Voting on income-contingent loans

Elena Del Rey* and María Racionero†

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Abstract

We consider risk-averse individuals who differ in two characteristics – ability to benefit from education and inheritance – and analyze higher education participation under two alternative financing schemes - tax subsidy and (risk-sharing) income-contingent loans. With decreasing absolute risk aversion a distributional university bias exists, with individuals with larger bequests being more likely to undertake higher education despite the fact that, according to the stylized financing schemes we consider, individuals do not pay up front any financial cost of education. We then determine which financing scheme arises when individuals are allowed to vote between schemes. If participation is relatively low the income-contingent scheme obtains a majority. The tax-subsidy scheme results only if participation is relatively large and relatively extreme assumptions are placed on relevant parameters.

Keywords: voting, higher education finance, income-contingent loans

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*Departament d’Economia, FCEE. Universitat de Girona. Campus de Montilivi, 17071 Girona (Spain). Tel.: +34 972 41 87 49. Fax: +34 972 41 80 32. E-mail: elena.delrey@udg.edu
†Research School of Economics, Australian National University, ACT0200 Canberra (Australia). Tel.: +61 2 61254466. Fax.: +61 2 61255124. E-mail: maria.racionero@anu.edu.au
1 Introduction

Higher education financing schemes that rely partly on contributions from students are being increasingly adopted, or expanded. One acknowledged problem of relying on cost-sharing by students is that liquidity constraints may negatively affect higher education participation. Even if mortgage-type loans are made available to overcome these liquidity constraints, education is often viewed as a risky investment, and deserving but risk averse individuals may decide not to take these loans. Funding schemes that rely on income-contingent loans, like the Australian Higher Education Contribution Scheme first instituted in 1989 or the more recent funding arrangements in the UK, provide insurance against uncertain educational outcomes. Income-contingent loans are hence supposed to partly, if not fully, overcome the negative effects of risk-aversion, and as such they have been generally regarded as an improvement on mortgage-type loans to enhance higher education participation. The assessment of income-contingent loans versus tax-subsidy schemes, which have been traditionally employed in many European countries to finance higher education, is less conclusive.

Financing schemes for higher education effectively differ in the way educational costs and risks are shared among the population. García-Peñalosa and Wälde (2000) and Del Rey and Racionero (2010) are among the few theoretical contributions in the literature that consider a relatively comprehensive set of higher education financing alternatives, including both tax-subsidy and income-contingent loans. In García-Peñalosa and Wälde (2000) individuals are assumed to differ only in inheritance, whereas in Del Rey and Racionero (2010) individuals differ only in ability. When individuals differ in inheritance, the social optimum implies that either none or all should study. When individuals differ in ability, it is possible to compute an optimum threshold ability level (i.e. an optimal level of participation in higher education) and assess whether different financing schemes yield insufficient or excessive participation. Indeed, Del Rey and Racionero (2010) focuses on participation, paying particular attention to the effects of the insurance and subsidy components of different financing schemes. Del Rey and Racionero (2010) shows that an income-contingent loan with risk-pooling can induce the optimal level of participation provided that it covers both financial costs of education and forgone earnings, lending theoretical support for extending the coverage of income-contingent loans to non-tuition costs of education. However, universal income-contingent loans of the risk-pooling type, where successful students are essentially responsible for the full cost of the education of their cohort, of the risk-sharing type, are relatively rare in reality. Tax-subsidy schemes, where the cost of education is financed by general taxes, have been historically common, particularly in Europe. However income-contingent loans of the risk-sharing type, where successful graduates contribute to a large extent to the cost of their education, possibly the full cost if there are no implicit subsidies, and basically the cost of the education of unsuccessful students is financed by general taxes, are being increasingly adopted or proposed (see Chapman (2006) for an overview of the international experience of income-contingent loans).

In this paper we focus on the tax-subsidy and risk-sharing income contingent loans
schemes. Contrary to García-Peñalosa and Wälde (2000) and Del Rey and Racionero (2010) we consider individuals that differ in two characteristics: ability and inheritance. In this sense the model follows De Fraja (2001), that incorporates both differences in parental income and ability, but departs in other respects: most notably, we incorporate income-contingent loans as a financing scheme option. We analyze participation under both schemes, paying particular attention to how individuals of different ability and inheritance fare under each. We use this information to study which financing scheme is preferred by a majority when individuals are able to vote between the two schemes.

Recent contributions in the political economy of higher education finance include De Fraja (2001), Anderberg and Balestrino (2008), and Borck and Wimbersky (2009). De Fraja (2001) considers two education policies - an admission test and a subsidy financed out of general taxation - and shows that both enhance equality of opportunity, but have ambiguous equity and efficiency effects. The ambiguous equity effects of the policies are reflected in the voting behavior of individuals: when voting on the extent of the subsidy a partial "ends against the middle' results. Anderberg and Ballestrino (2008) also consider tax-subsidy schemes in a model where endogenous credit constraints play a key role. They show that a voting equilibrium, if it exists, is such that voters in the two tails of the income distribution support a reduction, while the "middle-class" supports an expansion, of the education subsidy. Borck and Wimbersky (2009) study voting over higher education financing schemes in an economy with risk averse households who are heterogeneous in income. They consider four different systems (a traditional subsidy scheme, a pure loan scheme, income contingent loans and graduate taxes). Their numerical simulations suggest that majorities for income contingent loans include income-contingent loans in the menu of financing schemes. We consider however another dimension of individual heterogeneity, namely ability. The financing schemes that we study are somehow relatively inflexible, when compared to the flexible endogenous subsidy rates that Borck and Wimbersky (2009) consider, but we are able to obtain relatively clear and intuitive results, even if we do also rely on numerical simulations to illustrate different possibilities.

The paper is organized as follows. We first present the model and describe each financing scheme in section 2. In section 3 we determine the tax that is required under each financing scheme for a given participation level. In section 4 we analyze participation with risk neutral (benchmark) and risk averse individuals. In section 5 we characterize the voting outcome. We conclude in section 6.

2 The model

We consider an economy in which a continuum of individuals of mass $N$ live for 2 periods. Individuals differ in their ability $a$ and their initial wealth $b$ (bequest from their family, which we take so far as exogenously given), with $a \in [a, \overline{a}]$ and $b \in [\underline{b}, \overline{b}]$. That is, each individual is characterised by a pair $(a, b)$. We assume initially that ability and wealth are independently distributed in $[a, \overline{a}] \times [\underline{b}, \overline{b}]$. The marginal distributions are denoted by $F(a)$ with $F'(a) = f(a)$, and $H(b)$ with $H'(b) = h(b)$. 

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Individuals derive utility from consumption, \( c \), which is a function of bequest and earned income over the lifetime. We assume that this function is a von Neumann-Morgensten utility function \( u(c) \) with, for every \( c > 0 \), \( u'(c) > 0 \), \( u''(c) \leq 0 \), \( \lim_{c \to \infty} u'(c) = +\infty \), and 
\[
\frac{d}{dc} \left[ -\frac{u''(c)}{u'(c)} \right] < 0
\]
so that the utility function displays decreasing absolute risk aversion (DARA). In the simulations, we will use the constant relative risk aversion function 
\[
u(c) = \frac{c^{1-\sigma}}{1-\sigma}
\] (1)
where \( \sigma = -\frac{c u''(c)}{u'(c)} \) is the coefficient of relative risk aversion.

In the first period, the individual decides whether or not to undertake higher education. \( k \) is the per capita cost of education. Individuals who study forego earnings in the first period. In the second period all individuals work and earn income. If the individual has invested in education, her labour market income is given by \( w_H \) with probability \( p(a) \), and by \( w_L < w_H \) with probability \( 1 - p(a) \), with \( p(a) \in (0, 1) \), with \( p'(a) > 0 \) for all \( a \in [\underline{a}, \overline{a}] \). If an individual has not gone to university, then her income is given by \( w_L \) for sure.

There are three possible states: the individual studies and is successful, the individual studies and is unsuccessful or the individual does not study. We denote them by subscripts \( S \), \( U \) and \( N \) respectively. Labour supply is exogenous and is normalized to 1. Hence, the lifetime earned labour income of the individual is \( \delta w_H \), \( \delta w_L \) and \( (1 + \delta) w_L \), where \( \delta \) is the discount factor, for individuals \( S \), \( U \) and \( N \) respectively. We assume that \( \delta w_H > (1 + \delta) w_L \).

The government provides education free of charge in the first period and raises the necessary revenue in a manner that differs according to the financing scheme. A potentially different amount of individuals \( H_j \), where \( j \) represents the funding scheme, enroll in higher education in the first period. We focus on two financing schemes for higher education: tax-subsidy (\( TS \)) and (risk-sharing) income-contingent loan (\( IC \)).

- In the tax-subsidy system (\( TS \)), the cost of education is financed by general taxation in the second period. Therefore each individual pays \( H^{TS}k/N \) in present value terms, irrespective of her situation.

- We model the income-contingent loan (\( IC \)) as the risk-sharing income-contingent loan in Del Rey and Racionero (2010). All individuals who want to study borrow \( k \). Only those individuals who are successful have to repay the amount in full. However, a lump-sum tax, which amounts to \( (1 - p)H^{RS}k/N \) in present value terms, is levied on all individuals in order to raise the revenue needed to cover the education cost of unsuccessful students.
The timing of the model is the following: first, individuals vote for $TS$ or $IC$. Once the higher education financing scheme is chosen, by majority voting, they decide whether or not to participate. Finally, they contribute. We solve the model by backward induction, starting with the determination of the tax rates, given participation and the finance scheme.

3 Tax cost of alternative schemes

Let $a^{TS}(b)$ be the threshold ability level (i.e. the ability level of an individual who is indifferent between studying or not) of an individual with bequest $b$ for the tax-subsidy financing scheme. Under the tax-subsidy system the number of individuals who undertake higher education is

$$H^{TS} = \int_{\frac{b}{\pi}}^{\int_{\frac{a^{TS}(b)}}{f(a)h(b)dadb}},$$

and the lump-sum tax required to finance their education:

$$t^{TS} = k \int_{\frac{b}{\pi}}^{\int_{\frac{a^{TS}(b)}}{f(a)h(b)dadb}}.$$ (2)

With a (risk-sharing) ICL, all individuals who want to study borrow $k$ but only those students who are successful have to repay the amount in full. A lump-sum tax is levied on all individuals in order to raise the revenue needed to cover the education cost of unsuccessful students. As before, $a^{IC}(b)$ denotes the ability level of an individual with bequest $b$ who is indifferent between studying or not. Under the ICL system the number of individuals who undertake higher education is

$$H^{IC} = \int_{\frac{b}{\pi}}^{\int_{\frac{a^{IC}(b)}}{f(a)h(b)dadb}},$$

and the lump-sum tax required to finance their education:

$$t^{IC} = \frac{k}{N} \int_{\frac{b}{\pi}}^{\int_{\frac{a^{IC}(b)}}{(1-p(a))f(a)h(b)dadb}}.$$ (3)

4 Participation

Given the higher education finance scheme and anticipating the tax cost of participating, individuals decide whether or not to enrol. We first identify optimal participation, which we will use as a benchmark against which participation under each scheme will be compared.
4.1 Optimal participation

Focusing exclusively on efficiency, it is optimal that an individual studies when her expected earnings as a student net of the cost of her education exceed her earnings as a non-student. It is possible to determine a threshold ability level, $\hat{a}$, above which an individual should study and below which an individual should not study:

$$\delta [p(\hat{a}) w_H + (1 - p(\hat{a})) w_L] - k = (1 + \delta) w_L. \quad (6)$$

The optimal amount of graduates is $H^* = \int_{\hat{a}}^{\overline{a}} f(a) da$. Note that the optimal ability level is independent of family wealth $b$.

4.2 Tax-subsidy

Let $G^{TS}(a, b)$ denote the expected net utility gain from investing in higher education under the tax-subsidy scheme for an individual with ability $a$ and bequest $b$:

$$G^{TS}(a, b) \equiv (1 - p(a)) u(c_{U}^{TS}) + p(a) u(c_{S}^{TS}) - u(c_{N}^{TS}). \quad (7)$$

The expected net utility gain from investing in higher education increases with ability:

$$\frac{dG^{TS}(a, b)}{da} = p'(a) [u(c_{S}^{TS}) - u(c_{U}^{TS})] = p'(a) [u(b + \delta w_H) - u(b + \delta w_L)] > 0. \quad (8)$$

since $p'(a) > 0$ and $w_H > w_L$. More able individuals have higher expected utility from studying than less able individuals, and are hence more likely to choose higher education.

The threshold ability level of an individual with bequest $b$ for the tax-subsidy financing scheme, $a^{TS}(b)$, satisfies

$$G^{TS}(a^{TS}(b), b) = 0.$$

That is,

$$(1 - p(a^{TS})) u(b + \delta w_L - t^{TS}) + p(a^{TS}) u(b + \delta w_H - t^{TS}) = u(b + (1 + \delta) w_L - t^{TS}),$$

where $t^{TS}$ is the lump-sum contribution everyone makes to the financing of higher education.

**Proposition 1** If, for a bequest $b \in [\overline{b}, \overline{b}]$, there exists a level of ability $a^{TS} \in [\underline{a}, \overline{a}]$ such that $G^{TS}(a^{TS}, b) = 0$, then $a^{TS}$ is unique and the function $a^{TS}(b)$ is strictly decreasing in $b$.

**Proof.** From (8) we know that, if for some value of $b \in [\overline{b}, \overline{b}]$ there exists a level of ability $a^{TS} \in [\underline{a}, \overline{a}]$ such that $G^{TS}(a^{TS}, b) = 0$, then $a^{TS}$ is unique.

To determine that $a^{TS}(b)$ is strictly decreasing in $b$ we use the implicit function theorem:

$$\frac{\partial a^{TS}}{\partial b} = - \frac{\partial G^{TS}(\cdot)}{\partial b} \frac{\partial G^{TS}(\cdot)}{\partial a} < 0$$

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since, as shown above, \( \frac{\partial G_{TS}(\cdot)}{\partial a} > 0 \) (for a given level of \( b \) the expected net gain of investing in higher education increases with ability) and, on the other hand, \( \frac{\partial G_{TS}(\cdot)}{\partial b} > 0 \) due to the DARA assumption (for a given level of \( a \) a higher income individual is more willing to bear risk and invest in higher education):

\[
\frac{\partial G_{TS}(\cdot)}{\partial b} \equiv \left(1 - p (\hat{a})\right) u' (b + \delta w_L - t_{TS}) + p (\hat{a}) u' (b + \delta w_H - t_{TS}) - u (b + (1 + \delta) w_L - t_{IC})
\]

The fact that \( a_{TS} (b) \) is strictly decreasing in \( b \) means that there exists a distributional bias in university attendance: individuals with larger bequests are more likely to undertake higher education. This is similar to the result obtained by De Fraja (2001). It is worth noticing that this results in spite of the fact that, under the scheme considered, individuals do not pay up front any financial cost of education. As pointed out, the assumption of decreasing absolute risk aversion plays a key role. Investment in education is risky, and, if the decision maker has decreasing absolute risk aversion, she is more willing to bear risk if her bequest is higher; in other words, she requires a lower expected return in order to opt for an investment of a given riskiness.

It is easy to show that the threshold ability does not depend on \( b \) if individuals are risk neutral since \( b \), and \( t_{TS} \) as well, cancel out from the equation:

\[
G_{TS} (a, b) = (1 - p (\hat{a})) \delta w_L + p (\hat{a}) \delta w_H - (1 + \delta) w_L = G_{TS} (a).
\]

Let us denote the threshold ability level of risk neutral individuals under the tax-subsidy system \( \tilde{a}_{TS} \) (to distinguish it from the risk averse \( a_{TS} (b) \)).

**Proposition 2** Risk neutral individuals overinvest in education under TS: \( \tilde{a}_{TS} < \hat{a} \).

**Proof.** \( \hat{a} \) is given by (6) and \( \tilde{a}_{TS} \) is implicitly defined by \( G_{TS} (\tilde{a}_{TS}) = 0 \), which implies

\[
\delta \left[(1 - p (\tilde{a}_{TS})) w_L + p (\tilde{a}_{TS}) w_H \right] = (1 + \delta) w_L.
\]

Then,

\[
\delta [p (\hat{a}) w_H + (1 - p (\hat{a})) w_L] - k = \delta \left[(1 - p (\tilde{a}_{TS})) w_L + p (\tilde{a}_{TS}) w_H \right]
\]

and since

\[
\delta [p (\hat{a}) w_H + (1 - p (\hat{a})) w_L] > \delta \left[(1 - p (\tilde{a}_{TS})) w_L + p (\tilde{a}_{TS}) w_H \right]
\]

it follows that \( \tilde{a}_{TS} < \hat{a} \). ■

**Proposition 3** Risk aversion reduces participation for all income levels: \( a_{TS} (b) > \tilde{a}_{TS} \) \( \forall b \).
Proof. We evaluate $G^{T_S}(a, b)$ at $\hat{a}^{T_S}$, characterised implicitly by (10). Using this and (7) we obtain

$$G^{T_S}(\hat{a}^{T_S}, b) \equiv (1 - p(\hat{a}^{T_S})) u(b + \delta w_L - t^{T_S}) + p(\hat{a}^{T_S}) u(b + \delta w_H - t^{T_S})$$

$$- u(b + (1 + \delta) w_L - t^{T_S}) = (1 - p(\hat{a}^{T_S})) u(b + \delta w_L - t^{T_S}) + p(\hat{a}^{T_S}) u(b + \delta w_H - t^{T_S})$$

$$- u(b + \delta [(1 - p(\hat{a}^{T_S})) w_L + p(\hat{a}^{T_S}) w_H] - t^{T_S}) < 0$$

since, with risk aversion, the utility of expected income is higher than the expected utility. Since $\frac{\partial G^{T_S}(\cdot)}{\partial a} > 0$ and $G^{T_S}(a^{T_S}(b), b) = 0$, it turns out that $\hat{a}^{T_S} < a^{T_S}(b)$. Note that the above holds for any $b \in [\hat{b}, \bar{b}]$.

Since $\tilde{a}^{T_S} < \hat{a}$ and $\tilde{a}^{T_S} < a^{T_S}(b)$ it remains possible that participation is efficient under the tax subsidy scheme when individuals are risk averse. In contrast, it is not possible that $a^{T_S}(b) = \hat{a}$ for all $b$ since $a^{T_S}(b)$ is strictly decreasing. This would mean that below a certain threshold $\hat{b}$ individuals are under-represented, and above the threshold $\hat{b}$ individuals are over-represented in higher education.

Example

In order to illustrate how different degrees of risk aversion affect participation we represent in Figure 1 the efficient participation together with participation under the tax subsidy scheme when individuals are risk neutral and risk averse. In the later case, we use the CES utility specification (1) and assume a moderate degree of risk aversion ($\sigma = 1.5$). The low skilled wage is normalized to 1, the skilled wage is assumed to be 3 times as large and $\delta = .85$. We also set $p(a) = pa$, with $p = 1$ and calculate $t^{T_S}$ according to (3). Finally, the cost of higher education is assumed to equal 10% of the lifetime earnings of a low skilled worker $k = .1$. Also, both wealth and ability are assumed to be uniformly distributed in the population.
4.3 Income contingent loan

Let now $G^{IC}(a, b)$ denote the expected net utility gain from investing in higher education for an individual with ability $a$ and bequest $b$ under the risk sharing income contingent loan (IC) tax-subsidy scheme:

$$G^{IC}(a, b) \equiv (1 - p(a)) u(b + \delta w_L - t^{IC}) + p(a) u(b + \delta w_H - t^{IC} - k) - u(b + (1 + \delta) w_L - t^{IC})$$

The expected net utility gain from investing in higher education continues to increase with ability under this financing scheme:

$$\frac{\partial G^{IC} (.)}{\partial a} = p'(a) [u(b + \delta w_H - t^{IC} - k) - u(b + \delta w_L - t^{IC})] > 0$$

since $p'(a) > 0$ and $\delta (w_H - w_L) > k$ or, else, no individual would study. This also proves that if for some value of $b \in [\tilde{b}, \bar{b}]$ there exists a level of ability $a^{IC} \in [\underline{a}, \bar{a}]$ such that $G^{IC}(a^{IC}, b) = 0$, then $a^{IC}(b)$ is unique. To determine that $a^{IC}(b)$ is strictly decreasing in $b$ we use again the fact that $\frac{\partial G^{IC} (.)}{\partial b} > 0$ due to the DARA assumption and the implicit function theorem:

$$\frac{\partial a^{IC}}{\partial b} = -\frac{\frac{\partial G^{IC} (.)}{\partial b}}{\frac{\partial G^{IC} (.)}{\partial a}} < 0.$$
that \( G^{IC} (a^{IC}, b) = 0 \), then \( a^{IC} \) is unique and the function \( a^{IC} (b) \) is strictly decreasing in \( b \).

The threshold ability level under risk neutrality \( \hat{a}^{IC} \) satisfies:

\[
p (\hat{a}^{IC}) \delta (w_H - w_L) = w_L + p (\hat{a}^{IC}) k < w_L + k.
\]  

(13)

Since \( w_L + k < w_L \), it follows that \( \hat{a}^{TS} < \hat{a}^{IC} < \hat{a} \). On the one hand, higher education participation is lower than with the tax-subsidy system. This is due to the fact that the cost of education is partly subsidized by non-students but to a lesser extent than in the tax-subsidy system. At the same time, more individuals get educated than at the optimum since, in expected terms, students are only responsible for part of the cost of their education. Note that the expected cost of becoming educated is \( p (a) k \) (the tax part is paid irrespective of whether the individual studies or not) which is smaller than \( k \) (cost of self-finance).

**Proposition 5** Risk neutral individuals overinvest in education under IC, but less so than under TS: \( \hat{a}^{TS} < \hat{a}^{IC} < \hat{a} \).

Finally,

**Proposition 6** Risk aversion reduces participation for all income levels: \( a^{IC} (b) > \hat{a}^{IC} \) \( \forall b \).

**Proof.** \( \hat{a}^{IC} \) is characterised implicitly by

\[
(1 - p (\hat{a}^{IC})) \delta w_L + p (\hat{a}^{IC}) \delta w_H - k = (1 + \delta) w_L.
\]  

(14)

We now evaluate \( G^{IC} (a, b) \) at \( \hat{a}^{IC} \). Using (11) and (14), we obtain

\[
G^{IC} (\hat{a}^{IC}, b) \equiv (1 - p (a^{IC})) u (b + \delta w_L - t^{IC}) + p (\hat{a}^{IC}) u (b + \delta w_H - t^{IC})
- u (b + (1 + \delta) w_L - t^{IC})
= (1 - p (\hat{a}^{IC})) u (b + \delta w_L - t^{IC}) + p (\hat{a}^{IC}) u (b + \delta w_H - t^{IC})
- u (b + \delta [(1 - p (\hat{a}^{IC})) w_L + p (\hat{a}^{IC}) w_H] - t^{IC}) < 0
\]

since, with risk aversion, the utility of expected income is higher than the expected utility. As before, since \( \frac{\partial G^{IC}(b)}{\partial a} > 0 \) and \( G^{IC} (a^{IC} (b), b) = 0 \), it turns out that \( \hat{a}^{IC} < a^{IC} (b) \). The above holds for any \( b \in [\underline{b}, \bar{b}] \).

**Example**

It is unfortunately not possible to obtain an ordering of higher education participation under alternative financing schemes in general. Figure 2 represents the efficient participation together with the participation thresholds for both TS and IC for the benchmark
parameter specification described above. We observe that, although \( IC \) yields slightly lower participation, the difference is relatively small due to the fact that education costs are small. In both cases, the number of enrolled students is inefficiently low and more so for individuals with lower bequests.

![Figure 2: Participation with risk aversion: TS vs ICL (\( \sigma = 1.5 \))](image)

Increasing the coefficient for the degree of risk aversion (\( \sigma = 2.25 \)) makes the difference in participation between both schemes smaller (see Figure 3).

![Figure 3: Participation with risk aversion: TS vs ICL (\( \sigma = 2.25 \))](image)
5 Voting over the financing scheme

In this section we analyze the preferences of all individuals concerning the higher education financing scheme when they are able to anticipate both participation decisions and the corresponding tax rate.

5.1 Risk neutrality

For risk neutrality we obtained above that $\hat{a}^{TS} < \hat{a}^{IC} < \hat{a}$. Since participation is lower and graduates contribute more under $IC$, $t^{TS} > t^{IC}$. We are interested in identifying the decisive ability thresholds below which one financing scheme is preferred and above which the other one is preferred instead. Then, we can simply compare the number of individuals at each side of the threshold and conclude what the majority prefers. We are able to establish the following:

Proposition 7 Decisive individuals under risk neutrality: If $(t^{TS} - t^{IC})/k < p(\hat{a}^{IC})$, there exists a threshold $a' \in [\hat{a}^{TS}, \hat{a}^{IC}]$ below which individuals prefer $IC$ and above which individuals prefer $TS$. If $(t^{TS} - t^{IC})/k > p(\hat{a}^{IC})$, there exists a threshold $a'' > \hat{a}^{IC}$ below which individuals prefer $IC$ and above which individuals prefer $TS$.

Proof. First, below $\hat{a}^{TS}$, individuals do not study under any of the two systems: they prefer the risk-sharing income contingent loan because they pay less.

Second, in the region $[\hat{a}^{TS}, \hat{a}^{IC}]$ individuals study under $TS$ but do not study if the scheme is an $IC$ loan. They too prefer $IC$ if and only if

$$b + (1 + \delta) w_L - t^{IC} > (1 - p(a)) \left(b + \delta w_L - t^{TS}\right) + p(a) \left(b + \delta w_H - t^{TS}\right).$$

Hence

$$p(a) < \frac{w_L + t^{TS} - t^{IC}}{\delta (w_H - w_L)} = p(a').$$

(15)

Third, in the region above $\hat{a}^{IC}$ individuals study in both systems and they prefer the risk-sharing income contingent loan provided that

$$(1 - p(a)) \left(b + \delta w_L - t^{IC}\right) + p(a) \left(b + \delta w_H - t^{IC} - k\right) > (1 - p(a)) \left(b + \delta w_L - t^{TS}\right) + p(a) \left(b + \delta w_H - t^{TS}\right),$$

which can be rewritten as

$$t^{TS} - t^{IC} > p(a) k.$$  

Hence,

$$p(a) < \frac{t^{TS} - t^{IC}}{k} = p(a'').$$

(16)
How do \( p(a') \) and \( p(a'') \) compare?

\[
p(a') > p(a'') \iff \frac{w_L}{\delta(w_H - w_L) - k} > \frac{t^{TS} - t^{IC}}{k} = p(a'').
\]

From the condition for \( \hat{a}^{IC} \) in equation (14),

\[
p(\hat{a}^{IC}) = \frac{w_L}{\delta(w_H - w_L) - k},
\]

we can conclude that if \( p(\hat{a}^{IC}) > \frac{t^{TS} - t^{IC}}{k} = p(a'') \) then \( a'' \) is not in the region above \( \hat{a}^{IC} \). Then, all individuals above \( \hat{a}^{IC} \) prefer \( TS \) and there exists a threshold \( a' \in [\hat{a}^{IC}, \hat{a}^{TS}] \) below which individuals prefer \( IC \).

If, on the other hand, \( p(\hat{a}^{IC}) < \frac{t^{TS} - t^{IC}}{k} = p(a'') \), then some individuals above \( \hat{a}^{IC} \) prefer \( IC \) and all individuals below \( \hat{a}^{IC} \) also prefer \( IC \). To see this last point, note that we know that all individuals with \( p(a) > \frac{w_L + t^{TS} - t^{IC}}{\delta(w_H - w_L)} \) prefer \( TS \) in the region \([\hat{a}^{TS}, \hat{a}^{IC}]\).

We then need to show that \( \frac{w_L + t^{TS} - t^{IC}}{\delta(w_H - w_L)} \) is out of this region and hence everyone with ability in \([\hat{a}^{TS}, \hat{a}^{IC}]\) prefers \( IC \):

\[
p(a') = \frac{w_L + t^{TS} - t^{IC}}{\delta(w_H - w_L)} > \frac{w_L}{\delta(w_H - w_L) - k} = p(\hat{a}^{IC})
\]

if and only if

\[
w_L \delta (w_H - w_L) - w_L k + (t^{TS} - t^{IC}) \delta (w_H - w_L) - (t^{TS} - t^{IC}) k > \delta (w_H - w_L) w_L,
\]

or

\[
p(a'') = \frac{t^{TS} - t^{IC}}{k} > \frac{w_L}{\delta(w_H - w_L) - k} = p(\hat{a}^{IC})
\]

so it is always the case when \( p(\hat{a}^{IC}) < \frac{t^{TS} - t^{IC}}{k} \). The decisive individuals are given then by \( a'' \) such that \( p(a'') = \frac{t^{TS} - t^{IC}}{k} \).

### 5.2 Risk aversion

We now explore the relevant case where individuals are risk averse. In the region \([0, \hat{a}^{TS}(b)]\) individuals do not study under any of the two systems, and they prefer the risk-sharing income contingent loan because they pay less:

\[
u(b + (1 + \delta) w_L - t^{IC}) > u(b + (1 + \delta) w_L - t^{TS}).
\]
In the region \([\tilde{a}^{TS}(b), \tilde{a}^{IC}(b)]\) individuals study with the tax-subsidy scheme but do not study with the risk-sharing income contingent loan. They prefer \(IC\) if

\[
u(b + (1 + \delta w_L - t^{IC})) > (1 - p(a)) u(b + \delta w_L - t^{TS}) + p(a) u(b + \delta w_H - t^{TS}).
\]

We can thus define a threshold \(\tilde{a}'(b)\) such that individuals prefer not to study and pay the \(IC\) contribution to the cost of the education of unsuccessful students rather than study and pay the tax with \(TS\):

\[
p(\tilde{a}'(b)) \leq \frac{u(b + (1 + \delta w_L - t^{IC}) - u(b + \delta w_L - t^{TS})}{(u(b + \delta w_H - t^{TS}) - u(b + \delta w_L - t^{TS}))}
\]

Let \(G'(a, b)\) be the utility differential between studying with \(TS\) and not studying with \(IC\):

\[
G'(a, b) = (1 - p(a)) u(b + \delta w_L - t^{TS}) + p(a) u(b + \delta w_H - t^{TS}) - u(b + (1 + \delta) w_L - t^{IC}).
\]

\(G'(a, b)\) is increasing in \(a\). If we evaluate it at \(\tilde{a}^{IC}(b)\)

\[
G'(\tilde{a}^{IC}(b), b) = (1 - p(\tilde{a}^{IC}(b))) u(b + \delta w_L - t^{TS}) + p(\tilde{a}^{IC}(b)) u(b + \delta w_H - t^{TS}) - (1 - p(\tilde{a}^{IC}(b))) u(b + \delta w_L - t^{IC}) - p(\tilde{a}^{IC}(b)) u(b + \delta w_H - t^{IC}),
\]

we obtain two possibilities:

1. If

\[
G'(\tilde{a}^{IC}(b), b) > 0
\]

then, on the one hand, \(\tilde{a}'(b) < \tilde{a}^{IC}(b)\) and, on the other, every individual with wealth \(b\) and ability above \(\tilde{a}^{IC}(b)\) prefers \(TS\): expected utility of education is larger under \(TS\) for \(\tilde{a}^{IC}(b)\) and hence it is so for all \(a > \tilde{a}^{IC}(b)\). Then the decisive individual is \(\tilde{a}'(b)\).

2. If

\[
G'(\tilde{a}^{IC}(b), b) < 0
\]

then \(\tilde{a}'(b) > \tilde{a}^{IC}(b)\). Everyone in the region \([\tilde{a}^{TS}(b), \tilde{a}^{IC}(b)]\) prefers \(IC\) and some individuals above \(\tilde{a}^{IC}(b)\) prefer also \(IC\). There is then a second threshold \(\tilde{a}''(b) > \tilde{a}^{IC}(b)\) that becomes the relevant one.

The outcome that ultimately emerges, and whether the individuals with ability \(\tilde{a}'(b)\) or \(\tilde{a}''(b)\) are the decisive ones, depends on the particular combination of parameters. To shed more light on the role played by those parameters we next proceed to report some examples. Del Rey and Racionero (2010) noted that, with risk averse preferences, it is not possible to determine in general whether \(TS\) or \(IC\) induce higher participation. Nevertheless the simulations seem to suggest that higher participation with \(TS\) results for
most reasonable combinations of parameters and we concentrate hereafter on situations of this type.

It is worth noting the reason why it is important to characterize whether the relevant decisive ability threshold, which determines the support for IC versus TS, is \( \tilde{a}'(b) \) or \( \tilde{a}''(b) \). In the first case, support for the tax-subsidy scheme comes from those individuals who always study, irrespective of the financing scheme, and prefer to pay less, plus those individuals who study under the TS system but would not do so if offered IC loans instead. In the second case, support for the tax-subsidy scheme comes uniquely from a subset of the students who would study under both regimes (i.e., the relatively more able and wealthier). The key difference is that, while in the first case some of those who support TS would not access higher education if offered IC loans instead, in the second case all those who support TS study under both schemes and simply prefer to pay less.

In situations where participation in higher education with TS is below 50% the outcome of the choice between the two stylized schemes that we consider is trivial: the IC would be preferred. We concentrate next on examples where the combination of parameters adopted yields higher education participation in excess of 50%, which may be unrealistic in many countries yet required in the model to yield the choice non-trivial.

5.3 Example 1: the majority supports loans

We first provide an example where the returns to investing in higher education are large, the cost is high (20% of the lifetime earnings of a low skilled worker) and risk aversion is low (\( \sigma = 0.5 \)). The low skilled wage is normalized to 1, the skilled wage is assumed to be 4 times as large and \( \delta = 0.9 \). As before, \( p(a) = pa \), with \( p = 1 \) and \( t^{TS} \) is computed according to (3).
In this example, the threshold $\tilde{a}^{TS}$ is given by the red line (51% of the population study with the tax-subsidy scheme). Thus, those who never study do not have the majority of the vote. The threshold $\tilde{a}^{RS}$ is given by the green line, so all those who do not participate in higher education with the loan in fact support the loan scheme. So do some students of lower ability who choose to study under both schemes. Only those students above the threshold $\tilde{a}'$ (black line) support the tax subsidy scheme and they clearly do not hold the majority.

5.4 Example 2: the majority supports taxes

In this example, the tax subsidy scheme wins because participation in higher education is very large and those who participate prefer to pay less (i.e., $TS$). $TS$ also receives support from a few individuals who study only with the tax-subsidy scheme but would not do so if offered $IC$ loans instead. The skill premium in this example is still very large ($w_H = 3.5$), risk aversion is lower than before ($\sigma = 0.2$) and so is the cost of education (1% of the lifetime earnings of a low skilled worker). Everything else remains the same.

![Figure 5: Majority for TS](image)

This example highlights that the conditions necessary for the tax-subsidy to emerge are relatively extreme. In particular, very large levels of participation are required in order for the $TS$ scheme to receive a majority of votes.

6 Conclusions

We consider individuals who differ in two characteristics – ability to benefit from education and inheritance – and analyze higher education participation under two alternative
financing schemes - tax subsidy and (risk-sharing) income-contingent loans -, paying particular attention to the welfare achieved by individuals with different ability and wealth under each. We show that there exists a distributional bias in university attendance: individuals with larger bequests are more likely to undertake higher education. It is worth noticing that this results in spite of the fact that they do not pay in advance for their education: the assumption of decreasing relative risk aversion plays a crucial role in this result. We then study which financing scheme arises when individuals are allowed to vote between the two schemes. We do so both for the benchmark case of risk neutrality and for risk aversion. If participation is relatively low, then the income-contingent scheme obtains a majority. The tax-subsidy results only if participation is relatively large and relatively extreme assumptions are placed on relevant parameters.

The way in which we model the alternative financing schemes is arguably rather inflexible: in particular, the taxes individuals are required to pay to contribute to the cost of higher education are lump-sum, and are calculated from the budget constraint. As a result individuals do not vote on the tax (or subsidy) rate, as is the case in other contributions in the literature, but on the overall financing scheme.

References


