Economic growth and inequality patterns in the presence of costly technology adoption and uncertainty

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Abstract

We develop a stochastic endogenous growth model to explain the diversity in growth and inequality patterns and the non-convergence of incomes in transitional economies where an underdeveloped financial sector imposes an implicit, fixed cost on the diversification of idiosyncratic risk. In the model endogenous growth occurs through physical and human capital deepening, with the latter being the more dominant element. We interpret the fixed cost as a ‘learning by doing’ cost for entrepreneurs who undertake risk in the absence of well developed financial markets and institutions that help diversify such risk. As such, this cost may be interpreted as the implicit returns foregone due to the lack of diversification opportunities that would otherwise have been available, had such institutions been present.

The analytical and numerical results of the model suggest three growth outcomes depending on the productivity differences between the projects and the fixed cost associated with the more productive project. We label these outcomes as poverty trap, dual economy and balanced growth. Further analysis of these three outcomes highlights the existence of a diversity within diversity. Specifically, within the ‘poverty trap’ and ‘dual economy’ scenarios growth and inequality patterns differ, depending on the initial conditions. This additional diversity allows the model to capture a richer range of outcomes that are consistent with the empirical experience of several transitional economies.
1. Introduction

Since the late 1980s most transitional economies embarked on a series of structural, institutional and policy reforms. These reforms were mainly meant to remove repressive policies and started through the World Bank-sponsored structural adjustment programs (SAPs). While these reforms focussed on various sectors of the economy, the reforms that the current study is particularly concerned with are those with regard to the domestic financial sector. Among these domestic reforms include the privatisation and deregulation of the banking sector, the removal of interest rate and credit controls, and the deregulation of financial markets.

There has been a debate about the impact of these reforms on the economy. Krugman (1995), Edwards (1995) and Burki and Edwards (1995) observe that that despite widespread economic reforms, Latin American economies made little progress in terms of economic growth and poverty eradication. However, although De Gregorio and Guidotti (1995) also provide panel regression evidence consistent with the above observations for the Latin American countries, they argue that the negative relation between financial intermediation and economic growth might be reflective of the role played by the financial crises experienced by several countries in the region. Furthermore, a study on East African developing countries by the United Nations (2008) documents evidence showing that economic development improved with reforms in five countries, while it worsened in two countries.

The debate has not only been limited to the growth and poverty reduction effects of these reforms, but also their income-redistribution effects. Financial reforms are likely to improve income distribution if they improve the access of the poor to finance. However, it is possible that the benefits of reforms may disproportionately accrue to the rich and the more skilled who are able to promptly take the capital-intensive technological opportunities that are brought about by the reforms (Shahbaz and Islam, 2011). Bittencourt (2009) documents evidence that financial development, as measured by increased access to finance, improved income distribution in Brazil in the 1980s and the 1990s. Similarly, using a panel of 22 African countries, Batuo, et al. (2010) provide evidence that the financial development was associated with a reduction in the Gini coefficients in the period covering most of the financial reforms. However, Calderon and Serven (2003) establish that financial development worsens the income distribution. Similarly, Lopez (2004) documents panel econometric evidence that low and stable inflation and education are associated with a
decrease in inequality, while financial development and low government expenditure are associated with worsening levels of inequality.

The empirical evidence reviewed above shows that the growth and inequality experiences of transitional economies during and just after introducing structural and institutional reforms exhibit a great deal of diversity. Below we also present some descriptive and pairwise correlation evidence which seems to be supportive of this phenomenon. Firstly, we analyse the correlation between a measure of the financial reforms and growth in GDP for transitional economies in different regions. Secondly, we analyse trends in inequality in a selected group of transitional economies during and after reforms.¹

Measuring the level of financial reforms is a challenging issue. However, there is widespread consensus that financial reforms positively enhance financial development (see Klein and Olivei, 1999, De Gregorio, 1998). Thus, we use financial development as an indirect proxy for the financial reforms. A commonly used measure of financial development is the value of credit that banks provide to the private sector as a percentage of GDP.² In Figure 1 (a) we plot the correlation between the logarithm of this measure and GDP growth for the period 1986 – 2002.

As evident in Figure 1(a), the correlation between FD and GDP growth within this period had generally been positive for low income countries. However, when we break the low income countries according to regions, it becomes clear that the correlation varies from region to region. For East Asian and SSA developing countries, the correlation between financial development and GD growth exhibits a positive trend, while for Europe and Central Asia, Latin America and Caribbean, and Middle East and North Africa (MENA) developing countries, the correlation exhibits a negative trend.

In Figure 1(b), we analyse the behaviour of inequality as measured by the Gini coefficient for six transitional economies from the late 1980s to the early 2000s. The graphical plots of the Gini coefficient show that, on average, inequality has decreased for Brazil and Thailand, while it has increased for Argentina, Costa Rica, Uruguay and Croatia. Moreover, the plots show that some countries such as Thailand and Costa Rica are characterised by high fluctuations in inequality, while in other countries such as Argentina, Croatia and Uruguay the changes in inequality are much smoother.

¹ We do not perform this analysis region by region because of data constraints. Moreover, due to data constraints we could not look at countries in the Middle East North Africa (MENA) and sub-Saharan Africa (SSA) regions.
² Although stock market capitalisation as a percentage of GDP is also used as a measure of financial development, bank credit to the private sector is the most appropriate given that developing stock markets are very small, illiquid, inactive, and there is very little participation by the general public.
Figure 1(a): Correlation Between financial development and growth
Given the foregoing discussion, it becomes of interest to analyse why the growth and inequality experiences of countries and regions show such diversity. Furthermore, the differences in period-to-period fluctuations in inequality across countries are of research interest. Thus far, a number of explanations have been suggested for the increase of inequality across countries/regions. Studies such as Glomm and Ravikumar (1992), Glomm and Palumbo (1993), Ray and Streufert (1993), Galor and Mayer (2004), Chakraborty and
Das (2005), among others, emphasize the role of human capital accumulation as a source of persistence in inequality. Studies by Mokyr (1993), Greenwood and Yorukoglu (1997), Perente and Prescott (1994, 2004), Lahiri and Ratnasiri (2012), among others, propose on the other hand that non-convergence can be traced to differences in dates of technology adoption across nations. However, as discussed above, a monotonic increase in inequality is not the norm as far as the experience of some the countries in Figure 1(b) is concerned.

The current study, then, is motivated by the diversity in growth and inequality experiences of countries as well as the differences in period-to-period fluctuations of inequality across countries. We seek to answer the question why such differences exist, and why some countries continue to have adverse growth and inequality experiences despite empirical evidence of the positive impact of economic reforms (see Goldsmith, 1969, McKinnon, 1973, Shaw 1973, King and Levine, 1993, Fischer et al., 1996, Staehr, 2005). To this end we develop a model that explores growth and inequality in an environment where inadequate financial reforms impose a fixed cost on the undertaking of high-risk, high-return projects. Put differently, in the absence of well developed financial institutions, entrepreneurs face a ‘learning by doing’ cost of adopting technologies that have a high return on average, but are associated with risk. Specifically, the model developed is a simple two-period lived overlapping generations model where endogenous growth takes place through physical and human capital deepening. In the model, agents can invest in either one of the two projects available in the economy; one that is safe and less productive and another that is subject to uncertainty but more productive. The latter project is the one associated with the ‘learning-by-doing’ cost described above.

Based on some of the theoretical outcomes of the model as well as the results of numerical experiments, we are able to explain a variety of outcomes in a unified framework. Firstly, we show that the diversity in growth and inequality outcomes of transitional economies can be traced to the differences in initial productivities of projects and the cost of adoption across countries. To elaborate further on the nature of this adoption cost, it consists of the cost of ‘learning by doing’ as well as the cost of returns foregone due the risk diversification opportunities that would otherwise have been available, had financial markets and institutions been well developed. We emphasize that this cost is not necessarily limited to ‘monetary value’ paid, but also a ‘time component’ representing the time spent learning and acquiring knowledge that can only be gained experientially. Secondly, we show that persistence in cross-dynasty inequality can be traced to the delays in physical and human capital deepening by the poor dynasties due to their limited initial resource endowment. A feature of the model is that both growth and inequality are subject to fluctuations over time.
These fluctuations are, in part, due to the presence of uncertainty, but intrinsic to the model is the possibility of reversals in the growth process, even if uncertainty were to be absent.

The current study is related to a number of the studies. Firstly, like Greenwood and Jovanovic (1990) and Townsend and Ueda (2006) we explore the relationship between growth, inequality and finance. However, unlike these studies we do not explicitly model financial intermediation, but rather focus on the growth and inequality outcomes of physical and human capital deepening in an environment where inadequate financial reforms place a wedge on risk diversification. Secondly, like Atkinson and Stiglitz (1980), Townsend (1982), Bardhan et al. (2000) we indirectly explore the idea that cross-dynasty inequality is exacerbated by the fact that financial markets and institutions can facilitate risk diversification, risk pooling and risk sharing for rich dynasties thereby allowing them to take high-risk high-return investment opportunities at the expense of the poor. However, our study does not explicitly model risk diversification. We are only concerned about the implicit cost of diversifying risk (i.e. in the context of time and resources spent in acquiring knowledge of businesses and projects that are intrinsically risky but yield a high return) when financial markets and institutions are only partially reformed.

Thirdly, this study is related to several strands of literature that emphasize the role of human capital deepening in explaining growth and inequality (see Lahiri and Ratnasiri, 2012 and references therein). However, while they interpret their cost of human capital deepening in the form of costs of learning new technologies in the absence of risk, our interpretation is on learning of the type that occurs in the presence of risk. More specifically, the model has an AK structure and is somewhat similar to the technology-adoption model of Lahiri and Ratnasiri (2012), except that in our model the technology that is (on average) more productive is subject to uncertainty. Consequently, using our model, we are able to unearth some outcomes in addition to those of the former model. Specifically, the presence of uncertainty exacerbates the diversity of outcomes that emerge from the model.

To elaborate on this point, the benchmark model of this paper is a refinement of Lahiri and Ratnasiri (2012) and this refinement adds value in the sense that once having stripped the model of inessentials, additional and sharper insights emerge. In particular, the addition of uncertainty in the model helps interpret the experiences of some economies in a number of additional and distinct ways to those suggested in the model by Lahiri and Ratnasiri (2012). For instance, our model suggests that the high fluctuations in inequality in some economies discussed above such as Thailand and Costa Rica may be linked to the existence of a low cost of diversifying idiosyncratic risk when the average productivities of projects subject to uncertainty are high relative to safe projects. Secondly, unlike in Lahiri and
Ratnasiri (2012) where the ‘poverty traps’ and ‘dual economy’ scenarios each have a single type of equilibrium associated with a single set of productivity parameters and adoption cost, our model shows that each of these two scenarios has more than one type of equilibria, each associated with different sets of productivity parameters and adoption costs. Furthermore each of these equilibria shows its own unique growth and inequality patterns, thus suggesting the existence of a ‘diversity within diversity’. In summary, the addition of uncertainty in the model results in richer range of growth and inequality patterns, thus enhancing the model’s ability to account for the diverse patterns that are observed in the data.

The remainder of the paper is organised as follows. In Section 2, we describe the economic environment, particularly focussing on the theoretical implications of the general version of the model. In Section 3, we conduct some numerical experiments to illustrate the insights derived from the analysis presented in Section 2. Section 4 concludes the paper. The appendix presents technical details of the analysis in Section 2.

2. The Economic Environment

The economy consists of N two-period lived overlapping generations of agents whose wealth holdings are heterogeneous. Each agent is born with a unit of unskilled labour endowment that can earn them a subsistence wage, \( \bar{w} \). Apart from the subsistence wage, an agent born in period \( t \) also inherits wealth from their parents in the form of bequests. Time is discrete, with \( t = 0, 1, 2, ... \). The initial distribution of wealth is described by \( W(\cdot) \).

In each period \( t \), agents must decide which among two projects they should invest in. We refer to these projects, Project A and Project B. Investment in Project A is both cost-free and risk-free. However, this project less productive, giving a time-invariant return of \( \phi \). Project B has a stochastic return, and is, on average, more productive than Project A. However, the investment in Project B is subject to an exogenous investment cost, \( \delta \). Furthermore, the return on Project B is divided into parts. The first part of the return \( \eta > \phi \) is certain. The second part of the return \( \epsilon \) is subject to uncertainty and it depends on the type of shock that Project B is subjected to. If the shock is bad, and this occurs with the probability \( p \), then \( \epsilon = \epsilon_b < 0 \), while if the shock is good \( \epsilon = \epsilon_h > 0 \). The magnitude of \( \epsilon \) is such that the return to project B in the bad state is lower than the return from project A.

The economy’s output \( (Y) \) depends on capital \( (K) \) invested in each of the projects. The production functions \( F(K) \) assume a simple “AK” specification. More specifically we specify the production functions for project A and project B as \( F(K) = AK \) and \( F(K) = BK \).
respectively, where \( A \) and \( B \) are the respective total factor productivities associated with the projects, where \( A < B \). We assume that the total factor productivity parameters are time invariant. In the context of this model, \( K \) represents a composite good embodying both human and physical capital. However, we emphasize the dominance of the ‘human’ component, which can be interpreted as investment in higher level skill needed to invest in the more productive project.

In this case \( \delta \) can be conveniently interpreted as an implicit fixed cost that results due to the experiential learning required when entrepreneurs undertake risk in the absence of the risk diversification options offered when markets and institutions are well developed. This cost has a skill component, acquired through learning how to use Technology B, and a component that reflects the knowledge gained through the experience of reading the market conditions and making the best use of the technology in the given circumstances.

Apart from representing the cost of acquiring the higher level skill, \( \delta \) can also be interpreted as resulting from institutional and structural aspects of the economy. For instance, a country with poor legal institutions, agents may end up paying higher than necessary costs, in the form of bribes or search costs to acquire the higher level skill, resulting in a high \( \delta \) for the economy. Similarly an economy whose historical investment in education is very low may end up having a limited supply of educators relative to the demand for the services resulting in a higher \( \delta \).

Every period each generation faces a problem on whether they should invest in project A or project B. The choice of which project to invest in is not necessarily dependent on the project that their parents invested in. That is, offspring of parents who adopted B need not adopt A; it only depends on the magnitude of resources they inherit from their parents.

The agent does not consume in the first period of his life. The utility of the \( i \)th agent, in the event he/she invests in project A is described by:

\[
U(c^A_{it+1}, b^A_{it+1}) = \ln(c^A_{it+1}) + \theta \ln(b^A_{it+1})
\]

The preferences for agents who invest in Project B are described as follows:

\[
U(c^B_{it+1}, c^B_{h_it+1}, b^B_{it+1}, h^B_{h_it+1}) = p \ln(c^B_{it+1}) + (1 - p) \ln(c^B_{h_it+1}) + \theta_P \ln(b^B_{it+1}) + \theta(1 - p) \ln(b^B_{h_it+1})
\]

In equation (1), \( c^A_{it+1} \) and \( b^A_{it+1} \) denote period 2 consumption and bequests for agent \( i \) if he invests in Project A. In equation (2), \( c^B_{it+1} \) and \( b^B_{it+1} \) denote period 2 consumption and bequests.

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3 This type of preference structure is consistent with the idea that ‘consumption’ consists of household consumption which includes the consumption of the children. The agent therefore ‘consumes’ part of the consumption of his parents in the first period of his life and undertakes the consumption decision in the second period with his offspring in mind.
for agent $i$ if he invests in Project B, with superscripts $l$ and $h$ representing the nature of shock that the economy is subjected to. Subscript $l$ represents a bad shock while subscript $h$ denotes a good shock, where the probability of the bad shock is represented by $p$. In both equation (1) and equation (2), the parameter $\theta$ describes the extent of imperfect intergenerational altruism in the model.

Agents face different budget constraints depending on the project that they invest in. The budget constraint for agents that invest in Project A is as follows:

$$c_{it+1}^A = \phi (w + W_i) - b_{it+1}^A$$

(3)

where $W_i$ denotes resource endowment of the $i^{th}$ agent in period $t$ and all the other unknowns are as defined earlier. Resource endowments for agents depend on the project that their parents invested in. For agents whose parents invested in Project A, the endowment is given by $W_i = W_i^A = b_i^A$. The endowment of agents whose parents invested in Project B is given by:

$W_i = W_i^B = b_i^B$

Likewise, agents investing in Project B have the following state-contingent budget constraint:

$$c_{it+1}^B = (\eta + \varepsilon_i^x)(w + W_i) - b_{it+1}^{B,x} - \delta$$

(4)

where the superscript $x$ represents the nature of the shock (i.e. $l$ or $h$) that project B is subjected to.

Agent $i$’s problem is optimise his utility subject to his/her budget constraint. Agents investing in project A maximise equation (1) subject to constraint (3). This yields the following consumption and bequest plans:

$$c_{it+1}^A = \frac{\phi}{1 + \theta} [w + W_i]$$

(5)

$$b_{it+1}^A = \frac{\theta \phi}{1 + \theta} [w + W_i]$$

(6)

Alternatively, the optimal state-contingent plans for agents who invest project B depend on the sign of the shock that their parent faces. These are described by
The \( i^{th} \) agent will invest in project B iff.

\[
U^B(c_{it+1}^*, b_{it+1}^*) \geq U^A(c_{it+1}^*, b_{it+1}^*)
\]

where \( U^A \) and \( U^B \) denote the indirect utility functions for the agents investing in Project A and Project B respectively and the subscript * denotes the optimal choice of the variable in question. It can then be shown that this is equivalent to the following:

\[
\left[ (\eta + \varepsilon_i)(w + W_{it}) - \delta \right]^\nu \cdot \left[ (\eta + \varepsilon_h)(w + W_{it}) - \delta \right]^{(1-\nu)} \geq \phi[w + W_{it}]
\]

In equation (12), the LHS, represents the geometric average of the wealth that is accumulated when an agent invests in project B. The RHS gives the wealth that is accumulated by an agent who invests in project A.

Although we are not able to analytically solve equation (12) for the level of \( W_{it} \) (hereafter to be referred as \( W^* \)) that would equate the LHS to the RHS, there are reasons to believe that \( W^*_t \) exists. We provide an informal sketch of the proof here. Firstly, it is obvious from equation (12) that both the RHS and the LHS functions are continuous and increasing in \( W_{it} \). Secondly, since the adoption cost parameter \( \delta \) has the effect of shifting the LHS function downwards, there is a reason to believe the LHS is below the RHS at least for some low levels of \( W_{it} \). Thirdly, because project B is on average more productive than project A, it must be that the rate of increase of the LHS with respect to \( W_{it} \) is faster rate than that of the RHS and consequently the LHS will be above the RHS for some high level of \( W_{it} \). Assuming the above three points are satisfied, a \( W^* \) exists where the LHS function intersects with the RHS function. Given that there is a \( W^* \) that equates the RHS to the LHS, the following proposition can be made:
Proposition 1: There is an threshold level of initial endowment, $W^*$ that is required for an agent to invest in project B. This level of endowment is implicitly defined as the $W^*$ that solves equation (13):

$$
\left(\eta + \varepsilon_i\right)\left(\bar{w} + W^*\right) - \delta \right)^\phi \cdot \left[\left(\eta + \varepsilon_h\right)\left(\bar{w} + W^*\right) - \delta \right]^{(1-p)} = \phi \left[\bar{w} + W^*\right] \\

(13)
$$

An agent will invest in Project B iff. $W_i \geq W^*$

To gain more intuition about $W^*$, we carry out some comparative static analysis to examine how this threshold level of endowment changes with the parameters of the model. This is done by implicitly differentiating of $W^*$ with respect to each of the parameters in equation (13). Our results are intuitively plausible; we find that $W^*$ is decreasing in parameters: $\bar{w}$, $\eta$, $\varepsilon_h$ and increasing in parameters $\phi$, $\delta$, $p$, $\varepsilon_i$. 4

As in Lahiri and Ratnasiri (2012) and Khan and Ravikumar (2002), the outcomes of our model are independent of inclusion of borrowing to finance investment in project B. Firstly, this is because access to borrowing may not directly help the agents since the investment needed to undertake project B is a human-capital intensive activity. Secondly, even if the agents can access consumption loans, borrowing to invest in project may not be economically tenable. This is because our model is such that agents who want to borrow access the loans from those who are willing to lend, who in this case are those who have invested in project B. For the lenders to be willing to lend, they should at least earn an interest equal to the expected net return on project B, i.e. $(\eta + \varepsilon_i)\left(\eta + \varepsilon_h\right)^{(1-p)} - 1$. However, this is not possible given that agents who adopt B must also incur the adoption cost $\delta$ . As such it is not possible to the lender and the borrower to reach a deal.

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4 More specifically the following are the partial derivatives: 
\[
\frac{dW^*}{d\eta} = -\frac{pX^2Y + (1-p)X^2Z}{p(\eta + \varepsilon_i)XZ + (1-p)(\eta + \varepsilon_h)XY - YZ} < 0, \quad \frac{dW^*}{d\varepsilon_h} = -\frac{pX^2Y}{(1-p)XZ + (1-p)(\eta + \varepsilon_h)XY - YZ} < 0, \\
\frac{dW^*}{d\phi} = \phi \left[\frac{p(\eta + \varepsilon_i)XZ + (1-p)(\eta + \varepsilon_h)XY - YZ}{XYZ \ln(Z/Y)}\right] > 0, \\
\frac{dW^*}{d\delta} = \frac{X(Z+Y)}{p(\eta + \varepsilon_i)XZ + (1-p)(\eta + \varepsilon_h)XY - YZ} > 0, \quad \frac{dW^*}{dp} = \frac{XYZ}{\ln(Z/Y)} > 0.
\]

where $X = \bar{w} + W^*$, $Y = (n+\varepsilon_i)(\bar{w} + W^*_h) - \delta$, $Z = (n+\varepsilon_h)(\bar{w} + W^*_h) - \delta$. Since $\bar{w} + W^*_h > \bar{w} + W^*_h - \delta$, it is easy to see that $p(\eta + \varepsilon_i)XZ + (1-p)(\eta + \varepsilon_h)XY - YZ > 0$. 12
The dynamics of the model are described by the evolution of bequests overtime. This is
given by the following truncated system of first order difference equations:

\[
W_{it+1}^A = \gamma^A [W_t + W_{it}] \quad \text{for } W_t < W^* 
\]

and

\[
\begin{align*}
W_{it+1}^{B,l} &= \gamma^{B,l} [W_t + W_{it}] - \theta \delta / (1 + \theta), \quad \text{with probability } p \\
W_{it+1}^{B,h} &= \gamma^{B,h} [W_t + W_{it}] - \theta \delta / (1 + \theta), \quad \text{with probability } (1 - p)
\end{align*}
\]

where \( W_{it+1} = b_{it+1} \) in equilibrium, \( \gamma^A = \frac{\theta \phi}{1 + \theta} \), \( \gamma^{B,l} = \frac{\theta(\eta + \varepsilon_l)}{1 + \theta} \), \( \gamma^{B,h} = \frac{\theta(\eta + \varepsilon_h)}{1 + \theta} \), and \( W^* \) is defined in Proposition 1. Equations (12) and (13) highlight the importance of the
slopes, \( \gamma^A \), \( \gamma^{B,l} \), \( \gamma^{B,h} \) of the bequests function in determining the dynamics of the model.

These slopes are proportionally related to the productivities of the respective projects.\(^5\) Of
particular importance are the sizes of these slopes relative to the \( 45^{\theta} \) line, which has a slope
equal to 1.

To develop some intuition regarding how the economy evolves over time, we analyse a
‘deterministic’ version of the model in which project B is associated with a deterministic
return equal to a weighted average of the ‘low’ and ‘high’ shock cases. That is, in the case of
project B, we do not analyse the slopes of the two bequest functions individually, but we
focus on the weighted slope i.e. \( \gamma^{B,w} = p \gamma^{B,l} + (1 - p) \gamma^{B,h} \).\(^6\) Consequently, for project B,
equation (13) can be written in as a single weighted wealth function as follows:

\( W_{it+1}^{B,w} = \gamma^{B,w} [W_t + W_{it}] - \theta \delta / (1 + \theta) \). Combined with equation (12), then, this function
describes what happens ‘on average’ in the stochastic version of the economy.

Depending on the parameter representing the cost associated with project B, \( \delta \) and the
productivity parameters, \( \gamma^A \), \( \gamma^{B,w} \), three possible outcomes are possible. These
predictions can be labelled ‘poverty trap’, ‘dual economy’, and ‘balanced growth’,
respectively.

Within the poverty trap case there are two possibilities, characterized by Figures 1(a) and
1(b) respectively. In Figure 1(a) we have a situation in which agents above \( W^* \) adopt B,

\(^5\) Note that the productivity of project B depends on the shock that the economy is subjected to.

\(^6\) The analysis here is somewhat similar to that of Lahiri and Ratnasiri (2012). The deterministic version of our
model may in fact be regarded as a special case of the model of that paper.
However, given the dynamics of the system, they converge to the steady state associated with technology A. In Figure 1(b), even though the productivity of technology B is high enough so that the bequest functions of agents with wealth above $W^*$ have a slope greater than unity, the initial distribution is such that all agents have a wealth level below $W_{B}^*$, the unstable steady state associated with the technology B. As such, all dynasties in the economy converge to $W_{A}^*$, the stable steady state associated with Technology A. Because project A is the only one that exists in the long run, inequality always converges to zero irrespective of the conditions under which the poverty trap arises. However, long run growth rate is likely to differ depending on the productivity parameter of project A.

There are two cases in which a ‘dual economy’ arises, and these are characterized by Figures 2(a) and 2(b) respectively. Figure 2(a), as evident, is identical to Figure 1(b), but the initial distribution of the economy is such that there are some agents with a wealth level above that of the unstable steady state $W_{B}^*$. The dynasties of these agents experience continuous growth, while all dynasties with wealth level below $W_{B}^*$ converge to the stable steady state associated with technology A. This ‘dual economy’ is associated with a growth rate that is above that of the ‘poverty trap’ as well as high and persistent inequality.

In Figure 2(b) we have a different type of dual economy. Here the productivity associated with both technologies is relatively low, so that the slopes associated with them are below 1, and other parameters of the model are such that some agents in the initial distribution fall above $W^*$. The dynasties with initial wealth above $W^*$ then converge to the stable steady state associated with Technology B, while the remaining dynasties converge to the stable steady state associated with Technology A. Since all agents who invest in project B have a unique and stable steady state, inequality in the dual economy shown by Figure 2(b) is not as high as the inequality that results in the dual economy shown in Figure 2(a).

Finally, in Figure 3 we present the balanced growth case. Here, the productivity associated with both technologies is high, leading to slopes of bequest functions associated with them that are above unity. It is straightforward to see that the dynamics of the model imply complete adoption of the Technology B by all dynasties in the economy, regardless of the initial distribution characterizing the economy.

The inequality within the balanced growth economy increases sharply in the transition towards the long run and is persistent. This is because of two reasons. Firstly the fact that some agents invest in project A and some in project B means that the latter group will become rich and catching up will be take time. Secondly, in the stochastic version of this
economy, catching up is made more difficult due to the fact that project B is subject to uncertainty. For instance, agents of a particular dynasty may face a bad shock soon after their parents switch from project A to project B resulting in these agents falling back to project A. Since all agents eventually invest in project B, growth rate of the economy is driven by the productivity of this project and it is higher than the growth rate under the ‘poverty trap’ and the ‘dual economy’.

Figure 1(a): Poverty trap case 1.

Figure 1(b): Poverty trap case 2.
Figure 2(a): Dual economy case 1

Figure 2(b): Dual economy case 2

Figure 3: Balanced growth case
3. Numerical Experiments and Discussion

In this section we use numerical experiments to illustrate the intuition underlying the theoretical predictions that were reported in the preceding section. The initial distribution of wealth for the reported results is assumed to be lognormal with a mean of 2.5 and standard error of 0.4 and there are 501 agents. The parameter values used in this analysis for the three cases discussed above are reported in Table 1. Also reported are the slope/productivity parameters that result from these parameter choices.

Table 1: Parameter Values

<table>
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<tr>
<th>Case</th>
<th>φ</th>
<th>η</th>
<th>θ</th>
<th>ε_h</th>
<th>ε_l</th>
<th>p</th>
<th>δ</th>
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<th>(\gamma_{B,h})</th>
<th>(\gamma_{B,1})</th>
<th>(\gamma_{B,w})</th>
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<td>0.375</td>
<td>0.875</td>
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<td>Poverty Trap: Case 2</td>
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<td>2.25</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0.5</td>
<td>15</td>
<td>0.65</td>
<td>1.625</td>
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<td>1.125</td>
</tr>
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<td>Dual Economy: Case 1</td>
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<td>0.70</td>
<td>1.625</td>
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<tr>
<td>Balanced growth</td>
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<td>1</td>
<td>-1</td>
<td>1</td>
<td>0.5</td>
<td>15</td>
<td>1.125</td>
<td>2.100</td>
<td>1.100</td>
<td>1.600</td>
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Figure 4(a) illustrates the implications for project choice, inequality and economic growth for the ‘poverty trap’ case 1. Panel (a) of Figure 4(a) shows the number of agents that invest in project A or B in different time periods. As evident in panel (a), although both project A and project B co-exist in the initial stages, all agents in the economy eventually adopt the former project. Panel (b) illustrates the evolution of inequality within this ‘poverty trap’ economy over time. The inequality within this economy initially fluctuates at high levels and then decreases sharply in the transition process and eventually converges to zero. The initial, relatively high level of inequality is due to the fact that some agents adopt project A while others adopt project B. However, because project B is costly yet not very productive, agents who invest in this project find themselves unable to pass sufficient resources to their offspring to enable them to adopt project B and consequently the offspring switch to project A. This explains the rapid fall in inequality and the convergence of inequality to zero. The initial fluctuations in inequality reflect the uncertain nature of the wealth of the agents who initially adopt project B. Once project A is fully adopted, inequality stabilises. In panel (c) of Figure 4(a) we show the behaviour of growth. For the same reason as in the case of inequality, average growth is subject to fluctuations in the initial stages. However, growth sharply increases in the transition from below 0.7% and converges to a steady state of 1%.

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7 We choose parameters in such a manner that optimal consumption choice under the bad state is restricted to positive values i.e. \((\eta + \varepsilon_l) (\bar{w} + W_{l,1}) \geq \delta\).
Figure 4(b) illustrates the implications for project choice, inequality and economic growth for the ‘poverty trap’ case 2. Here the productivity of project B is above average while that of project A is below average. However, because project B is too costly relative to the endowment of most agents very few agents initially invest in project B. Moreover, because the few agents that invest in project B may face a bad shock, the entire economy ends up adopting project A in the steady state. This is evident in panel (a) of Figure 4(b). The transitional behaviour of growth and inequality for case 2 are quite similar to that of case 1. However, in case 2, the initial fluctuations in both growth and inequality are more pronounced that in case 1. Moreover, because the productivity of project B is above average, it takes more time for both growth and inequality to converge to their steady state equilibrium.

*Figure 4(a) Project choice, inequality and economic growth in the Poverty Trap case 1*
Figure 5(a) presents the numerical results for the ‘dual economy’ case 1. In this economy project A and project B co-exist in the steady state. Agents who adopt project A will be poorer while those that invest in project B will be richer. Thus, as shown in panel (b) of Figure 5(a), this economy is characterised by very high levels of inequality. Because of the stochastic nature of project B, project choice as well as the growth and inequality patterns of this economy are subject to fluctuations. The average growth rate for this economy is slightly above that of the ‘poverty trap’. This is due to the component of growth that is emanates from agents who invest in the more productive project B.

Figure 5(b) presents the numerical results for the ‘dual economy’ case 2. There are three notable differences between these results and those of case 1. Firstly, comparing panel (a) for the two cases it is clear that in case 1, more agents invest in project B than in case 2. The main reason for this is that in case 2 the productivity of project B is below average. However, in case 2 agents invest in this project because the cost of investing in the project is low relative to their initial endowment. Secondly, the fluctuations in growth in case 1 are much more than those in case 2. Case 2 is analogous to the deterministic version of the model presented in Figure 2(b) where both technologies are associated with a stable steady state while case 1 is analogous to the case presented in Figure 2(a) where technology B is associated with an unstable steady state. Therefore in case 2, there is a tendency for agents whose wealth is above $W^*$ to settle around an average which is represented by the non-
stochastic steady state in Figure 2 (b). Similarly, in case 1 agents with wealth sufficiently higher than the non-stochastic unstable steady state presented in Figure 2(a) experience continuous growth, albeit one that is subject to fluctuations. Thirdly, while in case 1 transitional inequality sharply increases before becoming very high and persistent, in case 2, inequality never reaches steady state as it keeps fluctuating between 0.19 and 0.8. This is because in case 2 no agent will experience continuous growth as all agents who invest in project B converge around the non-stochastic, stable steady state under that project (see Figure 2(b) in the analytical analysis, which represents the deterministic version of this economy). The fluctuations in equality as shown in Figure 5 are an empirical feature of several transitional economies (see for instance the inequality experiences of Costa Rica and Thailand in Figure 1(b)).

*Figure 5(a): Project choice, inequality and economic growth 'dual economy' case 1*
Figure 6 presents the numerical results for the ‘balanced growth’ case. In this case, the productivities of both projects are high enough to allow agents’ wealth to grow at a relatively high rate. Consequently, all agents in the economy eventually invest project B. Inequality in this economy increases in the transition process and remains persistently high. The high level and persistence of inequality is due to the fact that agents switch from project A to project B at different times. Furthermore, some of those that have switched to project B may be subjected to a negative shock resulting in their offspring reverting back to project A. Agents who switch to project B earlier and also receive a good shock will accumulate more wealth than those that remain in project A and those that invest in project B and face a bad shock. The steady state average growth rate for the ‘balanced growth’ economy is much higher than that of the ‘dual economy’ since both project are very productive. Furthermore, since only the stochastic project exists in the steady state, fluctuations in growth in the ‘balanced growth’ economy are much more pronounced than in the ‘dual economy’ case. The steady state project choice, growth and inequality outcomes of the ‘balanced growth’ are independent of cost of associated with adopting project B. However, the cost tends to affect transitional behaviour of the ‘balance growth’ economy. More specifically, the time taken for the economy to fully adopt project B, and for growth and inequality to converge to their steady state paths is increasing in the adoption cost.
Figure 6: Project choice, inequality and economic growth in the ‘balanced growth’ case

We carried additional experiments to analyse how the initial level of inequality, the parameters representing the probability of a bad shock, $p$ and the degree of altruism, $\theta$ affect the predictions of the stochastic version of the model. While the basic predictions of the model remain qualitatively the same, one additional observation is worth noting. The changes in the initial inequality and the degree of altruism $\theta$ tend to affect the period that it takes for growth and inequality to reach steady state levels. More specifically, the higher the level of initial inequality, the longer time period taken for growth and inequality to reach their steady states. This result also applies for the degree of altruism, $\theta$.

4. Concluding Remarks

The purpose of this paper was to examine the link between growth and inequality in transitional economies when risk diversification is constrained by insufficient financial reforms. The underlying motivation for this study stems partly from three empirical observations. Firstly, it is motivated by empirical evidence suggesting the existence of intra-country and cross-country non-convergence of incomes overtime. Secondly, it is motivated by the evidence presented in Section 1 that suggests that there is diversity in growth

\[\text{\footnotesize Note that theoretically, } p \text{ and } \theta \text{ affect the slope of the bequest function thus do not change the main predictions of the model.}\]
experiences of countries during and after implementation structural and institutional reforms, for instance financial reforms. Thirdly, it is motivated by the empirical observation that some countries are characterised by high fluctuations in inequality while others are characterised by much smoother changes in inequality.

We develop a simple endogenous growth model in which growth takes place through physical and human capital deepening, and agents are heterogeneous in their initial resource endowments. In the model, an agent faces the choice of investing in two projects, one that is safe but has low return and another that has high return but is subject to uncertainty. Furthermore, there is a cost associated with investment in the high return project. We interpret this cost as an implicit fixed cost that results due to experiential learning required by entrepreneurs when they take risk in the absence of well developed financial institutions and markets.

Depending on the initial productivity differences between the projects and the implicit fixed cost of investing in project B, three outcomes are possible. We characterize these as ‘poverty trap’, ‘dual economy’ and ‘balanced growth’. Further analysis of these three outcomes reveals that there exists a ‘diversity within diversity’ in the ‘poverty trap’ and ‘dual economy’ outcomes. More specifically, each of these two outcomes has more than one type of equilibria each associated with different sets of productivity parameters and adoption costs. Moreover, each of these equilibria shows its own unique growth and inequality patterns. Consequently, our model is able to account for the diverse growth and inequality patterns observed in the data. Further numerical results reveal that delays in convergence of growth and inequality to their steady state paths are related to initial inequality in resource endowment of agents as well as the degree of altruism.
Appendix 1:

Proof of Proposition 1

Agents invest in project B iff indirect utility of project B is greater than indirect utility of project A. This implies that agents invest in project B

\[ U^B(c^{B,l}_{it+1}, c^{B,h}_{it+1}, b^{B,l}_{it+1}, b^{B,h}_{it+1}) \geq U^A(c^{A,l}_{it+1}, b^{A}_{it+1}) \]

Substituting for the functional forms of the utility function we get,

\[ p \ln(c^{B,l}_{it+1}) + (1 - p) \ln(c^{B,h}_{it+1}) + \theta \ln(b^{B,l}_{it+1}) + \theta(1 - p) \ln(b^{B,h}_{it+1}) \geq \ln(c^{A,l}_{it+1}) + \ln(b^{A}_{it+1}) \]

Recognising that \( b^{A}_{it+1} = \alpha c^{A}_{it+1} \) we can substitute for \( b^{A}_{it+1} \) and simplify the resulting inequality to obtain the following:

\[ p \ln(c^{B,l}_{it+1}) + (1 - p) \ln(c^{B,h}_{it+1}) + \theta \ln(\alpha c^{B,l}_{it+1}) + \theta(1 - p) \ln(\alpha c^{B,h}_{it+1}) \geq \ln(c^{A,l}_{it+1}) + \ln(\alpha c^{A}_{it+1}) \]

Using laws of logarithms and then simplifying the above inequality we can obtain:

\[ p \ln(c^{B,l}_{it+1}) + (1 - p) \ln(c^{B,h}_{it+1}) \geq \ln(c^{A}_{it+1}) \]

Rewriting \( c^{A}_{it+1}, c^{B,l}_{it+1}, c^{B,h}_{it+1} \) in terms of their definitions in steady state equations (5), (7), (8) we obtain the following:

\[ p \ln \left[ \frac{(\eta + \epsilon_l)\left[\bar{w}+W_h\right] - \delta}{(1 + \theta)} \right] + (1 - p) \ln \left[ \frac{(\eta + \epsilon_l)\left[\bar{w}+W_h\right] - \delta}{(1 + \theta)} \right] \geq \ln \left[ \frac{\phi \left[\bar{w}+W_h\right]}{(1 + \theta)} \right] \]

Using laws of logarithms, we can simplify the above inequality to get the following:

\[ p \ln \left[ (\eta + \epsilon_l)(\bar{w}+W_h) - \delta \right] + (1 - p) \ln \left[ (\eta + \epsilon_l)(\bar{w}+W_h) - \delta \right] \geq \ln \left[ \phi (\bar{w}+W_h) \right] \]

Since \( \ln \) is a monotonic transformation, it must be that:

\[ \left[ (\eta + \epsilon_l)(\bar{w}+W_h) - \delta \right]^p \cdot \left[ (\eta + \epsilon_h)(\bar{w}+W_h) - \delta \right]^{(1-p)} \geq \phi [\bar{w}+W_h] \]

Since there is a level of endowment \( W^* \) that equates the RHS to the LHS, we can write:

\[ \left[ (\eta + \epsilon_l)(\bar{w}+W^*) - \delta \right]^p \cdot \left[ (\eta + \epsilon_h)(\bar{w}+W^*) - \delta \right]^{(1-p)} = \phi [\bar{w}+W^*] \]
References


