Sovereign Debt in the Euro Area: 
 a Model of Bailouts and Contagion*

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Abstract

We propose a model to explain the convergence after 2000 and subsequent divergence after 2008 of sovereign yields in the Euro Area, and the contagion among yields of the Euro periphery after the financial crisis of 2007-2008. The mechanism is based on the availability of bailouts by Euro Area institutions, whose lending capacity is a channel of contagion between the spreads of different countries: one country needing (or being perceived as close to needing) a bailout diminishes the prospects of another country receiving a bailout as well. We then investigate how the availability of bailouts affects the probability of a debt crisis and find the following main results: i) a bailout available with (near) certainty has the good features of a lender of last resort; ii) uncertainty arises as the bailout resources are shared between countries, which may create a severe moral hazard problem; iii) a small country has the highest potential for moral hazard: it can disproportionately increase its debt beyond its ability to repay, and effectively take exclusive control of the shared bailout resources.

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1 Introduction

Sovereign yields for the Euro Area were remarkably low and stable in the years 2001-2008 and subsequently diverged in many countries (Greece, Ireland, Italy, Portugal, Spain) in a way that many find excessive relative to the change in country fundamentals (for example, DeGrauwe and Ji (2013)). A lot of the discussion has focused on the “contagion” experienced by sovereign spreads of the Euro periphery after the financial crisis of 2007-2008 (for example, Constancio (2011))\(^1\).

In this paper, we propose a mechanism that can explain the “convergence” of the Euro Area (EA) spreads in 2000-2007 and the subsequent “excessive” divergence, and provides a novel channel for contagion.

The main idea is that since the start of the EA, government bond markets started pricing in an implicit bailout guarantee from the strong countries in the union, presumably on the grounds that the default of a country within the union would have severe spillovers over the union as a whole. Thanks to this guarantee, the spread of a country in the EA is necessarily lower than that of an otherwise identical stand-alone country. However, since the union does not have an unlimited bailout capacity, the spread dynamics are affected by the joint dynamics between the fundamentals of the country, the fundamentals of other countries in the area (that might also be in need of a bailout, thus exhausting the resources for bailout of the union), and, possibly, the overall bailout capacity of the EA, that might also become weaker in a downturn.

To make things as simple as possible, our model will involve two borrowing countries and a pool of resources of fixed size \(M\) that we call “the insurance”, which can be used to bail out a country that would otherwise default.

If the amount \(M\) is insufficient to prevent default, the resources of the insurance cannot

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\(^1\)Anecdotal evidence mentioned by Constancio, Vice President of the European Central Bank, includes the following: “When the sovereign crisis became more severe again and Moody’s downgraded Portugal on July 5, 2011, it cited among other factors developments in Greece. Moody’s believed that contagion from a default of Greece made it more likely that Portugal would require a second round of official financing. The downgrade of Portugal and, above all, the continuing fears of a Greek default apparently triggered a sell-off in Spanish and Italian government bonds. There had not been adverse data releases concerning the Spanish and Italian economies or budgetary situations around that time. By July 18, 2011 Italian government bond yields had increased by almost 100 basis points, while Spanish ones had increased by more than 80 basis points.”
be used. If both Country 1 and Country 2 need a bailout, they have to share the amount $M$, provided that this is enough to avoid both defaults. Possibly, $M$ could represent the spillover cost that the provider of the insurance would incur in case of a default. For example we could think the provider of the insurance as another country whose banking system is heavily exposed to the two borrower countries. Alternatively, the fixed limit $M$ could be due to the limited resources of the insurer.

In our model, a bailout may happen even in the absence of an ex-ante contract between the borrower countries and the provider of the insurance. The latter will ex-post offer a bailout in order to avoid the spillover costs from a default. This justifies why the markets were assuming an implicit guarantee even in times when no official mechanisms to avoid sovereign defaults were in place.

To provide intuition we first explore this mechanism in a simple 2-period model. Subsequently, we extend the analysis to a dynamic, infinite horizon model, based on the exactly solvable dynamic model for sovereign debt by Rochet (2006), which is able to replicate realistic values for the Debt-to-GDP ratio and the probability of default of a country. The introduction of the insurance complicates this model, and we solve it with recursive methods.

Finally, we use the more realistic dynamic model to analyze a classical question about bailouts: does the insurance cause a moral hazard problem so that countries are unable to repay their debt more often, or does it calm the markets and reduce spreads, thereby making crises less frequent?

The moral hazard problem has been discussed extensively in the literature devoted to the international bailouts in Latin America and East Asia in the 90’s (e.g. Calomiris (1998), Jeanne and Zettelmeyer (2001)). The coexistence of the pernicious effects associated with moral hazard and the beneficial effects associated with a lender of last resort have been studied extensively in connection with bank bailouts.

We find that if the insurance is available with a high degree of certainty (over 95% with our baseline parameterization), it has the classical beneficial effects of a lender of last resort: even if a country takes maximum advantage of the insurance to increase its level of borrowing and its debt, the incidence of crises decreases. A country’s ability to repay depends crucially on how much it can re-borrow from the markets when its debt is due. With the insurance markets lend more, and this is especially so when income is low, since in this case the insurance is in relative terms higher. The decrease in the incidence of crises happens entirely through the higher re-borrowing ability from the markets at low income, and the insurance
resources hardly ever need to be used.

However, the sharing mechanism that we describe in this paper can make the insurance uncertain, as one of the two countries can deplete the insurance, or the latter can be insufficient to bail out both. When the insurance is uncertain the moral hazard problem comes back with a vengeance.

First, even when the insurance is uncertain, a country can in general borrow more from the markets. But if next period the insurance has been depleted, the borrowing capacity from the market is lower again, which makes debt repayment more difficult. Second, we show that if the insurance limit is high enough, and if the probability that the insurance is available next period is in an appropriate range, the country can increase its borrowing capacity by essentially promising to use the insurance to repay when the latter is available, and default otherwise. In this case the probability of a crisis can rise to enormous levels.²

Finally, we show that when one of the countries sharing the insurance is small relative to the other, and is small relative to the insurance (in the sense that the resources of the insurance are much bigger than what the country may be able to borrow from the market), it can disproportionately increase its debt beyond its ability to repay, and effectively take exclusive control of the shared bailout resources, even as the bigger country is also trying to exploit the insurance and borrow as much as possible. This might shed some light on the puzzle of how a small country such as Greece can become so central and so dangerous.

This paper is organized as follows. Section 2 presents a simple 2-period model that qualitatively replicates the “structural break” observed in the Euro Area bond markets during the crisis, and the contagion effect. Section 3 presents a dynamic, infinite horizon model with realistic features, and solves the shared insurance problem within a reduced-form approach in which each country has access to a bailout with probability p. Section 4 explores how the insurance affects debt accumulation and the incidence of crisis in the framework of Section 3. Section 5 derives a full two-country solution of the shared insurance problem when both countries have the objective of maximizing their borrowing in each period. Section 6 concludes.

**Existing literature**

Part of the existing literature on sovereign spreads in EA emphasizes the role of a global

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²Policy might be able to mitigate this problem. For example, Jeanne and Zettelmeyer (2001) advocate making bailouts conditional on borrowing limits or other government policies. Forms of ex-ante conditionality might however be difficult to enforce, as the bailouts in the EA show.
risk aversion factor. With the financial crisis balance sheet-constrained investors developed a stronger preference for assets perceived as “safe havens” (the “flight-to-quality” effect). This behavior not only benefited sovereign securities as an asset class, but also introduced a higher degree of differentiation among sovereign securities. For example, an IMF study by Caceres, Guzzo and Segoviano (2010) finds that the flight-to-quality effect had a significant impact lowering German and, to some extent, other core Euro Area sovereign spreads during what they call “the financial crisis build-up”, between July 2007 and September 2008. However, this factor does not help explain the convergence of sovereign spreads in the years 2000-2007.

The role of the “flight-to-quality” effect, as well as that of the “flight-to-liquidity” effect, as drivers of sovereign spreads in EA, is also documented by Beber, Brandt and Kavajecz (2009).

“Behavioral” explanations for the spreads behavior in the period 2000-2007 include the “convergence trading” hypothesis (e.g. Arghirou and Kontalakis (2010)): according to this view, markets in this period were pricing in the expectation of a convergence between the economies in the European core and the economies in the periphery, while disregarding other possibilities.

DeGrauwe and Ji (2013) argue that the “disconnect” between spreads and fundamentals (in particular, Debt-to-GDP ratio) after 2008 would be evidence of multiple equilibria in the government bond markets. According to this view, member countries of monetary unions, that issue debt in a currency they do not control, may more easily fall victims of self-fulfilling prophecies. The mechanism would be similar to the one in the seminal paper by Calvo (1988): if investors start worrying that the Debt-to-GDP ratio of a country is excessive, or that a country might have a liquidity problem, they require a higher spread, which worsens the Debt-to-GDP ratio and exacerbates the liquidity problem.

Benzoni, Collin-Dufresne, Goldstein and Helwege (2014) develop a model of contagion between sovereign spreads, where the source of contagion is the existence of an underlying state of the economy affecting all the countries in the EA, about which agents are uncertain and have to update their beliefs.

Tirole (2014) is a recent reference about the optimality of bailouts and other forms of country solidarity, directly inspired by the Eurozone crisis.
2 A two-period model

2.1 Basic setup

We consider an endowment small open economy (SOE) that we simply call a “country” and two dates: $t = 0$, $t = 1$. At $t = 0$ the country borrows $B$, and at $t = 1$ the country needs to repay. Only non-contingent debt is available. A debt contract prescribes that if the country borrows $B$ at $t = 0$ it needs to repay $D$ at $t = 1$. The spread $x(B)$ is defined by the relationship $D = (1 + r_f + x(B))B$, where $r_f$ is the risk-free interest rate, that is exogenous and constant in this model.

The country’s income at $t = 1$, that we denote by $Y_1$, is stochastic and not affected by $B$ or $D$. Conditional on the information at $t = 0$, income at $t = 1$ is lognormally distributed, $Y_1 \sim LN(\mu, \sigma)$. The maximum amount of resources that the government can raise for debt repayment is a fraction $\phi$ of the country’s income, i.e. $\phi Y_1$. This is a model of non-strategic default: default occurs when these resources are not enough to repay the face value of debt $D$, in which case investors only recover $\phi Y_1$. Thus actual debt repayment at $t = 1$, that we denote by $R_1$, can be written as

$$R_1 = D - \phi \max\left(0, \frac{D}{\phi} - Y_1\right) \quad (1)$$

We assume that the lenders are risk-neutral\(^3\). Given this assumption, in equilibrium $D$ is given by the condition

$$E_0[R_1(D)] = (1 + r_f)B \quad (2)$$

where $E_0[.]$ denotes the expectation conditional on the information at $t = 0$.

Equation (1) shows that holding the country’s debt effectively means shorting a put option on its income. Therefore the relationship between $B$ and $D$ is

$$D - \phi P_Y\left(\frac{D}{\phi}\right) = (1 + r_f)B \quad (3)$$

where $P_Y(K)$ denotes the price of a (1-period) put option on the underlying $Y$ with strike $K$. Since $Y_1$ is by assumption lognormally distributed we have

$$P_Y(K) = e^{-r_f}\left(N(-d_2)K - N(-d_1)E[Y_1]\right) \quad (4)$$

\(^3\)Risk neutrality could be justified by the hypothesis that the SOE is uncorrelated with the rest of the world, so its risk could be fully diversified away. We make this assumption because we specifically want to focus on a model of the spreads based on default risk, rather than on risk aversion.
where \( N(\cdot) \) is the cumulative distribution function of the standard normal distribution and

\[
\begin{align*}
    d_1 &= \frac{1}{\sigma} \left[ \ln \left( \frac{E[Y_1]}{K} \right) + \frac{\sigma^2}{2} \right] \\
    d_2 &= \frac{1}{\sigma} \left[ \ln \left( \frac{E[Y_1]}{K} \right) - \frac{\sigma^2}{2} \right]
\end{align*}
\]

Suppose now that the country had access to an insurance which can contribute a maximum amount \( M \) when the country is unable to repay on its own, but will only do so if \( M \) is enough to avoid default. The repayment function at \( t = 1 \) would be

\[
R_1 = \begin{cases} 
    D & \text{if } Y_1 > D - \frac{M}{\phi} \\
    \phi Y_1 & \text{otherwise}
\end{cases}
\]

which can also be written as

\[
R_1 = D - \phi \max \left( 0, \frac{D - M}{\phi} - Y_1 \right) - M \mathbb{I}_{Y_1 < \frac{D - M}{\phi}}
\]

where \( \mathbb{I} \) is the indicator function. Hence the relationship between \( B \) and \( D \) is

\[
D - \phi \mathbb{P}_Y \left( \frac{D - M}{\phi} \right) - M \mathbb{B}_Y \left( \frac{D - M}{\phi} \right) = (1 + r_f)B
\]

where \( \mathbb{B}_Y(K) \) denotes the price of a binary put option on the underlying \( Y \) with strike \( K \). For a lognormally distributed \( Y \) it is \( \mathbb{B}_Y(K) = e^{-r_f} N(-d_2) \).

As shown in detail in Appendix A, any \( M > 0 \) increases the borrowing capacity and decreases the spread, notably keeping the spread finite for every sustainable borrowing level.

Figure 1 shows how the borrowing capacity depends on \( M \) (left panel) and compares the spreads with no insurance \((M = 0)\) and with insurance \( M = 5 \) (right panel). Here and in future numerical examples in this section we will use the following parameters: \( Y_1 \sim LN(\mu, \sigma) \) with \( \mu = \log(100) \) (the median value of \( Y_1 \) is 100 ) and \( \sigma = 0.5 \). Also, \( \phi = 0.1 \) and \( r_f = 0.04 \).

### 2.2 Two countries with a partial insurance

In this section we posit that two countries share a partial insurance. The insurer can bail out one or both countries if they are unable to repay on their own, but can contribute a maximum amount \( M \). If \( M \) is not enough to avoid both defaults, nobody will be bailed out and there will be default.
Figure 1: **Left panel:** Borrowing capacity as a function of the insurance $M$.  
**Right panel:** Spread as a function of borrowing level, with and without insurance.

This hypothetical situation is not dissimilar to what happens in the Eurozone. The European Stability Mechanism (ESM) was established in 2012 to provide assistance to Eurozone members in financial difficulty, with a maximum lending capacity of €500 billion. The ESM replaced two earlier similar EU funding programmes: the European Financial Stability Facility (EFSF) and the European Financial Stabilisation Mechanism (EFSM), that provided bailout loans to Ireland, Portugal and Greece. Although no such program was in place prior to 2010, it is reasonable to assume that the markets were anticipating that they would be created when needed.

We call $Y^{(1)}_t$ and $Y^{(2)}_t$ the income at time $t = 1$, $D^{(1)}_1$ and $D^{(2)}_1$ the face value of debt for country 1 and 2, and $\phi$ the parameter that governs recovery in case of default (we assume that $\phi$ is the same for the two countries). The repayment function by country 1 is

$$R_1 = \begin{cases} 
\phi Y^{(1)}_1 & \text{if } Y^{(1)}_1 < \frac{D^{(1)}_1 - M}{\phi} \\
\text{or } \left( \frac{D^{(1)}_1 - M}{\phi} \leq Y^{(1)}_1 < \frac{D^{(1)}_1}{\phi} \right) & \text{& } Y^{(1)}_1 + Y^{(2)}_1 < \frac{D^{(1)}_1 + D^{(2)}_1 - M}{\phi} \\
D^{(1)}_1 & \text{otherwise}
\end{cases}$$

The conditions in (10) mean that the country defaults either because it is unable to repay even with the help of the full insurance amount $M$, or because it has to share the insurance with the other country, and the insurance is not enough to cover both. We notice from
that repayment by Country 1 is a function of \( D^{(1)} \) and \( D^{(2)} \). Hence the conditions that determine \( D^{(1)} \) and \( D^{(2)} \) are intertwined

\[
E[R^{(1)}_1(D^{(1)}, D^{(2)})] = (1 + r_f)B^{(1)} \\
E[R^{(2)}_1(D^{(1)}, D^{(2)})] = (1 + r_f)B^{(2)}
\]

which makes it clear that there can be “contagion” between the two spreads. For example, as Country 2’s debt increases, it is more likely that Country 2 will need a bailout, which makes it less likely that also Country 1 will be able to receive a bailout, which in turn increases the spread of Country 1. Figure 2 provides a graphical illustration. Here we labeled

Case1 : \[ Y_1^{(1)} < \frac{D^{(1)} - M}{\phi} \]

Case2 : \[ \left( \frac{D^{(1)} - M}{\phi} \leq Y_1^{(1)} < \frac{D^{(1)}}{\phi} \right) \& \left( Y_1^{(1)} + Y_1^{(2)} < \frac{D^{(1)} + D^{(2)} - M}{\phi} \right) \]

Case3 : \[ \left( \frac{D^{(1)} - M}{\phi} \leq Y_1^{(1)} < \frac{D^{(1)}}{\phi} \right) \& \left( Y_1^{(1)} + Y_1^{(2)} > \frac{D^{(1)} + D^{(2)} - M}{\phi} \right) \]

Case4 : \[ Y_1^{(1)} > \frac{D^{(1)}}{\phi} \]

Figure 2: Default or no-default for Country 1 depending on \( Y_1 \) and \( Y_2 \)

More explicitly (11)-(12) can be written as

\[
Rec^{(1)}(D^{(1)}, D^{(2)}) + D^{(1)}P_{surv}^{(1)}(D^{(1)}, D^{(2)}) = (1 + r_f)B^{(1)} \\
Rec^{(2)}(D^{(1)}, D^{(2)}) + D^{(2)}P_{surv}^{(2)}(D^{(1)}, D^{(2)}) = (1 + r_f)B^{(2)}
\]
where \( Rec^i \) and \( P_{surv}^i \) are the expected recovery in default and the survival probability, respectively, for country \( i \). For country 1

\[
Rec^{(1)}(D^{(1)}, D^{(2)}) = P\left(\text{Case 1}\right) E_0 \left[ \phi Y_1^{(1)} \mid \text{Case 1} \right] \\
+ P\left(\text{Case 2}\right) E_0 \left[ \phi Y_1^{(1)} \mid \text{Case 2} \right] \\
P_{surv}^{(1)}(D^{(1)}, D^{(2)}) = P(\text{Case 3}) + P(\text{Case 4})
\]

(15)

(16)

The “positive” side of the contagion effect is that if a country has a low borrowing level, the other country, which can effectively rely on exclusive insurance, can borrow a lot at very low spreads. The “negative” side of contagion is that, especially at high levels of borrowing, a country’s spread can surge (or its borrowing level can become unsustainable) just because the other country increases its borrowing level.

The borrowing capacity as a function of \( M \) in a symmetric case in which the countries are identical, their future income is uncorrelated, and they both choose to borrow to their maximum capacity, is shown in Figure 3 (see Appendix A for the derivation). Here it is compared to the borrowing capacity in case one country had exclusive access to the insurance with the same limit \( M \), and to the case with no insurance. Clearly, when the borrowing level of one country is low, the borrowing capacity of the other country would tend to become as big as it would be if it had exclusive access to the insurance.

In Figure 4 we show the spread as a function of borrowing level with insurance \((M = 5)\) and without insurance \((M = 0)\). Here the distribution of income at time 1 is identical for the two countries and uncorrelated, \( Y_i \sim LN(\log(100), 0.5) \), \( i = 1, 2 \), and \( \phi = 0.1 \). We keep Country 2’s borrowing level fixed, \( B^{(2)} = 5 \) (a low borrowing level considering that the borrowing capacity of each country in the absence of insurance would be about 10.9 with these parameters). Clearly in the case with insurance the spread of Country 1 moves remarkably little with \( B^{(1)} \), especially compared with the situation without insurance.

Correlation also matters: as correlation is high, the countries tend to be unable to repay at the same time, and the insurance is more likely to become insufficient.

### 2.3 A simulation of the structural break

DeGrauwe and Ji (2013) show that the relationship between spreads and Debt-to-GDP ratio experienced a structural break around 2008. In particular they show that the positive relationship between spreads and Debt-to-GDP ratio, as well as the negative relationship...
between spreads and GDP growth rate, is very strong after the crisis but insignificant before the crisis. As they show, a standard Chow test confirms the structural break around 2008.

A simple exercise will show that in this model we are able to generate a similar result.

Consider two countries, Country 1 and Country 2. We can imagine that Country 2 is Greece and Country 1 is representative of a cross section of countries in the Euro area. We use the model described Section 2.2 to generate the spread experienced by Country 1 as three of its own characteristics change: Debt-to-GDP (uniformly distributed between 7% and
9%), expected time-1 income (uniformly distributed between 1 and 1.15) and time-1 income volatility (uniformly distributed between 10% and 20%) \(^4\). Income volatility is assumed to be an unobserved characteristic and therefore it accounts, in this simple exercise, for the fact that Debt-to-GDP ratio is an imperfect predictor of spread. We leave all characteristics of Country 2 fixed and generate a cross section of 400 spread observations for Country 1. To account for market imperfections not related to country fundamentals, we add a small noise component to the spread, uniformly distributed between 0 and 0.3%.

Subsequently we increase the Debt-to-GDP ratio of Country 2 by 33% – from 7.5% to 10% – an increase similar to that experienced by Greece between 2007 and 2010, and we recalculate the spreads for the same 400-observation sample: same Debt-to-GDP ratio and growth volatility for Country 1, same noise component. We can observe in Table 2 the dramatically increased sensitivity of spread on debt over GDP in the second set of observations, confirmed by a Chow test.

<table>
<thead>
<tr>
<th></th>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt-to-GDP Ratio</td>
<td>0.0024</td>
<td>0.0158***</td>
</tr>
<tr>
<td></td>
<td>[0.0028]</td>
<td>[0.0032]</td>
</tr>
<tr>
<td>Expected (t=1) GDP</td>
<td>-0.0021</td>
<td>-0.0188***</td>
</tr>
<tr>
<td></td>
<td>[0.0027]</td>
<td>[0.0031]</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.0035</td>
<td>0.1241</td>
</tr>
</tbody>
</table>

*** \(p=0.99\), ** \(p=0.95\), * \(p=0.90\)

Table 1: Simulation of the structural break

3 An Infinite-horizon model

In this section we present a dynamic, infinite-horizon model, that in its baseline version without insurance follows closely the model of Rochet (2006). The main new ingredient relative to the 2-period model is the ability of countries to re-borrow from the financial markets to repay their maturing debt. In subsection 3.1 we present the baseline model and

\(^4\)In this 2-period model, Debt-to-GDP ratio refers to the ratio between debt and expected income at time \(t=1\), since \(t=0\) income has no impact on the spread.
our calibration, which can reproduce fairly realistic values for a country’s Debt-to-GDP ratio and probability of default. In subsection 3.2 we add the insurance on top of the baseline model. Subsequently, in Section 4, we use this model to talk about how the insurance affects debt accumulation and the incidence of crises.

### 3.1 Baseline model

This model follows closely the model by Rochet (2006). The model rests on the following assumptions, most of which are natural extensions of those of the 2-period model of Section 2. Lenders are risk-neutral. All debt is short-term: if a country borrows $B_t$ at time $t$, it is required to repay $D_{t+1}$ at time $t+1$. In order to repay its debt, at $t+1$ the country can use at most a fraction $\phi$ of its current income $Y_{t+1}$, plus the amount that the market is willing to lend again at $t+1$. The country decides to repay whenever it can (no strategic default). Formally, repayment at time $t+1$ is

$$R_{t+1} = \begin{cases} D_{t+1} = (1 + r_f + x_t)B_t & \text{if } \phi Y_{t+1} + B^*_{t+1} > D_{t+1} \\ \phi Y_{t+1} & \text{otherwise} \end{cases}$$

(17)

where $B^*_{t+1}$ is the maximum amount the market is willing to lend at $t+1$.

Income $Y_t$ follows the following stochastic process

$$Y_t = g_t Y_{t-1}$$

(18)

where $g_t$ is i.i.d. and lognormally distributed, $g_t \sim LN(\mu_g, \sigma_g)$.

Appendix B shows how to solve for the spreads and the borrowing capacity in this model. In this section we will only summarize the main intuitions.

Thanks to the stationarity of the distribution of growth factor $g$, the spread at any time $t$ is a function (independent of $t$) of borrowing level as a fraction of $Y_t$:

$$x_t = x(b_t) \quad \text{with} \quad b_t \equiv \frac{B_t}{Y_t}.$$  

(19)

Intuitively, since the level of income has no effect on the distribution of future growth, a change in the level of income should allow one country to proportionally change its level of borrowing, without any effect on the probability of default. The same intuition applies to the borrowing capacity and the maximum debt:

$$B^*_t = b^* Y_t$$

(20)
\[ D_{t+1}^* = d^* Y_t \]  \hspace{1cm} (21)

with the maximum Debt-to-Income ratio \( d^* \) related to \( b^* \) by \( d^* = (1 + r_f + x(b^*))b^* \). Notice that \( D_{t+1} \), the debt due at \( t + 1 \), is determined at time \( t \) as a function of \( B_t \), therefore it is natural to express it as a fraction of time-\( t \) income.

In an infinite-horizon model, the borrowing capacity is a crucial ingredient to determine the default probability and hence the spreads: in order to know if the country will default tomorrow, we need to know what is the maximum amount it will be able to re-borrow. Equation (17) implies that default happens when

\[ Y_{t+1} < \frac{D_{t+1} - B_{t+1}^*}{\phi} \]  \hspace{1cm} (22)

Using the proportionality between borrowing capacity and income (20) and the definition of the growth factor (18), and expressing the face value of the debt \( D_{t+1} \) in terms of time-\( t \) income \( D_{t+1} \equiv d_{t+1} Y_t \), (22) can be translated into a condition on growth between \( t \) and \( t + 1 \), i.e. given \( d_{t+1} \) default happens when

\[ g_{t+1} < \frac{d_{t+1}}{\phi + b^*} \]  \hspace{1cm} (23)

As shown in Figure 5, the borrowing capacity increases with the mean of (the lognormal distribution of) the growth factor, \( \mu_g \), and decreases with its volatility \( \sigma_g \), reflecting the higher probability of falling below the default threshold when volatility increases.

As discussed by Rochet (2006), the prediction that default probability increases with income volatility is an advantage of this model over models of strategic default. In models of strategic default, pioneered by Eaton and Gersovitz (1982), a country repays to avoid the punishment of being excluded from the financial markets. Since volatility increases the option value of borrowing in the future, these models predict that more volatile countries default less often, a prediction at odds with empirical evidence\(^5\).

\(^5\)For example Catao and Kapur (2004) look at a set of 26 developing countries over the period 1970-2001 and find that all serial defaulters are highly volatile economies, with average output volatility about twice the sample average. Strategic default models are at odds with other stylized facts as well. First, default occurs by and large in bad times. For example, Reinhart-Rogoff (2009) document the link between sovereign defaults and financial crises. If default was purely a strategic decision, it would happen a lot also in good times. Second, strategic default, together with the absence of bankruptcy courts for sovereigns, would naturally imply zero recovery rates. This is clearly not in line with the stylized fact that default generates lengthy renegotiations (as documented for example by Benjamin-Wright (2008)) and that recovery rates are typically significantly higher than zero.
Figure 5: Borrowing capacity $b^*$ as a function of the growth parameter $\mu_g$ for constant growth volatility $\sigma_g = 0.1$ (left) and as a function of growth volatility $\sigma_g$ for constant growth parameter $\mu_g = 0.02$ (right).

Baseline parameter choice
We use $\mu_g = 0.02$ (2% average growth), $\sigma_g = 0.1$ (10% volatility), $\phi = 0.1$ (10% maximum primary surplus), and $r_f = 0.04$. With these inputs, the borrowing capacity $b^*$ is 39.8% of income, with corresponding Debt-to-GDP ratio $d^* = 43.4\%$, and the probability of default if the country borrows the maximum is 5.8\%.

A few comments about our choice of inputs. We set the income volatility 10%. A volatility of this magnitude is necessary to obtain a sizeable and realistic default probability. Although this value seems very high, what matters from the point of view of debt repayment is the distribution of the primary surplus, whose maximum attainable value is a constant fraction of income in this model. A 10% volatility seems much more realistic when referring to the primary surplus.

We set the maximum primary surplus at 10%. Eichengreen and Panizza (2014) document that very rarely the primary surplus of highly indebted countries exceeded 5% for an extended time, but also report several cases in which it was in excess of 10% for short periods.

Finally, some comments about the outputs of this model. As documented by Cohen and
Villemot (2012), most models in the literature, especially models of strategic default, predict minuscule Debt-to-GDP ratios (often below 10%) when calibrated to reproduce realistic default probabilities. This model, while predicting a maximum Debt-to-GDP ratio a bit lower than those observed in the real world, does a lot better in this respect.

3.2 A dynamic model with insurance

In this section we add an insurance with limit $M$ to our baseline infinite-horizon model. Given the technical complication that the insurance adds to the model, we start by considering a “one-country problem”, abstracting from the fact that the insurance is shared between two countries.

At $t+1$ the country has to repay the debt contracted at time $t$, using its primary surplus (up to $\phi Y_{t+1}$) and the amount it can borrow again from the markets, which we denote as $B_{mkt}^t(Y_{t+1})$. If the maximum primary surplus plus $B_{mkt}^t$ is insufficient to fully repay, the markets stop lending at all and the country can borrow at most $M$ from the bailout fund, provided that $M$ is enough to avoid default. If this is not the case, the country defaults, and returns only a fraction $\phi$ of its current income to creditors.

The problem is to find $B_{mkt}^t$ as a function of income. The complication is that, since $M$ is constant (hence the insurance as a fraction of income $m_t \equiv \frac{M}{Y_t}$ is time-dependent), the borrowing capacity from the market is not proportional to income any more, as in the model without insurance. When the insurance is large relative to income $B_{mkt}^t \gg b^* Y$. In contrast, when the insurance is small relative to income $B_{mkt}^t \approx b^* Y$. ($b^* Y$ would be the borrowing capacity in a world without insurance).

As derived in more detail in the Appendix, the function $B_{mkt}^t(Y)$ solves the equation

$$B_{mkt}^t(Y_t) = \frac{1}{1 + r_f} \max_{D} \left( P^{def}(D) E_t[\phi Y_{t+1} | Y_{t+1} < Y^{(def)}(D)] + (1 - P^{def}(D))D \right)$$

(24)

where $Y^{(def)}(D)$ is the threshold value of income below which there is default at $t+1$, given debt $D$, and $P^{def}(D) = P(Y < Y^{(def)}(D))$. $Y^{(def)}(D)$ can be found from the condition that the maximum attainable primary surplus at this income plus the maximum that can be borrowed either from the market or from the bailout fund is exactly equal to the face value of the debt

$$\phi Y^{(def)}(D) + \max(M, B_{mkt}^t(Y^{(def)}(D))) = D$$

(25)

Equation (24) says that the maximum I can borrow today is the present discounted value of
the maximum that in expectation I will be able to repay tomorrow, which in turn, through (25), depends on how much I will be able to re-borrow tomorrow.

We can obtain \( B^{(mkt)}(Y) \) with a recursive method. We start by making a first guess for \( B^{mkt}(Y) \), that we call \( B^{(1)} \)

\[
B^{(1)}(Y) = b^* Y
\]  

Equation (26) amounts to guessing that the borrowing capacity from the market is unaffected by the insurance at any income level. Notice that this would be strictly true at \( t + 1 \) if the insurance lasted only one period, i.e. if it was not available after \( t + 1 \). We insert this guess in the LHS of (24), and compute the borrowing capacity at time \( t \) as a function of income (the RHS). We call this function \( B^{(2)}(Y) \). We now insert \( B^{(2)}(Y) \) in the RHS of (24) and compute again the borrowing capacity on the RHS. (Notice that \( B^{(2)}(Y) \) would be the true borrowing capacity at \( t + 1 \) if the insurance was only available until \( t + 2 \)). We can proceed this way until we get to a function \( B^{(n)} \) that is sufficiently close to the condition \( B^{(n+1)}(Y) = B^{(n)}(Y) \). At each stage \( n \) of the iteration, \( B^{(n)}(Y) \) can be interpreted as the true borrowing capacity if the insurance lasted \( n \) periods.

Figure 6 shows the borrowing capacity as a function of income at each stage of the iteration, when we add an insurance \( M \) equal to 50% of initial income to the baseline infinite-horizon model. (We normalize initial income to 1, \( Y_0 = 1 \), and set \( M = 0.5 \). As can be seen from Figure 8, at every level of the iteration it is \( B^{(n)} = B^{(1)} = b^* Y \) above some threshold value of income. Appendix B shows that, at every level \( n \) of the iteration, this happens for

\[
Y \geq \frac{1}{b^*} \left( \frac{\phi + b^*}{d^*} \right)^{n-1} M
\]  

3.2.1 Shared insurance: a reduced-form approach

What happens when two countries share a partial insurance? As we already assumed in the two-period model, if the insurance limit \( M \) is insufficient to prevent both countries from defaulting, there will be no bailout, i.e. both countries will default.

The one-country problem above shows that borrowing capacity and spread depend on for how many periods the insurance is available. In the two-country problem the insurance can at any time cease to be available for one country just because the other country has depleted it. Knowing when that will happen also requires some information about the borrowing behavior of the other country, not only for the current period but also for all future periods.
However, suppose that we know the probability that one country has access to the insurance in each period, conditional on the insurance not having being used before (or, in other words, we know the conditional probability that “the other country” depletes the insurance in every period). Then the problem can be solved in a recursive way, similarly to the one-country problem.

For simplicity, assume that each period the conditional probability that one country has access to the insurance is a constant $p$. Time-$t$ investors know that when repayment time comes, at $t+1$, with probability $p$ the country will benefit from increased borrowing capacity from the markets like at time $t$ (in addition to the option of using the insurance right away), and with probability $1-p$ there will not be any insurance and the country will only be able to re-borrow a fraction $b^*$ of its income. As shown in Appendix A, the borrowing capacity from the market $B_{p}^{mkt}$ (with the subscript to highlight its dependence on $p$) solves

$$B_{p}^{mkt}(Y_t) = \frac{1}{1+r_f} \max_D \left\{ p \left( P^{def} E_t[\phi Y_{t+1} | Y_{t+1} < Y_p^{(def)}(D)] + (1-P^{def})D \right) + (1-p) \left( P^{def} E_t[\phi Y_{t+1} | Y_{t+1} < Y_0^{(def)}(D)] + (1-P^{def})D \right) \right\} \tag{28}$$

where $Y_p^{(def)}(D)$ is the threshold value of income below which there is default at $t+1$ given $D$. 

Figure 6: Borrowing capacity for level of iteration $n = [0, 20]$ (One-country problem).
in the scenario where the insurance is still available. \( P^{\text{def}} = P(Y_{t+1} < Y_p^{(\text{def})}(D)) \). \( Y_p^{(\text{def})}(D) \) satisfies a condition analogous to (25):

\[
\phi Y_p^{(\text{def})}(D) + \max(M, B_p^{(\text{mkt})}(Y_p^{(\text{def})}(D)))) = D
\]

(29)

\( Y_0^{(\text{def})}(D) \) is the default threshold if the insurance has already been depleted. It can be easily obtained from (23), by multiplying both sides by \( Y_t \):

\[
Y_0^{(\text{def})}(D) = \frac{D}{\phi + b^*}
\]

(30)

The one-country problem at the beginning of this section is a special case of the problem in (28) with \( p = 1 \). Our problem is to find \( B_p^{(\text{mkt})} \) that appears on the LHS of (28) and, through (29) also on the RHS. In the same recursive fashion in which we solved the \( p = 1 \) problem we can now solve this more general problem. Figure 7 shows the resulting borrowing capacity as a function of income for different values of \( p \). Figure 8 shows the spreads as a function of borrowing level for different values of \( p \).

The probability \( p \) that the insurance is depleted every period is the channel through which contagion operates. The borrowing capacity of one country changes very significantly when \( p \) changes even by a few percentage points, all fundamentals of the country unchanged. With insurance limit equal to 50% of income, the borrowing capacity is 51% of income with \( p = 0.9 \), 55% of income when \( p = 0.95 \) and 64% of income when \( p = 1 \). Also, the spread if the country borrows to the maximum changes dramatically: over 8% for \( p = 0.9 \), around 4.5% for \( p = 0.95 \) and less than 1.5% for \( p = 1 \).

4 Debt accumulation and debt crises

In this section we explore how the insurance affects borrowing, debt levels, and, most importantly, the incidence of debt crises and defaults, in the context of the dynamic model developed in Section 3. We define as “debt crisis” the situation in which a country is unable to repay by re-borrowing from the market, and needs a bailout. A default event is a debt crisis in which there is no available bailout or the bailout resources are insufficient.

We work within the reduced-form framework described in section 3.2.1, in which every period the insurance is available to one country with constant probability \( p \), conditional of not having been depleted before. Conditional on the insurance being available, a crisis occurs if income falls below the threshold \( Y_p^{cr} \), which is such that the maximum primary surplus
Figure 7: Borrowing capacity for different levels of $p$. $M$ is 50% of income.

Figure 8: Spread as a function of borrowing level for different levels of $p$. Borrowing level is expressed as a fraction of income and $M$ is 50% of income.
plus the maximum that can be re-borrowed from the markets at this income is just enough for full debt repayment:

$$\phi Y_p^{cr}(D) + B_p^{mkt}(Y_p^{cr}(D)) = D$$

(31)

Default occurs if income falls below the threshold $Y_p^{def}$, which solves (29). Conditional on the insurance not being available, crises and defaults coincide, and happen if income falls below the threshold $Y_0^{def}$ in (30). Crises and defaults coincide (hence $Y_p^{def} = Y_p^{cr}$) also when

$$M < B_p^{mkt}(Y_p^{def}(D)) = B_p^{mkt}(Y_p^{cr}(D))$$

(32)

i.e. when, at the income level where the borrowing capacity from the market is insufficient to repay, the maximum funding ability of the insurance is even lower. This appears clearly from the comparison between (31) and (29).

**Debt accumulation**

The borrowing capacity of a country unambiguously increases in the presence of the insurance. The country can borrow more at $t$ because in expectation it will be able to repay more at $t+1$: it might use the insurance resources in case of need, or otherwise, if the insurance is still available for the future, it will be able to re-borrow more from the markets. However, the spread for a given borrowing level is lower in the presence of the insurance. The effect on debt accumulation depends therefore on the behavior of the country: a prudent country that refrains from increasing its spending tends to accumulate less debt thanks to the lower spreads, whereas a profligate country that takes advantage of the insurance to spend to the maximum might accumulate more debt.

Here we focus our attention on the “profligate” type of country, that borrows to its maximum capacity. Empirical evidence supports the view that many countries borrow “because they can”, rather than for the more sound economic reason of smoothing consumption (see e.g. Levy-Yeyati (2009), and Kaminsky, Reinhart and Végh (2005))\(^7\). This type of country seems more suitable to describe the countries of the Euro periphery, that we have in mind in this paper.

We normalize initial income $Y_0$ to 1 and consider two values for the insurance limit $M$: a high value 0.5 and a low value 0.25. “High” and “low” are meant in relation to the current

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\(^7\)The main explanation for this phenomenon is the presence of political imperfections, such as governments with a short horizon, as in Habib and Rochet (2013), or altogether corrupt governments, as in Alesina, Campante and Tabellini (2008).
borrowing capacity from the market in the absence of insurance, which would be around 0.39.

Figure 9 (for the case $M = 0.5$ on the left panel and for $M = 0.25$ on the right panel) show how the borrowing level and the debt of a country that borrows to the maximum increases in the presence of the insurance, as a function of the probability $p$ that the insurance will be available next period.

We see that the maximum debt that a country can undertake is increasing in $p$, and in particular it is always higher than in the case without insurance. This is not fully obvious. The insurance could in principle lower the default probability (and hence the spread) so much that even while borrowing more a country could contract less debt. This is actually what happens in a 2-period model. In Section 2 we saw that without insurance, if the country borrows to its full capacity, debt is infinite. With insurance, borrowing capacity increases and debt always remains finite. This does not happen in the infinite-horizon model, where the ability to re-borrow from the markets at $t + 1$ keeps the spreads finite at $t$.

**Crises and Defaults**

Now that we have established that profligate countries borrow more and accumulate more debt in the presence of the insurance, we turn to the question we are most interested in: how does the insurance affect the probability of a debt crisis? From the point of view of time $t$, when our profligate country accumulates maximum debt $D^*$, the probability of a crisis at $t + 1$ is

$$ P_{cr} = pP(Y_{t+1} < Y_p^{cr}(D^*)) + (1-p)P(Y_{t+1} < Y_0^{def}(D^*)) $$  \hspace{1cm} (33) 

and the probability of default is

$$ P_{def} = pP(Y_{t+1} < Y_p^{def}(D^*)) + (1-p)P(Y_{t+1} < Y_0^{def}(D^*)) $$  \hspace{1cm} (34) 

The difference between the two is the probability of avoiding default thanks to the insurance.

Figure 10 (for $M = 0.5$ on the left panel and for $M = 0.25$ on the right panel) shows the probability of crises alongside the probability of default. Notice that with the low value of $M$ the probability of default is equal to the probability of a crisis. Indeed, with $M = 0.25$ condition (32) is satisfied at every $p$: even for $p = 0$ it is $B^{mkt}_p(Y_p^{cr}(D^*)) = 0.347$, bigger than $M$. 

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We find that when the probability $p$ that the insurance is available next period is high, the insurance reduces the probability of crises, which is noteworthy given that the profligate country tries to take maximum advantage of the insurance and borrow to the maximum. This happens for $p \gtrsim 95.5\%$ in case $M = 0.5$, and for $p \gtrsim 52.5\%$ for $M = 0.25$. On the other hand, for lower values of $p$ the probability of crises and even of defaults increases relative to the case of no insurance, and can reach very high values when the insurance is high.

To understand the result with high $p$, consider the extreme case in which the insurance is available with certainty, i.e. $p = 1$. A country can take more debt at $t$ thanks to the insurance, but it will be able to re-borrow more at $t+1$ to repay this debt, and in particular, as we see in Figure 8, it will be able to re-borrow more (relative to the world without insurance) when its income is low. This increase in market borrowing capacity at low income is what reduces the probability of crises.

Paradoxically, for $p = 1$ the country can borrow more, increase its debt and be able to repay more often thanks to the insurance, but will hardly ever need the insurance (unless the insurance is very big relative to the country’s income). Knowing that the insurance will be there for sure next period, markets are always willing to lend at least $\frac{M}{1+r_f}$. Moreover, with our baseline parameters, for $p = 1$ $B^{mk}(Y) > M$ for $Y \gtrsim 0.55M$. A bailout occurs only if the borrowing capacity is not enough to repay and $M$ is enough to fully repay, so it occurs only if income is small compared to the size of the insurance, $Y < 0.55M$, and debt is in the narrow range $\phi Y + M/(1 + r_f) < D < \phi Y + M$.

If $Y_t \gg 0.55M$ the insurer really becomes a lender of last resort that stabilizes the markets without (almost) ever needing to intervene.

Two effects contribute to increasing the probability of crises and defaults for $p$ lower than 1. First, the country can take more debt at $t$ thanks to the insurance, but as $p$ falls below 1 there is increasing probability $(1-p)$ that the insurance cease to be available at $t+1$, in which case its borrowing capacity is lower again thus the likelihood of default is much higher.  

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To identify this effect from numerical results, we rewrite (33) as

\[ P_{cr} = pP(cr|A) + (1-p)P(cr|NA) \]  

(35)

where $P(cr|A)$ and $P(cr|NA)$, shown in Figure 11 for $M = 0.5$ (left panel) and $M = 0.25$ (right panel), are the probabilities of a crisis next period conditional on the insurance being available and not available, respectively. We can check from the numerical values reported in Appendix C that the second term on the RHS of (35), $(1-p)P(cr|NA)$, is decreasing in $p$, (for any $p$, in case $M = 0.25$, and for $p > 0.7$, in case...
A second effect occurs only in the case of high $M$. For $M = 0.5$ this occurs in the range of $p$ between 0.75 and 0.9 (see Figure 10, left panel). In this range there is true moral hazard: by borrowing to its maximum capacity, the country essentially promises to use the insurance to repay whenever the insurance is available, and default when the insurance is not available. As we see from Figure 9, around $p = 0.7$ debt increases much faster in $p$ than the borrowing capacity from the market, so that in the range between $p = 0.75$ and $p = 0.9$ debt is very high relative to borrowing capacity, which makes it difficult to repay by re-borrowing from the market. At the same time, $M$ is bigger than the borrowing capacity from the market. This implies that (almost) the only chance for the country to repay is to use the insurance when available. For example, consider the case of $p = 0.75$. A country can take a debt equal to around 58% of its income but its borrowing capacity next period will be only around 45% of its income, if the latter does not change too much and the insurance is still available. Next period, even if the insurance is available, a crisis occurs if $Y < 1.08$. However, given the insurance $M = 0.5$, even in a crisis the country can still repay if its income is in the interval $0.83 < Y < 1.08$, which happens with 70% probability.

The probability of a crisis rises to enormous levels in this region (the peak is over 75% in case $M = 0.5$), as does the probability of default (around 25%). Even the probability of a crisis conditional on the insurance being available peaks at over 70%.

The main message of this section is that, while a certain or almost certain insurance reduces the incidence of crises, the sharing mechanism introduces uncertainty and can have counterproductive effects. While bailouts are meant to avoid defaults, we have shown that an uncertain insurance can increase the incidence of both crises and defaults.

$M = 0.5$). This occurs despite the fact that $P(cr|NA)$ is (weakly) increasing in $p$. 

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Figure 9: Borrowing capacity and debt accumulation as a function of $p$, for $M$ equal to 50% of income (left) and 25% of income (right).

Figure 10: Probability of crisis and default as a function of $p$, for $M$ equal to 50% of income (left) and 25% of income (right). The dotted line is the probability of default in case of no insurance.

5 Two profligate countries: full solution

In Section 4 we have obtained the borrowing capacity and the spread for one country using a reduced form approach in which all the information about the other country is incorporated.
in one parameter $p$, the probability that the other country does not deplete the insurance next period. In Section 5 we have looked at what the results obtained in Section 4 imply for debt accumulation and the frequency of crises, by focusing on the case of a “profligate” country whose objective is to maximize its borrowing every period. In this section we take a step further and assume that both countries are profligate. In this case the borrowing capacity of one country is only a function of the two income levels $Y^{(1)}$ and $Y^{(2)}$ for a given level of $M$. While in a more general case the borrowing capacity would also be a function of the debt level path chosen by the other country, the profligacy assumption pins down the debt level of the two countries in each period to a function of $Y^{(1)}$ and $Y^{(2)}$, too.

The borrowing capacities of the two countries solve

$$B^{* (1)}(Y^{(1)}, Y^{(1)}_t) = \frac{1}{1 + r_f} \max_{D_1} \left\{ P_{def}^{(1)} E_t[\phi Y_{t+1}^{(1)}|def^{(1)}] + D_1(1 - P_{def}^{(1)}) \right\} \quad (36)$$

$$B^{* (2)}(Y^{(2)}, Y^{(2)}_t) = \frac{1}{1 + r_f} \max_{D_2} \left\{ P_{def}^{(2)} E_t[\phi Y_{t+1}^{(2)}|def^{(2)}] + D_2(1 - P_{def}^{(2)}) + \right\} \quad (37)$$

This is a joint fixed point problem, since the probability of default $P_{def}^{(i)}$ for country $i = 1, 2$ also involves the arg max of the other country’s problem, $D^{*(i)}$, and both borrowing capacities $B^{*(i)}(Y^{(1)}_{t+1}, Y^{(2)}_{t+1})$. Indeed, whether or not one country defaults next period depends on how much it will be able to re-borrow, which also depends on whether the insurance is still available (i.e. on whether the other country is able to repay), which in turn depends on

Figure 11: Probability of crisis as a function of $p$, for $M$ equal to 50% of income (left) and 25% of income (right), conditional on the insurance being available or not.
the debt contracted by the other country and how much the other country can re-borrow. More details about this maximization problem are given in Appendix A.

In analogy with the baseline case in the previous sections, we use $M = 0.5$. The solution can be obtained in a recursive way similarly to what described in Section 4. Numerical results for the two surfaces can be found in Appendix C. Here we would like to briefly discuss two features that emerge from the solution: the advantage of the “leader” country toward the “follower” country, and the advantage of the “small” country toward the “big” country.

**The advantage of the “leader” vs the “follower”.** Although the two countries are in principle identical, their borrowing surfaces are different, and in particular depend on which country is the game “leader” and which is the “follower”. The leader, which can be thought as the country that is most aggressive in exploiting the insurance, decides how much to borrow first. The follower maximizes its borrowing given the action already taken by the leader and in general will be able to borrow less than the leader. We use the convention that Country 1 is the leader and Country 2 the follower. It is in general

$$B^*(1)(y, y') \geq B^*(2)(y', y)$$

(38)

**The advantage of the small country.** In spite of the leader’s advantage over the follower, when the follower’s income is: i) low relative to the leader’s income, ii) low enough that the insurance resources are much bigger than its borrowing capacity from the markets, then all the benefits of the insurance, in terms of increased borrowing capacity, shift to the follower.

As an illustration, consider the extreme case in which Country 2, the follower, has 0 income and Country 1 has positive income. For any debt level chosen by Country 1, the best response of Country 2 is to issue debt $M$. Indeed, even if $D_1$ is high, there is still some probability, maybe small, that Country 1 is able to repay without the help of the insurance, in which case Country 1 can avoid default by transferring the full amount of the insurance to its creditors. This gives a positive borrowing capacity to Country 1. Given that Country 1 has no ability to repay without the help of the insurance, this is clearly the strategy that maximizes its borrowing ability. On the other hand Country 1, despite being the leader, has lost any protection from the insurance given that Country 2 will for sure deplete it next period. As a result, Country 1’s borrowing capacity is reduced to $b^*Y_2$ (as in a world without insurance).
In other words, the ability of the small country to engage in moral hazard takes away any protection from the big country. This is one mechanism by which a small country such as Greece may have been able to borrow a lot more than bigger countries in the EA and almost taken full control of the bailout resources of the union.

The “small country effect” results in a non-monotonic and non-smooth behavior of the borrowing surface, as we see in Figure 12, representing sections of the surface for one country in which the income of ”the other” country is kept constant, and Figure 13, representing sections of the surface for one country in which its own income is kept constant.

Consider as an example the case in which the follower’s income is $Y^{(2)} = 0.4$. Given the follower’s small income relative to the insurance, the leader starts losing its advantage as a leader for $Y^{(1)} > 1.2$, resulting in a loss of borrowing capacity even as its income increases (see Figure 12, left panel, solid line). Correspondingly, at the same point $[Y^{(1)} = 1.2, Y^{(2)} = 0.4]$ the follower’s borrowing capacity jumps up (Figure 13, right panel, solid line).

Very similarly, consider the dash-dotted lines in the right panel of Figure 12 and left panel of Figure 13, showing the sections of the leader’s and follower’s surface for $Y^{(1)} = 1.4$. As the follower’s income grows beyond 0.6, its borrowing capacity decreases and correspondingly the leader’s income suddenly increases. Here, the follower is losing its small country advantage, and the leader is regaining its leader advantage.

**Probability of a crisis**

Figure 14 gives a visual representation of the probability of a crisis involving the leader (on the left) or the follower (on the right). Numerical values can be found in Appendix C.

In the region where both incomes are high, the crisis probability is around or below the crisis probability without insurance. In particular, in the regions in black in Figure 14, the crisis probability is between 4% and 5%, significantly smaller than the probability of a crisis without insurance for a country that always borrows the maximum (which is 5.8%). As in the discussion in Section 4, these are the areas where the insurance is unlikely to be depleted next period, as both countries are doing well, and the insurance induces the markets to lend more, especially in a downturn.

“Moral hazard” regions with very high crisis probability occur mostly when at least one income is small. When both incomes are small, it is the leader that can take control of the bailout resources, therefore there is a high probability of a crisis involving the leader.

The occasional small spots of lighter or darker color show the effect of shifts in borrowing
capacity as discussed above. It is intuitive that such shifts can leave a country that borrowed a lot in one period unable to re-borrow enough to repay in the next period, or the other way around.

Figure 12: Left (right) panel: sections of the leader’s (follower’s) borrowing capacity surface as its own income moves and the other country’s income is fixed.

Figure 13: Left (right) panel: sections of the leader’s (follower’s) borrowing capacity surface as its own income is fixed and the other country’s income moves.
Figure 14: Probability of a crisis over the next period for the leader (left) and the follower (right).

6 Conclusion

Summarize again the main findings.

ECB’s President Draghi “whatever it takes” speech in the summer of 2012 can be interpreted as causing a revision of beliefs toward a higher value of the insurance size $M$. There is wide consensus on the causal link between this speech and the subsequent dramatic decrease in sovereign spreads in the Euro periphery. While our current model only considers the case of fixed $M$ known with certainty, an extension with uncertain $M$ could be interesting to shed some light on these events.
References


Appendix A: Borrowing Capacity

Two-period model

We call $R_1(D)$ the repayment at $t = 1$ when debt is $D$. In the two-period model of Section 2, in which a country can access to an insurance with limit $M$, $R_1(D)$ is given by is given by (7). Expected repayment as a function of debt $D$ is

$$E[R_1(D)] = P \left( Y_1 < \frac{D - M}{\phi} \right) E \left[ \phi Y_1 | Y_1 < \frac{D - M}{\phi} \right] + P \left( Y_1 \geq \frac{D - M}{\phi} \right) D$$

that, more explicitly, can be written as

$$E[R_1(D)] = \int_0^{\frac{D - M}{\phi}} \phi Y_1 pdf(Y_1) dY_1 + \left( 1 - cdf \left( \frac{D - M}{\phi} \right) \right) D$$

where $pdf(.)$ and $cdf(.)$ are the probability density and the cumulative probability distribution, respectively, of $Y_1$ (income at $t = 1$).

The borrowing capacity of a country is the maximum that the market allows a country to borrow. This is related to the maximum that the country is expected to repay:

$$(1 + r_f)B^* = Max_D E[R_1(D)]$$

The value $D^*$ that corresponds to the maximum expected repayment is obtained by solving the first order condition

$$\frac{d(E[R_1(D)])}{dD} = 0$$

that, from (40), can be written more explicitly as

$$\left( 1 - cdf \left( \frac{D - M}{\phi} \right) \right) - \frac{M}{\phi} pdf \left( \frac{D - M}{\phi} \right) = 0$$

If we look at the two terms on the LHS of (43), the first is always positive. The second is zero for $M = 0$, confirming that in the absence of insurance expected repayment is always
increasing in $D$. For $M > 0$ the second term on the RHS of (43) is positive, so that for $M > 0$ (43) has an interior solution $D^*$, hence to a maximum borrowing capacity

$$ (1 + r_f)B^* = \int_0^{D^* - M \phi} \phi Y_1 pdf(Y_1) dY_1 + D^* \left( 1 - cdf \left( \frac{D^* - M \phi}{\phi} \right) \right) \quad (44) $$

**Two-period model: insurance shared by two countries**

If the insurance is shared by two countries, and can be used only if it is enough to avoid both defaults, the repayment of Country 1 is $R_1(D_1, D_2)$ is given by (10), where $D_1$ and $D_2$ is the debt of Country 1 and Country 2. Thus, assuming that the income of the two countries is uncorrelated, expected repayment is

$$ E[R_1(D_1, D_2)] = \int_0^{D_1 - M \phi} \phi Y_1 pdf(Y_1) dY_1 $$

$$ + \int_{D_1 - M \phi}^{D_1 \phi} \phi Y_1 pdf(Y_1) \left( D_1 + D_2 - M - \phi Y_1 \right) dY_1 $$

$$ + \left( cdf(Y_1) \left( \frac{D_1}{\phi} \right) - cdf(Y_1) \left( \frac{D_1 - M \phi}{\phi} \right) \right) \left( 1 - cdf(Y_1) \left( \frac{D_1 + D_2 - M - \phi Y_1}{\phi} \right) \right) D_1 $$

$$ + \left( 1 - cdf(Y_1) \left( \frac{D_1}{\phi} \right) \right) D_1 \quad (45) $$

where $pdf(.)$ and $cdf(.)$ are the probability density and the cumulative probability distribution for the income at time 1 of Country $i$, $i = 1, 2$. For a given $D_2 = \bar{D}_2$, the borrowing capacity of Country 1 is obtained as

$$ (1 + r_f)B^* = \max_{D_1} E[R_1(D_1, \bar{D}_2)] \quad (46) $$

The value $D_1^*$ that maximizes (46) solves the FOC

$$ 0 = -\frac{M \phi}{pdf(Y_1)} \left( \frac{D_1^* - M \phi}{\phi} \right) + \frac{D_1^*}{\phi} pdf(Y_1) \left( \frac{D_1^* - M \phi}{\phi} \right) cdf(Y_1) \left( \frac{D_2 - M \phi}{\phi} \right) $$

$$ - \frac{D_1^* - M \phi}{\phi} pdf(Y_1) \left( \frac{D_1^* - M \phi}{\phi} \right) cdf(Y_1) \left( \frac{D_2}{\phi} \right) $$

$$ + \int_{D_1 - M \phi}^{D_1 \phi} Y_1 pdf(Y_1) pdf(Y_1) \left( \frac{D_1^* + D_2 - M - \phi Y_1}{\phi} \right) dY_1 $$

$$ + \frac{1}{\phi} \left( pdf(Y_1) \left( \frac{D_1}{\phi} \right) - pdf(Y_1) \left( \frac{D_1 - M \phi}{\phi} \right) \right) \left( 1 - cdf(Y_1) \left( \frac{D_1 + D_2 - M - \phi Y_1}{\phi} \right) \right) $$

$$ - \frac{1}{\phi} \left( cdf(Y_1) \left( \frac{D_1}{\phi} \right) - cdf(Y_1) \left( \frac{D_1 - M \phi}{\phi} \right) \right) pdf(Y_1) \left( \frac{D_1 + D_2 - M - \phi Y_1}{\phi} \right) $$

$$ + \left( 1 - cdf(Y_1) \left( \frac{D_1^*}{\phi} \right) \right) \quad (47) $
A special case is if the two countries are identical and borrow the same amount. Then the maximum that each of them can borrow is $B^*$ such that

$$(1 + r_f)B^* = R_1(D^*, D^*)$$ (48)

where $D^*$ solves an equation analogous to (47), where we set $D_1^* = D_2 = D^*$.

**Infinite-horizon baseline model**

Analogously to the two-period model, we find $B^*_t$, the borrowing capacity at time $t$, as

$$(1 + r_f)B^*_t = M ax_{D_{t+1}} E_t[R_{t+1}(D_{t+1})]$$ (49)

using (22), (49) can be written as

$$(1 + r_f)B^*_t = M ax_{D_{t+1}} \left[ \int_0^{D_{t+1}-B^*_t} \phi Y_{t+1} pdf(Y_{t+1}) \, dY_{t+1} + D_{t+1}P(\phi Y_{t+1} + B^*_t > D_{t+1} | Y_t) \right]$$ (50)

where $pdf(\cdot)$ is the probability density function conditional on time-$t$ information, and, similarly $P(\cdot | Y_t)$ is the probability conditional on $Y_t$. The stationarity of the growth factors implies that $B^*_t$ is proportional to current income $Y_t$:

$$B^*_t = b^*Y_t$$ (51)

To see this, we can divide the LHS and RHS of (49) by $Y_t$, hence we integrate over the distribution of the time-stationary growth factor $g$. We obtain

$$(1 + r_f)b^* = M ax_d \left[ \int_0^{d-b^*} \phi g \, pdf(g) \, dg + d \times P((\phi + b^*)g > d) \right]$$ (52)

where $D_{t+1} \equiv dY_t$. We see from (52) that since the growth factor is stationary, any dependence on time disappears, so the $b^*$ that solves (52) must indeed be constant, i.e. $B^*_t$ must be proportional to current income.

Since

$$g < \frac{d - b^*g}{\phi} \Rightarrow g < \frac{d}{\phi + b^*}$$ (53)

we can write (52) as

$$(1 + r_f)b^* = M ax_d \left[ \int_0^{d-b^*} \phi g \, pdf(g) \, dg + d \times \int \left[ g > \frac{d}{\phi + b^*} \right] \right]$$ (54)
and defining $d' \equiv \frac{d}{\phi + b^*}$ we obtain

$$(1 + r_f)b^* = \max_{d'} \int_0^{d'} \phi g \text{pdf}(g) \, dg + d'(\phi + b^*) \times P(g > d')$$

hence $b^*$ can be found as the solution of the fixed-point equation

$$b^* = g(b^*)$$

where

$$g(b^*) = \frac{\max_{d'} \left[ \int_0^{d'} \phi g \text{pdf}(g) \, dg + d'(\phi + b^*) \times P(g > d') \right]}{1 + r_f}$$

In this model knowing the borrowing capacity is necessary to obtain spreads. For a given borrowing level $b_t$ by first finding the value $d'_{t+1}$ that solves

$$(1 + r_f)b_t = \int_0^{d'_{t+1}} \phi g \text{pdf}(g) \, dg + d'_{t+1}(\phi + b^*) \times P(g > d'_{t+1})$$

and correspondingly the required debt repayment (as a fraction of $Y_t$) $d_{t+1} = (\phi + b^*)d'_{t+1}$. At this point the spread $x_t$ is easily obtained by solving

$$d_{t+1} = (1 + r_f + x_t)b_t$$

**Infinite horizon model with insurance**

We assume that the insurance, with limit $M$, is available to one country with probability $p$ each period, conditional to being available in the previous period. (49) still holds. The country borrows at time $t$, when the insurance is available, and repays at $t + 1$, when the insurance will be available with probability $p$. If it is available, at $t + 1$ default occurs if $Y_{t+1} < Y_p^{(def)}(D)$ where $Y_p^{(def)}(D)$ is such that the maximum primary surplus plus the maximum that the country can borrow from the markets or from the insurance is equal to $D$, the face value of the debt:

$$\phi Y_p^{(def)}(D) + \max(M, B_p^{(mkt)}(Y_p^{(def)}(D))) = D$$

$B_p^{(mkt)}(Y)$ is the borrowing capacity from the market given income $Y$, in the assumption that the insurance will be available with probability $p$ next period. If the insurance is not
available at \( t + 1 \) (and will not be available henceafter), default occurs if \( Y_{t+1} < Y^{(def)}_0(D) \) where

\[
Y^{(def)}_0(D) = \frac{D}{\phi + b^*} \tag{61}
\]

Given that the country repays \( \phi Y_{t+1} \) in case of default, the expected repayment at \( t + 1 \) is

\[
E[R_1(D)] = p \left( P(Y_{t+1} < Y^{(def)}_0(D)) E_t[\phi Y_{t+1} | Y_{t+1} < Y^{(def)}_p(D)] + D \times P(Y_{t+1} \geq Y^{(def)}_p(D)) \right) \\
+(1 - p) \left( P(Y_{t+1} < Y^{(def)}_0(D)) E_t[\phi Y_{t+1} | Y_{t+1} < Y^{(def)}_0(D)] + D \times P(Y_{t+1} \geq Y^{(def)}_0(D)) \right) \tag{62}
\]

**Appendix B: Proof of eq(27)**

In Section 3.1 we described the recursive method to obtain the borrowing capacity in the infinite-horizon model, when in every period \( t \) the probability that the insurance is available is \( p \), conditional on being available at \( t - 1 \). In Figure 8 (for the special case \( p = 1 \)) we see that, at every level of the iteration \( n \), \( B^{(n)}(Y) = B^{(1)}(Y) = b^* Y \) above some threshold value of income, which we call \( Y_n \). Namely, above \( Y_n \), the borrowing capacity from the market is unaffected by the insurance. In this Appendix we show that it is

\[
Y_n = \frac{1}{b^*} \left( \frac{\phi + b^*}{d^*} \right)^{n-1} M \tag{63}
\]

and this is valid independent of \( p \). Clearly this is true for \( n = 1 \). We consider now \( n = 2 \).

\( B^{(2)} \) would be the true borrowing capacity at \( t \) if the insurance is available at \( t + 1 \) with probability \( p \) and is for sure unavailable after \( t + 1 \), i.e. . Clearly \( B^{(2)}(Y) = b^* Y \) for those values of \( Y \) for which the insurance is too small to ever be used at \( t + 1 \). This is the case if \( M \) is either insufficient to avoid default, or it is lower than the borrowing ability from the market. In particular, if \( M \) is lower than the borrowing ability from the market at the default threshold, then \( M \) can never be useful (for values of \( Y \) above the default threshold, \( M \) will always be lower than the borrowing ability from the market, and for values of \( Y \) below the default threshold, \( M \) will be insufficient to avoid default). Since the borrowing ability from the market at \( t + 1 \) is unaffected by the insurance by construction \( (B^{(2)} \) is the borrowing ability at \( t \) assuming that the borrowing ability at \( t + 1 \) is \( B^{(1)} \) it is

\[
Y^{\text{def}}_{t+1} = \frac{d^*}{b^* + \phi} Y_t \tag{64}
\]

The borrowing ability from the market at \( Y^{\text{def}}_{t+1} \) is \( b^* Y^{\text{def}}_{t+1} \). The condition \( M < b^* Y^{\text{def}}_{t+1} \)
translates into the condition for $Y_t$

$$Y_t > Y^{(2)} \equiv \frac{1}{b^*} \left( \phi + b^* \frac{d^*}{\phi} \right) M$$

(65)

Now suppose that we have proved (63) for $n$, i.e. that we have proved that $B^{(n)} = b^* Y$ for $Y > Y_n$. We prove it for $n + 1$. $B^{(n+1)}(Y)$ is the borrowing ability at $t$ if the insurance lasts (with positive probability) until $t + n$, i.e. if the corresponding borrowing ability at $t$ is $B^{(n)}$. If $Y_n$ is below the default threshold at $t + 1$ then for all practical purposes the borrowing capacity at $t + 1$ is unaffected by the insurance. This happens if

$$\frac{1}{b^*} \left( \phi + b^* \frac{d^*}{\phi} \right) M < \frac{d^*}{b^* + \phi} Y_t$$

(66)

which translates into

$$Y_t > Y^{(n+1)} = \frac{1}{b^*} \left( \phi + b^* \frac{d^*}{\phi} \right)^n M$$

(67)

In addition, if (67) is true, since it is always $\phi + b^* > d^*$, then it is also $Y_t > \frac{1}{b^*} \left( \phi + b^* \frac{d^*}{\phi} \right) M$, which means that $M$ is lower than the default threshold at $t + 1$. In sum, the insurance can never be used at $t + 1$ and the borrowing ability at $t + 1$ is unaffected by the insurance. Together, these two conditions imply that the insurance has no impact on $B^{(n+1)}$.

We proved (63) for the trivial case $n = 1$ and $n = 2$ and we proved that, if it is valid for $n$, it is valid for $n + 1$. Therefore (63) is valid for every $n$.

### Appendix C

The following tables contain all the numerical results discussed in Section 4 and 5. Table 2 and Table 3 contain results for Section 4. We report borrowing capacity, max debt, spread, crisis and default probability for a set of values of $p$, the probability that the insurance is available next period.

Table 4, 5, 6, and 7 contain all the numerical results discussed in Section 5. In particular, for a grid of values of the income of the two countries, Table 4 and 5 report the leader’s and follower’s borrowing surface, respectively; Table 6 and 7 report the crisis probability for the leader and follower, respectively.
A. Numerical results for Section 4

| $p$ | $B^*$ | $D^*$ | $x$ | $P_{cr}$ | $P_{def}$ | $P(cr|A)$ | $P(cr|NA)$ |
|-----|-------|-------|-----|----------|-----------|-----------|------------|
| 0   | 0.398 | 0.434 | 0.051 | 0.058    | 0.058     | 0.058     | 0.058      |
| 0.05| 0.399 | 0.436 | 0.053 | 0.063    | 0.060     | 0.061     | 0.063      |
| 0.1 | 0.400 | 0.437 | 0.054 | 0.067    | 0.061     | 0.062     | 0.068      |
| 0.15| 0.401 | 0.439 | 0.055 | 0.071    | 0.062     | 0.064     | 0.073      |
| 0.2 | 0.402 | 0.441 | 0.056 | 0.076    | 0.063     | 0.066     | 0.079      |
| 0.25| 0.404 | 0.443 | 0.057 | 0.081    | 0.064     | 0.068     | 0.085      |
| 0.3 | 0.405 | 0.445 | 0.058 | 0.087    | 0.066     | 0.071     | 0.094      |
| 0.35| 0.407 | 0.447 | 0.060 | 0.093    | 0.067     | 0.074     | 0.103      |
| 0.4 | 0.409 | 0.450 | 0.061 | 0.100    | 0.069     | 0.078     | 0.115      |
| 0.45| 0.411 | 0.454 | 0.064 | 0.108    | 0.071     | 0.082     | 0.130      |
| 0.5 | 0.413 | 0.457 | 0.066 | 0.118    | 0.074     | 0.088     | 0.149      |
| 0.55| 0.416 | 0.462 | 0.070 | 0.131    | 0.078     | 0.095     | 0.174      |
| 0.6 | 0.420 | 0.468 | 0.075 | 0.147    | 0.083     | 0.106     | 0.208      |
| 0.65| 0.424 | 0.476 | 0.083 | 0.172    | 0.091     | 0.124     | 0.261      |
| 0.7 | 0.430 | 0.489 | 0.099 | 0.219    | 0.107     | 0.160     | 0.356      |
| 0.75| 0.447 | 0.583 | 0.265 | 0.767    | 0.245     | 0.717     | 0.917      |
| 0.8 | 0.469 | 0.582 | 0.202 | 0.638    | 0.196     | 0.568     | 0.915      |
| 0.85| 0.491 | 0.581 | 0.145 | 0.372    | 0.147     | 0.277     | 0.913      |
| 0.9 | 0.512 | 0.581 | 0.094 | 0.118    | 0.099     | 0.030     | 0.910      |
| 0.95| 0.550 | 0.603 | 0.056 | 0.061    | 0.061     | 0.014     | 0.958      |
| 1   | 0.644 | 0.682 | 0.019 | 0.020    | 0.020     | 0.020     | 0.998      |

Table 2: Initial income $Y = 1, M = 0.5$. 
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
\textit{p} & \textit{B}^* & \textit{D}^* & \textit{x} & \textit{P}_{cr} & \textit{P}_{def} & \textit{P}_{cr}^1 & \textit{P}_{cr}^2 \\
\hline
0 & 0.398 & 0.434 & 0.051 & 0.058 & 0.058 & 0.058 & 0.058 \\
0.05 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.059 & 0.059 \\
0.1 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.059 & 0.059 \\
0.15 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.059 & 0.059 \\
0.2 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.059 & 0.059 \\
0.25 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.059 & 0.059 \\
0.3 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.058 & 0.059 \\
0.35 & 0.398 & 0.434 & 0.052 & 0.059 & 0.059 & 0.058 & 0.059 \\
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0.45 & 0.398 & 0.435 & 0.051 & 0.059 & 0.059 & 0.057 & 0.060 \\
0.5 & 0.398 & 0.435 & 0.052 & 0.058 & 0.058 & 0.056 & 0.061 \\
0.55 & 0.399 & 0.435 & 0.051 & 0.058 & 0.058 & 0.055 & 0.061 \\
0.6 & 0.399 & 0.436 & 0.051 & 0.058 & 0.058 & 0.055 & 0.063 \\
0.65 & 0.400 & 0.436 & 0.050 & 0.057 & 0.057 & 0.054 & 0.065 \\
0.7 & 0.401 & 0.437 & 0.050 & 0.057 & 0.057 & 0.052 & 0.067 \\
0.75 & 0.402 & 0.438 & 0.049 & 0.056 & 0.056 & 0.051 & 0.071 \\
0.8 & 0.404 & 0.440 & 0.049 & 0.055 & 0.055 & 0.050 & 0.077 \\
0.85 & 0.408 & 0.444 & 0.048 & 0.054 & 0.054 & 0.048 & 0.089 \\
0.9 & 0.414 & 0.449 & 0.047 & 0.053 & 0.053 & 0.046 & 0.111 \\
0.95 & 0.428 & 0.464 & 0.044 & 0.049 & 0.049 & 0.042 & 0.186 \\
1 & 0.483 & 0.518 & 0.034 & 0.038 & 0.038 & 0.038 & 0.583 \\
\hline
\end{tabular}
\caption{Initial income $Y = 1$, $M = 0.25$.}
\end{table}
### B. Numerical results for Section 5

**Leader’s borrowing surface**

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<td>0.053</td>
<td>0.054</td>
</tr>
</tbody>
</table>
### Follower’s crisis probability

| $Y^{(2)}$ | $Y^{(1)}$ | 0.4  | 0.6  | 0.8  | 0.9  | 1    | 1.1  | 1.2  | 1.3  | 1.4  | 1.6  | 1.8  | 2    | 2.2  | 2.4  | 2.6  | 2.8  | 3    |
|-----------|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.4       | 0.063     | 0.058| 0.056| 0.057| 0.058| 0.063| 0.083| 0.922| 0.893| 0.881| 0.860| 0.851| 0.861| 0.869| 0.870| 0.867| 0.866|
| 0.6       | 0.062     | 0.057| 0.058| 0.058| 0.057| 0.057| 0.075| 0.677| 0.462| 0.204| 0.155| 0.162| 0.163| 0.164| 0.168| 0.170| 0.174|
| 0.8       | 0.059     | 0.059| 0.059| 0.057| 0.057| 0.057| 0.061| 0.075| 0.099| 0.084| 0.262| 0.044| 0.071| 0.062| 0.062| 0.064| 0.064|
| 0.9       | 0.059     | 0.059| 0.059| 0.057| 0.057| 0.057| 0.062| 0.061| 0.059| 0.204| 0.047| 0.073| 0.064| 0.063| 0.063| 0.064| 0.066|
| 1         | 0.055     | 0.059| 0.058| 0.058| 0.058| 0.058| 0.061| 0.065| 0.073| 0.088| 0.121| 0.050| 0.072| 0.068| 0.067| 0.067| 0.067|
| 1.1       | 0.059     | 0.059| 0.059| 0.059| 0.059| 0.059| 0.064| 0.069| 0.080| 0.123| 0.065| 0.057| 0.066| 0.063| 0.062| 0.063| 0.062|
| 1.2       | 0.058     | 0.059| 0.059| 0.059| 0.059| 0.059| 0.063| 0.074| 0.090| 0.072| 0.063| 0.054| 0.056| 0.054| 0.054| 0.054| 0.054|
| 1.3       | 0.057     | 0.059| 0.059| 0.059| 0.059| 0.059| 0.065| 0.060| 0.041| 0.058| 0.051| 0.048| 0.052| 0.053| 0.052| 0.053| 0.051|
| 1.4       | 0.056     | 0.058| 0.058| 0.058| 0.058| 0.059| 0.057| 0.055| 0.051| 0.047| 0.047| 0.046| 0.046| 0.046| 0.047| 0.047| 0.046|
| 1.6       | 0.063     | 0.060| 0.059| 0.058| 0.058| 0.058| 0.056| 0.056| 0.054| 0.053| 0.049| 0.048| 0.046| 0.047| 0.045| 0.048| 0.048|
| 1.8       | 0.061     | 0.059| 0.059| 0.059| 0.058| 0.058| 0.057| 0.057| 0.054| 0.053| 0.050| 0.048| 0.048| 0.046| 0.049| 0.049| 0.046|
| 2         | 0.056     | 0.058| 0.059| 0.059| 0.058| 0.058| 0.057| 0.056| 0.054| 0.053| 0.049| 0.049| 0.051| 0.051| 0.051| 0.049| 0.049|
| 2.2       | 0.060     | 0.058| 0.059| 0.058| 0.058| 0.058| 0.057| 0.056| 0.054| 0.053| 0.049| 0.049| 0.051| 0.051| 0.051| 0.049| 0.049|
| 2.4       | 0.062     | 0.058| 0.058| 0.058| 0.057| 0.056| 0.056| 0.055| 0.054| 0.052| 0.052| 0.054| 0.054| 0.052| 0.053| 0.053| 0.051|
| 2.6       | 0.053     | 0.058| 0.059| 0.058| 0.057| 0.056| 0.056| 0.055| 0.054| 0.053| 0.053| 0.052| 0.052| 0.053| 0.053| 0.052| 0.052|
| 2.8       | 0.059     | 0.058| 0.058| 0.058| 0.056| 0.056| 0.056| 0.055| 0.054| 0.053| 0.053| 0.053| 0.053| 0.053| 0.053| 0.053| 0.053|
| 3         | 0.060     | 0.059| 0.058| 0.058| 0.057| 0.056| 0.055| 0.055| 0.054| 0.053| 0.053| 0.053| 0.053| 0.053| 0.053| 0.054| 0.054|