A Conditional Copula Test of Contagion

Vance L. Martin and Jessie Xiaokang Wang

University of Melbourne

This Version: November 4, 2011

Abstract

A general test of contagion is proposed based on a comparison of conditional copula distributions of financial returns in crisis and noncrisis periods. An important feature of the approach is that a range of contagion channels are tested including the usual channels that operate through the first and second moments, as well as channels that operate through higher order moments including coskewness and cokurtosis. The finite sample properties of the test are examined using a range of Monte Carlo experiments and compared with existing tests of contagion. The empirical application of the global financial crisis shows that the contagion occurs mainly through the fourth moments.

Key words: Contagion; copula; conditional distributions; Neyman’s smooth test; simulation.

JEL Classification: C12, C13, C14, C15
1 Introduction

An important feature of financial crises is that global asset markets become more integrated than they are during noncrisis periods. This empirical phenomenon is commonly referred to as contagion which is characteristic of the most recent global financial crisis, as well as previous crises including Argentina in 2001 to 2005, dot-com in 2000, Brazil in early 1999, Russia and the LTCM crises in the second half of 1998, and the East-Asian crisis of 1997 to 1998. For a recent analysis of financial crises and the definition of contagion, see Dungey, Fry, González-Hermosillo and Martin (2010).

The earliest tests of contagion consist of the correlation approach of King and Wadhwani (1994) and King, Sentana and Wadhwani (1994), the adjusted correlation statistic of Forbes and Rigobon (2002), the outlier models of Favero and Giavazzi (2002) and Pesaran and Pick (2007), and the probability model of Eichengreen, Rose and Wyplosz (1995, 1996). A feature of these tests is that contagion channels primarily operate through the conditional mean of the distribution of asset returns. Extensions of these methods to allow for contagion channels through higher order conditional moments consist of the co-exceedance test of Bae, Karolyi and Stulz (2003), the conditional coskewness and cokurtosis tests of Fry, Martin and Tang (2010), the copula tests of Rodriguez (2007), Busetti and Harvey (2011) and Harvey (2010), and the contagion test based on mutual jumps of Aït-Sahalia, Cacho-Diaz and Laeven (2010), which allows for higher order moment dependence through endogenous Poisson jumps referred to the Hawkes process. The advantage of basing the contagion test on a copula distribution is that all moments of the joint distribution are automatically specified with the choice of the copula function controlling the dependence structure of asset returns. The gain from adopting a distributional approach is that it yields tests with higher power in detecting contagion channels which do not operate solely through the conditional mean and which would potentially remain undetected by implementing the earlier class of contagion tests.

An important lesson in designing a test of contagion between two asset markets is that it...
is necessary to take into account the increase in the volatility of returns in the source asset market, otherwise the test can be biased towards accepting the hypothesis of contagion. This point is made forcefully by Forbes and Rigobon (2002) who show that tests of contagion based on unadjusted correlations increase in the presence of higher asset market volatility even where there is no contagion. This property is best understood by reinterpreting the Forbes and Rigobon adjusted correlation test as a Chow test of a change in the slope parameter relating asset returns during a crisis period (Dungey, Fry, González-Hermosillo and Martín (2005)). Viewed this way the test amounts to testing that the conditional mean of the asset returns distribution of the recipient asset market changes during a financial crisis.

The aim of the approach adopted in this paper is to develop a test for contagion that acts as an omnibus method for detecting contagion as well as being able to identify underlying conditional moment channels that contagion operates through. A feature of the approach is that a copula is also used to specify the joint distribution function of asset returns, however to circumvent potential problems arising from spurious contagion channels as a result of increases in asset return volatility during crises, a conditional copula is specified where conditioning is based on asset returns in the source asset market. This type of conditioning is potentially different to existing copula specifications where conditioning consists of allowing the dependence parameters in the copula to be a function of lagged returns (Rodriguez) or an exponentially weighted moving average filter (Busetti and Harvey (2011), Harvey (2010)).

Having specified and estimated a conditional copula model, a test of contagion is based on testing whether the copula conditional distribution function of asset returns has changed over the sample from the noncrisis period to the crisis period. Under the null hypothesis of no contagion, evaluating the noncrisis conditional distribution function using returns in the crisis period would yield a uniform random variable. To detect any departures from uniformity and hence the presence of contagion, a generalized uniform distribution initially proposed by Neyman (1937) and recently applied by Bera and Ghosh (2001) and Bera, Ghosh and Xiao (2010), is specified. A Lagrange multiplier test is then constructed which provides
an omnibus test of contagion as well as information on potential contagious channels.

The rest of the paper proceeds as follows. Section 2 introduces the background idea for the conditional copula test of contagion and briefly reviews the literature. Section 3 presents the method of modelling the conditional distribution using a conditional copula. The contagion test is derived in Section 4, whose finite sample properties are investigated in Section 5 using a range of Monte Carlo experiments. In Section 6, the proposed test is applied to the global financial crisis to test for contagion amongst equity markets. Concluding comments are provided in Section 7.

2 The Role of Conditioning In Contagion Tests

Let $x_{i,t}$ be the asset returns in the $i^{th}$ market in the noncrisis period and $y_{i,t}$ be the corresponding asset returns of the same market in the crisis period. The following linear model describes the relationship between the two asset markets during noncrisis and crisis periods respectively, where asset market 1 is the source and asset market 2 is the recipient

$$
x_{2,t} = \alpha_x + \beta_x x_{1,t} + v_{x,t}, \quad [\text{Noncrisis period}]
$$

$$
y_{2,t} = \alpha_y + \beta_y y_{1,t} + v_{y,t}, \quad [\text{Crisis period}]
$$

(1)

where $v_{x,t}$ and $v_{y,t}$ are iid disturbance terms corresponding to the recipient asset market which have zero means and respective variances $\sigma^2_{v,x}$ and $\sigma^2_{v,y}$, and $\alpha_x, \alpha_y, \beta_x$ and $\beta_y$ are unknown parameters. As asset market 2 is conditional on asset market 1, it follows that increases in volatility in asset market 1 during a crisis, $\text{var} \ (y_{1,t}) > \text{var} \ (x_{1,t})$, immediately result in an increase in volatility in asset market 2 (Forbes and Rigobon (2002)). If this increase in volatility occurs without changes in the fundamentals, as represented by the parameters remaining unchanged over the full period

$$
\alpha_x = \alpha_y, \beta_x = \beta_y, \sigma^2_{v,x} = \sigma^2_{v,y}.
$$

(2)
the increase in asset market volatility is not the result of contagion. For contagion to arise, the increase in volatility in asset market 2 needs to occur over and above the increase solely arising from market fundamentals (Dungey, Fry, González-Hermosillo and Martin (2010)).

The model in (1) admits three potential channels of contagion from asset market 1 (source) to asset market 2 (recipient). The first contagion channel is represented by a shift in the conditional mean, caused by for example, asset return movements in market 1 rising above a threshold, \( \kappa_1 \). Contagion of this form is given by

\[
\begin{align*}
\alpha_y &= \alpha_x, & |y_{1,t}| < \kappa_1, & \text{[No contagion]} \\
\alpha_y &> \alpha_x, & |y_{1,t}| > \kappa_1, & \text{[Contagion]}
\end{align*}
\]

which is representative of the contagion channels proposed by Favero and Giavazzi (2002). This type of contagion is interpreted as mean-shift contagion as it results in a positive shift in the conditional mean in the asset returns distribution of asset market 2. Letting \( F_{x} (x_{2|1}) \) and \( F_{y} (y_{2|1}) \) denote the conditional distribution functions in the noncrisis and crisis periods respectively, this form of contagion is highlighted in Figure 1(a) by a shift to the right of the conditional distribution function of asset returns in market 2.

The second contagion channel extends the linkage from the mean to the variance, whereby there is a shift in the conditional variance caused by movements in asset returns of market 1 rising above the threshold value, \( \kappa_2 \). Contagion of this form is given by

\[
\begin{align*}
\sigma^2_{v,y} &= \sigma^2_{v,x}, & |y_{1,t}| < \kappa_2, & \text{[No contagion]} \\
\sigma^2_{v,y} &> \sigma^2_{v,x}, & |y_{1,t}| > \kappa_2, & \text{[Contagion]}
\end{align*}
\]

This channel of contagion is interpreted as variance-shift contagion as it results in a positive shift in the conditional variance in the asset returns distribution of asset market 2. The effects of this contagion channel are highlighted in Figure 1(b) where the conditional distribution function of the returns in asset market 2 is now stretched out over its support compared to the noncrisis conditional distribution function.
Figure 1: The conditional distribution functions in noncrisis ($F(x_{2,t} | x_{1,t})$, blue) and crisis periods ($H(y_{2,t} | y_{1,t})$, red) for alternative channels of contagion. (a) Mean-shift contagion: $\alpha_x = 1, \alpha_y = 2; \beta_x = \beta_y = 1; \sigma_{v,x}^2 = \sigma_{v,y}^2 = 1$. (b) Variance-shift contagion: $\alpha_x = \alpha_y = 1; \beta_x = \beta_y = 1; \sigma_{v,x}^2 = 1, \sigma_{v,y}^2 = 2$. (c) Slope-shift contagion: $\alpha_x = \alpha_y = 1; \beta_x = 1, \beta_y = 2; \sigma_{v,x}^2 = \sigma_{v,y}^2 = 1$. 
The third contagion channel is

\[ \beta_y = \beta_x; \quad \text{[No contagion]} \]
\[ \beta_y > \beta_x; \quad \text{[Contagion]} \]

(5)

which is the more common form of contagion channel tested (Forbes and Rigobon (2002), Dungey, Fry, González-Hermosillo and Martin (2005)). During the crisis period, movements of the same order of magnitude in asset returns of the source market translate into even larger movements in asset returns of the recipient asset market. The effects of this type of contagion channel are highlighted in Figure 1(c) by a rotation of the conditional distribution of asset returns in market 2.

The earliest tests of contagion are based on detecting a change in the correlation between the returns in the crisis period \((\rho_y)\) and noncrisis periods \((\rho_x)\). Assuming that asset returns in both markets and in both periods are normality distributed, the joint density functions of asset returns in the noncrisis and crisis periods respectively, are

\[
f(x_1, x_2) = \frac{1}{2\pi\sigma_{x,1}\sigma_{x,2}\sqrt{1 - \rho_x^2}} \exp \left\{ -\frac{(x_1^2 - 2\rho_x x_1 x_2 + x_2^2)}{2(1 - \rho_x^2)} \right\}
\]

\[
h(y_1, y_2) = \frac{1}{2\pi\sigma_{y,1}\sigma_{y,2}\sqrt{1 - \rho_y^2}} \exp \left\{ -\frac{(y_1^2 - 2\rho_y y_1 y_2 + y_2^2)}{2(1 - \rho_y^2)} \right\},
\]

(6)

where \(\sigma_{x,i}^2\) and \(\sigma_{y,i}^2\) are the variances of asset returns in market \(i\) during the noncrisis and crisis periods respectively. Forbes and Rigobon (2002) show that an increase in the variance of asset market 1 from the noncrisis to the crisis periods, \(\sigma_{y,1}^2 > \sigma_{x,1}^2\), results in an increase in the correlation coefficient

\[
\rho_y > \rho_x,
\]

(7)

even where the restrictions in (2) still hold. To circumvent this problem Forbes and Rigobon propose an adjusted correlation coefficient test. An alternative way to approach the problem
is to note that as the restrictions in (2) relate to the conditional distribution, another way to test for contagion is to use the conditional distribution and not the joint distributions (6). Using the definition of a conditional distribution and assuming normal marginal distributions for $x_1$ and $y_1$, the pertinent conditional density functions in the noncrisis and crisis periods are respectively

$$f(x_2 | x_1) = \frac{f(x_1, x_2)}{f(x_1)} = \frac{1}{\sqrt{2\pi\sigma^2_{v,x}}} \exp \left( -\frac{(x_2 - \alpha_x - \beta_x x_1)^2}{2\sigma^2_{v,x}} \right)$$

$$h(y_2 | y_1) = \frac{h(y_1, y_2)}{h(y_1)} = \frac{1}{\sqrt{2\pi\sigma^2_{v,y}}} \exp \left( -\frac{(y_2 - \alpha_y - \beta_y y_1)^2}{2\sigma^2_{v,y}} \right),$$

where the parameters of the conditional distribution are as defined in the model given by (1). This suggests that a Chow test of $\beta_x = \beta_y$ for example, provides a test of slope contagion (Dungey, Fry, González-Hermosillo and Martin (2005)).

Natural extensions of the Chow test based on the restriction $\beta_x = \beta_y$ in (8), consist of tests of intercept contagion based on the restriction $\alpha_x = \alpha_y$, and variance contagion based on the restrictions and $\sigma^2_{v,y} = \sigma^2_{v,x}$. This suggests that a more general method of testing for contagion is to compare the conditional distribution functions in the noncrisis and crisis periods given in (8), as this automatically takes into account all potential changes in the conditional moments arising from contagion. The asset return model in (1) is based on a simple static linear regression model with normal error terms. This model can be extended through the inclusion of additional dynamics which provides a more general framework in which to model contagion. In doing so, it is necessary to extend the information set to include not only contemporaneous information arising asset returns in other markets, but also lagged information which includes the history of returns in all markets. This suggests that tests of contagion are based on the conditional distribution function, with the hypotheses specified.
as

\[ H_0 : \ F(x_2 \mid x_1, \Omega) = H(y_2 \mid y_1, \Omega) \]
\[ H_1 : \ F(x_2 \mid x_1, \Omega) \neq H(y_2 \mid y_1, \Omega), \]

where \( F(\cdot) \) and \( H(\cdot) \) denote the noncrisis and crisis conditional distributions respectively and \( \Omega \) represents lagged information. To estimate the conditional distribution, the approach adopted here is to use a conditional copula as is discussed next.

3 A Conditional Copula Distribution

Copulas are useful in constructing multivariate models of asset returns as they provide a convenient way to combine univariate marginals into a joint distribution. The fundamental theorem that supports all copula-based analysis is Sklar’s Theorem which can be extended for conditional distributions (Patton (2001)).

**Theorem 1** (Sklar’s Theorem for Continuous Conditional Distributions) *Let \( F_{1,2} \) be a two dimensional conditional distribution function of some random vector \( x = (x_1, x_2) \), and let \( \Omega \) be some conditioning set. Now \( F_{1,2} \) has the following unique conditional copula representation

\[ F_{1,2}(x_1, x_2 \mid \Omega) = C(F_1(x_1 \mid \Omega), F_2(x_2 \mid \Omega)) \Omega, \]

where \( F_1 \) and \( F_2 \) denote respectively the conditional marginal distribution of the random variables \( x_1 \) and \( x_2 \), and \( C \) is a conditional copula function which captures the dependence of \( x \). If \( F_1 \) and \( F_2 \) are continuous, then \( C \) is unique.*

The implication of Sklar’s Theorem is that for continuous bivariate distribution functions \( F_{1,2} \), the univariate marginals and the bivariate dependence structure decompose with the dependence structure represented by the copula function \( C \). The following corollary is attained from (10).
Corollary 2  Let $F_{1,2}$ be a two dimensional conditional distribution function with continuous marginals $F_1, F_2$ and conditional copula $C$ (satisfying (10)). Then, for any $u = (u_1, u_2)$ in $[0, 1]^2$

$$C (u_1, u_2|\Omega) = F_{1,2} \left( F_1^{-1} (u_1|\Omega), F_2^{-1} (u_2|\Omega) |\Omega \right),$$

(11)

where $F_i^{-1}$ is the inverse of $F_i$, while $u_1 = F_1(x_1|\Omega)$ and $u_2 = F_2(x_2|\Omega)$.

Based on the definition of a joint conditional distribution function, the conditional distribution function is redefined as follows

$$F (x_2| x_1, \Omega) = \int_{-\infty}^{x_2} f (s_2| x_1, \Omega) ds_2 = \int_{-\infty}^{x_2} \frac{f_{1,2}(x_1, s_2| \Omega)}{f_1(x_1| \Omega)} ds_2$$

$$= \frac{1}{f_1(x_1| \Omega)} \int_{-\infty}^{x_2} f_{1,2}(x_1, s_2| \Omega) ds_2$$

$$= \frac{1}{f_1(x_1| \Omega)} \frac{\partial F_{1,2}(x_1, x_2| \Omega)}{\partial x_1}.$$

(12)

Using (10) to replace $F_{1,2}(x_1, x_2| \Omega)$ by $C$

$$F (x_2| x_1, \Omega) = \frac{\partial C (F_1(x_1| \Omega), F_2(x_2| \Omega)| \Omega)}{\partial x_1} \frac{1}{f_1(x_1| \Omega)}$$

$$= \frac{\partial C (F_1(x_1| \Omega), F_2(x_2| \Omega)| \Omega)}{\partial F_1(x_1| \Omega)},$$

(13)

where the last step uses the definition $f_1(x_1| \Omega) = \partial F_1(x_1| \Omega)/\partial x_1$. Alternatively, as $u_i = F_i(x_i| \Omega)$ and $\partial u_i = \partial F_i(x_i| \Omega) (i = 1, 2)$, then

$$F (x_2| x_1, \Omega) = \frac{\partial C (u_1, u_2| \Omega)}{\partial u_1}.$$

(14)
To estimate the unknown parameters of the model the joint density function $f_{1,2}$ is expressed in terms of the copula by using (10)

$$f_{1,2}(x_1, x_2 | \Omega) = \frac{\partial^2 F_{1,2}(x_1, x_2 | \Omega)}{\partial x_1 \partial x_2}$$

$$= \frac{\partial^2 C(F_1(x_1 | \Omega), F_2(x_2 | \Omega)|\Omega)}{\partial x_1 \partial x_2}$$

$$= \frac{\partial^2 C(F_1(x_1 | \Omega), F_2(x_2 | \Omega)|\Omega)}{\partial F_1(x_1 | \Omega) \partial F_2(x_2 | \Omega)} \frac{\partial F_1(x_1 | \Omega)}{\partial x_1} \frac{\partial F_2(x_2 | \Omega)}{\partial x_2}$$

$$= c(F_1(x_1 | \Omega), F_2(x_2 | \Omega)|\Omega) \times f_1(x_1 | \Omega) \times f_2(x_2 | \Omega), \quad (15)$$

where $c$ is the copula density given by

$$c(F_1(x_1 | \Omega), F_2(x_2 | \Omega)|\Omega) = \frac{\partial^2 C(F_1(x_1 | \Omega), F_2(x_2 | \Omega)|\Omega)}{\partial F_1(x_1 | \Omega) \partial F_2(x_2 | \Omega)}.$$

or, equivalently by using $u_i = F_i(x_i | \Omega)$

$$c(u_1, u_2 | \Omega) = \frac{\partial^2 C(u_1, u_2 | \Omega)}{\partial u_1 \partial u_2}. \quad (17)$$

Equation (15) shows that the joint density $f_{1,2}$ is expressed as the product of the copula density $c$, and the univariate marginal densities $f_1$ and $f_2$. This suggests the following 2-step estimator to compute the unknown parameters of the joint density (Genest, Ghoudi and Rivest (1995)):

**Stage 1** Estimate the marginal distributions (parametrically or nonparametrically).

**Stage 2** Estimate the parameters of the copula density given the estimates of the marginal distributions in the first stage, by solving

$$\hat{\theta} = \arg \max_{\theta} \ln L(\theta)$$

$$= \arg \max_{\theta} \left( \sum_{t=1}^{T} \ln c(\hat{u}_{1,t}, \hat{u}_{2,t}; \theta | \Omega) \right), \quad (18)$$
where \( \hat{u}_{1,t} = \hat{F}_{1,t}(x_{1,t} | \Omega) \) and \( \hat{u}_{2,t} = \hat{F}_{2,t}(x_{2,t} | \Omega) \), which are the estimated conditional marginal distributions. Define \( F(x_2 | x_1, \Omega) \) and \( H(y_2 | y_1, \Omega) \) as the conditional distribution functions for the noncrisis and crisis periods respectively, a test can be constructed by testing for equality of the conditional distributions.

To complete the specification of the model it is necessary to specify the form of the copula density \( c \) in (16) as well as the marginal distributions \( F_1 \) and \( F_2 \).

### 3.1 Copula Function Specification

Two copula functions are investigated, namely the Gaussian copula and the Student t copula. For other types of copulas and their properties, see Nelsen (1999). The Gaussian copula is one of the more popular forms which allows for dependence through the correlation parameter thereby serving as a benchmark in the analysis. The Student t copula represents a generalization of the normal copula by allowing for tail-dependence through the degrees of freedom parameter.

#### 3.1.1 Gaussian Copula

The Gaussian copula belongs to the class of elliptical copulas. Define \( \Phi_{1,2} \) as the standard bivariate normal distribution with Pearson’s correlation parameter \( \rho \in [-1, 1] \), and let \( \Phi^{-1} \) represent the inverse of the univariate standard normal distribution. Using Sklar’s theorem in (10), the two-dimensional Gaussian copula is

\[
C(u_1, u_2; \rho) = \Phi_{1,2}(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho)
\]

\[
= \Phi^{-1}(u_1) \Phi^{-1}(u_2) \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \times \exp \left\{ -\frac{(s_1^2 - 2\rho s_1 s_2 + s_2^2)}{2(1-\rho^2)} \right\} ds_1 ds_2.
\]
Roncalli (2002) demonstrates that the Gaussian copula can be equivalently represented as

\[
C(u_1, u_2; \rho) = \int_{-\infty}^{u_1} \Phi \left( \frac{\Phi^{-1}(u_2) - \rho \Phi^{-1}(s_1)}{\sqrt{1 - \rho^2}} \right) ds_1.
\]  (20)

Using equation (14), the copula representation of the conditional distribution is

\[
F(x_2 | x_1, \Omega) = \frac{\partial C(u_1, u_2; \rho)}{\partial u_1} = \Phi \left( \frac{\Phi^{-1}(u_2) - \rho \Phi^{-1}(u_1)}{\sqrt{1 - \rho^2}} \right),
\]  (21)

where \(u_i = F_i(x_i), i = 1, 2\). Denoting the marginal density functions as \(\phi\) and the joint density by \(\phi_{1,2}\), the density of the Gaussian copula is

\[
c(u_1, u_2; \rho) = \frac{\partial^2 \phi_{1,2}(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho)}{\partial u_1 \partial u_2} = \frac{\phi_{1,2}(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho)}{\phi(\Phi^{-1}(u_1)) \phi(\Phi^{-1}(u_2))},
\]  (22)

where

\[
\phi_{1,2}(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho) = \frac{1}{2\pi \sqrt{1 - \rho^2}} \exp \left( -\frac{\Phi^{-1}(u_1)^2 + \Phi^{-1}(u_2)^2 - 2\rho \Phi^{-1}(u_1) \Phi^{-1}(u_2)}{2(1 - \rho^2)} \right)
\]

and

\[
\phi(\Phi^{-1}(u_1)) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \Phi^{-1}(u_1)^2 \right)
\]

\[
\phi(\Phi^{-1}(u_2)) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \Phi^{-1}(u_2)^2 \right).
\]
The density of the Gaussian copula is further expressed as

\[
c(u_1, u_2; \rho) = \frac{1}{\sqrt{1 - \rho^2}} \exp \left( \frac{\Phi^{-1}(u_1)^2 + \Phi^{-1}(u_2)^2 - 2\Phi^{-1}(u_1)\Phi^{-1}(u_2)}{2(1 - \rho^2)} \right),
\]

which is used to construct the log-likelihood in equation (18).

### 3.1.2 Student t Copula

The Student t copula as with the Gaussian copula belongs to the elliptical copulas. This copula is defined as

\[
C(u_1, u_2; \nu, \rho) = G_{v,2}(G^{-1}(u_1), G^{-1}(u_2); \rho, v)
\]

\[
= \int_{-\infty}^{u_1} \int_{-\infty}^{u_2} \frac{1}{2\pi\sqrt{1 - \rho^2}} \left(1 + \frac{s_1^2 - 2\rho s_1 s_2 + s_2^2}{v(1 - \rho^2)}\right)^{-\frac{\nu + 2}{2}} ds_1 ds_2,
\]

where \(G_{v,2}\) is the bivariate Student t cdf with \(v\) degree of freedom and correlation parameter \(\rho\), and \(G^{-1}\) denotes the inverse of the corresponding univariate Student t distribution. As is shown by Roncalli (2002), the Student t copula can be equivalently expressed as

\[
C(u_1, u_2; \nu, \rho) = \int_{-\infty}^{u_1} G_{v+1} \left( \left( \frac{v + 1}{v + [G^{-1}_v(u_1)]^2} \right)^{1/2} \frac{G^{-1}_v(u_2) - \rho G^{-1}_v(u)}{\sqrt{1 - \rho^2}} \right) ds_1.
\]

As a result, the Student t copula representation of the conditional distribution is

\[
F(x_2 | x_1, \Omega) = \frac{\partial C(u_1, u_2; \rho, v)}{\partial u_1} = G_{v+1} \left( \left( \frac{v + 1}{v + [G^{-1}_v(u_1)]^2} \right)^{1/2} \frac{G^{-1}_v(u_2) - \rho G^{-1}_v(u)}{\sqrt{1 - \rho^2}} \right),
\]
where \( u_i = F_i(x_i), i = 1, 2 \). Denoting the marginal density functions as \( g \) and the joint density by \( g_{1,2} \), the density of the Student t copula is defined as

\[
c(u_1, u_2; \rho, v) = \frac{\partial G_{1,2}(G^{-1}(u_1), G^{-1}(u_2); \rho, v)}{\partial u_1 \partial u_2} = \frac{g_{1,2}(g^{-1}(u_1), g^{-1}(u_2); \rho, v)}{g(g^{-1}(u_1)) g(g^{-1}(u_2))},
\]

(27)

where

\[
g_{1,2}(G^{-1}(u_1), G^{-1}(u_2); \rho, v) = \frac{1}{2\pi \sqrt{1-\rho^2}} \left( 1 + \frac{G^{-1}(u_1)^2 - 2\rho G^{-1}(u_1) G^{-1}(u_2) + G^{-1}(u_2)^2}{v(1-\rho^2)} \right)^{-\frac{v+2}{2}},
\]

\[
g(G^{-1}(u_1); v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi} \Gamma\left(\frac{v}{2}\right)} \left( 1 + \frac{G^{-1}(u_1)^2}{v} \right)^{-\frac{v+1}{2}},
\]

\[
g(G^{-1}(u_2); v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi} \Gamma\left(\frac{v}{2}\right)} \left( 1 + \frac{G^{-1}(u_2)^2}{v} \right)^{-\frac{v+1}{2}}.
\]

The copula density is further expressed as

\[
c(u_1, u_2; \rho, v) = \frac{\Gamma\left(\frac{v}{2} + 1\right) \Gamma\left(\frac{v}{2}\right)}{\sqrt{1-\rho^2} \Gamma\left(\frac{v+1}{2}\right)^2} \left( 1 + \frac{G^{-1}(u_1)^2 - 2\rho G^{-1}(u_1) G^{-1}(u_2) + G^{-1}(u_2)^2}{v(1-\rho^2)} \right)^{-\frac{v+2}{2}} \left( 1 + \frac{G^{-1}(u_1)^2}{v} \right)^{-\frac{v+1}{2}} \left( 1 + \frac{G^{-1}(u_2)^2}{v} \right)^{-\frac{v+1}{2}},
\]

(28)

which is used to construct the log-likelihood in equation (18).
3.2 Marginal Distribution Specification

A number of approaches can be used to model the marginal distributions in (15). One approach adopted is non-parametric by using the empirical distribution function defined as

\[
\hat{F}_1(x_1) = \frac{1}{T_x} \sum_{t=1}^{T_x} 1(x_{1,t} \leq x_{1,t}), \quad t < T_x,
\]

\[
\hat{F}_2(x_2) = \frac{1}{T_x} \sum_{t=1}^{T_x} 1(x_{2,t} \leq x_{2,t}), \quad t < T_x,
\]

where \(x_{1,N}\) and \(x_{2,N}\) represent the non-crisis data ordered from the lowest to the highest value. An alternative approach is to include conditioning information into the marginal distributions by adopting a parametric specification. For example, to allow for lagged information arising from time-varying volatility, a GARCH model can be used to model the marginal distributions. If the error terms in the mean equations are assumed to be normal, the conditional marginal distributions are also normal

\[
\hat{F}_1(x_1 | \Omega) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi h_{1,t}}} \exp\left( -\frac{s_1^2}{2h_{1,t}} \right) ds_1,
\]

\[
\hat{F}_2(x_2 | \Omega) = \int_{-\infty}^{x_2} \frac{1}{\sqrt{2\pi h_{2,t}}} \exp\left( -\frac{s_2^2}{2h_{2,t}} \right) ds_2,
\]

where \(h_{1,t}\) and \(h_{2,t}\) are the GARCH conditional variances.

4 Testing for Contagion

The general approach adopted to test for contagion in (9) consists of estimating the copula conditional distribution in (8) for the noncrisis data \((x_{1,t}, x_{2,t})\) and evaluating the estimated conditional distribution based on the crisis data \((y_{1,t}, y_{2,t})\). Formally this involves defining
the random variable \( z \) as

\[
z = F(y_1, y_2) = \int_{-\infty}^{\infty} f(s|y_1, y_2) ds.
\]

(33)

which is the non-crisis conditional distribution evaluated using the crisis data \((y_1, y_2)\). If the conditional distributions are the same in the non-crisis and crisis periods, the random variable \( z \) should have a uniform distribution in the interval \((0, 1)\), equivalent to a test of uniformity of \( z \) in \((0, 1)\) with density

\[
q(z) = 1, \quad 0 < z < 1.
\]

(34)

As a result, a test of \( H_0 \) in (9) is equivalent to a test of uniformity of \( z \) in \((0, 1)\).

To construct a test of the generalization of the uniform distribution originally proposed by Neyman (1937) is adopted

\[
q(z) = \eta(\theta) \exp \left[ \sum_{j=1}^{k} \theta_j \tau_j(z) \right], \quad 0 < z < 1.
\]

(35)

where \( \theta_j, j = 1, 2, \ldots, k \), are unknown parameters, and \( \eta(\theta) \) is a normalising constant that ensures the probability density function integrates to one. To identify alternative channels of contagion for various moments the functions \( \tau_j(z) \) are chosen to be orthogonal based on the null hypothesis that the conditional distributions are equal in the non-crisis and crisis periods.

To identify alternative channels of contagion the functions \( \tau_j(z) \) are chosen to be orthogonal based on the null hypothesis that the conditional distributions are equal in the non-crisis and crisis periods.
the normalised Legendre polynomials, where the first five polynomials are given by

\begin{align}
\pi_0(z) &= 1 \\
\pi_1(z) &= \sqrt{12} \left( z - \frac{1}{2} \right) \\
\pi_2(z) &= \sqrt{5} \left( 6 \left( z - \frac{1}{2} \right)^2 - \frac{1}{2} \right) \\
\pi_3(z) &= \sqrt{7} \left( 20 \left( z - \frac{1}{2} \right)^3 - 3 \left( z - \frac{1}{2} \right) \right) \\
\pi_4(z) &= 210 \left( z - \frac{1}{2} \right)^4 - 45 \left( z - \frac{1}{2} \right)^2 + \frac{9}{8}. \tag{36}
\end{align}

The polynomials have the property that they are normalized as

\[ \int_0^1 \pi_i(z) \, dz = 0, \quad i \geq 1, \]

and are orthogonal

\[ \int_0^1 \pi_i(z) \pi_j(z) \, dz = \begin{cases} 1 & : \ i = j \\ 0 & : \ i \neq j. \end{cases} \]

As no contagion corresponds to the uniform distribution in (35), \( q(z) = 1 \), an overall test of no contagion is equivalent to testing the corresponding null and alternative hypotheses

\begin{align}
H_0 & : \quad \theta_1 = \theta_2 = \cdots = \theta_k = 0 \quad \text{(no contagion)} \\
H_1 & : \quad \text{At least one } \theta_j \neq 0. \quad \text{(contagion)} \tag{37}
\end{align}

Individual tests of contagion corresponding to each of the first \( k \) moments in (35), are conducted by testing the restriction \( \theta_j = 0, \forall j \leq k \). As a preliminary investigation of the ability of (35) to identify contagion channels corresponding to alternative moments of the conditional distribution, the following simulation experiment is performed. The results of five experiments are given in Figure 2. In all experiments the noncrisis distribution is \( x \sim \)
For comparison Figures 2(a) to 2(e) provide plots of the first five polynomials in (36).

In the first experiment the non-crisis and crisis distributions are the same, so the crisis distribution is also \( N(0, 1) \). Figure 2(f) gives the histogram of \( q(z) \) in (33) using

\[
z = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{s^2}{2} \right) ds,
\]

where \( y \) are \( T_y = 2000 \) simulated observations from a standardized normal distribution. The histogram of \( q(z) \) is flat showing that the null hypothesis of no change in the distribution in the crisis period is not rejected.

In the second experiment there is a mean shift whereby the crisis distribution is now \( N(0.5, 1) \). Simulating the crisis data from \( N(0.5, 1) \) and evaluating (38) results in an upward sloping histogram for \( q(z) \) in Figure 2(g) which matches the shape of the normalised Legendre polynomial in Figure 2(b). Extending the exercise to allow for a variance shift whereby the crisis distribution becomes \( N(0, 2) \), the histogram of \( q(z) \) in Figure 2(h) now displays a U-shape, again matching the shape of the normalised Legendre polynomial in Figure 2(c).

Experiment 4 allows for a change in skewness in the crisis period whereby the crisis distribution is now based on the standardized gamma distribution with parameters \( \alpha = 4 \) and \( \beta = 10 \). The histogram of \( q(z) \) in Figure 2(i) matches the shape of the normalised Legendre polynomial in Figure 2(d) exhibiting a mode below 0.5 and an antimode about 0.5.

In the final experiment contagion results in a change in the fourth moment during the crisis period by specifying the standardized Student t distribution

\[
f(y) = \frac{\Gamma \left( \frac{\nu + 1}{2} \right)}{\sigma_y \sqrt{\pi (\nu - 2)} \Gamma \left( \frac{\nu}{2} \right)} \left( 1 + \frac{y^2}{\sigma_y^2 (\nu - 2)} \right)^{-\left( \frac{\nu + 1}{2} \right)},
\]

where the degrees of freedom parameter is \( \nu = 5 \). The effect of an increase in kurtosis results in the histogram of \( q(z) \) in Figure 2(j) having a mode at 0.5 and interior antimodes, as does
the normalised Legendre polynomial given in Figure 2(e).

To implement the contagion test, alternative strategies exist using either the Wald or Lagrange Multiplier (LM) approaches. In the present context, the LM statistic has the advantage that it is relatively easy to evaluate as it does not require estimating the parameters \( \theta_j \) in (35) as all parameters are evaluated under the null (Bera and Ghosh (2001); and Bera, Ghosh and Xiao (2010)). Letting \( \theta_0 \) represent the parameter vector under \( H_0 \), the LM statistic is

\[
LM = G(\theta_0)' I(\theta_0)^{-1} G(\theta_0),
\]

where \( G(\theta) \) is the gradient vector of the log likelihood and \( I(\theta) \) is the information matrix.

To construct the LM statistic, define the log-likelihood function for the crisis data as

\[
\ln L(\theta) = \sum_{t=1}^{T_y} \ln \left( q(z_t) \right)
= T_y \ln \eta(\theta) + \sum_{j=1}^{k} \theta_j \sum_{t=1}^{T_y} \pi_j(z_t),
\]

where \( z \) is defined in (33). The gradient vector is

\[
G(\theta) = T_y \frac{\partial \ln \eta(\theta)}{\partial \theta_j} + \sum_{t=1}^{T_y} \pi_j(z_t), \quad j = 1, \ldots, k,
\]

which reduces under the null to

\[
G(\theta_0) = \sum_{t=1}^{T_y} \pi_j(z_t),
\]

as \( \frac{\partial \ln \eta(\theta)}{\partial \theta_j}\big|_{\theta=\theta_0} = 0 \). To derive this result, observe that the normalising constant is

\[
\eta(\theta) = \frac{1}{\int_0^1 \exp \left[ \sum_{j=1}^{k} \theta_j \pi_j(z) \right] dz}.
\]
Figure 2: Let $x$ and $y$ denote the asset returns in the noncrisis and crisis periods respectively. In the noncrisis period, $x$ is $N(0,1)$ in all five cases. The distributions of $y$ in the crisis period are: ($f$) $N(0,1)$, no contagion; ($g$) $N(0.5,1)$, mean shift; ($h$) $N(0,2)$, variance shift; ($i$) Standardized gamma distribution, skewness contagion; and ($j$) Standardised Student $t$ with 5 degrees of freedom, kurtosis contagion.
Taking logarithms and differentiating with respect to $\theta_j$ gives

$$\frac{\partial \ln \eta (\theta)}{\partial \theta_j} = - \int_0^1 \pi_j (z) \exp \left[ \sum_{j=1}^k \theta_j \pi_j (z) \right] dz \int_0^1 \exp \left[ \sum_{j=1}^k \theta_j \pi_j (z) \right] dz.$$ \hspace{1cm} (44)

Evaluating under the null

$$\left. \frac{\partial \ln \eta (\theta)}{\partial \theta_j} \right|_{\theta=\theta_0} = - \int_0^1 \pi_j (z) dz = 0, \quad j = 1, 2, \cdots, k,$$ \hspace{1cm} (45)

where the last step follows from the property of the standardized Legendre polynomials in (36).

The information matrix is

$$I (\theta) = \mathbb{E} \left[ - \frac{\partial^2 \ln \eta (\theta)}{\partial \theta \partial \theta'} \right] = T_y \frac{\partial^2 \ln \eta (\theta)}{\partial \theta_j \partial \theta_t},$$ \hspace{1cm} (46)

which simplifies under the null hypothesis to

$$I (\theta_0) = T_y \left. \frac{\partial^2 \ln \eta (\theta)}{\partial \theta_j \partial \theta_t} \right|_{\theta=\theta_0} = T_y I_k,$$ \hspace{1cm} (47)

where $I_k$ is a $k \times k$ identity matrix. This last step follows from the orthogonality property of the Legendre polynomials and the definition of $\eta (\theta)$ in (43).

Combining equations (40), (42) and (47), the LM statistic has the following form

$$\Psi_k^2 = \sum_{j=1}^k V_j^2,$$ \hspace{1cm} (48)

where

$$V_j = \frac{1}{\sqrt{T_y}} \sum_{t=1}^{T_y} \pi_j (z_t).$$ \hspace{1cm} (49)
As \( T_x \to \infty \) and \( T_y \to \infty \), \( \Psi_k^2 \) is asymptotically distributed as chi-square with \( k \) degrees of freedom

\[
\Psi_k^2 \xrightarrow{d} \chi_k^2.
\]  

(50)

Individual tests of contagion are performed by testing each parameter \( \theta_j \) in (35). The first four of these tests are denoted as \( V_j^2, j = 1, 2, 3, 4 \), which under the null hypothesis are asymptotically chi-square with one degree of freedom

\[
V_j^2 \xrightarrow{d} \chi_1^2, \quad j = 1, 2, 3, 4.
\]  

(51)

Given the properties of the generalized uniform distribution discussed in Figure 2, it follows that these tests can be used to identify channels of contagion through the first four moments.

In practice \( z \) in (33) is replaced by \( \widehat{z} \) where the latter reflects that the noncrisis conditional distribution \( F(x_2|x_1, \Omega) \), is estimated by a copula

\[
\widehat{z} = \hat{F}(y_2|y_1, \Omega)
\]

\[
= \frac{\partial C \left( \hat{F}_1(y_1|\Omega), \hat{F}_2(y_2|\Omega) \right) \mid \Omega}{\partial \hat{F}_1(y_1|\Omega)},
\]  

(52)

where \( \hat{F}_1(y_1|\Omega) \) and \( \hat{F}_2(y_2|\Omega) \) are the noncrisis marginals evaluated using the crisis data.

For the Gaussian copula in (21) or the Student t copula in (26), \( \hat{\mu}_i = \hat{F}_i(y_i|\Omega) \ i = 1, 2 \), the marginal distributions are expressed as

\[
\hat{F}_1(y_1) = \frac{1}{T_x} \sum_{t=1}^{T_x} 1(x_{1,t} \leq y_{1,j}), \quad j < T_y,
\]  

(53)

\[
\hat{F}_2(y_2) = \frac{1}{T_x} \sum_{t=1}^{T_x} 1(x_{2,t} \leq y_{2,j}), \quad j < T_y.
\]  

(54)
in the case of the nonparametric expressions in (29) and (30) or

\[
\hat{F}_1(y_1 | \Omega) = \int_{-\infty}^{y_1} \frac{1}{\sqrt{2\pi h_{1,t}}} \exp\left(-\frac{s_1^2}{2h_{1,t}}\right) ds_1,
\]

(55)

\[
\hat{F}_2(y_2 | \Omega) = \int_{-\infty}^{y_2} \frac{1}{\sqrt{2\pi h_{2,t}}} \exp\left(-\frac{s_2^2}{2h_{2,t}}\right) ds_2,
\]

(56)

using the parametric distribution functions in (31) and (32).

As the parameters of the copula are estimated using the noncrisis data, Bera, Ghosh and Xiao (2010) show that whilst this does not have an effect on the asymptotic distribution under the null hypothesis, it can however cause potential size distortions in the finite samples. One approach to circumvent this problem is to choose the crisis period according to the rule

\[ T_y = \lambda \sqrt{T_x}, \]

where \( \lambda \) is a parameter which is adjusted in the Monte Carlo simulations next.

5 Monte Carlo Results

To examine the sampling properties of the proposed contagion tests, a range of Monte Carlo experiments are presented. In all of the experiments, moments up to \( k = 4 \) in (48) are considered. By way of comparison the simulation results of the adjusted correlation test proposed by Forbes and Rigobon (2002) (FR) are also reported. The results are presented for three DGPs. The first consists of a linear model with normal disturbances where the contagion channel operates through the conditional mean. The second DGP is also based on a linear regression model but with Student t disturbances where the contagion channels operate through the change of tail dependence. The third DGP is based on an MGARCH model where contagion channels operate through the conditional mean and the conditional variances.

All Monte Carlo experiments are conducted using the software Gauss 7.0. The number of replications is 5000 for all the experiments. The sample size for the noncrisis period is \( T_x = 1000 \) which represents approximately four years of daily data. To allow for sufficient
number of observations in the crisis period \( T_y = 250 \). This corresponds to setting \( \lambda = 7 \) in the rule \( T_y = \lambda T_x \) to control for potential size distortions.

### 5.1 DGP: Linear Model with Normal Errors

The first DGP used in the Monte Carlo experiments is motivated by the contagion model of Forbes and Rigobon (2002), consisting of a linear model with normal errors

\[
\begin{align*}
x_{2,t} &= \alpha_x + \beta_x x_{1,t} + v_{x,t}, \\
y_{2,t} &= \alpha_y + \beta_y y_{1,t} + v_{y,t},
\end{align*}
\]

where the first equation represents the noncrisis period, the second represents the crisis period, and \( v_{x,t} \) and \( v_{y,t} \) are independent normally distributed random variables with zero mean and constant variances. The source asset market is assumed to be normally distributed as \( x_{1,t} \sim N(0, \sigma^2_{x,1}) \) in the noncrisis period and \( y_{1,t} \sim N(0, \sigma^2_{y,1}) \) in the crisis period. The marginal distributions are estimated nonparametrically using equations (29) and (30), while the dependence structure is based on the Gaussian copula.

#### 5.1.1 Size

Table 1 reports the size properties of the contagion tests under the restriction of no contagion as \( x = y = 0 \). \( \alpha_x = \alpha_y = 0.05; \beta_x = \beta_y = 0.5; \sigma^2_{x,1} = \sigma^2_{y,1} = 1 \) in (57). The size is based on the 5% asymptotic \( \chi^2_1 \) critical values in the case of the individual copula tests and FR test, while the size of the joint copula test is based on the 5% asymptotic \( \chi^2_5 \) critical values. Two experiments are reported. In the first experiment the variances in the noncrisis and crisis periods are equal, whilst in the second experiment the variance doubles during the crisis period to allow for heteroskedasticity. The results show that all tests are robust to the presence of heteroskedasticity with empirical sizes close to the nominal size of 24.
Size properties of alternative contagion tests using the linear model DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on the 5% asymptotic critical value. The parameter settings in equation (57) is: $\alpha_x = \alpha_y = 0.05; \beta_x = \beta_y = 0.5$.

<table>
<thead>
<tr>
<th>Intensity Copula Tests</th>
<th>$V^2_1$</th>
<th>$V^2_2$</th>
<th>$V^2_3$</th>
<th>$V^2_4$</th>
<th>$\Psi^2_4 = \sum_{j=1}^{4} V^2_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{x,1}^2 = \sigma_{y,1}^2 = 1$</td>
<td>0.072</td>
<td>0.060</td>
<td>0.054</td>
<td>0.052</td>
<td>0.054</td>
</tr>
<tr>
<td>$\sigma_{x,1}^2 = 1, \sigma_{y,1}^2 = 2$</td>
<td>0.070</td>
<td>0.064</td>
<td>0.052</td>
<td>0.050</td>
<td>0.050</td>
</tr>
</tbody>
</table>

5%. Only the individual test $V^2_1$, is slightly oversized (7.2% and 7.0%).

5.1.2 Power

The size-adjusted power results of the contagion tests are given in Tables 2 (shift contagion) and 3 (slope contagion) in the case of the constant variance experiment. Two types of power comparisons are performed. For the shift contagion experiment the intercept of the crisis period increases according to

$$\alpha_y = \{0.05, 0.1, 0.2, 0.3, 0.4, 0.5\}.$$  

The results in Table 2 show that the power function of the joint copula test of contagion is monotonically increasing in the contagion parameter $\alpha_y$. Further inspection of this table shows that the power of this comes from the conditional mean contagion channel in $V^2_1$. As to be expected the $FR$ test exhibits no power in detecting this form of contagion with power between 6% and 8% for all values of the contagion parameter $\alpha_y$.

In the slope contagion experiment the slope during the crisis period in (57) increases according to

$$\beta_y = \{0.5, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5\}.$$  

The results in Table 3 show that the power function of the joint copula test of contagion is
Table 2:

Power properties of alternative contagion tests using the linear model DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on size adjusted critical values of 5%. The parameter settings in equation (57) is: $\alpha_x = 0.05, \beta_x = \beta_y = 0.5$.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Copula Tests</th>
<th>$FR$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_1^2$</td>
<td>$V_2^2$</td>
</tr>
<tr>
<td>$\alpha_y = 0.05$</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>$\alpha_y = 0.10$</td>
<td>0.148</td>
<td>0.044</td>
</tr>
<tr>
<td>$\alpha_y = 0.20$</td>
<td>0.570</td>
<td>0.066</td>
</tr>
<tr>
<td>$\alpha_y = 0.30$</td>
<td>0.954</td>
<td>0.096</td>
</tr>
<tr>
<td>$\alpha_y = 0.40$</td>
<td>1.000</td>
<td>0.238</td>
</tr>
<tr>
<td>$\alpha_y = 0.50$</td>
<td>1.000</td>
<td>0.434</td>
</tr>
</tbody>
</table>

monotonically increasing in the contagion parameter $\beta_y$, reaching power of 100% for $\beta_y = 1.4$.

Inspection of this table shows that the power of this test comes from the second moment $V_2^2$, and the fourth moment $V_4^2$, with no power coming from the odd-moment versions of the copula test, $V_1^2$ and $V_3^2$. In contrast to the previous power experiment in Table 2, the $FR$ test is the more powerful test with power reaches 100% power for $\beta_y > 0.05$. This result is a reflection of the property that the FR test is primarily designed to detect contagion channels through the second moment. In contrast there is some loss of power in using the copula test relative to the $FR$ test in this situation as the former is a joint test of all channels of contagion.

5.2 DGP: Linear Model with Student t Errors

In the second experiment higher order contagion channels are investigated by extending the DGP in the first experiment by replacing the normality assumption by a Student t distribution where $\gamma_x$ is the degrees of freedom parameter in the noncrisis period, and $\gamma_y$ is the degrees of freedom parameter in the crisis period. Returns in the source asset market are assumed to be Student t with zero mean and constant degree of freedom over the whole period. The marginal distributions are estimated nonparametrically by using equations (29)
Table 3:
Power properties of alternative contagion tests using the linear model DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on size adjusted critical values of 5%. The parameter settings in equation (57) is: $\alpha_x = \alpha_y = 0.05, \beta_x = 0.5$.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Copula Tests</th>
<th>$FR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1^2$</td>
<td>$V_2^2$</td>
<td>$V_3^2$</td>
</tr>
<tr>
<td>$\beta_y = 0.5$</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>$\beta_y = 1.0$</td>
<td>0.074</td>
<td>0.532</td>
</tr>
<tr>
<td>$\beta_y = 1.1$</td>
<td>0.088</td>
<td>0.792</td>
</tr>
<tr>
<td>$\beta_y = 1.2$</td>
<td>0.086</td>
<td>0.964</td>
</tr>
<tr>
<td>$\beta_y = 1.3$</td>
<td>0.112</td>
<td>0.996</td>
</tr>
<tr>
<td>$\beta_y = 1.4$</td>
<td>0.100</td>
<td>1.000</td>
</tr>
<tr>
<td>$\beta_y = 1.5$</td>
<td>0.088</td>
<td>1.000</td>
</tr>
</tbody>
</table>

and (30), while a Student t copula is used for modelling the dependence structure.

5.2.1 Size

Table 4 reports the size properties of the contagion tests under the restriction of no contagion

$$\alpha_x = \alpha_y = 0.05; \quad \beta_x = \beta_y = 0.5; \quad \gamma_x = \gamma_y = 25.$$  

The joint copula test $\Psi_4^2$ is over-sized with a size of 9.7%. Inspection of the individual copula test results shows that this is due to the copula tests corresponding to the first two moments, $V_1^2$ and $V_2^2$ where the sizes are 11% and 8.4%, respectively. The other two copula tests are reasonably well behaved with empirical sizes of 6.2% and 6.8%. The $FR$ test is slightly undersized with a size of 4.5%.

5.2.2 Power

The size-adjusted power properties of the contagion tests are given in Tables 5. To model the change of tail dependence of asset returns in the crisis period, the degrees of freedom parameter of the error term $v_{y,t}$ is allowed to be time-varying in the crisis period according
Table 4:
Size properties of alternative contagion tests using the linear model DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on the 5% asymptotic critical value. The parameter settings in equation (57) is: $\alpha_x = \alpha_y = 0.05, \beta_x = \beta_y = 0.5, \gamma_x = \gamma_y = 25$.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Copula Tests</th>
<th>$FR$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_1^2$</td>
<td>$V_2^2$</td>
</tr>
<tr>
<td>$x = y = 0$:05</td>
<td>0.075</td>
<td>0.079</td>
</tr>
<tr>
<td>$x = y = 0$:5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x = y = 25$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

to

$$\gamma_{y,t} = 5 + \frac{20}{1 + \xi |y_{1,t}|},$$

where $\xi$ is an intensity parameter controlling the level of contagion operating through the change of tail dependence. Values of $\xi$ are chosen to increase according to

$$\xi = \{0, 1, 5, 10, 15, 20, 50, 100, 300, 500, \infty\},$$

where $\xi = 0$ represents the case where there is no contagion ($\gamma_{y,t} = \gamma_x = 25$) and $\xi = \infty$ represents the case where the degrees of freedom parameter $\gamma_{y,t}$, approaches the lower bound 5 for all $t$. This choice of lower bound is set to 5 which ensures the first four moments exist. The results are given in Table 5. The power of the joint copula test of contagion is monotonically increasing in the intensity parameter $\xi$ with power reaching 40% for $\xi = \infty$. Further inspection of this table confirms that the power of the copula test is mainly coming from the fourth moment $V_4^2$. In contrast, the $FR$ test exhibits relatively low power in detecting this form of contagion. This is due to the fact that the FR test is designed to identify power in the second moment. Thus the test can not detect the contagion arising from the fourth moment such as cokurtosis.
Table 5:

Power properties of alternative contagion tests using the linear model DGP with sample sizes \( T_x = 1000 \) and \( T_y = 250 \), based on size adjusted critical values of 5\%. The parameter settings in equation (57) is: \( \alpha_x = \alpha_y = 0.05 \), \( \beta_x = \beta_y = 0.5 \), \( \gamma_x = 25 \).

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Copula Tests</th>
<th>( FR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi = 0 )</td>
<td>0.050 0.050 0.050 0.050</td>
<td>0.050 0.050</td>
</tr>
<tr>
<td>( \xi = 1 )</td>
<td>0.052 0.060 0.052 0.071</td>
<td>0.061 0.052</td>
</tr>
<tr>
<td>( \xi = 5 )</td>
<td>0.054 0.097 0.059 0.145</td>
<td>0.121 0.076</td>
</tr>
<tr>
<td>( \xi = 10 )</td>
<td>0.053 0.128 0.061 0.217</td>
<td>0.180 0.087</td>
</tr>
<tr>
<td>( \xi = 15 )</td>
<td>0.055 0.150 0.061 0.264</td>
<td>0.208 0.097</td>
</tr>
<tr>
<td>( \xi = 20 )</td>
<td>0.054 0.165 0.063 0.270</td>
<td>0.233 0.109</td>
</tr>
<tr>
<td>( \xi = 50 )</td>
<td>0.058 0.210 0.065 0.356</td>
<td>0.303 0.121</td>
</tr>
<tr>
<td>( \xi = 100 )</td>
<td>0.055 0.230 0.066 0.407</td>
<td>0.348 0.147</td>
</tr>
<tr>
<td>( \xi = 300 )</td>
<td>0.056 0.253 0.069 0.443</td>
<td>0.389 0.144</td>
</tr>
<tr>
<td>( \xi = 500 )</td>
<td>0.056 0.261 0.069 0.436</td>
<td>0.398 0.152</td>
</tr>
<tr>
<td>( \xi = \infty )</td>
<td>0.059 0.263 0.068 0.458</td>
<td>0.396 0.169</td>
</tr>
</tbody>
</table>

5.3 DGP: Multivariate GARCH

The third and final experiment to allow for the effects of contagion channels through the conditional variances, the DGP is extended to allow for MGARCH conditional heteroskedasticity. The following DGP consists of a multivariate GARCH model (MGARCH) extends upon the previous DGP by allowing for additional channels of contagion. The noncrisis model is given by

\[
\begin{align*}
x_{1,t} &= \sqrt{h_{1,t}} z_{1,t}, \\
h_{1,t} &= (1 - g_1 - g_2) + g_1 h_{1,t-1} z_{1,t-1}^2 + g_2 h_{1,t-1}, \\
x_{2,t} &= \sqrt{h_{2,t}} z_{2,t}, \\
h_{2,t} &= (1 - g_1 - g_2) + g_1 h_{2,t-1} z_{2,t-1}^2 + g_2 h_{2,t-1},
\end{align*}
\]  

(59)
and the crisis model is given by

\[
\begin{align*}
y_{1,t} &= \sqrt{h_{1,t}} z_{1,t}, \\
h_{1,t} &= (1 - g_1 - g_2) + g_1 h_{1,t-1} z_{1,t-1}^2 + g_2 h_{1,t-1}, \\
y_{2,t} &= \beta y_{1,t} + \sqrt{h_{2,t}} z_{2,t}, \\
h_{2,t} &= (1 - g_1 - g_2) + g_1 h_{2,t-1} z_{2,t-1}^2 + g_2 h_{2,t-1} + \delta_1 h_{1,t-1} z_{1,t-1}^2 + \delta_2 h_{1,t-1},
\end{align*}
\tag{60}
\]

where \( h_{i,t} (i = 1, 2) \) is the conditional variance of \( x_{i,t} \) given previous information, \( z_{i,t} \) are iid distributed standard normal variables, \( z_{i,t} \sim N(0, 1) \) and \( E[z_{1,t} z_{2,t}] = 0 \). The first contagion channel is through the conditional mean with parameter \( \beta \), as in the linear DGP in the first experiment. The second contagion channel operates through the conditional variance equation, which is controlled by the parameters \( \delta_1 \) and \( \delta_2 \). Under the null hypothesis of no contagion, the restrictions on the MGARCH model are

\[
H_0 : \beta = \delta_1 = \delta_2 = 0,
\tag{61}
\]

which results in both asset returns having independent GARCH(1,1) processes. The marginal distributions are estimated based on GARCH(1,1) processes with the pertinent conditional distributions given by equations (31) and (32). As the estimated degrees of freedom parameter for the Student t copula is found to be over 50, in the simulations the Gaussian copula is adopted for modelling the dependence structure.

5.3.1 Size

To examine the size properties of the tests, the restrictions under the null in (61) are imposed. The remaining parameters in (59) and (60) are set at \( g_1 = 0.05 \) and \( g_2 = 0.9 \). The results in Table 6 show that the joint copula test \( \Psi^2_4 \), is slightly over-sized in terms of the nominal size of 5%. Inspection of the individual copula tests shows that this result is due to \( V^2_2 \) and \( V^2_4 \).
Table 6:

Size properties of alternative contagion tests using the MGARCH DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on an asymptotic size of 5%. The parameter settings in equation (60) is: $\beta = \delta_1 = \delta_2 = 0$, $g_1 = 0.05$, $g_2 = 0.9$.

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Copula Tests</th>
<th>$FR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1^2$</td>
<td>$V_2^2$</td>
<td>$V_3^2$</td>
</tr>
<tr>
<td>0.053</td>
<td>0.098</td>
<td>0.047</td>
</tr>
</tbody>
</table>

where the sizes are 9.8% and 6.9%, respectively. The other two individual copula tests are well behaved in the presence of GARCH conditional heteroskedasticity with empirical sizes of 5.3% and 4.7%. The $FR$ test is over-sized with a size of 11.6%.

5.3.2 Power

To investigate the power properties of the copula test, two transmission mechanisms of contagion are considered. The first contagion channel is through the mean $\beta > 0$, with $\delta_1 = \delta_2 = 0$. The range of parameter values chosen for $\beta$ are

$$\beta = \{0.0, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5\},$$

where the first element $\beta = 0$ represents the null hypothesis of no contagion which also corresponds to the noncrisis period. The results on the power of the tests are presented in Table 7. The joint copula test displays monotonic increases in power, reaching 95% power at $\beta = 1.0$. As with the power results for the linear DGP experiment, the power result of the joint copula test are inherited from the good power properties of $V_2^2$. The individual copula test $V_4^2$ also displays good power although at a slightly lower rate. Not surprisingly $V_1^2$ and $V_3^2$ display relatively lower power as these tests are designed to identify contagion channels in the odd order moments. Comparing the joint copula and $FR$ test results shows that the latter exhibits higher power for this DGP which again is indicative of the $FR$ test being designed to identify power in the second moment, whereas the copula test experiences some
loss of power as it is a joint test.

The good power properties of the FR test immediately stem from the property that the correlation between the asset returns during the crisis period is a function of the parameter $\beta$ according to

$$
Corr (y_1, y_2) = \frac{Cov (y_1, y_2)}{\sqrt{\sigma^2_{y_1} \sigma^2_{y_2}}} = \frac{\beta^2}{\sqrt{1 + \frac{\delta_1 + \delta_2}{1 - g_1 - g_2} + \beta^2}},
$$

as $\sigma^2_{y,1} = E[h_{1,t}] = 1$, $\sigma^2_{y,2} = E[h_{2,t}] + \beta^2 E[h_{1,t}] = 1 + (\delta_1 + \delta_2) (1 - g_1 - g_2)^{-1} + \beta^2$, and

$$
Cov (y_1, y_2) = E[(y_{1,t} - \mu_{y,1})(y_{2,t} - \mu_{y,2})] = E \left[\left(\sqrt{h_{1,t}z_{1,t}}\right) \left(\beta y_{1,t} + \sqrt{h_{2,t}z_{2,t}}\right)\right] = E \left[\left(\sqrt{h_{1,t}z_{1,t}}\right) \left(\beta \sqrt{h_{1,t}z_{1,t}} + \sqrt{h_{2,t}z_{2,t}}\right)\right] = E \left[\beta h_{1,t}z_{1,t}^2 + \sqrt{h_{1,t}z_{1,t}} \sqrt{h_{2,t}z_{2,t}}\right] = \beta E[h_{1,t}]
$$

where $\mu_t$ denotes the mean of asset returns. As the correlation in the noncrisis period is $Corr (x_1, x_2) = 0$, which is obtained by setting $\beta = 0$ in (62) it follows that increases in $\beta$ directly cause the correlation in the crisis period to increase.

The second channel of contagion investigated is through the variance equation ($\delta_1, \delta_2 > 0$) with $\beta = 0$. The parameter values chosen are

$$
\delta_1, \delta_2 = \{0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30\},
$$

where $\delta_1 = \delta_2 = 0$ is the null of no contagion, equivalent to the noncrisis period. The results of the second power experiments given in Table 8 demonstrate the main advantages of the joint copula test as it identifies the higher order contagion occurring in the DGP. In contrast,
Table 7:

Power properties of alternative contagion tests based on the parameter of $\beta$ using the MGARCH DGP with sample sizes $T_x = 1000$ and $T_y = 250$, based on size adjusted critical values of 5%. The parameter settings in equation (60) is: $\delta_1 = \delta_2 = 0$, $g_1 = 0.05, g_2 = 0.9$.

<table>
<thead>
<tr>
<th>Conditional Moments</th>
<th>$FR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1^2$</td>
<td>0.00</td>
</tr>
<tr>
<td>$V_2^2$</td>
<td>1.00</td>
</tr>
<tr>
<td>$V_3^2$</td>
<td>1.10</td>
</tr>
<tr>
<td>$V_4^2$</td>
<td>1.20</td>
</tr>
<tr>
<td>$\Psi_4 = \sum_{j=1}^4 V_j^2$</td>
<td>1.30</td>
</tr>
<tr>
<td>$V_2^2$</td>
<td>1.40</td>
</tr>
<tr>
<td>$V_4^2$</td>
<td>1.50</td>
</tr>
</tbody>
</table>

even though the $FR$ test is shown in the previous experiment to have greatest power in the direction of changes in the correlation coefficient of asset returns, it does not detect the contagion in the DGP arising from the second moment and the fourth moment. The poor power properties of the $FR$ test in this case immediately follow from the expression for the crisis period correlation in (62) which is $\text{Corr}(y_1, y_2) = 0$ whenever $\beta = 0$, regardless of the values of $\delta_1$ and $\delta_2$.

Closer inspection of the individual copula tests shows that these tests correctly identify that contagion operates through the even moments. The good power results of the copula test $V_2^2$, stem from the property that the crisis return variance of asset 2 for the special case of $\beta = 0$ is $\sigma_{y,2}^2 = E[h_{2,t}] = 1 + (\delta_1 + \delta_2) (1 - g_1 - g_2)^{-1}$, which is an increasing function of $\delta_1$ and $\delta_2$, provided that $g_1 + g_2 < 1$. In comparison the noncrisis variance is simply $\sigma_{x,2}^2 = E[h_{2,t}] = 1$, which is obtained by setting $\delta_1 = \delta_2 = 0$ in the expression for $\sigma_{y,2}^2$.

A similar result occurs for the fourth moment. In the case of cokurtosis for example, during the crisis period cokurtosis, subject to the restriction $\beta = 0$, is given by

$$Cokurtosis(y_1^2, y_2^2) = 1 + \frac{\text{Cov}(h_{1,t}, h_{2,t})}{\delta_1 + \delta_2},$$

(64)
as

\[
E \left[ \left( \frac{y_{1,t} - \mu_{y,1}}{\sigma_{y,1}} \right)^2 \left( \frac{y_{2,t} - \mu_{y,2}}{\sigma_{y,2}} \right)^2 \right] = \frac{1}{\sigma_{y,1}^2 \sigma_{y,2}^2} E \left[ \left( \sqrt{h_{1,t}} z_{1,t} \right)^2 \left( \sqrt{h_{2,t}} z_{2,t} \right)^2 \right] \\
= \frac{1}{\sigma_{y,1}^2 \sigma_{y,2}^2} E \left[ h_{1,t} h_{2,t} z_{1,t}^2 z_{2,t}^2 \right] \\
= \frac{1}{\sigma_{y,1}^2 \sigma_{y,2}^2} E \left[ h_{1,t} h_{2,t} \right] \\
= \frac{1}{1 + \frac{\delta_1 + \delta_2}{1 - g_1 - g_2}} \left( \text{Cov} \left( h_{1,t}, h_{2,t} \right) + E \left[ h_{1,t} \right] E \left[ h_{2,t} \right] \right),
\]

which uses the previous results \( \sigma_{y,1}^2 = E \left[ h_{1,t} \right] = 1 \) and \( \sigma_{y,2}^2 = E \left[ h_{2,t} \right] = 1 + (\delta_1 + \delta_2) (1 - g_1 - g_2)^{-1} \), together with the equality \( \text{Cov} \left( h_{1,t}, h_{2,t} \right) = E \left[ h_{1,t} h_{2,t} \right] - E \left[ h_{1,t} \right] E \left[ h_{2,t} \right] \). Upon rearranging this expression and using the definitions of \( E \left[ h_{1,t} \right] \) and \( E \left[ h_{2,t} \right] \) again gives (64). As \( \delta_1, \delta_2 > 0 \), from equation (60), \( h_{2,t} \) is positively related to \( h_{1,t} \), so \( \text{Cov} \left( h_{1,t}, h_{2,t} \right) > 0 \) and the second term in (64) is greater than unity. As cokurtosis in the noncrisis period is \( \text{Cokurtosis} \left( x_{1,t}^2, x_{2,t}^2 \right) = 1 \), which is obtained by imposing the additional restrictions \( \delta_1 = \delta_2 = 0 \) in (64), it follows that increases in \( \delta_1 \) and \( \delta_2 \) directly cause a higher cokurtosis in the crisis period, which is reflected through the copula test \( V_4^2 \). Finally, the lack of power in the third moment copula test \( V_3^2 \), follows from the property that coskewness is zero in noncrisis and crisis periods as the distribution of \( z_t \) is symmetric.

6 Empirical Application

The conditional moment copula test is now applied to testing for contagion amongst equity markets during the recent global financial crisis. Three equity markets are investigated: UK (FTSE 100), US (S&P 500) and EU (Euro stoxx 50). The data are daily beginning 30 July 2004 and ending 3 March 2009, a total of 1198 observations. Equities are expressed as percentage daily returns according to \( 100 \times \ln \left( P_t / P_{t-1} \right) \) where \( P_t \) is the equity index. Plots of the returns are given in Figure 3. All data are obtained from Datastream.
Table 8:

Power properties of alternative contagion tests based on the parameters of \((\delta_1, \delta_2)\) using the MGARCH DGP with sample sizes \(T_x = 1000\) and \(T_y = 250\), based on size adjusted critical values of 5%. The parameter settings in equation (60) is: \(\beta = 0, g_1 = 0.05, g_2 = 0.9\). 

<table>
<thead>
<tr>
<th>(\delta_1 = \delta_2)</th>
<th>(V_1^2)</th>
<th>(V_2^2)</th>
<th>(V_3^2)</th>
<th>(V_4^2)</th>
<th>(\Psi_4^2 = \sum_{j=1}^{4} V_j^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.050</td>
<td>0.050</td>
<td>0.050</td>
<td>0.050</td>
<td>0.050</td>
</tr>
<tr>
<td>0.05</td>
<td>0.059</td>
<td>0.793</td>
<td>0.099</td>
<td>0.751</td>
<td>0.887</td>
</tr>
<tr>
<td>0.10</td>
<td>0.076</td>
<td>0.919</td>
<td>0.108</td>
<td>0.912</td>
<td>0.976</td>
</tr>
<tr>
<td>0.15</td>
<td>0.062</td>
<td>0.961</td>
<td>0.136</td>
<td>0.955</td>
<td>0.992</td>
</tr>
<tr>
<td>0.20</td>
<td>0.085</td>
<td>0.980</td>
<td>0.106</td>
<td>0.971</td>
<td>0.996</td>
</tr>
<tr>
<td>0.25</td>
<td>0.097</td>
<td>0.970</td>
<td>0.115</td>
<td>0.971</td>
<td>0.992</td>
</tr>
<tr>
<td>0.30</td>
<td>0.099</td>
<td>0.985</td>
<td>0.127</td>
<td>0.975</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Table 9 reports the moments and co-moments of the daily returns. Two forms of coskewness are computed

\[
\text{Coskewness}_1 = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{y_{i,t} - \hat{\mu}_{y,i}}{\hat{\sigma}_{y,i}} \right)^{1} \left( \frac{y_{j,t} - \hat{\mu}_{y,j}}{\hat{\sigma}_{y,j}} \right)^{2}, \quad i \neq j,
\]

\[
\text{Coskewness}_2 = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{y_{i,t} - \hat{\mu}_{y,i}}{\hat{\sigma}_{y,i}} \right)^{2} \left( \frac{y_{j,t} - \hat{\mu}_{y,j}}{\hat{\sigma}_{y,j}} \right)^{1}, \quad i \neq j.
\]

And the cokurtosis is computed as

\[
\text{Cokurtosis} = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{y_{i,t} - \hat{\mu}_{y,i}}{\hat{\sigma}_{y,i}} \right)^{2} \left( \frac{y_{j,t} - \hat{\mu}_{y,j}}{\hat{\sigma}_{y,j}} \right)^{2}, \quad i \neq j.
\]

The coefficients of skewness and kurtosis show that the daily equity returns exhibit skewness and heavy tails. The Jarque-Bera test (JB) also rejects the null hypothesis that returns are normally distributed. A further inspection of Table 9 shows that equity returns have lower means and higher volatilities during the crisis period compared to the noncrisis period, a result that is inconsistent with risk averse agents who construct optimal portfolios based on the first two moments of the returns distribution. However, further inspection of Table
9 shows a switch in skewness from negative to positive in the crisis period, reflecting that investors have a preference for positive skewness and are prepared to trade-off this for lower average returns for a given level of risk (Harvey and Siddique (2000), Fry, Martin and Tang (2010)).

Table 9 also shows a change in the dependence relationships amongst the equity returns, with correlation, coskewness and cokurtosis all increasing during the crisis period. As with skewness, a higher level of coskewness represents investors preference for switching from assets with lower expected return to assets with lower risk (Kraus and Litzenberger (1976)). The increase in cokurtosis in the crisis period shows that the degree of dependence in volatility amongst equity markets increases during financial crises.

Table 9:

<table>
<thead>
<tr>
<th></th>
<th>Non-crisis period</th>
<th>Crisis period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UK</td>
<td>US</td>
</tr>
<tr>
<td>Moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.047</td>
<td>0.039</td>
</tr>
<tr>
<td>Variance</td>
<td>0.481</td>
<td>0.385</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.416</td>
<td>−0.093</td>
</tr>
<tr>
<td>JB</td>
<td>173.739*</td>
<td>35.331*</td>
</tr>
<tr>
<td>Co-moments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation</td>
<td>0.694</td>
<td>0.819</td>
</tr>
<tr>
<td>Coskewness1</td>
<td>−0.137</td>
<td>−0.312</td>
</tr>
<tr>
<td>Coskewness2</td>
<td>−0.212</td>
<td>−0.342</td>
</tr>
<tr>
<td>Cokurtosis</td>
<td>2.651</td>
<td>3.332</td>
</tr>
</tbody>
</table>

In Table 10 two tests of contagion tests are presented based on the copula test and the Forbes-Rigobon (FR) test. The conditional moment copula test uses GARCH(1,1) estimated marginal distributions, with a Gaussian copula to model the dependence structure. The joint copula test ($\Psi_4^2$) shows significant contagion at the 5% level amongst all three equity markets.
Figure 3: The percentage log-returns of daily equity index data in three markets: UK (FTSE100), US (S&P 500) and Eurozone (Euro Stoxx 50). Sample period begins 30 July 2004 and ends 3 March 2009.
with the exception of contagion from the EU to the UK where it is significant at the 10% level and from the US to the EU where the p-value is 0.376. Inspection of the individual copula tests ($V^2_1$ to $V^2_4$) show that contagion mainly arises through the fourth moment with all channels significant at the 5% level with the exception of contagion from the US to the EU where it is statistically significant at the 10% level. For the other three individual copula tests ($V^2_1$, $V^2_2$ and $V^2_3$), the test statistics are all statistically insignificant except for contagion through the second moment from the UK to the US.

In comparison to the copula-based tests of contagion, the FR test finds no evidence of contagion. Interestingly, the FR statistics for all combinations of potential contagion channels are negative, indicating a decrease in the conditional correlation, having adjusting for heteroskedasticity, during the crisis period.

Table 10:
Contagion tests of equity index returns in three markets: UK (FTSE 100), US (S&P 500) and Eurozone (Euro Stoxx 50). P-values in parentheses. A * denotes statistically significant at the 5% level.

<table>
<thead>
<tr>
<th>Host</th>
<th>UK to:</th>
<th>US to:</th>
<th>EU to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>US</td>
<td>EU</td>
<td>UK</td>
</tr>
<tr>
<td>$V^2_1$</td>
<td>1.617</td>
<td>1.746</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(0.186)</td>
<td>(0.972)</td>
</tr>
<tr>
<td>$V^2_2$</td>
<td>9.896*</td>
<td>0.431</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.512)</td>
<td>(0.704)</td>
</tr>
<tr>
<td>$V^2_3$</td>
<td>0.014</td>
<td>2.580</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>(0.904)</td>
<td>(0.108)</td>
<td>(0.650)</td>
</tr>
<tr>
<td>$V^2_4$</td>
<td>7.142*</td>
<td>13.359*</td>
<td>9.432*</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.000)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\Psi^2_4$</td>
<td>18.670*</td>
<td>18.115*</td>
<td>9.749*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.045)</td>
</tr>
<tr>
<td></td>
<td>(1.000)</td>
<td>(1.000)</td>
<td>(1.000)</td>
</tr>
</tbody>
</table>
7 Conclusions

The aim of this paper has been to propose a general test of contagion based on conditional copulas. An important advantage of this approach is that it not only can be used to identify a range of channels of contagion, but can be used to detect specific contagious channels operating through various moments of the distribution.

The properties of the proposed test were investigated using a range of Monte Carlo experiments, with the results compared to the sampling properties of the FR adjusted correlation test. The advantages of adopting the copula based test were highlighted in experiments where the true channels of contagion came from the first moment or higher order moments such as the fourth moment. In those cases, where the contagion operated through the second moment the FR test nevertheless exhibited superior power to the copula test, but this result also reflected the nature of the copula test, which are based on jointly modeling all moments of the distribution, compared to the former test which specifically focuses on the second moment. The copula test exhibited superior power to the FR test in the case where the contagion operated through the change of tail dependence. In the case where the contagion came from the fourth moment, the copula test exhibits good power, while the FR test fails to detect this type of contagion.

Applying the copula testing framework to investigating the presence of contagion amongst global equity markets during the recent global financial crisis, shows that contagion occurs mainly through the fourth moments. This contrasts with the results of the Forbes and Rigonbon test which is not able to detect any evidence of contagion as it focusses only on changes in conditional correlations.

References


