A realized approach to estimate conditional alphas

This paper proposes a two-step procedure to back out the realized alpha of a given stock from high-frequency returns. The first step estimates the realized factor loadings of the stock, whereas we retrieve the conditional alpha by estimating the conditional expectation of the stock return in excess over the realized risk premia. In particular, consider the following factor model for returns:

\[ r_{i,t} = \alpha_{i,t} + \sum_{k=1}^{K} \beta_{i,k,t} F_{k,t} + \epsilon_{i,t}, \]

where \( F_t = (F_{1,t}, \ldots, F_{K,t}) \) denote a vector of observable factors at the high frequency. For instance, \( F_t \) would include the S&P 500 index and its powers within the context of the CAPM with higher-order moments, whereas one could employ exchange-traded funds (ETFs) based on size and book-to-market considerations to proxy for the Fama-French three-factor model.

To estimate the realized factor loadings, we must first orthogonalize the factors \( F_t \) by taking linear combinations of the intraday returns on the risk factors, namely, \( \tilde{F}_{m,t} = B_t F_{m,t} \) with \( \tilde{F}_{k,m,t} \perp \tilde{F}_{\kappa,m,t} \) for all \( 1 \leq k \leq \kappa \leq K \) as well as for every instant \( m \) within day \( t \) (or week, whatever). Note that the rotation matrix \( B_t \) is known even if it changes every day and hence it is possible to recover the original (daily) factor loadings \( \beta_{i,k,t} \) from the daily loadings of the orthogonal factors \( \tilde{\beta}_{i,k,t} \). We estimate the latter using the standard realized beta approach, yielding a realized loading for each orthogonal factor given by \( \tilde{\beta}_{i,k,t}^{(M)} \) and so a realized risk premium of

\[ \sum_{k=1}^{K} \beta_{i,k,t}^{(M)} \tilde{F}_{k,t} = \sum_{k=1}^{K} \beta_{i,k,t}^{(M)} F_{k,t}. \]

By subtracting the realized risk premia from the individual stock return, we find the realized counterpart of \( \epsilon_{i,t} = \alpha_{i,t} + \epsilon_{i,t} \), that is to say, \( \epsilon_{i,t}^{(M)} = r_{i,t} - \sum_{k=1}^{K} \beta_{i,k,t}^{(M)} F_{k,t} \). Identification of the conditional alpha results from the fact that the conditional expectation of \( \epsilon_{i,t} \) is zero, whereas \( \alpha_{i,t} \) is measurable in the information set. It thus follows that \( \alpha_{i,t} = \mathbb{E}(\epsilon_{i,t}|Z_t) \), where \( Z_t \) is the vector of state variables. The second step of the procedure then amounts to estimating \( \alpha_{i,t}^{(M)} = \mathbb{E}(\epsilon_{i,t}^{(M)}|Z_t) \) using kernel methods. Note that there is an extensive list of state variables to include in \( Z_t \) if we pay attention to the conditional alpha-beta literature. This means that we should think about employing dimension-reduction techniques by imposing either an additive or a single-index dependence structure.
To do

1. characterization of the error in the realized beta estimation

2. estimation of conditional alpha with known factor loadings

3. estimation of conditional alpha with realized factor loadings

4. (asymptotic and/or bootstrap-based) confidence bands for conditional alpha

5. test for the null of constant (or zero) alpha for all $t$

6. test for the null of negative (or positive) alpha for all $t$

7. propose a framework to analyze persistence in the conditional alpha