Estimation of operational risks using a semi-parametric approach*

Ainura Tursunalieva† and Param Silvapulle‡

Abstract

This paper employs a semiparametric framework for estimating 99.9 per cent operational risks (ORs) and expected shortfalls (ESs) of some US businesses. The latter is a coherent risk measure, while the former is not. Since the severity loss distributions are heavy right-tailed, the peaks over threshold method, which is known as POT, is used. To implement this method, we estimated the threshold parameter by adopting the approach proposed by Huisman et al. [2001]. The Generalised Pareto Distribution (GPD) is then fitted to the data in the right tail. In contrast to the previous studies that estimated the population mean by the simple sample mean, this paper used the method proposed by Johansson [2003] which exploits the property of the tail index to estimate the population mean. The estimates of 99.9 per cent ORs and ESs of five business line and three event type losses, range from $1,361 mln to $12,547 mln and from $1,683 mln to $20,350 mln, respectively.

Keywords: Heavy-tailed distribution, Generalized Pareto distribution, Semi-parametric method, Value-at-Risk, Expected Shortfall

JEL Classification: C13, C14, C46

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*This research is supported by ARC Discovery Project (DP0878954).
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1 Introduction

Modelling and estimating operational risks of internationally active banks and financial institutions have become very popular in the recent literature on quantifying and managing risks. Following the recent global financial crisis, the Basel Committee on Banking Supervision [2010], BCBS (2010) hereafter, has focused on some practical challenges associated with operational risk measurement framework.

BCBS (2010) emphasises the importance of the reliable estimation of operational risk which determine the appropriate level of capital requirement to cover the unexpected large losses. Operational risk is defined as the unexpected loss at the 99.9 per cent confidence level. Severity distributions of operational losses display heavy right tails. Another common feature of the loss data is that the large loss events are rare, and thus the data in the high quantile regions of the loss distributions is scarce. Therefore, the main focus is on modelling the tail behavior of operational losses, which is difficult to establish with the very small number of observations. To this end, the Extreme Value Theory (EVT) provides the basis for modelling the extreme losses (see, for example, Embrechts et al. 1997). In the peaks over threshold (POT) methodology, which is used in this paper, a parametric model is fitted to the right tail of the distribution (see for example, Smith 2003). We use Generalised Pareto Distribution (GPD) to model the losses exceeding a sufficiently high threshold parameter, which is estimated by the method proposed by Huisman et al. [2001]. Because of the reliance of application of POT method on the threshold parameter estimate, we assess its robustness by a method based on the mean excess function.

Although the far end of the right tail has only a small number of observations, fitting the extreme value distribution is attractive, because this distribution has only one
or two parameters. Furthermore, once the parametric distribution is estimated, one can estimate the high quantile related measures even if there is no observed data fall in the far end of the high quantile regions. Despite the Basel Committee on Banking Supervision [1996] approving the quantile based value at risk (VaR) measures for calculating capital requirements for banks, researchers showed subsequently that the VaR is not a coherent risk measure. VaR does not satisfy the sub-additivity condition, whereas an alternative risk measure known as expected shortfall (ES) satisfies this condition and therefore is a coherent risk measure; see, for example Artzner et al. [1999], Kusuoka [2001], Acerbi and Tasche [2002]. In this paper, we compute the 99.9 per cent OR and ES for five business lines and three event types occurred in the United States.

The structure of the paper is as follows: Section 2 briefly discussed the methodology used for modelling the tails and high quantile of heavy-tailed distribution. Section 3 presents the method for estimating the population mean of the heavy-tailed loss distribution. In Section 4, we discuss the operational loss data, empirical estimation of ORs and ESs and their results. The final Section 5 concludes the paper.

2 Peaks Over Threshold method and Generalised Pareto Distribution

The POT method is used to analyse the excesses over a high threshold $u$: $|X_i - u| X_i > u$, $i = 1, ..., n$. A model for extreme losses is based on the parametric form of the limit distribution of excesses over the threshold.

To explain the main idea of this approach, we denote data $X_1, X_2, \ldots, X_n$ and the corresponding excesses $Y_1, Y_2, \ldots, Y_n$ over the threshold $u$, where $Y_i = X_i - u$, for
$i = 1, \ldots, n$. We assume, the observations $X_1, X_2, \ldots, X_n$ are i.i.d. with distribution function $F \in MDA(H_\xi)$ for some $\xi \in \mathbb{R}$, where $MDA(H_\xi)$ is in the maximum domain of attraction of GPD with the shape parameter $\xi$. For a selected threshold, say $u_n$, and the excesses $Y_1, Y_2, \ldots, Y_n$, asymptotically for $n$ large, a GPD $G_{\xi, \beta}$ with $\xi \in \mathbb{R}$ and $\beta > 0$ has the following tail distribution:

$$
\tilde{G}_{\xi, \beta}(x) = \begin{cases} 
(1 + \frac{\xi}{\beta}x)^{-1/\xi}, & \xi \neq 0, \\
\exp(-\frac{x}{\beta}), & \xi = 0,
\end{cases}
\quad x \in D(\xi, \beta),
$$

(1)

where the support of the GPD is:

$$D(\xi, \beta) = \begin{cases} 
[0, \infty) & \text{if } \xi \geq 0, \\
[0, -\beta/\xi) & \text{if } \xi < 0.
\end{cases}
$$

The distribution $F$ is heavy tailed when $\xi \geq 0$ with the tail index $\alpha = \frac{1}{\xi}$. When $\alpha < 2$, $F$ is heavy-tailed and is in the domain of attraction of a stable distribution, $\bar{F}(x) = x^{\frac{1}{\alpha}} L(x)$ with unspecified slowly varying function $L(\cdot)$. When $\alpha \geq 2$, $F$ is in the domain of attraction of a normal distribution. The tail index is one of the key measures in modelling extremes as it determines the tail behavior of $F$. The properties of the tail index clearly depend on the choice of $k$, which in turn, determines the threshold value. A suitable $k$ should be chosen so that the estimated parameters of the GPD are robust with respect to shifts in the threshold parameter estimate.

If the threshold is robust and sufficiently large, then the approximation of $G_{\xi, \beta}(x)$ by a GPD is more reliable, but it can be unstable if $k$ is very small. Once a threshold $u_n$ has been selected, the shape $\xi$ and scale $\beta$ parameters can be estimated
by maximum likelihood method. The question then is how to choose a suitable $k$.

Several methods were proposed in the literature for estimating the optimal $k$ (Dekkers and de Haan [1993], Hall and Welsh [1985], Beirlant et al. [1996a], Beirlant et al. [1996b], Draisma et al. [1999]). The detailed study conducted by Gomes and Oliveira [2001] found that the double bootstrap method of Danielsson et al. [2001] for estimating a positive tail index is preferable to other methods. This method is based on minimising the asymptotic mean squared error (AMSE) of the estimated tail index for a given $k$. However, the tail index estimated by minimising the AMSE suffers from the small-sample bias. In an extensive simulation study, Huisman et al. [2001] have shown that the estimator based on the weighted average of Hill estimators is unbiased in small samples. In what follows, we discuss the process of choosing an optimal threshold based on this estimator.

2.1 A weighted average of Hill estimators

To define the well-known and widely used Hill index [Hill, 1975], consider the order statistics $X_{n,1} \leq X_{n,2} \leq \ldots \leq X_{n,n}$ of the i.i.d. random variables $X_1, \ldots, X_n$. For a selected threshold $X_{n,n-k+1}$ (or $u_n$), there are only $k$ extreme observations lie in the farthest right tail, and use these $k$ number of observations to define the Hill index estimator as follows.

$$\gamma_k = \frac{1}{k} \sum_{i=1}^{k} \ln(X_{n,n-i+1}) - \ln(X_{n,n-k})$$ (2)

The tail index is estimated by $\frac{1}{\gamma_k}$. The main idea of this method is that if a random variable has a Pareto distribution then the log of this variable will have an exponential
distribution with the estimated tail index. The asymptotic normality of the Hill estimator was established by many authors. See, for example, Häusler and Teugels [1985], Beirlant and Teugels [1987], and Beirlant et al. [2004]. As was mentioned before, the accuracy of the Hill estimator depends on the sample size $n$, and the number of extreme value statistics $k$.

In order to improve the conventional Hill index estimator defined in (2), Huisman et al. [2001] proposed a tail index estimator which, in fact, is the weighted average of Hill estimators for various values of $k$. To outline this method, consider the following model:

$$\gamma(k) = \beta_0 + \beta_1 k + \epsilon(k), \quad (3)$$

where $k = 1, \ldots, \kappa$.

Traditionally, the optimum $k$ is chosen first and then the tail index is estimated using (2). In the present method, however, the Hill index $\gamma(k)$ is estimated for every $k$ from 1 to $n/2$, and the estimated tail index is used as the dependent variable in (3). The property of the tail index exploited in developing the new estimator is that the Hill index estimator $\gamma(k)$ is unbiased only when $k \rightarrow 0$. Therefore, based on this property, the estimated $\beta_0$ is an unbiased estimator of the tail index. However, the OLS estimate of tail index is inefficient due to two reasons: (i) the variance of $\gamma(k)$ is not constant for different $k$, and thus the error term is heteroscedastic, and (ii) an overlapping data problem exists due to the way in which $\gamma(k)$ is constructed. To overcome these problems, Huisman et al. [2001] suggest to use a weighted least squares method, which is given as follows.
Multiplying (3) by the weight \( w(k) \), we obtain,

\[
\gamma(k)w(k) = \beta_0^{wls} w(k) + \beta_1^{wls} k w(k) + u(k)
\]

(4)

where \( w(k) \) is defined in a vector form as \( W = (\sqrt{1}, \sqrt{2}, \ldots, \sqrt{k}) \) and \( u(k) = \epsilon(k)w(k) \).

The new tail index estimator \( \hat{\beta}_0^{wls} \) is both unbiased and efficient. See Huisman et al. [2001] for details.

The estimator of \( k \) is chosen by minimising the distance between \( \gamma(k) \) and \( \hat{\beta}_0^{wls} \) which is obtained from (4). As a goodness of fit tests for \( k \) we use two graphical-based tools: the plot of the empirical mean excess function and the Hill plot, which are explained in the following section in detail.

### 2.2 Mean excess function and Hill plot

The mean excess function (MEF) proposed by Davison and Smith [1990] is widely used in practice to identify whether data exhibits a heavy-tailed behavior. This plot is based on the following properties: if the distribution is subexponential the MEF tends to infinity, if the distribution is exponential the MEF is a constant and if the distribution is normal the MEF tends to zero.

The MEF of a rv \( X \) with finite expectation and the right endpoint \( x_F \) is defined as follows:

\[
F_u(x) = P(X - u|X > u) \approx G_{\xi,\beta}(x), \ 0 \leq x \leq x_F
\]

(5)

\( F_u(x) \) is the excess df of the rv \( X \) over the threshold \( u \). The MEF \( e(u) \) for a random
variable $X$ with Pareto distribution $G_{\xi, \beta}$ is given as follows:

$$e(u) = E(X - u | X > u) = \frac{\beta + \xi u}{1 - \xi}, \ u \in D(\xi, \beta), \ \xi < 1,$$

(6)

hence $e(u)$ is linear in $u$. This suggests that the threshold should be selected so that $e_n(x)$ is approximately linear for $x \geq u$. The empirical counterpart of the MEF, used in data analysis, is given as follows:

$$\hat{F}_u = \frac{\sum_{i=1}^n X_{i,n} \mathbf{1}_{X_{i,n} > u} - u}{\sum_{i=1}^n X_{i,n} \mathbf{1}_{X_{i,n} > u}},$$

(7)

This empirical version is plotted against the values of $u = x_{i,n}$ for $k = 1, ..., n - 1$. Further details on the MEF can be found in Embrechts et al. [1997].

Another popular approach for choosing $k$ in practice is the Hill plot. Traditionally, Hill estimators, found by using (2) and inverted, are plotted together with the 95%-confidence bounds of the estimate. Using both the ME plot and the Hill plot to select the optimal threshold does not always provide a unique choice, however these two plots can be used as the additional tools for choosing the suitable $k$.

### 2.3 Maximum Likelihood Estimation of parameters and quantiles of GDP

Recall that the observations $X_1, X_2, \ldots, X_n$ are iid with the distribution function $F$. Assume that $F$ is GPD with two parameters $\xi$ and $\beta$ and the density function is:

$$f(x) = \frac{1}{\beta}(1 + \frac{x}{\beta})^{-\frac{1}{\xi} - 1}, \ x \in D(\xi, \beta).$$

(8)
The log-likelihood is:

\[ \ell((\xi, \beta); X) = -n \ln \beta - \left( \frac{1}{\xi} + 1 \right) \sum_{i=1}^{n} \left( 1 + \frac{\xi}{\beta} X_i \right). \]  \hspace{1cm} (9)

With the parameter estimates of the GPD, an estimator of the quantile \( x_p \) can be computed as follows:

\[ \hat{x}_p = u + \frac{\hat{\beta}}{\xi} \left( \left( \frac{n}{k} (1 - p) \right)^{-\xi} - 1 \right). \]  \hspace{1cm} (10)

Since the operational risk (OR) is defined as the difference between the \((1 - p)\) quantile and the mean of the distribution, we need to estimate the mean. We discuss the method that is specific for estimating the mean of heavy-tailed distribution in Section 3.

### 2.4 Risk measures: Value at Risk and Expected Shortfall

The main characteristic of a risk measure for calculating operational risk capital charge is its ability to provide a reasonably accurate measure of the capital charge. It is known that popular measures of risk such as VaR and/or ES may underestimate the risk of a loss variable with heavy tails. The VaR at the 99.9 per cent confidence level is the upper 99.9 percentile of a loss distribution. By definition, VaR measures the quantile of the distribution, and disregards extreme losses beyond the VaR. On the other hand, the ES at 99.9 per cent confidence level is defined as follows

\[ ES = E\{X_i|X_i > VaR\} \]
where VaR is the 99.9% OR.

In what follows, we state the four most important properties that a coherent risk measure should satisfy. Consider a set $V$ of real-valued random variables. A function $\rho : V$ is called a coherent risk measure if it satisfies the following four conditions:

1. monotonous: $X \in V, X \geq 0 \implies \rho(X) \leq 0$,

2. sub-additive: $X, Y, X + Y \in V \implies \rho(X + Y) \leq \rho(X) + \rho(Y)$,

3. positively homogeneous: $X \in V, hX \in V, \implies \rho(hX) = h\rho(X)$,

4. translation invariant: $X \in V, a \in \mathbb{R} \implies \rho(X + a) = \rho(X) - a$

A sub-additivity measure is a desirable property for calculating the capital adequacy requirements. The sub-additivity axiom does not hold for VaR and therefore it is not a coherent risk measure [Acerbi and Tasche, 2001]. On the other hand, ES satisfies the above four conditions, including the sub-additivity axiom, and hence it is a coherent risk measure. This means that if we compute a risk measure for a bank that has a number of branches, then a sum of individual ESs will constitute an upper bound for the ES of the bank. Since OR is calculated as a measure of quantile of the loss distribution, OR does not have this property.
3 Estimating the mean of heavy-tailed distribution

Let’s assume that the observations \( X_1, X_2, \ldots, X_n \) are i.i.d. with the distribution function \( F \), where

\[
\bar{F}(x) := 1 - F(x) = cx^{-1/\xi} = (1 + x^{-\delta}L(x)),
\]

for \( \xi \in (0, 1), \delta > 0 \), with unspecified slowly varying function \( L(\cdot) \). For \( \xi \in (1/2, 2) \), the sample mean \( \bar{X} \) converges to a stable distribution and the parameters of stable distributions are difficult to estimate [Durrett, 1996]. Johansson [2003] proposed the following estimator for \( \mu \), for which the tail is assumed to start at some threshold level \( u_n \) and \( \bar{F} \) is an estimate of the tail distribution function.\(^1\) In this paper, we use method proposed by Johansson [2003]. We fit GPD to the excesses over the threshold \( u_n \) and this means that for high threshold \( \hat{F}_u(y) = \hat{G}_{\hat{\beta}, \hat{\xi}}(y) \), then \( E[X] \) can be estimated as follows:

\[
\hat{E}[X] := \int_0^u xd\hat{F}_n(x) + \int_{u_n}^\infty xd\hat{F}(x) = \frac{1}{n} \sum X_i 1_{X_i \leq u} + \int_{u_n}^\infty x \frac{N}{n\hat{\beta}} \left( 1 + \xi \frac{x - u}{\hat{\beta}} \right)^{-1-1/\hat{\xi}} dx \]

\[
= \frac{1}{n} \sum X_i 1_{X_i \leq u} + \hat{p} \left( u + \frac{\hat{\beta}}{1 - \hat{\xi}} \right), \text{ for } \hat{\xi} \in (0, 1)
\]

\(^1\)A similar procedure was proposed by Peng [2001], where one parameter model is fitted to the tail. The estimator of \( \mu \) is based on the Hill index and is designed for distributions with the tail index above one.
where $\hat{p} = n/k$. This estimator of the mean is unbiased with an easily estimated variance; see Johansson [2003] for details.

4 Data

This paper analyses nominal operational losses occurred in the United Stated between May 1984 and May 2008. For comparison, we also report losses expressed in real terms using January 1997 level of the Consumer Price Index. Table 1 reports some descriptive statistics for losses across five business lines and three event types occurred in the United States. For all type of losses the mean is pulled in the direction of extremely large losses, indicating that these loss distributions are heavy skewed to the right. The skewness for all types of losses is above 5.4. Internal Fraud event type has the largest number of losses, about 30% of all losses. Trading and Sales business line has the most extreme losses. The standard deviation ranges from 69 for External Fraud to 399 for Trading and Sales.

4.1 Estimation of Operational Risks: an application to the business line Internal Fraud

To illustrate the process of estimating Operational Risk we discuss the estimation methods step by step for the Internal Fraud event type in detail. Figure 1 shows the plot of the time series of operational losses for this business line. The estimated threshold is $65.7$ mln., and the way in which this parameter estimated is discussed in the next paragraph. The pdf plot in Figure 2 shows clearly that the distribution of Internal Fraud event type losses is heavily skewed to the right. The pattern of
the Mean Excess Function presented in Figure 3 is not linear, confirming that the
distribution of *Internal Fraud* losses is indeed heavy tailed.

The first step is to choose the number of excesses $k$. We calculate the weighted
average of Hill estimators using (4) which is found to be 0.78. By minimising the
distance between this weighted average and the Hill estimators for different values
of $k$, we estimated the optimal $k$ that produces the corresponding Hill estimator as
close as possible to the weighted average of Hill estimators of 0.78. Such $k$ is found
to be 98. We also observe the Hill plot in Figure 4 that shows that the choice of the
threshold $u_n$ is somewhere above 84, while the ME-plot for $k = 98$ in Figure 5 shows
that ME function is approximately linear. From both the graphical tool based tests,
we can infer that the threshold is 98. The loss value of $65.7$ mln corresponds to this
optimum $k$. The tail index of the loss distribution of the *Internal Fraud* event type
is computed as $1/0.78 = 1.3$.

To obtain the OR, we estimate the mean and the 99.9 per cent quantile using the
equations (14) and (10) respectively. The estimated mean is 123.9, which is about 47
per cent higher than the sample mean of 84.3. The estimated quantile is 9308, and
hence the OR is 9184. The ES estimate is 12610 which is about 36 per cent higher
than the estimated OR.

### 4.2 Discussion of the results

While the sample means reported in Table 1 range from $23.6$ mln. to $93$ mln.,
the estimated adjusted means reported in Table 2 range from $27.0$ mln. to $405.9$
mln for all five business lines and three event types. All of the estimated adjusted
mean estimates are found to be larger than their simple sample means. The closer
the tail index to one the larger the difference between the adjusted mean and the sample mean of loss data with heavy right-tailed distribution. For the two business lines *Retail Brokerage* and *Trading and Sales* with the tail indexes of 1.0 and 1.1 respectively, the adjusted means are three times greater than their corresponding sample means. On the other hand, when the tail index is between 1.1 and 1.4 the adjusted mean is about 50 per cent larger than the sample mean. This difference is less than 20 per cent when the tail index is above 1.5. Note that if the tail index is close to 2.0 then the loss distribution is very close to normal, for which the simple sample average is the best estimate of the mean of the distribution.

As expected, the estimated values of ESs are greater than those of ORs. The difference between ESs and ORs is also larger when the tail index is close to one. For two *Retail Brokerage* and *Trading and Sales* losses with the tail index of 1.0 and 1.1 respectively, the estimated ES is about 60 per cent greater than corresponding OR. When the tail index is between 1.1 and 1.4 the ES is greater than the OR by 35-50 per cent. This difference is below 30 per cent when the tail index is above 1.5.

5 Conclusion

For banks and other financial institutions obtaining the reliable operational risk estimates is becoming an increasingly important issue. This paper presents two alternative risk measures of operational loss data of US businesses classified into five business lines and three event types. We estimate the 99.9 per cent OR and ES using a semiparametric approach. To ensure the reliability of the estimated risk measures, we employ statistical methods associated with heavy-tailed distributions. Since the operational losses are heavy right-tailed, we pay great attention to estimating the
threshold parameter by adopting the approach proposed by Huisman et al. [2001], along with the two graphical tool based tests. Once the threshold is estimated, the Generalised Pareto Distribution is fitted to the data in the far end right tail and and its shape and scale parameters were estimated. These parameter estimates are used to obtain the estimates of the quantile and the mean of the loss distribution, and hence the 99.9 per cent OR estimate.

The fact that for all types of losses the estimated adjusted means are larger than their sample means implies that the expected losses can be greatly underestimated. The adjusted mean was computed by taking into account the information about the degree of tail thickness. The difference between the adjusted mean and the sample mean increases as the estimated tail index approaches one. However, when the tail index is close to two then the loss distribution is normal, and hence this difference is very small.

The estimates of ORs and ESs of five business line and three event type losses, range from $1,388 mln to $12,952 mln and from $1,710 mln to $20,756 mln, respectively. It is clear that ESs are larger than ORs. Since ES is a coherent risk measure it is considered as a more reliable coherent risk estimator. Furthermore, the difference between OR and ES estimates is larger when the tail index is close to one (ES is about 60 per cent larger than OR). When the tail index is close to two ESs are about 10-20 per cent larger than ORs.

This paper provides practical examples to show the importance of making use of statistical measures associated with the heavy-tailed nature of loss distributions in the estimation of high quantile, 99.9 per cent OR, as well as a coherent risk measure ES based on which the capital charge requirements are calculated.
Figure 1: *Internal Fraud* losses in mln USD. The threshold line is equal to $65.7$ mln
Figure 2: PDF plot of Internal Fraud losses
Figure 3: Mean Excess Function of *Internal Fraud* losses excluding losses exceeding $65.7\text{mln.}$. 
Figure 4: Hill plot for *Internal Fraud* losses.
Table 1: Descriptive statistics of operational losses.

<table>
<thead>
<tr>
<th>Loss type</th>
<th>Sample size</th>
<th>Mean</th>
<th>Median</th>
<th>Skewness</th>
<th>Min ×100</th>
<th>Max</th>
<th>St. dev.</th>
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<tr>
<td>Retail Brokerage</td>
<td>269</td>
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<td>9.0</td>
<td>3.0</td>
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<td>93.0</td>
<td>8.7</td>
<td>8.3</td>
<td>2.5</td>
<td>4,400.0</td>
<td>399.0</td>
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<td>83.4</td>
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<td>360.1</td>
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<td>5.4</td>
<td>2.8</td>
<td>1,600.0</td>
<td>185.5</td>
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