more likely the higher the loan-to-collateral ratio, hence $\frac{\partial \Pi_{t+k}}{\partial L_t K_t} < 0$, and the lower the exogenous source of risk, and therefore $\frac{\partial \Pi_{t+k}}{\partial \epsilon_x} > 0$ for $x \in \{0,1,...,t+k\}$. Given the pricing of loans, the market leverage of the financial intermediary can then be defined as $Lev_t = \frac{B_t}{Q_t L_t}$.

Assume further that the economy has experienced a prolonged boom until period $t$ characterised by several positive realisations of the exogenous source of risk. During the boom period, intermediaries expanded their balance sheets by increasing their debt $B_t$ to issue loans $L_t$ backed by relatively less collateral, such that $\frac{L_t}{K_t}$ has increased. However, even though the size of the intermediary’s balance sheet expanded during the boom, its market leverage increased only mildly since $Q_t$ remained high as default ratios are expected to remain low. The length of the boom period and the maturity mismatch play important roles in this regard. The longer the boom lasts, the stronger the balance sheet expansion of the financial sector and the more old, maturing (and potentially safe) loans have been replaced by new loans backed by less collateral.

The economy enters period $t+1$. During this period, the intermediary does not issue any new loans, does not pay out dividends, and cannot raise equity. The financing gap of the financial intermediary in period $t+1$ can be written as

$$Gap_{t+1} = (\gamma Q_{t+1} L_t + B_t) - (\gamma Q_{t+1} L_t + (1-\gamma)L_t \Pi_{t+1})$$

stating that the intermediary receives income from the repayment of maturing loans and selling non-maturing loans, while it has to repay its outstanding debt $B_t$ and purchases non-matured loans. The financing gap simplifies to

$$Gap_{t+1} = B_t - (1-\gamma)L_t \Pi_{t+1}$$
since the financial intermediary is the sole buyer of financial assets. Assume that the economy experiences a sudden bust in period $t+1$ such that $\epsilon_{t+1}$ and $\Pi_{t+k}$ for $k \geq 1$ are low. Next, consider two cases. First, the case in which loans are short-term, $\gamma = 0$, and there is no maturity mismatch. Then, the intermediary is solvent if $L_t \Pi_{t+1} > B_t$ and the financing gap is negative or put differently, the net worth is positive. In this case, there is no rigidity with respect to the intermediary’s balance sheet. Given a positive net worth, the intermediary is completely free to choose its new debt - and if one considers new investments, then also its new loans, the loan-to-collateral ratio on new loans, and hence its leverage.

In contrast, consider next the case in which loans are long-term, $0 < \gamma < 1$, and the intermediary runs a maturity mismatch. In this case, the financing gap is $B_t - (1 - \gamma)L_t \Pi_{t+1}$ which is likely to be positive, in particular if profits are low. The amount of new debt $B_{t+1}$ has to cover at least this gap. The intermediary’s leverage is then given by $Lev_{t+1} = \frac{B_{t+1}}{Q_{t+1} \gamma L_t}$ where the price of the non-matured loans $\gamma L_t$ is

$$Q_{t+1} = E_t \left[ \sum_{k=2}^{\infty} \Lambda_{t+1,t+k} \gamma^{k-2} (1 - \gamma) \Pi_{t+k} \left( \frac{L_t}{R_t}, \{\epsilon_t\}_{t=0} \right) \right]$$

which is low due the fall in $\Pi_{t+k}$ for $k \geq 1$. The leverage is higher the lower the price of non-matured loans, $\frac{\partial Lev_{t+1}}{\partial Q_{t+1}} < 0$, and the larger the initial stock of debt, $\frac{\partial Lev_{t+1}}{\partial B_t} > 0$, which has to be serviced. The new price of loans $Q_{t+1}$ itself is lower, the higher the initial loan-to-collateral ratio, $\frac{\partial Q_{t+1}}{\partial (L_t K_t)} < 0$, since borrowers are more likely to default. Hence, if the adverse shock is large enough, the intermediary’s leverage is going to increase, especially if the financial sector is unable to deleverage by raising equity or selling loans to an outsider.

The above comparative statics illustrate that the preceding boom, during which the intermediary expanded its balance sheet, increased the likelihood of a sharp spike in the intermediary’s leverage after a sudden bust and the increase in the financial sector’s leverage may cause further problems between the intermediary and its creditors, potentially leading to a creditor run. In the case of a maturity mismatch, the intermediary is unable to work against these effects in the short run since the intermediary is not entirely free to choose the structure of its balance sheet, but instead the amount of new debt $B_{t+1}$, overall loans in period $t+1$, and the associated loan-to-collateral ratios are partly set by the legacy assets and liabilities. In this sense, the maturity mismatch also plays an important role in the bust since it increases the rigidity of the financial sector’s balance sheet.

This simple example can be used to interpret Figure (1). During the run-up to the Great Recession, intermediaries expanded their balance sheets by issuing more loans and acquired more debt, but their market leverage increased only slowly since prices were elevated. However, after the sudden bust, intermediaries’ leverage spiked sharply since they held a portfolio of non-performing loans and the prices of these loans fell faster than they were able to decrease their debt burden, causing further problems between intermediaries and creditors. Next, these ideas are incorporated into an infinite-horizon, discrete-time production economy.
4 Real Model

4.1 Household

Following Greenwood, Hercowitz and Huffman (1988), the representative household values consumption $C_t$ and dislikes labor $H_t$ captured by the flow utility

$$U(C_t, H_t) = \log \left( C_t - \chi \frac{H_t^{1+\phi}}{1+\phi} \right)$$

where $\phi$ represents the inverse Frisch elasticity of labor supply. The household chooses contingent plans for consumption, labor supply, and savings, in the form of short-term and riskless bonds $B^H_t$, so as to maximize lifetime utility, while discounting the future at the rate $\beta^H$. Taking prices, wages, and interest rates as given, the household solves the problem

$$V^H (B^H_{t-1}, S_t) = \max_{C_t, H_t, B^H_t} \left\{ U(C_t, H_t) + \beta^H \mathbb{E} \left[ V^H (B^H_t, S_{t+1}) \right] \right\}$$

subject to

$$C_t + \frac{B^H_t}{R_{t+1}} \leq w_t H_t + B^H_{t-1} + T_t$$
$$S_{t+1} = \Gamma(S_t)$$

where $w_t$ is the real wage and $R_{t+1}$ is the real interest rate on short-term bonds between period $t$ and $t+1$ which is known in period $t$. $\Gamma(.)$ denotes the law of motion for aggregate state variables $S_t$ and the expectation $\mathbb{E} [.]$ is formed conditional on the information set $S_t$ at time $t$. Moreover, the household receives lump-sum transfers $T_t$ from firms, described in more detail below. The solution to the above problem gives the inter- and intratemporal optimality conditions

$$1 = \mathbb{E} [\Lambda_{t,t+1}] R_{t+1}$$
$$w_t = \chi H_t^\phi$$

where

$$\Lambda_{t,t+1} = \beta^H \left( \frac{C_t - \chi \frac{H_t^{1+\phi}}{1+\phi}}{C_{t+1} - \chi \frac{H_{t+1}^{1+\phi}}{1+\phi}} \right)$$

is the household’s stochastic discount factor.

4.2 Good Production

A representative good producer operates according to a Cobb-Douglas production function

$$Y_t = A_t K_t^\alpha H_t^{1-\alpha}$$  (1)
combining labor $H_t$ supplied by the household with aggregate capital $K_t$ supplied by entrepreneurs in period $t-1$ in order to produce the good $Y_t$. The only source of aggregate risk in the model enters via the technology level $A_t$

$$A_t = e^{a_t}$$
$$a_t = \rho_a a_{t-1} + \epsilon_t^a$$
$$\epsilon_t^a \sim N(0,\sigma^2_a)$$

where $\epsilon_t^a$ is termed the technology shock. Factor markets are competitive, and, as a result, all available factors are employed and pay their marginal products, which are given by

$$w_t = (1 - \alpha) \frac{Y_t}{H_t}$$
$$r^K_t = \alpha \frac{Y_t}{K_t}$$

where $r^K_t$ is the rental rate per unit of capital. The role of financial intermediation is to transform the household’s short-term savings $B^H_t$ into investment in the economy’s aggregate capital stock $K_{t+1}$ which enters the production function of the good producer in period $t + 1$. The two types of agents which fulfill this role, the financial intermediary and entrepreneurs, are at the heart of the model and described in detail next. The intermediation chain is summarized in Figure (2).

![Figure 2: Intermediation chain.](image)

### 4.3 Entrepreneurs

First, the evolution of long-term debt and the entrepreneurs’ default decision are introduced for a given newly issued loan. The next section outlines the agency problem between entrepreneurs and the financial intermediary which determines the amount of newly issued loans.

#### 4.3.1 Stochastic Maturity and Default

An entrepreneur who acquires a loan in period $t$ is termed a ‘new’ entrepreneur in that period (highlighted by the superscript new). There is a unit mass of new entrepreneurs and since all new entrepreneurs turn out to be identical, I omit entrepreneur-specific notation. A new entrepreneur has net worth $N_t$ and acquires a long-term, collateralized, and defaultable loan $Q_t L_t^{new}$ from the financial intermediary (both net worth and loan value are determined shortly).
Combining $N_t$ and $Q_t L_{t}^{new}$, the entrepreneur purchases $K_{t+1}^{new}$ units of capital

$$Q^K_t K_{t+1}^{new} = N_t + Q_t L_{t}^{new}$$

where $Q^K_t$ denotes the price of capital and $Q_t$ the price of long-term loans. The underlying collateral of the loan are the units of capital $K_{t+1}^{new}$ which an entrepreneur purchases and the face value of the loan is $L_{t}^{new}$ which the entrepreneur has to repay to the intermediary. The capital is lent to the good producer and the returns on the entrepreneurs’ investments are risky, since the amount of capital is chosen at least one period in advance.

In order to keep the model tractable and the number of aggregate state variables to a minimum, long-term debt is introduced as follows. Each loan has an exogenous stochastic maturity and matures with probability $1 - \gamma$ in the next period. An entrepreneur with a non-maturing loan dating from period $t$ does not have to make a payment to the financial intermediary, but receives the rental rate $r^K_{t+k}$ per unit of capital in period $t + k$ for $k \in \{1, 2, ..., \infty\}$ from the good producer. These profits are transferred lump-sum to the household. The underlying collateral $K_{t+1}^{new}$ of a non-maturing loan stays constant as the household refurbishes depreciated capital $\delta K_{t+1}^{new}$ where $\delta$ is the rate of depreciation. Moreover, the face value $L_{t}^{new}$ remains unchanged as an entrepreneur does not have to make a payment to the financial intermediary if the loan does not mature.

If a still outstanding loan, initially given out in period $t$, matures in period $t + k$, then the entrepreneur receives the rental rate $r^K_{t+k}$ per unit of capital and sells the remaining capital for the price $Q^K_t$. Additionally, the profits of a maturing loan are hit by an idiosyncratic shock $\omega$, with $\omega \sim U[1 - b, 1 + b]$ which is normalized to have mean unity and is drawn independently across time and entrepreneurs. The overall profits are

$$\omega R^K_{t+k} Q^K_t K_{t+1}^{new}$$

where $R^K_{t+k} = \left(\frac{Q^K_{t+k} (1-\delta) + r^K_{t+k}}{Q^K_{t+k-1}}\right)$. An entrepreneur compares these profits to the face value of debt $L_{t}^{new}$ and decides whether to default on its obligation to repay or not. An entrepreneur defaults if $L_{t}^{new}$ is larger than the entrepreneur’s profits $\omega R^K_{t+k} Q^K_t K_{t+1}^{new}$. Or put differently, if the idiosyncratic shock $\omega$ is lower than a threshold level $\omega_{t+k|t}$ in period $t + k$ which depends on the aggregate profits to capital $R^K_{t+k} Q^K_t K_{t+1}^{new}$ and the loan-to-collateral ratio $\frac{L_{t}^{new}}{K_{t+1}^{new}}$ for a loan issued in period $t$.

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4 This assumption ensures that the probability of default of very old entrepreneurs does not approach one since their underlying capital and therefore their profits do not approach zero.

5 This assumption implies no default considerations before a loan matures which makes the problem tractable and aggregation feasible as shown in Appendix A.1.

6 One could also add an idiosyncratic shock on the profits of a non-maturing loan. However, this would not change the model since these profits are transferred lump-sum to the household and would therefore wash out in the aggregate if such an idiosyncratic shock has mean unity. For simplicity, it is therefore omitted in the above description.
Default Decision. An entrepreneur from period $t$ whose loan matures in $t + k$ where $k \in \{1, 2, ..., \infty\}$ defaults iff

$$\omega < \omega_{t+k|t}$$

where

$$\omega_{t+k|t} = \frac{L_{t}^{new}}{R_{t+k}^{K}Q_{t+k-1}^{K}K_{t+1}^{new}}$$

Equation (2) highlights that the model incorporates vintage-specific default thresholds $\omega_{t+k|t}$ which depend on the face value of debt $L_{t}^{new}$ and the underlying collateral $K_{t+1}^{new}$ from period $t$. Loans issued in different periods under distinct states of the economy can therefore have different default thresholds when maturing in the same period $t + k$. The following simple example illustrates this point.

Example. If the loan-to-capital ratios of loans issued in periods $t - 2$ and $t - 1$ are such that

$$\frac{L_{t-2}^{new}}{K_{t-1}^{new}} > \frac{L_{t-1}^{new}}{K_{t-1}^{new}},$$

and these loans are maturing in period $t$, then their default thresholds in period $t$ relate according to

$$\omega_{t|t-2} = \frac{L_{t-2}^{new}}{R_{t-2}^{K}Q_{t-1}^{K}K_{t-1}^{new}} > \frac{L_{t-1}^{new}}{R_{t-1}^{K}Q_{t-1}^{K}K_{t+1}^{new}} = \omega_{t|t-1}$$

The loan with a higher loan-to-capital ratio also has a higher default threshold, because both receive the same aggregate profits $R_{t}^{K}Q_{t-1}^{K}$ per unit of capital.

If an entrepreneur defaults, then $(1 - \mu)\omega R_{t+k}^{K}Q_{t+k-1}^{K}K_{t+1}^{new}$ is recovered by the financial intermediary where $\mu$ is the fraction of profits lost and reflects the costs of default. If an entrepreneur does not default, $L_{t}^{new}$ is repaid to the financial intermediary and the remaining profits $\omega R_{t+k}^{K}Q_{t+k-1}^{K}K_{t+1}^{new} - L_{t}^{new}$ are equally split among new entrepreneurs in period $t + k$ which ensures that all new entrepreneurs start with the same amount of net worth. New entrepreneurs consist of all entrepreneurs whose loans matured (whether they defaulted or not), ensuring that the number of entrepreneurs stays constant. An interpretation of the above specification is that firms face long-term projects during which their capital remains fixed and they take on long-term debt in order to finance these projects. The model is therefore in line with the observation that firms infrequently change their capital stock and borrowing. Figure (3) summarizes the described set-up of long-term debt under stochastic maturity and default.
4.3.2 Risk-shifting problem

A new entrepreneur cannot only invest in the described project, which I term the good project, but also into a second project which is called the bad project. A project is chosen when a loan is taken up and an entrepreneur cannot switch projects for the duration of the loan contract. The properties of the bad project are as follows.

**Bad Project.** The good and bad project only differ in the properties of the distribution from which the idiosyncratic shock is drawn which hits an entrepreneur’s profits when his loan matures. The profits under the bad project are denoted $\tilde{\omega} R_{t+k}^K Q_{t+k-1}^K K_{t+1}^{new}$ with $\tilde{\omega} \sim U[\tilde{k} - \tilde{c}, \tilde{k} + \tilde{c}]$ which is independent across entrepreneurs and time. It is assumed that $\tilde{c} > b$ and $\tilde{k} < 1$, i.e. $\tilde{\omega}$ has a higher variance and a lower mean compared to $\omega$.

In the model’s equilibrium under reasonable parameter values which ensure that the bad project is sufficiently different from the good project, it can be shown that the above assumptions ensure that lending to entrepreneurs, who invest in the bad project, is a negative net present value investment for the financial intermediary. In contrast, a leveraged entrepreneur may prefer the bad project over the good project, due to the possibility to take up a loan under limited liability and due to the higher variance (and therefore the higher potential upside) of the bad project. Under the assumption that the debt contract cannot be conditioned on the type of the project, the financial intermediary would therefore never agree to contracts for which entrepreneurs have the incentive to invest into the bad project. Hence, any agreement must satisfy the incentive compatibility (IC) constraint which demands that the entrepreneur’s objective function under the good project $V_{t}^{good}$ is at least as large as the objective function under the bad project $V_{t}^{bad}$, i.e.

$$V_{t}^{good} \geq V_{t}^{bad} \quad (3)$$

Knowing that any debt contract with the financial intermediary has to satisfy the IC constraint at the time when a contract is negotiated, inequality (3) directly enters as a constraint into a new entrepreneur’s decision problem. Given the net worth from maturing entrepreneurs $N_t$ and the price of new debt $Q_t$, a risk-neutral, new entrepreneur chooses...
the amount of debt $L_{t+1}^{new}$ to take on and the units of capital $K_{t+1}^{new}$ to purchase.\footnote{The net worth of a new entrepreneur is given by $N_t = (1 - \gamma) \sum_{j=1}^{\infty} \int_{\tilde{\omega}_{i(t-j)}}^{1} \left( \omega R_{t+k}^K Q_{t+k-1}^{K_{t+1}} - L_{t+1}^{new} \right) d\Phi(\omega)$.} The complete decision problem can be written as

$$\max_{K_{t+1}^{new}, L_{t+1}^{new}} V_t^{good} = \mathbb{E} \left[ \sum_{k=1}^{\infty} \left( 1 - \gamma \right) \int_{\tilde{\omega}_{t+k}}^{1} (\omega R_{t+k}^K Q_{t+k-1}^{K_{t+1}} - L_{t+1}^{new}) d\Phi(\omega) + \gamma r_{t+k} K_{t+1}^{new} \right]$$

subject to

$$Q_t^K K_{t+1}^{new} = N_t + Q_t L_{t+1}^{new}$$

$$\tilde{\omega}_{t+k|t} = \frac{L_{t+1}^{new}}{R_{t+k}^K Q_{t+k-1}^{K_{t+1}}} \text{ for } k \in \{1, 2, \ldots, \infty\}$$

$$V_t^{good} \geq V_t^{bad}$$

$$S_{t+1} = \Gamma(S_t)$$

where $\Phi(\omega)$ is the c.d.f. for $\omega$ and $V_t^{bad}$ is defined equivalent to $V_t^{good}$, taking into account the properties of the idiosyncratic shock $\tilde{\omega}$ under the bad project stated above. The entrepreneur’s objective function $V_t^{good}$ captures both the profits if the loan matures and the entrepreneur does not default (the first term in the curly bracket) as well as the profits received until the loan matures (the second term in the curly bracket). Constraint (5) is the entrepreneur’s budget constraint. The constraints summarized in (6) define the future default thresholds $\tilde{\omega}_{t+k|t}$ and constraint (7) gives the IC constraint. For the chosen calibration which ensures that the bad project is sufficiently different to the good project, it can be shown that the IC constraint is always binding, since the derivative of $V_t^{good}$ with respect to $L_{t+1}^{new}$ is positive at the point where $V_t^{good}$ and $V_t^{bad}$ intersect. The entrepreneur would therefore prefer to borrow more, but cannot, because a higher amount of debt would induce the entrepreneur to switch to the bad project. Proposition 1 argues that this reduces the entrepreneur’s problem to just one equilibrium condition which largely simplifies the problem.

**Proposition 1.** Since the IC constraint (7) is always binding, the solution to the entrepreneur’s decision problem is given by $L_{t+1}^{new}$ where

$$0 = \left( \frac{N_t + Q_t L_{t+1}^{new}}{Q_t^K} \right) J_t - L_{t+1}^{new} G_t + \frac{Q_t^K (L_{t+1}^{new})^2}{(N_t + Q_t L_{t+1}^{new})} S_t$$

and $J_t$, $G_t$, and $S_t$ are three ‘forward-looking’ auxiliary variables defined in Appendix A.2.

**Proof:** See Appendix A.1.

Based on the calibration in section 4.7, Figure (4) plots the entrepreneur’s objective functions $V_t^{bad}$ and $V_t^{good}$ against the amount of real new debt $Q_t L_{t+1}^{new}$ raised. The left of Figure (4) shows that $V_t^{bad}$ intersects $V_t^{good}$ exactly once from below. The intersection determines the amount of debt $Q_t L_{t+1}^{new}$ a new entrepreneur can raise and the relative movements of the two objective functions over the business cycle change this amount. For example, during a recession, both $V_t^{bad}$ and $V_t^{good}$ decrease, but $V_t^{bad}$ decreases less relative to $V_t^{good}$, because the bad project offers a higher upside and therefore
becomes relatively more attractive to the entrepreneur when profits to capital are low and default thresholds high. The risk-shifting problem therefore becomes more severe and credit to new entrepreneurs is rationed. The right of Figure (4) illustrates this point. The opposite holds during a boom when profits are high, default thresholds are low, and the good project becomes relatively more attractive because it offers a higher expected profit than the bad project. Incentives to risk-shift therefore decrease in a boom and new entrepreneurs are allowed to raise more debt.

Figure 4: Credit Rationing. Left: Based on the calibration of the model as outlined in section 4.7, the objective functions \( V^{\text{good}} \) and \( V^{\text{bad}} \) are plotted against \( Q_t L_t^{\text{new}} \). All state variables are at their stochastic steady state values and all endogenous variables are held constant while \( L_t^{\text{new}} \) is varied. The range of \( L_t^{\text{new}} \) is chosen to cover a three standard deviation of \( L_t^{\text{new}} \) from its steady state value (based on a simulation of the model). Right: Additionally, the objective functions \( V^{\text{good}} \) and \( V^{\text{bad}} \) are plotted for which all state variables are again at their steady state values, except for the technology shock which is two standard deviations below its mean, representing an economy in a recession.

4.4 Financial Intermediary

The role of the representative financial intermediary is to transform short-term and riskless debt \( B_t \) into long-term and risky loans \( L_t \), given out to entrepreneurs. The financial intermediary is perfectly diversified and invests into the whole market portfolio of loans \( L_t \) which is defined recursively and encompasses all outstanding loans

\[
L_t = L_t^{\text{new}} + \gamma L_{t-1}
\]

I assume that the financial intermediary values shareholders’ flow utility of real dividends \( D_t \) which is denoted as

\[
U(D_t) = \log(D_t)
\]
Given the aggregate state variables $S_t$, its outstanding debt and interest $B_{t-1}R_t$, and loans $L_{t-1}$ from last period, the financial intermediary chooses new short-term debt $B_t$, loans $L_t$, and dividends $D_t$ every period to maximize life-time shareholder utility, discounted at the rate $\beta^F$. Taking prices and interest rates as given, the financial intermediary solves

"Unconstrained Problem".

$$V(S_t, B_{t-1}R_t, L_{t-1}) = \max_{D_t, L_t, B_t} \{ U(D_t) + \beta^F \mathbb{E}[V(S_{t+1}, B_tR_{t+1}, L_t)] \}$$

subject to

$$D_t + Q_tL_t + B_{t-1}R_t \leq B_t + R^L_{t-1}Q_{t-1}L_{t-1}$$

$$S_{t+1} = \Gamma(S_t)$$

where $Q_t$ is the price and $R^L_t$ the return per loan in period $t$. Constraint (8) is a budget constraint, stating that the amount of new debt $B_t$ and the profits on last period’s loans $R^L_{t-1}Q_{t-1}L_{t-1}$ have to cover at least the payout of dividends $D_t$, new loan investment $Q_tL_t$, and outstanding debt and interest $B_{t-1}R_t$. The above problem implies that the financial intermediary never raises equity, but always issues a positive amount of real dividends. This assumption is reasonable, given that the model is mainly used to analyze the boom period before a financial crisis occurs, during which intermediaries can retain earnings to build up their capital buffer, and the bust, during which it is difficult to raise equity. The solution to the intermediary’s “Unconstrained Problem” is given by two intertemporal optimality conditions

$$\frac{1}{D_t} = \beta^F \mathbb{E}\left[ \frac{1}{D_{t+1}} \right] R_{t+1}$$

$$\frac{1}{D_t} = \beta^F \mathbb{E}\left[ \frac{1}{D_{t+1}} R^L_{t+1} \right]$$

I define $R^L_t$ to be the return on the whole market portfolio of loans which is given by

$$R^L_t = \frac{\gamma Q_t + (1-\gamma) \frac{1}{Q_{t-1}} \left\{ \sum_{j=1}^{\infty} (1 - \Phi(\omega_{t|t-j})) \gamma^{j-1} L_{t-j}^{\text{new}} + (1-\mu) \sum_{j=1}^{\infty} \left[ \frac{\sum_{k=0}^{\infty} \omega_k}{\gamma^{j+1}} \int_{1-b}^1 \omega_{t-j} R^K_{t-1} Q_{t-1}^{K_1} K_{t-j-1}^{\text{new}} d\Phi(\omega) \right] \right\}}{Q_{t-1}}$$

$R^L_t$ captures both returns from non-maturing loans $\gamma Q_t$, as well as returns from maturing loans across all vintages. The latter consist of repaid loans (the first term in the curly bracket) and the recovery from defaulted loans (the second term in the curly bracket). It seems to be rather difficult to account for all loans in the economy. There is an infinite number of outstanding loans with vintage-specific debt $L_{t-1}^{\text{new}}$ and underlying collateral $K_{t-1}^{\text{new}}$ and hence vintage-specific default thresholds $\omega_{t|t-j} = \frac{L_{t-1}^{\text{new}}}{R^K_{t-1} Q_{t-1}^{K_1} K_{t-j-1}^{\text{new}}}$ in period $t$. However, due to the exogenous stochastic maturity assumption, three aggregate state variables can account for $R^L_t$. Proposition 2 formalizes this claim.
Proposition 2. The aggregate state variables

\[ L_t = L^{\text{new}}_t + \gamma L_{t-1} \]
\[ K_{t+1} = K^{\text{new}}_{t+1} + \gamma K_t \]
\[ x_t = \frac{(L^{\text{new}}_t)^2}{K^{\text{new}}_t} + \gamma x_{t-1} \]

can account for all outstanding loans \( L_t \), their combined collateral \( K_{t+1} \), and all vintage-specific loan-to-capital ratios via the auxiliary variable \( x_t \). Given the return on capital and the price per loan \( Q_t \) in period \( t \), the profits per loan \( R^K_t Q_{t-1}^K \) can be expressed in terms of only these three aggregate state variables.

Proof: See Appendix A.1.

The auxiliary variable \( x_t \) takes an intuitive form. It accounts for all vintage-specific loan-to-capital ratios since it is updated with the loan-to-capital ratio \( \frac{L^{\text{new}}_t}{K^{\text{new}}_{t+1}} \) of newly issued loans each period, weighted by the face value \( L^{\text{new}}_t \) of these loans. One can therefore interpret \( x_t \) as a loan risk indicator. Further, by dividing \( x_{t-1} \) by all outstanding loans \( L_{t-1} \) and current profits to capital \( R^K_t Q_{t-1}^K \), one can derive a weighted default threshold of all outstanding loans

\[ \omega_t \equiv \frac{x_{t-1}}{L_{t-1} R^K_t Q_{t-1}^K} = \sum_{k=1}^{\infty} \frac{k L^{\text{new}}_t}{L_{t-1}} \omega_{t-k} \]

where \( \frac{L^{\text{new}}_t k^{-1}}{L_{t-1}} \) for \( k \in \{1, 2, ..., \infty\} \) is the remaining fraction of a vintage of loans relative to all outstanding loans and \( \omega_{t-k} \) for \( k \in \{1, 2, ..., \infty\} \) is the default threshold for vintage \( t-k \) in period \( t \).

The model gives a realistic representation of the financial sector’s balance sheet. Table (1) illustrates the time dependence of the intermediary’s asset portfolio in period \( t \) and how it intuitively relates to the aggregate state variables \( L_t \), \( K_{t+1} \), and \( x_t \). For each vintage of loans, the amount of outstanding loans, their remaining collateral, and their default threshold in period \( t \) are shown. For example, with respect to the vintage of loans issued in period \( t-2 \) an amount \( \gamma^2 L^{\text{new}}_{t-2} \) remains outstanding in period \( t \), backed by collateral \( \gamma^2 K_{t-1}^{\text{new}} \), and the default threshold in period \( t \) for the maturing loans of this vintage is given by \( \omega_{t-2} \). Table (1) highlights that loans which were issued in the past are going to remain on the financial sector’s balance sheet for a longer period of time and that the conditions under which a loan was initially issued in the past determines its payoff in the present.

<table>
<thead>
<tr>
<th>Loans issued in period</th>
<th>( t )</th>
<th>( t-1 )</th>
<th>( t-2 )</th>
<th>( t-3 )</th>
<th>...</th>
<th>State Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans outstanding in period ( t )</td>
<td>( L^{\text{new}}_t )</td>
<td>( \gamma L^{\text{new}}_{t-1} )</td>
<td>( \gamma^2 L^{\text{new}}_{t-2} )</td>
<td>( \gamma^3 L^{\text{new}}_{t-3} )</td>
<td>...</td>
<td>( \sum_{k=0}^{\infty} \gamma^k L^{\text{new}}_{t-k} = L_t )</td>
</tr>
<tr>
<td>Collateral remaining in period ( t )</td>
<td>( K^{\text{new}}_{t+1} )</td>
<td>( \gamma K^{\text{new}}_{t} )</td>
<td>( \gamma^2 K^{\text{new}}_{t-1} )</td>
<td>( \gamma^3 K^{\text{new}}_{t-2} )</td>
<td>...</td>
<td>( \sum_{k=0}^{\infty} \gamma^k K^{\text{new}}<em>{t+1-k} = K</em>{t+1} )</td>
</tr>
<tr>
<td>Default threshold in period ( t )</td>
<td>-</td>
<td>( \omega_{t-1} )</td>
<td>( \omega_{t-2} )</td>
<td>( \omega_{t-3} )</td>
<td>...</td>
<td>( \sum_{k=0}^{\infty} \gamma^k \frac{(L^{\text{new}}<em>{t-k})^2}{K^{\text{new}}</em>{t-k+1}} = x_t )</td>
</tr>
</tbody>
</table>

Table 1: **Balance Sheet Financial Intermediary.** Illustration of the financial intermediary’s assets in period \( t \).
4.4.1 Occasional Financial Crises

Next, I introduce occasional financial crises into the above framework. The trigger for a crisis and the actions during a crisis are chosen such that the model captures the following sequence of events.

(i) The financial sector’s assets drop in value which ...

(ii) ... increases the financial sector’s market leverage (see Figure 1).

(iii) Creditors are worried about repayment and restrict their funding to the financial sector which ...

(iv) ... decreases the intermediaries’ demand for financial assets and leads to a contraction of new lending.

In accordance with this sequence of events, a financial crisis is triggered if the intermediary’s choices under the “Unconstrained Problem” are such that its leverage $\frac{B_t}{Q_t L_t}$ in period $t$ exceeds the threshold $\kappa$. If that is the case, the financial intermediary additionally faces the constraint

$$B_t \leq \tau(S_t) \quad (11)$$

which states that the amount of new debt $B_t$ cannot exceed an upper limit $\tau(S_t)$. This upper limit is a function of aggregate state variables (described in more detail in the calibration of the model), but taken as given by the financial intermediary. The constraint (11) directly enters into the intermediary’s decision problem and the intermediary solves “Constrained Problem”.

$$V(S_t, B_{t-1} R_t, L_{t-1}) = \max_{D_t, L_t, B_t} \left\{ U(D_t) + \beta F E[V(S_{t+1}, B_t R_{t+1}, L_t)] \right\}$$

subject to

$$D_t + Q_t L_t + B_{t-1} R_t \leq B_t + R^L_t Q_{t-1} L_{t-1}$$

$$B_t \leq \tau(S_t) \quad (12)$$

$$S_{t+1} = \Gamma(S_t)$$

The solution to the problem is given by the two intertemporal optimality conditions

$$\frac{1}{D_t} - \lambda_t = \beta F E \left[ \frac{1}{D_{t+1}} \right] R_{t+1}$$

$$\frac{1}{D_t} = \beta F E \left[ \frac{1}{D_{t+1}} R^L_{t+1} \right] \quad (13)$$

where $\lambda_t$ is the lagrange multiplier on the debt constraint in (12), which is positive if $\tau(S_t)$ is sufficiently low and the intermediary is therefore funding-constrained. I define a financial crisis or a creditor run to be a period in which the financial intermediary’s leverage under the “Unconstrained Problem” exceeds the threshold $\kappa$ and the intermediary has to solve the “Constrained Problem” instead. Creditor runs are linked to the balance sheet of the financial intermediary.
since they only occur if the intermediary’s unconstrained choices are such that its leverage exceeds the threshold $\kappa$ which is consistent with the mentioned empirical evidence by Gorton (1988). In order to ensure that a financial intermediary is not caught in a spiral of constant creditors runs, I further assume that creditor runs cannot occur for two consecutive periods. This assumption ensures stability of the model and is in line with the observation that creditor runs generally occur within a small time window.

### 4.4.2 Risk and Liquidity Premia

The two Euler equations in (13) can be combined to an arbitrage equation

$$E \left[ \beta F \frac{D_t}{D_{t+1}} R_t \right] = E \left[ \beta F \frac{D_t}{D_{t+1}} \right] R_{t+1} + \lambda_t D_t$$

Denote the stochastic discount factor $M_{t,t+1} = \beta F D_t D_{t+1}$ and excess returns $R^X_{t+1} = R^L_{t+1} - R_{t+1}$. The expected excess return can then be divided into a classic “risk-premium” and a second component which is termed a “liquidity premium”

$$E \left[ R^X_{t+1} \right] = \frac{\lambda_t D_t}{E \left[ M_{t,t+1} \right]} - \frac{\text{Cov}(M_{t,t+1}, R^X_{t+1})}{E \left[ M_{t,t+1} \right]}$$

(14)

If $\lambda_t > 0$, the intermediary is funding constrained and the liquidity premium is positive which raises the expected excess return. This is in line with the observation that expected excess returns are generally high when an economy experiences a financial crisis during which intermediaries are funding-constrained (Muir, 2014).

### 4.5 Capital Producers and Resource Constraint

Capital good producers undertake real investment. Given the price of capital $Q^K_t$, capital good producers maximize their profits by choosing the economy-wide units of investment $I_t$

$$\max_{I_t} \{Q^K_t I_t - \Phi(I_t, K_t)\}$$

subject to

$$\Phi(I_t, K_t) = I_t + \frac{\zeta}{2} \left( \frac{I_t - \delta K_t}{K_t} \right)^2 K_t$$

where $I_t = K_{t+1} - (1-\delta)K_t$ and $\Phi(I_t, K_t)$ reflects quadratic adjustment costs which follows the parsimonious specification in He and Krishnamurthy (2014). The above problem gives the intratemporal optimality condition

$$Q^K_t = 1 + \frac{\zeta}{K_t} \left( \frac{I_t}{K_t} - \delta \right)$$

I complete the description of the real model by stating the resource constraint
\[ Y_t = C_t + D_t + I_t + \frac{\zeta}{2} \left( \frac{I_t - \delta K_t}{K_t} \right)^2 K_t + (1 - \gamma)\mu \sum_{j=1}^{\infty} \left[ \int_{1-b}^{\infty} \omega R_t^K Q_t^K \gamma \sum_{k=1}^{\infty} \mu_{t+j} K_t^{\text{new}} d\Phi(\omega) \right] \]

implying that the final output good is used as consumption, as conversion into capital goods, and for covering real default costs.

### 4.6 Equilibrium Conditions and Numerical Solution

Given the optimality conditions to the agents’ decision problems and the clearing of goods, labor, and debt markets, the equilibrium conditions of the model for \( t = 0, 1, \ldots, \infty \) are listed in Appendix A.2, given an initial state \( S_0 \). The endogenous variables can be separated into a vector of non-state variables \( X_t \) and a vector of state variables \( S_t \). \( X_{t+1} \) is unknown in period \( t \) and \( S_t = \{ \mathbf{S}_t, \mathbf{\hat{S}}_t \} \) comprises both exogenous state variables \( \mathbf{S}_t \) and endogenous state variables \( \mathbf{\hat{S}}_t \).

\( \mathbf{S}_t = \{ \rho_a a_t - 1 + \epsilon_a t \} \) includes the technology shock \( \epsilon_a t \) and the probability distribution of this shock is known to all agents.

The realization \( \epsilon_a t+1 \) is unknown in period \( t \). \( \mathbf{\hat{S}}_t = \{ K_t, L_{t-1} - 1, x_{t-1}, R_t B_{t-1}, r_{t-1} \} \) collects the endogenous state variables and \( \mathbf{\hat{S}}_{t+1} \) is known in period \( t \). Overall, the model has six state variables. One is linked to the shock, three arise from the defaultable loan contracts, \( K_t, L_{t-1} \), and \( x_{t-1} \) (as defined in Proposition 2), \( R_t B_{t-1} \) accounts for the outstanding debt of the financial intermediary, and \( r_{t-1} \) is an indicator variable which is equal to one if there has been a creditor run in the previous period and zero otherwise.

**Definition 1.** A competitive general equilibrium is a solution of the model which is given by a set of policy functions \( \mathbf{\hat{S}}_{t+1} = f_{\mathbf{S}}(S_t) \) and \( X_t = f_X(S_t) \) which satisfy the model equations listed in Appendix A.2 for \( t = 0, 1, \ldots, \infty \) in the relevant state space.

Numerical methods are used to solve for the policy functions \( f_{\mathbf{S}}(S_t) \) and \( f_X(S_t) \). However, instead of relying on local perturbation methods which are commonly used to solve dynamic stochastic general equilibrium models, I solve the model using projection methods which give a global nonlinear solution of the model. I rely on such methods for two reasons. First, projection methods allow to integrate occasionally-binding constraints easily. Here, the amount of debt a financial intermediary can raise is occasionally constrained when a creditor run occurs. The intermediary’s problem is therefore solved sequentially. In a first step, the “Unconstrained Problem” is solved and if the intermediary’s leverage exceeds the threshold \( \kappa \) under this problem, the “Constrained Problem” is solved instead. And second, the solution method captures precautionary behavior by agents linked to whether a financial crisis may occur in the future due to the realization of future shocks. I give a detailed description of the solution algorithm in Appendix A.3.

### 4.7 Calibration

The model is calibrated to quarterly frequency for the U.S. economy. The structural parameters are listed in Table (2) and I discuss the calibration of the non-standard parameters next.
<table>
<thead>
<tr>
<th>Agents</th>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
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<td>Household</td>
<td>Discount factor HH</td>
<td>$\beta^H$</td>
<td>0.99</td>
<td>Literature</td>
</tr>
<tr>
<td></td>
<td>Inv. Frisch elasticity</td>
<td>$\phi$</td>
<td>0.5</td>
<td>Literature</td>
</tr>
<tr>
<td></td>
<td>Rel. utility weight</td>
<td>$\chi$</td>
<td>1.6</td>
<td>Normalization: $H \sim 1$ in steady state</td>
</tr>
<tr>
<td>Good Producer</td>
<td>Effective capital share</td>
<td>$\alpha$</td>
<td>0.3</td>
<td>Literature</td>
</tr>
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<td></td>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
<td>Literature</td>
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<td></td>
<td>Std. technology shock</td>
<td>$\sigma_a$</td>
<td>0.72%</td>
<td>Util.-adj. TFP series (Fernald, 2014)</td>
</tr>
<tr>
<td></td>
<td>Persist. technology shock</td>
<td>$\rho_a$</td>
<td>0.93</td>
<td>Util.-adj. TFP series (Fernald, 2014)</td>
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<td>Intermediary</td>
<td>Discount factor FI</td>
<td>$\beta^F$</td>
<td>0.987</td>
<td>Impulse response matching</td>
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<td></td>
<td>Leverage threshold</td>
<td>$\kappa$</td>
<td>0.49</td>
<td>Frequency Crises: $\sim 2.4%$</td>
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<td></td>
<td>Debt constraint</td>
<td>$\tau$</td>
<td>0.91</td>
<td>Severity Crises: $\frac{\Delta \text{GDP,Fin,Rec.}}{\Delta \text{GDP,Ave,Rec.}} \sim 1.12$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Boissay, Collard and Smets (2013, 2015)</td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>Width support of $\omega$</td>
<td>$b$</td>
<td>1</td>
<td>$\sigma^2_\omega = \frac{1}{3}$, BGG (1999)</td>
</tr>
<tr>
<td></td>
<td>Mean of $\tilde{\omega}$</td>
<td>$\bar{b}$</td>
<td>0.9</td>
<td>$E_t[R_{t+1}^L] &lt; 1 \forall t$ under bad project</td>
</tr>
<tr>
<td></td>
<td>Default costs</td>
<td>$\bar{c}$</td>
<td>1.55</td>
<td>Annual Default Rate $\sim 3%$</td>
</tr>
<tr>
<td></td>
<td>Stochastic maturity loan</td>
<td>$\gamma$</td>
<td>0.908</td>
<td>Bernanke, Gertler and Gilchrist (1999)</td>
</tr>
<tr>
<td>Capital Producer</td>
<td>Capital adjustment cost</td>
<td>$\zeta$</td>
<td>3</td>
<td>He and Krishnamurthy (2014)</td>
</tr>
</tbody>
</table>

Table 2: **Calibration.** Calibration of structural parameters.

The persistence and standard deviation of the technology shock are obtained by estimating an AR(1) process to the linearly detrended utilization-adjusted log-TFP-series by Fernald (2014) for the second half of the post-WWII period (1980Q1-2014Q2), giving $\rho_a = 0.93$ and $\sigma_a = 0.72\%$.

The difference between the household’s discount rate $\beta^H$ and the financial intermediary’s discount rate $\beta^F$ determines the leverage of the financial intermediary - the model’s main indicator of financial stability. The literature tends to calibrate this difference to match an empirical counterpart for the intermediary’s leverage. However, this approach faces the challenge that the financial intermediary sector is a very heterogeneous sector, encompassing for example hedge funds, commercial banks, and investment banks. Such financial institutions show quite distinct leverage ratios. It is therefore a challenge to find a one-size-fits-all-calibration. Moreover, besides the overall level of leverage, what is particularly important is how leverage behaves over the business cycle, as emphasized in the work of Adrian and Shin (2010), which in the end determines how the economy transits from a standard business cycle period into a financial crisis. In this regard, the impulse response of leverage to the model’s aggregate source of risk (the technology shock) governs how financial stability and aggregate risk interact and is crucial for the results which follow. I therefore obtain empirical evidence in this respect and match the model’s implied impulse response to this evidence by choosing the difference between $\beta^H$ and $\beta^F$. The intuition behind this calibration approach is that the difference between $\beta^H$ and $\beta^F$ not only determines the overall level of leverage, but also the shape of the impulse response of leverage. The larger the gap between $\beta^H$ and $\beta^F$, the higher the intermediary’s leverage. Moreover, the higher the initial leverage, the more strongly leverage increases after an adverse shock which decreases the value of assets - i.e. small changes in prices lead to large changes in leverage if leverage is initially high.
Moreover, after an adverse shock which decreases the value of assets, it also takes longer for leverage to reach again its initial level the more leveraged the intermediary is under constraints which prevent the sector from deleveraging quickly (such as restrictions on raising equity in the short run). Hence, by choosing the difference between $\beta^H$ and $\beta^F$, both level and impulse response of leverage are determined.\footnote{It should be noted that this calibration approach differs from Christiano, Eichenbaum and Evans (2005) in two aspects. First, Christiano, Eichenbaum and Evans (2005) estimate their model to minimize the distance between VAR and model responses to a monetary policy shock. Here, the fact that the model is solved globally complicates such an exercise and therefore the difference between $\beta^H$ and $\beta^F$ is calibrated to roughly match the initial sign and turning point of the empirical impulse response. Second, Christiano, Eichenbaum and Evans (2005) aim to match impulse responses of standard macroeconomic variables. Here, the fact that the model is solved globally complicates such an exercise and therefore the difference between $\beta^H$ and $\beta^F$ is calibrated to roughly match the initial sign and turning point of the empirical impulse response.}

The residual of the estimated AR(1) process for the utilization-adjusted TFP series gives a quarterly series of structural technology shocks $\{\hat{\epsilon}_t\}$ from 1980Q1 to 2014Q2. An impulse response function of the market leverage of financial institutions $\hat{\text{Lev}}_t$ to this shock series is obtained with a version of the method of local projections outlined in Jordà (2005).\footnote{As in Figure (1), I use data on commercial and investment banks. See Appendix A.8 for a description of the data and derivations.} Using the generalized method of moments (GMM), I simultaneously estimate the following system\footnote{GMM requires the choice of a weighting matrix and instruments. Regarding the weighting matrix, a Newey-West correction for heteroskedasticity and autocorrelation is used. The instruments are the regressors in (15) and (16).}.

$$\log(TFP)_t = \alpha_0 + \rho_0 \log(TFP)_{t-1} + \hat{\epsilon}_t$$ \hspace{1cm} (15)

$$\hat{\text{Lev}}_{t-1+k} - \hat{\text{Lev}}_{t-1} = \beta^k_0 + \beta^k_1 \hat{\epsilon}_t + \epsilon_{t+k} \quad \text{for} \quad k \in \{1, 2, ..., 20\}$$ \hspace{1cm} (16)

where $\beta^k_1$ gives the reaction of leverage to a technology shock at horizon $k$.\footnote{The system is estimated jointly in order to avoid a generated regressor problem. Such a problem arises when estimating the series $\{\hat{\epsilon}_t\}$ first and then using this series to derive impulse response functions. Newey and McFadden (1994) show that one can obtain consistent estimates by simultaneously estimating all equations, using GMM.} The coefficients are normalized to the obtained standard deviation $\sigma_{\hat{\epsilon}} = 0.72\%$ and the result is shown in Figure (5) for which the standard error of $\beta^k_1$ at horizon $k$ is used to obtain the confidence bands. Following a positive technology shock, leverage initially declines and then rises over time, turning positive after around eight quarters.\footnote{In (16), $\hat{\text{Lev}}_{t-1}$ and $\beta^k_0$ are used as a proxy for the path that leverage would have followed in the absence of a shock $E_{t-1} \left[ \hat{\text{Lev}}_{t-1+k} \right]$. The results are robust to controlling for additional lags of leverage. Given the limited length of the time series, up to three lags are included in this robustness check. These additional results are not reported, but are available upon request.} By calibrating $\beta^F$ for a given $\beta^H$, I find that an asset-to-equity ratio of around 2 gives a model implied impulse response that matches well the initial sign and the turning-point of the obtained empirical impulse response. This calibration gives a slightly lower leverage than observed among commercial and investment banks, which operate under asset-to-equity ratios above 5 in normal times (see for example Figure (1) in this regard), but matches well with other parts of the intermediary sector - such as hedge funds which show ratios of around 2 (Ang, Gorovyy and van Inwegen, 2011).

The leverage threshold $\kappa$ governs the frequency of crises in the model and I choose $\kappa$ to match an annual frequency of around 2.4%, based on calculations in Boissay, Collard and Smets (2015).\footnote{Using the specification in (16), I confirm that the initial decline in market leverage after a positive technology shock is due to an immediate increase in the market equity of financial institutions, whereas book values respond more slowly. Moreover, real GDP increases following a positive technology shock. These results are in line with the model’s impulse responses presented in section 4.8 and are available upon request.} When a creditor run occurs, a financial crises occurs
intermediary is restricted in the amount of new debt $B_t$ to raise. According to equation (11), the financial intermediary cannot take on more new debt than $\tau(S_t)$. I choose $\tau(S_t)$ such that the intermediary is always able to repay its outstanding debt, but has less new debt $B_t$ available to finance dividend payouts and newly issued loans compared to its unconstrained choices, i.e.

$$\tau(S_t) = \tau(D_t^\text{unc} + Q_t^\text{unc}L_t^{\text{new,unc}}) + B_{t-1}R_t - R_L^tQ_{t-1}L_{t-1} + Q_t\gamma L_{t-1}$$

where the superscript $\text{unc}$ describes the intermediary’s choices under the “Unconstrained Problem”. The parameter $0 < \tau < 1$ determines the severity of a financial crisis. Boissay, Collard and Smets (2013) show that for 14 OECD countries the drop in real GDP from peak to trough is around 11.8% bigger during financial recessions ($-6.61\%) than during average recessions ($-5.91\%)$ based on HP-filtered data. I choose $\tau$ such that the ratio between financial and average recessions in a simulation of the model matches this empirical finding, giving $\tau = 0.91$.\(^{16}\)

The following parameters govern the long-term debt contract between the financial intermediary and entrepreneurs. The parameter $b$ controls the variance of the idiosyncratic shock $\omega$, received by an entrepreneur with a maturing loan on its profits. I choose $b = 1$ which is convenient as it gives a lower bound of zero on the idiosyncratic shock and therefore ensures that the default thresholds are always above this lower bound. Given $b = 1$, the variance of the idiosyncratic shock is $\frac{1}{3}$ which is close to the variance of the idiosyncratic shock of 0.28 as chosen by Bernanke, Gertler and Gilchrist (1999). The mean and the variance of the idiosyncratic shock $\tilde{\omega}$ under the bad project are chosen to match two targets: an annual default rate of around 3% in steady state (as in Bernanke, Gertler and Gilchrist, 1999) and to ensure that the

\(^{16}\)Following Boissay, Collard and Smets (2013), I define a recession as a year in which the percentage change in output is among the 10.2% lowest changes, from the peak of the previous year to the trough of the current year, which is consistent with the fact that recessions are observed around 10.2% of the time in the data. A recession during which a creditor run occurs is defined as a financial recession and a recession during which no creditor run occurs is defined as a non-financial recession.
financial intermediary prefers not to lend to entrepreneurs who invest in the bad project, because the expected return
on such an investment is below one and therefore implies a negative net present value investment for the intermediary.
The fraction of profits which is lost in case of default $\mu$ is directly taken from Bernanke, Gertler and Gilchrist (1999)
and set to 0.12. English, van den Heuvel and Zakrjesk (2012) document that the average maturity of assets of U.S.
commercial banks is around 4.5 years, of liabilities around 0.4 years, with a ratio between the two of 10.88. I normalize
the maturity of short-term debt to one quarter and choose $\gamma$ to give an average maturity of long-term debt of 10.88 quarters.

The stochastic steady state resulting from this calibration is given in Appendix A.5 and the accuracy of the solution
is shown in Appendix A.4. Next, I analyze the dynamics of the model with respect to the technology shock.

### 4.8 Impulse Response Functions

Figures (6) and (7) show impulse response functions to a one standard deviation positive technology shock, starting from
the stochastic steady state of the model. For these responses, a financial crisis does not occur as the intermediary’s leverage
is sufficiently below the threshold $\kappa$. Following a positive technology shock, output $Y_t$ and consumption $C_t$ increase. And
due to consumption smoothing and the increase in the real wage, the household’s savings increase, the real interest rate
$R_{t+1}$ declines, and the household chooses to work more $H_t$.

![Impulse Response Functions](image)

**Figure 6: Impulse Response Functions.** IRFs to a one standard deviation positive technology shock, starting at
stochastic steady state of the model.

After a positive technology shock, the profits to capital $R^K_t Q^K_{t-1}$ increase which decreases the probability of default
of all outstanding loans as indicated by the weighted default threshold $\varpi_t$. Since new entrepreneurs are also less likely to
risk-shift, they can borrow more in absolute terms, $Q_t L_t^{new}$, in relative terms to their collateral $\frac{L_t^{new}}{K_{t+1}}$, and the face value
of their loans $L_t^{new}$ increases. The loan risk indicator $x_t$ therefore rises.

Figure 7: Impulse Response Functions. IRFs to a one standard deviation positive technology shock, starting at stochastic steady state of the model.

Following a positive technology shock, the financial intermediary expands its balance sheet. Given the reduced incentives to risk-shift and the lower real interest rate, the intermediary takes on more debt $B_t$ to increase its new lending. The overall stock of outstanding loans $L_t$ and the economy’s capital stock $K_{t+1}$ therefore rise. The lower real interest rate
and the increased profitability of its loan portfolio, $R_t L_{Q-1}$, induce the intermediary to pay out more dividends $D_t$. The intermediary’s leverage $\frac{B_t}{Q_t L_t}$ initially decreases since all of its loans are increasing in value. Over time, when the price of loans $Q_t$ returns and the intermediary continues to take on more debt to issue new loans, its leverage increases. The impulse response of leverage therefore roughly matches the empirical impulse response shown in Figure (5).

### 4.9 Financial Crises

In principle, a financial crisis can break out at any point in time, if an adverse technology shock is sufficiently large. Hence, nothing in the model restricts crises to occur out of booms or recessions. However, financial crises are more likely to happen if certain preconditions are met. In order to understand these preconditions, I analyze the typical behavior of endogenous variables and the aggregate shock around financial crises. First, the model is simulated for 500,000 periods. Then, I collect the sequences of endogenous variables and shocks in a window of 30 quarters before and 20 quarters after a financial crisis. Figures (8) and (9) plot period by period the median, 33rd, and 66th percentile across these sequences for each variable with respect to windows in which only one financial crisis occurs. In what follows, the median path for each variable is referred to as the “typical path” around a crisis.\(^{17}\)

The first row in Figure (9) shows the typical behavior of the technology shock $\epsilon_a$ and the technology level $a_t$. A typical build-up period leading to a financial crisis is characterized by an elevated technology level. When the technology level starts to decline, the probability that a crisis will occur in the next quarter has already started to increase. A typical crisis is initiated by a relatively moderate adverse change in the technology level which occurs within a one-year window. After the first adverse changes in the technology level, the probability that a crisis will occur within the next quarter strongly increases. The median shock which triggers a crisis is a negative 1.58 standard deviation shock. Typical financial crises therefore occur out of boom periods and the model does not need to rely on extremely large adverse shocks in order to initiate financial crises.

Output $Y_t$, the capital stock $K_{t+1}$, and hours $H_t$ all increase in the build-up period and decrease once a crisis is triggered. Additionally, the model depicts an endogenous credit risk cycle. A lending boom occurs before a typical crisis as more loans $Q_t L_{new}^t$ with higher loan-to-capital ratios $\frac{L_{new}^t}{K_{new}^t}$ are issued which are both strongly decreasing in the bust. The overall stock of loans $L_t$ and the loan risk indicator $x_t$ therefore increase in the boom and over time more risky loans accumulate on the intermediary’s balance sheet. Whereas the weighted default threshold $\bar{\omega}_t$ on all outstanding loans decreases in the boom, this trend is quickly reversed in the bust, when the intermediary suddenly holds a portfolio of non-performing loans and current and future default probabilities sharply increase.

\(^{17}\)The median path does not necessarily represent the path around one particular crisis. In order to give an example of a path around one particular crisis, in Appendix A.6 the behavior of variables around the crisis is reported, for which the shock sequence preceding a crisis is closest to the median shock sequence in Figure (8). This exercise is in the spirit of Fry and Pagan (2011) with respect to the literature on sign restrictions. However, the motivation for conducting this exercise is different. Fry and Pagan (2011) suggest to report the impulse responses of the model which is closest to the median impulse response, which is obtained across a range of models, since the median impulse response does not imply the identification of one unique model. Fry and Pagan (2011) term this suggested impulse response the median target response. In contrast, here, the model is identified, but the paths leading to financial crises can take different shapes, based on a simulation of the model. The “median target financial crisis” is reported to give an example of one particular path.
The financial intermediary sees its profits on loans $R^t L_{t-1}$ rise in the boom and pays out more dividends $D_t$. As the household smooths consumption by saving in the boom period, the real interest rate $R_{t+1}$ declines. The decline in the interest rate and the easing of the risk-shifting problem induce the intermediary to expand its balance sheet by taking on more debt $B_t$ to issue more loans. The intermediary’s leverage $\frac{B_t}{Q_t L_t}$ increases slowly in the boom and then sharply spikes during the crisis, which is consistent with the data shown in Figure (1). The path of leverage is linked to the impulse responses shown in the last section. During a prolonged boom, the medium-term response of leverage to a positive technology shock dominates and leverage slowly increases following a persistently elevated technology level. In a bust, the short-run response of leverage dominates and leverage increases following a negative technology shock. The rise of leverage in the bust is due to the drop in the value of the whole market portfolio of loans, while the intermediary still has to service its increased debt burden. The intermediary cannot deleverage by raising equity or by selling its assets to an outsider. Hence, the financial sector is unable to restructure its balance sheet quickly and holds a portfolio of non-performing loans during a crisis. The combination of these effects leads to an increase in the intermediary’s leverage above the threshold $\kappa$ under the “Unconstrained Problem” and creditors run. A creditor run further amplifies the initial increase in leverage since the value of loans $Q_t$ drops even further when the intermediary is funding-constrained - resulting in a sharp spike of leverage during a crisis. Further, the risk-free interest rate $R_{t+1}$ drops during a crisis due to the quantity-restriction in the short-term debt market associated with a creditor run. While the intermediary is funding-constrained, the expected excess return $\mathbb{E}[R^X_{t+1}]$ sharply increases due to a positive “liquidity-premium” as shown in equation (14) which is consistent with empirical evidence (Muir, 2014).
 Besides playing a key role during a financial crisis when the financial sector finds itself with a portfolio of non-performing long-term loans, the maturity mismatch is also important for understanding the build-up period. The longer the boom lasts, the more old, maturing loans are replaced by new loans backed by less collateral and therefore the stronger the expansion of the intermediary’s balance sheet and leverage. Due to the maturity mismatch, this process takes some time—a financial sector cannot change the structure of its balance sheet quickly and a typical crisis does not build up over night.

Figure 9: Typical Financial Crises. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.
4.10 Non-Financial Recessions

In order to understand how financial crises differ from non-financial recessions (defined in footnote 16), I repeat the exercise of the last section around non-financial recessions. Figures (24), (25) and (26) in Appendix A.7 plot the typical median paths around non-financial recessions and financial crises. Compared to financial crises, the rise and decline in output associated with non-financial recessions is less pronounced. In line with the findings on financial crises, non-financial recessions are triggered by a relatively moderate adverse shock. The median shock which triggers a crisis is a 1.61 standard deviation negative technology shock.

Compared to financial recessions, the role played by the financial sector is quite different with respect to non-financial recessions. Non-financial recessions are not preceded by strong increases in credit \( L_t \), the intermediary’s debt \( B_t \), leverage \( B_t Q_t L_t \), or the loan risk indicator \( x_t \), which contrasts the behavior of these variables in the build-up towards financial crises. During non-financial recessions, the weighted default threshold \( \omega_t \), the intermediary’s leverage, and the expected excess return \( E[R_{t+1}] \) do not sharply increase as they do during financial crises. Further, the drop in the intermediary’s funding \( B_t \), its dividend payout \( D_t \), new loans \( Q_t L_t^{new} \), loan-to-capital ratios \( \frac{L_t^{new}}{K_t+1} \), and the value of loans \( Q_t \) are stronger during financial crises. Overall, this comparison shows that it is the financial sector’s balance sheet and its lending behavior which play crucial roles during the boom-bust cycle around financial crises. In this regard, non-financial recessions are different.

4.11 Predicting Financial Crises

How well does the model perform in predicting and replicating financial crises that were observed ex-post? In order to test the model in this regard, I again obtain a series of structural technology shocks \( \{\hat{\epsilon}_a t\} \), given the estimated AR(1) process to the utilization-adjusted TFP-series, and feed the model with this series.

The result is shown in Figure (10) which plots the path of the technology level \( a_t \), the probability that a financial crisis will occur in the next quarter, and the path of the intermediary’s leverage \( \frac{B_t}{Q_t L_t} \). During the boom period of 2000...
to 2005, the technology level increases step-by-step and so does the intermediary’s leverage. This boom period is followed
by declines in the technology level which initiate a crisis and lead to a creditor run when the intermediary’s leverage goes
beyond the threshold \( \kappa \) (indicated by the red, dotted line). While the probability that a crisis will occur during the next
quarter remains close to zero for most of the sample, it starts to increase around 2005 and jumps up in the quarters that
follow. So even though the information that is given to the model is quite limited, the model does a remarkable job in
predicting the Great Recession and replicating the occurrence of a financial crisis in 2008Q3.

4.12 Cyclicality of Market Leverage

Starting with the work of Adrian and Shin (2010), the cyclicality of financial institutions’ leverage has been discussed as an
indicator of the financial sector’s procyclical risk-taking. Adrian and Shin (2010) show that the leverage of certain financial
institutions is procyclical based on book value data\(^{18}\). However, book values may not always represent current market
prices and the procyclicality of leverage may therefore be misrepresented when using book value data\(^{19}\). I therefore com-
pute the cyclicality of market leverage and compare the empirical results to the model implied cyclicality of market leverage.

Based on data for U.S. commercial and investment banks, the correlation between market leverage and real GDP is
obtained\(^{20}\). Two samples are considered, one including and one excluding the Great Recession. The results are reported
in Table (3). Market leverage is mildly countercyclical. However, the hypothesis of a zero correlation cannot be rejected.

<table>
<thead>
<tr>
<th></th>
<th>1980Q1-2007Q4</th>
<th>1980Q1-2014Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Real GDP} )</td>
<td>-0.03 (0.7210)</td>
<td>-0.12 (0.1439)</td>
</tr>
</tbody>
</table>

Table 3: Correlations. Correlations between market leverage and an indicator of economic activity. Asterisks denote statistical significance at the 1\%[***], 5\%[**], and 10\%[*] confidence level for testing the hypothesis of no correlation against the alternative of non-zero correlation. P-values are in parenthesis.

The model’s implied cyclicality of market leverage is in line with this empirical evidence. Based on a simulation of
500,000 periods, the correlation between output \( Y_t \) and leverage \( \frac{B_t}{QL_t} \) is close to zero (0.001). This acyclicality of leverage is
due the impulse response of leverage and output following a technology shock as shown in Figures (6) and (7). After
a positive technology shock, output \( Y_t \) increases, while leverage decreases in the short run. In the medium run, leverage
increases, while output is still above its steady state value. Output and leverage are therefore negatively correlated in
the short run, but positively correlated in the medium run - explaining the overall correlation which is close to zero.
However, the initial negative response of leverage after a positive technology shock does not imply that leverage decreases
during a prolonged boom. As shown in Figure (9), following a prolonged elevated technology level, leverage increases as
the medium-term impulse response of leverage dominates the short-run response.

\(^{18}\)The exercises in this section are repeated for book leverage ratios and reported in Appendix A.9. The results confirm that leverage ratios for commercial and investment banks are procyclical based on book value data.

\(^{19}\)See also Appendix A.8.1 for a short discussion on the use of book vs. market data.

\(^{20}\)The logged indicators of economic activity and the market leverage are detrended using a Hodrick-Prescott-Filter (a smoothing parameter of 1600 for quarterly and 129,600 for monthly data is applied following Ravn and Uhlig, 2002).
5 Nominal Model

The real version of the model outlined so far takes a simplified view of the economy in the sense that all debt contracts are written in real terms and monetary policy would be neutral. In what follows, I extend the model to include price rigidities, nominal debt contracts, and monetary policy. Within this extended version, inflation changes the real value of debt. Inflation expectations therefore play a crucial role in determining long-term nominal lending and one can additionally analyze the impact of monetary policy on financial stability.

5.1 Monetary Policy

The nominal interest rate is controlled by a monetary authority according to a policy rule of the type suggested by Taylor (1993). Empirical studies find that for several decades prior to the Great Recession the conduct of monetary policy by the U.S. Federal Reserve is well approximated by such a rule (Clarida, Gali and Gertler, 2000), excluding the period highlighted by Taylor (2007) prior to the Great Recession. Monetary policy and price stickiness are introduced as in Gertler and Karadi (2011). The monetary authority controls the nominal interest rate on short-term debt \(i_t\) according to

\[
i_t = (1 - \rho^m) \left(i^{SS} + \phi_x \log \Pi_t + \phi_y \log \frac{Y_t}{Y^*_t} + \rho^m i_{t-1} + \epsilon^m_t\right)
\]

where \(i^{SS}\) is a constant, \(\Pi_t\) is the rate of inflation from period \(t-1\) to \(t\), \(Y^*_t\) is the natural (flexible price) level of output, and \(0 < \rho^m < 1\) reflects the desire of the monetary authority to smooth interest rates. I follow Gertler and Karadi (2011) and use minus the price markup as a proxy for the output gap. In what follows, I turn off the technology shock and the only source of aggregate risk in the model enters via equation (17), where \(\epsilon^m_t \sim N(0, \sigma_m^2)\) is a monetary policy shock. The real interest rate is given by the Fischer equation

\[
R_t = \frac{(1 + i_{t-1})}{\Pi_t}
\]

Debt contracts, both short-term debt between the household and the financial intermediary as well as long-term debt between the financial intermediary and entrepreneurs, are now written in nominal terms. Hence, the real value of debt has to be adjusted for changes in the rate of inflation. In order to keep the description brief, I do not outline the model again, but state the modified model equations in Appendix A.2. Following Gertler and Karadi (2011) and Woodford (2003), I consider the limit of the economy as it becomes cashless.

5.2 Sticky Prices

Another step in the production process is introduced to include price stickiness. The output produced by the good producer via equation (1) is now termed an intermediate good and denoted \(Y^m_t\). Before the intermediate good is used as a consumption good or for conversion into capital goods, it is repackaged andreassembled. Retailers purchase and repackage the intermediate good and sell their output to a final good producer who assembles the retailers’ output and
sells the final good $Y_t$. During this process, retailers are subject to price rigidities which follow a similar specification as in Christiano, Eichenbaum and Evans (2005). The final output good $Y_t$ is a CES-composite of a continuum of mass unity of differentiated retail firms’ output

$$Y_t = \left( \int_0^1 Y_{j,t}^{\epsilon-1} \, dj \right)^{1/(\epsilon-1)}$$

(18)

where $Y_{j,t}$ is the produced output of retailer $j$. The final output producer minimizes its costs

$$\min_{Y_{j,t}} P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} \, dj$$

subject to equation (18). The minimization problem yields the following demand function for retail goods

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{\epsilon} Y_t$$

(19)

A fraction $\theta$ of retailers cannot adjust their price every period. Firms that are allowed to adjust, choose their optimal price $P_t^*$ solving the following problem

$$\max_{P_t^*} \mathbb{E} \left[ \sum_{i=0}^{\infty} \theta^i \Lambda_{t,t+i} \left\{ \frac{P_t^*}{P_{t+i}} - P_{m,t+i} \right\} Y_{j,t+i} \right]$$

subject to equation (19). $P_t$ is the price level and $P_{m,t}^*$ the intermediate good price in period $t$. The first-order condition to this problem can be written in recursive form, using the two auxiliary ‘forward-looking’ variables $F_t$ and $Z_t$ as stated in Appendix A.2. In comparison to the real model, the nominal version has an additional state variable, i.e. last period’s price dispersion $\Delta_{t-1}$.

### 5.3 Calibration

The calibration is adjusted in order to achieve the same targets as in Table (2).$^{21}$ The parameters in addition to the real model are summarized in Table (11). Most of the parameters are standard and not further discussed. The probability of price adjustments is calibrated to an average price frequency of 14.5 months, as documented in Kehoe and Midrigan (2014) based on micro-price data and excluding temporary price changes.

$^{21}$The only parameters which are re-calibrated are $\kappa = 0.51$ and $\chi = 1.06$.  

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The calibration of a new parameter which deserves motivation is the standard deviation of the monetary policy shock $\sigma_m$. Stating the obvious, this shock comes as a surprise to the economy with respect to the information set one period in advance. A calibration of the standard deviation of this surprise therefore requires a proxy for the economy’s expectation about the nominal interest rate and hence the monetary policy’s reaction function one quarter in advance. Federal Funds future contracts allow to derive such a proxy. Using 30-day Federal Funds futures from January 1989 until December 2007, I derive a time series of the economy’s expected Federal Funds rate one quarter ahead. Appendix A.10 describes the data in more detail and states calculations. The difference between the expected and the realized Federal Funds rate then gives an estimate for the monetary policy surprises and $\sigma_m$ is calibrated to the standard deviation of this surprise series, giving $\sigma_m = 0.29\%$. This calibration approach has the advantage that it does not depend on a specific policy rule for the entire sample, but instead allows for changes in the policy rule over time, as long as the market changed its expectations accordingly.

Again, I obtain empirical evidence on the response of the model’s main indicator of financial stability (market leverage) to the model’s aggregate source of risk (the monetary policy shock). A suitable monetary policy identification technique in this respect is a recent approach by Gertler and Karadi (2015). The main idea of this approach is to combine the high-frequency monetary policy identification with a vector-autoregressive (VAR) model and use surprises obtained from Federal Funds future contracts as external instruments within a VAR. This technique has three advantages: (i) monetary policy shocks are unanticipated, (ii) no timing restrictions are imposed, and (iii) one can trace out the response of variables at longer horizons (while allowing them to interact with other variables in the VAR). I estimate a reduced-form VAR(p)-model

$$X_t = A_0 + \sum_{j=1}^{p} A_j X_{t-j} + u_t \quad (20)$$

where $A_0$ is a vector of constants, $A_k \forall k \geq 1$ are coefficient matrices, and the vector $X_t$ comprises the Federal Funds rate (F), the (log) consumer price index, (log) real industrial production, and the market leverage of financial intermediaries.\(^{22}\) The structural form of the VAR(p)-model can be obtained by pre-multiplying (20) by $S^{-1}$ and the reduced-form errors $u_t$ are related to the structural shocks $\epsilon_t$ via $u_t = S\epsilon_t$. Structural shocks are assumed to have a unit variance and the

\(^{22}\text{Again, data on commercial and investment banks is used (see Appendix A.8 for details).}\)
variance-covariance matrix of the reduced form errors is denoted

$$\sum = E[u_t u_t'] = E[SS']$$

In order to identify the column $s$ in matrix $S$ which relates each element in $u_t$ to the structural monetary policy shock denoted $\epsilon_F^t$, an external instrument series $Z_t$ is used. In this regard, a series of surprise changes in the contemporaneous month’s Federal Funds rate is obtained. This series is derived using Federal Funds future contracts. As for the calibration of $\sigma_m$, these future contracts allow to obtain a proxy for the market’s expectation of changes in the Federal Funds rate. A surprise change can then be derived by comparing the settlement price of a future contract shortly before and after an FOMC meeting.\(^{23}\) The conditions for a valid instrument are that the instrument is correlated with the structural shock which relates to the Federal Funds rate, but uncorrelated with all the other structural shocks, i.e.

$$E[Z_t \epsilon_t^F] \neq 0$$
$$E[Z_t \epsilon_t^q] = 0$$

where $q \neq F$ and one can argue that both conditions are satisfied for this external instrument series. Further, denote $s^q \in s$ to be the response of $u_t^q$ to a unit-increase in $\epsilon_t^q$ as well as $s^F \in s$ to be the response of $u_t^F$ to a unit-increase in $\epsilon_t^F$. The identification of the ratio $s^q \overline{s^F}$ then proceeds in two stages. First, $u_t^F$ is projected on $Z_t$ and the fitted values $\hat{u}_t^F$ from this regression are obtained which isolate the variation in $u_t^F$ which is due to the structural shock $\epsilon_t^F$. Second, $u_t^q$ is projected on $\hat{u}_t^F$

$$u_t^q = s^q \overline{s^F} \hat{u}_t^F + \epsilon_t$$

giving a consistent estimate of the ratio $s^q \overline{s^F}$ under the instrument conditions. The ratio $s^q \overline{s^F}$ can then be entangled with the help of the variance-covariance matrix of the reduced-form errors, and one can obtain consistent estimates of $s^q$, $s^F$, and hence $s$ (see Appendix A.11 for details). Given the vector $s$, one is able to compute the impulse responses of $X_t$ with respect to the structural shock $\epsilon_t^F$.

Monthly data from 1979M7 to 2007M12 is used and a VAR-model for $p = 12$ is estimated.\(^{24,25}\) Figures (12) and (13) plot impulse response functions to a 100 basis points contractionary monetary surprise, using surprises around scheduled and unscheduled FOMC meetings (Figure 12) and surprises around scheduled FOMC meetings only (Figure 13).\(^{26}\) The latter

\(^{23}\)Surprises in the current month’s Federal Funds rate are used as an instrument series which is derived using a 30-minute window around FOMC meetings. If multiple meetings occur within one month, I derive the combined shock for that month by taking the sum across the individual meetings. In this regard, I thank Peter Karadi and Mark Gertler for sharing their data, which is based on Gürkaynak, Sack and Swanson (2005).

\(^{24}\)The choice for starting point of the sample follows the reasoning in Gertler and Karadi (2015) and coincides with the beginning of Paul Volcker’s tenure as Federal Reserve chair. The end point is chosen to exclude the Great Recession. A timing issue arises since the external instrument series $Z_t$ is available for a shorter time span (1989-2007) than the time frame on which the VAR is estimated (1979-2007). Following Gertler and Karadi (2015), the VAR is first estimated on the longer sample, the reduced form errors from this estimation are restricted to the shorter sample, and then used to identify the vector $s$.

\(^{25}\)The lag length of the VAR is chosen as suggested by the Hannah-Quinn information criterion.

\(^{26}\)Confidence bands are computed via a wild bootstrap as in Mertens and Ravn (2013) and Gertler and Karadi (2015). This bootstrapping procedure has the advantage that generated regressor problems are avoided as the step associated with the identification via the instrument is included in the bootstrapping procedure, leading to valid confidence bands under heteroskedasticity and strong instruments. 10,000 bootstrap
addresses the concern that the surprise series and therefore the responses are reflecting the release of private information by the Federal Reserve which is particularly a problem around unscheduled FOMC meetings.\textsuperscript{27} The instrument series used for Figure (13) therefore excludes surprises around unscheduled FOMC meetings.

Following a contractionary monetary policy surprise, the Federal Funds Rate increases, while industrial production and the consumer price index decrease - given a price puzzle. Importantly, market leverage increases in the short run for both instrument series and crosses the x-axis after around 10 months or 3-4 quarters (in Figure 13). Hence, financial intermediaries are subject to short-term interest rate risk and an unanticipated increase in the short-term interest rate pressures their balance sheet and leads to an increase in their market leverage in the short run.\textsuperscript{28} Again, given $\beta^H$, I calibrate $\beta^F$ to match the initial sign and turning-point of the empirical impulse response, i.e. an initial increase followed by a decrease after 3-4 quarters. Again, this calibration approach gives an asset-to-equity ratio of around 2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{impulse_response.png}
\caption{Impulse Response Functions. IRF to a 100 basis points contractionary monetary policy surprise. Sample: 1979M1-2007M12. Instrument series: Surprises around scheduled and unscheduled FOMC meetings obtained from future contracts with respect to the current month’s Federal Funds Rate. F-stat: 25.15. Robust F-stat: 25.82. 95\% confidence bands are shown.}
\end{figure}

repetitions are used to obtain the impulse responses.

\textsuperscript{27}The F-statistic and the robust F-statistic from the first stage of the identification are well above the threshold of 10, when using surprises around scheduled and unscheduled FOMC meetings, as suggested by Stock, Wright and Yogo (2002). However, when using the instrument series comprising surprises around scheduled FOMC meetings only, these values drop below 10 and therefore problems associated with weak instruments cannot be excluded.

\textsuperscript{28}In separate VAR-specifications, I confirm that the initial increase in market leverage is due to an immediate decrease of the market equity of financial institutions following a contractionary monetary policy surprise, while book values are responding more slowly. These additional results are in line with the impulse responses presented in section 5.4 and are available upon request.
Figure 13: **Impulse Response Functions.** IRF to a 100 basis points contractionary monetary policy surprise. Sample: 1979M1-2007M12. Instrument series: Surprises around scheduled FOMC meetings obtained from future contracts with respect to the current month’s Federal Funds Rate. F-stat: 5.35. Robust F-stat: 3.89. 95% confidence bands are shown.

### 5.4 Impulse Response Functions

Figures (14) and (15) show impulse response functions to a one standard deviation contractionary monetary policy shock, starting from the stochastic steady state of the model. Output $Y_t$, consumption $C_t$, hours $H_t$, and inflation $\Pi_t$ decrease.

When analyzing the dynamics for entrepreneurs, one can differentiate between outstanding and new loans. Since the real profits on capital $R^K_t Q^K_{t-1}$ decrease and the real burden of outstanding debt increases due to lower inflation, default probabilities on entrepreneurs’ loans, which were given out prior to the shock, increase. To illustrate this point, Figure (15) shows the evolution of the default threshold for the vintage of loans given out one period prior to the contractionary monetary policy shock $\mathcal{W}_{i|0} = \frac{L_{0|0}^{new}}{R^K_t Q^K_{t-1} K^{new}_{t-1} \prod_{k=1}^t (\Pi_k)}$, where zero indicates the period prior to the shock. However, lower profits for entrepreneurs also intensify the risk-shifting problem. The real amount of credit issued to new entrepreneurs $Q_t L_{t|1}^{new}$ is rationed in order to ensure that they invest into the good project and the loan-to-capital ratio on newly issued loans $\frac{L_{t|1}^{new}}{K_{t+1}}$ declines. Due to the contraction in lending, the economy’s capital stock $K_{t+1}$ decreases. In contrast to outstanding loans, newly issued loans show an initial decline in default probabilities, as highlighted by the evolution of the default threshold for the vintage of loans given out in the period when the shock realizes $\mathcal{W}_{i|1} = \frac{L_{1|1}^{new}}{R^K_t Q^K_{t-1} K^{new}_{t} \prod_{k=2}^t (\Pi_k)}$.

The model therefore confirms the asymmetric response in default probabilities on new vs. outstanding loans, as shown empirically by Jiménez et al. (2009, 2014).

When analyzing the intermediary’s balance sheet, it is again important to take into account how inflation changes the real value of nominal contracts, since both the intermediary’s liabilities and assets are denoted in nominal terms. To
illustrate the impact of inflation, the responses of debt $B_t$ and loans $L_t$ are shown both in real and nominal terms. After a contractionary monetary policy shock, the nominal interest rate increases and the intermediary reduces its nominal debt, given this price increase. However, due to the decrease in inflation, the real value of debt $B_t$ actually increases.

On the asset side, the financial intermediary faces a decline in the profits of its portfolio of real loans $R^L_t Q_{t-1}$ due to the increase in default thresholds on outstanding loans. However, if a loan does not default, the real value of repayment actually increases due to lower inflation. Even though the issuance of new loans $Q_t L_{t+1}^{\text{new}}$ contracts, the stock of outstanding

Figure 14: Impulse Response Functions. IRFs to a one standard deviation contractionary monetary policy shock, starting at stochastic steady state of the model.
loans \( L_t \) increases in real terms, which is again due to lower inflation. Since the real value of outstanding loans increases, loan-to-capital ratios on outstanding loans rise as well. The loan risk indicator \( x_t \) therefore increases, even though new loans are issued with lower loan-to-capital ratios \( \frac{L_{new}}{K_{t+1}} \).

Given these countervailing effects, the intermediary initially reduces the amount of dividends paid out to shareholders \( D_t \), since the decline in the intermediary’s profits, the increased cost of funding, and the rise in the real value of the intermediary’s debt dominate. Leverage \( \frac{B_t}{Q_{t-1}} \) initially increases, because the whole portfolio of loans drops in value. Over time, the price of loans \( Q_t \) returns and the intermediary reduces its real debt burden, leading to a decrease in the intermediary’s leverage over time. The path of leverage therefore roughly matches the empirical evidence shown above - an initial increase with a decrease after around four quarters. However, the following decline in leverage is more persistent than in the data which is due to the model’s inherent persistence coming from the stochastic maturity set-up.

\[
\text{Figure 15: Impulse Response Functions. IRFs to a one standard deviation contractionary monetary policy shock, starting at stochastic steady state of the model.}
\]

5.5 Financial Crises

The analysis in section 4.9 is repeated for the nominal model. Figures (16) and (17) show the typical behavior of the nominal economy around financial crises. The first row in Figure (17) plots the behavior of the monetary policy shock \( \epsilon^n_t \) and the nominal interest rate \( i_t \). A typical build-up period leading to a financial crisis is characterized by a longer period of expansionary monetary policy and at the end of this period the probability that a crisis will occur in the next quarter has already strongly risen. A crisis is initiated by a few contractionary monetary policy shocks. The nominal interest rate therefore steadily declines in the run-up and then increases. Hence, both expansionary and contractionary monetary policy can increase financial instability and it is a U-shaped pattern of the policy target rate which is particularly likely to
adversely affect financial stability (see below for a discussion in this regard). As in the real model, financial crises occur out of boom periods and are initiated by moderate adverse changes in the nominal interest rate. The median shock which triggers a crisis is a 1.19 standard deviation positive monetary policy shock.

Output $Y_t$, capital $K_t$, hours $H_t$, and inflation $\Pi_t$ all increase in the build-up period and then sharply decrease. The rise and fall of inflation $\Pi_t$ influences the dynamics of the intermediary’s balance sheet. In order to isolate the impact of inflation, debt $B_t$ and loans $L_t$ are again shown in real and nominal terms.\(^{29}\) In nominal terms, the intermediary strongly expands its balance sheet - taking on more debt to issue more loans since incentives to risk-shift decrease in good times and the agency problem is relaxed. However, due to the rise of inflation in the boom, the real amount of outstanding loans is constantly adjusted downwards. The total amount of loans $L_t$ in real terms therefore remains fairly constant in the boom, even though more loans $Q_t L_t^{\text{new}}$ are issued. Inflation also lowers the real burden of debt $B_t$, but because the expansion in nominal terms overweighs, the real amount of debt slightly increases in the boom. And since the real price of loans $Q_t$ remains fairly constant, the intermediary’s leverage $\frac{B_t}{Q_t L_t}$ increases in the boom.

Figure 16: Typical Financial Crises. Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.

In contrast to the real model, the loan risk indicator $x_t$ slightly decreases in the boom, since loan-to-capital ratios of outstanding loans fall and the rise of loan-to-capital ratios on newly issued loans $\frac{L_t^{\text{new}}}{K_t+1}$ cannot overturn this effect. However, in contrast to the real model, $x_t$ sharply rises in the bust, since the rate of inflation $\Pi_t$ drops and loan-to-capital ratios on outstanding loans increase. The weighted default threshold $\omega_t$ behaves in a similar way. Compared to the real model, $\omega_t$ increases more strongly in the bust due to the change in inflation which raises default thresholds in addition to

\(^{29}\)In order to convert real into nominal series, a time series of the price level is obtained, the log of this series is linearly detrended, and then used to convert real into nominal series.
the fall in profits on capital.

In contrast to the real model, the rise in the value of loans $Q_t$, the intermediary’s profits $R^t_t Q_{t-1}$, and dividends $D_t$ is less pronounced in the nominal model due to the changes in inflation and the readjustment of real loans. As in the real model, the behavior of leverage is consistent with the data as shown in Figure (1), and expected excess returns $\mathbb{E} [R^X_{t+1}]$ spike because of the liquidity-premium.

Figure 17: **Typical Financial Crises.** Event window around financial crisis at Quarter = 0. Based on a simulation of 500,000 periods. Median, 33rd, and 66th percentiles are shown.
5.6 Monetary Policy & Financial Stability

The model predicts that a U-shaped pattern of monetary policy is particularly likely to increase financial instability. The behavior of the nominal interest rate therefore takes a similar shape as the counterparts in the U.S. before the Great Recession and in Japan in the late 1980s/early 1990s as shown in Figure (18). However, the increases in the nominal interest rate which initiate a financial crisis in the model are much larger than the step-by-step increases in the nominal interest rates as seen in Figure (18). The model therefore offers a reason why central banks raise nominal interest rates in small steps. Small steps avoid to put too much pressure on the balance sheets of financial institutions.

![Figure 18: Monetary Policy. Left: U.S. Monetary Policy, Federal Funds Target Rate. Right: Japanese Monetary Policy, Discount rate. Source: Fred Database Federal Reserve Bank of St. Louis.](image)

However, while small increases in the target rate may not be able to cause a crisis, they still put additional pressure on financial intermediaries’ balance sheets. The model therefore highlights the dilemma of central banks when trying to “lean against the wind” using monetary policy. In the short run, contractionary monetary policy may increase financial instability instead of decreasing it. A trade-off between price and financial stability may therefore arise during boom periods, when rising prices are pressuring the economy and increases in the target rate can work against such price pressures, but may also increase financial instability in the short run. Moreover, since periods of low interest rates intensify the adverse impact of contractionary monetary policy on financial stability, they should be avoided by central banks if financial stability concerns outweigh other motives.
6 Conclusion

This paper develops a calibrated macroeconomic model which includes financial intermediation and occasional financial crises. The paper presents a mechanism for how financial crises can occur out of boom periods and why they can be initiated by relatively moderate adverse shocks. The financial sector’s balance sheet and its lending behavior play key roles in this regard. Financial crises occur out of boom periods because during good times, financial institutions expand their balance sheets. They take on more debt to issue loans that are backed by less collateral, as agency problems between the financial sector and its borrowers are relaxed in good times. After a sudden bust, current and future default probabilities of loans strongly increase, the financial sector holds a portfolio of non-performing loans, and is left with an increased debt burden. These effects pressure the financial sector’s balance sheet and lead to a rationing of credit to intermediaries. In accordance with empirical evidence, the financial sector’s market leverage increases slowly during boom periods and spikes sharply during busts when asset prices collapse. Moreover, financial crises are rare events, break out in the midst of a credit-intensive boom, and are more severe than non-financial recessions. The model behaves differently around the latter, as non-financial recessions are not preceded by an expansion of the financial sector’s balance sheet or a credit-boom. Even though the model is stylized, it does a remarkable job in predicting and replicating the occurrence of the Great Recession, when confronted with a series of structural shocks. The paper shows additionally how to introduce the core mechanism and occasional financial crises into a standard New-Keynesian dynamic general equilibrium model - a model which is commonly used at central banks around the world. Within this extended framework, a U-shaped behavior of the nominal interest rate is particularly likely to lead to financial instability - a pattern which preceded recent financial crises in the U.S. and Japan.

The paper is part of an evolving literature which aims to integrate financial frictions, (endogenous) risk, and occasional financial crises into standard macroeconomic models. The paper particularly focuses on three aspects, which I consider to be of first-order importance in this regard. First, the modeling of the financial sector’s balance sheet and the financial sector’s lending behavior are in accordance with current institutional realities and empirical evidence. In particular, I show how to introduce long-term defaultable debt into a general equilibrium model in a parsimonious way which gives a realistic representation of the financial sector’s balance sheet. Second, financial crises arise endogenously and the likelihood of a financial crisis varies over the business cycle. Financial crises are unlikely to break out at any point in time, but rather if certain preconditions are met. Third, in order to capture the nonlinear dynamics characteristic to episodes of financial distress, a nonlinear global solution of the model is obtained. The nonlinear behavior of the economy is particularly reflected in the way default and leverage behave during financial busts. Several avenues for future research come to mind. First, it would be useful to develop macroeconomic models which consider an even more detailed balance sheet of the financial sector (introducing mortgages and different types of contracts like fixed and variable rate loans). Second, given standard macroeconomic models which include financial crises, are there policy interventions which can improve welfare (macro-prudential policies)? And third, new approaches and applications to solving models nonlinerly are needed such that nonlinear global solutions can become part of the standard toolkit of central banks.
References


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A Appendix

A.1 Derivations

A.1.1 Incentive Compatibility Constraint

The entrepreneur’s objective function (4) under the good project can be rewritten as

\[ V_{good}^t = (1 - \gamma) E \left[ \sum_{j=1}^{\infty} \Lambda_{t,t+j} \left\{ \gamma^{j-1} R_{t,j}^K Q_{t,j}^K K_{new}^t \int_{\omega_t}^{\omega_{t+j}} (\omega - \omega_{t+j}) d\Phi(\omega) + \frac{\gamma_j}{1 - \gamma} r_{t+j}^K K_{new}^t \right\} \right] \]

since

\[ L_{new}^t = R_{t,j}^K Q_{t,j}^K K_{new}^t \omega_{t+j} \text{ for } j \geq 1 \]

where cumulative distribution function and partial expectation under the uniform distribution for \( \omega \sim U[1 - b, 1 + b] \) are

\[ \Phi(\omega_{t+j}^t) = \frac{\omega_{t+j}^t - (1 - b)}{2b} \quad (22) \]

\[ \int_{\omega_{t+j}^t}^{1+b} \omega d\Phi(\omega) = \frac{1}{4b} \left( (1 + b)^2 - \omega_{t+j}^t \right) \quad (23) \]

and

\[ \int_{\omega_{t+j}^t}^{1+b} (\omega - \omega_{t+j}^t) d\Phi(\omega) = \frac{1}{2b} \left( \frac{1}{2} (1 + b)^2 + \frac{1}{2} \omega_{t+j}^t - \omega_{t+j}^t (1 + b) \right) \]

Using these definitions, one can show that the value function can be expressed as

\[ V_{good}^t = (1 - \gamma) \left( K_{new}^t J_{t}^g - L_{new}^t G_{t}^g + \frac{(L_{new}^t)^2}{K_{new}^t S_{t}^g} \right) \]

where \( J_{t}^g, G_{t}^g, \) and \( S_{t}^g \) are three ‘forward-looking’ auxiliary variables which are defined as

\[ J_{t}^g = E \left[ \Lambda_{t,t+1} \left\{ \frac{(1 + b)^2}{4b} R_{t+1}^K Q_{t+1}^K + \frac{\gamma}{1 - \gamma} \right\} \right] \]

\[ G_{t}^g = E \left[ \Lambda_{t,t+1} \left\{ \frac{(1 + b)}{2b} - \gamma G_{t+1}^g \right\} \right] \]

\[ S_{t}^g = E \left[ \Lambda_{t,t+1} \left\{ \frac{1}{4b} R_{t+1}^K Q_{t+1}^K + \gamma S_{t+1}^g \right\} \right] \]

One can show that the value function under the bad project has a similar representation. Taking these representations together, the continuously binding IC constraint can be expressed as in equations (30)-(33) in Appendix A.2.
A.1.2 Return on Market Portfolio of Loans

The return on the market portfolio of loans is given by equation (10) and repeated here

\[ R_L^t = \gamma Q_t + (1 - \gamma) \frac{1}{\pi_{t-1}} \left\{ \sum_{j=1}^{\infty} \left( 1 - \Phi(\omega_{t|t-j}) \right) \gamma^{j-1} L_{t-j}^{new} + (1 - \mu) \sum_{j=1}^{\infty} \left[ \int_{1-b}^{\infty} \omega R_t^{K} Q_{t-1}^{K} \gamma^{j-1} K_{t-j+1}^{new} d\Phi(\omega) \right] \right\} Q_{t-1} \]

Using the definition of the cumulative distribution function in (22), one can show that

\[ \sum_{j=1}^{\infty} \left( 1 - \Phi(\omega_{t|t-j}) \right) \gamma^{j-1} L_{t-j}^{new} = \left( \frac{1 + b}{2b} \right) L_{t-1} - \frac{1}{2b R_t^{K} Q_{t-1}^{K}} x_{t-1} \]

where

\[ L_{t-1} = L_{t-1}^{new} + \gamma L_{t-2} \]
\[ x_{t-1} = \frac{(L_{t-1}^{new})^2}{K_{t_{new}}^{new}} + \gamma x_{t-2} \]

and using the definition of the partial expectation given in (23), one can show that

\[ \sum_{j=1}^{\infty} \left[ \int_{1-b}^{\infty} \omega R_t^{K} Q_{t-1}^{K} \gamma^{j-1} K_{t-j+1}^{new} d\Phi(\omega) \right] = \frac{1}{4b} \frac{1}{R_t^{K} Q_{t-1}^{K}} x_{t-1} - \frac{(1-b)^2}{4b} R_t^{K} Q_{t-1}^{K} K_t \]

where

\[ K_t = K_t^{new} + \gamma K_{t-1} \]

Taking these together, the profits per loan can be written as

\[ R_L^t Q_{t-1} = \gamma Q_t + (1 - \gamma) \left\{ \frac{1 + b}{2b} - (1 - \mu) \frac{(1-b)^2}{4b} R_t^{K} Q_{t-1}^{K} K_t - \frac{x_{t-1}(1 + \mu)}{4b R_t^{K} Q_{t-1}^{K} L_{t-1}} \right\} \]

Using a similar reasoning, one can show that the evolution of net worth for new entrepreneurs can be expressed as

\[ N_t = (1 - \gamma) \left\{ \frac{(1+b)^2}{4b} R_t^{K} Q_{t-1}^{K} K_t - \frac{1 + b}{2b} L_{t-1} + \frac{x_{t-1}}{4b R_t^{K} Q_{t-1}^{K}} \right\} \]
A.2 Model Equations

This section outlines the model equations for **both the real and the nominal model** in a parsimonious way. The real version of the model is obtained by setting $\Pi_t = P_t^m = \Delta_t = 1 \ \forall \ t$ and omitting the subsection “Monetary Policy & Price Stickiness”.

**Household & Good Producer**

\[
\mathbb{E} [\Lambda_{t,t+1} R_{t+1}] = 1 \quad (24)
\]

\[
\Lambda_{t,t+1} = \beta \left( \frac{1}{\Pi_t} \right) \quad (25)
\]

\[
(1 - \alpha) P_t^m Y_t = \chi H_t^{1+\phi} \quad (26)
\]

\[
Y_t = \Delta_t e^a K_t^a H_t^{1-\alpha} \quad (27)
\]

\[
a_t = \rho_a a_{t-1} + \epsilon_t^a \quad (28)
\]

**Entrepreneurs**

\[
Q_t^K K_{t+1}^{new} = N_t + Q_t L_{t+1}^{new} \quad (29)
\]

\[
0 = \left( K_{t+1}^{new} J_t - L_{t+1}^{new} G_t + \left( L_{t+1}^{new} \right)^2 S_t / K_{t+1}^{new} \right) \quad (30)
\]

\[
J_t = \mathbb{E} \left[ \Lambda_{t,t+1} \left\{ \left( (1+b)^2 - (\bar{k} + \bar{c})^2 \right) R_t^K Q_t^K + \gamma J_{t+1} \right\} \right] \quad (31)
\]

\[
G_t = \mathbb{E} \left[ \Lambda_{t,t+1} \left\{ \left( \frac{1+b}{2b} - \frac{\bar{k} + \bar{c}}{2c} \right) \frac{1}{\Pi_t} + \gamma \frac{1}{\Pi_t} G_{t+1} \right\} \right] \quad (32)
\]

\[
S_t = \mathbb{E} \left[ \Lambda_{t,t+1} \left\{ \left( \frac{1}{4b} - \frac{1}{4c} \right) \frac{1}{\Pi_t^2} R_t^K Q_t^K + \gamma \frac{1}{\Pi_t^2} S_{t+1} \right\} \right] \quad (33)
\]

\[
N_t = (1 - \gamma) \left\{ \left( \frac{1+b}{4b} - R_t^K Q_{t-1}^K K_{t-1} \right) L_{t-1} \frac{1}{\Pi_t} + \left( \frac{x_{t-1}}{4b \Pi_t^2} R_t^K Q_{t-1}^K \right) \right\} \quad (34)
\]

**Financial Intermediary**

\[
D_t + Q_t L_t + B_{t-1} R_t = B_t + \frac{R_t^L}{\Pi_t} Q_{t-1} L_{t-1} \quad (35)
\]

\[
\mathbb{E} \left[ \frac{R_{t+1}}{D_{t+1}} \right] + \lambda_t = \mathbb{E} \left[ \frac{1}{D_{t+1}} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] \quad (36)
\]

\[
L_t = L_{t+1}^{new} + \gamma L_{t+1} \quad (37)
\]

\[
K_{t+1} = K_{t+1}^{new} + \gamma K_t \quad (38)
\]

\[
x_t = \left( L_{t+1}^{new} \right)^2 / K_{t+1}^{new} + \gamma x_{t-1} / \Pi_t^2 \quad (39)
\]

\[
R_t^L Q_{t-1} = \gamma Q_t + (1 - \gamma) \left\{ \left( \frac{1+b}{2b} - (1-\mu) \frac{1}{L_{t-1}} \right) R_t^K Q_{t-1}^K K_t - \frac{x_{t-1}}{4b \Pi_t^2} \right\} \quad (40)
\]

\[\text{49}\]
Occasional Financial Crises

If $\frac{B_t}{Q_t L_t} \leq \kappa$ or $r_{t-1} = 1$

$$\lambda_t = 0$$

If $\frac{B_t}{Q_t L_t} > \kappa$ and $r_{t-1} = 0$

$$B_t = \tau(D_t^{unc} + Q_t^{unc} L_t^{new,unc}) + B_{t-1} R_t - \frac{R_t}{\Pi_t} \Pi_{t-1} + Q_t \frac{L_{t-1}}{\Pi_t}$$

Capital Producers and Resource Constraint

$$Q_t^K = 1 + \zeta \left( \frac{K_{t+1}}{K_t} - 1 \right)$$ (41)

$$Y_t = C_t + D_t + K_{t+1} - (1 - \delta) K_t + \frac{\zeta}{2} \left( \frac{K_{t+1}}{K_t} - 1 \right)^2 K_t +$$

$$(1 - \gamma) \mu \left( \frac{1}{4b} \frac{x_{t-1}}{\Pi_t^2} R_t^K Q^{K}_{t-1} - \frac{(1-b)^2}{4b} R_t^K Q^K_{t-1} K_t \right)$$ (43)

$$R_t^K Q^{K}_{t-1} = \left( Q_t^K (1 - \delta) + \alpha P_t^m \frac{Y_t}{\Delta_t K_t} \right)$$ (44)

Monetary Policy & Price Stickiness (only present in the nominal model)

$$i_t = (1 - \rho^m) \left( i^{SS} + \phi_s \log \Pi_t + \phi_d \log \left( \frac{\epsilon - 1}{\epsilon P_t^m} \right) \right) + \rho^m i_{t-1} + \epsilon_t^m$$ (45)

$$\frac{F_t}{Z_t} = \frac{\epsilon - 1}{\epsilon - \left( \frac{1 - \theta \Pi_t^{-1}}{1 - \theta} \right)^{\frac{1}{\sigma}}}$$ (46)

$$F_t = P_t^m Y_t + E \left[ \theta \Lambda_{t,t+1} \Pi_{t+1}^t F_{t+1} \right]$$ (47)

$$Z_t = Y_t + E \left[ \theta \Lambda_{t,t+1} \Pi_{t+1}^t Z_{t+1} \right]$$ (48)

$$\Delta_t = \left( 1 - \theta \left( \frac{1 - \theta \Pi_t^{-1}}{1 - \theta} \right)^{-\frac{1}{\sigma}} + \theta \frac{\Pi_t}{\Delta_t-1} \right)^{-1}$$ (49)

$$R_t = \frac{(1 + i_{t-1})}{\Pi_t}$$ (50)
A.3 Solution Technique

Given the definition of a solution in Section 4.6, I describe next how the policy functions \( \hat{S}_{t+1} = f_S(S_t) \) and \( X_t = f_X(S_t) \) are obtained using a projection algorithm. Broadly, this involves three choices. First, one has to choose a grid on which the model is solved. Second, one has to decide on the parametrization of the policy functions. Third, given an initial parametrization, one has to choose an iteration procedure. These three choices structure the description below.

**Grid.** The model is solved on a Smolyak sparse grid due to the curse of dimensionality (Bellman, 1961). The construction of the Smolyak sparse grid works as follows. The grid points in the space \([-1, 1]\) for each state variable are obtained by tensor-products of nested sets of Chebyshev extrema and the application of the Smolyak rule for a given level of approximation which controls how many of these tensor-products are included in the grid (see also Malin, Krueger and Kubler, 2011). I select 5 for the level of approximation, giving 2433 grid points for the five state variables (excluding \( r_{t-1} \)). Next, the grid points are transformed from the space \([-1, 1]\) into the relevant space given the model’s calibration. In order to determine the relevant space, I solve the model first with a third-order perturbation method around a deterministic steady state (ignoring the occasionally-binding constraint), simulate it for 500,000 periods, and choose the 2.5%- and 97.5%-percentiles as the lower \( L_B \) and upper \( U_B \) bounds for each state variable. A linear transformation \((x + 1)(\frac{(U_B-L_B)}{2}) + L_B\) is used to transform each grid point \( x \) from \([-1, 1]\) into \([L_B, U_B]\), giving the full set of grid points \( j = 1, ..., M \) in the relevant state space. The grid on which the model is solved remains fixed during the iteration on the policy functions (see Maliar and Maliar (2015) for an alternative method).

**Parametrization of policy functions.** I parametrize several non-state variables using third-order ordinary polynomials. In particular, let \( X_t^p \) be a parametrized variable where \( X_t^p \in \{ K_{t+1}^{new}, \Delta, R_{t+1}, J_t, S_t, G_t \} \) and let \( r_t \) be an indicator function determining whether there is a creditor run in period \( t \), in which case \( r_t = 1 \), or not, such that \( r_t = 0 \). Then, \( X_t^p(S_t) \) is parametrized using the piecewise flexible form with separate coefficients \( \beta_{r_t=0}^X \) and \( \beta_{r_t=1}^X \)

\[
X_t^p(S_t) = (1-r_t)\beta_{r_t=0}^X T(S_t) + r_t\beta_{r_t=1}^X T(S_t)
\]

where \( T(S_t) \) is a vector collecting the basis functions. The number of grid points is larger than the number of coefficients. Hence, the outlined solution algorithm does not give an exact solution on the grid points such as in collocation methods (see for example Malin, Krueger and Kubler, 2011).31

**Iteration.** Given an initial guess for the coefficients \( \beta_{r_t=0}^X \) and \( \beta_{r_t=1}^X \), the iteration proceeds as follows.

1. Obtain the vector collecting the basis functions \( T(S_{t,j}) \) for a given grid point \( j \). Assume that there is no creditor run

---

30 I slightly perturb the model by adding a small risk-premium on debt for the financial intermediary which depends on the level of debt. This gives a unique portfolio choice in a deterministic steady state and allows to solve the model with standard perturbation methods around this deterministic steady state (see for example Schmitt-Grohe and Uribe (2003) for similar techniques with respect to small-open economy models). The third-order perturbation solution also serves as a first guess for coefficients in the parametrized policy functions. During the iteration to obtain a global solution, the risk-premium is sequentially reduced until it reaches zero (see also chapter 5.9. on “Homotopy Methods” in Judd, 1998, in this regard). I thank Fabrice Collard for a discussion on this topic.

31 A collocation method cannot be applied since the number of grid points for which a creditor run occurs may change during the iteration procedure.
in period \( t \) and calculate the parametrized variables \( X_{t,j}^P(S_{t,j}; r_t = 0) \) via (51). Substitute \( X_{t,j}^P(S_{t,j}; r_t = 0) \) into the set of equations summarized in Appendix A.2. Using the equilibrium conditions, solve for the rest of the variables and set the lagrange multiplier \( \lambda_t = 0 \). Check whether the financial intermediary’s leverage exceeds the threshold \( \kappa \). If so, go back to the beginning, set \( r_t = 1 \), use the parametrization \( X_{t,j}^P(S_{t,j}; r_t = 1) \) instead, and additionally add the constraint \( B_t = \tau(S_{t,j}) \). This separates the grid points into two sets: a set of points for which there is no creditor run and a set for which there is a creditor run. When there is a creditor run, \( K_{t+1}^{\text{neg}} \) is not parametrized, but given by the equilibrium conditions. Instead, the intertemporal equation (13) is used to check that the lagrange multiplier \( \lambda_t \) is positive.

2. Having solved for all period \( t \) variables at grid point \( j \), one obtains next period’s endogenous state variables \( \hat{S}_{t+1,j} \).

In order to approximate integrals arising from expectation operators in intertemporal equations, I use nine Hermite-Gaussian quadrature nodes and weights for period \( t + 1 \). This gives next period’s exogenous state variables \( \overline{S}_{t+1,j,i} \) and therefore \( S_{t+1,j,i} \) for each node \( i \in \{1, 2, ..., 9\} \) in period \( t + 1 \) and each grid point \( j \). Assume that there is no creditor run in period \( t + 1 \) at node \( i \) for grid point \( j \). Use again the initial parametrization \( X_{t+1,j,i}^P(S_{t+1,j,i}; r_{t+1} = 0) \) and solve for the rest of the variables at node \( i \) in period \( t + 1 \). If \( r_t = 0 \), check whether the financial intermediary’s leverage in period \( t + 1 \) exceeds threshold \( \kappa \). If so, use the parametrization \( X_{t+1,j,i}^P(S_{t+1,j,i}; r_{t+1} = 1) \) instead, and additionally add the constraint \( B_{t+1} = \tau(S_{t+1,j,i}) \).

3. Having solved for all period \( t + 1 \) variables, compute the expectation integrals appearing in the model’s equations.

The intertemporal equations give an estimate \( X_{t,j}^P(S_{t,j}) \) for each of the initially parametrized variables \( X_{t,j}^P(S_{t,j}) \) at each grid point \( j \). A fixed-point is obtained when the initially assumed value \( X_{t,j}^P(S_{t,j}) \) is equal to \( X_{t,j}^P(S_{t,j}) \).

4. Until a fixed point is reached, iterate over the coefficients \( \beta_0^X \) and \( \beta_1^X \) in the policy functions (51). Collect the vectors of basis functions \( T(S_{t,j}) \) for all grid points for which there is no creditor run in period \( t \), combine them in a matrix \( T_0(S_t) \) and project these on the obtained estimates \( X_{t,j}^P(S_{t,j}) \), giving

\[
\hat{\beta}_0^X \equiv (T_0(S_t)'T_0(S_t))^{-1}T_0(S_t)'X_{t,j}^P, r_t = 0
\]

where \( X_{t,j}^P, r_t = 0 \) is a vector collecting the obtained estimates \( X_{t,j}^P(S_{t,j}) \) for which there is no creditor run. Repeat the same for all grid points for which there is a creditor run in period \( t \), resulting in

\[
\hat{\beta}_1^X \equiv (T_1(S_t)'T_1(S_t))^{-1}T_1(S_t)'X_{t,j}^P, r_t = 1
\]

5. Compute the coefficients \( \beta_0^{X'} \) and \( \beta_1^{X'} \) which are used for the next iteration via

\[
\beta_0^{X'} = (1 - \xi)\hat{\beta}_0^X + \xi \beta_0^X
\]
\[
\beta_1^{X'} = (1 - \xi)\hat{\beta}_1^X + \xi \beta_1^X
\]
where $0 < \xi < 1$ is a dampening parameter which makes convergence more likely ($\xi = 0.1$ is used).

6. Check for convergence and end iteration if

\[
\frac{1}{6} \sum_{X^P_t \in A} \frac{1}{M - Mb} \sum_{j=1}^{M} \left| \frac{X^P_{t,j} - X^P_{t,j|r_t=0}}{X^P_{t,j|r_t=0}} \right| < \eta
\]

and

\[
\frac{1}{5} \sum_{X^P_t \in A'} \frac{1}{Mb} \sum_{j=1}^{M} \left| \frac{X^P_{t,j} - X^P_{t,j|r_t=1}}{X^P_{t,j|r_t=1}} \right| < \eta
\]

7. where $A = \{K_{t+1}^{new}, \frac{1}{D_t}, R_{t+1}, J_t, S_t, G_t\}$, $A' = A \setminus K_{t+1}^{new}$, and $Mb$ denotes the number of binding coefficients which may change while iterating. $\eta = 5^{-04}$ is used.\(^{32}\)

Given the policy functions $X^P_t(S_t)$, one can use the model’s equations to obtain the full set of policy functions $\hat{S}_{t+1} = f_{\hat{g}}(S_t)$ and $X_t = f_X(S_t)$ for a solution of the model as given in Definition 1.

### A.4 Accuracy

Besides having an accurate solution on the grid points, I check whether the solution is also accurate at other points in state space. The accuracy of the solution is confirmed by analyzing the absolute residual equation errors

\[
\left| \frac{X^P_{t} - X^P_{t}}{X^P_{t}} \right|
\]  

(52)

in a simulation of the model as suggested by Judd (1992), see also Bocola (2015) for a similar analysis. $X^P_{t}$ is obtained as described in Appendix A.3. I simulate the model for 500,000 periods and compute the decimal log of the absolute residual equation errors for each period $t$ and for each $X^P_t \in \{K_{t+1}^{new}, \frac{1}{D_t}, R_{t+1}, J_t, S_t, G_t\}$. In Figure (19), I plot histograms of those errors for each parametrized variable, where red lines indicate means.

\(^{32}\)I find that setting $\eta$ to a lower value does not strongly increase the accuracy of the solution away from the grid points or change any of the results, but results in an increase in computational time.
The means all lie around -3.5. Given the reasoning in (Judd, 1992), one can consider the solution to be accurate. Since I am particularly interested in obtaining an accurate solution at points for which the occasionally-binding constraint holds and a financial crisis occurs, I report the decimal log of the absolute residual equation errors at those points separately in Figure (20). As can be seen, the solution is still very accurate in the state space where financial crises occur which lies far away from the steady state of the model. Moreover, the lagrange multiplier $\lambda_t$ on the debt constraint is always positive during financial crises. I conclude that the outlined solution algorithm gives an accurate nonlinear global solution of the model, both in the state space in which “standard business cycles” occur, as well as in the region in which the economy experiences a financial crisis.
A.5 Stochastic Steady State

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$C_{SS}$</td>
<td>1.77</td>
<td>Asset-to-Equity-Ratio</td>
<td>$Q_{SS}L_{SS} - D_{SS}$</td>
</tr>
<tr>
<td>Hours</td>
<td>$H_{SS}$</td>
<td>0.96</td>
<td>Dividends</td>
<td>$D_{SS}$</td>
</tr>
<tr>
<td>Output</td>
<td>$Y_{SS}$</td>
<td>2.13</td>
<td>Return to loans</td>
<td>$R^L_{SS}$</td>
</tr>
<tr>
<td>Capital</td>
<td>$K_{SS}$</td>
<td>13.86</td>
<td>Loans</td>
<td>$L_{SS}$</td>
</tr>
<tr>
<td>Return capital</td>
<td>$R^K_{SS}$</td>
<td>1.02</td>
<td>Net worth</td>
<td>$N_{SS}$</td>
</tr>
<tr>
<td>New loans</td>
<td>$Q_{SS}L^\text{new}_{SS}$</td>
<td>0.18</td>
<td>Price long-term debt</td>
<td>$Q_{SS}$</td>
</tr>
</tbody>
</table>

Table 4: Stochastic Steady State. Value of endogenous variables at the stochastic steady state of the real model.

A.6 Median Target Financial Crisis

In the spirit of Fry and Pagan (2011), this section reports the “Median Target Financial Crisis”. The paths around this crisis are obtained by selecting the crisis, for which the sequence of shocks is ‘closest’ to the median shock sequence in Figure (8) before a financial crisis occurs. The closest sequence of shocks is the one which minimizes a distance criterion.

Denote $\tilde{\epsilon}_{t-k}$ the size of the shock $k$ quarters before the financial crisis which occurs at time $t$, $\text{med}(\tilde{\epsilon}_{t-k})$ the median and $\text{std}(\tilde{\epsilon}_{t-k})$ the standard deviation across the shock sequences $k$ quarters before a crisis. The “Median Target Financial Crisis” is the one which minimizes $\sum_{k=0}^{30} \left( \frac{\tilde{\epsilon}_{t-k} - \text{med}(\tilde{\epsilon}_{t-k})}{\text{std}(\tilde{\epsilon}_{t-k})} \right)^2$. The shocks occurring after the crisis are set to zero and the paths around the “Median Target Financial Crisis” are compared to the median paths in Figures (8) and (9).
### Figure 21: Median Target Financial Crisis vs. Typical Financial Crises.
Event window around “Median Target Financial Crisis” (red, dotted line) or median path of the typical financial crisis (blue, solid line) at Quarter = 0.

### Figure 22: Median Target Financial Crisis vs. Typical Financial Crises.
Event window around “Median Target Financial Crisis” (red, dotted line) or median path of the typical financial crisis (blue, solid line) at Quarter = 0.
Figure 23: Median Target Financial Crisis vs. Typical Financial Crises. Event window around “Median Target Financial Crisis” (red, dotted line) or median path of the typical financial crisis (blue, solid line) at Quarter = 0.

A.7 Typical Non-Financial Recessions

Figure 24: Typical Non-Financial Recessions vs. Financial Crises. Event window around non-financial recession (red, dotted line) or financial crisis (blue, solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
Figure 25: Typical Non-Financial Recessions vs. Financial Crises. Event window around non-financial recession (red, dotted line) or financial crisis (blue, solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.

Figure 26: Typical Non-Financial Recessions vs. Financial Crises. Event window around non-financial recession (red, dotted line) or financial crisis (blue, solid line) at Quarter = 0. Based on a simulation of 500,000 periods. Median paths are shown.
A.8 Market Leverage

A.8.1 Book vs. Market Leverage

The literature frequently reports leverage ratios of financial institutions which are obtained using only book value data. However, such data depends on specific accounting rules - in particular if and how often financial institutions need to update their balance sheets to current market prices. Hence, a mapping to variables in economic models is incomplete. In contrast, market leverage - which can be defined for example as the market value of assets divided by the market value of equity - does have an immediate economic meaning and corresponds to the equivalent definitions in economic models. The market value of assets, liabilities, or equity is the expected present discounted value of future cash flows. Market leverage therefore gives the (present value) share that belongs to creditors relative to shareholders of an institution’s future cash flows. In what follows, I explain how to obtain the necessary data and what assumptions are made to approximate the market leverage of financial institutions.

A.8.2 Data Description

I largely follow Adrian and Brunnermeier (2011) in the construction of a measure of the financial sector’s market leverage, approximated by data on commercial and investment banks. Equity and balance sheet data for commercial banks are collected from the CRSP/Compustat database. Commercial banks are indicated by the SIC Codes 60, 61, and 6712. From CRSP, monthly share prices and number of outstanding shares for commercial banks with these SIC Codes are chosen. From Compustat, quarterly total assets and total liabilities for commercial banks with the mentioned SIC Codes are collected. The quarterly data are converted into monthly data using linear interpolation. The two datasets are merged via the PERMNO identifier. Using the GVKEY identifier, I repeat this exercise for the following selected investment banks: Bear Stearns, Citigroup, Credit Suisse, Goldman Sachs, HSBC, JP Morgan, Lehmann Brothers, Merrill Lynch, and Morgan Stanley.

A.8.3 Definition

The market equity of a firm is defined as its share price multiplied by the number of shares outstanding. However, the market leverage of a firm cannot be computed exactly since it requires either data on the market value of assets or the market value of liabilities - both of which are only observed in terms of their book values. In order to approximate the market leverage, one can either assume that

\[ \text{Market Assets} \approx \text{Book Assets} \]  

(53)

or

\[ \text{Market Liabilities} \approx \text{Book Liabilities} \]  

(54)
Given one of these assumptions, the market leverage of institution $j$ is then approximated by

\[ \text{Market Leverage}_j \approx \frac{\text{Book Assets}_j}{\text{Market Equity}_j} \]  

(55)

or

\[ \text{Market Leverage}_j \approx \frac{\text{Market Equity}_j + \text{Book Liabilities}_j}{\text{Market Equity}_j} \]  

(56)

The financial sector’s market leverage is then given by

\[ \text{Sector Market Leverage} \approx \frac{\sum \text{Book Assets}_j}{\sum \text{Market Equity}_j} \]  

(57)

or

\[ \text{Sector Market Leverage} \approx \frac{\sum \text{Market Equity}_j + \sum \text{Book Liabilities}_j}{\sum \text{Market Equity}_j} \]  

(58)

where the sums are taken over the remaining observations. For example, at the end of 2010, the number of remaining observations are 424 given definition (57) and 407 given definition (58). The difference in the number of observations is due to the availability of balance sheet data. Over the sample used for the empirical analyzes, the data availability is better with respect to book assets than book liabilities. I therefore primarily work with definition (57) and check robustness using definition (58). All of the results in the paper are robust to using definition (58) and are available upon request.

Figures (27) and (28) plot real book assets and real market equity (both in log units) around the Great Recession.

![Book Assets](image)

**Figure 27: Book Assets.** The graph shows the evolution of real book assets for different types of U.S. financial institutions around the Great Recession.
Figure 28: **Market Equity.** The graph shows the evolution of real market equity for different types of U.S. financial institutions around the Great Recession.

### A.9 Cyclicality of Book Leverage

This section repeats the exercise in section 4.12 for book leverage ratios (see section 4.12 for a description). Book leverage is defined as

\[
\text{Book Leverage} = \frac{\text{Total Book Assets}}{\text{Total Book Assets} - \text{Total Book Liabilities}}
\]

and quarterly balance sheet data on both commercial and investment banks is again used. Based on book value data, leverage is procyclical and significant at the 1% confidence level for 1980Q1-2014Q2.

<table>
<thead>
<tr>
<th></th>
<th>1980Q1-2007Q4</th>
<th>1980Q1-2014Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>0.28</td>
<td>0.38 ***</td>
</tr>
<tr>
<td></td>
<td>(0.2464)</td>
<td>(0.0063)</td>
</tr>
</tbody>
</table>

Table 5: **Correlations.** Correlations between market leverage and an indicator of economic activity. Asterisks denote statistical significance at the 1%[***], 5%[**], and 10%[*] confidence level for testing the hypothesis of no correlation against the alternative of non-zero correlation. P-values are in parenthesis.

### A.10 Monetary Policy Surprises

30-day Federal Funds future contracts are used to obtain a series of the economy’s expectation about future nominal interest rates. These contracts are traded at the Chicago Mercantile Exchange and historical pricing series are obtained. The timing and pricing of these contracts works as follows. A 30-day Federal Funds Future contract traded in month \( k - x \) for \( x \geq 0 \) is a bet on the average Federal Funds rate for month \( k \). Given the price in month \( k - x \), the expected Federal Funds rate in month \( k - x \) for the Federal Funds rate in month \( k \) can be obtained via

\[
E_{k-x}[\text{mean (FFR}_k\text{)]} = 100 - \text{Price}_{k-x}
\]
for $x \geq 1$. The monthly frequency is converted into a quarterly frequency as follows. The price for some month $k$ is derived on the mid-day in the mid-month in the quarter prior to month $k$. For example, for January, February, and March, the price is obtained on the 15th of November prior to those months. A quarterly series is then obtained by averaging over the related three expected Federal Funds rates. The difference between the derived series of expectations and the quarterly average of the realized Federal Funds rate gives a series of quarterly monetary policy surprises which is used to calibrate the standard deviation of the monetary policy shock.

### A.11 Derivation Monetary Policy Identification

The description on how to separate $s^F$ from the ratio $\frac{s^q}{s^f}$ follows footnote 4 in Gertler and Karadi (2015) and is here repeated for completeness. Partition $u_t$, $S$, and the reduced form error variance-covariance matrix $\sum = E[u_t' u_t] = E[SS']$ as

\[
\begin{align*}
  u_t &= [u_t^F u_t^q]' = [u_{1t} u_{2t}]' \\
  S &= [s \ s_q] = [S_1 \ S_2] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \\
  \sum &= \begin{bmatrix} \sum_{11} & \sum_{12} \\ \sum_{21} & \sum_{22} \end{bmatrix}
\end{align*}
\]

$s^F$ is then identified via

\[
(s^F)^2 = s_{11}^2 = \sum_{11} - s_{12} s_{12}'
\]

where

\[
s_{12} s_{12}' = \left( \sum_{21} - \frac{s_{21}}{s_{11}} \sum_{11} \right)' Q^{-1} \left( \sum_{21} - \frac{s_{21}}{s_{11}} \sum_{11} \right)
\]

and

\[
Q = \frac{s_{21}}{s_{11}} \sum_{11} \frac{s_{21}'}{s_{11}} - \left( \sum_{21} \frac{s_{21}'}{s_{11}} + \frac{s_{21}}{s_{11}} \sum_{21}' \right) + \sum_{22}
\]