Moving Towards a Single Labour Contract: Transition vs. Steady-State*

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[In Progress – Comments Welcome]

Abstract

This paper analyzes the optimal design of a single open-ended contract (SOEC) and studies the political economy of moving towards such a SOEC in a dual labour market. We compare two economic environments: one with flexible entry-level jobs and high employment protection at long tenure, and another with a SOEC featuring employment protection levels that increase smoothly with tenure. For illustrative purposes, we specialize the discussion of such choices to Spain. A SOEC has the potential of bringing big time efficiency and welfare gains in a steady-state sense. We also identify winners and losers in the transitional path of such a reform and analyze its political support.

Keywords: Single contract; Employment Protection; Dualism; Labor Market Reform

JEL codes: H29, J33, J65

1 Introduction

Employment protection legislation (EPL) has been rationalized on several grounds. These range from strengthening the bargaining power of workers to avoiding moral hazard by employers or improving worker’s employer-sponsored training. Yet, one of the most important reasons for having EPL is to increase job stability of risk-averse workers in order to insure them against dismissals. In several countries, particularly in Southern Europe, labour markets exhibit a high degree of dualism: workers under open-ended (permanent) contracts

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1See Booth and Chatterji (1989, 1998).
enjoy very stringent EPL while those under fixed-term (temporary) contracts enjoy little or even none. In particular, permanent contracts bear mandated severance payment that increase with tenure, typically subject to a cap. This is usually measured in terms of days of wages per years of service (d.w.y.s.), which are lower for dismissals due to fair (economic) reasons than those deemed unfair. In contrast, due to their short-term duration, temporary contracts are hardly ever destroyed although they are subject to a fixed termination cost (again in terms of d.w.y.s.) which is quite lower than permanent workers’ redundancy pay (see Cahuc et al., 2012). As noted by Blanchard and Landier (2002), lacking enough wage flexibility, the large gap in severance costs between these types of workers makes employers reluctant to transform temporary contracts into permanent ones.

Thus, temporary contracts create a “revolving door” with workers rotating between temporary jobs and unemployment. Bentolila et al. (2012) have pointed out that the discontinuity (the so-called “wall”) created by conventional EPL schemes in dual labour markets has negative consequences for unemployment, human capital and innovation since it leads to excessive turnover (Blanchard and Landier, 2002), excessive wage pressure (Bentolila et al., 1994), low investment by firms in employer-sponsored training schemes for temporary workers (Cabrales et al., 2014), and the adoption of mature rather than innovative technologies (Saint-Paul, 2002).

This has triggered a heated debate on redesigning dual employment protection, leading to policy initiatives in Southern Europe defending the suppression of the firing-cost gap once and for all. To achieve this goal, a key policy advice in these proposals is to introduce a single open-ended contract (SOEC hereafter) for new hires, at the same time that most temporary contracts are abolished – the exception being replacement contracts for maternity or sickness/disability leaves. The key feature of SOEC is that it has no ex ante time limit (unlike fixed-term contracts) and that severance payments smoothly increase with seniority (unlike current indefinite term contracts where the same indemnity per year of service applies from the start). In this fashion, a SOEC would provide a sufficiently long entry phase and a smooth rise in protection as job tenure increases. The rationale for the gradually increasing severance pay could be that the longer a worker stays in a given firm, the larger is her/his loss of specific human capital and the psychological costs suffered in case of dismissal – a negative externality that firms should internalize.

However, despite being high on the European political agenda, most SOEC proposals so far have been fairly deceptive. As a result, several design and implementation problems need to be worked out before SOEC becomes operative. In this paper, we take a first step towards addressing the following pending issues. First, as mentioned earlier, there is a general agreement that

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3 See Blanchard and Tirole (2003). Further, as argued by Ignacio García Pérez and Osuna (2014), even if a SOEC implies lower redundancy pay per year of service than current EPL for permanent contracts, the increase in job tenure that it may bring in can lead to a “Laffer curve effect” whereby the total compensation received by a dismissed worker is larger under SOEC than under the current dual EPL system.

4 See Chapter 4 of the OECD 2014 Employment Outlook devoted to this topic.
SOEC should feature mandated severance payments that increase smoothly with tenure; but little is known about the exact tenure profile of the contract. Secondly, there is general agreement that SOEC would benefit the functioning of labour markets; but little is known about the magnitude of the allocational and welfare improvements that would result. Thirdly, in the context of dual labour markets, there is suspicion that a non-negligible number of insiders would lose from the policy change and, thus, would oppose the reform; but not much is known about the relevance of this argument, i.e., the political strength of insiders, the size of their welfare losses, and whether an appropriately designed transition towards the new steady-state could limit their losses.

We tackle these questions by developing an equilibrium search and matching model, which we use to investigate the effects of introducing a SOEC in a dual labour market. In our model, risk averse workers demand insurance, a feature that enables us to compute the optimal tenure profile according to some aggregate welfare criterion. More specifically, our model is one where young and older workers coexist and the former become older at a given rate. Both receive severance pay but differ with respect to the use they can make of this compensation. So, while it is assumed that young workers consume it upon reception (say, because of binding credit constraints associated to lower job stability; see Crossley and Low, 2014), older workers are allowed to buy annuities in order to smooth their consumption until retirement. Furthermore, since our focus is on the political economy of the reform introducing SOEC, steady-state comparisons as well as transitional dynamics are considered.

Optimality is defined both in terms of the welfare of a newborn in a steady state and of average welfare across the current population when taking into account the transition from a dual EPL system to SOEC. While the former criterion accounts for the introduction of a SOEC in a retroactive manner, without looking at the transition from the extant regulation to the SOEC, the latter implies a non-retroactive regulation change that also takes the transition into account.

For illustrative purposes, the model is calibrated to the Spanish labour market before the Great Recession, when the unemployment rate was similar to the EU average rate, namely about 8%. We choose Spain because it has been often considered as the epitome of a dual labour market (see e.g., Dolado et al., 2002). Yet, the methodology proposed here could be extended to other dual labour markets, like France or Italy. Specifically, once the parameters are calibrated to reproduce targets prior to the crisis, we compute the optimal tenure profile of redundancy pay according to the two above-mentioned criteria. A key element in our setup is that unemployment benefits are financed by a payroll tax and that severance pay has in part a layoff tax nature due the existence of red-tape cost associated to litigation procedures, etc. In this fashion, our analysis is related to Blanchard and Tirole (2008)’s discussion of whether the contribution rate—the ratio between layoff taxes and unemployment (UI) benefits—should

\[5\] In our model, young workers should be interpreted as prime-age workers (workers aged 25 to 54). Correspondingly, older workers are those aged 55 to 65 years old. We use this terminology for simplicity, and in keeping with standard OLG models where young agents are those who work and old agents are those who consume savings and get a pension.
be greater than, equal or lower than one, depending on the nature of the deviation from their benchmark model where risk-averse workers can be insured by risk-neutral firms.

Indeed, Blanchar and Tirole (2008) is one the forerunners of this paper. We differ from their analysis in that their focus is essentially normative and does not provide actual figures that would inform specific labor market policies, whereas ours is positive in that we are interested in modelling key features of a notoriously dysfunctional labour market, as is the case of Spain. Further, while their analysis is static, ours involves rich dynamics and considers the transition from dual EPL to SOEC. A second related paper is Ignacio García Pérez and Osuna (2014) who also look at the effects of moving towards a SOEC in the context of the Spanish labour market. The main differences between our approach and theirs is that: (i) they impose a given tenure profile in a SOEC rather than deriving it, and (ii) workers are risk neutral in their setup whereas they are risk averse and value consumption smoothing in ours. Finally, there is a recent paper by Boeri et al. (2013) which proposes a rationale for mandatory severance pay increasing with tenure on the basis that financing initial investment in training through wage deferrals is not sustainable if employers cannot commit not to dismiss workers who have invested in training. As before, their model is again one where agents are risk neutral and they do not derive specific tenure profiles.

Our main findings are that when comparing steady states, by avoiding the excess worker turnover rate implied by dual EPL, SOEC can lead to a large reduction in nonemployment (early retirement and unemployment rates), higher wages, lower payroll taxes and lower job destruction rates both for young and older workers. Further, along the transition from a dual EPL system to SOEC we identify winners and losers from the reform as well as the fraction of the population supporting it.

The rest of the paper is structured as follows. Section 2 lays out the main ingredients of the model. Section 3 proceeds to calibrate the model to the Spanish labour market prior to the Great Recession. Section 4 present the results of the simulations we carry out involving the optimal tenure profile of SOEC on the basis of several welfare criteria, comparison of steady states and the transition phase when replacing dual EPL by SOEC. We also discuss the trade-off between unemployment insurance and employment protection which is present in our model. Section 5 concludes. An Appendix presents our numerical methodology for computing steady states and transition paths.

2 The model

This section presents our search and matching model. The model is a variant of Mortensen and Pissarides (1994), which we accommodate to: (i) provide a role for insurance, (ii) allow workers to have different tenure at their job and (iii) obtain tractability outside the steady-state.
2.1 Economic environment

Time is discrete and runs forever. The economy may not be in steady-state and thus we need to keep track of calendar time. This is indexed by the subscript \( t \).

**Workers**

The economy is populated by a continuum of risk-averse workers who work and then retire from the labor market. Workers derive utility from consumption \( c_t > 0 \) according to the per-period CRRA utility function

\[
    u(c_t) = c_t^{1-\sigma} - \frac{1}{1-\sigma}
\]

The coefficient of relative risk-aversion \( \sigma \) is strictly positive and ensures that \( u'(c_t) > 0 \) and \( u''(c_t) < 0 \). A coefficient of one makes this utility function logarithmic.

It is assumed that workers face incomplete asset markets and that there is no storage technology. We abstract from savings to keep the computational task manageable, but we will nevertheless allow for some form of insurance (details follow).

**Production**

Production is carried out by a continuum of firms. A firm is a small production unit with only one job, either filled or vacant. There is a per-period cost \( k > 0 \) of having a vacant job. Firms enter and leave the market freely and maximize the sum of profit streams discounted by the interest rate \( r \), which is exogenous and fixed.

Workers and firms come together via search. There is an aggregate Cobb-Douglas matching function with constant returns-to scale:

\[
    m(v_t, u_t) = A \psi v_t^{1-\psi} u_t
\]

where \( v_t \) and \( u_t \) are the number of vacancies and job-seekers, respectively. \( \psi \in (0, 1) \) is the elasticity of the number of contact to the number of job-seekers and \( A \) is a matching-efficiency parameter. Thus, the job-finding probability for workers \( \theta_t q(\theta_t) \), where \( q(\theta_t) = A \theta_t^{-\psi} \), is increasing in labour-market tightness \( \theta_t \equiv v_t / u_t \). Likewise, the vacancy-filling probability for firms is given by \( q(\theta_t) \), such that \( q'(\theta_t) < 0 \).

A worker-firm match is characterized by its idiosyncratic productivity \( z \), which takes discrete values and evolves according to a Markov process with transition matrix \( \Pi = (\pi_{z,z'}) \). Every worker-firm pair starts at the highest productivity level, which is normalized to 1.

Anticipating on the design of employment protection schemes, we denote by \( \tau \) the tenure of a given worker-firm match. In our applications, we impose a cap on tenure at a value \( T \). Thus,
the law of motion for $\tau$ is: $\tau' = \min\{\tau + 1, T\}$. Observe that, as a result, there are (at least) two state variables for every worker-firm pair: tenure ($\tau$) and productivity ($z$).

**Young vs. older workers**

The working life span is uncertain, and each period a fraction of newborns enters the labor market to maintain the size of the workforce at a constant unit level. Within the workforce, we distinguish young ($y$) workers from older ($o$) workers. As in Castaneda et al. (2003), it is assumed that retirement and aging occur stochastically: at the end of each period, young workers become older with probability $\gamma$ and older workers retire with probability $\chi$.

There are two key differences between young workers and older workers. First, following job loss, young workers keep searching for new jobs whereas older workers abandon the labor market until they fully retire (“early retirement”). To rationalize older workers’ preference for leisure, we add a leisure component $\ell > 0$ to their momentary utility function. Second, older workers are allowed to buy an annuity from firms upon separation from the job. In so doing, they can increase their consumption until reaching full retirement.

It is appropriate here to comment on these two assumptions. The annuity payment we allow for implies that one needs to keep track of older workers’ employment history after a job loss since this capitalizes into the annuity scheme. However, because they stop searching for jobs, the distribution of older unemployed workers across tenure levels in the previous job is irrelevant for the vacancy-posting decision of firms. Conversely, young unemployed workers are homogeneous in that they are prevented from capitalizing their employment history into annuities. Thus, although somewhat extreme, these two assumptions allow us to provide a role for insurance while maintaining feasibility for computations outside the steady-state.

**Government-mandated programs**

The government runs two labour market programs: unemployment insurance and employment protection schemes. The provision of these programs is financed by the proceeds of a payroll tax $\kappa$. Importantly, we assume that the government balances its budget in every period.

The unemployment insurance program consists in providing a constant-level benefit $b$ to the nonemployed. There is no monitoring technology, and thus older workers can collect $b$ after a job loss, although they stop searching for jobs.

Employment protection is introduced in the form of government-mandated severance payments. In line with a long-established literature (e.g. Bertola and Rogerson, 1997), it is assumed that severance payments have two components: (i) the transfer to the worker and (ii) a firing tax representing red-tape costs involved in the dismissal procedure. The total severance payment is

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6The leisure component is a simple modelling device to increase the value of nonemployment for older workers. We introduce it to ensure that low-productivity matches with an old workers are destroyed at very long tenure.
(possibly) a function of tenure \( \tau \) and is denoted by the function \( \phi(\tau) \). Workers receive a fraction \( \upsilon \) of the severance package while \((1 - \upsilon)\) is the fraction of red-tape costs.\footnote{The tax component of severance payment is meant to capture costly administrative procedures inherent to the implementation of the policy scheme. This is hence a deadweight loss for the economy and cannot be used to balance the government budget. As a result, the payroll tax \( \kappa_t \) is used to finance the provision of unemployment benefits, but not the employment protection scheme.}

### Disposable income

Having described the environment, we are now in a position to describe the income workers receive (and consume) in the different states of the labour market. In employment, they bargain over the surplus of the match and obtain a wage \( w_i(z, \tau) \), where \( i \in \{y, o\} \) (details follow). Notice that the wage can be contracted upon \( z \) and \( \tau \), the age of the worker \( i \), and that it may depend on calendar time \( t \).

In nonemployment, young workers collect unemployment benefits \( b \). As mentioned earlier, lacking access to annuity schemes, they consume the transfer component of severance payment \( \upsilon \phi(\tau) \) entirely upon separation. Older workers, on the other hand, can buy an annuity upon receipt of the severance pay and collect the proceeds until they retire. As a result the total disposable income of older unemployed workers is

\[
\tilde{b}(\tau) = b + \frac{1}{1 - (1 + r)^{-1/\chi}} \frac{r}{1 + r} \upsilon \phi(\tau)
\]

namely, the payoff of an actuarially fair annuity associated with \( \upsilon \phi(\tau) \) (the transfer component of severance payment) where \( 1/\chi \) is the expected number of periods until full retirement.

### 2.2 Bellman equations

We formulate workers’ and firms’ decision problems in recursive form. For workers, let \( R_t \) denote the value of being retired, which we specify exogenously (see Section 3). Let \( U^y_i \) (resp. \( W^y_i \)) denote the value of being nonemployed (resp. being employed), where \( i \in \{y, o\} \) indexes the age of the worker.

While nonemployed, a young worker enjoys a flow income \( b \), remains in the current age category with probability \((1 - \gamma)\) and either finds a job with probability \( \theta_t q(\theta_t) \), or stays nonemployed. Otherwise he/she becomes old with probability \( \gamma \) and the asset value becomes \( U^o_{t+1}(0) \):

\[
U^y_i = u(b) + \frac{1}{1 + r} \left[ (1 - \gamma) \left[ \theta_t q(\theta_t) W^y_{t+1} (1, 0) + (1 - \theta_t q(\theta_t)) U^y_{t+1} \right] + \gamma U^o_{t+1}(0) \right]
\]

where \( W^y_{t+1} (1, 0) \) denotes a young worker’s asset value of being employed at the entry productivity level and no tenure. An old unemployed worker who had tenure \( \tau \) in his/her previous job has flow income \( \tilde{b}(\tau) \) and enjoys a per-period utility of leisure \( \ell \). He/she remains in early
The value of employment for older workers, on the other hand, is given by:

\[
U_i^o (\tau) = u (b(\tau)) + \frac{1}{1+r} ((1-\chi)U_{i+1}^o(\tau) + \chi R_{i+1})
\]  

(5)

While employed at a job with productivity \(z\) and tenure \(\tau\), a young worker consumes the wage \(w_i^y (z, \tau)\) and his/her job is subject to productivity shocks. In the event of job destruction, the value of young workers becomes \(\bar{U}_i^y (\tau) = U_i^y + u (b + v\phi(\tau)) - u(b)\): the worker consumes the severance payment (as a function of previous tenure) in the period immediately after dismissal. Therefore, \(W_i^y (z, \tau)\) satisfies:

\[
W_i^y (z, \tau) = u (w_i^y (z, \tau)) + \frac{1}{1+r} \left( (1-\gamma) \sum_{z'} \pi_{z,z'} \max \left\{ W_{i+1}^y (z', \tau'), \bar{U}_{i+1}^y (\tau') \right\} 
+ \gamma \sum_{z'} \pi_{z,z'} \max \left\{ W_{i+1}^o (z', \tau'), U_{i+1}^o (\tau') \right\} \right)
\]  

(6)

The value of employment for older workers, on the other hand, is given by:

\[
W_i^o (z, \tau) = u (w_i^o (z, \tau)) + \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \left\{ W_{i+1}^o (z', \tau'), U_{i+1}^o (\tau') \right\} + \chi R_{i+1} \right)
\]  

(7)

As for firms, let \(V_i\) denote the value of holding a vacant position and denote by \(J_i^z\) the value of having a filled job, where \(i \in \{y, o\}\) is the age of the worker who is currently employed. Just like the worker, the firm forms expectations over future values of productivity and age. In the event of job destruction, the value of a firm is that of having a vacant position minus the severance package which includes the transfer to the worker and the firing tax. For a young worker, the severance package is exactly \(\phi(\tau)\), while this is replaced by the expected discounted value of the annuity payment \(\Phi(\tau)\) when the worker is older. Hence:

\[
J_i^y (z, \tau) = z - (1 + \kappa_i)w_i^y (z, \tau) + \frac{1}{1+r} \left( (1-\gamma) \sum_{z'} \pi_{z,z'} \max \left\{ J_{i+1}^o (z', \tau'), V_{i+1} - \phi (\tau') \right\} 
+ \gamma \sum_{z'} \pi_{z,z'} \max \left\{ J_{i+1}^o (z', \tau'), V_{i+1} - \Phi (\tau') \right\} \right)
\]  

(8)

\[
J_i^o (z, \tau) = z - (1 + \kappa_i)w_i^o (z, \tau) + \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \max \left\{ J_{i+1}^o (z', \tau'), V_{i+1} - \Phi (\tau') \right\} 
+ \chi V_{i+1} \right)
\]  

(9)
The value of a vacancy, on the other hand, is given by:

\[ V_t = -k + \frac{1}{1+r} \left( q(\theta_t)J_{t+1}^{y}(1,0) + (1-q(\theta_t))V_{t+1} \right) \]  

where, as in equation (4), labour-market tightness in period \( t \) determines the probability of employment in period \( t+1 \).

### 2.3 Wage setting

Following much of the literature, we assume that wages are set by Nash bargaining. Let \( \beta \in (0,1) \) denote the bargaining power of the worker. Wages are given by:

\[ w^y_t(z,\tau) = \arg\max_w \left( W^y_t(z,\tau;w) - \tilde{U}^y_t(\tau) \right)^\beta \left( J^y_t(z,\tau;w) - V_t + \phi(\tau) \right)^{1-\beta} \]  

\[ w^o_t(z,\tau) = \arg\max_w \left( W^o_t(z,\tau;w) - U^o_t(\tau) \right)^\beta \left( J^o_t(z,\tau;w) - V_t + \Phi(\tau) \right)^{1-\beta} \]

for all \((z,\tau)\). For future reference, it is useful to write the first-order condition associated with the above maximization problems. That is,

\[ (1-\beta) \frac{1+\kappa_t}{J^y_t(z,\tau) - V_t + \phi(\tau)} = \beta \frac{u'(w^y_t(z,\tau))}{W^y_t(z,\tau) - \tilde{U}^y_t(\tau)} \]  

\[ (1-\beta) \frac{1+\kappa_t}{J^o_t(z,\tau) - V_t + \Phi(\tau)} = \beta \frac{u'(w^o_t(z,\tau))}{W^o_t(z,\tau) - U^o_t(\tau)} \]

On the one hand, the numerator in the left-hand side of equations (13) and (14) is the effect of a marginal reduction in the wage for the firm, which increases profit streams by \( 1+\kappa_t \). On the other hand, the effect of a marginal increase in the wage on the utility of the worker depends on the value of the wage, because of diminishing marginal utility of consumption (right-hand side of the equations). Thus, unlike the canonical search and matching model, our model features nontransferable utilities between agents. This implies that we cannot solve for the joint surplus of the match in order to obtain the wage functions.\(^8\)

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\(^8\)Another implication is that Lazear’s (1990) “bonding critique” is not applicable to our setup. That is, workers and firms cannot fully neutralize the transfer component of severance payments because they differ as to their valuation of a reallocation of payments over time; see Lalé (2014) for a discussion in a similar context.
2.4 Separation decisions

Associated with the maximization of the asset value functions of employment, there are productivity thresholds for separation decisions. Let $\xi^\gamma_i(\tau)$ (resp. $\xi^\omega_i(\tau)$) denote the productivity cutoff for a match with a young (resp. old) worker with tenure $\tau$. The threshold $\xi^\gamma_i(\tau)$, with $i \in \{y, o\}$, is the value of $z$ that makes both parties indifferent between keeping the job alive and dissolving the match. Since private bargains are efficient, $\xi^\gamma_i(\tau)$ can be recovered by using either the value functions of the worker or that of the firm. That is,

$$W^\gamma_i(\xi^\gamma_i(\tau), \tau) = U^\gamma_i(\tau), \quad J^\gamma_i(\xi^\gamma_i(\tau), \tau) = V_i - \phi(\tau)$$  \hspace{1cm} (15)$$

and

$$W^\omega_i(\xi^\omega_i(\tau), \tau) = U^\omega_i(\tau), \quad J^\omega_i(\xi^\omega_i(\tau), \tau) = V_i - \Phi(\tau)$$  \hspace{1cm} (16)$$

Due to the nonstandard problem for wages, it is also relevant to define separation decisions in relation to the reservation wage of the worker and that of the firm. Let $w^\gamma_i(z, \tau)$ denote the lowest possible wage that a worker of age $i$ and current tenure $\tau$ would accept in a job with productivity $\tau$. These reservation wages solve:

$$u(w^\gamma_i(z, \tau)) = U^\gamma_i(\tau) - \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \max \left\{ W^\gamma_i(z', \tau'), U^\gamma_i(\tau') \right\} \right)$$

$$+ \chi \sum_{\tau'} \pi_{z,\tau'} \max \left\{ W^\omega_i(z', \tau'), U^\omega_i(\tau') \right\}$$  \hspace{1cm} (17)$$

$$u(w^\omega_i(z, \tau)) = U^\omega_i(\tau) - \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \max \left\{ W^\omega_i(z', \tau'), U^\omega_i(\tau') \right\} + \chi R_{t+1} \right)$$  \hspace{1cm} (18)$$

Similarly, the highest possible wage that the firm would pay to this worker, $\bar{w}^\gamma_i(z, \tau)$, is given by

$$\bar{w}^\gamma_i(z, \tau) = \frac{1}{1 + \kappa_i} \left[ z - V_i + \phi(\tau) + \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \max \left\{ J^\gamma_i(z', \tau'), V_{t+1} - \phi(\tau') \right\} \right) \right.$$ \hspace{1cm} (19)

$$+ \chi \sum_{\tau'} \pi_{z,\tau'} \max \left\{ J^\omega_i(z', \tau'), V_{t+1} - \Phi(\tau') \right\} \left. \right]$$

$$\bar{w}^\omega_i(z, \tau) = \frac{1}{1 + \kappa_i} \left[ z - V_i + \Phi(\tau) + \frac{1}{1+r} \left( (1-\chi) \sum_{\tau'} \pi_{z,\tau'} \max \left\{ J^\omega_i(z', \tau'), V_{t+1} - \Phi(\tau') \right\} \right) \right.$$ \hspace{1cm} (20)

$$+ \chi V_{t+1} \left. \right]$$
A separation occurs when: \( \bar{w}^i_j (z, \tau) < w^i_j (z, \tau) \). Thus, one can determine whether the productivity threshold \( \bar{\psi}^i (\tau) \) is larger than current productivity \( z \) by comparing \( \bar{w}^i_j (z, \tau) \) and \( w^i_j (z, \tau) \). Notice that, in equations (17)–(20), reservation wages depend on calendar time \( t \) only through the outside option of agents and through the payroll tax \( \kappa_t \).

### 2.5 Flow equations

Using labour market tightness \( \theta_t \) and separation decisions \( \bar{z}^y_t (\tau) \) and \( \bar{z}^o_t (\tau) \), we are in a position to write the flow equations that govern the evolution of population distributions in the labour market. Let \( \lambda^y_t (z, \tau) \) (resp. \( \lambda^o_t (z, \tau) \)) denote the population of young (resp. older) workers employed at a job with current productivity \( z \) and with tenure \( \tau \) at time \( t \). Likewise, let \( \mu^y_t \) (resp. \( \mu^o_t \)) denote the population of young (resp. older) unemployed workers. Note that for older unemployed workers we need to keep track of the tenure variable.

In employment, new hires are given by:

\[
\lambda^y_{t+1} (1, 0) = \theta_t q (\theta_t) (1 - \gamma) \mu^y_t
\]

while employment in on-going jobs (\( \tau' > 0 \)) evolves according to:

\[
\lambda^y_{t+1} (z', \tau') = \sum_z \chi \{ z' \geq \bar{z}^y_{t+1} (\tau') \} \pi_{z, \tau'} (1 - \gamma) \lambda^y_t (z, \tau)
\]

\[
\lambda^o_{t+1} (z', \tau') = \sum_z \chi \{ z' < \bar{z}^o_{t+1} (\tau') \} \pi_{z, \tau'} (\gamma \lambda^y_{t+1} (z, \tau) + (1 - \chi) \lambda^o_{t+1} (z, \tau))
\]

As for the evolution of the nonemployment pool, we have

\[
\mu^y_{t+1} = \bar{\mu}^y + (1 - \theta_t q (\theta_t)) (1 - \gamma) \mu^y_t + (1 - \gamma) \sum_z \chi \{ z' < \bar{z}^y_{t+1} (\tau') \} \pi_{z, \tau'} \lambda^y_t (z, \tau)
\]

where \( \bar{\mu}^y = \chi \bar{\lambda}^y \) is the mass of new entrants in every period. Among the old nonemployed with tenure level \( \tau \) at the time of being dismissed from the previous job, the law of motion is:

\[
\lambda^o_{t+1} (\tau) = \gamma \mu^y_t \chi \{ \tau = 0 \} + (1 - \chi) \mu^o_t (\tau)
\]

\[
+ \sum_z \chi \{ z' < \bar{z}^o_{t+1} (\tau') \} \pi_{z, \tau'} (\gamma \lambda^y_t (z, \tau) + (1 - \chi) \lambda^o_t (z, \tau))
\]

The term with \( \gamma \mu^y_t \chi \{ \tau = 0 \} \) accounts for the fact that a young unemployed worker who becomes old enters the pool of older workers with no tenure accumulated in the previous job.

\footnote{That is, with our stochastic life-cycle there are \( \frac{\tau}{\bar{\lambda}^y} \) older workers in the workforce. A fraction \( \chi \) of them retires every period, and the same number of individuals enters the labour market to keep the size of the workforce at a constant level.}
Finally, given that the size of the workforce is equal to one in every period $t$, it follows that

$$\sum_{\tau} \sum_{z} (\lambda^{y}_t(z, \tau) + \lambda^{o}_t(z, \tau)) + \sum_{\tau} \mu^{o}_t(\tau) + \mu^{y}_t = 1$$ \hspace{1cm} (26)

### 2.6 Free-entry

A free-entry condition holds in every period of the model, thus making firms exhaust the present discounted value of job creation net of the vacancy-posting cost. That is, $V_t = 0$ for all $t$. Using equation (10), labour market tightness in period $t$ is pinned down by

$$\frac{k}{q(\theta_t)} = \frac{1}{1 + r} f^y_{t+1}(1, 0)$$ \hspace{1cm} (27)

Notice that the right-hand side of the equation, i.e. the present discounted value of filling a vacant position, depends on calendar time $t + 1$ only. Using this insight, it follows that the outside options of agents in period $t$ are fully determined once value functions in period $t + 1$ are known.

### 2.7 Balanced budget condition

Finally, since the government balances the budget of the unemployment insurance system period by period, the payroll tax satisfies

$$\kappa_t \sum_{\tau} \sum_{z} (w^{y}_t(z, \tau) \lambda^{y}_t(z, \tau) + w^{o}_t(z, \tau) \lambda^{o}_t(z, \tau)) = b \left( \sum_{\tau} \mu^{o}_t(\tau) + \mu^{y}_t \right)$$ \hspace{1cm} (28)

for all $t$. Notice that workers and firms need to know $\kappa_t$ to set wages, and wages in turn affect the revenues raised by the tax.

### 2.8 Steady-state

When all features of the economic environment (policy parameters, preferences, etc.) remain constant, and because there is no aggregate shock, the economy reaches a steady-state after a possibly long period of time. Dropping the time subscript to simplify notations, we define a steady-state as follows:

Definition. A steady-state equilibrium is a list of value functions $(U^y, U^o(\tau), W^y(z, \tau), W^o(z, \tau), J^y(z, \tau), J^o(z, \tau))$, a list of wage functions $(w^y(z, \tau), w^o(z, \tau))$, a set of rules for separation decisions $(\pi^y(\tau), \pi^o(\tau))$, a value for labour market tightness $\theta$ and for the payroll tax $\kappa$, and a distribution of workers across employment status, productivity levels and tenure $(\mu^y, \mu^o(\tau), \lambda^y(z, \tau), \lambda^o(z, \tau))$ such that:
1. Agents optimize: Given $\theta$, $\kappa$ and the wage functions $(w^y(z, \tau), w^o(z, \tau))$, the value functions $U^y, U^o(\tau), W^y(z, \tau), W^o(z, \tau), J^y(z, \tau), J^o(z, \tau)$ satisfy equations (4) – (9), respectively.

2. Separation: Given the list of value functions $(U^y, U^o(\tau), W^y(z, \tau), W^o(z, \tau), J^y(z, \tau), J^o(z, \tau))$, the separation decisions $\bar{z}^y(\tau), \bar{z}^o(\tau)$ satisfy equations (15) and (16), respectively.

3. Nash-bargaining: Given $\theta$, $\kappa$ and the list of value functions $(U^y, U^o(\tau), W^y(z, \tau), W^o(z, \tau), J^y(z, \tau), J^o(z, \tau))$, the wage functions $w^y(z, \tau), w^o(z, \tau)$ solve equations (13) and (14), respectively, in matches where $z \geq \bar{z}^i(\tau)$ and $i \in \{y, o\}$.

4. Free-entry: Given $J^y(1, 0)$, labour market tightness $\theta$ is the solution to equation (27).

5. Balanced budget condition: Given the wage functions $(w^y(z, \tau), w^o(z, \tau))$ and distribution of workers across states of nature $(\mu^y, \mu^o(\tau), \lambda^y(z, \tau), \lambda^o(z, \tau)) J^y(1, 0)$, $\kappa$ is the solution to equation (28).

6. Time-invariant distribution: Given $\theta$ and the rules for separation decisions $(\bar{z}^y(\tau), \bar{z}^o(\tau))$, the distribution $\mu^y, \mu^o(\tau), \lambda^y(z, \tau), \lambda^o(z, \tau)$ is invariant for the law of motion described in equations (21)–(26).

Appendix A.1 describes our methodology for computing the steady-state equilibrium just described.

3 Calibration and steady-state analysis

This section describes our calibration and characterizes the steady-state of the benchmark economy. We select parameters to reproduce a set of informative data moments for Spain in 2006-2007, i.e. just before the outbreak of the Great Recession.

3.1 Calibration procedure

The model period is set to a quarter. We specialize idiosyncratic productivity $z$ as a two-state symmetric Markovian process: one high productivity state (which we have already normalized to 1), and another low productivity state $1 - \bar{z}$, with $\bar{z} \in (0, 1)$. Thus, denoting the transition probability by $\pi$, we have:

$$z \in \{1 - \bar{z}; 1\}, \quad \Pi = \begin{bmatrix} 1 - \pi & \pi \\ \pi & 1 - \pi \end{bmatrix}$$

(29)
Furthermore, we specify the lifetime utility of being retired, $R$, as the (time-invariant) value of consuming a constant retirement pension $b_r$, and enjoying the added value of leisure $\ell$ forever. That is, 

$$R = \frac{1 + r}{r} (u(b_r) + \ell).$$

Overall, the model has 15 parameters, namely \{r, \sigma, \gamma, \chi, T, \psi, \beta, \nu, A, k, b, b_r, \ell, z, \pi\}. The first eight parameters are set outside the model while the remaining seven are calibrated.

### Parameters set externally

The first eight rows of Table 1 report parameter values set outside the model. The chosen interest rate is set at $r = 0.01$ to yield an annual interest rate of about 4 percent. The coefficient of relative risk aversion in (1) is $\sigma = 2$, which is a standard value in the literature. The demographic probabilities are set at $\gamma = 1/120$ and $\chi = 1/40$ to match the expected durations of the first (“young”) and second (“old”) phase of a worker’s life cycle. This choice is motivated by our interpretation of young workers as those aged 25–54, and older workers as those aged 55–64.\(^{10}\)

We set the cap on tenure, $T$, equal to 120 model periods, i.e., 30 years.\(^{11}\) Following standard practice in the literature (see Petrongolo and Pissarides, 2001), the unemployment elasticity of the number of matches and worker’s bargaining power are set to $\psi = \beta = 0.5$. Finally, we draw upon results reported by Bentolila et al. (2012) for Spain, and set the fraction of red-tape costs at $1 - \nu = 0.5$.

[Table 1 about here.]

### Calibrated parameters

The remaining seven rows in Table 1 show the parameters set within the model to match the following moment conditions obtained from the Spanish Labour Force Survey for 2006-2007: (1) the average unemployment spell for young workers is 2.5 quarters, i.e., 7.5 months; (2) the replacement rate of unemployment benefits, defined as the ratio between the benefit payment $b$ and the average wage $\tilde{w}$, is 58 percent; (3) the replacement rate of pension benefits, defined as $b_r/\tilde{w}$, is 81 percent (see OECD, 2007);\(^{12}\) (4) labour market tightness $\theta$ is normalized to unity; (5) the quarterly job destruction rate for temporary jobs is 7.5 percent (see Ignacio García Pérez and Osuna, 2014); (6) the quarterly job destruction rate for permanent jobs is 2.1 percent (see Ignacio García Pérez and Osuna, 2014); (7) the non-employment rate among workers aged 55-64 is 50 percent.

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\(^{10}\)This is also consistent with the observation that workers aged 55–64 account for about 25 percent of the working-age population in Spain.

\(^{11}\)That is, with a deterministic lifecycle, no worker (including the young) would ever reach the maximum tenure level.

\(^{12}\)Observe that we do not seek to finance the provision of pensions. Retirement income is a parameter which we use to specify the lifetime utility of retirees. We keep it unchanged across our numerical experiments.
As can be observed in the lower part of Table 1, the fitted values are very close to the targets. Note that the calibrated productivity process implies that matches operating at the lower productivity level $1 - \bar{z}$ are only about half as productive as high-productivity matches. Moreover, the calibrated value for $\pi$ implies that idiosyncratic match productivity switches on average after 3.3 years.

**Benchmark severance payments**

The crux of our analysis relates to the severance pay function. We follow Bentolila et al. (2012) and Ignacio García Pérez and Osuna (2012) in computing a function of job tenure that stands similar to EPL in Spain prior to the reform undertaken in February 2012. As the latter authors do, we specify it as a function of the average annual wage in the labour market, rather than as a function of individual tenure and productivity. The reason for making this assumption is that it makes it easier to solve the model because we do not need any knowledge on the wage profile when specifying $\phi(\tau)$.

In particular, we use the following pieces of information to compute $\phi(\tau)$. It is assumed that all workers start with a fixed-term (“temp”) contract that lasts at most two years. The termination costs are 8 d.w.y.s., so that this represents 2.2 percent ($= 8/365$) in terms of average yearly wage. If the worker is promoted to a permanent (“perms”) contract then he/she is entitled to 45 days of wages p.y.o.s. since joining the firm, with a maximum of 3.5 annual wages, under an unfair dismissal which represented most of the dismissals until 2012. Thus, a promoted worker who loses her/his job after one year is entitled to 37 percent ($= 3 \times 45/365$) of average yearly wage. Using these observations, the severance cost function used in the benchmark economy for workers of type $i$ ($i \in \{y, o\}$) and tenure $\tau$ (in quarter) is computed as follows:

$$\phi(\tau) = \begin{cases} 
(8/365) \times \tilde{w} \times \tau, & 1 \leq \tau \leq 8 \\
(45/365) \times \tilde{w} \times \tau, & 9 \leq \tau \leq 113 \\
(45/365) \times \tilde{w} \times 113, & \tau > 113 
\end{cases}$$

Figure I depicts this function with tenure (in quarters) in the horizontal axis and a multiple of the average annual wage in the vertical axis.

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13 In this reform, severance pay for unfair dismissals of permanent workers went down from 45 to 33 d.w.p.s. while termination costs to temporary workers went up from 8 to 12 d.w.p.y. (see Ignacio García Pérez and Osuna (2014) for details).

14 Observe that the average wage is an equilibrium outcome of the model, not a pre-specified parameter. Thus, upon computing the benchmark equilibrium, we add an outside loop to iterate over the average wage used to specify the severance payment function.
3.2 Benchmark economy

In this section we provide some insights of how our benchmark economy works. Given the two productivity levels, jobs with high productivity \((z = 1; \text{ recall that it is the level at which all newly-matched worker-firm pairs start})\) will never be destroyed for any tenure, irrespectively of whether the worker is young or old. In contrast, some of the low-productivity jobs are destroyed, depending on tenure and on whether the worker is young or old. More specifically, jobs of young workers who move from high productivity to low productivity are destroyed if the current job tenure is 26 quarters (6.5 years) or less. Conversely, if tenure is 27 quarters or longer, the worker-firm pair continues to operate at the low productivity level. For older workers, low-productivity jobs are not destroyed if the tenure is 114 quarters (about 28 years) or less. In other words, only if the worker has reached a tenure of 115 quarters or more are low-productivity matches destroyed, leading to early retirement by older dismissed workers.

As regards wages, Figure 2 depicts the wage-tenure profile for young (upper panel) and older workers (lower panel). In each panel solid and dashed lines represent the wages of high and low-productivity workers, respectively. As can be seen, due to the shape of the redundancy pay function, there is a dip in the wage profile at the end of the first two years of job tenure. The insight is that workers are willing to accept lower wages in exchange for higher future entitlements to severance payments (“permanent job”).

[Figure 2 about here.]

The reason why the wage curve for young workers is rather flat is that their outside option when becoming unemployed is to consume their severance payment immediately, and hope to be matched soon to a new high-productive job. This means that, given concavity of the utility function, another period of tenure only buys them a marginal one-time increase in consumption. Hence, they bargain for high wages even if their tenure is short because the incentive to increase their job tenure is not that high, unless they become old. By contrast, the wage profile for older workers is steeper because their outside option when unemployed is to consume the annuity and enjoy the fixed utility from leisure. The key is that another period of tenure buys them a marginal increase in consumption every period that follows in case of early retirement. Hence, they accept lower wages if their tenure is short. Then, as the tenure increases, their bargaining position rises and so do wages. At period 115 in the benchmark model, there is no option value of higher severance payments in the future, because they are capped. At that point, low-productive matches are destroyed and the severance payment is paid out. High-productive jobs remain intact until they are either hit by a bad shock or the worker retires.

Table 2 in turn reports the simulated values of the main outcome variables in our model economy which, whenever possible, are then compared to their realized counterparts. The latter are obtained from the Spanish Labour Force Survey (EPA) and the Wage Structure Survey (ESS), and are calculated as averages over 2006-2007. As mentioned earlier, data for young
(resp. older) workers correspond to those aged 25-54 (resp. 55-64). As can be observed, in general the fit is quite reasonable. The simulated unemployment rate for young workers (7.8%) fits the data perfectly. The fit is also good as regards the other variables. So, the simulated non-employment (early retirement) for older workers is 48.3% in the model against 50% in the data. Weighting these two simulated rates by their population shares, yields an aggregate non-employment rate of 17.9%, which is quite close to the 17.2% observed in the data. Regarding wage differentials, we find a gap in favour of older workers of 16 percent \( = \frac{0.85}{0.73} - 1 \) which compares reasonably well with the gap observed in ESS, namely 14.4% in 2006-2007 (€1,522 and €1,741 for older workers). Finally, the simulated payroll tax (satisfying the budget constraint) is 12.68%, which again fares reasonably well against the 14.95% (which results from adding those components of employers’ and workers’ social security contributions related to unemployment benefits).

\[ \text{Table 2 about here.} \]

4 Numerical experiments

This section contains the main results of the paper. We use our model economy as a laboratory to discuss the effects of moving from a dual labour market, as the one in Spain, towards a SOEC designed according to some welfare criterion. In the first subsection below, we define the type of SOEC we consider in the sequel while in the second subsection we present the welfare criteria that underlie the choice of this contract. Finally, we analyze the consequences of shifting from benchmark EPL to SOEC and discuss how robust are these effects to changes in some of the key parameters determining the optimal tenure profile of SOEC.

4.1 Preliminaries

To define a SOEC, we start by exploring a relatively simple class of severance payment functions, namely a subset of piecewise linear functions of tenure. Specifically, we specify severance payments as

\[ \phi(\tau) = \begin{cases} 0 & \text{if } \tau \leq \tau_0 \\ g \times \tau & \text{otherwise} \end{cases} \]

where \( \tau_0 \) is the minimum tenure requirement for eligibility to severance payment, and \( g \) measures the rate of return to each year of tenure, conditional on eligibility. Thus, this class of severance payment function is easily interpretable. Observe that smoothness requires that there

\[ ^{15}\text{Computed as one minus the ratio of employment and population among those aged 55–64.} \]
is no jump at the time of gaining eligibility. Smoothness of the severance pay function is consistent with workers’ preference for smooth consumption.

The steady-state comparison is between the initial and new steady-state, indexed by $t_0$ and $t_1$, respectively. The more interesting thought experiment is the following: we introduce the severance payment function that exists in $t_1$ for every new match that is being formed at time $t_0$. Agents in matches that already exist at time $t_0$ now decide whether to remain matched or to separate. Along the transition path, the tax rate adjusts. This affects all matches in the economy, not only those with the new contract. The welfare comparison taking into account the transition is the one that measures the effect of taking the right decision after $t_0$. More formally:

**Definition.** Let $W_0$ denote lifetime utility of a worker in steady-state $t_0$ before the $t_1$ severance pay function is introduced, and let $W_1$ be the lifetime utility of the same worker when the $t_1$ severance pay function is introduced. The comparison of $W_0$ and $W_1$ measures the individual welfare change for the worker. In consumption equivalent units (CEU), the welfare change is measured by:

$$\zeta = \left( \frac{W_1}{W_0} \right)^{\frac{1}{\sigma}} - 1$$

Hence, lifetime consumption multiplied by $1 + \zeta$ would make the worker indifferent between the two situations.

From the previous expression, we can measure $\zeta$ for each individual of the benchmark economy. This allows us to compute: (i) the fraction of individuals with $\zeta > 0$, namely the proportion of “winners” from the reform, which gives a sense of whether there would be enough political support for the policy change, and (ii) a set of relevant statistics (e.g., average, minimum, maximum etc.) of $\zeta$ using the distribution of agents at time $t_0$, i.e., the current generation of workers.

Notice that in computing the distribution of $\zeta$, it is unclear that we can compare for each individual worker the effect of having the same age, productivity and tenure but living in steady-state $t_1$ rather than in $t_0$ since it could be sometimes optimal not to be employed in $t_1$ with the same age/productivity/tenure as in $t_0$. Thus, the only steady-state to steady-state comparison which is always well-defined is for newborn workers.

Another issue that deserves some discussion is the role of employers in the computation of welfare. We abstract from them for two reasons. First, given that workers are risk-averse and entrepreneurs are risk-neutral, it would be difficult to aggregate heterogeneous preferences. Secondly, unlike workers, the measure of employers is not fixed across steady-states (there are $v + 1 - u$ of them). Due to these technical difficulties, we leave entrepreneurs out of the welfare evaluation and interpret firms in our setup just as a modelling device to endogenize wages and vacancies.

$^{16}\zeta = \exp \left( \frac{1}{1-\sigma} (W_1 - W_0) \right) - 1$ if $\sigma = 1$, i.e., when the utility function is logarithmic
4.2 Welfare objectives

Our goal in this section is to compute which is the tenure profile of dismissal costs, i.e., the shape of \( \phi(\tau) \), which maximizes welfare. We denote it as \( \phi^*_k(\tau) \) where \( k \) indexes the chosen criterion. We consider two welfare criteria. The first one, denoted as Objective I, is given by the lifetime utility of young unemployed workers \( U^y \), that is the newborn population, which is compared in the two steady states. Therefore, this initial comparison does not take the transition path into account. Accordingly, the preferred tenure profile of SOEC satisfies

\[
\{\phi^*_I(\tau)\}_{\tau=1}^\infty = \arg\max \{U^y\}
\]

where \( U^y \) is defined in (4). The second criterion, denoted as Objective II, is the average welfare across the current population when taking into account the transition. Thus, Objective II considers welfare for the current generation which is alive when the unified contract with the selected profile of severance pay is introduced at \( t = 1 \). The key assumption is that workers who are already employed at \( t = 1 \) retain the EPL of the old contract whereas only newly-formed matches at \( t = 1, 2, \ldots \) are subject to a new contract. Specifically,

\[
\{\phi^*_II(\tau)\}_{\tau=1}^\infty = \arg\max \left\{ \sum_{\tau} \sum_z \left( \lambda_y(t_0,z,\tau) W^y(t_0,z,\tau) + \lambda_o(t_0,z,\tau) W^o(t_0,z,\tau) \right) \right. \\
+ \sum_{\tau} \mu_o(t_0,\tau) U^o(t_0,\tau) + \mu_y(t_0,\tau) U^y(t_0,\tau) \right\}
\]

where the distribution is from the benchmark economy, i.e., the time-invariant distribution just before the government enforces the \( t_1 \) contract for new hires.

4.2.1 Objective I

Figure 3 displays the optimal tenure profile of the SOEC under Objective I, \( \phi^*_I(\tau) \), together with the benchmark EPL profile. As can be inspected, the outcome of maximizing welfare of newborn population yields an optimal tenure profile of SOEC which looks remarkably smoother than mandated severance pay in the benchmark economy. It starts slightly below benchmark EPL, becoming the same after about five quarters. Later on, it increases smoothly until it reach about 0.8 times of the average annual wage after 60 quarters of tenure (15 years), levelling off afterwards. The intuition behind this result is simple. On the one hand, since the new contract maximizes \( U^y \), currently-employed young workers prefer this SOEC because it maximizes their outside option (and raises wages). On the other hand, older workers are not directly affected since their value of unemployment \( U^o \) is determined by the contract that was previously in place. Putting both considerations together, the new contract improves welfare for newborns and currently-employed young workers, and a priori it leaves currently-old workers indifferent.
Table 3 presents a comparison of the changes in our main endogenous variables between the two steady states as a result of introducing this SOEC. The overall nonemployment rates plummets from almost 18% in the benchmark economy to 2% under SOEC. Wages rise, as a result of the increase in the outside option, while the job destruction rate for young workers with low tenure (“temp”) goes down dramatically since many jobs that were not continued in the benchmark case when the EPL gap hit, are now saved. Notice that the job destruction rate for older workers also goes down. The reason why older workers benefit from this SOEC is because lower severance pay for longer tenures (see Figure 3) implies their jobs get destroyed at a much lower rate than in the benchmark economy. In the latter, older workers with long tenure got laid off when productivity switched from high to low productivity level. Likewise, productivity falls because the cutoffs go down so that less-productive workers that would lose their jobs under existing EPL now remain in the firm. Overall, despite the reduction of severance pay in days of wages, especially in the first two years of tenure, the lifetime utility of a newborn increases by almost 15 CEU, precisely because job tenure increases a lot.

4.2.2 Objective II

Figure 4 depicts the optimal SOEC under Objective II whereas Table 5 reports the corresponding changes in the main endogenous variables as a result of introducing this new contract. [TO BE COMPLETED]

4.2.3 Severance payments vs. Unemployment benefits

In the sequel we compute the optimal contract that would result from varying two key parameters in the model: $\sigma$ (risk aversion) and $b$ (unemployment benefits, UB) under Objective I. With regard to $\sigma$ we consider two alternative values to our benchmark value of $\sigma = 2$, namely, (i) $\sigma = 1$ (log case) and (ii) $\sigma = 5$. Regarding $b$, we also consider two variations in relation to the benchmark replacement rate of 58%, namely, (i) 40%, and (ii) 70%. For each of these alternative values of $\sigma$ and $b$, the model is re-calibrated to match the same moments as in the benchmark economy.

Figures 5 and 6 display the new optimal contracts for the above-mentioned values of $\sigma$ and $b$, respectively, where to make differences more visible we focus on the first three years of
tenure. The main finding is that SOEC is fairly robust to variations in the degree of risk aversion. By contrast, SOEC depends considerably on the replacement rate of UB so that more generous UB implies a lower optimal tenure profile and vice versa. This is explained by the fact that in our setup the government uses two instruments (UB and severance pay) instead of only one. This issue is especially relevant because the value function for older workers clearly shows that there is substitutability between the two policy instruments. Our results show that EPL can be used for the purpose of making matched individuals internalize the social effects of their separation decisions (less tax revenues, more congestion externalities, more unemployment to finance) but since less stringent EPL implies higher job creation and destruction rates, the counterpart of lower job protection is higher worker protection though more generous EPL. This option has given rise to the so-called Flexicurity model in countries like Denmark (see Boeri et al., 2012).

Digging deeper on the relationship between severance pay and UB as two alternative ways of providing insurance against the risk of becoming unemployed, we next compare the implications of a change in the tenure profile of FC costs to a change in the generosity of UBs. In particular, we conduct the following exercise: (i) find the severance pay schedule that maximizes expected welfare according to Objective I and save the optimal level $U_y^*$; (ii) return to the severance pay function considered in the benchmark economy and change the generosity of UBs such that expected welfare is exactly equal to $U_y^*$.

Table 6 shows the main results of this simulation exercise. [TO BE COMPLETED]

5 Conclusion [TBC]
References


A Numerical appendix

This appendix details our numerical methodology to compute steady-states and transition paths of the model economy presented in Section 2.

A.1 Computing steady-states

In keeping with Subsection 2.8, we drop the time subscript to indicate that the economy is in steady-state. A steady-state is nontrivial to compute because the continuation values in certain labor market states are unknown. Specifically, we need to solve for $U^y$, $W^y(z,T)$, $W^o(z,T)$, $J^y(z,T)$ and $J^o(z,T)$, as well as $w^y(z,T)$ and $w^o(z,T)$. The algorithm is as follows:

1. Solve for $W^o(z,T)$, $J^o(z,T)$ and $w^o(z,T)$ using the following steps:
   
   (a) Set initial guesses $\hat{W}^o(z,T)$, $\hat{J}^o(z,T)$, $\hat{w}^o(z,T)$, where we use $\sim$ to indicate a guess.
   
   (b) Compute the reservation wage of the worker $w^o(z,T)$ and that of the firm $\bar{w}^o(z,T)$ associated with $\hat{W}^o(z,T)$ and $\hat{J}^o(z,T)$ using equations (18) and (19).
   
   (c) If $w^o(z,T) \leq \bar{w}^o(z,T)$, then solve for the Nash-bargained wage $w$ using the associated first-order condition:
   
   $\frac{\beta}{1 + \kappa} \left[ z - (1 + \kappa)w + \frac{1 - \chi}{1 + r} \sum_{z'} \pi_{z,z'} \max \left\{ \hat{J}^o(z',T), \Phi(t) \right\} + \Phi(T) \right]$
   
   $= \frac{1 - \beta}{u'(w)} \left[ u(w) + \frac{1 + r}{1 + r} \left( (1 - \chi) \sum_{z'} \pi_{z,z'} \left\{ \hat{W}^o(z,T), U^o(T) \right\} + \chi R \right) \right]$
   
   and update $\hat{w}^o(z,T)$ using this value. This is a nonlinear equation, which we solve using the bisection method. If, on the other hand, $\bar{w}^o(z,T) < w^o(z,T)$, set $\hat{w}^o(z,T) = \frac{1}{2} (\bar{w}^o(z,T) + w^o(z,T))$.
   
   (d) Update $\hat{W}^o(z,T)$ and $\hat{J}^o(z,T)$ using equations (7) and (9).
   
   (e) If initial and updated guesses for value functions and wages are close enough, then we are done. Otherwise, go back to step (1a).

2. Solve for $W^o(z,\tau)$, $J^o(z,\tau)$ and $w^o(z,\tau)$ recursively from $\tau = T - 1$. That is:

   (a) Compute the reservation wage of the worker $w^o(z,\tau)$ and that of the firm $\bar{w}^o(z,\tau)$ using equations (18) and (19). Note that the continuation values only involve $\tau + 1$, which allows to compute $w^o(z,\tau)$ and $\bar{w}^o(z,\tau)$.

   (b) If $w^o(z,\tau) \leq \bar{w}^o(z,\tau)$, then solve for the Nash-bargained wage using the first-order condition (14). The continuation values in this equation depend on $\tau + 1$ only, and the outside option of the worker $U^o(\tau)$ is pre-determined.

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17Observe that $U^o(T)$ is completely determined, as per equation (5).
(c) Compute the value functions $W^o(z, \tau)$ and $J^o(z, \tau)$ from equations (7) and (9).

3. Solve for $U^y, W^y(z, \tau), J^y(z, \tau)$ and $w^y(z, \tau)$ using the following steps:

(a) Set an initial guess for $\hat{U}^y$.

(b) Solve for $W^y(z, T), J^y(z, T)$ and $w^y(z, T)$ using a methodology similar to (1b). That is,
   i. Set initial guesses $\hat{W}^y(z, T), \hat{J}^y(z, T)$ and $\hat{w}^y(z, T)$.
   ii. Use the analogon of step (1b) to obtain the reservation wage of the worker and the reservation wage of the firm.
   iii. Use the analogon of step (1c) to update the wage. Observe that $\hat{U}^y$ is used as the outside option of the worker in the Nash bargain.
   iv. Update $\hat{W}^y(z, T)$ and $\hat{J}^y(z, T)$ using equations (6) and (8).
   v. Iterate until convergence.

(c) Solve for $W^y(z, \tau), J^y(z, \tau)$ and $w^y(z, \tau)$ recursively from $\tau = T - 1$ using a methodology similar to step (1). Again, observe that knowledge of $\hat{U}^y$ is required to compute the Nash-bargained wage.

(d) Use the Bellman equation of a young unemployed worker to update $\hat{U}^y$. If the initial and the updated guess are close enough, then we are done. Otherwise, go back to step (3a) using the updated $\hat{U}^y$.

The algorithm above takes into account the fact that, in a steady-state, $U^y, W^y(z, T), W^o(z, T), J^y(z, T)$ and $J^o(z, T)$ are the solution to an infinite-horizon problem, while the other value functions associated with employment solve a standard finite-period ($T$) problem and $U^o(\tau)$ is completely determined.

A steady-state also features an equilibrium tuple $(\theta, \kappa)$. Thus, the algorithm is nested into outer loops to iterate on $(\theta, \kappa)$. We fix the payroll tax $\kappa$, solve for labour market tightness $\theta$, and then update $\kappa$ until convergence. In the benchmark economy, the loop for $\theta$ is skipped owing to our calibration procedure, but we have an outer loop to iterate on the average wage $\bar{w}$.

A.2 Computing transition paths

A transition path between periods $t_0$ and $t_1$ involves a sequence of value functions, wage functions, rules for separation decisions, labour market tightness, the payroll tax, and the distribution of workers across employment status, productivity levels and tenure such that:

- Conditions (1)–(5) defined in Subsection 2.8 (optimal decisions, Nash-bargaining, free-entry and balanced budget) hold every period;

- Condition (6) (Time-invariant distribution) is replaced by adequacy between labor market stocks in every period and the flow equations defined in Subsection 2.3.
During the transition towards a new steady-state equilibrium, computations at time $t$ are simplified in that all continuation values depend on time $t+1$. That is, the transition path eliminates the infinite horizon problem that arises in steady-state. There is, however, a problem specific to the transition path; namely, it requires knowledge of the time-path of $\kappa_t$ (denoted by $(\kappa_t)_{t=t_0}^{t_1}$).

In addition, we have an additional state variable for employed workers and the old unemployed, $c \in \{0, +\}$, indicating whether their current labor market status pertains to the contract that prevailed at time $t_0$ ($c = 0$) or to the contract introduced after $t_0$ ($c = +$).

The structure of our model implies that, instead of storing the sequence for all these objects, we need “only” the distribution of agents at $t_0$ and the sequences $(\theta_t)_{t=t_0}^{t_1}$, $(w^y_t(z, \tau, c))_{t=t_0}^{t_1}$, $(w^o_t(z, \tau, c))_{t=t_0}^{t_1}$, $(p^y_t(\tau, c))_{t=t_0}^{t_1}$, $(p^o_t(\tau, c))_{t=t_0}^{t_1}$ to check that a time-path $(\kappa_t)_{t=t_0}^{t_1}$ is valid. Our methodology to compute those is as follows:

1. Compute the steady-state of the economy in period $t_1$.
2. Plug the initial severance payment function (that of $c = 0$) into the outside option of agents at time $t_1$. Compute the wage and value functions of being in a match at time $t_1$ with the outside option set by the $c = 0$ contract.
3. Guess a path for the payroll tax $(\widehat{\kappa}_t)_{t=t_0}^{t_1}$.
4. Solve for value functions, wages, separation decisions and labour market tightness recursively from $t_1 - 1$ until $t_0$ as follows:
   (a) Compute labor market tightness consistent with free-entry at time $t$ and store it.
   (b) Compute the value of searching for a new job at time $t$, $U^y_t$. Note that, in every period of the transition path, a young unemployed worker can only find a job with the $c = +$ contract applying to this job.
   (c) Solve for the wage functions of older and younger workers at time $t$ and store them. Then compute the associated value functions. Finally, compute the separation decisions at time $t$ and store them.
5. Set the initial distribution of agents to the time-invariant distribution that obtains in the steady-state before $t_0$.
6. Using $(\theta_t)_{t=t_0}^{t_1}$, $(w^y_t(z, \tau, c))_{t=t_0}^{t_1}$, $(w^o_t(z, \tau, c))_{t=t_0}^{t_1}$, $(p^y_t(\tau, c))_{t=t_0}^{t_1}$, $(p^o_t(\tau, c))_{t=t_0}^{t_1}$ and the flow equations described in Subsection 2.5 compute the evolution of the distribution during the time path. Each period, compute the realized payroll tax $(\kappa_t)_{t=t_0}^{t_1}$ implied by the balanced budget condition.
7. If $(\widehat{\kappa}_t)_{t=t_0}^{t_1}$ and $(\kappa_t)_{t=t_0}^{t_1}$ are close enough, then we are done. Otherwise, go back to step (3).
To ensure that the payroll tax obtained at the end of the transition path coincide with the steady-state $t_1$ payroll tax, we allow for a very large number of periods between $t_0$ and $t_1$. In our application, we set the number of period to 2,000. After 1,750 periods, the number of older workers still employed in the $t_0$ contract is less than $10^{-6}$. 
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>$r$</td>
<td>0.01</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Matching function</td>
<td>$\psi$</td>
<td>0.5</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Bargaining power</td>
<td>$\beta$</td>
<td>0.5</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Red-tape cost</td>
<td>$\upsilon$</td>
<td>0.5</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>2</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Retirement prob.</td>
<td>$\chi$</td>
<td>1/40</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Prob. becoming old</td>
<td>$\gamma$</td>
<td>1/120</td>
<td>set outside</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Matching function</td>
<td>$A$</td>
<td>0.40</td>
<td>job finding prob.</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>Vacancy cost</td>
<td>$k$</td>
<td>0.623</td>
<td>normalization of $\theta$</td>
<td>1.00</td>
<td>1.00</td>
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<td>Unempl. benefits</td>
<td>$b$</td>
<td>0.4011</td>
<td>replacement rate</td>
<td>0.58</td>
<td>0.58</td>
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<tr>
<td>Retirement income</td>
<td>$b'$</td>
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<td>replacement rate</td>
<td>0.81</td>
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<td>Non-empl. utility</td>
<td>$\ell$</td>
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<td>non-empl. old</td>
<td>0.50</td>
<td>0.48</td>
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<tr>
<td>Productivity process</td>
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<td>0.493</td>
<td>job destr perm</td>
<td>0.021</td>
<td>0.019</td>
</tr>
<tr>
<td>Productivity process</td>
<td>$\pi$</td>
<td>0.0750</td>
<td>job destr temp</td>
<td>0.075</td>
<td>0.073</td>
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Table 2. Benchmark model economy: Comparison with the data

<table>
<thead>
<tr>
<th>Description</th>
<th>Model</th>
<th>Data</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate (young, in %)</td>
<td>7.8</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>Early retirement rate (old, in %)</td>
<td>48.3</td>
<td>50.0</td>
<td>part of calibr.</td>
</tr>
<tr>
<td>Non-Employment rate (all, in %)</td>
<td>17.9</td>
<td>17.2</td>
<td></td>
</tr>
<tr>
<td>Average wage (young)</td>
<td>0.68</td>
<td>1,522</td>
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</tr>
<tr>
<td>Average wage (old)</td>
<td>0.73</td>
<td>1,743</td>
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<tr>
<td>Average productivity (young)</td>
<td>0.85</td>
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<td></td>
</tr>
<tr>
<td>Average productivity (old)</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job destruction rate (temp, in %)</td>
<td>7.3</td>
<td>7.5</td>
<td>part of calibr.</td>
</tr>
<tr>
<td>Job destruction rate (perm, in %)</td>
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<td>2.1</td>
<td>part of calibr.</td>
</tr>
<tr>
<td>Job finding rate (in %)</td>
<td>40.0</td>
<td>40.0</td>
<td>part of calibr.</td>
</tr>
<tr>
<td>Share of new jobs becoming perm (in %)</td>
<td>50</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Min Tenure: low-prod. job remains</td>
<td>22</td>
<td></td>
<td>= 5.5 years</td>
</tr>
<tr>
<td>Payroll tax (in %)</td>
<td>12.68</td>
<td>14.95</td>
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</table>
Table 3. Welfare-maximizing severance payments

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Objective I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment rate (young, in %)</td>
<td>7.82</td>
<td>2.00</td>
</tr>
<tr>
<td>Nonemployment rate (old, in %)</td>
<td>48.3</td>
<td>2.00</td>
</tr>
<tr>
<td>Nonemployment rate (all, in %)</td>
<td>17.9</td>
<td>2.00</td>
</tr>
<tr>
<td>Equilibrium tax rate $\kappa$ (%)</td>
<td>12.7</td>
<td>1.05</td>
</tr>
<tr>
<td>Average wage (young)</td>
<td>0.68</td>
<td>0.79</td>
</tr>
<tr>
<td>Average wage (old)</td>
<td>0.73</td>
<td>0.79</td>
</tr>
<tr>
<td>Average productivity (young)</td>
<td>0.85</td>
<td>0.77</td>
</tr>
<tr>
<td>Average productivity (old)</td>
<td>0.80</td>
<td>0.76</td>
</tr>
<tr>
<td>Job destruction rate (temp, in %)</td>
<td>7.28</td>
<td>0.00</td>
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<tr>
<td>Job destruction rate (perm, in %)</td>
<td>1.88</td>
<td>0.70</td>
</tr>
<tr>
<td>Job finding prob (in %)</td>
<td>40.0</td>
<td>40.3</td>
</tr>
<tr>
<td>Lifetime utility of a newborn ($U^y$)</td>
<td>-37.9</td>
<td>-25.67</td>
</tr>
<tr>
<td>Severance payment after 2 years (in days of wages)</td>
<td>64</td>
<td>305</td>
</tr>
<tr>
<td>Severance payment after 5 years (in days of wages)</td>
<td>900</td>
<td>770</td>
</tr>
<tr>
<td>Severance payment after 20 years (in days of wages)</td>
<td>3600</td>
<td>1800</td>
</tr>
</tbody>
</table>
**Figure 1.** Benchmark severance payments

NOTE: The plot shows the benchmark severance payment function as defined in equation (30). Severance payments are expressed in terms of the average yearly wage.
Figure 2. Wage function for young (top) and older (bottom) workers

NOTE: The plot shows the wage function in the benchmark economy, in high-productivity (solid line) and low-productivity (dashed line) matches. Wage functions are plotted as a function of tenure (τ). In the benchmark economy, low-productivity matches with a young worker and tenure under 22 quarters (inclusive) are destroyed; low-productivity matches with an older worker and tenure over 100 quarters (inclusive) are destroyed.
Figure 3. Benchmark vs. welfare-maximizing severance payments: Objective I
NOTE: The blue line shows the benchmark severance payment function as defined in equation (30). The red line shows the welfare-maximizing severance payment function as defined by objective I.
Figure 5. Welfare-maximizing severance payments: Different degrees of risk-aversion

NOTE: The blue line shows the benchmark severance payment function as defined in equation (30). The red, black and purple lines shows the welfare-maximizing severance payment function as defined by objective I obtained after varying the coefficient of risk aversion $\sigma$. 

![Graph showing different severance payment functions for varying degrees of risk aversion.](image)