Revealed preferences for diamond goods

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Abstract

When consumers care not only about the intrinsic consumption component but also for the value of commodities, it can be rational to purchase products as they become more expensive. We modify the standard revealed preference conditions to allow for so-called diamond effects. We show that the alternative revealed preference conditions are refutable, and that it is possible to capture the fraction of a consumer's marginal willingness to pay arising from preferences for the value associated with a good. We also present a first application of the method to observational data from the Russian Longitudinal Monitoring Survey, and obtain intuitively plausible results.

JEL Classification: C18, D03, D11, D12

Keywords: revealed preferences; diamond effects; price-dependent preferences

1 Introduction

Economics researchers are typically confronted with a tradeoff between the (psychological) realism of their models and the ability of the models to describe and estimate the underlying structure of consumers’ decisions (Rabin, 2013). For instance, it is well known that consumers care not only about the quantity of their purchases but also for the value associated with various commodities (Ng, 1987; Mandel, 2009). However, allowing for price-dependent preferences generally weakens the testable implications of a model (unless additional assumptions are made on the functional form of utility functions, for instance). Here we present a model in which consumers are allowed to care about the value of a purchase (diamond effect). We present a corresponding revealed preference characterization, which enables us to test rationality in the presence of diamond effects. We also present a first (nonparametric) empirical application of our model with diamond effects.

Diamond goods and price-dependent preferences Demand analysis typically treats consumers as ‘rational’ or ‘optimizing’ agents who maximize their utility by purchasing the commodities they like. The rationality assumption enables researchers to estimate welfare-related measures (such as cost-of-living indices) and demand functions for various commodities on the basis of real expenditure data. The assumption that consumers ‘maximize’ their utility is therefore crucial. In most applications, the utility functions are rather strictly defined: it is assumed that consumers care about the quantity of their purchase. However, Ng (1987, 1993) argues that sometimes a good is purchased for its value rather than for its intrinsic consumption effect.

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Jewelry is probably the most intuitive example. A diamond is not always purchased for its size. Its value may be more important as a means to please a loved one. Similarly, an art collection is prized for its value rather than the number of pieces in the collection (Mandel, 2009). More generally, various goods can have some degree of this so-called diamondness. When individuals treat their friends to dinner, go shopping in expensive clothing stores, or acquire a collection of wines or cigars, we can reasonably argue that these people care about the value of their purchase. Testing the nature of commodities to see whether they can be described as diamond goods is important. Ng (1987) shows that the rules for optimal taxation are completely different in a setting with diamond goods. From a theoretical perspective, taxing these goods increases government revenues without imposing an overly large burden on consumers. Moreover, failure to take diamond effects into account can lead to biased results for rationality, welfare, and demand (Heffetz and Shayo, 2009).

The preferences that we consider here are a special case of price-dependent preferences. Beside diamondness, preferences can depend on prices because of conspicuous consumption (Veblen, 1899) or quality considerations (Scitovsky, 1945). The distinction between diamond goods and Veblen goods is rather formal. For diamond goods, prices affect utility directly because consumers care about the value of the commodities they possess. For Veblen goods, prices affect utility indirectly. As Bagwell and Bernheim (1996) argue, it is 'status' rather than 'price' that enters the utility function for such consumers, and visibility of the expenditure plays an important role. In such a setting, the production side of the economy generally needs to be investigated as well. Here we focus on creating a test of rational behavior when consumers are allowed to care about the value of commodities, either directly (the true diamond effect) or indirectly. We do not formally disentangle the diamond and Veblen effects. However, we interpret our results using information from the visibility index of Heffetz (2011), which evaluates various commodities on the basis of their visibility to society.

To date, there has been little research that both incorporates price-dependent preferences in a standard model of consumer behavior and tests the modified model on the basis of observational data. One reason is that it is difficult to disentangle non-budget-constraint from budget-constraint price effects. Moreover, the way in which non-budget-constraint prices impact on decisions is not observed.

Heffetz and Shayo (2009) deal with this issue by setting up an experiment in which distinct prices are presented to respondents: relative prices that monitor the choice set and visual price stickers that capture non-budget-constraint price effects. However, in observational data sets, which are typically used for demand and welfare estimation, this type of information is unavailable.

Basmann et al. (1988) try to elicit Veblen effects (measured as elasticity of the marginal rate of substitution with respect to total expenditures) from observational data by estimating a Fechner-Thurstone direct utility function. This utility function has both quantity and price as arguments, so price-dependent preferences can be incorporated. However, much structure is imposed on the utility functions. This has two potential drawbacks. First, if the method rejects rationality, it is uncertain whether the individual was truly irrational or whether an incorrect specification of the utility function was imposed. Second, to estimate the utility functions it is assumed that consumers are homogeneous in terms of preferences. To deal with these problems, we follow a nonparametric (revealed preference) approach.

**Revealed preference** To obtain a test of rationality without having to specify individual utility functions, we use the revealed preference approach. Revealed preference models in the tradition of Samuelson (1938), Afriat (1967), Diewert (1973), and Varian (1982) define refutable conditions that need to hold in order for a consumer to be rational. The conditions are derived from a finite set of observables: price and quantity in-

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1 Note that even more generally, prices can also impact on consumption decisions beyond their effects through budget constraints or preferences. Chiappori (1988, 1992) and Apps and Rees (1988) developed a collective model of consumption to describe consumption decisions by households. The collective model is theoretically attractive for studying joint decision-making because it allows different members to have different preferences and lets intra-group bargaining power vary over time. Interestingly, variation in bargaining power may be driven by variation in the market prices of commodities.
formation at different points in time. Under the assumption of preference homogeneity over time, revealed preference theory allows testing of the transitivity of preference relations without imposing a functional form on the utility functions. Another attractive feature of this methodology is that consumption decisions of different agents can be analyzed independently, thereby fully recognizing that different agents can have different tastes.

Here we argue that letting preferences depend on both quantity and expenditure does not automatically preclude a revealed preference test. However, we need to modify the test. Standard revealed preference conditions are unable to take diamond effects into account. It is typically assumed that individuals care about quantity (or at least the intrinsic characteristics of the good\footnote{Blow et al. (2008) provide a revealed preference analysis of characteristics models.}) but not about the total value of a purchase. Failure to model additional price effects can lead to incorrect conclusions on the rationality of consumers. Consumer choices that seem irrational according to the standard test may be rationalizable if diamond effects are taken into account, and vice versa.

Unfortunately, the theory of revealed preferences and the conjecture that preferences depend on value or price are difficult to reconcile (Bilancini, 2011; Frank and Nagler, 2012). Revealed preference theory requires a finite data set of consumption choices under different price regimes while maintaining a constant preference ordering. If the preference ordering itself is influenced by prices, revealed preference theory becomes useless because we cannot compare different consumption bundles over time.\footnote{Pollak (1977) provides an insightful overview of the modeling of price-dependent preferences. The author considers two distinct ways in which price-dependent preferences are analyzed: the unconditional approach and the conditional approach. According to the unconditional approach, economic agents express their preferences not only over quantities but also over price–quantity pairs (Kalman, 1968; Piccione and Rubinstein, 2008). In this case, welfare conclusions are still possible (there exists a homogeneous preference ordering defined over quantities and prices) but the model cannot be tested on the basis of data from standard consumption surveys, in which individuals choose quantities and not price–quantity pairs. Because data on choices over price–quantity pairs (at any moment in time) are generally unavailable, the conditional approach seems more popular in empirical work. Following this approach, the preference orderings (defined over quantities) are conditional on prices. A popular functional specification for the utility function — which accounts for preference-shifting parameters such as price — is the generalized Fechner-Thurstone utility function. However, this utility function does not allow for welfare comparisons between periods in which preference-shifting parameters (i.e. prices) take different values.} However, we focus on preferences for value, so that price only enters the utility function through total expenditure on a commodity. Therefore, a homogeneous preference ordering can still be defined over different quantities and values of goods. This gives a new set of (refutable) revealed preference conditions that can be used when commodities have some degree of diamondness, or to identify the diamondness of commodities.

**Contribution** This paper makes theoretical, methodological, and empirical contributions to the existing literature.

First, at a theoretical level we present a model in which consumers are allowed to care about both the quantity and value associated with commodities. Our model builds on the framework developed by Ng (1993) but generalizes by allowing for more than one ‘diamond’ good. Moreover, we introduce a parameter that captures the diamondness of commodities, which is the marginal willingness to pay for value. Capturing the diamondness of a commodity in a single parameter is in accordance with research by Rabin (2013), who supports a PEEM (portable extension of existing models) approach, whereby one parameter is added to existing economic models to incorporate insights from the behavioral literature. By letting the diamondness of a commodity vary between 0 and 1, we can move from a model with traditional preferences (consumers only care about quantity) to the model of Ng (1987), which treats a commodity as a pure diamond good (consumers only care about the value associated with the pure diamond good). If the diamondness lies strictly between 0 and 1, the marginal willingness to pay for additional units of consumption stems from both the intrinsic utility associated with the good and its value.

Second, at a methodological level we present the corresponding revealed preference characterization.
We show that, conditional on some level of diamondness, rationality can still be tested in a meaningful way. Moreover, the different characterizations are generally non-nested. In other words, increasing diamondness associated with one or multiple goods does not automatically relax the revealed preference conditions. This implies that we can construct ‘bounds’ on the diamondness associated with various commodities.

Finally, at an empirical level we apply our revealed preference tests for rationality to a data sample from the Russian Longitudinal Monitoring Survey (RLMS). To the best of our knowledge, this is the first empirical revealed-preference test of diamond effects.

In Section 2, we define rationality and preferences for value and introduce our diamondness parameter. In Section 3, we present the revealed preference characterizations that allow us to test rationality for different specifications of the diamondness vector. These characterizations contain conditions that can be implemented using (mixed integer) linear programming techniques. We also discuss the non-nestedness of our characterizations. This section ends with a discussion of standard measures of empirical performance in the revealed preference literature. In Section 4, we briefly discuss our sample taken from the RLMS. Section 5 presents rationality results under different specifications of the diamondness vector. We also show that the appropriate degree of diamondness depends on the individual and the product at hand. Section 6 concludes.

2 Theory

We present a model in which consumers derive utility from both the goods they consume and the expenditure on these commodities. Because our focus is on diamond effects, we let commodity value enter the utility function of consumers.\(^4\) The price of a commodity can only affect preferences via expenditure on this commodity. Therefore, it remains possible to have a homogeneous preference ordering over time (provided that the social environment is rather stable), so welfare comparisons and the revealed preference approach are still valid. This contrasts with the conditional approach to more general price-dependent preferences, whereby preferences are conditional on prices, and prices do not necessarily enter the utility function via expenditure.

Ng (1987) introduced a model in which one good is assumed to be a (pure) diamond good, that is, the market value of this commodity enters the consumer utility function. In a later paper, Ng (1993) also allowed for mixed diamond goods, whereby both the market value and the intrinsic consumption component of one (mixed) diamond good enter the utility function. Unfortunately, in the framework of Ng (1987, 1993) there is only one pure or mixed diamond good, and other goods are standard goods. Here we present a framework in which, in principle, all goods can be characterized by some degree of diamondness, and diamondness (expressed on a continuum between 0 and 1) can be measured in monetary terms.

2.1 Modeling diamondness

To construct a testable model with diamond effects, we build on the model of Ng (1993), which is itself a generalization of the original Ng (1987) framework. The model of Ng (1993) assumes that utility can be derived from both the quantity and market price value of one commodity. We extend this framework to assess preferences for value associated with multiple goods, and we introduce a parameter to capture the relative importance of the marginal willingness to pay for value.

Suppose that we have a data set \(S = \{P_t, Q_t | \forall t \in T\}\) consisting of \(|T|\) observations. For each observation \(t\), this data set contains information on the observed quantity vector \(Q_t \in \mathbb{R}^{|N|}_{++}\) as chosen by consumers and the corresponding price vector \(P_t \in \mathbb{R}^{|N|}_{++}\). Let \(M_t \in \mathbb{R}^{|N|}_{++}\) represent the vector of expenditures on \(|N|\) commodities in period \(t\) (i.e. \(M_t\) consists of elements \(M_{tn} = P_{tn}Q_{tn}\)). We assume that utility functions take

\(^4\)Other studies in which price enters the utility function as a factor of quantity include those by Ng (1987, 1993), Weber (2002), Deng and Ng (2004), Mandel (2009), and Engström (2011).
the form \( U(Q, M) \), which means that consumers can derive utility from quantity on one hand and from the value of a purchase on the other hand:

\[
\begin{align*}
\max_Q U(Q, M) \\
\text{s.t.} \quad \lambda_t : P_t'Q \leq y_t \\
\forall n \in N : M^n = P^n_t Q^n.
\end{align*}
\]

We obtain testable implications by deriving the first-order conditions associated with the above problem:

\[
\forall n \in N : \frac{\partial U(Q, M)}{\partial Q^n} + \frac{\partial U(Q, M)}{\partial P^n_t Q^n} P^n_t = \lambda_t P^n_t. 
\tag{1}
\]

This expression clearly shows that the total marginal utility from an additional unit of good \( n \) is composed of the marginal utility associated with the good itself, \( \frac{\partial U(Q, M)}{\partial Q^n} \), and the marginal utility associated with the value of this good, \( \frac{\partial U(Q, M)}{\partial P^n_t Q^n} \). The second term follows directly from the diamond effect. We define diamondness as the marginal willingness to pay for value. To this end, we divide the marginal utility from additional expenditure, \( \frac{\partial U(Q, M)}{\partial P^n_t Q^n} \), by the marginal utility from one unit of income, \( \lambda_t \). In this way, we obtain a measure for the diamond effect in monetary terms:

\[
\theta^n = \frac{1}{\lambda_t} \frac{\partial U(Q, M)}{\partial P^n_t Q^n}.
\]

Interestingly, we let the diamondness parameter \( \theta^n \) be commodity-specific. We allow for the fact that certain commodities are more likely to trigger diamond effects than others. The diamondness parameter can take any value between 0 and 1. Suppose first that \( \theta^n = 1 \), in other words, the marginal utility from an additional unit of income stems purely from the diamond effect. In this case, good \( n \) is a pure diamond good (Ng, 1987). Consumers derive utility only from expenditure on this good and not from its quantity. Second, suppose that \( \theta^n = 0 \), in other words, additional expenditure on good \( n \) has no direct impact on the utility of the consumer. In this case, good \( n \) is a standard good, in the sense that additional income only impacts utility because the consumer can purchase larger amounts of good \( n \). Finally, we allow the diamondness weight \( \theta^n \) to take any value between 0 and 1, \( \theta^n \in [0, 1] \), which accounts for the possibility that goods are purchased both for their intrinsic value and for their monetary value.

Interestingly, the proposed model encompasses the neoclassical consumption model when \( \theta^n = 0 \) for all \( n \in N \). It can easily be verified that the second term in Eq. (1) (i.e. the diamond effect) would drop out for all commodities. The proposed model therefore fits within the PEEM framework of Rabin (2013). Indeed, our model extends the neoclassical counterpart only insofar as the newly proposed diamondness vector deviates from 0. At the other extreme, our model encompasses the situation in which all commodities are purchased only for their value, when \( \theta^n = 1 \) for all \( n \in N \). In this case, it is always possible to construct a (consistent) preference ordering such that utility increases with total expenditure.

Assume now that \( \theta \in \mathbb{R}^{\left| N \right|} \) is a vector containing the \( |N| \) diamondness parameters. Utility-maximizing behavior conditional on the diamondness vector is described in Problem 1.
Problem 1 Optimization problem OPT – θ:

\[
\begin{align*}
\max_Q & \quad U(Q, M) \\
\text{s.t.} & \quad \lambda_t : P_t'Q \leq y_t \\
\forall n \in N & : \begin{cases} 
    M^n = P^n_t Q^n \\
    \theta^n = \frac{1}{\lambda_t} \frac{\partial U(Q, M)}{\partial Q^n}.
\end{cases}
\end{align*}
\]

Revealed preference theory then proceeds by looking for some utility function \( U \) such that the observed consumption pattern solves Problem 1. Definition 1 links rationalizability with \( \theta – \text{diamondness} \) for consistency with the above program.

Definition 1 Consider a data set \( S = \{P_t, Q_t \mid \forall t \in T\} \). We say that \( S \) is rationalizable with \( \theta – \text{diamondness} \) if there exists a utility function \( U \) defined over quantity vector \( Q \) and expenditure vector \( M \) such that \( \{Q_t; \forall t \in T\} \) solves optimization problem 1.

3 Methodology

In the previous section we formulated a generalization of the original models by Ng (1987, 1993). We also introduced a parameter to capture the diamond effect. In this section, we first show how nonparametric (revealed preference) conditions can be derived to test the rationalizability concepts. The revealed preference approach is particularly attractive in the current setting. It avoids putting additional structure on the utility function \( U(\cdot) \). This guarantees that recovery of the diamondness vector is independent of the functional form of a utility function. Moreover, it rules out issues related to unobserved heterogeneity across consumers, because each consumer is analyzed separately. Next, we show that different specifications of the diamondness vector are generally non-nested. Finally, we discuss standard revealed preference measures that allow us to test the empirical performance of our proposed method.

Revealed preference methodology We first present the revealed preference test for consistency with a standard utility function of the form \( U(Q) \). This standard revealed preference test was developed by Afriat (1967), Diewert (1973), and Varian (1982). Specifically, the data set \( S = \{P_t, Q_t \mid \forall t \in T\} \) is said to be rationalizable if there exists a utility function \( U(Q) \) such that

\[
Q_t = \arg \max_Q U(Q) \text{ s.t. } P_t'Q \leq y_t.
\]

No further functional form restrictions are imposed on \( U(\cdot) \). Afriat’s Theorem, a central result in the revealed preference literature, then provides a rationality test.

Proposition 1 Consider a data set \( S = \{P_t, Q_t \mid \forall t \in T\} \). The following conditions are equivalent:

1. The data set \( S \) is rationalizable.

2. For all decision situations \( t \in T \) and for all commodities \( n \in N \), there exist utility numbers \( u_t \) and (Lagrange) multipliers \( \lambda_t \in \mathbb{R}_{++} \) such that for all \( t, v \in T \),

\[
u_t - u_v \leq \lambda_t P_t' (Q_t - Q_v).\]
3. For all decision situations \( t \in T \) and for all commodities \( n \in N \),

\[
S = \{ P_t; Q_t | \forall t \in T \} \text{ satisfies the GARP.}
\]

Statement 2 contains the so-called Afriat inequalities. Observed behavior can be rationalized by the standard utility-maximization model if and only if the data are consistent with the Afriat inequalities. Moreover, these conditions are also equivalent to stating that \( S = \{ P_t; Q_t | \forall t \in T \} \) is consistent with the generalized axiom of revealed preference (GARP). First, the GARP constructs revealed preference relationships \( R_{t,v} = 1 \) (observation \( t \) is revealed preferred over observation \( v \)) if \( P_t Q_t \geq P_v Q_v \). If \( Q_v \) was affordable at observation \( t \) but not chosen, it must be the case that the consumer preferred \( Q_t \) over \( Q_v \). Second, transitivity is imposed on \( R_t \), such that \( R_{t,v} = 1 \) if \( R_{t,v} = 1 \) and \( R_{v,w} = 1 \). Finally, the GARP requires that \( P_v Q_t \geq P_v Q_v \) when \( R_{t,v} = 1 \). This is because if bundle \( Q_t \) is revealed preferred over bundle \( Q_v \), it should be chosen over \( Q_v \) when it is affordable. In other words, if \( Q_v \) was chosen, it can be inferred that \( Q_t \) was not affordable.

We now propose modified revealed preference conditions that can be used to test consistency with Definition 1.

### 3.1 Revealing preferences for diamond goods

We develop a test for rationalizability for a given vector of diamondness weights \( \theta \). We investigate whether it is possible to construct a well-behaved utility function \( U \), with both values \( \mathbf{M} \) and quantities \( \mathbf{Q} \) as arguments, such that the observed consumption pattern \( \{ Q_t; \forall t \in T \} \) solves Problem 1. We start from the concavity of the utility function. For all \( t \) and \( v \), we must have that

\[
U(Q_t, M_t) - U(Q_v, M_v) \leq \sum_{n} \frac{\partial U(Q_{t,v}, M_{t,v})}{\partial Q_{t,v}^n} (Q_t^n - Q_v^n) + \sum_{n} \frac{\partial U(Q_{t,v}, M_{t,v})}{\partial P_{t,v}^n} (P_t^n Q_t^n - P_v^n Q_v^n).
\]

The first-order conditions without restrictions on the diamondness vector were presented in Eq. (1). They imply that

\[
\lambda_t P_t^n = \frac{\partial U(Q, M)}{\partial Q^n} + \frac{\partial U(Q, M)}{\partial P_t^n} P_t^n.
\]

We can formulate the marginal utilities in monetary terms by dividing both terms by \( \lambda_t \). This gives shadow prices \( p_t^n \) and \( q_t^n \) such that

\[
p_t^n = p_t^n + q_t^n P_t^n
\]

\[
q_t^n = \frac{1}{\lambda_t} \frac{\partial U(Q, M)}{\partial Q^n}
\]

and

\[
q_t^n = \frac{1}{\lambda_t} \frac{\partial U(Q, M)}{\partial P_t^n}.
\]

We also set \( u_t = U(Q_t, M_t) \) and \( u_v = U(Q_v, M_v) \). Finally, our definition of \( \theta^n \) implies that \( q_t^n = \theta^n \). By combining the above restrictions, we obtain the necessary conditions for rationalizability with \( \theta - \text{diamondness} \). In Appendix A, we show that these conditions are also sufficient. We can now formulate Proposition 2.

**Proposition 2** Consider a data set \( S = \{ P_t; Q_t | \forall t \in T \} \). The following conditions are equivalent:

1. The data set \( S \) is rationalizable with \( \theta - \text{diamondness} \).
2. For all decision situations \( t \in T \) and all commodities \( n \in N \), there exist utility numbers \( u_t, (\text{Lagrange}) \) multipliers \( \lambda_t \in \mathbb{R}_{++} \), and shadow prices \( p_t \in \mathbb{R}^{\left|N\right|} \) and \( \Psi_t \in \mathbb{R}^{\left|N\right|} \) such that for all \( t, v \in T \),

\[
\begin{align*}
\forall t, v \in N : \{ P^n_v &= p^n_v + \Psi^n_v \cdot P^n_v \\
\Psi^n_v &= \theta^n \} \quad \text{(Statement 3)}
\end{align*}
\]

\[
\begin{align*}
&\quad u_t - u_v - \lambda_t \sum_{n} p^n_v \cdot (Q^n_t - Q^n_v) + \lambda_v \sum_{n} \Psi^n_v \cdot (P^n_t Q^n_t - P^n_v Q^n_v) \\
&\quad \forall n \in N : \{ P^n_v = p^n_v + \Psi^n_v \cdot P^n_v \} \quad \text{(Statement 2)}
\end{align*}
\]

3. For all decision situations \( t \in T \) and for all commodities \( n \in N \), there exist shadow prices \( p_t \in \mathbb{R}^{\left|N\right|} \) and \( \Psi_t \in \mathbb{R}^{\left|N\right|} \) such that for all \( t, v \in T \),

\[
\begin{align*}
S &= \{ p_t, \Psi_t, Q_t, M_t | \forall t \in T \} \text{ satisfies the GARP} \\
\forall n \in N : \{ P^n_v = p^n_v + \Psi^n_v \cdot P^n_v \} \quad \text{(Statement 3)}
\end{align*}
\]

Statement 1 gives the definition of rationality when the magnitude of the diamond effect is given by \( \theta \). Statement 2 presents inequalities that allow us to test the presumption of rationalizability with \( \theta - \text{diamondness} \). The conditions are similar in nature to the well-known Afriat inequalities. However, there are two main differences. First, prices are not observed. We use shadow prices \( p^n_v \) and \( \Psi^n_v \) to capture the marginal willingness to pay for quantity and value, respectively. Second, there is an additional set of \( |T| \cdot |N| \) conditions, which state that the sum of the marginal willingness to pay for an additional unit of some good and the marginal willingness to pay for value associated with this good (multiplied by the market price) should equal the respective market price. Statement 3 provides us with an alternative test of rationalizability based on the GARP conditions.

Statements 2 and 3 are easily implementable. Notice first that both \( p^n_v \) and \( \Psi^n_v \) are determined as soon as \( \theta^n \) is specified. Conditional on the vector \( \theta \), the conditions in Statement 2 are linear in \( u_t, u_v, \lambda_t, \lambda_v \). Therefore, we can use simple linear programming techniques to implement this test. Statement 3 contains the alternative GARP formulation, which is particularly convenient. When \( \theta^n \) is specified, on one hand, Statement 3 simply requires verification of a set of combinatorial restrictions. It is no longer necessary to formulate (and solve) a programming problem. On the other hand, Statement 3 enables us to check rationalizability even if \( \theta \) is unspecified before the analysis. This enables us to endogenize the diamondness parameter \( \theta^n \) (as well as shadow prices \( p^n_v \) and \( \Psi^n_v \)), and consequently provides bounds on the feasible set of diamondness values.

A GARP-based test with unknown diamondness vector can be implemented using a linear programming problem with integer variables.

As a side note, we point out that this framework is also useful for analysis of bad commodities. The standard neoclassical model stipulates that consumers should not spend their budget on a bad commodity, which reduces their (intrinsic) utility. However, when preferences depend on value, rational consumers can purchase additional units of a bad commodity, as long as the marginal utility from its value exceeds the (negative) intrinsic marginal utility. Testing whether a commodity \( n \) is bad is now easy. We can simply modify the requirement

\[
p^n_v < 0
\]

in the revealed preference characterization in Proposition 2. The (negative) marginal utility due to quantity is then offset by the (positive) marginal utility from value if \( \Psi^n_v > 1 \).
3.2 Independence

One of our main contributions is a rationality test when consumers have preferences for value. As argued above, we can test for rationality conditional on a diamondness vector \( \theta \), or recover the set of diamondness weights that allows rationalization of (the largest fraction of) the data. At this point, it is worth noting that the different rationality tests (corresponding to different specifications of diamondness) are generally non-nested.

Indeed, it is possible that a data set can violate the predictions of the classical model while it is rationalizable with strictly positive diamondness for particular commodities. Likewise, a data set can be consistent with the classical model while it is not rationalizable for some strictly positive specification of \( \theta^n \). This suggests that we can construct meaningful bounds on the diamondness. There is one exception, however. The specification in which all goods are treated as ‘pure’ diamond goods cannot reject rationality (i.e. setting all diamondness parameters to \( \theta = 1 \) can always trivially rationalize the data). An empirical solution to this problem (discriminatory power) is discussed in Section 5. Intuitively, discriminatory power captures the strength of a test, that is, its capacity to reject rationality when confronted with random choice behavior.

To demonstrate the non-nestedness between most of the specifications, we use two examples. The first involves a data set that is not rationalizable when all goods are standard goods but is rationalizable when the diamondness weight associated with one good is strictly positive.

**Example 1** Consider a data set with \( |T| = 2, |N| = 2 \), market price vector \( P = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \), and quantity vector \( Q = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \).

We first show that this data set violates the standard Afriat inequalities. In particular, we have

\[
[P'_1Q_1 = 10] > [P'_1Q_2 = 6] \Rightarrow u_1 > u_2 \\
[P'_2Q_2 = 10] > [P'_2Q_1 = 6] \Rightarrow u_2 > u_1,
\]

which is a contradiction.

Next, we show that the data set is consistent with a model in which \( \theta^1 = 1 \) and \( \theta^2 = 0 \). Specifically, we have

\[
[P'_1Q_1 + P'_1Q_1^2 = 10] > [P'_2Q_2^1 + P'_2Q_2^2 = 4] \Rightarrow u_1 > u_2 \\
[P'_2Q_2 + P'_2Q_2^2 = 10] < [P'_1Q_1^1 + P'_1Q_1^2 = 12] \Rightarrow u_2 > u_1,
\]

which is feasible.

The second example is a data set that is rationalizable when all goods are standard goods but is not rationalizable when the diamondness weight associated with one particular good is strictly positive.

**Example 2** Consider a data set with \( |T| = 2, |N| = 3 \), market price vector \( P = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \end{pmatrix} \), and quantity vector \( Q = \begin{pmatrix} 1 & 9 & 10 \\ 9 & 3 & 0 \end{pmatrix} \).

We first show that this data set is consistent with the standard Afriat inequalities. In particular, we have

\[
[P'_1Q_1 = 49] > [P'_1Q_2 = 15] \Rightarrow u_1 > u_2 \\
[P'_2Q_2 = 45] < [P'_2Q_1 = 51] \Rightarrow u_2 > u_1,
\]
which is feasible.

Next, we show that the data set cannot be rationalized with $\theta^1 = 0, \theta^2 = 1, \text{and } \theta^3 = 0$. Specifically, we have

\[
[P_1^1 Q_1^1 + P_1^2 Q_2^1 + P_1^3 Q_3^1 = 49] > [P_2^1 Q_2^1 + P_2^2 Q_2^2 + P_1^3 Q_3^2 = 18] \Rightarrow u_1 > u_2
\]

\[
[P_2^1 Q_2^1 + P_2^2 Q_2^2 + P_2^3 Q_2^3 = 45] > [P_1^1 Q_1^1 + P_2^2 Q_2^1 + P_2^3 Q_3^1 = 42] \Rightarrow u_2 > u_1,
\]

which is a contradiction.

### 3.3 Measures of empirical performance

We conclude this section with an overview of empirical performance measures that allow us to assess various specifications of the diamondness vector. We focus on three commonly used criteria in revealed preference analysis: pass rates, power, and predictive success. On the basis of these measures, we will be able to critically examine our revealed preference characterizations for real data.

An attractive feature of the revealed preference framework is that each consumer can be analyzed separately. In other words, we can run an independent test for the rationality of each consumer in the presence or absence of preferences for value. The test gives a positive response (value of 1) if the individual’s behavior can be rationalized by a well-behaved utility function, and a negative response (value of 0) if the individual's behavior cannot be rationalized, conditional on some specification of the diamondness vector. By computing the average response, we obtain the pass rate $r$, which is the fraction of observed data sets that can be rationalized. Hence, a pass rate of 1 indicates that each individual has chosen consistently with the specified model, whereas a pass rate of 0 indicates that no agent is rational according to the model.

Given that our purpose is to compare the empirical performance of specifications that impose various degrees of diamondness, we also need a measure for the strength of our tests. It is easy to show why such a measure is necessary. A characterization in which all diamondness parameters are set to 1 would impose no meaningful restrictions. The underlying explanation is that in such a case, a preference ordering $U(Q, M)$ can be constructed that increases with total expenditure $\sum M^x$ and that trivially rationalizes the observed expenditure.\(^5\) Furthermore, even if some goods were modeled as standard goods, the strength of our test could be influenced by the choice of the parameters. To control for this, we compute the power $d$ associated with different specifications of the model (for a review of power measures, see Andreoni et al. (2011)). Specifically, we construct data sets by simulation, and test whether these randomly generated data sets satisfy the revealed preference conditions associated with the model. Discriminatory power then equals one minus the average pass rate for random data sets. The nonparametric conditions should enable us to reject rationality for random, inconsistent choices.

For robustness, we use two different power measures: the bootstrap power index and the power index of Bronars (1987). First we consider the bootstrap procedure. For each consumer and for each wave, we simulate a new set of budget shares (spent on 14 commodities) by drawing one set from the distribution of observed sets of budget shares in the sample.\(^6\) These budget shares are then multiplied by the budget associated with the respective consumer and wave to create new commodity bundles. In this way, we create 8200 random data sets (82 respondents for 100 iterations). An attractive feature of this bootstrap procedure is that the newly generated data sets still reflect a minimal amount of realism. However, because the different choices within one simulated data set do not come from the same individual with identical preferences, these data sets should not satisfy the conditions of our models. Results for the bootstrap index are provided in the main text.

\(^5\)In this respect, Bilancini (2011) and Frank and Nagler (2012) also noted that any pattern of choices can be rationalized by a (non-restricted) utility function of the form $U(Q, P)$.

\(^6\)This approach refers to the so-called bootstrap power calculation, discussed by Andreoni et al. (2011).
Second, we calculate the power measure of Bronars (1987). For each consumer and each wave, the Bronars procedure simulates a new set of budget shares by drawing shares from the uniform distribution. This method takes none of the available information on observed bundles into account. Following Cherchye et al. (2009), who applied the Bronars procedure to a similar sample from the RLMS, we modify the estimation strategy to take the large number of instances of zero expenditure into account. In practice, we impose a condition whereby each simulated budget share should not exceed the relative number of instances of zero expenditure in the data. In this way, we control for commodity groups that are not frequently purchased.

Our aim is to decide on which specification is most attractive in terms of empirical performance. Therefore, we apply the measure of predictive success $p$ proposed by Selten (1991) and discussed in more detail by Beatty and Crawford (2011). An elegant feature of this measure is that it combines pass rates ($r$) and discriminatory power ($d$). Predictive success is defined as

$$p = r - (1 - d).$$

Higher predictive success indicates that the model is better able to distinguish between observed behavior (which is supposed to be rational according to the model) and random, simulated behavior (which is supposed to violate the conditions of the model). Hence, predictive success of 1 would suggest that each observed data set is consistent with the model, whereas each random data set violates the conditions of the model. Predictive success of $-1$ is the worst possible outcome, and predictive success of 0 implies that the model cannot discriminate between real and random data sets. Hence, more positive predictive success scores are desirable.

### 4 Data

We apply our revealed preference tests to consumer data from the RLMS from 1994 to 2006, with the exception of 1997 and 1999. These 11 waves correspond to the second collection phase of the RLMS data (Phase II). We assume that the preferences of each respondent are sufficiently stable over time to construct the revealed preference conditions.

We restrict our attention to data sets for single individuals who do not receive any unemployment benefits. Furthermore, we only consider individuals who report expenditure for the 11 waves. Finally, we focus on individuals who were house and car owners during the full observation period. This yields a sample of 82 individuals. By conditioning on house and car ownership, we can exclude ‘large’ decisions on durable goods from the analysis.\(^7\) The reason is straightforward: we want to make a clear distinction between decisions driven by the diamond effect on one hand and intertemporal portfolio decisions on the other hand. Since the focus of this paper is on the former, we only consider nondurable commodities.

The nondurable commodities that we consider here are bread, potatoes, vegetables, fruit, meat, dairy products, alcohol, tobacco, food outside the home, clothes, car fuel, wood fuel, gas fuel, and luxury products. This grouping follows Cherchye et al. (2009), who conducted a similar revealed preference application to the RLMS data.

The prices of these aggregates are weighted (geometric) means of the prices associated with various detailed subgroups of goods. For instance, the price of alcohol is a weighted mean of the prices of vodka, liquor, and beer. We therefore have that the aggregate price $P^n_v$ for some commodity group $n$ in period $v$ is equal to

$$P^n_v = \prod_{k} (p^n_k)^{w^k},$$

\(^7\)We thus implicitly assume that decisions on these nondurable commodities and large decisions on durables are weakly separable. Although this assumption is contestable, it is quite common in applied static demand analysis. Moreover, interpersonal variation in durable decisions is not an issue because we analyze each agent separately.
where index $k$ denotes subgroups of products that belong to the aggregate commodity $n$, and the weights $w^k$ are determined by the average expenditure share of $k$ relative to expenditure on commodity $n$. By construction, these weights sum to one. The aggregation of prices $p^k_v$ in $P^n_v$ (and quantities $q^k_v$ in $Q^n_v$) is not uncontroversial. Consider an aggregate commodity $Q^n_v$ comprising a good $q^h_v$ and a good $q^l_v$:

$$Q^n_v = \frac{p^l_v q^l_v + p^h_v q^h_v}{P^n_v} = \frac{p^l_v q^l_v + p^h_v q^h_v}{(p^l_v)^w (p^h_v)^{1-w}}.$$

From Eq. (2), we can show that $Q^n_v$ increases with $p^l_v$ if the budget share of $l$ is greater than $w$, and that $Q^n_v$ increases with $p^h_v$ if the budget share of $h$ is greater than $1 - w$. This produces a positive relationship between the aggregate prices and quantities that is not due to the diamond effect, but simply stems from the aggregation. To deal with this issue, we rely on the Hicks Composite Commodity Theorem, which was first discussed by Leontief (1936) and Hicks (1946) and further developed by Gorman (1953). The theorem states that commodities $Q^n_v$ and aggregate prices $P^n_v$ can be treated in the same way as goods $q^l_v$ and $q^h_v$ and unobserved prices $p^l_v$ and $p^h_v$, provided that the relative prices $p^h_v/p^l_v$ remain stable across observations $v$ (i.e., $p^h_v/p^l_v = \alpha$). In Appendix B we show that the Hicks Theorem still holds in a setting with diamond goods. Moreover, our data show strong correlations between the prices of various subgroups that belong to the same aggregate.

For the revealed preference analysis, we restrict our attention to real prices. We divide all nominal prices by the average price level in each period. In this way, we avoid a situation in which price changes due to inflation impact on the consumption decisions of consumers. The (implicit) assumption that there is no money illusion seems standard in many studies involving price-dependent preferences.

To limit the number of parameters to be estimated in the empirical application, we let diamondness vary across seven commodity groups: food at home (bread, potatoes, vegetables, fruit, meat, dairy products), alcohol, tobacco, food outside the home, clothing, fuel, and luxuries. Moreover, food at home and fuel can be distinguished from alcohol, tobacco, food outside the home, clothing, and luxuries on the basis of a visibility ranking created by Heffetz (2011). To create this ranking, Heffetz (2011) used information from 480 interviews on the visibility of various commodities. The main question in their survey was whether respondents would notice if another household spent more than average on some commodity (e.g., jewelry and watches). Respondents were also asked how much time it would take to notice this greater-than-average spending pattern. In this way, the commodities (not brands) that are most visible to society were expected to obtain a high rank.

Tobacco (ranked 1st), clothing (ranked 3rd), jewelry (ranked 5th), food outside the home (ranked 7th), and alcohol (ranked 8th) are all among the top 10 most visible commodities according to the Heffetz (2011) ranking. Therefore, we assign these commodities to the visible category. Food at home (ranked 14th) and gasoline (ranked 21st) were ranked considerably lower. These commodities are assigned to the invisible category.

In the empirical application, we start from a setting in which all goods (both visible and invisible) are assumed to be standard goods, that is, utility is only derived from the quantities consumed. We investigate whether the behavior of agents is rational according to the standard GARP test. In the next step, we examine whether (and to what extent) the behavior of agents can be described by a model that allows for strictly positive diamondness weights. If the latter specifications outperformed the former, we could not reject the hypothesis that at least some degree of diamondness is required. Moreover, the method allows us to elicit preferences

---

8A similar issue arises when the aggregate $Q^n_v$ consists of a low-quality good $q^l_v$ and a high-quality good $q^h_v$. When a consumer spends significantly more money on the high-quality good, $Q^n_v$ increases with $p^h_v$. This is a quality effect rather than a diamond effect.

9Lewbel (1996) used a parametric framework to show that correct aggregation is possible even under weaker conditions. Specifically, the generalized aggregation theorem of Lewbel (1996) only requires that the evolution of relative prices (i.e., $p^h_v/p^l_v = \alpha$) in an aggregate $n$ is independent of the aggregate price level $P^n_v$.
for value associated with different goods, that is, we allow for heterogeneity across different commodities. Although we do not disentangle direct and indirect preferences for value (i.e., conspicuous consumption), information on the visibility of commodities provides us with an interesting interpretation of the results.

5 Application

We first test rationality conditional on various specifications of the diamondness of commodities. We compare specifications on the basis of (average) pass rates, power, and predictive success. We also interpret the results using the visibility index of Heffetz (2011). We expect that stronger preferences for value will correspond to more visible commodities.

Second, we compute predictive success at the individual level. This gives insight into interpersonal heterogeneity in preferences for value. We assess different specifications of diamondness on the basis of the distribution of predictive success (across the sample).

In a final step, we investigate the marginal willingness to pay for value associated with each commodity separately. This gives insight into heterogeneity in diamondness across commodities. Specifically, it is possible to bound the marginal willingness to pay for value using our revealed preference approach.

5.1 Testing for rationality with fixed diamondness

Table 1 presents the pass rates and power estimates associated with different specifications of the model. The rows represent different degrees of diamondness associated with food at home and fuel, and the columns represent different degrees of diamondness associated with alcohol, tobacco, food outside the home, clothing, and luxury commodities. A more detailed decomposition per commodity is provided in Section 5.3. Recall that when one particular $\theta^n$ equals 0, the respective commodity is valued for its intrinsic consumption component only. By contrast, when $\theta^n$ equals 1, the commodity is specifically prized for its value.

<table>
<thead>
<tr>
<th>Diamondness of visible goods</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamondness of less visible goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.561</td>
<td>0.573</td>
<td>0.610</td>
<td>0.634</td>
<td>0.634</td>
</tr>
<tr>
<td></td>
<td>(0.481)</td>
<td>(0.474)</td>
<td>(0.466)</td>
<td>(0.460)</td>
<td>(0.460)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.549</td>
<td>0.585</td>
<td>0.610</td>
<td>0.659</td>
<td>0.671</td>
</tr>
<tr>
<td></td>
<td>(0.408)</td>
<td>(0.399)</td>
<td>(0.387)</td>
<td>(0.379)</td>
<td>(0.380)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.671</td>
<td>0.720</td>
<td>0.707</td>
<td>0.732</td>
<td>0.793</td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td>(0.310)</td>
<td>(0.293)</td>
<td>(0.280)</td>
<td>(0.279)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.732</td>
<td>0.780</td>
<td>0.829</td>
<td>0.866</td>
<td>0.890</td>
</tr>
<tr>
<td></td>
<td>(0.230)</td>
<td>(0.210)</td>
<td>(0.186)</td>
<td>(0.164)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>1</td>
<td>0.841</td>
<td>0.890</td>
<td>0.890</td>
<td>0.927</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.113)</td>
<td>(0.076)</td>
<td>(0.039)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

Table 1: Pass rates (power estimates) for less visible goods (fuel and food at home) and visible goods (food away from home, clothes, luxuries, tobacco, and alcohol)

The top left result in Table 1 corresponds to the neoclassical utility maximization model (testable with GARP). The behavior of approximately 56% of the consumers can be rationalized by a well-behaved utility function of the form $U(Q)$. The bottom right result corresponds to the model in which all commodities are prized for their value. Not surprisingly, this revealed preference model imposes no testable restrictions, as a
result of which all the data sets are rationalized. The other results are more interesting. By varying the relevant parameters, very different pass rates and (bootstrap) power estimates are obtained.

<table>
<thead>
<tr>
<th>Diamondness of less visible goods</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamondness of visible goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.042</td>
<td>0.047</td>
<td>0.076</td>
<td>0.094</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td>[−0.022; 0.106]</td>
<td>[−0.015; 0.110]</td>
<td>[0.015; 0.137]</td>
<td>[0.033; 0.155]</td>
<td>[0.033; 0.154]</td>
<td></td>
</tr>
<tr>
<td>0.051</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[−0.102; 0.016]</td>
<td>[−0.073; 0.042]</td>
<td>[−0.059; 0.054]</td>
<td>[−0.017; 0.093]</td>
<td>[−0.004; 0.106]</td>
<td></td>
</tr>
<tr>
<td>0.061</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[−0.067; 0.061]</td>
<td>[−0.031; 0.090]</td>
<td>[−0.059; 0.061]</td>
<td>[−0.046; 0.070]</td>
<td>[0.018; 0.126]</td>
<td></td>
</tr>
<tr>
<td>0.024</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[−0.101; 0.024]</td>
<td>[−0.070; 0.050]</td>
<td>[−0.040; 0.070]</td>
<td>[−0.019; 0.080]</td>
<td>[0.009; 0.094]</td>
<td></td>
</tr>
<tr>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[−0.014; 0.037]</td>
<td>[−0.040; 0.046]</td>
<td>[−0.076; 0.008]</td>
<td>[−0.068; −0.001]</td>
<td>[0; 0]</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Predictive success for less visible goods (fuel and food at home) and visible goods (food away from home, clothes, luxuries, tobacco, and alcohol).

The predictive success results in Table 2 summarize our findings in Table 1. We investigate which specifications are empirically supported. The predictive success of the standard model amounts to 0.042. Increasing the diamondness associated with visible consumption generally improves the predictive success results, whereas increasing the diamondness associated with less visible consumption lowers the predictive success scores. In particular, the highest predictive success is obtained when visible commodities have almost full diamondness (i.e., for \( \theta \) ranging from 0.75 to 1) and the less visible commodities have no diamondness. The corresponding predictive success more than doubles the GARP result.

We finally investigate whether the differences in predictive success are statistically significant. To this end, we apply a procedure described by Demuynck (2014) that allows us to construct 95% confidence bounds around mean predictive success scores. Demuynck (2014) showed how confidence bounds can be computed on the basis of mean pass rates, mean power, and individual pass and power results across the sample. On one hand, the predictive success of neither the standard model nor the specifications giving higher diamondness to less visible goods is statistically different from 0. On the other hand, the mean predictive success associated with specifications that attribute higher diamondness to visible goods is statistically different from 0. Intuitively, this means that the modified model can describe the observed decisions while it rejects most of the random behavior.

In Appendix C we report power estimates and predictive success results when random bundles are simulated using the Bronars (1987) approach. These results mainly confirm our earlier findings. Summarizing, we can not reject the hypothesis that consumers care about the value of their purchase, in other words, that diamond effects are present. Moreover, the diamondness weights increase with the Heffetz (2011) visibility score. This indicates that consumer preferences for value largely stem from conspicuous consumption incentives.

### 5.2 Towards heterogeneous preferences for value

One of the attractive features of our revealed preference methodology is that it allows us to test the rationality of different consumers independently. Thus, we can take into account the preference heterogeneity of consumers. Some consumers have stronger preferences for value than others do. In such cases, the diamond
effect would be absent for the first type but important for the second type. Therefore, we not only test for diamondness on the basis of mean predictive success scores but also investigate the distribution of predictive success conditional on different diamondness vectors.

Table 3 presents the distribution of the predictive success measure across our sample. We report predictive success quantiles for four characterizations: (1) $\theta_{lessvis} = \theta_{vis} = 0$ (the GARP test), (2) $\theta_{lessvis} = 0$ and $\theta_{vis} = 0.75$, (3) $\theta_{lessvis} = 0$ and $\theta_{vis} = 1$, and (4) $\theta_{lessvis} = 1$ and $\theta_{vis} = 0$. We investigate these specifications from an individual perspective.

First, we see that the median value across the sample is always positive. Positive predictive success implies that a specification is able to distinguish between random (irrational) behavior and observed (rational) behavior. We find that the observed choices of at least 50% of the consumers in our sample can be described (and empirically distinguished from random behavior).

<table>
<thead>
<tr>
<th>Individual $p$</th>
<th>Min</th>
<th>Q20</th>
<th>Q40</th>
<th>Median</th>
<th>Q60</th>
<th>Q80</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = [0; 0]$</td>
<td>$-0.800$</td>
<td>$-0.441$</td>
<td>$-0.217$</td>
<td>$0.160$</td>
<td>$0.317$</td>
<td>$0.501$</td>
<td>$0.770$</td>
</tr>
<tr>
<td>$\theta = [0; 0.75]$</td>
<td>$-0.790$</td>
<td>$-0.400$</td>
<td>$0.119$</td>
<td>$0.240$</td>
<td>$0.330$</td>
<td>$0.491$</td>
<td>$0.730$</td>
</tr>
<tr>
<td>$\theta = [0; 1]$</td>
<td>$-0.790$</td>
<td>$-0.391$</td>
<td>$0.119$</td>
<td>$0.235$</td>
<td>$0.320$</td>
<td>$0.491$</td>
<td>$0.740$</td>
</tr>
<tr>
<td>$\theta = [1; 0]$</td>
<td>$-0.950$</td>
<td>$0$</td>
<td>$0.040$</td>
<td>$0.085$</td>
<td>$0.120$</td>
<td>$0.191$</td>
<td>$0.500$</td>
</tr>
</tbody>
</table>

Table 3: Distribution of predictive success with $\theta = [\theta_1, \theta_2]$ such that $\theta_1$ captures the diamondness of less visible commodities and $\theta_2$ the diamondness of more visible commodities.

Second, although characterization 4 is the most general (at least 80% of the respondents are characterized by a non-negative predictive success score), it is also the least precise. The highest possible predictive success score is (only) 0.5. Characterization 1 (GARP) is the most precise (because the highest predictive success scores are achieved) but it is unable to successfully describe the decisions of at least 40% of the respondents. Characterizations 2 and 3, which attribute higher levels of diamondness to more visible goods, seem well suited to describe the observed decisions. These specifications successfully characterize at least 60% of the respondents and they can produce very precise descriptions. To gain more insight, we investigate the density functions of the predictive success conditional on the four characterizations (Fig. 1).

We first consider characterization 4. The mass of consumers who can be described by the model is substantial. However, the density function peaks at a predictive success score that is barely greater than $p = 0$. Hence, characterization 4 cannot discriminate much better between observed (rational) and simulated (irrational) behavior than a random coin toss. Characterization 1 seems somewhat more precise. It identifies an important mass of consumers at both ends of the predictive success distribution. Hence, characterization 1 presents an accurate description of a considerable fraction of the respondents. Unfortunately, the mass of consumers who can be described by the model ($p > 0$) is hardly greater than the mass of consumers who cannot be described by the model ($p < 0$). Compared to this specification, the density functions associated with characterizations 2 and 3 have more mass at the higher predictive success levels. This provides empirical support for the notion that diamond effects are associated with more visible commodities.

5.3 Identification of diamondness

Up to now we have used constant diamondness weights within the subgroups of visible and less visible goods. We now go one step further by splitting the visible subgroup into alcohol, tobacco, food away from home, clothing, and luxuries. We analyze if, and to what extent, the number of consumers who behave rationally evolves as a function of diamondness for each product. The diamondness of less visible goods is set to $\theta = 0$ (which is empirically supported by the results in Table 2) or $\theta = 0.25$ (for robustness).

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10 We focus on the visible subgroup for two reasons. First, in the previous subsections we could reject the hypothesis that consumers have strong preferences for value associated with commodities in the less visible subgroup. Second, the power estimates in Table 1
Figure 1: Predictive success densities with $\theta = [\theta_1, \theta_2]$ such that $\theta_1$ captures the diamondness of less visible commodities and $\theta_2$ the diamondness of more visible commodities.
Figure 2 shows the number of rational consumers as a function of the diamondness of alcohol, tobacco, food away from home, clothing, and luxuries. For completeness, we also consider the number of rational consumers as a function of the diamondness of all visible goods.

For the first graph, the diamondness of less visible goods is set to 0. We find that the number of rational consumers improves by 3 if the diamondness of restaurant visits is increased to 1, and by 2 if the diamondness of alcohol is increased to 1. Clothing and luxuries have a smaller impact on the rationality results. For the second graph, the diamondness of less visible goods is set to 0.25. We then find that the number of rational consumers improves by 3, 3, 4, and 3 if the diamondness of restaurant visits, alcohol, clothing, and luxuries, respectively, is set to 1. We do not find evidence that the consumers in our sample have preferences for value associated with tobacco.

Finally, we consider all visible goods jointly. As expected from Table 1, 46 consumers are rational (56.1%) when $\theta = 0$, whereas 52 consumers are rational (63.4%) when $\theta = 1$, conditional on all others goods being standard goods. The results in Figure 2 allow us to investigate this evolution more thoroughly. Specifically, they show that pass rates do not monotonically increase with the diamondness of one commodity (or group of commodities). This corresponds to our earlier non-nestedness result: it is possible that decisions are rationalizable given lower diamondness values and not rationalizable for higher diamondness values. For example, at $\theta = 0.169$, the number of individuals that pass the test decreases. Indeed, we can identify a respondent whose marginal willingness to pay for value lies in $[0; 0.169] \cup [0.603; 1]$. Similar observations at $\theta = 0.571$ and $\theta = 0.768$ correspond to respondents whose marginal willingness to pay for value is bounded from above by 0.571 and 0.768, respectively.

6 Conclusion

We incorporated preferences for value in the revealed preference framework. We defined diamondness as the marginal willingness to pay for the value associated with commodities. Strictly positive diamondness implies that consumers derive utility from the value of goods, not just from the quantity consumed.

We first generalized the model of Ng (1993) by allowing for more than one diamond good. Moreover, we let the degree of diamondness vary on a scale from 0 to 1. Interestingly, the newly proposed diamondness parameter measures the diamond effect in monetary terms. By extending the neoclassical model of rationality with one (set of) parameter(s), our approach fits the PEEM research agenda supported by Rabin (2013).

Next, we constructed revealed preference conditions that can be used to verify assumptions on the diamondness of commodities. An attractive feature of the revealed preference methodology is that it imposes minimal restrictions on the form of utility functions. Moreover, each agent can be analyzed separately so that debatable preference homogeneity assumptions can be avoided. We showed that the revealed preference approach produces refutable conditions even if preferences depend on value. In this respect, we also argued that the different characterizations (corresponding to different assumptions for the diamondness vector) are generally non-nested. The method can produce meaningful bounds on the diamondness associated with particular commodities for particular consumers.

Finally, we applied our nonparametric test of preferences for value to a data sample from the RLMS. To the best of our knowledge, this is the first application of revealed preference that specifically incorporates preferences for value. We found that the predictive success of a standard GARP test is significantly less than that of alternative specifications that set strictly positive marginal willingness to pay for value. To interpret our results, we investigated the relationship between marginal willingness to pay for value and the visible power of a commodity to society. On the one hand, the hypothesis that less visible commodities have strong diamondness was rejected. It is unlikely that individuals have preferences for value associated with food con-confirm that discriminatory power is more or less constant for various assumptions on the diamondness of visible goods. When analyzing visible goods, we can therefore restrict our attention to pass rates.
Figure 2: Pass rates as a function of diamondness.

Pass rates as a function of diamondness, conditional on $\theta = 0$ for less visible consumption

Pass rates as a function of diamondness, conditional on $\theta = 0.25$ for less visible consumption
sumed at home and fuel. This might explain why Heffetz and Shayo (2009), who focused on food, did not find significant non-budget-constraint price effects in their experiment. On the other hand, we could not reject the hypothesis that consumers have strong preferences for value associated with more visible commodities, such as clothing. Following the argument of Ng (1987), special taxation rules may be appropriate. Furthermore, we found much variation in the rationality results across the sample. This clearly shows that choices and preferences are very diverse, and that homogeneity assumptions on preferences for value are probably unrealistic. Finally, we set out rationality results for all respondents as a function of the diamondness of alcohol, tobacco, food away from home, clothing, and luxuries. We found that pass rates generally increase with the diamondness of all visible commodities apart from tobacco. For particular individuals, we could also establish meaningful bounds on their marginal willingness to pay for value.

There are different avenues for further research. First, although the current results are already interesting, we intend to apply the model and associated revealed preference conditions to a larger sample. Second, modeling of preferences for value could be applied in a collective setting that covers household decisions. Various other factors (such as the affection between two partners) may have an impact on the consumption of so-called diamond goods. Finally, we restricted our attention to testing and identifying the extent to which value affects consumption decisions beyond its effect on budget constraints. To elicit the precise motivations that underlie preferences for value, we could focus on a more limited set in which different goods (or brands) are differentiated on the basis of price and possibly other characteristics signaling quality or financial status.

References


A Proof of Proposition 2

Proof.

• We first prove that Condition 1 implies Condition 2. Consider the following (necessary) first-order condition for the optimization of $\text{OPT} - \theta$:

$$
\frac{\partial U}{\partial Q^n} + \frac{\partial U}{\partial (P^nQ^n)} \cdot P^n_t \leq \lambda P^n_t,
$$

where $\frac{\partial U}{\partial Q^n}$ and $\frac{\partial U}{\partial (P^nQ^n)}$ are subderivatives of the (concave) utility function with respect to $Q^n$ and $P^nQ^n$, respectively. The inequalities are replaced with equalities if the quantities $Q^n$ are strictly positive.

Moreover, the concavity of the utility function gives

$$
u_t - u_v \leq \sum^n \frac{\partial U_v}{\partial Q^n_v} \cdot (Q^n_t - Q^n_v) + \sum^n \frac{\partial U_v}{\partial (P^n_vQ^n_v)} \cdot (P^n_t Q^n_t - P^n_v Q^n_v).
$$

Finally, we replace $p^n_v = \frac{\partial U_v}{\partial Q^n_v} \lambda_v$ and $\mathcal{P}^n_v = 1 - \frac{p^n_v}{P^n_v}$ such that $\mathcal{P}^n_v \geq \frac{\partial U_v}{\partial (P^n_vQ^n_v)} \frac{\lambda_v}{\lambda_v}$. For strictly positive quantities $Q^n$, we have that $\mathcal{P}^n_v = \frac{\partial U_v}{\partial (P^n_vQ^n_v)} \lambda_v$. Consistency with Eq. (3) thus requires consistency with

$$
u_t - u_v \leq \lambda_v \sum^n p^n_v \cdot (Q^n_t - Q^n_v) + \lambda_v \sum^n \mathcal{P}^n_v \cdot (P^n_t Q^n_t - P^n_v Q^n_v)
$$

(4)

$$
P^n_v = p^n_v + \mathcal{P}^n_v \cdot P^n_v \ \forall n \in N.
$$

This concludes the necessity part.

• We then prove that Condition 2 implies Condition 1 based on Varian (1982). We start from the observation that

$$
U(Q, M) \leq U_v + \lambda_v \sum^n p^n_v \cdot (Q^n - Q^n_v) + \lambda_v \sum^n \mathcal{P}^n_v \cdot (P^n Q^n - P^n_v Q^n_v).
$$

We then select the minimum of all overestimates according to

$$
U(Q, M) = \min \{ U_v + \lambda_v \sum^n p^n_v \cdot (Q^n - Q^n_v) + \lambda_v \sum^n \mathcal{P}^n_v \cdot (P^n Q^n - P^n_v Q^n_v) \}.
$$
This formulation should be such that any \((Q,M)\) for which \(P'_tf_t \geq P'_tQ\) implies that \(U(Q_t,M) \geq U(Q,M)\).

First, it is important to understand that \(U(Q,v,M_v) = U_v\) for \(v = 1, \ldots, T\). Indeed, for some \(t\) we have that

\[
U(Q,v,M_v) = U_t + \lambda_t \sum_{n} p^n_t \cdot (Q^n_v - Q^n_t) + \lambda_v \sum_{n} P^n_v \cdot (P^n_v Q^n_v - P^n_t Q^n_t)
\]

If this inequality were strict, we would have that

\[
U_v - U_t > \lambda_t \sum_{n} p^n_t \cdot (Q^n_v - Q^n_t) + \lambda_v \sum_{n} P^n_v \cdot (P^n_v Q^n_v - P^n_t Q^n_t),
\]

which contradicts the Afriat inequalities. Hence, \(U(Q,v,M_v) = U_v\).

Second, any \((Q,M)\) for which \(P'_tf_t \geq P'_tQ\) must be consistent with

\[
U(Q,M) = \min_v (U_v + \lambda_v \sum_{n} p^n_v \cdot (Q^n - Q^n_v) + \lambda_v \sum_{n} P^n_v \cdot (P^n_v Q^n_v - P^n_t Q^n_v))
\]

which implies that

\[
U_t + \lambda_t \sum_{n} p^n_t \cdot (Q^n - Q^n_t) + \lambda_v \sum_{n} P^n_t \cdot (P^n_t Q^n_t - P^n_v Q^n_v)
\]

The first inequality follows from the definition of \(U(Q,M)\), and the second inequality follows from

\[
U_t + \lambda_t \sum_{n} P^n_t \cdot (Q^n - Q^n_t) + \lambda_v \sum_{n} P^n_v \cdot (P^n_v Q^n_v - P^n_t Q^n_v)
\]

This concludes the sufficiency part.

\section{The Hicks Composite Commodity Theorem in the diamond setting}

In this proof, we show that the Hicks Composite Commodity Theorem also applies in a setting with diamond goods. In other words, the testable conditions in Proposition 2 give the same results regardless of whether the restrictions are applied to aggregates with unit values or to subgroups with specific prices, provided that there is no relative price variation within the aggregates.

\textbf{Proof.} We start from aggregate prices, which are constructed as

\[
P^n_v = (p^n_v) \cdot \left( \frac{P^n_v}{p^n_v} \right) \cdot (1 - w),
\]

and from the assumption that there is no (relative) price variation within subgroups of products, that is,

\[
\frac{p^n_h}{p^n_v} = \alpha.
\]
First, we show that this information enables us to express $Q_v$, $P^l_v$, and $P^h_v$ in terms of constants $\beta^l = \alpha^{w-1}$ and $\beta^h = \alpha^w$, which are invariant across periods $v$:

$$Q_v = \beta^l q^l_v + \beta^h q^h_v,$$
$$P^l_v = \beta^l P^l_v,$$
$$P^h_v = \beta^h P^h_v.$$

To see this, we use that $p^h_v = p^l_v$ and $P^n_v = (p^l_v)^w (p^h_v)^{-w}$:

$$Q_v = \frac{p^l_v q^l_v + p^h_v q^h_v}{P^n_v} = \frac{p^l_v q^l_v + p^h_v q^h_v}{(p^l_v)^w (p^h_v)^{-w}} = \frac{p^l_v q^l_v + \alpha p^l_v q^h_v}{(p^l_v)^w (\alpha p^l_v)^{-w}} = \frac{q^l_v + \alpha q^h_v}{\alpha^{1-w}} = \alpha^{w-1} q^l_v + \alpha^w q^h_v = \beta^l q^l_v + \beta^h q^h_v.$$

$$P^l_v = (p^l_v)^w (p^h_v)^{-w} = (p^l_v)^w (\alpha p^l_v)^{-w} = \alpha^{1-w} p^l_v.$$

$$P^h_v = (p^l_v)^w (p^h_v)^{-w} = (p^h_v)^w (\alpha p^h_v)^{-w} = \alpha^{-w} p^h_v.$$

Next, we reformulate the first-order conditions for all subgroups $k \in \{h, l\}$ that belong to aggregate $n$:

$$P^k_v = p^k_v + \pi^k_v \cdot P^k_v \iff \beta^k P^k_v = p^k_v + \pi^k_v \cdot P^k_v \iff P^k_v = \frac{p^k_v}{\beta^k} + \frac{\pi^k_v}{\beta^k} \cdot P^k_v.$$

Hence, we can conclude that for fixed $\theta^n = \theta^h \cdot \theta^l$ such that $\pi^k_v = \pi^l_v = \pi^h_v$,

$$\frac{p^h_v}{\beta^h} = \frac{p^l_v}{\beta^l}.$$

We can simply redefine $p^k_v = \frac{p^h_v}{\beta^h} = \frac{p^l_v}{\beta^l}$.

Finally, we can show the equivalence between the inequalities in Proposition 2 (Statement 2) applied to the subproducts on one hand and the inequalities in Proposition 2 (Statement 2) applied to the aggregates on the other hand.
\[ u_t - u_v \leq \lambda_v p_v^1(q_t^h - q_v^h) + \lambda_v p_v^2(q_t^l - q_v^l) + \lambda_v p_v^1(q_t^h - q_v^h) + \lambda_v p_v^2(q_t^l - q_v^l) + \lambda_v p_v^1(q_t^h - q_v^h) + \lambda_v p_v^2(q_t^l - q_v^l) + \lambda_v p_v^1(q_t^h - q_v^h) + \lambda_v p_v^2(q_t^l - q_v^l) \]

\[ \Leftrightarrow u_t - u_v \leq \lambda_v p_v^1(\beta h^1 q_t^h - \beta h^1 q_v^h) + \lambda_v p_v^2(\beta h^2 q_t^h - \beta h^2 q_v^h) + \lambda_v p_v^1(\beta h^1 q_t^h - \beta l^1 q_v^h) + \lambda_v p_v^2(\beta h^2 q_t^h - \beta l^2 q_v^h) + \lambda_v p_v^1(\beta l^1 q_t^l - \beta h^1 q_v^h) + \lambda_v p_v^2(\beta h^2 q_t^h - \beta l^2 q_v^h) + \lambda_v p_v^1(\beta l^1 q_t^l - \beta l^1 q_v^h) + \lambda_v p_v^2(\beta h^2 q_t^h - \beta l^2 q_v^l) \]

\[ \Leftrightarrow u_t - u_v \leq \lambda_v p_v^1(Q_t^l - Q_v^l) + \lambda_v p_v^2(Q_t^l - Q_v^l) + \lambda_v p_v^1(Q_t^l - Q_v^l) + \lambda_v p_v^2(Q_t^l - Q_v^l) + \lambda_v p_v^1(Q_t^l - Q_v^l) + \lambda_v p_v^2(Q_t^l - Q_v^l) \]

\[ \begin{array}{cccccc}
    \text{Diamondness of visible goods} & 0 & 0.25 & 0.5 & 0.75 & 1 \\
    0 & 0.561 & 0.573 & 0.610 & 0.634 & 0.634 \\
       & (0.452) & (0.445) & (0.442) & (0.440) & (0.443) \\
    0.25 & 0.549 & 0.585 & 0.610 & 0.659 & 0.671 \\
       & (0.379) & (0.366) & (0.358) & (0.354) & (0.357) \\
    0.5 & 0.671 & 0.720 & 0.707 & 0.732 & 0.793 \\
       & (0.303) & (0.287) & (0.272) & (0.264) & (0.262) \\
    0.75 & 0.732 & 0.780 & 0.829 & 0.866 & 0.890 \\
       & (0.219) & (0.191) & (0.166) & (0.150) & (0.147) \\
    1 & 0.841 & 0.890 & 0.890 & 0.927 & 1 \\
       & (0.160) & (0.126) & (0.080) & (0.037) & (0) \\
\end{array} \]

Table 4: Pass rate (and Bronars power estimates) for less visible goods (fuel and food at home) and visible goods (food away from home, clothes, luxuries, tobacco, and alcohol)
Most of our earlier conclusions remain valid. First, the predictive success scores generally increase as a function of diamondness for visible goods.

None of the predictive success scores are statistically different from 0 at the 5% level. At the 10% level, however, we find that (only) some characterizations (those that attribute high diamondness to visible goods) have a predictive success score that is statistically different from 0.

<table>
<thead>
<tr>
<th>Diamondness of less visible goods</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
</tr>
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<tr>
<td>0</td>
<td>0.013</td>
<td>0.018</td>
<td>0.052</td>
<td>0.075</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>[−0.050; 0.076]</td>
<td>[−0.043; 0.078]</td>
<td>[−0.008; 0.112]</td>
<td>[0.015;0.134]</td>
<td>[0.016;0.137]</td>
</tr>
<tr>
<td>0.25</td>
<td>−0.073</td>
<td>−0.048</td>
<td>−0.033</td>
<td>0.012</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>[−0.131; −0.014]</td>
<td>[−0.105;0.009]</td>
<td>[−0.089;0.024]</td>
<td>[−0.044;0.068]</td>
<td>[−0.028;0.083]</td>
</tr>
<tr>
<td>0.5</td>
<td>−0.026</td>
<td>0.007</td>
<td>−0.020</td>
<td>−0.005</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>[−0.089;0.036]</td>
<td>[−0.052;0.065]</td>
<td>[−0.079;0.038]</td>
<td>[−0.062;0.053]</td>
<td>[0.002;0.108]</td>
</tr>
<tr>
<td>0.75</td>
<td>−0.050</td>
<td>−0.029</td>
<td>−0.005</td>
<td>0.016</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>[−0.112;0.013]</td>
<td>[−0.087;0.030]</td>
<td>[−0.059;0.049]</td>
<td>[−0.034;0.065]</td>
<td>[−0.007;0.080]</td>
</tr>
<tr>
<td>1</td>
<td>0.001</td>
<td>0.016</td>
<td>−0.029</td>
<td>−0.036</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>[−0.049;0.051]</td>
<td>[−0.025;0.057]</td>
<td>[−0.071;0.012]</td>
<td>[−0.070;−0.001]</td>
<td>[0;0]</td>
</tr>
</tbody>
</table>

Table 5: Predictive success based on the Bronars method for less visible goods (fuel and food at home) and visible goods (food away from home, clothes, luxuries, tobacco, and alcohol)