Dispersed information in FX trading: a martingale representation.

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September 20, 2015,
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Motivation

· **Macro view:** Exchange rate disconnect puzzle - macroeconomic fundamentals do not matter for currency pricing.

· **Finance view:** Exchange rates are asset prices and volatility can to some extent be explained by liquidity, but erratic price movements prominent. Can be fitted quite well by stochastic volatility diffusion (Merton, 1973; Eraker et al., 2003).

Most studies assume econometrician and agent have the same information: reasonable?

Engel and West (2005) show that many standard macro models actually imply that exchange rate is an infinite recursion of future fundamentals.
Addressing the exchange rate disconnect:

- If information about macro conditions is dispersed:
  - Will this information be aggregated into the exchange rate?
  - Could new information about past economic conditions matter for the exchange rate?
- Can the jump diffusion framework help connect exchange rates and macroeconomic conditions?
Contribution:

- Model where **dispersed information** and **gradual discovery** obscures link between macro and FX rate.
- Micro-founded macro setup that converges, $t \to 0$ to a **time-varying jump diffusion** with an economic interpretation of coefficients.
- High-frequency jump-detection on an intraday time scale.
- Agent expectation errors on macro news releases match well the high frequency jump path.
- Exchange rate dynamics with marked **changes in average expectations**.
Existing attempts to reconnect exchange rates with fundamentals

- Parameter instability
  (Rossi, 2006; Sarantis, 2006; Beckmann et al., 2009)

- Bubbles made by irrational or optimistic traders
  (De Long et al., 1990; Hellwig et al., 2006)

- Order flow portfolio shifts
  (Lyons, 1997; Evans and Lyons, 2002; Evans and Rime, 2010)

- *Rational confusion*
  (Bacchetta and Wincoop, 2006)
Dispersed information: Agents face two flows of information: private observations of their micro environment, and occasional arrival of public news about economic conditions.

Higher order expectations: risk premium determined by how agents form expectations.

Define:

Market wide risk premium: \( \bar{\rho}_t := \int E_t^i[s_{t+1}]di - s_t - (r_t - r_{t*}) \),

Foreign exchange demand: \( FX_{t,i}^d = \alpha_s (E_t^i \Delta s_{t+1} - (r_t - r_{t*})) + h_t^i \),

\( h_t \) denotes individual hedging/fundamental demand.

Public Information: \( \Omega(t) \)

Microinformation: \( \mathcal{F}_t = \{ \Omega(t), \omega_t^i(t) \} \),

where \( \omega_t^i = f_t + \xi_t + \epsilon_t^i \). \( \xi_t \) is a common error component measuring the overall signal dispersion and \( \epsilon_t^i \) is idiosyncratic noise.

Price relevant fundamentals: \( f_t = \beta X_t, X_t = X_{t-1} + \epsilon_f^t \)
Assumptions about the economy

- **Dispersed private information:**
  Agents receive continuous flow of info about their surroundings:
  \[ \mathcal{S}^i(t) = \{ \Omega(t), \omega^i(t) \} \], where
  \[ \omega^i = f_t + \zeta_t + \epsilon^i \].
  \( \zeta_t \) is a common error component measuring the overall signal dispersion and \( \epsilon^i \) is idiosyncratic noise.

- **Arrival of public information about current macroeconomic conditions**
  At random times public news about the current (or past) macroeconomic conditions arrive.

- **Homogenous conditional expectations:**
  Agents are ex-ante identical: given the same information they will have the same expectations.
  \[ E_t^i \left[ r^*_{t+1} - r_t + 1 - \frac{1}{\alpha_s} \int_I h^i_{t+1} di \right] = E[d(f_{t+1})|\mathcal{J}_t^i]. \]

- **Efficient Markets:**
  Exchange rate is a martingale difference process w.r.t. public information.
  \[ E \left[ E(s_{t+\tau} - s_t) | \Omega(t) \right] = 0, \text{ for all } t. \]
Formation of expectations

Public information: $\Omega(t)$

Agent $i$ Information: $\mathcal{F}^i(t) = \{\Omega(t), \omega^i(t)\}$, $\omega^i_t = f_t + \xi^i_t$

Mapping from macro to exchange rate: $d(f_t): F \rightarrow s$

Proposition (Exchange rate expectations.)

Define an equilibrium expectation formation rule as a rule which, if followed by all agents of the exchange market, will be mutually consistent. Then one such rule is given by:

$$E^i_t(s_{t+1}) = E[d(f_{t+1})|\mathcal{F}^i(t)] + E[\lambda^i d(f_{t+2})|\mathcal{F}^i(t)]$$  (1)

where $\lambda^i$ is the agents estimated correlation between his signal flow and the average market signal flow.
Exchange rate dynamics

\[ \Delta s_{t+1} = s_{t+1} - s_t \]

\[ = \left( r^*_{t+1} - r_{t+1} - \frac{1}{\alpha_s} \hat{h}_{t+1} \right) - \left( r^*_t - r_t - \frac{1}{\alpha_s} \hat{h}_t \right) \]

\[ + \bar{E}_{t+1} [d(f_{t+2})] - \bar{E}_t [d(f_{t+1})] \]

\[ + \bar{E}_{t+1} [\lambda_i d(f_{t+3})] - \bar{E}_t [\lambda_i d(f_{t+2})] \]

= change in fundamentals + revision of (average) expectations

(Where \( h_t \) denotes individual hedging demand.)
Dispersed information in FX trading: a martingale representation.
Convergence to a jump diffusion:
Part I - Private information

Proof.

Define: \( \Delta s_{t+1} \equiv \Delta(f_{t+1}|\Omega(t)) + E_{t+1}g(f_{t+2}) \).

\( E(g(\cdot)) \): aggregate expectation revision between \( t \) and \( t+1 \). \( f_t \) converges to a Brownian motion, so

\[
\Delta(f_{t+1}) \xrightarrow{\Delta t \to 0} (\sigma^f(t)B^f(t))
\]

\[
\int_I E[g(f_{t+2})|\Omega(t), \omega^i(t)] \xrightarrow{\Delta t \to 0} \int_I E[g(\sigma^f(t)B^f(t))|\Omega(t), \sigma^f(t)B^f(t)]
\]

\[
+ \sigma^\xi(t)B^\xi(t) + \epsilon^i_t \]

\[
\equiv G_t^\xi \sigma^f(t)B^g(t)
\]

\[
\Delta s_{t+1} \xrightarrow{\Delta t \to 0} D_t \sigma^f(t)B^f(t) + G_t^\xi \sigma^f(t)B^g(t) \equiv \sigma^s(t)B^s(t),
\]

for all \( t \) such that \( \Omega(t^+) = \Omega(t^-) \), that is, for all \( t \) at which there are no news arrivals.
When new public information arrives: Step two

Proof.

Consider $s_t$ at any news arrival time $\tau$. Then,

$$\Delta s_{\tau^+} = D(f_{\tau^+}) + E_{\tau}g(f_{\tau^+}) = D(f_{\tau^+}) + \int_I E[g(f_{\tau^+})|G(\tau), \bar{S}^p(\tau), h(\tau), \omega^i(\tau)]di.$$

$$m_t^j = E(f_t|\mathcal{H}_t^j), m_t^{je} = \int_I E\left[m_{\tau}^{ji}|\Omega(\tau^-), \omega^i(\tau^-)\right]di, \text{ std. news, } n_t^j = \frac{m_t^j - m_t^{je}}{\sigma(m_t^j)}.$$

$$\int_I E\left[g\left(f(\tau^+)^+\right)|\Omega(\tau^-), \omega^i(\tau^-), m_{\tau}^j\right]di = G_{\tau}(n_t^j) \equiv \kappa(\tau).$$

Hence,

$$ds(\tau) = s(\tau^+) - s(\tau^-) = \kappa(\tau).$$

$E(s_{t+\tau}) - s_t = \bar{E}\left[g(f_{t+\tau}) - s_t|\mathcal{G}_t, \mathcal{B}_t^p, \omega_t^i, \mathcal{H}_t\right] = 0.$ Thus $\kappa(t) = G_t(n_t^j)$, while the jump intensity is the arrival rate of news $n_t$. 

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Dispersed information in FX trading: a martingale representation.
Theorem (Convergence to a Jump-Diffusion)

In the presence of continuous dispersed information and news releases, the standard macroeconomic models of exchange rates converge to a standard time varying parameter jump diffusion.

\[
\Delta s_{t+1} \xrightarrow{\Delta t \to 0} ds(t) = \sigma^s(t)B^s(t) + \kappa(t)dq(t).
\]

where
- \(\sigma^s(t)\) is a measure of current informational dispersion in the economy directly related to the variance of the error on agents' private signals on fundamentals
- \(\kappa(t)\) measures the aggregate expectation revision at time \(t\).
- \(q(t)\) is the jump intensity parameter of the process
- \(\lambda^s(t)\) is the arrival rate of public news.
Using well-known asymptotic results (Barndorff-Nielsen and Shephard, 2004):

- \( RV_{t+1}(\Delta) \rightarrow_{\Delta \rightarrow 0} \int_t^{t+1} \sigma^2(\tau)d\tau + \sum_{0<\tau \leq t} \kappa^2(\tau) = QV \)

- \( RV_{t+1}(\Delta) - BV_{t+1}(\Delta) \rightarrow \sum_{0<\tau \leq t} \kappa^2(\tau) = \text{Cumulative jump activity over } \Delta \)

\[
\frac{RV_{t+1}(\Delta) - BV_{t+1}(\Delta)}{cTQ_{t+1}(\Delta)} \overset{D}{\rightarrow} N(0, 1)
\]

This is combined with threshold jump estimation (Mancini, 2009), using a linearly interpolated hourly jump threshold, to extract hourly squared jumps and per minute jump path for the EURUSD exchange rate from 2001-2011.
Data:

Exchange rate:
- One minute round-the-clock bid and ask quotes for the dollar pr euro (EURUSD) exchange rate
- from Thompson Reuters high frequency quotes
- span from January 2001 to December 2011
- total of a bit more than 5, 700 000 midquote observations.

Public news:
- Macro news releases and market expectations from Briefing inc. for 83 regularly released series from the Thompson Reuters market expectations survey.

Following Andersen et al. (2003):

\[ \text{news}_t = \sum_{j=1}^{J} \frac{\text{released}_j - \text{market expectation}_j}{\text{std. err. (released)_j}} (1000), \]

- 3718 hour-observation with news arrival,
- 573 hour-observations with arrival of significant news
Jumps and news releases

Figure: Median standardized surprises/news and jumps

(a) Hourly Jumps

(b) Median std.ized surprise
Jumps and news releases

Figure: Jumps (blue) and News (red) oct-des 2004
What drives jumps?
- Parametric estimation of Jump distribution

- The model implies that jumps are driven by a time varying Poisson hazard according to how much new information becomes public.

- Exchange rate jump process is modeled as
  \[ \kappa(t)q(t) = k(t)q(\lambda(t)), \text{ where } \lambda(t) = \alpha_0 e^{|Z|\beta}. \]

- Jump Intensity parameters are estimated by Maximum likelihood from the extracted jump indicator series.

- Using the extracted jump path, the jump distribution process is modeled as \( \kappa(t) \sim N(Z_t\beta, \Sigma) \), estimated by maximum likelihood.
Conditional Hazard and jump distribution

Table: News impact on jump intensity: $\lambda(t)$, and jumps: $N(Z\beta)$

<table>
<thead>
<tr>
<th>Measure</th>
<th>Intensity</th>
<th>Tstat</th>
<th>Height</th>
<th>Tstat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto Sales</td>
<td>0.64</td>
<td>4.21</td>
<td>0.08</td>
<td>0.25</td>
</tr>
<tr>
<td>Core CPI</td>
<td>0.44</td>
<td>2.34</td>
<td>0.07</td>
<td>1.16</td>
</tr>
<tr>
<td>Home Sales</td>
<td>0.37</td>
<td>1.05</td>
<td>-0.50</td>
<td>-8.82</td>
</tr>
<tr>
<td>Initial Claims</td>
<td>0.27</td>
<td>2.94</td>
<td>0.12</td>
<td>0.39</td>
</tr>
<tr>
<td>Mich sentiments</td>
<td>0.27</td>
<td>1.22</td>
<td>2.47</td>
<td>16.11</td>
</tr>
<tr>
<td>Nonfarm Payroll</td>
<td>0.57</td>
<td>3.20</td>
<td>-0.47</td>
<td>-1.59</td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.34</td>
<td>1.81</td>
<td>0.09</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Table: Kullback–Leibler divergence w.r. to random walk

<table>
<thead>
<tr>
<th>KLIC</th>
<th>Ex. Rate</th>
<th>Diffusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownian motion</td>
<td>0.050</td>
<td>0.019</td>
</tr>
<tr>
<td>Brownian motion with drift</td>
<td>-0.027</td>
<td>-0.003</td>
</tr>
<tr>
<td>GBM</td>
<td>-1.798</td>
<td>0.006</td>
</tr>
<tr>
<td>BM fixed hazard</td>
<td>-0.030</td>
<td></td>
</tr>
<tr>
<td>BM tv hazard</td>
<td>0.240</td>
<td></td>
</tr>
<tr>
<td>BM tv hazard, N(news)</td>
<td>0.240</td>
<td></td>
</tr>
<tr>
<td>Jump Diffusion (tv diffusion $\sigma$)</td>
<td>0.243</td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- A jump-diffusion coefficients can be given economically meaningful interpretations in the current setting.
- Dispersed information can explain why news on past conditions matter for expectations on future.
- New information on the macro economy seems to matter for jump process.
- Time varying hazard model has better pseudo out of sample fit than the random walk.
Dispersed information in FX trading: a martingale representation.
Figure: Decomposition of the realized variance

(a) Realized Variance

(b) Bi-power Variation

(c) Significant Jumps
Likelihoods

\[ m := ae^{z'\beta} \]

- Poisson: \( \ln L = y_i \ln(m) - m \ln(y!) \)
- Binomial: \( \ln L = -m(1 - y_i) + y_i \ln(1 - e^{-m}) \)
- Altered Binomial: \( \ln L = -m(1 - y_i) - \left( e^{-m} + \frac{e^{-2m}}{2} + \frac{e^{-3m}}{3} \right) \)

3rd order MacLaurin approximation of \( \ln(1 - e^{-m}) \)

Back to Parameter Recovery.
Proof of the consistency of expectations formation: Part 1

By definition,

$$E_i^t(s_{t+1}) = E[r^*_t + r_{t+1} - \frac{1}{\alpha_s}\tilde{h}_t|\tilde{S}^i(t)] + E[\tilde{E}_{t+1}s_{t+2}|\tilde{S}^i(t)].$$

By substituting for $s_{t+2}$,

$$E_i^t(s_{t+1}) = E[r^*_t - r_{t+1} - \frac{1}{\alpha_s}\tilde{h}_{t+1}|\tilde{S}^i(t)]$$

$$+ E_t\left[\tilde{E}_t[r^*_t - r_{t+2} - \frac{1}{\alpha_s}\tilde{h}_{t+2}|\tilde{S}^i(t)]\right]$$

$$+ E_t[\tilde{E}_{t+1}\tilde{E}_{t+2}s_{t+3}|\tilde{S}^i(t)]$$

$$= E\left[d(f_{t+1})|\tilde{S}^i(t)\right] + E\left[\tilde{E}_{t+1}d(f_{t+2})|\tilde{S}^i(t)\right] + E_t[\tilde{E}_{t+1}s_{t+3}|\tilde{S}^i(t)]$$
Part 2

Assume all agents form expectations according to:

\[ E^i_t(s_{t+1}) = E[d(f_{t+1})|\mathcal{F}^i(t)] + E[\lambda^i d(f_{t+2})|\mathcal{F}^i(t)] \]

The average expectation in the economy will then be given by,

\[ \bar{E}_t s_{t+1} = \int_I E^i_t(s_{t+1}) di = \int_I E[d(f_{t+1})|\mathcal{F}^i(t)] + E[\lambda^i d(f_{t+2})] di \]

\[ = \int_I E[d(f_{t+1})|\mathcal{F}^i(t)] + E_t^i \left[ \frac{\sigma_t^\omega^2}{\sigma_t^\omega^2 + \sigma_t^\varepsilon^i} d(f_{t+2}) \right] di \]

\[ = \bar{E}_t [d(f_{t+1})] + \bar{\lambda}_t \bar{E}_t [d(f_{t+2})] \]
The exchange rate in \( t + 1 \) will then be given by

\[
\begin{align*}
    s_{t+1} &= r^*_t - r_t + \frac{1}{\alpha_s} \bar{h}_t + \bar{E}_{t+1}[s_{t+2}] \\
    &= r^*_t - r_t + \frac{1}{\alpha_s} \bar{h}_t + \bar{E}_{t+1}[d(f_{t+2})] + \bar{E}_t[\lambda^i d(f_{t+3})].
\end{align*}
\]

Therefore, the agents following this expectations rule will on average turn out to be correct.

Back to [Expectations](#).
Appendix

Likelihoods

References

Proposition (Private information is diffusive)

The stream of private information received by agents, $\omega_t^i$, evolve according to a noisy Brownian motion with variance $\sigma^\omega(t)$

$$\omega^i(t) \xrightarrow{D} \sum_{j \in J} \sigma_j B(\tau_j) + \epsilon^i(t) \xrightarrow{D} \sigma^\omega(t)B^\omega(t) + \epsilon^i(t),$$

where $\sigma^\omega = \sqrt{\sigma_t^f + \sigma_t^\xi}$

Back to [Convergence to J-D](#).
Proof: Step I

We have:

- \( \omega^i_t = f_t + \xi_t + \epsilon^i_t \)
- \( f_t = \beta X_t \),
- \( X_t = X_{t-1} + \epsilon_t \),

\( \rightarrow \) \( X \) is the cumulation of past impulses,

\[
X_t = \sum_{s=0}^{t} \epsilon_s \rightarrow f_t = \sum_{s=0}^{t} \beta \epsilon^f_s \equiv M^f_t
\]

\[
\xi_t = \xi_{t-1} + \epsilon^\xi_t \Rightarrow \xi = \sum_{s=0}^{t} \epsilon^\xi_s \equiv M^\xi_t
\]

Then,

\[
\omega^i_t = \sum_{s=0}^{t} \beta \epsilon^f_s + \sum_{s=0}^{t} \epsilon^\xi_s + \epsilon^i_t \equiv M^\omega_t + \epsilon^i_t
\]
Consider each partition $M_j^\omega$ of $M_t^\omega$ for which $\epsilon_t^\xi$ has invariant variance $\sigma_j^\omega$,

$$M_j^\omega = \sum_{s=t_{j,k}}^{t_{j,k}+\tau_j} \beta \epsilon_s^f + \sum_{s=t_{j,k}}^{t_{j,k}+\tau_j} \epsilon_s^\xi = \sum_{s=t_{j,k}}^{t_{j,k}} \epsilon_s^j,$$

where $\epsilon_s^j$ is distributed with mean zero and variance $\sigma_j$. Each such partition will, by Donsker's invariance principle, converge to a Brownian motion with variance $\sigma_j^i \tau_j$ as the sampling length diminishes,

$$M_j = \sum_{s=t_{j,k}}^{t_{j,k}+\tau_j} \epsilon_s^j \xrightarrow{D} \sigma_j B(\tau_j), \quad M_t = M_t^f + M_t^\xi \bigcup M_j = \sum_j M_j$$

Note: $B(\tau_j) \overset{\text{dist.}}{=} B(t+\tau_j) - B(t), \quad \rightarrow M_t \xrightarrow{D} \sum_j \sigma_j^\omega \left[ B(t_j + \tau_j) - B(t_j) \right]$.

where, for time-ordered $j$, $t_{j,k} + \tau_j = t_{j,k+1}$. 
Combining the previous results:

\[ \omega^i(t) = M^\omega_t + \varepsilon^i_t \xrightarrow{\Delta t \to 0} \sum_{j \in J} \sigma_j B(\tau_j) + \varepsilon^i(t) \]

\[ = \sum_{j,k} \sigma^i_{j,k} \left[ B(t_{j,k+1}) - B(t_{j,k}) \right] + \varepsilon^i(t) \xrightarrow{\dim(J) \to \infty} \sigma^\omega(t) B^\omega(t) + \varepsilon^i(t) \]

\[ = \sigma^f(t) B^f(t) + \sigma^\xi(t) B^\xi(t) + \varepsilon^i(t), \]

where,

\[ \sigma^\omega(t) = \sqrt{\sigma^f(t)^2 + \sigma^\xi(t)^2} \]

is the limit process of \( \{\sigma_j\} \), as the number of elements in \( J \) increases.

\[ \rightarrow \omega^i_t = \sigma^\omega(t) B(t) + \varepsilon^i(t). \]

Hence, the flow of private information is a time-varying volatility diffusive process.
Some descriptive stats

Table: Summary statistics for hourly EURUSD Realized Volatilities, Bi-power variations and Jumps

|        | price | RV    | BV    | Jumps^2 | news  | \(\sqrt{RV}\) | \(\sqrt{BV}\) | \(\sqrt{Jump^2}\) | |news| |
|--------|-------|-------|-------|---------|-------|---------------|---------------|------------------|--------|
| Mean   | 1.06  | 7.05E-05 | 10.3E-05 | 1.77E-05 | 0.29  | 0.006         | 0.007         | 0.003            | 0.66   |
| Median | 1.08  | 1.9E-05  | 2.44E-05 | 4.93E-06 | 0     | 0.004         | 0.005         | 0.002            | 0.56   |
| Max    | 1.36  | 0.006   | 0.01  | 1.6E-03  | 3.79  | 0.08          | 0.10          | 0.04             | 3.79   |
| Min    | 0.84  | 0       | 0     | 1.16E-09 | -1025 | 0             | 0             | 3.4E-05         | 0      |
| Std.Dev.| 0.15  | 0.0002  | 0.0003 | 5.8E-05 | 0.89  | 0.006         | 0.007         | 0.003            | 0.60   |
| Skewness| 0.07  | 11.6    | 10.9  | 16.7    | 0.11  | 2.7           | 3.4           | 1.37             |
| Kurtosis| 1.6   | 253     | 241   | 395     | 4.07  | 16.3          | 15.7          | 27.6             | 5.37   |
| Obs    | 11156 | 10651   | 9103  | 2642    | 822   | 10651         | 9103          | 2642             | 822    |

The difference in observation between the various components of the squared price are due to the need of several consecutive price observations for estimation of the integrated volatility (BV), and the jump activity. Any hour for which less than 5 prices are reported is omitted, hence the reduction from price to RV. The BV requires a sufficient number of recorded price triplets, whereas the jump series requires sequences of four prices recorded (robustified estimators).


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