FINANCIAL MARKET IMPERFECTIONS AND LABOUR MARKET OUTCOMES *

Alireza Sepahsalari†
alireza.sepahsalari.09@ucl.ac.uk

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Abstract

This paper investigates the importance of credit market frictions at establishment level, which disrupts firms access to external financing, on the labour market outcomes. In this paper, I build a tractable search and matching model of labour market with firm dynamic and heterogeneity in productivity and size, in which I introduce a frictional credit market. The idiosyncratic shocks create a motive for labour and capital to move from low productivity firms to high productive ones. But the existence of collateral constraints limits the ability of high productive firms to finance their investment and therefore affect their behaviour in labour market. In the presence of collateral constraints, productive firms have a sluggish upward adjustment as they need to grow their asset stock gradually. But on the other hand, firms with falling productivity are hesitant to lay off their workforce. Since the firms hiring decision entails an irreversible search cost, there is an option value for falling productivity firms to postpone their lay off decision in the case of a rise in future productivity. I show, first, that the interaction of search and financial frictions slow down the reallocation of labour and capital and therefore prolong the recession when a financial shock happens. Second, I find that the introduction of financial frictions enhances the ability of the model to explain the fluctuation and persistence observed in output and labour market flows.

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†University College London
1 Introduction

The previous collapse of financial markets during the 2007/2008 recession was accompanied by large
deteriorations in the labour market. In UK, while GDP and labour productivity fell by 2.5% and 3.1%
respectively the unemployment rate increased by 3% according to National Office of Statistics (ONS).

Figure 1 shows the quarterly monetary financial institution lending to private non-financial firms. Apart
from a small drop in the beginning of 90s, the total lending to non-financial corporation increases
quite rapidly from 60s till previous recession when it experiences a fall by more than 30%.

![Figure 1: Quarterly amount of monetary financial institution lending to private non-financial corporations (in sterling millions) seasonally adjusted (from Bank of England).](image)

Moreover, the Credit Condition Survey results show that the availability of credit has fallen by
about 40% between 2007 and 2009 and there has been a slight recovery after that as it is depicted
in figure 2. Although the credit has dried largely during the previous crisis but the condition has
been different across the firm distribution. Firms who managed to increase their debt during the
previous crisis have also increase their level of employment. In contrast, firms who faced a decline in
their debt also decreased their employment. This is depicted in figure 3. This graph shows the mean
log change in employment from 2007 for two groups of firms. As it is depicted in the figure, both
groups are increasing their employment from 2005 to 2007 by the same amount while after the 2007
the employment grows by more than 5% among the increasing debt group of firms while it goes down by around 4% between the decreasing debt group of firms. This correlation suggests the importance of financial market imperfections for the labour market outcomes.

![Figure 2: The overall change to availability of credit provided to the corporate sector (CCS)](image)

How does financial tightening affect the aggregate economy? How does it distort unemployment rate, job creation, and the transition rates from and to unemployment? And how different are those impacts across firms? In order to answer the aforementioned questions, this paper develops a tractable search and matching model of labour market with firm dynamic and heterogeneity in productivity and size, in which I introduce a frictional credit market where firms borrowing is subject to a collateral constraint.

In this environment, the idiosyncratic shocks create a motive for labour and capital to move from low productive firms to high productive ones. But the existence of collateral constraints limits the ability of high productive firms to finance their investment and therefore affect their behaviour in labour market. In the presence of collateral constraints, productive firms have a sluggish upward adjustment as they need to grow their asset stock gradually. But on the other hand, firms with falling productivity are hesitant to lay off their workforce. This is because the hiring decision entails an irreversible search cost, therefore there is an option value for falling productivity firms to postpone their lay off decision in the case of a rise in future productivity. In this environment, the interaction of labour and financial market frictions reduces the reallocation of resources form unproductive to productive firms.
What is the impact of a financial shock? Productive firms are further constrained by a credit tightening and therefore hire even less workers. Unproductive firms are either less constrained or not constrained. They face a lower cost of keeping their workers which comes from a general equilibrium effect on reducing wages and a lower search cost of hiring since they face less competition from productive firms. Therefore they are less willing to shrink and delay further their liquidation decision. Consequently, labour market friction causes delay in layoff decision of firms and capital market tightening interact with labour market friction and generate an even a longer delay in reallocation of labour and capital.

The impact of financial tightening is in contrast with a TFP shock. A negative productivity shock reduces the value of all firms and has cleansing effect. It results in unproductive firms exiting the market and shifts the distribution of workers to the more productive firms and increase the reallocation. While the credit tightening has the opposite effect caused by the interaction of financial and search frictions. And this effect is different across the distribution of firms depending on the productivity and wealth of firms.

Figure 3: FAME: UK annual firm level data. The blue line shows corresponds to firms who increased their debt from 2007 to 2009 and the red line corresponds to firms who decreased their debt from 2007 to 2009.
Related Literature

This paper is related to several strands of literature. First, it relates to a growing literature on the impact of financial frictions in macroeconomics, especially on the labour market outcomes. The modelling of financial frictions in this paper is similar to Nobuhiro Kiyotaki and John Moore (1997) where borrowing is restricted by a collateral constraint. But I abstract from the feedback effects of the asset prices to the collateral constraint. Jermann and Quadrin (2012), J.Buera, Jaef, and Yongseok Shin (2015) and Zetlin-Jones and Shourideh (2014) use the same way of modelling for financial market. Jermann and Quadrin (2012) look at the impact of debt versus equity financing on the dynamic of real and financial variables. Zetlin-Jones and Shourideh (2014) investigates the importance of external financing for privately and publicly held firms and study the significance of financial shocks in output. The main difference of this paper and theirs is that this work introduces a frictional labour market to an economy with frictional financial market and heterogeneous firms and look at the interaction of these two frictions when a financial tightening is happening. I show that since the response of constrained and unconstrained firms are different to a reduction of collateral constraint, their behaviour change differently in the labour market and that generates novel implications for how the credit shock affects the aggregate economy.

Although J.Buera, Jaef, and Yongseok Shin (2015) also features the interaction of labour and financial market but in their paper entrepreneurs hire in a centralized and competitive market where the labour market frictions are introduced by an interferes with the re-entry of unemployed workers to that market. The salient difference between this paper and their work is to have a frictional labour market in the spirit of Diamond-Mortensen-Pissarides, where job finding is taking place in a frictional environment and the degree of these frictions depend on the ratio of total available vacancies to the number of job seekers which is an endogenous object. This environment let me study the difference between the intensive and extensive margins of firms hiring as well as their entry and exit decisions when the market condition changes. Zanetti (2011) and Garin (2015) also study the impact of financial shocks to the labour market in aggregate level while they are abstracting away from firms heterogeneity. Instead here I emphasis on the fact that a financial crisis is a boom time for unconstrained firms and that is caused by the interaction of financial and labour market frictions.

This paper is also related to another strand literature which introduces search and matching frictions to the models of firm dynamics. Acemoglu and Hawkins (2010) and Elsby and Michaels (2013) introduce the notion of firm size to the model of (Mortensen Pissaridis) by a decreasing return to scale production technology for firms. The first paper assumes that wages are determined by continuous bargaining between workers and firms and shows that response of unemployment rate and market tightness are considerably more persistent than in the Mortenses pissardis benchmark. Elsby and Michaels (2013) instead unifies the approaches of Mortensen and Pissarides (1994) with the empirical findings of Steven J. Davis and Schuh (1996) by providing an analytical framework. However, following Kaas and Kircher
this paper uses a competitive search mechanism in the labour market. The block recursive property of the competitive search enables me to compute the out of steady state dynamics of the model without using approximation methods. Kaas and Kircher (2014) applies a convex cost of vacancy posting in their framework and show that firms can achieve faster growth by offering higher wages. But Schaal (2015) shows that the introduction of time varying idiosyncratic volatility to a model with heterogeneous firms, competitive search and decreasing return to scale can improve the fit of search and matching models to explain the business cycle moments though the uncertainty alone cannot account for the persistence observed in the previous crisis. The contribution of my paper to this literature is to introduce a new dimension of financial markets to this literature and look at the interaction of frictions in labour and credit market which results in a greater degree of misallocation of resources in the case of a financial tightening.

This paper is organised as follow. In section 2 I explain the model and show the equivalence joint surplus maximization of firms and workers in decentralized market with planner problem. In section 3, I calibrate the model and explain the policy functions of firms. In section 4, I analyse and discuss the impact of a credit shock and compare it with a productivity shock and section 5 is the conclusion.

2 Model

In order to study the impact of financial imperfections on labour market outcomes, I introduce the notion of firm size to a search dynamic model where heterogeneous firms have decreasing return to scale production technology and their borrowing is subject to a collateral constraint. The endogenous hire, layoff, entry and exit decisions of firms in this environment would let me study the impact of financial tightening on the labour market outcomes. This paper builds on the directed search framework of Kaas and Kircher (2014) and Schaal (2015). The block recursive property of this framework makes its numerical solution tractable by reducing the dimensionality of this problem since the firms policy functions can be solved regardless of the distribution of employment across firms. I elaborate on the block recursive property below and discuss the numerical solution further.

2.1 Preferences, population and technology

Time is discrete and all agents discount future at rate $\beta$. The economy consists of a continuum of workers and firms. The mass of workers is normalized to one. Workers are all risk neutral and infinity lived. A worker supplies one unit of labour per period when employed and receives unemployment benefit when unemployed. Firms are risk neutral and they are large relative to workers in the sense that each firm employs a continuum of workers. Firms are subject to idiosyncratic productivity shock $y$ which is governed by a Markov process, $\pi(y|y')$, on a finite state space $\mathcal{Y}$ and $z$ is the aggregate or common component of the productivity. In each period, a firm produce output $zyf(l,k)$ where $l$ and
$k$ are the amount of labour and capital used in the production and $f$ is a twice differentiable, strictly increasing and concave function satisfying Inada condition. 

Each firm pays a fixed cost of operation each period which is required to obtain a non-trivial exit margin. New entrants are endowed with an initial level of asset $b_0$. They pay setup cost $\bar{C}$ and draw an initial productivity level $y_0 \in \mathcal{Y}$ with probability $\pi_0(y_0)$. Firms are subject to an exogenous death shock, $d_0 > 0$ which is the lower bond of endogenous exit decision of firm, $d$. In that case, all workers are laid off to unemployment and firm loses its asset stock. Also, $\mu_0$ is an exogenous separation shock which is the lower bond of the layoff decision of firms $\mu$.

2.2 Financial market

Firms working capital, $k$ is equal to the firm asset, $b$ plus funds that are either borrowed or rented, $x$. However, I assume that firm’s access to financial market is limited by a collateral constraint and this limit is proportional to the firm’s asset holding, $x \leq \lambda b$. The main idea behind the financial friction is that firms can use their financial asset as collateral in order to rent capital. The variable $\lambda$ represents the case with which a firm can obtain capital using its asset as collateral and also it represents the ability of financial market to reallocate capital across different firms. Since the lending contract is not perfectly enforceable, one way to rationalize the use of such a collateral constraint is to assume that in the case of a default the creditor can seize $\frac{1}{\lambda}$ fraction of firms asset. If the firm is not borrowing constrained, it’s investment in capital is optimal.

2.3 Labour market

Job search is directed. When labour market opens, firms announce contract to attract workers. Firms who have posted contracts with the identical values compete in the same submarket. Workers observe the value of contracts offered by firms and direct their search toward the most attractive offers. Therefore firms and workers form submarkets. Each submarket is characterized by a market tightness $\theta(\omega)$, where $\theta$ is the vacancy to unemployment ratio. This represents the relative supply and demand for jobs, as it determines the probability of a match. In each submarket workers find job with probability $q(\theta)$, where $q : [0, \infty] \rightarrow [0, 1]$: the higher the value of $\theta$, It is easier for a worker to find a job, so $q$ is a strictly increasing function: $q' > 0$. In contrast, the higher the ratio of vacancy to unemployment, it is more difficult for firms to fill their vacancies. We denote the probability that a firm fills a vacancy by $p(\theta)$, where $p : [0, \infty] \rightarrow [0, 1]$ is a strictly decreasing function, $p' < 0$. Since matching is always in pairs, matching probability of workers must be consistent with those of firms, in particular, it must be the case that $q(\theta) = \theta p(\theta)$. We also require the standard assumptions hold: $q$ is twice continuously differentiable, strictly concave and has a strictly decreasing elasticity. The fact that we express the matching probability in terms of the ratio of firms to workers $\theta$ and not the number of workers and

\[^1 f_1(0, k) = \infty, f_1(\infty, k) = 0, f_k(l, 0) = \infty \text{ and } f_k(l, \infty) = 0\]
firms effectively means that we assume a matching technology that is constant returns. As the number of workers and vacancies doubles, the number of matches doubles, yet the matching probabilities remain unchanged. Moreover, I assume that firms post a mass of vacancies, so a low of large number applies and there is no uncertainty regarding the number of workers they recruit. This means that a firm that posts \( v \) vacancies employ exactly \( v p(\theta) \) workers.

### 2.4 Contracts

Contracts states what firms promise to workers. For now, I assume that contracts are complete, state-contingent and there is a full commitment for both workers and firms. Formally a contract is a set of state contingent wages and retention probabilities offered by a firm:

\[
C = \left( w_{t+j}, \phi_{t+j} \right)_{j=0}
\]

Where each element of the contract at time \( t+j \) is contingent on the entire history of shocks \( (y_{t+j}) \)

### 2.5 Timing

The timing is illustrated in figure ???. Each period is divided into three stages. At the first stage of period \( t \), new firms enter to the market and immediately after idiosyncratic productivities are revealed. In the second stage, after the realization of productivities, firms make a decision about possibly exiting at the end of period. Also firms make a decision about how much they want to borrow or lend, they pay the wage bill and operation cost and produce at the end of this stage. In the last stage, firms decide how much asset they want to save for the next period and how many workers they want to hire or layoff. Then the labour market opens, firms post their vacancies and matches are formed.

### 2.6 Workers Problem

An unemployed worker with value of unemployment \( U \), try to choose the contract which maximizes his expected utility of unemployment. Therefore a worker look at the trade-off between the value of contract \( C \) and the likelihood of getting that contract, \( q(\theta(C)) \). The more attractive offers are more difficult to get. If the unemployed worker is successful to get the contract, it will continue with the value of an employed worker, while if he is unsuccessful, he will receive an unemployment benefit \( h \), and will search again next period. Therefore the value of unemployment can be written as follow:

\[
U = \max_C \quad h + \beta \left\{ q(\theta(C))(1 - d(y))E_y W(C, y') + (1 - q(\theta(C))(1 - d(y)))U \right\}
\] (1)

Note that a worker chooses the contract such that it maximizes his expected value of applying for a job:
\[ \omega = \max_C q(\theta(C))(1 - d(y))(E_y W(C, y') - U) \]  

(2)

Therefore the problem of an unemployed worker can be rewritten as:

\[ U = h + \omega + \beta U \]  

(3)

An employed worker with contract \( C \) who is working in a firm with productivity \( y \) receive his wage \( w \) and next period if the firm does not exit and he is not laid off, he will continue as an employed worker while otherwise he will become unemployed with probability \( \phi(y') \) and start looking for a job in the next period.

\[ W(C, y) = w + \beta \left\{ (1 - \phi(y)U + \phi(y)E_y W(C, y')) \right\} \]  

(4)

### 2.7 Firms problem

A firm enters period \( t \) with a stock of asset \( b \) it has accumulated, a stock of workers \( l \) it has employed in the past as well as the contracts signed with these workers \( C_\tau \). Note that, employed workers hired by the firm might be on different contracts and therefore each worker is indexed by \( \tau \in [0, l] \) and a contract \( C_\tau \). After the realization of idiosyncratic productivity \( y \), the firm pays the wage bill and fixed cost of operation, makes a decision about how much it wants to borrow and produce the output. Next, the firm makes a decision about how much asset it wants to save for the next period and labour adjustment. Therefore, it chooses the number of new hires as well as the contract it wants to offer them and post the vacancies in the labour market. Finally firms who have wants to shrink, layoff their labour force and firms who have decided to exit, leave the market. The following is firm problem:

\[ J(\{C_\tau\}_{\tau \in [0,l]}, l, b, y) = \max_{d,v, C, x,b,y'} yz f(l,k) + (1 - \sigma)k - (1 + r)(k - b) - b' - W_b - c.v - c_f + \beta (1 - d)E_y J(\{C_\tau\}_{\tau \in [0,l]}, l', b', y') \]  

subject to:

\[ l' = (1 - \mu)l + p(\theta(C))v \]  

(6)

\[ x \leq \lambda b \]  

(7)

\[ \omega = \max_C q(\theta(C))(1 - d(y))(E_y W(C, y') - U) \]  

(8)
Constraint 6 shows the law of motion for firm employment. The number of employed workers at the beginning of next period is equal to those who are not separated this period plus new hires. Constraint 7 is a collateral constraint and shows firms restriction in the financial market. Once a firm knows its productivity it decides how much it wants to borrow to form the working capital. But the maximum borrowing is restricted to a fraction of firm asset holding. And the last constraint, equation 8 is the workers participation constraint. It specifies the minimum utility that a contract has to offer in order to attract the worker to apply for the firm.

An entrants firm faces a similar problem. It enters the economy with initial endowment $b_0$ and no labour force, only if the expected value of entry covers the entry cost. The free entry condition implies that at steady state a potential new entrant is indifferent between entering or not and makes no profit.

$$\sum_y \pi(y)J_0(b_0, 0, y) \leq C$$ (9)

This condition implicitly determines the expected value of job search $\omega$ and therefore the relationship between the value of contract and job filling rate.

### 2.8 Joint surplus maximization

The joint surplus maximization of a firm and its workers satisfies the following Bellman equation:

$$G(b, l, y) = \max_{d, \mu, \theta, v, x, b'} yzf(l, k) + (1-\sigma)k - (1+r)(k-b) - b' - hl - \omega[l + \frac{1}{\theta}v] - c_v - c_f + \beta(1-d)E_yG(b', l', y')$$ (10)

Where this problem is subject to 6, 7 and 8. The interpretation of this problem is straightforward. The joint surplus includes the firms flow of output plus its undepreciated working capital net of the cost of borrowing, cost of vacancy posting, fixed cost of operation, saving for next period and the cost of workers. These workers are the current stock of employment $l$ and unemployed workers $\frac{1}{\theta}v$ who are applying to get a job at this firm.

The full commitment assumption and transferability of utility considerably simplify this problem. The transferability of utility implies that wages are just an internal transfer between the workers and firms and do not appear in the joint surplus maximization problem. Therefore to solve the decentralized market problem one does not need to keep track of the evolution of distribution of wages within a firm. Moreover, since the planner solution is identical to the one of decentralized economy, the cohort specific layoff probability chosen by planner is the same as per period layoff decision of a firm. 2

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2This is explained in appendix A
2.9 Entry and firm distribution dynamic

Free entry of firms implies that given the aggregate state of economy, the expected surplus of new firm is equal to the entry cost.

\[ \sum_y \pi(y)G_0(b_0, 0, y) \leq C \] (11)

In the equilibrium a constant measure of firms \( N_0 \) enter the market in every period. The number of entrant firms is such that the aggregate resource constraint holds with equality

\[ \sum_y N(y) \left[ l + \frac{1}{\theta}v \right] \leq 1 \] (12)

This constraint says the total number of employed and unemployed workers tied to all firms has to be equal to the unit mass of workers in the economy. Here \( N(y) \) is the distribution of all firms, \( l \) is the stock of employed workers and \( \frac{1}{\theta}v \) is the number of unemployed applying for a job at the firm. In other words, firms entry follow a residual of the resource constraint. In every period a firm with productivity \( y \) and employment \( l \) attracts \( \frac{1}{\theta}v \) job seekers according to its policy function and a number of new entrants \( N_0 \) enter to the market to absorb the remaining of job seekers.

The law of motion for firms show how the number of different firm types changes from period \( t \) to \( t + 1 \). The evolution of firm type depends on the idiosyncratic productivity draw governed by a Markov process and the firms exit decisions.

\[ N_{t+1}(y) = [1 - d(y)]\pi(y)N_t(y) \] (13)

Also, the distribution of workforce within a firm adjust given the hiring and layoff decisions made by the firm. If a firm decides to fire its labour force, it post no vacancy and choose a \( \mu > \mu_0 \).

\[ l_{t+1}(y) = [1 - \mu(y)]l_t(y) + p(\theta)v \] (14)

2.10 Definition of equilibrium

A competitive block-recursive equilibrium consists of the joint surplus of workers and firms, a utility promised to workers, a distribution of incumbent firms and a distribution of new entrants.

\[ [G(b, l, y), \omega, N(b, l, y), N_0(b_0, l, y)] \]

such that:

1. Firms decisions (exit, borrowing/lending, saving, hiring and layoff strategies) are optimal. That is
\( G(.) \) is the value function and \( d, \mu, \theta, v, x \) and \( b \) are the optimal policy functions.

2. Workers search strategies are optimal. \( \omega \) and \( U \) are optimal for the workers.

3. Aggregate resource constraints holds: \( \sum_y N(y) \left[ l + \frac{1}{y}v \right] = 1 \)

4. \( T(N(b, l, y), N_0(b_0, 0, y), \eta) = N(b_0, l_y) \)

5. Complementary slackness condition holds: \( \sum_y \pi(y_0)G(b_0, 0, y) = \bar{C} \)

Condition (1) and (2) state that all firm decisions are optimal. Condition (3) states the total number of employed and unemployed workers must be equal to one which is the total number of workers in the economy. Condition (4) states that the optimal state of economy must be such that the optimal actions of firms and workers cause this state to be reproduced in each period and condition(5) explains that the entering firms must be willing to enter.

**Proposition 1** When there is positive entry of firms, a stationary competitive search equilibrium exists and is unique.

Proposition 1 shows that the existence of a solution to the joint surplus maximization of workers and firms and free entry condition always exists when the cost of entry is sufficiently low. Unfortunately, the existence of a full block recursive equilibrium with positive entry is difficult to prove under various different parametrization of this model. Therefore I check this condition numerically when I solve the model.

Usually in the search and matching models a hiring firm needs to know the workers utility value of unemployment and that often depends on the distribution of other firms which is potentially an infinite dimensional object. But in my framework following [Menzio and Shi (2010)], the competitive search and free entry condition make this problem easier. Homogeneity on the workers side means that the unemployed workers are indifferent between applying to new entrants and incumbent firms. Therefore new entry of firms adjust to equate the cost of entry to the benefit of entry regardless of the distribution of existing firms. Therefore free entry condition pins down a unique \( \omega \) independent of the distribution of incumbents and accordingly a measure of new firms enter to the economy to clear the labour market. Also one should note that, only the aggregate states of the economy can potentially enter the workers utility. For example, in my setup, if the common component of productivity \( z \) or the measure of imperfection in financial market \( \lambda \) changes, then \( \omega \) should adjust such that in the new steady state the benefit and cost of entry are equal to each other. Moreover, this implies that distribution of new entry is going to be adjusted in the new steady state and interestingly since the distribution of new
entrants depends on the distribution of incumbents, this adjustment will be sluggish. I will explore this property more in my numerical exercises.

**Proposition 2** A competitive search equilibrium is socially optimal.

*(The planner problem and joint surplus maximization are equivalent.)*

This proposition establishes that the block recursive equilibrium is constrained efficient. This extends the results of [Kaas and Kircher (2014)] in an environment with financial market and collateral constraint and guarantees that the joint surplus maximization of workers and firms coincide with the planners problem. This implies that given a level of financial constraint, all decisions made by a firm are exactly up to the socially optimal level. This result is even true if the firm can only commit to promised wages. This is because a firm can set future wages following any sequence of idiosyncratic shocks equal to the unemployment income plus the expected payoff of applying for a job, \( h + \omega \). In this case the worker does not have any incentive to unilaterally leave the firm as he is exactly compensated by what he will get in the alternative. Moreover since the flow surplus of any retained worker is equal to his value in alternative, the firm problem in this case is identical to planner problem. All this results hold on the risk neutrality of workers and firms and a slight degree of risk aversion on either side would result in a different specification.

### 2.11 Optimal firm decisions

In what follows I take the benchmark calibration of the model and discuss the optimal decision of firms in the space of asset and labour \((b,l)\) for a relatively low productive firm in the second decile of productivity distribution and a high productive firm in the 8th decile. A firm with low level of asset might decide to leave the market as it may not be able to afford the fixed cost of operation under a tight collateral constraint where it cannot freely borrow to increase its production and cover its expenses. That can be seen in the left down corner of both panels in figure [3]. But a similar firm with a higher level of asset will lay in the layoff area where the firms decide to decrease it’s size to reduce the labour cost but still stay in the market.

Second, separation tends to occur more in big unproductive firms. The separation area in the left panel is much larger than the one on the right panel. Taking into account \( \mu_0 \), the firm decides to lay off workers if the expected value of keeping the labour is negative next period:

\[
E_y \frac{\partial G(b', l', y')}{\partial l} < 0
\]

Moreover, given a level of productivity separation is also a decreasing function of asset. A high asset holder firm has potentially a higher borrowing limit because of collateral constraint and therefore is able to finance a larger number of labour forces. In other words, the marginal productivity of labour
is higher in a firm with higher level of asset and the firm tends to layoff less and stays larger.

Third, in contrast, hiring mostly occurs in small and productive firms since they have a high marginal product of labour. These firms expand as long as the expected marginal value of increasing the labour force is higher than expected cost of hiring which is sum of two costs as it is shown in the RHS of below equation: expected cost of posting the vacancy plus the expected value that a firm offers to the applicants for the job.  

\[
\beta(1-d)\mathbb{E}_y \frac{\partial G(b',l',y')}{\partial l} > \frac{c}{p(\theta)} + \frac{\omega}{\theta p(\theta)}
\]

As it is shown on the right panel of figure 4, the hiring decision of a firm is an increasing function of its asset holding. A productive firm with low level of asset which has a high marginal productivity of labour cannot afford to grow large quickly since hiring is costly and the firm does not have enough assets to borrow against it. It implies that this firm has to expand gradually by increasing its stock of asset which let him in turn to collateralize more debt and relax the borrowing constraint finally. This upward adjustment of productive firms becomes more sluggish if the collateral constraint becomes tighter.

And finally for all values between the RHS of first and second inequalities firms are inactive in labour market. If the marginal benefit of expanding firm is below its marginal cost but still positive, the firm neither lays off nor hires. Although firms tend to hire more if they are holding more asset but they tend to lay off much less with higher level of asset and as a result the inactivity area also widen with asset for both low and high productive firms

\[ \frac{1}{p(\theta)} \text{ is the expected time for filling a vacancy} \]
The labour market optimal behaviour of firms heavily depends on their level of asset holdings and how tight the financial constraint is. In a perfect financial market all these policy function would be horizontal since the firm can go on debt without any constraint. Therefore the firms level of wealth would not affect its behaviour in the labour market.

3 Calibration

3.1 Functional forms

The parametrization of the model is as follows: The production function is Cobb-Douglas with two factors of production which are labour and capital $f(l,b) = l^\alpha k^\gamma$ where $\alpha$ and $\gamma$ are the output elasticities of capital and labor, respectively. To introduce the notion of firm size, this production function is decreasing return to scale and therefore $\alpha + \gamma < 1$. Following Menzio and Shi (2010), I pick the CES matching functions

$$q(\theta) = \theta(1 + \theta^\gamma l)^{-1/\gamma l}, \quad p(\theta) = q(\theta)/\theta = (1 + \theta^\gamma l)^{-1/\gamma l}$$

where $\gamma_l$ is the elasticity of matching function. To parametrize $\gamma_l$, following Schaal (2015), I estimate matching function $q$ by non-linear least square using the job finding rates constructed by Smith (2011). For $\theta$, I use the measure of vacancy over unemployment provided by ONS vacancy survey/labour force survey\(^5\). The estimate obtains $\gamma_l = 1.27$ and the fit is reasonably good $R^2 = 92.5$.

The idiosyncratic productivity follows an AR(1) process

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\(^4\)The data is constructed by Jennifer C Smith and is available on her webpage at https://sites.google.com/site/jcsmithecon/data

\(^5\)The measure of vacancy over number of unemployed workers can be found at http://www.ons.gov.uk/ons/datasets-and-tables/index.html
\[ y_t = \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, 1) \]

I use the log version of above equation to make sure that idiosyncratic productivity remains positive and discretise the productivity process using Tauchen method with ten grid points.

### 3.2 Calibration strategy

Calibration proceed in two steps. In the first step, I set the pre-calibrated parameters exogenously. These are those parameters which either have direct counterpart in the data, or have been widely used by other studies. In the second step, I use the simulated method of moments to match the targeted moments.

The time period is a quarter. I set the quarterly interest rate \( r \) to match an annual interest rate of 5 percents \( (1 + 5\%)^{1/4} \approx 1.2\% \). Following Riddick and Whited (2009) I set the depreciation rate of capital equal to 0.0375 which corresponds to an annual depreciation rate of 15 percent. I also set the discount factor equal to 0.98.

The parameters left to be calibrated are the following: the home production \( h \), the cost of posting vacancy \( c \), the fixed cost of operation \( c_f \), the cost of entry \( \bar{C} \), the collateral constraint, \( \lambda \), and the persistence and standard deviation of idiosyncratic shock \((\rho_y, \epsilon_y)\).

All the parameters are jointly estimated in equilibrium using the method of moments but we can identify which parameter is mostly related to which target. To inform the estimation of the output elasticities of labour and capital \((\alpha, \gamma)\), I target an annual capital output ratio of 2.4 and a labour share of production of 0.65. To discipline the calibration of labour market parameters, \((h, c)\) I target the historical average of unemployment-employment (UE) rate of 0.30 and employment-unemployment (EU) rate of 0.012 using the time series constructed by Smith (2011). Since operation cost \( c_f \) is the main determinant of exit, I target an average firm size of 15.1 as in the Business Population Estimates of 2007. To target the average debt to total asset in the UK economy, the standard deviation of size-weighted net debt to total asset ratio of private firms and the corporate total asset over total output, I use the parameters \( \lambda, \rho_y \) and \( \epsilon_y \).
### Pre-calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.980</td>
<td>Discount factor</td>
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<tr>
<td>$r$</td>
<td>0.012</td>
<td>Interest rate (%5 annual)</td>
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<tr>
<td>$\sigma$</td>
<td>0.0375</td>
<td>Depreciation rate of capital (%15 annual, Riddick and Whited)</td>
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<tr>
<td>$\gamma_l$</td>
<td>1.27</td>
<td>Matching function elasticity parameter</td>
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### Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>0.653</td>
<td>Labour factor in production function</td>
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<tr>
<td>$\gamma$</td>
<td>0.278</td>
<td>Capital factor in production function</td>
</tr>
<tr>
<td>$h$</td>
<td>0.861</td>
<td>Unemployment income</td>
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<tr>
<td>$c$</td>
<td>1.623</td>
<td>Vacancy posting cost</td>
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<tr>
<td>$c_f$</td>
<td>0.946</td>
<td>Fixed operating cost</td>
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<tr>
<td>$C$</td>
<td>33.516</td>
<td>Entry cost</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>4.261</td>
<td>Financial friction</td>
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<tr>
<td>$\rho_y$</td>
<td>0.92</td>
<td>Persistence of idiosyncratic productivity</td>
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<tr>
<td>$\epsilon_y$</td>
<td>0.0305</td>
<td>Standard deviation of idiosyncratic productivity</td>
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### Targeted Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>Description</th>
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<tr>
<td>$K/Y$</td>
<td>2.3</td>
<td>0.65</td>
<td>Capital to output ratio</td>
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<tr>
<td>Agg. labour share</td>
<td>0.30</td>
<td>0.012</td>
<td>Unemployment-employment rate</td>
</tr>
<tr>
<td>$UE$</td>
<td>0.30</td>
<td>0.012</td>
<td>Employment-unemployment rate</td>
</tr>
<tr>
<td>$EU$</td>
<td>15.1</td>
<td>3.5</td>
<td>Average firm size (labour)</td>
</tr>
<tr>
<td>Mean debt-to-assets ratio</td>
<td>3.5</td>
<td>Corporate total Asset over GDP</td>
<td></td>
</tr>
<tr>
<td>STD debt-to-assets ratio</td>
<td>0.54</td>
<td></td>
<td></td>
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<tr>
<td>Market tightness</td>
<td>0.46</td>
<td>the ratio total vacancies to unemployed workers</td>
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4 Financial Shocks

In this section I describe the impact of a financial shock as modelled by a tightening of financial constraint at the macroeconomic level and use the model to study at a deeper level how these shocks affect firms in the cross section. Moreover, I analyse the difference between financial and aggregate productivity shocks and explain the implication of these shocks in the cross section of firms.

I model the financial shock as an aggregate shock to $\lambda$. I suppose that there is an unanticipated drop in the collateral constraint, $\lambda_t$. But once the shock hits $\lambda$, everyone in the economy perfectly know its deterministic path afterward. Using the credit condition survey, I choose the size of drop so that it immediately results in a 45% drop in available credit. Following the initial fall, $\lambda_t$ reverts to its initial level gradually at a rate of 9.5% per quarter. This choice of a mean reversion rate of 9.5% is chosen so as to match the speed of recovery in the net available credit from CCS. The left panel of figure 5 shows the change in availability of credit and the right panel depicts the path of collateral constraint $\lambda$ after the shock. To proceed further, I first discuss the policy functions of a high and low productive firm to explain what are the partial and general equilibrium effects of a financial tightening on the firms behaviour in the labour market. Then I compute the impulse response path of economy as it reverts back to the steady state under perfect foresight.

![Figure 5: Credit tightening](image)
Policy function- Figure 6 and 7 illustrate how a financial tightening affects the employment strategy of firms in partial equilibrium keeping the value of unemployment and the market tightness fixed and in general equilibrium for the firms in the 2\textsuperscript{nd} and 8\textsuperscript{th} decile of productivity distribution. The solid lines depict the exit, layoff, inaction and hiring thresholds before the shocks. The dashed lines represent how these thresholds are affected when the shock hits.

As this is depicted in the figures, the optimal decisions of firms are changing with their level of asset holdings. A firm with low level of asset might decide to leave the market as it may not be able to afford the fixed cost of operation under a tight collateral constraint where it cannot freely borrow to increase its production and cover its expenses. That can be seen in the left down corner of both panels in figure 6. But a similar firm with a higher level of asset will lay in the layoff area where the firms decide to decrease its size to reduce the labour cost but still stay in the market. If the firm would have even higher level of asset it may decide to be inactive in the labour market or increase its labour force depending on its optimal policy function. So the labour market behaviour of firms heavily depends on their level of asset holdings and how tight the financial constraint is. In a perfect financial market all these policy function would be horizontal and firms wealth or its accessibility to financial market would not affect its behaviour in the labour market.

The partial equilibrium effect of a decline in $\lambda$ is evident. When the collateral constraint becomes tighter, working capital might fall below the optimal level of capital for some firms. This reduces the marginal value of employment and therefore shrinking the hiring area for the firms who are employing. Moreover, since the value of employment has gone down the firms are less eager to keep workers and that widens the layoff region. Also, a fall in marginal productivity makes firms at the margin of profitability unable to cover the fixed cost of operation and therefore it shifts the exit threshold to the right.

However after partial equilibrium effect, the forces in labour market lead to adjust the value of unemployment and market tightness. As firms increase layoff and reduce hiring, the total number of
unemployed works goes up in the economy. This is also exacerbated by an increase in the number of exiting firms and a decrease in the number of entrants. A higher number of unemployed workers increase the competition among the workers to find a job and therefore reduce the probability of job finding and the value of unemployment. But on the other side of the market, firms find it easier to hire workers as the expected cost of filling a vacancy diminishes. Lower value of unemployment means that firms are facing a lower cost of keeping their labour. That shift the layoff threshold to the left and among the lower productive firms this effect is stronger for higher asset holders as it is depicted in the left panel of the figure 7. Also lower expected cost of filling a vacancy means that firms are more eager to increase their hiring. This also shift the hiring threshold to the left and this effect is also stronger for wealthier firms as it is depicted in the right panel of the same figure.

The total effect of financial tightening is dominated by partial equilibrium effect. I should also highlight that even if after general equilibrium effect the firms had a wider hiring and inaction areas one could not make the conclusion that this shock increases the employment. Financial tightening reduces the value of firms causing an important decline in the number of entrant firms. That is why the tightness and the value of unemployment fall and let the incumbent to grow and shift the thresholds to the left after partial equilibrium effect. The reason behind the strength of general equilibrium effect is the entry condition. This means that at steady state the value of entrants should remain equal to the cost of entry and that necessities a strong reaction of general equilibrium effect. Therefore a fall in the value of entrants must be compensated by a decline in the cost of hiring which let the incumbent firms grow larger.

**Impulse responses**- Figure 8 depicts the impulse responses of labour market outcomes and figure 9 displays the impulse responses of other aggregate variables as log deviation from the steady state. In what follows I will discuss what this impulse response exercise tell us about the nature of a financial shock and the implications in the labour market.
First, the impulse to the economy generates roughly 12% decline in output and labour productivity on impact. The collateral shock generates a recession by causing a larger degree of misallocation compared to the steady state. Constrained firms, who are experiencing a positive idiosyncratic productivity shock during the collateral shock, experience a much slower upward adjustment as they need to build up their asset stock gradually to be able to collateralize their future debt. Therefore, these firms post less vacancy and hire fewer workers. Moreover, a general equilibrium effect amplifies this degree of misallocation. A fall in collateral constraint reduces the demand for labour from good firms and results in lower cost of keeping labour by firms. Therefore, unconstrained firms that are experiencing a negative shock to their idiosyncratic productivity during the collateral shock, shrink more slowly along the impulse path and delay their liquidation. Both of the above effects generate a higher degree of misallocation and reduces the labour productivity and total output.

Second, the financial shock results in a large fall in total vacancies and hiring. This is caused by a reduction in hiring of constrained firms, a fall in number of new entrants who do not find it any more profitable to enter the market under a tighter constraint and also an increase in exit. These three effects together largely dominates the mild increase in hiring by unconstrained incumbents. Third, the fall in job creation and hiring is accompanied by an increase in layoff upon the impact of shock. These two together generates a high unemployment rate which can roughly explains 50% of the increase in UK unemployment we observed in the previous crisis. Moreover, after the shock, since the incumbents and new entrants need to accumulate asset gradually, it takes a longer time for firms to increase their employment to the steady state level and therefore prolong the recovery time. Although the model is quite successful in generating persistent unemployment rate but the financial shock alone is not sufficient enough to explain the persistence of unemployment during 2007-2009 recession. This shows that introduction of imperfect financial market and credit tightening to search and matching models can provide an additional propagation mechanism and improve the performance of these models along different dimensions.

Fourth, entry and exit react quite sharply to the credit tightening. A reduction in the number of entrants and an increase in the exiting firms reduce the total number of firms. But to clear the labour market, the general equilibrium effect reacts strongly by reducing the workers compensation and the number of entrants goes back to or above its steady state level quite quickly.

Finally, regarding to aggregate variables, the ratio of debt to asset and labour to asset both fall on impact after the shock, while the recovery of second one is more sluggish than first one. This shows that firms increase their labour force slower than their debt. Furthermore, the financial tightening results in a 41% fall on impact in total debt of firms which is comparable to 35% decline in total lending to private non-financial corporation observed in UK.
Figure 8: Labour market responses to a 45% credit tightening. (The series are presented in log deviation from steady state. The time period is a quarter and the shock hits at $t = 0$. UE is calculated as the gross flow from $U$ at $t - 1$ to $E$ at $t$, divided by total number of unemployed workers at $t - 1$. EU is calculated as the gross flow from $E$ at $t - 1$ to $U$ at $t$, divided by the total number of employed workers at $t - 1$.)
Figure 9: Macroeconomic outcomes responses to a 45% credit tightening.
4.1 Comparing the effect of productivity and financial shock

In this section I compare the impacts of financial shocks to productivity shocks in the model. This exercise provides intuition for the difference in mechanisms at work in each case. It also provides a benchmark to compare the magnitude and persistence of the effects across different variables. In order to be able to compare the impacts of these two shocks, I consider a permanent decrease in productivity and a permanent decline in collateral constraint such that in each case the output per worker falls by 4.7% on impact as observed between 2008Q1 and 2009Q1.

Figure 10: Response to a permanent decrease in collateral constraint (Transition path is shown in log deviation from steady state. The collateral constraint shock is matched to produce 4.7% fall in labour productivity)

Figure 10 shows the transition path in log deviation between the two steady states caused by a
21.6% tightening of collateral constraint and figure 14 shows the same after a 4.6% drop of aggregate productivity. I first discuss the implications of an aggregate productivity shock and then compare it with a financial shock.

A fall in aggregate productivity reduces the joint surplus of firms and workers. That demotivates new firms to enter the market as with lower productivity they may not be able to cover the entry cost. Moreover, firms with low productivity may not be able to afford the cost of operation anymore and decide to exit the market. Therefore the total number of firms and the market tightness go down and it becomes more difficult for unemployed workers to find a job which reduces the UE transition rate. A lower job finding rate for unemployed workers means that the value of unemployment is lower now. As a result, firms might find it more profitable to keep their workers if the value of unemployment falls more than the labour productivity. Productive firms may even find it profitable to post more vacancy and hire more workers in response to the fall in value of unemployment and increase in probability of job filling. That is why the total number of vacancies and hiring may spike upon the shock but both converge to a lower level at the new steady state. As times go on, the layoff rate which dropped initially recovers gradually and the number of exiting firms converges to a lower level in the new steady state.

In Appendix D I show the change in the policy functions of firms in the 2nd and 8th decile of productivity distribution after the partial and general equilibrium effects caused by a fall in aggregate productivity. And then I compare the transition path of the economy between the two steady states with an economy without financial frictions but otherwise identical.

Comparing the effect of two shocks with a similar drop in labour productivity, one can observe different responses of labour market variables and other aggregate variables. A financial shock triggers a much higher unemployment rate compared with a productivity shock. This is caused by a higher degree of layoff as well as a dipper fall in job creation and hiring. Firms who face a tighter financial constraint need to downsize immediately and reduce their cost of hiring in order to be able to afford the operation cost and stay in the market. But as time goes on, they increase their asset to be able to collateralize more debt under a tighter financial constraint. This increase in debt lets constrained firms reduce the lay off and in turn increase the job creation and hiring later. Instead as a result of a productivity shock, the unemployment increase to a lower lever upon impact by a rise in the exit of firms in the lower part of productivity distribution and gradually converges to the new steady state. Hiring and job creation also steadily move to their new levels which are lower than the current steady state. The high response of labour market variables including unemployment rate, job creation, hiring and layoff are caused through two channels. The first one is a an increase in the number of exiting firms and the second one is the labour adjustment mainly by productive firms who want to stay in the market but to do so, they need to reduce their labour cost in a condition where they cannot borrow as much as before. Therefore they considerably increase their layoff, reduce their job creation and hiring and try to increase their stock of collateralizable asset to be able to survive in an economy with tighter financial
constraint. Instead low productive firms which are not constrained do not increase the layoff and tend to hoard more labour when the value of unemployment and therefore the cost of labour is lower. In contrast, in the case of a productivity shock, all firms are symmetrically affected by the reduction in productivity.

Regarding to aggregate variables, the total output produced by firms falls more in the case of a productivity shock. The existence of frictions amplifies the impact of a 4.6% fall in aggregate productivity and total output falls almost by 7% caused by a reduction in total number of firms as well as a fall in the output produced by the existing firms. Instead, in the case of a financial shock the drop in output is slightly more than 3%. This is also caused by an increase in the number of exiting firm and also a fall in the production of constrained firms. Figure 16 and figure 17 contain the response of debt.
to asset ratio and labour to asset ratio. In the case of a financial shock, the ratio of debt to asset falls. This is initially caused by a fall in debt but it also converges to a lower level because firms increase their stock of asset to overcome the tighter constraint. In contrary, a fall in productivity means that the firms are in average less constrained than before because they have a lower optimal level of capital. Therefore they reduce both their debt and asset where here the reduction in asset is more than debt and therefore the ratio of debt to asset goes up. The ratio of labour to asset decreases in the case of a collateral tightening since firms tend to accumulate more assets. But a productivity shock, which triggers a bigger fall in the value of unemployment, also motivates the existing firms in the market to increase their labour force and therefore the ratio of labour to asset goes up in that case. The reason behind this results is that the cost of borrowing is exogenously set and does not change under any of the shocks while a general equilibrium effect in the labour market reduce the cost of labour where this reduction is more in the case of a productivity shock.

The most important difference between a financial and productivity shock comes from the impact of a tighter collateral constraint on the allocation of resources. While a productivity shock has cleansing effect and shift the resources toward the more productive firms, in contrast a financial shock hits the productive firms who want to grow faster by taking more debt. A tighter constraint delays the job creation and hiring of these firms till they gradually grow their stock of asset against which they can take more debt. But a financial shock has also an important effect on the unproductive firms who do not want to grow any further or are in their process of liquidation. A reduction in demand for labour force from productive firms reduces the cost of maintaining the current labour force for the unproductive firms. Therefore they are more eager to shrink at a slower rate and gamble on the hope of regaining their productivity. This delay in reallocation of resources which is caused by a more sluggish upward adjustment of productive firms and a more sluggish downward adjustment of unproductive firms affect the productivity of labour force and further deepen the crisis. In contrast, in the case of a productivity shock, the resources shift toward the more productive firms caused by an exit of firms in the left tail of productivity distribution.

5 Conclusion

In this paper, I have developed a dynamic model with firm heterogeneity and frictions in both labour and capital markets. The model features directed search in the labour market and frictions in capital market are governed by collateral constraints. I use this framework to analyse the role of credit tightening on labour market outcomes, such as: unemployment rate, job creation, hiring, layoff, entry and exit of firms. I calibrate the model to UK moments and show that credit tightening can have a large and persistent effect on labour market outcomes and explain 58% of the observed increase in unemployment rate for the recession of 2008-2010. Then I compare the impact of a productivity shock with credit tightening. A fall in aggregate productivity results in exit of firms with low productivity and affects
the rest of firms symmetrically. In contrast, a credit tightening, affect the productive growing firms and makes their path of upward adjustment more sluggish. While at the same time unproductive firms shrink slower. This causes a delay in reallocation of resources from unproductive to productive firms which further deepen the crisis.
References


A  Proofs

Proof. of Proposition 1
Given a production function, $f(l,k)$, which is continuous, differentiable and strictly concave in $l$ and $k$, the value function of entrant firms $G(b_0,0,y)$ is continuous and decreasing in $\omega$. This implies that the expected value of entry is continuous and decreasing in $\omega$. Therefore for a fixed cost of entry, equation [1] can have at most one solution for any $\bar{C} > 0$ and this solution exists provided that $\bar{C}$ is sufficiently small. Note that, since $f_l(0,k) = \infty$ some entrants find it optimal to hire workers since $G_l(b_0,0,y)$ is higher than cost recruiting and $\omega$. ■

Proof. of Proposition 2
The proof has three steps. Following Kaas and Kircher (2014) first, I show how to derive the joint surplus maximization of firms and workers. Second I discuss the planner problem. And finally I show that the planner problem is identical to joint surplus maximization in the decentralized economy.

Step One: Joint surplus maximization of firm and workers
The joint surplus of a firm and it’s workers can be written as:

$$G \left( \{C\}_{\tau \in [0,l]}, l, b, y \right) \equiv J \left( \{C\}_{\tau \in [0,l]}, l, b, y \right) + \sum_{\tau=0}^{l-1} l_{\tau} [W(C_{\tau}, y) - U]$$

(15)

Using 3 and 4 the worker surplus satisfies:

$$W(C_{\tau}, y) - U = w_{\tau} - h - \omega + \beta \phi_{\tau}(y) E_{y,z} [W(C, y) - U]$$

Now substituting this equation and 16 into 15.
\[ G(.) = \max_{d,v,\theta,C,x,b'} \left\{ yzf(l,k) + (1 - \sigma)k - (1 + r)(k - b) - b' - \sum_{\tau=0}^{l-1} l_{\tau}w_{\tau} - c.v - c_f + \beta(1 - d)E_{y,J(.)} \right\} + \sum_{\tau=0}^{l-1} l_{\tau} \left[ w_{\tau} - b - \omega + \beta \phi_{\tau} E_{y,[W(C_{\tau},y') - U]} \right] \]

\[ = \max_{d,v,\theta,C,x,b'} \left\{ yzf(l,k) + (1 - \sigma)k - (1 + r)(k - b) - b' - [h + \omega]l - c.v - c_f + \beta(1 - d)E_{y,J(.)} + \beta \sum_{\tau=0}^{l-1} l_{\tau} \phi_{\tau} E_{y,[W(C_{\tau},y') - U]} \right\} \]

\[ = \max_{d,v,\theta,C,x,b'} \left\{ yzf(l,k) + (1 - \sigma)k - (1 + r)(k - b) - b' - hl - \omega[l + \frac{1}{\theta} v] - c.v - c_f + \beta(1 - d)E_{y,J(.)} + (1 - d) \beta \sum_{\tau=0}^{l} l_{\tau} \phi_{\tau} E_{y,[W(C_{\tau},y') - U]} \right\} \]

Here maximization is always subject to (6a) and (7) and the third equation makes use of

\[ (1 - \delta)l_{\tau+} = \phi_{\tau}(y)l_{\tau} \]

and

\[ \omega \frac{1}{\theta} v = \beta(1 - d)p(\theta)vE[W(C_{\tau},y') - U] \]

This equation shows the joint surplus maximization of firm and workers in a decentralized economy.

**Step two: Planner problem**

At every period and for every firm type, the planner decides, how much to borrow or lend \( x \), how much fund to save \( b' \), an exit probability \( d \geq d_0 \), a separation rate \( \mu \geq \mu_0 \), the number of vacancies \( v \), and a matching probability \( \theta \). Also the planner decides the mass of new entrants \( N_0 \geq 0 \)

The sequential planning problem is:

\[ \max_{d,\mu,v,\theta,C,x,b',N_0} \sum_{t \geq 0} \beta^t \psi_t \left\{ -CN_0 + \sum_y N(y) \left[ yzf(l,k) + (1 - \sigma)k - (1 + r)(k - b) - b' - hl - c_f - c.v \right] \right\} \]

subject to the law of motion for firms (13) workers (14) aggregate resource constraint (12) and financial constraint (7)
I first write the recursive formulation of the planner problem and then prove the equivalence between
the recursive and sequential formulation. Where $\beta^t \psi_t \eta$ is the multiplier on resource constraint 12.
Therefore let $P(b, l, y)$ denotes the recursive formulation of the social value of a firm with $b$ stock of
asset, $l$ stock of employment and idiosyncratic productivity $y$.

$$P(b, l, y, z) = \max_{d, \mu, \beta, v, x, b'} yzf(l, k) + (1 - \sigma)k - (1 + r)(k - b) - b' - hl - \eta[l + 1/\theta v] - c.v - cf + \beta(1 - d) \mathbb{E}yP(b', l', y')$$

(18)

Lemma 1:

a) For any given multipliers $\eta$ there exists value functions $P : \mathbb{R}_+ \times B \times Y \to \mathbb{R}$ satisfying the system
of the system of recursive equations 18.

b) If $(N, l, v, \theta, \delta, \mu, x, b')$ is a solution of the planning problem 17 then the corresponding firm policies
also solve problem 17 and the complementarity slackness condition

$$\sum_y \pi(y_0)G(b_0, 0, y, z) \leq C, \quad N_0 \geq 0$$

is satisfied. Conversely, if this vector solves for every firm problem 17 with multiplier $\eta$, and if
the complimentary slackness and resource constraint hold for the $z$, then the is a solution of the
planning problem 18

Proof. of Lemma 1

Part a) The RHS in the system of equations in 18 defines an operator $T$ which maps a sequence of
bounded functions $P = (P_t)_{t \geq 0}$, with $P_t : [0, \bar{l}] \times [0, \bar{b}] \times Y \times Z \to \mathbb{R}$ such that $||P|| \equiv \sup_t ||P_t|| < \infty$,
into another sequence of bounded functions $\tilde{P} = (\tilde{P}_t)_{t \geq 0}$ with $||P|| \equiv \sup_t ||\tilde{P}_t|| < \infty$. Here $\bar{l}$ and $\bar{b}$ are
sufficiently large such that the decisions for labour ($l$) and capital ($k$) never binds for any $l \in [0, \bar{l}]$ and $b \in [0, \bar{b}]$. The existence of $\bar{l}$ and $\bar{b}$ follows from Inada condition for $f$: the marginal product of
an additional worker or unit of capital must be negative for any $y \in Y$, for all $l' \geq \bar{l}$ or $b' \geq \bar{b}$ with
sufficiently large $\bar{l}$ or $\bar{b}$; hence no hiring will occur beyond $\bar{l}$ and no investment will happen beyond $\bar{b}$.
Because the operator satisfies Blackwells sufficient conditions, it is a contraction in the space of
bounded function sequences $P$. Hence, the operator $T$ has a unique fixed point which is a sequence of
bounded functions.

Part b) Take first a solution $S$ of the planning problem, and write $\beta^t \psi_t \eta \geq 0$ for the multipliers on the
aggregate resource constraint. Then $S$ maximizes the Lagrange function

$$\mathcal{L} = \max_{d, \mu, \beta, v, x, b', N_0, N_0} \beta^t \psi_t \left\{ - \mathbb{C}N_0 + \sum_y N(y) [yzf(l, k) + (1 - \sigma)k - (1 + r)(k - b) - b' - cf - c.v \right\}$$
For each individual firm, this problem is the sequential formulation of the recursive problem 18 with multipliers $\eta$. Hence, firm policies also solve the recursive problem; furthermore, the maximum of the Lagrange function is the same as the sum of the social values of entrant firms plus the social values of firms which already exist at $t = 0$, namely,

$$
\mathcal{L} = \max_{N_0} \sum_{t \geq 0} \beta_t \psi_t N_0 \left[ -C + \sum_{y_a} \pi_0(y) G_t(b_0, 0, y) \right] + \sum_{t} \phi_t \sum_{y} N(y) G_0(b, l, y)
$$

This also proves that the complementary-slackness condition describes optimal entry.

Step three: The equivalence of planner and decentralized market problem

Provided that $\eta = \omega$, the firm joint surplus maximization in decentralized market 10 is identical to planner problem 18. The only difference is that the firm commit to cohort specific retention probabilities, while planner chooses an alike separation probability for all workers within a firm every period. But both problems have the same answer as both are the dynamic optimization decision of a single decision maker with the same states and payoff functions. This means that the solution to both problems regarding to, investment, saving, vacancy posting, choosing the submarket, lay off and exit are identical. Moreover, the free entry conditions in both problems are the same. When $\eta = \omega$, the value of entry in 10 is the same as 18 and therefore given the aggregate resource constraint the measure and distribution of entrants are the same in both problems.

A.1 FOCs of joint surplus maximization
B Calibration and numerical implementations

This section explains the numerical implementation of the model solution and other numerical exercises in the paper.

B.1 Computing the stationary equilibrium

To solve the model numerically I follow Hopenhayn and Rogerson (1993) and construct my algorithm based on the definition of equilibrium in 2.10. The problem of 10, 6, 7 and 8 define two nested fixed point problem that must be solved to find the equilibrium. The value functions are solved on a \( nl \times nb \times ny \) grid where the number of grid points in my baseline calibration is the following: \( (nl = 60, nb = 60, ny = 10) \)

The algorithm consists of three steps:

The first step, uses condition (5) to find the unique value of \( \omega \) that is consistent with free entry in equilibrium. For any given \( \omega \) one can compute the value functions and check if the expected value of entry is equal to the cost of entry. This yields firm value functions \( G(b, l, y) \), as well as the policu functions \( d, \mu, \theta, v, x \) and \( b \).

The second step, find \( N \) up to a scale factor. One can use the policy functions and Markov process to compute the transition function \( T \). A stationary equilibrium requires a pair \( (N, N_0) \) such that \( N \) is a fixed point of \( T(N, N_0, \omega) \). Given \( \omega \) and \( N_0 \) the operator \( T \) has a fixed point.

The third step, determines the scale factor \( N_0 \). Once a fixed \( \bar{N} \) has been found, the equilibrium must also satisfy the aggregate resource condition

\[
\sum_y N(y) \left[ l + \frac{1}{\theta} v \right] = 1
\]

The left hand side of this equation is linearly homogenous in \( N_0 \) and the right hand side is constant, therefore there must be a unique \( N_0 \) that satisfies this equation.
C Data description

C.1 UK aggregate data

- Output is taken from the ONS (office for national statistics). I use quarterly and seasonally adjusted GDP in Pound Sterling from 2000Q1 to 2012Q4.

- Productivity is seasonally adjusted and taken from ONS. It measure as output per worker $\frac{Y}{L}$ over the period 2000Q1 to 2012Q4.

- Unemployment rate is quarterly, seasonally adjusted and constructed by ONS from Labour Force Survey over the period 2000Q1 to 2012Q4.

- The number of vacancies is quarterly, seasonally adjusted and constructed by ONS from Vacancy Survey over the period 2000Q1 to 2012Q4. The total number of vacancies excludes Agriculture, Forestry and Fishing.

- The time series of unemployment to employment (UE) and employment to unemployment (UE) transition rates are taken from Smith (2011) over the period 2000Q1 to 2012Q4.

- The total lending to private non-financial corporations are in Pound Sterling, quarterly, seasonally adjusted and taken from bank of England data set.

C.2 Credit Conditions Survey

The first Credit Conditions Survey was conducted in 2007 Q2 and additional questions have been included since 2007 Q4. A full set of results is available in Excel on the Banks website at: www.bankofengland.co.uk/publications/Pages/other/monetary/creditconditions.aspx

Positive balances indicate that lenders, on balance, reported/expected demand/credit availability/defaults to be higher than over the previous/current three-month period, or that the terms and conditions on which credit was provided became cheaper or looser respectively. Where the survey balances are discussed, descriptions of a significant change refer to a net percentage balance greater than 20 in absolute terms, and a slight change refers to a net percentage balance of between 5 and 10 in absolute terms. Survey balances between 0 and 5 in absolute terms are described as unchanged.

To calculate aggregate results, each lender is assigned a score based on their response. Lenders who report that credit conditions have changed a lot are assigned twice the score of those who report that conditions have changed a little. These scores are then weighted by lenders market shares. The results are analysed by calculating net percentage balances the difference between the weighted balance of lenders reporting that, for example, demand was higher/lower or terms and conditions were
Figure 12: Data are shown in log deviation from the first quarter of 2008 when the first rise in unemployment has been observed. The time series of EU and UE has been available till the first quarter of 2010.

Q: How has the proportion of loan applications from medium/large PNFCs being approved changed?
Figure 13: The overall change in proportion of loan applications from medium/large PNFCs being approved

C.3 FAME

I obtain data on UK firms from the FAME data set created by the Bureau van Dijk through UCL library website. Fame combines the filed balance sheet data at Company House. I restrict my attention to the firms located in UK and construct a balance panel where at each year I can observe, total employment, total asset and total debt of a firm. I define total debt, as the sum of short term and long term debt for each firm. I exclude firms with negative asset (three observations). I obtain data on UK firms from the FAME data set created by the Bureau van Dijk through UCL library website. Fame combines the filed balance sheet data at Company House. I restrict my attention to the firms located in UK and construct a balance panel where at each year I can observe, total employment, total asset and total debt of a firm. I define total debt, as the sum of short term and long term debt for each firm. I exclude firms with negative asset (three observations). I exclude firms with negative asset (three observations). I exclude firms with negative asset (three observations). [EXPLAIN HOW YOU EXCLUDE FIRMS WITH TOO MUCH DEBT] my final data set contains 9872 firms which I can follow them and observe the three aforementioned variables from 2005 to 2011.
D Productivity Shocks

D.1 Policy functions

Figure 14: Firms optimal strategy after financial tightening (partial equilibrium effect)

Figure 15: Firms optimal strategy after financial tightening (general equilibrium effect)
Figure 16: Response to a permanent decrease in collateral constraint (Transition path is shown in log deviation from steady state. The collateral constraint shock is matched to produce 4.7% fall in labour productivity)

[POLICY FUNCTIONS AFTER AN AGGREGATE PRODUCTIVITY SHOCK]
Figure 17: Response to a permanent decrease in aggregate productivity (Transition path is shown in log deviation from steady state. The aggregate productivity shock is matched to produce 4.7% fall in labour productivity)