Time Varying Volatility and the Origins of Financial Crises

Omar Rachedi†

JOB MARKET PAPER

September 4, 2014

Abstract

I document that financial crises coincide with the reversal of a prolonged period of low aggregate volatility. I argue that shocks to the volatility of total factor productivity are a source of financial instability, and account for both the build-up of risk and the burst of crises. I develop a DSGE model with an occasionally binding borrowing constraint - in which housing serves as collateral - and a frictional housing market. The search frictions determine the speed of housing transactions and make the loan-to-value (LTV) ratio to depend on the collateral liquidity. In this environment volatility shocks affect the frequency of fire sales by changing the liquidity of the collateral. In the model times of low volatility boost housing investment. The higher collateral liquidity relaxes the LTV ratio and triggers a credit expansion. If households’ leverage rises enough, a sudden volatility spike dries up the liquidity of housing, lowers the LTV ratio and turns the credit boom into a bust. In a quantitative exercise I feed into the model the stochastic volatility of the U.S. Solow residual. I find that the interaction of time varying volatility and a frictional housing market increases the frequency of financial crises by 47% and the corresponding output drop by 30%. Volatility shocks also generate time variations in LTV ratios, providing a foundation for financial shocks.

Key Words: Stochastic Volatility, Housing Market, Collateral Liquidity, Occasionally Binding Borrowing Constraint, Search Frictions, Non-Linear Dynamics.

*I am indebted to Andres Erosa, Matthias Kredler, Claudio Michelacci, Salvador Ortigueira and Hernan Seoane for their guidance. I also thank Javier Fernandez Blanco, Davide Debortoli, Antonia Diaz, Luis Franjo, Alessandro Galesi, Pedro Gomes, Belen Jerez, Tim Kehoe, Iacopo Morchio, Gabriel Perez-Quiros, Alessandro Peri, Giorgio Primiceri, Vincenzo Quadrami, Carlos Ramirez, Manuel Santos, Carlos Thomas, Roine Vestman, and presentation participants at Universidad Carlos III de Madrid, Stockholm University and the Workshop on Dynamic Macroeconomics in Vigo for helpful suggestions.

†Universidad Carlos III de Madrid, Department of Economics, Calle Madrid 126, 28903, Getafe (Madrid), Spain. E-mail: orachedi@eco.uc3m.es, web: https://sites.google.com/site/omirachedi
1 Introduction

Can changes in the volatility of the business cycle affect the frequency of financial crises? The financial instability hypothesis of Minsky (1992) conjectures that periods of low aggregate uncertainty can generate a crisis ex-post, a phenomenon that Brunnermeier and Sannikov (2013) refer to as the volatility paradox.

In this paper I document a new stylized fact on financial crises which is consistent with the volatility paradox. I find that the volatility of total factor productivity (TFP) is around 10% below trend over the two years before a financial crisis, before peaking up to 13% above trend amidst the crisis.

To explain this fact I argue that shocks to the volatility of TFP are a source of financial instability, and account for both the build-up of risk and the burst of financial crises. I develop a non-linear DSGE model in which volatility shocks interact with an occasionally binding collateral constraint - where housing serves as collateral - and a frictional housing market. On the one hand, in the model financial crises happen when the collateral constraint binds, forcing households to deleverage and fire sell their housing stock. The constraint becomes binding with an endogenous probability that depends on the optimal choices of the households. On the other hand, the search frictions determine the speed of housing transactions and make the loan-to-value (LTV) ratio to depend on the collateral liquidity. Households can borrow at a higher LTV when the housing market is more liquid. In this environment volatility shocks affect the frequency of fire sales by changing the liquidity of the collateral.

In a quantitative exercise I feed into the model the stochastic volatility of the Solow residual of the U.S. economy, which I estimate with Bayesian techniques. I numerically solve for the equilibrium using global methods and find that the interaction of volatility shocks and a frictional housing market increases the frequency of financial crises by 47% and the corresponding output drop by 30%.
I show that financial crises are characterized by deflationary spirals à la Fisher (1933) in both the house price and the LTV ratio, a novel mechanism which amplifies the severity of a downturn. The initial drop in the LTV ratio forces households to deleverage, generating a decline in both house prices and housing liquidity, which has the final effect of decreasing even further the LTV ratio in a perverse deflationary loop.

The mechanism of the paper works through changes in the liquidity of housing, which determines the maximum LTV ratio at which households can access credit. In the model periods of low fluctuations foster housing investment. As more households look for a house, sellers are more likely to meet with a buyer. The higher liquidity of housing relaxes the LTV ratio and generates a credit and an investment boom which reinforce each other. This spiral builds up systemic risk because the economy becomes fragile to the realizations of adverse shocks at high levels of households’ leverage. Indeed, the dynamics of the model are non-linear. As in Mendoza (2010), Bianchi (2012) and Bianchi and Mendoza (2013), negative shocks generate only mild recessions at low levels of leverage. Instead, when households are highly indebted, a sudden peak in volatility can dry up the liquidity of housing and lower the LTV ratio down to the point that the borrowing constraint becomes binding. Agents are then forced to fire sell their housing triggering a debt deflationary spiral in both the house price and the LTV ratio which turns the credit boom into a bust.

The frictional housing market creates a direct link between the liquidity of the collateral and households’ borrowing capacity, a novel mechanism in the literature of DSGE models with financial frictions. While standard models à la Kiyotaki and Moore (1997) usually assume that the LTV ratio is exogenous, in

---

1The role of the collateral liquidity is already pointed out in Del Negro et al. (2011) and Kiyotaki and Moore (2012). These papers exogenously impose the degree of collateral illiquidity, ruling out any feedback effect between households’ credit conditions and their borrowing margin.
this paper the ratio is endogenous. The link between housing liquidity, collateral values and the LTV ratio follows Brunnermeier and Pedersen (2009), in which market liquidity directly determines households’ funding liquidity, that is, the ease at which households can access new loans. In the model a house has a high collateral value if lenders can sell it both quickly and at a high price in case they seize it. Through this channel, changes in the liquidity of the housing market alter the value of the collateral asset and affect households’ borrowing capacity. In equilibrium the LTV ratio is the ratio between the reservation price of a house - the value of a house on sale that is expected to be sold in the future and does not yield any dividend or utility to its owner - and the fundamental value of housing. The wedge between these two prices widens in illiquid markets because houses are expected to be on sale for a longer time. From this perspective, this paper follows the contributions of Fostel and Geanakoplos (2008) and Geanakoplos (2010) on the importance of endogenizing the LTV ratio to match the dynamics of macroeconomic and financial variables.

The frictional housing market is also a crucial feature to propagate volatility shocks into the real economy. Indeed, search frictions generate adjustment costs and partial irreversibilities in housing investment. On the one hand, households incur search costs whenever they look for a house. On the other hand, agents are forced to sell their properties at a discounted price when the housing market is highly illiquid. Hence housing investment is expensive to reverse. As shown in Bloom (2009), in this environment agents become more cautious in uncertain times. Intuitively, agents reduce their investment propensity to avoid incurring the costs of frequently adjusting the housing stock. Instead, when volatility is low, housing investment spikes and the housing market heats up. In the quantitative analysis I show that volatility shocks barely change the frequency of financial crises if the housing market is perfectly liquid.
This paper contributes to the literature focusing on the real effects of volatility shocks, such as Justiniano and Primiceri (2008), Bloom (2009) and Fernandez-Villaverde et al. (2011). While these papers focus on business-cycle fluctuations, I emphasize the role of stochastic volatility in generating financial crises. I show that changes in the exogenous volatility of productivity may generate sharp movements in the endogenous volatility of output and credit amidst a financial crisis when households’ leverage is high enough.

The presence of the volatility shocks and its effect of house prices relates to the literature on asset pricing with long-run risk. Bansal and Yaron (2004) find that the stochastic volatility of consumption growth in a model of long-run risk can account for the time variation of risk premia. The importance of aggregate volatility is also stressed in Bansal et al. (2014) and Nakamura et al. (2014), who point out that changes in macroeconomic risk have a long-run component that affects asset prices. I complement this literature by studying the role of stochastic volatility into a production economy. Although I consider standard CRRA preferences, I show that volatility shocks can still generate sharp fluctuations in house prices around a financial crisis.

Finally, this paper provides a quantitative theory of financial crises which sheds lights on the debate on the causes of the last recession. For instance, Gilchrist and Zakrajsek (2012) and Jermann and Quadrini (2012) show that the recent crisis has been driven by a large negative financial shock that generated a credit crunch and consequently a sharp drop in investment and employment. These results have consolidated the view that the cause of the recent crisis is the disruption of credit supply due to the breakdown of the financial intermediaries.

\footnote{What is the interpretation of the volatility shocks? Carvalho and Gabaix (2013) show that the changes from the manufacturing sector towards the service and the financial sector can account for the movements in the volatility of U.S. macroeconomic variables over the last decades. Alternatively, Bloom (2009) documents that time variations in aggregate volatility are correlated with changes in the cross-sectional dispersion of firms’ growth rates. Christiano et al. (2014) shows that shocks to the dispersion of firms’ productivity are an important source of business cycle fluctuations.}
Yet, this explanation is at odds with the empirical evidence provided by Mian and Sufi (2009, 2011), who find that the housing market in the U.S. started to slump around 2006, much earlier than the collapse of Bear Stearns and Lehman Brothers. In this vein, the deterioration of the balance sheet of the households - rather than the one of banks - has triggered the Great Recession.

To reconcile these different views, I propose a mechanism which is only based on real shocks, and especially on innovations to the volatility of TFP. Importantly, this mechanism works entirely through variations in credit demand. In the model there is no bank. However, volatility shocks drive changes in the liquidity of housing, which affect households’ collateral values and eventually modify the LTV ratio. This mechanism makes households’ leverage to move over time even when the fundamental value of housing does not change. Hence, the interaction of volatility shocks and a frictional housing market generates dynamics in the LTV ratio that are observationally equivalent to a financial shock, although they entirely hinge on credit demand. This result suggests that financial shocks should not necessarily be interpreted as if they were originated in the financial sector, and could rather be caused by shifts in credit demand.

1.1 Related Literature

This paper is connected to three strands of literature. First, I complement the empirical evidence provided by Reinhart and Rogoff (2009), Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et al. (2013a,b) on the dynamics of macroeconomic variables around financial crises. These authors show that financial crises are actually credit booms gone bust. I document that although aggregate volatility does not display strong comovements with recessions, it is characterized by large swings around financial crises. Second, this paper contributes to the debate on the recent house price boom and bust. The
global savings glut is often referred to as the main cause of the house price boom. For instance, Justiniano et al. (2014) show that the global savings glut accounts for around one fourth of the increase in U.S. house prices in the early 2000’s. Yet, Favilukis et al. (2013) argue that the boom and bust in the housing market is explained by financial development in the mortgage market. While there is a burgeoning evidence on the improvements in financial markets in recent years, it is harder to understand the reversal of the process of financial development amidst the financial crisis. In my model movements in the LTV ratio - due to changes in housing liquidity - provide a rationale to both the process of financial development and its reversal. The relaxation of credit conditions in the mortgage market can be explained by the high liquidity of the housing market in the 2000’s. Analogously, the liquidity freeze around the crisis can account for the reversal of the process of financial development. Third, this paper relates to the literature on role of search frictions in the housing market, which follows the contribution of Wheaton (1990). For example, Diaz and Jerez (2013) show that a model with a frictional housing market can reproduce the volatility of house prices. I add to this literature by showing that changes in housing liquidity affect the frequency of financial crises.

2 Evidence on Volatility and Financial Crises

In this Section I document a new stylized fact on the dynamics of aggregate volatility around financial crises. I find that crises coincide with the reversal of a long period of low aggregate fluctuations.

---

3 There is also a recent literature that studies the role of search frictions in over-the-counter financial markets building on Duffie et al. (2005, 2007). See Rocheteau and Weill (2011) for a review of this literature.

4 The basic setting of the housing market in my paper grounds on Ungerer (2013), which shows that monetary policy affect the leverage of the economy through an endogenous borrowing margin that depends on housing market liquidity. Instead, I focus on the link between volatility, housing market liquidity and financial crises. I also emphasize the dynamics of Fisherian deflation affecting the loan-to-value ratio.
2.1 Data on Volatility and Financial Crises

I build a panel of 20 developed countries from 1980 until 2013 to understand how volatility is related to financial crises. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. For any of these countries, I consider an indicator of aggregate volatility at an annual frequency. Namely, I consider the stochastic volatility of the series of total factor productivity (TFP). I compute the series of TFP $z_t$ for each country using data from the Penn World Tables. Then, I posit that in each country TFP follows a first-order autoregressive process with stochastic volatility

\[ z_t = \rho_z z_{t+1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0, 1) \]  

**\( (1) \)**

\[ \sigma_t = (1 - \rho_\sigma) \bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta \epsilon_{\sigma,t}, \quad \epsilon_{\sigma,t} \sim N(0, 1) \]

where $\rho_z$ denotes the persistency of the level equation of TFP, $\rho_\sigma$ is the persistency of the volatility equation, $\bar{\sigma}$ is the long-run mean of the volatility of TFP and $\eta$ captures the degree of stochastic volatility in the process. $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ denote the innovations to the level and volatility of TFP, respectively. I assume that both $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are independent to each other.

Since the innovations $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are unknown to the econometrician, I need to apply a filter to the data to estimate the parameters of the process. In this framework the Kalman filter is unsuitable because it applies only to linear series, while here the shocks to the volatility enter non-linearly in the level equation of TFP. I evaluate the likelihood of this process by appealing to the Sequential Importance Sampling particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et al. (2011). The estimation
of the stochastic volatility of TFP closely follows Born and Pfeifer (2013). I use Bayesian techniques to estimate the likelihood of the process of productivity. I elicit some unrestrictive priors, and after deriving the likelihood of the process for some given parameters with the SIS particle filter, I maximize the posterior likelihood using a random walk Metropolis-Hastings algorithm with 20000 replications, out of which the first 5000 represent burn-in draws. Finally, I recover the historical distribution of the volatility of TFP using the backward-smoothing routine of Godsill et al. (2004).

I also consider different measures of volatility as robustness checks. First, following Bloom (2009), I proxy aggregate volatility with the logarithm of the variance of daily stock returns within a year. Second, I compute the volatility of quarterly GDP growth over a moving window of 20 quarters.

Finally, I take the dates of financial crises from multiple sources, that is, Bordo et al. (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of recessions are instead given by the OECD recession indicators. Overall, the panel covers 29 events of financial crises and 118 events of recessions. I report the dates of crises and recessions by country and all the sources of the data in Appendix A.

2.2 The Dynamics Around Crises and Recessions

This Section studies the dynamics of volatility around financial crises and recessions. For any country and for any financial crisis and recession, I take the series of aggregate volatility in a time window of nine years around the event of interest.

5I report the computational algorithm and the details on the priors in Appendix E.3.
that is, from four years before either the financial crisis or the recession up to four years afterwards. Then, I consider the series defined by the average observations across events for any year of the window as the typical pattern around financial crises and recessions. For example, to define the typical level of volatility the year preceding a financial crisis, I take the volatility of the Solow residual one year before each of the 30 financial crises of my sample and then compute the mean.

Figure 1 displays the typical dynamics of aggregate volatility around financial crises and recessions. The figure documents that aggregate volatility asymmetrically varies around crises and recessions. While there are negligible deviations from trend during recessions, the behaviour of volatility around crises is characterized by large swings. Crises tend to be preceded by years in which volatility is around 10% below trend and the burst of the crisis pushes volatility up to around 15% above trend.

Figure 2 shows that aggregate volatility maintains the same dynamics around financial crises and recessions even when it is measured as the variance within a year of daily stock returns or when I use the variance of quarterly GDP growth rates over a moving window of 20 quarters. I also find a similar dynamics when either considering the median observations of the deviations of the Solow residual from trend around crises and recessions or ruling out the last financial crisis episodes, see Appendix B.

I argue that this evidence points out a new stylized fact on the dynamics of volatility around financial crises which is consistent with the volatility paradox of Brunnermeier and Sannikov (2013). Appendix E provides panel data evidence showing that changes in volatility predict the burst of financial crises, even when controlling for other macroeconomic variables, country characteristics, and country and year fixed effects.
Figure 1: Aggregate Volatility around Crises and Recessions.

The figure plots the average values of the deviations from the trend of the stochastic volatility of countries’
total factor productivity around recessions and financial crises (9 year window). The continuous line
indicates the dynamics around financial crises, while the dashed line refers to recession. The dates of
financial crises are taken from Reinhart and Rogoff (2009). Recessions are derived from the OECD
recession indicators.

Figure 2: Different Measures of Aggregate Volatility.

(a) Volatility of Stock Returns

(b) Volatility of GDP Growth

Note: The figure plots the dynamics of different measures of aggregate volatility around recessions and
financial crises (9 year window). In Panel (a) the volatility is measured as the variance of daily stock
market returns within a year. In Panel (b) the volatility refers to the variance of quarterly GDP growth
rates computed over a moving window of 20 quarters. The continuous line indicates the dynamics around
financial crises, while the dashed line refers to recession. The dates of financial crises are taken from
Reinhart and Rogoff (2009). Recessions are derived from the OECD recession indicators.
2.3 Volatility Shocks and the Housing Market

The previous analysis points out that changes in the level of aggregate volatility tend to coincide with the building up of a financial crisis and its burst. What is the mechanism behind this result? In this Section I shed light on the relationship between volatility and housing markets. I run a structural VAR model, in which I compute the response of house price, the quantity of house sold and a measure of liquidity of the housing market to an unexpected increase in volatility.

The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three variables on the housing markets related to price, quantity and liquidity.

I borrow the volatility indicator from Bloom (2009). This variable identifies a number of large and arguably exogenous peaks of stock market volatility, and is defined such that it equals 1 in each of these dates and zero otherwise. These dates coincide with events like the assassination of Kennedy, the Arab-Israeli War, the Gulf War and the 9/11 attack. The identification restriction posits that within a month the volatility indicator reacts only to the level of the S&P stock returns, but not to any of the aforementioned macroeconomic variable. The presence of the stock returns allows me to disentangle volatility shocks from any change in the level of stock market data. As housing market variables, I consider the median sales price of new one family homes sold\(^6\), the number of new one family homes sold and the months supply provided by the Census Bureau. The latter is the ratio of houses for sale to houses sold and measures the number of months a house for sale is expected to last on the market. Hereafter I refer to this variable as the time on the market.

The benchmark ordering of the VAR considers the level of S&P 500 returns

\(^6\)Results do not change when using the Conventional Mortgage Home Price Index, see Appendix B.1.
Figure 3: Volatility Shocks and the Housing Market.

(a) Volatility  
(b) House Price  
(c) House Sales  
(d) Time on the Market

Note: VAR estimated from January 1963 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline. The time on the market is measured by the monthly supply of homes, that is the ratio of houses for sale to houses sold. This statistic provides an indication of how many months the current inventory are expected to keep on sales if no additional new houses were built.

and the indicator of volatility first, then the interest rate, the consumer price index and the house price index, and finally the quantities with the industrial production, the level of sold houses and the time of the market. Figure 3 reports the response of house prices, the number of houses sold and the time on the market to a positive one standard deviation shock to volatility. Panel (b) shows that house prices respond very sluggishly to an increase in volatility, and start declining only around 10 months after the realization of the shock. At the peak, the response is around $-0.001\%$ below the baseline, which gives an annualized
rate of $-1.21\%$. Instead, an increase in volatility reduces the number of houses sold at peak by around $-0.0065\%$ on a monthly basis, which gives an annualized rate of $-8.08\%$. Finally, a volatility shock raises the expected time on the market of a house in sale by $0.005\%$ on a monthly basis, which corresponds to an annualized rate of $6.17\%$. This evidence suggests that volatility shocks do affect the housing market, mostly through changes in the number of houses sold and the time on the market of a house on sale.

This evidence is consistent with the dynamics of GDP growth volatility and housing market liquidity over the last decades. Figure 4 shows that the volatility of GDP growth rates has decreases starting from the 1980’s, a phenomenon which is known as the Great Moderation. Stock and Watson (2002) document that in those years the standard deviations of GDP, consumption and investment have decreased by 41%, 38% and 22%, respectively. This trend has been partially reversed during the last recession. Figure 5 displays that the behavior of the housing market liquidity comoves with the volatility of the macroeconomic environment. Periods of low fluctuations are characterized by a low time on the market, while turbulent periods - such as the oil crises in the 1970’s and the Great Recession - have a much lower liquidity. Interestingly, the last period of the Great Moderation coincides with a historical low in the time on the market of a house of sale at around 3.5 months. Amidst the Great Recession, the time on the market peaked up to around 12 months.
Figure 4: U.S. GDP Growth Rate

Note: The figure plots the quarterly series of US GDP growth rate from 1970Q1 until 2013Q4. The series is computed as the first difference of the log real GDP.

Figure 5: Time on the Market of Houses on Sales in the U.S.

Note: The figure plots the quarterly series of the time on the market of houses on sale from 1963Q1 until 2013Q4. The time on the market series is given by the month supply of new one family houses from the Census Bureau.
3 The Model

3.1 Environment

In the economy there is a continuum of identical families that consist of a continuum of members. Although members live in different dwellings, there is perfect risk-sharing within the family. Families access a production function which assembles labor and housing to produce a consumption good. The technology is subject to aggregate productivity shocks with stochastic volatility.

Family members trade real estate properties on a frictional market, such that there is a probability that a house on sale will not be matched with a buyer.

Families borrow from foreign lenders, and lack of commitment to repay debt. If families renege on debt, lenders seize their housing stock. To avoid the repudiation of debt, lenders impose a constraint on families’ borrowing capacity. In equilibrium, families cannot borrow more than the collateral value of housing.

The role of housing is threefold: it provides utility services, it is a production input and it acts as the collateral asset.

3.1.1 Timing

Every period is split into four different stages. In the first one families observe the current realizations of the shocks. In the second one families borrow from the foreign lenders. This stage serves as a rationale for having in equilibrium a borrowing constraint that depends on current values of families’ collateral. In the third stage production takes place and family members trade real estate properties on a frictional housing market. Finally, in the fourth stage a fraction of homeowners is hit by a mismatch shock and forced to leave the houses, which become vacant.
3.2 Families

The economy is populated by a continuum of families $i \in [0, 1]$. Each family consists of a continuum of ex-ante identical infinitely-lived members of measure one. Each member lives in a different dwelling and can own at most one house. Although family members individually trade real estate on the housing market, they pool their revenues within the family.

Each family maximizes the sum of their members’ life-time utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, l_{i,t}, h_{i,t})$$

where $\beta$ is the time discount factor of family members, $c_{i,t}$ denotes the consumption of the family, $l_{i,t}$ is the level of leisure and $h_{i,t}$ is the level of housing services which is assumed to be proportional to the stock of occupied housing.

Families access a decreasing return to scale technology that uses labor force $n_{i,t}$, rented at the equilibrium wage $w_t$, and the stock of occupied housing $h_{i,t}$ to produce a homogeneous consumption good, as follows

$$y_{i,t} = e^{z_t} F(n_{i,t}, h_{i,t}).$$

The consumption good $y_{i,t}$ is sold on a frictionless market, and is the numeraire of the economy. The production function is subject to an aggregate productivity
shock \( z_t \), which follows an autoregressive motion with stochastic volatility

\[
    z_t = \rho_z z_{t-1} + e^{\sigma_t} \epsilon_{z,t} \tag{4}
\]

\[
    \sigma_t = (1 - \rho_\sigma) \bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta \epsilon_{\sigma,t} \tag{5}
\]

where \( \rho_z \) denotes the persistence of the level of productivity, \( \rho_\sigma \) is the persistence of the volatility of productivity, \( \bar{\sigma} \) is the long-run mean of volatility and \( \eta \) captures the degree of stochastic volatility of the process. When \( \eta = 0 \), the process reduces to a standard autoregression motion. Finally, \( \epsilon_{z,t} \) and \( \epsilon_{\sigma,t} \) denote the innovations to the level and volatility of productivity. I assume that they are i.i.d. following normal distributions \( N(0, \sigma_{\epsilon_z}) \) and \( N(0, \sigma_{\epsilon_\sigma}) \), respectively.

I appeal to this specification for aggregate productivity because the dynamics over time of the level and the volatility of aggregate productivity are pinned down by two different shocks, \( \epsilon_{z,t} \) and \( \epsilon_{\sigma,t} \), respectively. The two different sources of uncertainty, one related to the level and the other one linked to volatility, allows me to disentangle the role of volatility shocks and their contribution to the quantitative results of the model.\(^7\)

### 3.3 The Housing Market

In the model houses are either occupied or vacant. Each family \( i \in [0, 1] \) has a fraction of \( h_{i,t} \) members which occupy a house and a fraction of \( v_{i,t} \) members which own a house that does not fit to their needs. I refer to the latter as vacant housing. I assume that vacant houses cannot be used as a production input, do not provide utility services and cannot be pledged as collateral. I further consider a fixed unit supply of houses.\(^8\)

7E.g., in a GARCH model a unique shock drives the dynamics over time of both the level and the volatility of the process. I refer to Fernández-Villaverde et al. (2011) for further discussion on the topic.

8Davis and Heathcote (2007) find that the trend and volatility of US house prices are mostly driven by fluctuations in the price of land. Liu et al. (2013) show that fluctuations in land prices are a driving force.
Real estate properties are traded on a frictional housing market. The search frictions capture in a reduced form the fact that matching in the housing market is time consuming. On one side of the market, each family has \( v_{i,t} \) members who own vacant housing, which are put up on sale. On the other side of the market, there are \( 1 - h_{i,t} \) members which do not occupy a house and seek to buy one on the frictional market. Family members exercise a search effort \( s_{i,t} \) - in units of time - in order to match with a seller. I assume that every unit of search effort comes at a monetary cost \( \kappa s_{i,t}^2 \). The ratio between the total amount of buyers (measured in efficiency units) to the total supply of houses on sale defines the tightness of the housing market

\[
\theta_t = \frac{\int_0^1 (1 - h_{i,t}) s_{i,t} \, di}{\int_0^1 v_{i,t} \, di}.
\]  

(6)

A high market tightness \( \theta_t \) indicates that the housing market is hot, that is, there are more buyers than sellers.

Following Wheaton (1990), the aggregate number of successful matches \( m_t \) is defined by a constant return to scale Cobb-Douglas matching function

\[
m_t = \left( \int_0^1 (1 - h_{i,t}) s_{i,t} \, di \right)^{1-\gamma} \left( \int_0^1 v_{i,t} \, di \right)^\gamma.
\]  

(7)

where \( \gamma \in (0, 1) \). Upon a match, the transaction price of the house \( q_{f,t} \) is defined by a Nash bargaining problem, which I describe in Section 3.6. The matching function (7) stipulates that not all the houses supplied to the market are matched to a buyer. Indeed, the probability at which family members sell houses is

\[
P_{s,t} = \frac{m_t}{\int_0^1 v_{i,t} \, di} = \theta_t^{1-\gamma}
\]

of business cycle. In this vein, the housing stock in fixed supply of my model can be interpreted as land.
which is increasing in the market tightness $\theta_t$. The probability at which family members meet with buyers raises in hot housing market because there is a disproportionately larger amount of buyers exerting a high effort. Instead, the probability that a family member meets with a seller equals

$$P_{b,t} = \frac{m_t}{\int_0^1 (1 - h_{i,t}) s_{i,t} di} = \theta_t^{-\gamma}.$$  

The probability of buying a house negatively depends on the tightness of the market. In a hot market, there are much more buyers than sellers, and any given family member is less likely to meet with a seller.

In this environment a family member manages to sell its house only with a probability $P_{s,t}$. With the remaining probability $1 - P_{s,t}$, the house keeps being on sale on the future period. Since vacant houses cannot be used either as production input and as collateral asset, I can define the reservation value of a house $q_{r,t}$ - the value of a house on sale that does not yield any utility service or dividend to the owner - when the frictional market opens as

$$q_{r,t} = P_{s,t} q_{f,t} + (1 - P_{s,t}) \mathbb{E}_t \left[ \Lambda_{t+1} q_{r,t+1} \right]. \quad (8)$$

Equation (8) stipulates that the reservation value of housing depends on the liquidity of the housing market, the housing price and the continuation value of a vacant house. On the one hand, vacant houses have no reservation value when their selling probability in any future period goes to zero. On the other hand, the reservation value of vacant houses equals the fundamental value of houses - as priced by the frictional market - when the current frictional market is perfectly liquid, that is, $P_{s,t} = 1$. Notice that the reservation value $q_{r,t}$ is the actual value of houses put up on sale by the family members which sell their shelters. As long as the frictional market is partially illiquid, then $q_{r,t} \leq q_{f,t}$, and the relevant house
price for a seller is lower than the relevant house price for a buyer. Hence, the structure of the housing market endogenously generates a bid-ask spread $q_{f,t} - q_{r,t}$ which depends on the liquidity of the frictional market.

Finally, I assume that a fraction $\psi$ of homeowners is hit by a mismatch shock after that trading in the housing market has taken place. Sellers cease to occupy their own dwelling, which adds to the stock of vacant housing that is carried over the next period. The laws of motion of occupied housing and vacant housing are

$$h_{i,t+1} = (1 - \psi) \left( h_{i,t} + P_{s,t} s_{i,t} (1 - h_{i,t}) \right)$$

$$v_{i,t+1} = (1 - P_{s,t}) v_{i,t} + \psi \left( h_{i,t} + P_{s,t} s_{i,t} (1 - h_{i,t}) \right)$$

3.4 Borrowing Constraint

At the beginning of each period families observe the realizations of the aggregate shocks and then decide how much to borrow $d_{i,t+1}$. Families borrow from risk-neutral foreign investors, which inelastically supply funds at the gross interest rate $R$. Families need also to purchase a fraction $\nu$ of the labor cost $w_{i,t} n_{i,t}$ in advance of production. Hence, they receive a working capital loan from the foreign investors. Working capital loans are repaid within the period and do not carry interest payments.

Families lack full commitment and can immediately decide to renege on their debt. In such a case, the lenders seize the housing stock $h_{i,t}$. Under the further assumptions that financial contracts are not exclusive, families can renege on

---

9The presence of the mismatch shock is often assumed in the literature of search frictions in the housing market, and dates back to Wheaton (1990). The shock allows to have always some vacant house in equilibrium. The mismatch shock can be interpreted by job mobility across locations, which forces homeowners to sell their real estate before relocating to a new city. Also a change in the number of people within a family could force homeowners to sell their house and buy a different one. The mismatch shock is analogous to the exogenous separation shock used in the search models of the labor market, see Pissarides (2000).

10This assumption is consistent with the analysis of Mendoza and Quadrini (2009) and Warnock and Warnock (2009) on the effects of US foreign capital inflows on the Treasury bill interest rates since mid 1980’s.
their debt only at the beginning of each period and there is no additional penalty in repudiating the debt. Appendix D.1 shows that in equilibrium the collateral constraint equals

\[
\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq q_{r,t} h_{i,t} \frac{q_{f,t}}{q_{r,t} h_{i,t}} .
\]  

(11)

Collateral Value of Housing

As in Iacoviello (2005), families’ borrowing capacity is determined by the collateral value of the housing. In Iacoviello (2005) the collateral value of housing equals to an exogenous fraction of its fundamental value. In my model the collateral value of families’ housing stock is always lower than its fundamental value, as long as the housing market is not perfectly liquid, and there is a spread between the house price \(q_{f,t}\) and housing reservation value \(q_{r,t}\).

Multiplying and dividing the right-hand side of the constraint by the price of occupied housing \(q_{f,t}\), the constraint becomes

\[
\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq q_{r,t} h_{i,t} \frac{q_{f,t}}{q_{r,t} h_{i,t}} .
\]  

(12)

Maximum Loan-to-Value Ratio

Fundamental Value of Housing

Equation (12) shows that the collateral value of agents depends on the fundamental value of their housing stock, multiplied by a factor which defines the maximum LTV ratio. Standard models usually assume that the degree of pledgeability of the collateral is an exogenous parameter. Instead, in this framework the LTV ratio is endogenous and depends on the liquidity of the housing market. When the housing market liquidity freezes out, the low probability of selling vacant houses raises the wedge between the price of occupied housing \(q_{f,t}\) and the one of vacant housing \(q_{r,t}\). As the liquidity margin shrinks, the LTV ratio decreases. Therefore, Equation (12) defines the direct link throughout which the liquidity of the housing market endogenously determines agents’ borrowing capacity. In this
environment the LTV ratio moves over time because of changes in the liquidity of the housing market.

3.5 Decentralized Equilibrium

The families use output net of the labor cost $z_t F (n_{i,t}, h_{i,t}) - n_{i,t} w_t$, the revenues from supplying labor $(1 - l_{i,t}) w_t$, the new amount of borrowing $\frac{d_{i,t+1}}{R}$ and the revenues from selling houses $q_{f,t} P_{b,t} s_{i,t}$, to finance consumption $c_{i,t}$, the searching cost $\kappa s_{i,t}^2$, the repayment of debt $d_{i,t}$, and the purchases of new occupied houses $q_{f,t} P_{b,t} s_{i,t} (1 - h_{i,t})$. Therefore, families’ budget constraint reads

$$
\begin{align*}
  c_{i,t} + \kappa s_{i,t}^2 + q_{f,t} P_{b,t} s_{i,t} (1 - h_{i,t}) + d_{i,t} &= \left[ e^{z_t} F (n_{i,t}, h_{i,t}) - n_{i,t} w_t \right] + \ldots \\
  \ldots + (1 - l_{i,t}) w_t + q_{f,t} P_{s,t} v_t + \frac{d_{i,t+1}}{R}.
\end{align*}
$$

(13)

Hereafter, I focus on a symmetric competitive equilibrium. Since families are all ex-ante identical and there is no source of idiosyncratic uncertainty, families face the same budget and borrowing constraint, and take identical optimal choices. Therefore, I drop the subscripts from all the variables of the model.

The states of the families’ problem are given by its stock of occupied houses $h_t$, the level of debt $d_t$, aggregate bond holdings $D_t$, the aggregate stock of occupied houses $H_t$ and finally the level and volatility of productivity, $z_t$ and $\sigma_t$. Since the stock of housing is in fixed supply, families do not need to take in account the stock of vacant houses $v_t$.

As long as prices depends on the aggregate level of bond holdings, and optimal decisions depend on current and future prices, families have to forecast also future aggregate bond holdings. I denote by $\Gamma_D (H, D, z, \sigma)$ the law of motion of aggregate bond holding $D$ perceived by any family, and $\Gamma_H (H, D, z, \sigma)$ is the law of motion of the aggregate stock of housing occupied by the families $H$. Then,
the individual maximization problem is

\[
V(h, d; H, D, z, \sigma) = \max_{c, l, n, s, d'} \left\{ U(c, l, h') + \beta \mathbb{E}_{z', \sigma'|z, \sigma} \left[ V(h', d'; H', D', z', \sigma') \right] \right\}
\]

s.t.  \[c + d + C_h = e^z F(n, h) + \frac{d'}{R} + G_h\] (14)

\[C_h = \kappa s^2 + q_f(H, D, z, \sigma) P_b(H, D, z, \sigma) s (1 - h)\] (15)

\[G_h = q_f(H, D, z, \sigma) P_s(H, D, z, \sigma) v\] (16)

\[h' = (1 - \psi) \left( h + P(H, D, z, \sigma) s (1 - h) \right)\] (17)

\[\frac{d'}{R} + \nu w(H, D, z, \sigma) n \leq q_r(H, D, z, \sigma) h\] (18)

\[D' = \Gamma_D(H, D, z, \sigma)\] (19)

\[H' = \Gamma_H(H, D, z, \sigma)\] (20)

subject to the law of motion for the TFP shocks as described by Equation (4). Equation (14) denotes the budget constraint, Equation (15) defines the total cost of trading housing \(C_h\), Equation (16) is instead the total gain from trading housing \(G_h\), Equation (17) denotes the law of motion of occupied houses, Equation (18) is the borrowing constraint and Equations (19) - (20) stipulate the perceived laws of motion for total bond holdings and occupied housing. Note that in the symmetric equilibrium \(n_t = 1 - l_t\).

Upon observing the states of the economy, agents decide the optimal policy on consumption \(\hat{c}(h, d; H, D, z, \sigma)\), working hours \(\hat{n}(h, d; H, D, z, \sigma)\), the search effort in the housing market \(\hat{s}(h, d; H, D, z, \sigma)\), and the amount of resources to borrow from the foreign investors \(\hat{d}'(h, d; H, D, z, \sigma)\). In equilibrium, the perceived level of aggregate bond holdings \(\Gamma_D(H, D, z, \sigma)\) has to coincide with the individual policy \(\hat{d}'(h, d; H, D, z, \sigma)\), and the same applies for the law of motion of the aggregate stock of occupied housing \(\Gamma_H(H, D, z, \sigma)\). Appendix C reports the definition of equilibrium and the first-order conditions of the problem.
3.6 Nash Bargaining

The price $q_{f,t}$ of an occupied house $h_{i,t}$ which is sold on the frictional market is determined through the following Nash bargaining problem

$$q_{f,t} \equiv \arg \max_{q_{f,t}} \left\{ S(q_{f,t})^\zeta B(q_{f,t})^{1-\zeta} \right\}$$  \hspace{1cm} (21)

s.t. $S(q_{f,t}) = q_{f,t} - \mathbb{E}_t [\Lambda_{t+1} q_{r,t+1}] \geq 0$  \hspace{1cm} (22)

$B(q_{f,t}) = V^H_t - q_{f,t} \geq 0$  \hspace{1cm} (23)

where $S(q_{f,t})$ is sellers’ surplus in case of a transaction, $B(q_{f,t})$ denotes buyers’ surplus, $\zeta$ is sellers’ bargaining power, and $V^H_t$ is the fundamental value that families attribute to a new unit of occupied housing. The expected future price of vacant houses is the outside opportunity for a family member that does not manage to sell its house.

In the symmetric competitive equilibrium each family has the same fundamental value of occupying a house and therefore the identity of the buyer does not matter on the specification of the house price. Indeed, in equilibrium the price of a occupied house is

$$q_{f,t} = \zeta V^H_t + (1-\zeta) \mathbb{E}_t [\Lambda_{t+1} q_{r,t+1}] .$$  \hspace{1cm} (24)

Families’ fundamental value of housing can be derived using the envelope condition on the optimal stock of occupied housing, which yields

$$V^H_t = \psi \mathbb{E}_t \left[ \Lambda_{t+1} \left( P_{s,t+1} q_{f,t+1} + (1-P_{s,t+1}) q_{r,t+1} \right) \right] + \ldots$$

$$\ldots + (1-\psi) \mathbb{E}_t \left[ \Lambda_{t+1} \left( V^H_{t+1} + \frac{U_{h_{t+1}}}{U_{c_{t+1}}} + \epsilon_{z_{t+1}} F_{h_{t+1}} + \frac{\phi_{t+1}}{U_{c_{t+1}}} q_{r,t+1} \right) \right]$$  \hspace{1cm} (25)

where $Y_{x_t}$ denote the derivatives of the function $Y(\cdot)$ with respect the term $x_t$. 

25
and \( \phi_t \) is the Lagrange multiplier associated to the borrowing constraint. The fundamental value of a marginal house bought by a family member can be interpreted as follows. First, with probability \( \psi \) the new homeowner is hit by the mismatch shock and forced to sell the house. The house is successfully matched with a buyer with a probability \( P_{s,t+1} \) and keeps being on sale with the remaining probability. If the family member is not hit by the mismatch shock, then she will effectively occupy the house over the following period. In this case, the family receives the utility service from occupying the house, uses the house as an input in the production function and gains the marginal productivity \( e^{zt+1} F_{ht+1} \). Moreover, the family enjoys the continuation value of owning the house \( V_{H_t+1} \). Finally, the ownership of an additional house increases the collateral value of families’ housing stock, relaxing its borrowing constraint. Thereby, families can access a larger loan, increase consumption and raise its utility level by \( \frac{\phi_{t+1}}{U_{c,t+1}} q_{r,t+1} \).

3.7 Characterization of the Decentralized Equilibrium

**Proposition 1.** In a steady-state equilibrium, the LTV ratio \( \frac{q_{r,t}}{q_{f,t}} \) positively depends on the liquidity of the housing market. **Proof.** See Appendix D.2.

In this model, the LTV ratio is endogenous and depends on the liquidity of the housing market. When the market heats up, the ratio increases and therefore families’ borrowing capacity is relaxed. Analogously, a liquidity freeze tightens the LTV ratio, decreasing families’ borrowing capacity. This result implies that the observed movements in maximum LTV ratios could be partially accounted for by changes in the liquidity of the housing market.

**Proposition 2.** The house price \( q_{f,t} \) negatively depends on the current shadow value of families’ borrowing constraint. **Proof.** See Appendix D.3.

When families become borrowing constrained, they decrease the level of search
effort on the housing market generating a fire sale spiral which is detrimental for house prices $q_{f,t}$. In this environment fire sales negatively affect both families’ collateral value and their LTV ratio, and therefore the fire sale spiral gets propagated even further.

**Proposition 3.** The tightness of the housing market equals family members’ search effort.

The behaviour of the frictional housing market is starkly simplified in a symmetric competitive equilibrium. Indeed, in such an equilibrium every member opts for the same level of search effort, implying that $\int_0^1 (1 - h_{i,t}) s_{i,t} di = (1 - h_t) s_t$. As a result, the equilibrium market tightness becomes

$$\theta_t = \frac{(1 - h_t) s_t}{v_t} = s_t$$

since the total housing stock is an unitary fixed supply. The tightness of the housing market entirely depends on the search effort exerted by buyers. Hence, the housing market is hot as long as the level of effort is high. This result further implies that in each period the probability of selling a house is $P_{s,t} = \theta_t^{1-\gamma} = s_t^{1-\gamma}$. Hot housing market are characterized by a high level of effort from buyers and a high probability of selling a house. The opposite applies in cold markets.

Instead, the probability of buying a house equals $P_{b,t} = \theta_t^{-\gamma} = s_t^{-\gamma}$. Given this probability, the total amount of houses bought by a family member becomes $P_{b,t} s_t (1 - h_t) = s_t^{1-\gamma} (1 - h_t)$, which is increasing in the level of search effort exerted by family members.

**Implication 1.** The frictional housing market generates partial irreversibilities in housing investment.
The price at which families purchase housing $q_{f,t}$ is higher than the expected price at which they sell houses $q_{r,t}$. As in Duffie et al. (2005), the bid-ask spread depends on the presence of the search frictions. This spread implies that the housing investment is partial irreversible (i.e., the marginal gain of disinvestment is lower than the marginal cost of investment) and the degree of irreversibility fluctuates over time as a function of housing market liquidity. Investment is less irreversible in hot housing market.

Implication 2. Partial irreversibilities in investment together with the presence of a decreasing to scale production function allow volatility shocks to have real effects: an increase in volatility freezes housing investment.

Partial irreversibilities in investment coupled together with a decreasing return to scale production function make changes in volatility to have real effects. When it is expensive to reverse investment, family members become cautious and lower their search effort in uncertain times. Thus, a high volatility reduces the liquidity of the housing market. Instead, in a stable macroeconomic environment, agents increase their search effort and the housing market heats up. Decreasing returns to scale are key for this result. Caballero (1991) shows that a higher uncertainty decreases investment only in environment in which asymmetric adjustment costs interact with either imperfect competition or decreasing returns to scale technologies. If profits are convex in demand or costs, then a higher uncertainty actually rises expected profits leading to an investment boom.

4 Quantitative Analysis

I calibrate one period of the model to correspond to one quarter. To understand the quantitative relevance of the link between volatility, liquidity and financial
crises, I estimate the shocks to both the level and the volatility of the aggregate total factor productivity of the U.S. economy using quarterly data from 1947Q2 until 2013Q4. The level and volatility shocks are estimated using Bayesian Sequential Monte Carlo methods.

I calibrate most of the parameters of the model to the values either estimated or used in previous papers. The main parameters which I calibrate to an empirical targets is the cost of searching effort in the housing market. Indeed, in Section 3.7 shows that the probabilities of buying and selling a house in the frictional market depend on the search effort exerted by the families. If the effort was costless, then the search frictions would be offset by an infinitely amount of search effort exerted by family members, and the liquidity of the housing market would be perfect. Therefore, I calibrate the cost of search effort to match the long-run mean of the time on the market of a house on sale.

The model is solved numerically using global methods, which do not rely on approximations based on Taylor expansions around the steady state. Although the algorithm is much more time intensive, it preserves the non-linear dynamics of the model. I refer to Appendix E.4 for the details on the algorithm.

### 4.1 Estimating the Volatility of Total Factor Productivity

In the model the ultimate source of the build-up of risk and burst of financial crises is given by shocks to the level and the volatility of TFP. To understand the quantitative relevance of this mechanism, I take the actual series of level shocks and volatility shocks to TFP from the data. Namely, I derive the Solow residual of the U.S. economy using quarterly data on output, capital and labor from 1947Q2 until 2013Q4 and apply a one sided HP filter with parameter $\lambda = 1600$.\(^{11}\)

\(^{11}\)The estimated series of Solow residual has to be considered just a proxy of the concept of productivity implied by my model. I refer to Appendix E.2 for further discussions on this issue and all the details on the computation of the Solow residual.
As in Section 2, I estimate the process using Bayesian methods. I elicit some unrestrictive priors, and after deriving the likelihood of the process for some given parameters with the Sequential Importance Sampling (SIS) particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et al. (2011), I maximize the posterior likelihood using a random walk Metropolis-Hastings algorithm with 25000 replications, out of which the first 5000 represent burn-in draws. Finally, I recover the historical distribution of the time varying volatility by appealing to the backward-smoothing routine of Godsill et al. (2004).

4.2 Estimation Results

I elicit Beta priors centred around 0.90 for both the autocorrelation coefficients of the level equation $\rho_z$ and the volatility equation $\rho_\sigma$. The implicit assumption is that both the level and the volatility are known to be highly persistent over time. For the degree of stochastic volatility $\eta$, I elicit a Gamma prior with mean 0.315 and standard deviation 0.03 following the posterior estimate of Born and Pfeifer (2013), who derive the stochastic volatility of the Solow residual of the U.S. economy using data from 1970 on. Finally, I define a uniform distribution between $-11$ and $-3$ for the long-run log volatility $\bar{\sigma}$.

Table 1 reports the results of the estimation. I report the median and the 5-th and 95-th quantiles of the posterior distribution of each parameter. I find strong evidence of persistence in both the level and volatility equation. Especially the latter is important. Indeed, the mechanism of the model relies on the existence of a prolonged period of low volatility which fosters a boom in investment and credit, and makes the leverage of the economy to reach high values. Since the autocorrelation of the volatility equals 0.7949, the model is able to reproduce long periods of both high and low aggregate fluctuations. The process is also
characterized by a high degree of stochastic volatility. A one standard deviation increase in the volatility shocks raises the volatility of the innovation to the level of TFP by 

\[(e^{\eta} - 1) \times 100 = 32.4\%\].

The 90% confidence bands show that all the parameters are tightly estimated. Appendix E.3.3 reports further details on the estimation output.

Table 1: Estimation of the Stochastic Volatility of TFP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
<th>5 Percent</th>
<th>95 Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_z)</td>
<td>Beta</td>
<td>0.90</td>
<td>0.10</td>
<td>0.8137</td>
<td>0.7500</td>
<td>0.8734</td>
</tr>
<tr>
<td>(\rho_\sigma)</td>
<td>Beta</td>
<td>0.90</td>
<td>0.10</td>
<td>0.7949</td>
<td>0.6071</td>
<td>0.9065</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Gamma</td>
<td>0.315</td>
<td>0.03</td>
<td>0.2805</td>
<td>0.2362</td>
<td>0.3267</td>
</tr>
<tr>
<td>(\bar{\sigma})</td>
<td>Uniform</td>
<td>-7.00</td>
<td>2.30</td>
<td>-5.3869</td>
<td>-5.5792</td>
<td>-5.2130</td>
</tr>
</tbody>
</table>

Note: \(\rho_z\) denotes the autocorrelation parameter of the level equation, while \(\rho_\sigma\) is the autocorrelation of the volatility equation. \(\eta\) captures the degree of stochastic volatility in the process, and \(\bar{\sigma}\) denotes the long-run log volatility.

4.3 Calibration Exercise

Most of the parameters of the model are targeted to values estimated or used in previous papers. In particular, the calibration closely follows Bianchi and Mendoza (2013). The crucial parameter which is calibrated is the cost of exerting search effort on the housing market, which determines the behavior of the tightness of the market and eventually the level of housing liquidity.

I consider the following utility function for the families

\[ U(c_{i,t}, l_{i,t}, h_{i,t}) = \left( \frac{c_{i,t}^{\xi} h_{i,t}^{1-\xi}}{c_{i,t}^{\xi} h_{i,t}^{1-\xi} - \mu \frac{(1-l_{i,t})^{1+\omega}}{1+\omega}} \right)^{1-\delta} - 1 \]
The parameters $\delta$, $\mu$ and $\omega$ denote the risk aversion, the degree of disutility from working and the inverse of the Frisch elasticity of family members. The parameter $\xi$ governs the substitutability between consumption and housing. This utility function belongs to the class of preferences introduced in Greenwood et al. (1988), and rules out any wealth effect on the labor supply, which would counter-factually lead to an increase in labor supply during a crisis. I set the disutility of work as $\mu = \alpha_n$ to have mean hours that equal 1. Then, the Frisch elasticity is defined as $1/\omega = 1$ and I set the risk aversion $\delta = 2$ as in Bianchi and Mendoza (2013), whereas $\xi = 0.76$ as in Davis and Ortalo-Magné (2011), who find that the share of households’ expenditure in housing is constant over time around a value of 24$. The subjective time discount factor is set to the standard value at the quarterly frequency of $\beta = 0.99$.

I stipulate a decreasing return to scale production function

$$y_t = e^{zt} n_t^{\alpha_n} h_t^{\alpha_h}$$

where $\alpha_n + \alpha_h < 1$. The parameter $\alpha_h$ is calibrated to match the ratio of housing stock value over the GDP. Using data from the Flow of Funds from 1952Q1 until 2013Q4, the ratio of the market value of the real estate of the private nonfinancial sector over GDP is 2.24. In the model, this average is matched by a value of $\alpha_h = 0.11$. Instead, the labor share is set to the standard value of $\alpha_n = 0.64$. Overall, the returns to scale of the technology sum up to 0.75. Finally, the productivity process $z_t$ inherits the data generator process estimated in the previous Section.

I calibrate the gross real interest rate to $R = 1.0065$, that is the value that Bianchi and Mendoza (2013) estimate for the average of the ex-post real interest

---

12The Cobb-Douglas function in consumption expenditures reflects the fact that expenditure shares on housing are constant over time and across metropolitan areas, see Davis and Ortalo-Magné (2011).
rate on three months Treasury Bills over the last three decades. Instead, the working capital coefficient is set to \( \nu = 0.17 \). To compute this value, I use firms’ M1 money holdings to proxy for their working capital. Since two-thirds of the total M1 stock are held by firms, M1 accounts on average for 16% of annual GDP over the period 1959Q1-2013Q4, and the 0.64 labor share defined above, I set \( \nu = (2/3) \times 0.16/0.64 = 0.17 \).

### Table 2: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disutility from work</td>
<td>( \mu = \alpha_n )</td>
<td>Normalization</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>( \omega = 1 )</td>
<td>Bianchi and Mendoza (2013)</td>
</tr>
<tr>
<td>Substitutability consumption/housing</td>
<td>( \xi = 0.76 )</td>
<td>Davis and Ortalo-Magné (2011)</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \delta = 2 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Time discount</td>
<td>( \beta = 0.99 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Share labor</td>
<td>( \alpha_n = 0.64 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Share housing</td>
<td>( \alpha_h = 0.11 )</td>
<td>Ratio real estate value over GDP</td>
</tr>
<tr>
<td>Gross real interest rate</td>
<td>( R = 1.0065 )</td>
<td>Average return Treasury Bills</td>
</tr>
<tr>
<td>Working capital parameter</td>
<td>( \nu = 0.17 )</td>
<td>Ratio M1 over GDP held by firms</td>
</tr>
<tr>
<td>Mismatch shock</td>
<td>( \psi = 0.0278 )</td>
<td>Ngai and Tenreyro (2014)</td>
</tr>
<tr>
<td>Sellers’ matching function parameter</td>
<td>( \gamma = 0.21 )</td>
<td>Genesove and Han (2012)</td>
</tr>
<tr>
<td>Sellers’ bargaining power</td>
<td>( \zeta = \gamma )</td>
<td>Hosios’ condition</td>
</tr>
<tr>
<td>Cost searching effort</td>
<td>( \kappa = 0.78 )</td>
<td>Average TOM house on sale</td>
</tr>
</tbody>
</table>

Note: The table report the calibrated value of all the parameters of the model, except for the DGP of the technology shock. TOM refers to the expected time on the market.

Finally, I calibrate the parameters characterizing the dynamics of the housing
market as follows. I define the mismatch shock to be equal to $\psi = 0.0278$ to match the average stay in a house of 9 years reported by Ngai and Tenreyro (2014). The parameter of the matching function which refers to the houses supplied to the market by the real estate sector is set to $\gamma = 0.21$ following the value estimated in Genesove and Han (2012). The bargaining power of the seller is set such as $\zeta = \gamma$ so that the Hosios (1990) condition holds. Finally, the monetary cost of exerting searching effort in the frictional market is calibrated to match the average time on the market of a house on sale using data from 1963Q1 until 2013, which is 6.21 months. In this way, I find a value of $\kappa = 0.78$.

4.4 Quantitative Results

4.4.1 Frequency and Severity of Financial Crises.

I compare the quantitative performance of the model under the benchmark calibration with three alternative economies. In the first one, which I refer to as the “Search Frictions” economy, there is a frictional housing market but I shut down the stochastic volatility of the model. Thereby, in this environment TFP follows a standard AR(1) process and level shocks provide the only source of exogenous variation. In the second alternative, which I refer to as the “Stochastic Volatility” economy, I consider a stochastic volatility for TFP but I shut down the channel of liquidity. I consider an infinitely efficient matching function which turns the frictional housing market into a Walrasian one. In the third alternative, which I refer to as the “Level Competitive” economy, I shut down both the stochastic volatility of TFP and the search frictions. This environment has only level shocks to TFP and a Walrasian housing market. Note that the addition of stochastic volatility does not alter the unconditional mean of volatility. The presence of a stochastic volatility implies only a time varying pattern of volatility around its unconditional mean. The level of risk is the same in all the scenarios I compare.
Table 3 reports the results of the model on the frequency and the severity of financial crises, on a sample of simulated data over 10,000 periods. I compare the frequency and severity of financial crises implied by the four economies with the actual moments recovered from U.S. data. Consistently with the empirical literature on crises, I define a financial crisis in the model as the state in which aggregate credit falls down by more than one standard deviation. According to this definition, the savings and loan crisis in the mid 1980’s was not a financial crisis. Hence, over the last century, there have been two financial crises, in 1929 and in 2007. As measures of the severity of financial crises, I consider the cumulative drop of output growth and credit growth during a financial crisis.

Table 3: Results

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Search Frictions</th>
<th>Stochastic Volatility</th>
<th>Level Competitive</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Crises</td>
<td>1.09%</td>
<td>0.80%</td>
<td>0.72%</td>
<td>0.67%</td>
<td>2.00%</td>
</tr>
<tr>
<td>Output Drop</td>
<td>−2.46%</td>
<td>−2.13%</td>
<td>−2.01%</td>
<td>−1.89%</td>
<td>−3.65%</td>
</tr>
<tr>
<td>Credit Drop</td>
<td>−6.28%</td>
<td>−4.91%</td>
<td>−4.54%</td>
<td>−4.32%</td>
<td>−7.40%</td>
</tr>
</tbody>
</table>

Note: The output drop and credit drop refer to the fall in output growth and credit growth, respectively, upon the realization of a financial crisis. In the model, a financial crisis correspond to the state in which aggregate credit falls down by more than one standard deviation. The “Benchmark” refers to an economy with stochastic volatility and a frictional housing market. The “Search Frictions” refers to an economy with a frictional housing market and without stochastic volatility. The “Stochastic Volatility” refers to an economy with stochastic volatility and without a frictional housing market. The “Level Competitive” refers to an economy with neither stochastic volatility nor a frictional housing market.

The first column of Table 3 shows that the interaction of a frictional housing market and time varying volatility generates a frequency of financial crises of 1.03%, a drop in output of −2.46% and a drop in credit of −6.26%. Hence, the benchmark model accounts for between 51% of the observed frequency of crises.
It also accounts for 67% of the fall in output and almost 85% of the drop in credit. When I consider the “Level Competitive” economy, in which the housing market is perfectly liquid and the volatility of TFP is constant over time, the frequency of crises equals 0.67% with a drop in output of −1.89% and a credit crunch that equals −4.32%. Thus, the interaction of volatility shocks and search frictions in the housing market raises the probability of experiencing a crisis by around 47%. Volatility shocks and search frictions boost also the drop of output by around 30% and the fall in credit by 45%.

The results of the “Search Frictions” and “Stochastic Volatility” disentangle the contribution of each of the two features on the performance of the benchmark model. Search frictions increase the frequency of crises by almost 16%, while the time varying volatility raises the frequency by 4%. Therefore, the interaction of these two features accounts for the remaining 27% of the difference in the probability of experiencing a crisis between the benchmark economy and the “Level Competitive” one. The same applies for the severity of crises. Search frictions amplify the drop in output by around 13%, while the time varying volatility amplifies the severity of crises by 6%. The interaction of the two features explain the remaining 11% of the decline in GDP growth experienced in the benchmark economy. As far as the drop in credit is concerned, the interaction of the stochastic volatility and the frictional housing market account for a 26% of the differences between the benchmark model and the “Level Competitive” economy.

These results point out that either search frictions or volatility shocks can improve the performance of the model, although falling short in reproducing the characteristics of crises observed in the data. Instead, the interaction of a frictional housing market and volatility shocks accounts for a large fraction of the probability of experiencing a crisis, and its corresponding drop in output and
creidt. This result also adds to the literature on the real effects of a time varying volatility, such as Justiniano and Primiceri (2008), Bloom (2009) and Fernandez-Villaverde et al. (2011), by pointing out that a frictional housing market is an appealing modeling feature for significantly amplifying the propagation of volatility shocks into the economy.

4.4.2 Dynamics of Aggregate Productivity around Financial Crises.

Table 3 shows that allowing aggregate productivity to have a stochastic volatility increases the occurrence of financial crises by around 47%. What is the contribution of the volatility shocks to productivity in the build-up of risk and the burst of financial crises? I plot the dynamics of the level and volatility of TFP around a crisis, as implied by both the model and the data on the 2007 crisis, in Figure 6 to understand the relevance of either type of shocks.

The graphs show that the period preceding a crisis is characterized by a high level and a low volatility of productivity. In the data, the level of productivity is 7% above than its long-run mean three years before a crisis, while the volatility of productivity is around 10% below mean. The model is able to reproduce a similar dynamics, although it slightly over-estimates the magnitude of the swings: the level of productivity is 11% above than its long-run mean three years before a crisis, while the volatility of productivity is around 15% below mean. This prolonged period of high level of productivity with low volatility generates a credit and investment boom which reinforce each other, raising the equilibrium LTV ratio and ultimately boosting households’ leverage. In this way, these realizations of high productivity and low volatility stimulate a build-up of systemic risk in the economy. Indeed, a joint 10% drop in the level of productivity and a rise in volatility of around 22% turn the borrowing constraint into binding and trigger a financial crisis. Hence, a crisis coincides with both a slump
in the level of productivity and a sudden spike in the volatility of productivity which follow a prolonged period of high productivity with low volatility. The sharp rise in volatility amidst a crisis is crucial to originate a sudden freezing of housing liquidity and fire sales in both the house price and the LTV ratio. Overall, Figure 6 indicates that both the level shocks to productivity and the volatility shocks to productivity are a source of financial instability and account for both the build-up of risk and the burst of crises.

Figure 6: Dynamics of Aggregate Volatility around Financial Crises.

(a) Level of Aggregate Productivity

(b) Volatility of Aggregate Productivity

Note: The figure plots the average values of the deviations from the long-run mean of the level $z_t$ (Panel a) and the volatility $\sigma_t$ (Panel b) of total factor productivity in a 9 year window around financial crises. The solid line denotes the dynamics implied by the model, whereas the dashed line denotes the dynamics in the data. A financial crisis is defined as the state in which aggregate credit drops down by more than one standard deviation.
I plot the dynamics of house price and the loan-to-value ratio around financial crises in Figure 7 to understand the role of the changes in households’ collateral values. Financial crises are preceded by an inflationary spiral in both the house prices and the LTV ratio which relaxes households’ credit limit and raises the level of leverage and therefore systemic risk in the economy. Afterwards, the inflationary spirals are abruptly reversed into deflationary spirals amidst the burst of the financial crisis.

Figure 7: House Price and Loan-to-Value Ratio around Financial Crises.

(a) House Price

(b) Loan-to-Value Ratio

Note: The figure plots the average values of the deviations from the long-run mean of the house price $q_{f,t}$ (Panel a) and the loan-to-value ratio $q_{r,t}$ (Panel b) in a 9 year window around financial crises. The solid line denotes the dynamics implied by the model, whereas the dashed line denotes the dynamics in the data. A financial crisis is defined as the state in which aggregate credit drops down by more than one standard deviation.
In the model a financial crisis is preceded by a boom in house price of around 5% above mean while in the data house prices reach a peak of almost 10% over the two years preceding a financial crisis. The onset of a financial crisis is characterized also by a rally in the LTV ratio, which spikes up around 10% above mean. Then, in the model, both the house price and the LTV ratio start plummeting the year before the crisis, and both reach their trough exactly at the onset of the crisis. In particular, a financial crisis coincides with a house price which is 10% below mean and a LTV ratio which is 15% below mean. So, in two years the house price collapses by around 15% while the LTV ratio drops down by a larger extent, around 25%. This result underlies the key role of the novel mechanism of this paper - the endogenous boom and bust in the LTV ratio - in accounting for the frequency and the severity of financial crises.

### 4.4.3 Time Variation in the Loan-to-Value Ratio.

In this Section I study the time variation in the LTV ratio $q_{r,t}/q_{f,t}$ implied by the model. Table 4 reports the standard deviation of the LTV ratio in the data and in the four economies I consider. I report the average standard deviation of the LTV ratio together with the standard deviations conditional on whether the economies is in normal times or in crisis times. To derive the data counterpart of the LTV ratio of my model, I follow Jermann and Quadrini (2012). First, I log-linearize the collateral constraint defined in Equation (12). I assume that the collateral constraint is always binding and derive the series of the LTV ratio as the residual once I substitute each variable with its observable counterpart in the data. I take data on employment, wage, total liabilities and the market value of real estate. In this way, I obtain the value of the LTV ratio over time, a series which Jermann and Quadrini (2012) refer to as an exogenous financial shock.\(^\text{13}\)

\(^{13}\)The standard deviation of the series does not change in case either I consider only the data on real estate and liabilities of the household sector or if I consolidate the household sector together with the non-financial
Table 4: Standard Deviation LTV Ratio

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Search Frictions</th>
<th>Stochastic Volatility</th>
<th>Level Competitive</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1.73%</td>
<td>1.16%</td>
<td>0%</td>
<td>0%</td>
<td>3.48%</td>
</tr>
<tr>
<td>Normal Times</td>
<td>1.50%</td>
<td>1.01%</td>
<td>0%</td>
<td>0%</td>
<td>3.41%</td>
</tr>
<tr>
<td>Crisis Times</td>
<td>4.02%</td>
<td>2.35%</td>
<td>0%</td>
<td>0%</td>
<td>5.52%</td>
</tr>
</tbody>
</table>

Note: The LTV ratio is the ratio between the reservation value and the actual price of housing, $\frac{q_{r,t}}{q_{f,t}}$. Crisis Times refer to the year preceding and the one following a financial crisis. In the model, a financial crisis correspond to the state in which aggregate credit falls down by more than one standard deviation. The “Benchmark” refers to an economy with stochastic volatility and a frictional housing market. The “Search Frictions” refers to an economy with a frictional housing market and without stochastic volatility. The “Stochastic Volatility” refers to an economy with stochastic volatility and without a frictional housing market. The “Level Competitive” refers to an economy with neither stochastic volatility nor a frictional housing market.

Table 4 shows that as long as in the model the housing market is perfectly liq-

uid and $q_{r,t}$ equals $q_{f,t}$, the LTV ratio is constant and equals 1. This is case in all the economies without search frictions. Instead, once I allow for a frictional hous-

ing market, the LTV ratio changes over time. In the “Search Frictions” economy, the standard deviation of the ratio is 1.16%. It equals 1.01% in normal times and it peaks up to 2.35 in crisis times. When I add volatility shocks, the standard deviation becomes 1.73%, with a value of 1.50% in normal times and 4.02% in crisis times. These results show that volatility shocks amplify the variation in the borrowing margin by around 55% on average, and account for roughly half of the observed standard deviation of the borrowing margin. Moreover, Table 4 shows that although the model falls short in accounting for the volatility of LTV ratios in normal times, it provides a much better approximations in crisis times. The benchmark model accounts for around 73% of the standard deviation of LTV
ratios amidst a financial crisis. Indeed, volatility shocks do not generate much variation in LTV ratios in good times. Instead, when the households’ borrowing constraint becomes binding, changes in the level and volatility of TFP trigger a Fisherian deflation spiral in the house price and housing liquidity which amplifies the fluctuations in the LTV ratio.

Overall Table 4 shows that volatility shocks can be accounted for as a possible foundation of the financial shocks à la Jermann and Quadrini (2012), especially in crisis times. Hence, this model provides a quantitative theory of time varying LTV ratios which can be tested using data on housing market liquidity.

Moreover, in the model the changes in LTV ratios are driven by credit demand motives, because is no role for credit supply in the form of banks. The implications of this result are twofold. First, this evidence suggests that financial shocks should not necessarily be interpreted as if they were originated in the financial sector. Second, the findings of this paper can help reconciling different views on the cause of the last recession. Indeed, through the lenses of this paper, the fact the drop in investment, credit and employment amidst the Great Recession can be accounted for by a large negative financial shock - as shown in Jermann and Quadrini (2012) and Gilchrist and Zakrajsek (2012) - is not necessarily counterfactual with respect to the possibility that the recent credit crunch has been triggered by a severe contraction in credit demand due to the deterioration of households’ balance sheets, as documented by Mian and Sufi (2009, 2011).

5 Concluding Remarks

In this paper I document a new stylized fact on the dynamics of aggregate volatility around financial crises consistent with the financial instability hypothesis of Minsky (1992) and the volatility paradox of Brunnermeier and Sannikov (2014).
I find that the volatility of TFP is around 10% below trend over the two years preceding a financial crisis, before peaking up to 13% above trend amidst the burst of the crisis.

I propose a model in which shocks to the volatility of total factor productivity are a source of financial instability and account for both the build-up of risk and the burst of financial crises. Volatility shocks are propagated into the real economy by the liquidity of housing, which in the model is determined by search frictions in the real estate market. The liquidity of the housing market determines households’ maximum LTV ratio. Households can access to a higher LTV ratio when the housing market is more liquid.

I show that volatility shocks affect the frequency of crises by changing the liquidity of the collateral. Moreover, financial crises are characterized by deflationary spirals in both the house price and the LTV ratio, a novel mechanism which amplifies the magnitude of the credit crunch. Volatility shocks are the main source of variation of the LTV ratios, and provide a rationale for the financial shocks à la Jermann and Quadrini (2012).

Finally, in this paper the whole dynamics of credit boom and bust does not hinge on the presence of banks. The build-up of risk and the burst of crises is driven only by credit demand motives. Yet, the model generates dynamics in the LTV ratios which are observationally equivalent to a financial shock due to a breakdown in credit supply. This evidence supports the findings of Mian and Sufi (2009, 2011), who point out that the deterioration of the balance sheet of the households, rather than the one of the financial intermediaries, has triggered the Great Recession. These results also warn policy-makers in interpreting shifts in LTV ratios as entirely driven by changes in credit supply.
References


**Mendoza, E., and V. Quadrini.** 2010. Financial Globalization, Financial Crises


A Data

A.1 Aggregate Volatility and Financial Crises

I build a panel of 20 developed countries from 1980 until 2013. Extending the panel back to the 60’s or 70’s does not alter the results because in those years the 20 developed countries under investigation experienced almost no financial crisis. The countries covered are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States.

Financial Crises: I take the dates of financial crises from multiple sources, that is, Bordo et al. (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. I follow most of the dating procedure used in Schularick and Taylor (2012) and Jorda et al. (2013b).

Recessions: The dates of recessions are instead given by the OECD recession indicators. For the United States, I follow the dates provided by NBER. The dates of crises and recessions by country are reported in Table A.1.

Total Factor Productivity: I take the series of TFP from the Penn World Tables 8.0. TFP is computed as the residual of real GDP minus the capital stock times the complement to one of the share of labour compensation on GDP minus the total level of labor force (employment times average annual hours worked by persons engaged) multiplied by the share of labour compensation. The nominal variables are normalised at constant 2005 national prices.

Stock Market Volatility: The measure of aggregate volatility is based on the volatility of stock market returns. For each of the 20 countries of the panel, I consider the representative stock market index, I take daily returns and compute a measure of dispersion (either the variance or the interquantile range) within a period (either a year or
### Table A.1: The Dates of Financial Crises and Recessions

<table>
<thead>
<tr>
<th>Country</th>
<th>Financial Crises</th>
<th>Recessions</th>
</tr>
</thead>
</table>

Note: The dates of financial crises come from Bordo et al. (2001), Caprio and Klingebiel (2003), Reinhart and Rogoff (2009), Laeven and Valencia (2012), Schularick and Taylor (2012), Jorda et al. (2013b). Financial crises are defined as credit crunches in which the financial sector experiences large losses and bank runs, that eventually lead to a spike in bankruptcies, forced merged and government intervention. The dates of recessions come from OECD recession indicators.

The stock market indexes are the following: MSCI for Australia, MSCI for Austria, MSCI for Belgium, TSX for Canada, MSCI for Denmark, MSCI for Finland, MSCI for France, DAX for Germany, ATHEX for Greece, MSCI for Ireland, MSCI for Italy, NIKKEI for Japan, MSCI for Netherlands, MSCI for Norway, MSCI for Portugal, MSCI for Spain, MSCI for Sweden, MSCI for Switzerland, FTSE for the UK, DJIA for the US. The source of the data is Datastream.

**Credit to the Private Nonfinancial Sector:** I take the series on private credit from the “Long Series on Total Credit and Domestic Bank Credit to the Private Nonfinancial Sector” of the Bank for International Settlements. For each country, I take the adjusted for breaks nominal quarterly series. I take the series in which the lending sector is any sector and the borrowing sector is the private nonfinancial sector. Real values are
derived by dividing the credit series by the CPI. Annual observations are computed by averaging the quarterly values within a year.

**Gross Domestic Product:** I take the series of real GDP for the United States from the Bureau of Economic Analysis, series ID GDPC1. For all the other countries, I take the series of nominal GDP from the “Main Economic Indicators” database of the OECD. I compute the real series by dividing the nominal GDP series by the CPI.

**House Prices:** Real house prices are mostly taken from the International House Price database of FED Dallas, which is borrowed from Mack and Martinez-Garcia (2011). For Austria, Greece and Portugal, I have taken the quarterly series of house prices from the Property Price Statistics of the Bank for International Settlements (BIS). For Austria, I consider the series of “Residential Property Prices, All Flats (Vienna), per square meter”, for Greece I consider the series of “Residential Property Prices, All Flats (Other Cities), per dwelling”, and for Portugal I consider the series. The real annual prices are taken by deflating with the according CPI series the nominal series, which has been aggregated at the annual level by taking the average over the four quarterly observations per year. For Portugal, I take the monthly series from the Property Price Statistics of the BIS, considering the series of “Residential Property Prices, All Dwellings, per square meter”. The annual series is computed by taking the average over the twelve observations per year.

### A.2 SVAR: Volatility Shocks and the Housing Market

The VAR is estimated using with monthly data from January 1963 until December 2013 on the level of S&P 500 returns, an indicator of volatility, the Federal Funds Rate, the consumer price index, industrial production and three variables on the housing markets related to price, quantity and liquidity. Each series but the volatility indicator is taken in logarithm and detrended with a band-pass filter that removed frequencies below 18 months and above 96 months. The VAR includes a set of 12 lags.

**S&P 500 returns:** I take the logarithmic returns of the series of S&P 500 Stock Price Index provided by S&P Dow Jones Indices LLC.
Indicator of Volatility: The indicator of volatility is borrowed by Bloom (2009). The measure of volatility is an indicator function which equals one in the events in which the VIX index (or the volatility of daily returns within a month in case the VIX data is not available) is at least 1.65 standard deviations above its long run trend, as proxied by the HP-filtered trend.

Federal Funds Rate: The series is the Effective Federal Funds Rate provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is FEDFUNDS.

Consumer Price Index: The series is the Consumer Price Index for All Urban Consumers: All Items provided by the Bureau of Labor Statistics. The FED-FRED indicator code is CPIAUCSL.

Industrial Production: The series is the Industrial Production Index provided by the Board of Governors of the Federal Reserve System. The FED-FRED indicator code is INDPRO.

House Price: The series is the Median and Average Sales Prices of New Homes Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is MSPNHSUS. In the robustness checks, I also use the series of the Conventional Mortgage Home Price Index provided by Freddie Mac, which starts in January 1975.

Quantity of Houses Sold: The series is the Number of Houses Sold provided by the Census Bureau. The series refers to new, single-family houses only. The FED-FRED indicator code is HSN1F.

Liquidity of the Housing Market: The series is the Monthly Supply of Home provided by the Census Bureau. The series refers to new, single-family houses only. The series indicates the expected time of the market of houses put up on sale. The FED-FRED indicator code is MSACSR.
B Dynamics around Crises and Recessions

Figure B.1 plots the dynamics around financial crises and recessions of credit growth, GDP growth, the house price growth and the level of the Solow residual. Panel (a) of Figure B.1 shows that credit growth is much more volatility around financial crises than around recessions. Moreover, financial crises are preceded by a credit boom in which credit grows around 2% above trend. The trend is reversed upon the burst of the crisis, after which credit growth becomes highly negative. Instead, the dynamics around recessions do not present sizeable deviations from the long-run mean of credit growth. An analogous dynamics characterize also the GDP, the house price growth and the level of the Solow residual, as depicted in Panel (b), (c) and (d). This evidence supports the view of Reinhart and Rogoff (2009), Mendoza and Terrones (2012), Schularick and Taylor (2012), and Jorda et al. (2013a,b) that financial crises are booms gone bust.

Figure B.1: Dynamics around Crises and Recessions.

Note: The figure plots the median values of cross-country annual growth rates of real credit to the private non-financial sector (Panel a), real GDP growth rates (Panel b), real house price growth (Panel c) and the level of the Solow residual (Panel d) measured in log differences from the long-run mean - around recessions and financial crises (9 year window). The continuous line indicates the dynamics around financial crises, while the dashed line refers to recessions. The dates of financial crises are taken from Reinhart and Rogoff (2009). Recessions are derived from the OECD recession indicators.
Figure B.2 shows that the dynamics of volatility around financial crises and recessions are not altered when either computing volatility as the median values of the deviations of the Solow residual from the trend (instead of the mean as in Figure 1), or when excluding the recent financial crises episodes.

Figure B.2: Different Measures of Aggregate Volatility.

Note: The figure plots the dynamics of aggregate volatility around financial crises and recessions (9 year window). The continuous line indicates the dynamics around financial crises, while the dashed line presents the dynamics around recession. In Panel (a) aggregate volatility is measured as the median values of the deviations from the trend of the stochastic volatility of countries’ total factor productivity. In Panel (b) aggregate volatility is measured as the median values of the deviations from the trend of the stochastic volatility of countries’ total factor productivity over the period 1980-2006, therefore excluding the recent financial crisis. The dates of financial crises are taken from Reinhart and Rogoff (2009). Normal recessions are derived from the OECD recession indicators.

B.1 SVAR and the House Price

Figure B.3 shows that the impulse response functions of the housing market variables do not change even when considering a different measure of the house price, that is, the CMHPI series from Freddie Mac.
Figure B.3: Volatility Shocks and the Housing Market.

(a) Volatility  
(b) House Price

(c) House Sales  
(d) Time on the Market

Note: VAR estimated from January 1975 to December 2013. The dashed lines are 1 standard-error bands around the response to a volatility shock. The coordinates indicate percent deviations from the baseline.

C Characterization of the Equilibrium

C.1 Definition of Decentralized Equilibrium

In this environment, a recursive decentralized equilibrium is defined by the individual value function $V(h, d; H, D, z, \sigma)$ and optimal policy functions $\{\hat{c}(h, d; H, D, z, \sigma), \hat{n}(h, d; H, D, z, \sigma), \hat{s}(h, d; H, D, z, \sigma), \hat{d}'(h, d; H, D, z, \sigma)\}$, pricing functions for occupied housing $q_f(H, D, z, \sigma)$, vacant housing $q_r(H, D, z, \sigma)$ and labor $w(H, D, z, \sigma)$, probabilities of selling and buying a house $P_s(H, D, z, \sigma)$ and $P_b(H, D, z, \sigma)$, and a perceived law of motion for aggregate bond holdings $\Gamma_D(H, D, z, \sigma)$ and occupied housing $\Gamma_H(H, D, z, \sigma)$ such that:

1. Given the pricing functions $q_f(H, D, z, \sigma)$, $q_r(H, D, z, \sigma)$ and $w(H, D, z, \sigma)$, the
The probability of selling and buying a house, \( P_s (H, D, z, \sigma) \) and \( P_b (H, D, z, \sigma) \), and the law of motions of aggregate bond holdings \( \Gamma_D (H, D, z, \sigma) \) and aggregate occupied housing \( \Gamma_H (H, D, z, \sigma) \), the families’ problem is solved by \( V (h, d; H, D, z, \sigma) \), \( \hat{n} (h, d; H, D, z, \sigma), \hat{s} (h, d; H, D, z, \sigma) \), \( \hat{d}' (h, d; H, D, z, \sigma) \). 

2. The housing markets clear, the probability of buying a house is 
\[
P_b (H, D, z, \sigma) = \frac{\hat{s} (h, d; H, D, z, \sigma) [(1 - h) \hat{s} (h, d; H, D, z, \sigma)]^{1 - \gamma} (1 - h)^\gamma}{(1 - h) \hat{s} (h, d; H, D, z, \sigma)},
\]
the probability of selling a home is 
\[
P_s (H, D, z, \sigma) = \frac{[(1 - h) \hat{s} (h, d; H, D, z, \sigma)]^{1 - \gamma} (1 - h)^\gamma}{1 - h},
\]
where the prices of occupied and vacant housing are determined by Equation (24) and (8), respectively. 

3. The labor market clears at the equilibrium wage \( w (H, D, z, \sigma) \). 

5. The perceived law of motion of aggregate bond holdings coincide with the actual one, that is, \( \Gamma_D (H, D, z, \sigma) = \hat{d}' (h, d; H, D, z, \sigma) \). 

6. The perceived law of motion of the aggregate stock of occupied houses coincide with the actual one: \( \Gamma_H (H, D, z, \sigma) = (1 - \psi) (h + P_b (H, D, z, \sigma) \hat{s} (h, d; H, D, z, \sigma) (1 - h)) \).

### C.2 First Order Conditions

The first order conditions of the problem yield the optimal choices on the supply of working hours, the number of workers to hire, housing investment and borrowing:

\[
w_t = \frac{U_{lt}}{U_{ct}},
\]
\[
z_t F_{nt} = w_t \left[ 1 + \frac{\phi \mu}{U_{ct}} \right]
\]
\[ q_{f,t} + \frac{2\kappa s_t}{P_{h,t}(1 - h_t)} = \psi \mathbb{E}_t \left[ \lambda_{t+1} \left( P_{s,t+1} q_{f,t+1} + (1 - P_{s,t+1}) q_{r,t+1} \right) \right] + \ldots \]
\[ \ldots + (1 - \psi) \mathbb{E}_t \left[ \lambda_{t+1} \left( V_{t+1}^H + \frac{U_{h,t+1}}{U_{c,t+1}} + e^{zt+1} F_{h,t+1} + \frac{\phi_{t+1}}{U_{c,t+1}} q_{r,t+1} \right) \right] \] (C.3)

\[ U_{c,t} = \beta R \mathbb{E}_t \left[ U_{c,t+1} \right] + \phi_t \] (C.4)

where \( Y_{x,t} \) denotes the derivatives of the function \( Y(\cdot) \) with respect the term \( x_t \), and \( \phi_t \) is the Lagrange multiplier associated to the borrowing constraint of the families.

The Equation (C.1) is the standard condition for the optimal labor supply. Instead, the optimal labor demand (C.2) is distorted by the presence of the Lagrange multiplier associated to the borrowing constraint \( \phi_t \). In the states in which the borrowing constraint binds, the multiplier \( \phi_t \) is positive, and the shadow price of the borrowing constraint defines a wedge above the marginal cost. Hence, when a family is borrowing constrained, the cost of hiring labor force de-facto increases, forcing the families to reduce the number of workers hired and the overall level of production.

The Equation (C.3) represents the equilibrium conditions for the search effort on the frictional market. It stipulates that in equilibrium the overall cost of searching for a house equal its marginal gain. The cost is the sum of the searching cost and the house price. The gain is the sum of the production dividends, the utility services received from occupying the house, the extra amounts of resources obtained by relaxing the borrowing constraint with an additional unit of collateral and the continuation value of owning a house. This term also considers the event in which the member is hit by a mismatch shock and forced to sell the house.

Finally, the Equation (C.4) characterizes the optimal choices of bonds. Again, the borrowing constraint adds an extra-financing cost \( \phi_t \) which increases the actual repayment cost. Therefore, in the states in which the borrowing constraint binds, households de-facto incur in an interest rate that is above the one charged by foreign investors.
D Proofs

D.1 Equilibrium Borrowing Constraint

The derivation of the equilibrium borrowing constraint closely follows Bianchi and Mendoza (2013). The borrowing constraint arises in equilibrium as an incentive compatibility constraint which grounds on a limited enforceability of debt, that is, families lack of commitment to repay their debt. I consider the incentive compatibility constraint which yields zero expected profits for the lenders in case they seize families’ collateral, and ensures that families do not default. I consider the following environment:

1. Loans are signed with lenders in a competitive environment;
2. Financial contracts are not exclusive;
3. There is no informational friction between lenders and families;
4. Families borrowing during the first stage of each period of the model, that is, just after the realization of the shocks, and before production takes place;
5. Families lack of commitment in repaying the debt only during the first stage of the problem;
6. If families renege on their debt, the stock of occupied housing \( h_{i,t} \) is seized by the lenders at the beginning of the fourth stage, that is, defaulting families can still use their stock of occupied housing for production and enjoy its utility services;
7. Lenders immediately sell the liquidated housing to a real estate sector at the beginning of the fourth stage;
8. The real estate sector consists of a continuum of real estate agencies;
9. Each family owns a diversified stake in the real estate sector;
10. There is free entry in the real estate sector, which is further perfectly competitive;
11. The real estate sector buys the liquidated houses from the lenders and puts them up on sale on the frictional market;
12. The real estate sector do not use the stock of liquidated houses either as a production input or as a collateral asset, and does not enjoy any utility service of housing;

13. After reneging on debt, families can immediately access again financial market at no penalty, and can purchase again its housing stock at competitive prices.

In this environment, in case a family defaults on its current level of debt, the lenders lose an amount of resources that equals $\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t}$, and gain $q_{r,t} h_{i,t}$ from selling the liquidated housing to the real estate sector. Hence, in equilibrium lenders will not require a collateral value larger than $q_{r,t} h_{i,t}$.

On the other hand, from a family perspective, the gain of defaulting equals $\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t}$ while its cost is $V_{i,t}^H h_{i,t}$, that is, the value that families attribute to the stock of housing seized by the lenders. Since $V_{i,t}^H h_{i,t} \geq q_{r,t} h_{i,t}$, families will always decide to repay back their debt. Thus, the borrowing constraint

$$\frac{d_{i,t+1}}{R} + \nu w_t n_{i,t} \leq q_{r,t} h_{i,t}$$

ensures that lenders do not make ex-ante profits on a defaulting family and that families do not default in equilibrium. In this way, the real estate sector does not operate on an equilibrium path.

### D.2 Proof of Proposition 1.

The loan-to-value ratio $\frac{q_{r,t}}{q_{h,t}}$ equals

$$\frac{q_{r,t}}{q_{f,t}} = P_{s,t} + (1 - P_{s,t}) E_t [\Lambda_{t+1} q_{r,t+1}]$$

In a steady-state equilibrium, the loan-to-value ratio equals

$$\frac{q_r}{q_f} = \frac{P_s}{1 - (1 - P_s) \beta}$$
since \( \Lambda_{t+1} = \beta \frac{U_{c,t+1}}{U_{c,t}} \), and \( U_{c,t} = U_c \) along the steady-state. Thus, the derivative of the loan-to-value ratio with respect to a change in the current level of the liquidity of the frictional housing market, measured in terms of probability of selling a house is

\[
\frac{\partial q_r}{\partial P_s} = \frac{1 - \beta}{[1 - (1 - P_s) \beta]^2} > 0 \quad \forall \beta \in (0, 1), P_s \in (0, 1)
\]

D.3 Proof of Proposition 2.

I use the equation of house price \( q_{w,t} \) given by the condition \((24)\) to characterize the expected equity premium associated to the investment in housing

\[
E_t [R_{l+1}^{ep}] = E_t [R_{l+1}^{h} - R]
\]

where \( R_{l+1}^{h} = \frac{e^{z_{t+1} F_{h,t+1} + q_{f,t+1}}}{q_{f,t}} \) denotes the cum-dividend return on housing investment. The equity premium reads

\[
E_t [R_{l+1}^{ep}] = \frac{1}{E_t [\Lambda_{t+1}]} \left\{ \begin{array}{c}
\phi_t \frac{U_{c,t}}{U_{c,t+1}} \\
\frac{\Lambda_{t+1} \Delta q_{f,t+1}}{q_{f,t+1}} \\
- E_t [\Lambda_{t+1} \Delta q_{f,t+1} \frac{Z_{t+1}^{H}}{q_{f,t+1}}] \\
- E_t [\Lambda_{t+1} \Delta q_{f,t+1} \frac{\phi_{t+1}}{U_{c,t+1} q_{f,t+1}}] \\
- E_t [\Lambda_{t+1} \Delta q_{f,t+1} \frac{\phi_{t+1}}{U_{c,t+1} q_{f,t+1}}]
\end{array} \right\}
\]

where

\[
\Omega_{t+1} = \zeta \psi E_t \left[ \Lambda_{t+1} \Delta q_{f,t+1} (1 - P_{s,t+1}) \left( 1 - \frac{q_{r,t+1}}{q_{f,t+1}} \right) \right]
\]

The formula above highlights that the premium, and therefore the house price, depends on collateral values and search frictions. Indeed, in standard asset pricing conditions, the equity return depends only the level of risk, that is, the covariance between families’
stochastic discount factor and the equity premium. Here, the equity premium is also increasing in the current Lagrange multiplier associated to the borrowing constraint $\phi_t$ and the search frictions as measured by the margin of the borrowing constraint. On one hand, when the borrowing constraint binds, the equity premium rises, and the house price $q_{f,t}$ declines. Thus, borrowing constrained families that are forced to fire sales depress the current house price. On the other hand, when the future probability of selling houses in the frictional market decreases, tightening the borrowing margin, the equity premium rises and therefore the house price declines. So, a liquidity freeze lowers the house price. In either case, there is also an indirect effect. The high equity return in the states in which the borrowing constraint binds and the liquidity of the housing market is low tends to be associated by disproportionately higher levels of families’ marginal utility of consumption. This comovement further depresses the house price.
E Supplementary Appendix

E.1 The Role of Time-Varying Volatility

Although the behavior of volatility is related with the realizations of financial crises, the relationship could be statistically insignificant once controlling for other macroeconomic variables. Therefore, I provide panel data estimates in which I check the role of volatility once considered together with a wide set of covariates. Namely, I run an OLS linear probability model where the dependent variable is a dummy that equals 1 in the year of a financial crisis. The benchmark regression then considers as independent variables four lags of volatility, house price growth, credit growth and GDP growth. I also include countries’ characteristics such as the ratio of population with age above 65 years and the degrees of openness and financial development. Finally, I always control for country and year fixed effects. As in Fogli and Perri (2013), I include the share of the population aged 65 or above because Jaimovich and Siu (2009) show that demographic factors do affect the level of volatility in G7 countries. The presence of countries’ openness is due to Di Giovanni and Levchenko (2009), who show instead that volatility is positively affected by the degree of trade openness of a country. Instead, Bekaert et al. (2006) show that a high degree of financial development is positively related to lower consumption growth volatility. Finally, the presence of country and fixed effects controls for unobserved country characteristics and international aggregate shocks that could affect the frequency of financial crisis. The results reported in Column (1) of Table E.2 show the coefficient of the third lag of volatility is negative and significant at the 5% confidence level. Hence, a low level of volatility is related with the realization of a financial crisis three years afterwards. In addition, a one percent decrease of volatility below its long-run mean increases the probability of experiencing a crisis three years afterwards by 6.34%. Therefore, the effect of low volatility on the frequency of financial crisis is both statistically and economically significant. Also the two years lag of house price growth is statistically significant, though just at the 10% level. In this case, a one percent increases of real estate price above its long-run mean increases the probability
of experiencing a crisis three years afterwards by 45.10%. Thus, Column (1) supports the hypothesis that both low volatility and house price boom are related with financial crises, even once controlling for other covariates. In Column (2) I define the product of volatility and house price growth as a new variable to test whether the interaction of low fluctuations and a real estate boom is also related with the burst of a crisis. I add the new interaction term to the set of covariates of Column (1). Column (2) shows that the third lag of volatility keep being significant at the 5% level, and its point estimate has just slightly decreased. Surprisingly, the addition of the interaction makes the role of house price growth per se to be statistically insignificant. Instead, the third lag of the interaction is significant at the 5% level. A one percent decrease of the level of the interaction below its long-run mean increases the probability of experiencing a crisis three years afterwards by 51.74%. Note that these results are robust even when controlling for further covariates, such as the level of interest rates in Column (3) and the inflation rate in Column (4).

Table E.3 replicates the same analysis using as dependent variable a dummy that equals 1 in the dates of recessions. The results show that in this case there is no significant relationship between low level of volatility, house price booms and the probability of experiencing a recession.

E.2 Computation of the Solow Residual

I approximate total factor productivity of the U.S. economy with the series Solow residual computed following Cooley and Prescott (1995) and especially Rios-Rull and Santaulalia-Llopis (2010).

E.2.1 Data

To derive the Solow residual, I take the following series from 1948Q1 until 2012Q4:

- \( UCI = RI + CP + NI + GE \) is the unambiguous capital income, in which

  1. \( RI \) is rental income from Table 1.12 of the National Income and Product
Accounts of the Bureau of Economic Analysis (NIPA-BEA)

2. $CP$ is corporate profits from Table 1.12 of NIPA-BEA

3. $NI$ is net interests from Table 1.12 of NIPA-BEA

4. $GE$ is current surplus of government enterprises from Table 1.12 of NIPA-BEA

- $DEP$ is consumption of fixed capital and equals the difference between Gross National Product and Net National Product from Table 1.7.5 of NIPA-BEA

- $UI = UCI + DEP + CE$ is the unambiguous income, in which
  
  1. $CE$ is compensation of employees from Table 1.12 of NIPA-BEA

- $ACI = \frac{UCI}{DEP} (PI + Tax - Subs + BCTP + SDIS)$ is the ambiguous capital income, where
  
  1. $DEP$ is consumption of fixed capital from Table 1.7.5 of NIPA-BEA
  2. $PI$ is proprietor’s income from Table 1.12 of NIPA-BEA
  3. $Tax$ is taxes on production from Table 1.12 of NIPA-BEA
  4. $Subs$ is subsidies from Table 1.12 of NIPA-BEA
  5. $BCTP$ is business current transfer payments from Table 1.12 of NIPA-BEA
  6. $SDIS$ is the statistical discrepancies, which accounts for statistical wedges between the series of net national product and national income, from Table 1.12 of NIPA-BEA

- $GNP$ is the gross national product from Table 1.7.5 of NIPA-BEA

- Private Fixed Assets is from Table 1.1 and Table 1.2 of the Fixed Asset Tables of the Bureau of Economic Analysis (FAT-BEA)

- Depreciation is the depreciation of private fixed assets from Table 1.3 of FAT-BEA

- Employment is the series ID CES0000000001 from the Current Establishment Survey of the Bureau of Labor Statistics (CES-BLS)

- Average weekly hours is the series ID PRS85006023 from CES-BLS
E.2.2 Methodology

The Solow residual $S_t$ is derived as follows

$$S_t = \log z_t - \text{constant} - g * t$$

where

$$\log z_t = \log y_t - \alpha_n \log n_t - \alpha_k \log k_t$$

and $g$ denotes the long-run growth rate of the economy, $y_t$ denotes the gross domestic product of the economy, $n_t$ is labor and $k_t$ is capital, and here approximates the housing stock of my model. First, I need to derive the share of labor $\alpha_n$ in the production function. To do so, I assume that $\alpha_n + \alpha_h = 1$, that is, the production function is a constant return to scale. Later on, I discuss the implications of these choices on the interpretation of the Solow residual of my model.

The dynamics of labor share $\alpha_{n,t}$ over time can be computed as

$$\alpha_{n,t} = 1 - \left( \frac{UCI_t + DEP_t + ACI_t}{GNP_t} \right)$$

To derive the Solow residual, I consider the mean of this sequence of labor shares, which I denote as the labor share $\alpha_n = 0.6761$. Now, to derive the Solow residual, beside the measure of GDP $y_t$, I need to construct a series for labor and one for capital. I construct the series of aggregate hours as follows. I take the series of employment and I multiply it for the average weekly hours. Then, I approximate capital with the chain-type quantity index from Table 1.2 in the FAT-BEA and the current-cost net stock in 2009 from Table 1.1 in FAT-BEA. The derived Solow residual is displayed in Figure E.4

A very important disclaimer in this exercise is that the Solow residual I compute is only an approximation of the concept of Solow residual implied by my model. Indeed, the derivation of the Solow residual implemented above requires that markets are perfectly competitive and the production function is a constant return to scale. Instead, in my model either the Cobb-Douglas is characterised by decreasing returns to scale.
or there is monopolistic competition among firms. These features are required in order to let an increase in the volatility of TFP to negatively affect the investment rate of the economy.

E.3 Estimation of the Stochastic Volatility

I closely follow the Web Appendix of Born and Pfeifer (2013) to describe the Sequential Particle Filter and the Particle Smoother to estimate the stochastic volatility of the Solow residual of the U.S. economy.

E.3.1 The Sequential Particle Filter

I consider the following data generator process for the dynamics of

\[
    z_t = \rho_z z_{t-1} + e^{\sigma_t} \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0,1)
\]

\[
    \sigma_t = (1 - \rho_\sigma) \bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta \epsilon_{\sigma,t}, \quad \epsilon_{\sigma,t} \sim N(0,1)
\]

(E.5)
where $\rho_z$ and $\rho_\sigma$ denote the persistence of the level and volatility equation, respectively, $\bar{\sigma}$ is the unconditional mean of the volatility of the process and $\eta$ captures the degree of stochastic volatility of the process. Even when there is no uncertainty on the value of the parameters of the model, there is a filtering problem which to understand whether the observed residual to the level equation is driven by innovations to that equation itself $\epsilon_{z,t}$ or it is due by changes in the stochastic volatility $e^{\sigma_t}$. Since the latter appears non-linearly, I cannot appeal to the Kalman filter to disentangle the two components.

Hence, I use the Sequential Particle filter introduced in Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et al. (2011). Given the data generator process (E.5) for the Solow residual $z_t$, an initial value $z_0$, and using the normality of the innovations $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$, I can factorise the likelihood of the vector stacking the observables $z_t$ up to time $T$, $z^T$, as

$$p(z^T; \Theta) = \prod_{t=1}^{T} \int p(z_t \mid z^{t-1}; \Theta) \, d\sigma_t = \int \frac{1}{e^{\sigma_0} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{z_1 - \rho z z_0 e^{\sigma_0}}{e^{\sigma_0}} \right)^2} \, d\sigma_0 \times \ldots$$

$$\cdots \times \prod_{t=2}^{T} \frac{1}{e^{\sigma_t} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{z_t - \rho z z_{t-1} e^{\sigma_t}}{e^{\sigma_t}} \right)^2} \, d\sigma_t \quad (E.6)$$

where $\Theta$ denotes the vector of the parameters of the data generator process (E.5).

Since there is no analytical counterpart of the likelihood (E.6), the idea is to approximate it numerically with a large number of random draws using Monte Carlo methods. Fernandez-Villaverde and Rubio-Ramirez (2007) and Fernandez-Villaverde et al. (2011) propose an approximation of the filtering density $p(\sigma_t \mid z^T; \Theta)$ through a distribution derived from simulating and re-sampling from the vector of the observable data $z^T$ as follows

$$p(\sigma_t \mid z^T; \Theta) \approx \sum_{i=1}^{N} \omega_i^t \delta_{\sigma_t}(\sigma_t) \quad (E.7)$$

where $i = 1, \ldots, N$ denotes the number of particles used in the approximation, $\delta$ is the
Dirac delta function and $\omega^i_t$ is the relative (probability) weight of each particle $\sigma^i_t$ such that $\sum_{i=1}^{N} \omega^i_t = 1$ and $\omega^i_t \geq 0$. Then, the conditional probability is continuously update from a period $t$ to the following on $t+1$ using sequential resampling methods. Given the data generator process (E.5) and an initial value $p(\sigma_0 \mid z^0; \Theta) = p(\sigma_0; \Theta)$, it is possible to derive a new conditional probability $p(\sigma_1 \mid z^0; \Theta) = p(\epsilon_1) p(\sigma_0 \mid z^0; \Theta)$. In this way, I iteratively compute a sequence of particles $\{\sigma^i_{t+1} \mid t\}_{i=1}^N$ using $N$ draws $\{\sigma^i_t \mid t\}_{i=1}^N$ from the conditional density $p(\sigma_t \mid z^t; \Theta)$ and the random draw of the exogenous shocks $\epsilon^i_t \sim N(0, 1)$. In order for the algorithm to converge to the try likelihood, it is required to re-sample from the simulated particles to update them at every step. Basically, at every step the conditional probability $p(\sigma^t \mid z^{t-1}; \Theta)$ is updated to $p(\sigma^t \mid z^t; \Theta)$. The re-sampled particles $\{\sigma^i_{t|t} \}_{i=1}^N$ are drawn by a distribution in which to every simulated particle $\{\sigma^i_{t-1} \}_{i=1}^N$ is attached a resampling probability

$$\omega^i_t = \frac{p(z_t \mid z^{t-1}, \sigma^i_t \mid t-1; \Theta)}{\sum_{i=1}^{N} p(z_t \mid z^{t-1}, \sigma^i_t \mid t-1; \Theta)}$$ (E.8)

As shown in Fernandez-Villaverde and Rubio-Ramirez (2007), the likelihood can be consistently estimated by

$$p(z^T; \Theta) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{e^{\sigma_0^0 / \sqrt{2\pi}}} e^{-\frac{1}{2} \left( \frac{(z_t - \rho z_{t-1})^2}{\sigma^0_t} \right)} \times \ldots \times \prod_{t=2}^{T} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{e^{\sigma^i_{t-1} / \sqrt{2\pi}}} e^{-\frac{1}{2} \left( \frac{(z_t - \rho z_{t-1})^2}{\sigma^i_{t-1}} \right)}$$ (E.9)

The estimation procedure is carried out through 25000 draws from the random walk Metropolis-Hasting algorithm, in which the first 5000 represent the burn-in draws. For each draw, I use $N = 10000$ particles. Throughout the implementation of the algorithm, the acceptance rate has been set to around 30%. 
E.3.2 The Particle Smoother

Following Born and Pfeifer (2013), I derive the historical distribution of the volatility of TFP using the backward-smoothing algorithm introduced in Godsill et al. (2004). The idea is to sample from the smoothing density

\[
p(\sigma^T | z^T; \Theta) = p(\sigma_T | z^T; \Theta) \prod_{t=1}^{T-1} p(\sigma_t | \sigma_{t+1:T}, z^T; \Theta)
\]

(E.10)

where

\[
p(\sigma_t | \sigma_{t+1:T}, z^T; \Theta) = p(\sigma_t | \sigma_{t+1}, z^T; \Theta) = \frac{p(\sigma_t | z^T; \Theta) f(\sigma_{t+1} | \sigma_t)}{p(\sigma_{t+1} | z^T)}
\]

\[
\propto p(\sigma_t | z^T; \Theta) f(\sigma_{t+1} | \sigma_t)
\]

(E.11)

where \(f(\cdot)\) refers to the law of motion of the stochastic volatility defined in Equation (E.5). Hence, this algorithm applies in a backward way from the last observations of the estimated series of stochastic volatility towards the first observation.

The conditional smoothing density \(p(\sigma_t | \sigma_{t+1}, z^T; \Theta) \propto p(\sigma_t | z^T; \Theta) f(\sigma_{t+1} | \sigma_t)\) can be the approximated by

\[
p(\sigma^t | \sigma_{t+1}, z^t; \Theta) \approx \sum_{i=1}^{N} \omega^i_{t|t+1} \delta_{\sigma^i_t} (\sigma_t)
\]

(E.12)

where

\[
\omega^i_{t|t+1} = \frac{\omega^i_t f(\sigma_{t+1} | \sigma^i_t)}{\sum_{j=1}^{N} \omega^j_t f(\sigma_{t+1} | \sigma^j_t)}
\]

(E.13)

given the weights of the filtering step \(\omega^i_t\). The algorithm starts from the very last observation by deriving a random sample \(\tilde{\sigma}^i_T\) from the conditional distribution \(p(\sigma_T | z^T)\) using weights \(\omega^i_T\). Then, it goes recursively in backward fashion at each date \(t\) to derive new random draws \(\tilde{\sigma}^i_t\) using the weights defined in (E.13).

To derive the historical distribution of the stochastic volatility of TFP, I use \(N = 10000\) particles in the posterior smoothing algorithm.
E.3.3 Estimation Output

This Section presents the output of the estimation of the stochastic volatility for the Solow residual of the U.S. economy. Figure E.5 shows the level of the estimated stochastic volatility of TFP together with one standard deviation error bands. The graph shows that volatility has dramatically decreased starting from the 1980’s onwards. Although the level of volatility has kept at lower levels throughout the last two decades, volatility has experienced several peaks. For example, one of the last peaks of volatility is dated on 2006Q2, which is consistent with the mechanism presented in this paper. Indeed, the burst of financial crisis requires a prolonged period of low volatility which is then followed by a sudden increase in volatility, which does not necessarily have permanent.

Figure E.5: Estimated Stochastic Volatility of the Solow Residual

![Figure E.5](image)

Note: The figure plots the estimated stochastic volatility of the Solow residual of the U.S. economy - using quarterly data from 1948Q2 until 2012Q4 - together with one standard deviation confidence bands. The dashed line denotes the unconditional mean of the volatility.

Figure E.6 and Figure E.7 show the estimated innovations to the level and volatility of TFP $\epsilon z, t$ and $\epsilon \sigma, t$, respectively. The two Figures above show that in the fourth
quarter of 2006, the series of TFP of the U.S. economy has experienced a large negative innovation to the level together with a large positive innovation to the variance.

Figure E.6: Estimated Innovations to the Level of the Solow Residual

![Graph of estimated innovations to the level of the Solow residual of the U.S. economy from 1948Q2 to 2012Q4.]

Note: The figure plots the estimated innovations to the level of the Solow residual of the U.S. economy - using quarterly data from 1948Q2 until 2012Q4.

Figure E.7: Estimated Innovations to the Volatility of the Solow Residual

![Graph of estimated innovations to the stochastic volatility of the Solow residual of the U.S. economy from 1948Q2 to 2012Q4.]

Note: The figure plots the estimated innovations to the stochastic volatility of the Solow residual of the U.S. economy - using quarterly data from 1948Q2 until 2012Q4.
The next Figures show the estimated series of the level and innovations to volatility, the innovations to the level of the Solow residual, the dynamics of the Metropolis-Hastings algorithm in the resampling of parameters of the stochastic volatility process. I further display evidence supporting the fact that the resampling of the parameters has converged toward their ergodic distribution.

Figure E.8: MCMC Draws

Note: The figure plots the 25000 draws from a random walk Metropolis-Hastings algorithm for the estimation of the parameters of the stochastic volatility process for the Solow residual. The first panel refers to $\rho_z$, the persistence of the level equation. The second panel refers to $\rho_\sigma$, the persistence of the volatility equation. The third panel refers to $\eta$, the degree of stochastic volatility. The last panel refers to $\bar{\sigma}$, the long-run volatility.
Figure E.9: Convergence of the MCMC Draws

Note: The figure plots the mean of 20000 draws (burn-in excluded) from a random walk Metropolis-Hastings algorithm for the estimation of the parameters of the volatility process for the Solow residual. The first panel refers to $\rho_z$, the persistence of the level equation. The second panel refers to $\rho_\sigma$, the persistence of the volatility equation. The third panel refers to $\eta$, the degree of stochastic volatility. The last panel refers to $\bar{\sigma}$, the long-run volatility.

Figure E.10: Prior and Posterior Distribution for $\rho_z$

Note: The figure plots the prior distribution (dashed line) and the posterior distribution (solid line) of $\rho_z$, the persistence of the level equation. The posterior distribution has been computed over 20000 draws derived from a random walk Metropolis-Hastings algorithm.
Figure E.11: Prior and Posterior Distribution for $\rho_\sigma$

Note: The figure plots the prior distribution (dashed line) and the posterior distribution (solid line) of $\rho_\sigma$, the persistence of the volatility equation. The posterior distribution has been computed over 20000 draws derived from a random walk Metropolis-Hastings algorithm.

Figure E.12: Prior and Posterior Distribution for $\eta$

Note: The figure plots the prior distribution (dashed line) and the posterior distribution (solid line) of $\eta$, the degree of stochastic volatility. The posterior distribution has been computed over 20000 draws derived from a random walk Metropolis-Hastings algorithm.
Figure E.13: Prior and Posterior Distribution for $\bar{\sigma}$

Note: The figure plots the prior distribution (dashed line) and the posterior distribution (solid line) of $\bar{\sigma}$, the persistence of the level equation. The posterior distribution has been computed over 20000 draws derived from a random walk Metropolis-Hastings algorithm.

### E.4 Computational Algorithm

The computational algorithm finds a solution for a dynamic system composed of 8 equations in 8 unknowns, that is

$$
\left( c_t - \mu \frac{(1 - l_t)^{1+\omega}}{1 + \omega} \right)^{-\delta} = \beta R \mathbb{E}_t \left[ \left( c_{t+1} - \mu \frac{(1 - l_{t+1})^{1+\omega}}{1 + \omega} \right)^{-\delta} \right] + \phi_t \tag{E.14}
$$

$$
\mu (1 - l_t)^{\omega} = w_t \tag{E.15}
$$

$$
z_t \alpha_n (1 - l_t)^{\alpha_n - 1} \eta_t^{\alpha_h} = w_t \left( 1 + \frac{\phi_t \nu}{(c_t - \mu \frac{(1 - l_t)^{1+\omega}}{1 + \omega})^{-\delta}} \right) \tag{E.16}
$$

$$
q_{f,t} + \frac{2 \kappa e_t}{P_{b,t} (1 - h_t)} = V_t^H \tag{E.17}
$$

$$
q_{f,t} = \zeta V_t^H + (1 - \zeta) \mathbb{E}_t [\Lambda_{t,t+1} q_{r,t+1}] \tag{E.18}
$$

$$
q_{r,t} = P_{s,t} q_{f,t} + (1 - P_{s,t}) \mathbb{E}_t [\Lambda_{t,t+1} q_{r,t+1}] \tag{E.19}
$$

$$
\frac{d_{t+1}}{R} + \nu w_t (1 - l_t) \leq q_{r,t} h_t \tag{E.20}
$$

$$
c_t + d_t + \kappa e_t^2 + q_{f,t} P_{b,t} e_t (1 - h_t) = z_t (1 - l_t)^{\alpha_n} h_t^{\alpha_h} + \frac{d_{t+1}}{R} + q_{f,t} P_{s,t} v_t \tag{E.21}
$$
which consists of the four first-order conditions, the price determination in the Nash bargaining problem, the definition of the housing reservation value, the borrowing constraint and the budget constraint. The unknowns are $c_t, l_t, e_t, d_{t+1}, q_{f,t}, q_{r,t}, w_t, \phi_t$, while the state variables are $h_t, d_t, z_t, \sigma_t$. Notice that

$$V_t^H = \psi \mathbb{E}_t \left[ A_{t+1} \left( P_{s,t+1} q_{f,t+1} + (1 - P_{s,t+1}) q_{r,t+1} \right) \right] + \ldots$$

$$\cdots + (1 - \psi) \mathbb{E}_t \left[ A_{t+1} \left( V_{t+1}^H + z_{t+1} F_{ht+1} + \frac{\phi_{t+1}}{U_{ct+1}} q_{r,t+1} \right) \right]$$

$$P_{b,t} = \frac{((1 - h_t) e_t)^{1-\gamma} (v_t)^\gamma}{(1 - h_t) e_t}$$

$$P_{s,t} = \frac{((1 - h_t) e_t)^{1-\gamma} (v_t)^\gamma}{v_t}$$

The algorithm works through the approximation of the three following functions

$$\Gamma_1 = V_t^H$$

$$\Gamma_2 = \mathbb{E}_t \left[ \left( c_{t+1} - \mu \frac{(1 - l_{t+1})^{1+\omega}}{1+\omega} \right)^{-\delta} \right]$$

$$\Gamma_3 = \mathbb{E}_t \left[ A_{t,t+1} q_{r,t+1} \right]$$

First, I guess a value for the approximated functions $\Gamma_1, \Gamma_2, \Gamma_3$ on a two-dimensional grid for the endogenous states $h_t, d_t$ for each of the grid points of the exogenous states $z_t, \sigma_t$. I obtain values outside the grid through bilinear interpolation. Then, I find the value of the 8 unknowns that solve the system \([E.14] - [E.21]\) at each of the grid points and for each of the values of the two exogenous states. In all the grid points, I check whether the borrowing constraint binds - which would imply that $\phi_t > 0$ - or not - such that $\phi_t = 0$. The solution of the dynamic system are then used to recover the implied values of the functions $\Gamma_1, \Gamma_2, \Gamma_3$. I keep iterating until the guesses for $\Gamma_1, \Gamma_2, \Gamma_3$ evaluated at the grid points coincide with the values of the functions recovered by the optimal choices that solve the dynamic system \([E.14] - [E.21]\).
Additional References


Table E.2: Volatility and Financial Crises - OLS Linear Probability Model

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1) Crises</th>
<th>(2) Crises</th>
<th>(3) Crises</th>
<th>(4) Crises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility_{t-1}</td>
<td>0.0065</td>
<td>0.0122</td>
<td>0.0218</td>
<td>0.0119</td>
</tr>
<tr>
<td></td>
<td>(0.0241)</td>
<td>(0.0243)</td>
<td>(0.0244)</td>
<td>(0.0244)</td>
</tr>
<tr>
<td>Volatility_{t-2}</td>
<td>0.0331</td>
<td>0.0349</td>
<td>0.0292</td>
<td>0.0340</td>
</tr>
<tr>
<td></td>
<td>(0.0262)</td>
<td>(0.0265)</td>
<td>(0.0263)</td>
<td>(0.0266)</td>
</tr>
<tr>
<td>Volatility_{t-3}</td>
<td>-0.0635**</td>
<td>-0.0551**</td>
<td>-0.0489*</td>
<td>-0.0547**</td>
</tr>
<tr>
<td></td>
<td>(0.0255)</td>
<td>(0.0258)</td>
<td>(0.0256)</td>
<td>(0.0260)</td>
</tr>
<tr>
<td>Volatility_{t-4}</td>
<td>0.0252</td>
<td>0.0247</td>
<td>0.0295</td>
<td>0.0199</td>
</tr>
<tr>
<td></td>
<td>(0.0222)</td>
<td>(0.0224)</td>
<td>(0.0226)</td>
<td>(0.0227)</td>
</tr>
<tr>
<td>House Price Growth_{t-1}</td>
<td>-0.2934</td>
<td>-0.2226</td>
<td>-0.0898</td>
<td>-0.2209</td>
</tr>
<tr>
<td></td>
<td>(0.2248)</td>
<td>(0.2284)</td>
<td>(0.2286)</td>
<td>(0.2300)</td>
</tr>
<tr>
<td>House Price Growth_{t-2}</td>
<td>0.4510*</td>
<td>0.4308</td>
<td>0.2344</td>
<td>0.4507</td>
</tr>
<tr>
<td></td>
<td>(0.2701)</td>
<td>(0.2732)</td>
<td>(0.2760)</td>
<td>(0.2760)</td>
</tr>
<tr>
<td>House Price Growth_{t-3}</td>
<td>-0.0040</td>
<td>0.0628</td>
<td>0.0978</td>
<td>0.0613</td>
</tr>
<tr>
<td></td>
<td>(0.2525)</td>
<td>(0.2587)</td>
<td>(0.2574)</td>
<td>(0.2608)</td>
</tr>
<tr>
<td>House Price Growth_{t-4}</td>
<td>0.0455</td>
<td>-0.0025</td>
<td>0.0121</td>
<td>-0.0019</td>
</tr>
<tr>
<td></td>
<td>(0.2130)</td>
<td>(0.2160)</td>
<td>(0.2159)</td>
<td>(0.2185)</td>
</tr>
<tr>
<td>House Price Growth × Volatility_{t-1}</td>
<td>-0.2257</td>
<td>-0.2283</td>
<td>-0.1790</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2557)</td>
<td>(0.2562)</td>
<td>(0.2568)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility_{t-2}</td>
<td>-0.1511</td>
<td>0.0723</td>
<td>-0.1611</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2598)</td>
<td>(0.2624)</td>
<td>(0.2611)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility_{t-3}</td>
<td>-0.5174**</td>
<td>-0.4357*</td>
<td>-0.5099**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2515)</td>
<td>(0.2515)</td>
<td>(0.2530)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility_{t-4}</td>
<td>0.0278</td>
<td>0.0224</td>
<td>0.0221</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2331)</td>
<td>(0.2308)</td>
<td>(0.2348)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.3598</td>
<td>0.3694</td>
<td>0.3708</td>
<td>0.3702</td>
</tr>
<tr>
<td>Observations</td>
<td>552</td>
<td>552</td>
<td>551</td>
<td>552</td>
</tr>
</tbody>
</table>

Note: The dependent variable is a dummy variable which equals one at the starting dates of financial crises. Volatility is the volatility of countries’ stock market index, and equals the variance of daily returns within a year. All regressions include country and fixed effects, and control for four lags of real credit growth and real GDP growth, a measure of openness (the sum of import and export ratio to GDP), a measure of financial development (share of private credit by deposit money bank to GDP), and the share of population with more than or equal to 65 years. Column (3) includes four lags of the real interest rate. Column (4) includes four lags of the inflation rate. Standard errors are reported in brackets. ***, ** and * denote significance at the 1%, 5% and 10%, respectively.
Table E.3: Volatility and Recessions - OLS Linear Probability Model

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1) Recessions</th>
<th>(2) Recessions</th>
<th>(3) Recessions</th>
<th>(4) Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility(_{t-1})</td>
<td>0.0131</td>
<td>0.0047</td>
<td>0.0125</td>
<td>0.0082</td>
</tr>
<tr>
<td></td>
<td>(0.0423)</td>
<td>(0.0429)</td>
<td>(0.0434)</td>
<td>(0.0428)</td>
</tr>
<tr>
<td>Volatility(_{t-2})</td>
<td>-0.0352</td>
<td>-0.0369</td>
<td>-0.0346</td>
<td>-0.0292</td>
</tr>
<tr>
<td></td>
<td>(0.0460)</td>
<td>(0.0467)</td>
<td>(0.0469)</td>
<td>(0.0467)</td>
</tr>
<tr>
<td>Volatility(_{t-3})</td>
<td>0.0090</td>
<td>0.0067</td>
<td>0.0051</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.0448)</td>
<td>(0.0455)</td>
<td>(0.0456)</td>
<td>(0.0456)</td>
</tr>
<tr>
<td>Volatility(_{t-4})</td>
<td>0.0018</td>
<td>0.0070</td>
<td>-0.0026</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
<td>(0.0391)</td>
<td>(0.0400)</td>
<td>(0.0402)</td>
<td>(0.0399)</td>
</tr>
<tr>
<td>House Price Growth(_{t-1})</td>
<td>-0.4280</td>
<td>-0.5305</td>
<td>-0.5569</td>
<td>-0.5314</td>
</tr>
<tr>
<td></td>
<td>(0.3951)</td>
<td>(0.4034)</td>
<td>(0.4077)</td>
<td>(0.4033)</td>
</tr>
<tr>
<td>House Price Growth(_{t-2})</td>
<td>0.3980</td>
<td>0.4308</td>
<td>0.3872</td>
<td>0.3456</td>
</tr>
<tr>
<td></td>
<td>(0.4746)</td>
<td>(0.4826)</td>
<td>(0.4921)</td>
<td>(0.4841)</td>
</tr>
<tr>
<td>House Price Growth(_{t-3})</td>
<td>-0.6039</td>
<td>-0.6126</td>
<td>-0.6241</td>
<td>-0.6879</td>
</tr>
<tr>
<td></td>
<td>(0.4438)</td>
<td>(0.4568)</td>
<td>(0.4590)</td>
<td>(0.4574)</td>
</tr>
<tr>
<td>House Price Growth(_{t-4})</td>
<td>0.3594</td>
<td>0.3508</td>
<td>0.4511</td>
<td>0.4313</td>
</tr>
<tr>
<td></td>
<td>(0.3742)</td>
<td>(0.3815)</td>
<td>(0.3851)</td>
<td>(0.3833)</td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-1})</td>
<td>0.5197</td>
<td>0.4642</td>
<td>0.5506</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4517)</td>
<td>(0.4569)</td>
<td>(0.4504)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-2})</td>
<td>0.2146</td>
<td>0.1297</td>
<td>0.2457</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4588)</td>
<td>(0.4680)</td>
<td>(0.4580)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-3})</td>
<td>0.2176</td>
<td>0.2091</td>
<td>0.2244</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4441)</td>
<td>(0.4486)</td>
<td>(0.4437)</td>
<td></td>
</tr>
<tr>
<td>House Price Growth × Volatility(_{t-4})</td>
<td>0.0584</td>
<td>0.0032</td>
<td>0.0096</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.4117)</td>
<td>(0.4116)</td>
<td>(0.4118)</td>
<td></td>
</tr>
</tbody>
</table>

R\(^2\) | 0.1815 | 0.2938 | 0.2934 | 0.2941

Observations | 552 | 551 | 552 | 551

Note: The dependent variable is a dummy variable which equals one at the starting dates of recessions. Volatility is the volatility of countries’ stock market index, and equals the variance of daily returns within a year. All regressions include country and fixed effects, and control for four lags of real credit growth and real GDP growth, a measure of openness (the sum of import and export ratio to GDP), a measure of financial development (share of private credit by deposit money bank to GDP), and the share of population with more than or equal to 65 years. Column (3) includes four lags of the real interest rate. Column (4) includes four lags of the inflation rate. Standard errors are reported in brackets. ***, ** and * denote significance at the 1%, 5% and 10%, respectively.