Managing a Conflict – Mediation, Litigation, and Informational Punishment*

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PRELIMINARY AND INCOMPLETE

Abstract

We analyze optimal mediation by an ex-ante uninformed mediator who mediates between two parties who are facing each other in court. Therefore the outside option for both parties is a litigation trial which is entered if either refuses mediation. In addition to a standard Myerson (1982) coordination mechanism we introduce tighter participation constraints, by allowing participants to publicly veto mediation and forcing the opponent into the contest.

Litigation as the outside option is modelled as an all-pay contest in which the two contestants need to provide evidence in order to convince the court about them being right. In the litigation game contestants are heterogeneous with respect to the distribution of their cost of evidence production. The realization of this cost is their private information.

As contestants are only heterogeneous if they play the outside option, strategically referring contestants back to court on-path is the only screening device of the mediator.

We use a mechanism design approach in which the mediator solves this twofold problem: (1) it derives the optimal information structure that fulfils incentive compatibility, before (2) scaling it such that everyone participates.

We show it is possible to substantially reduce both the probability of litigation and as a consequence legal expenditures through mediation. Perhaps surprisingly, it is not only the case that low-cost types do not gain from the mediation option, but due to informational punishment the ex-ante strongest player and type can actually be worse off compared to a situation in which the mediation process is not allowed.

KEYWORDS: Conflict resolution, Mediation, All-pay contest, Mechanism design without transfers, Bayes' plausible signals

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1. Introduction

While the purpose of concepts such as the “rule of law” has been widely accepted in many societies, they usually involve a very costless process. Courts and lawyers need to be paid, witnesses need to be reimbursed and both producing and evaluating evidence is a time consuming procedure. This might be one of the reasons, why “alternative dispute resolution” (ADR) is used in many modern societies as a way to avoid costly litigation. Nonetheless, each party in the dispute should have the possibility to enforce litigation, as not to jeopardize the “rule of law”.

One particular way of ADR is that of mediation. Mediation usually involves a neutral third-party to whom the parties talk to and who then proposes a resolution contract to both parties. Given parties accept the mediators proposal, the conflict is resolved, if they reject, mediation is considered to have failed and both parties continue with litigation.

The goal of mediation is to a large extend to reduce legal expenditures and to settled for an efficient agreement. The option to not only quit mediation but to then enter the litigation procedure as an outside option complicates the problem as litigation itself is a strategic and belief-dependent process.

Disputes often involve asymmetric information on both sides for example on the availability (or verifiability) of evidence supporting the parties’ claims. Already interestingly enough, the information on the other party’s cost to make their case has an influence on one’s own evidence production. If evidence is costly to produce one would try to produce as little as possible, but enough to win the case. In addition, private information further complicates the decision whether to accept the mediator’s proposal. If you knew you are in litigation against a high-cost opponent, you might expect more from litigation and need to be rewarded for that by the mediator.

For the mediator, however, the presence of private information has ambiguous effects. On the one hand it introduces an additional constraint as some realized costs demand an information rent to be paid to one or the other party to get mediation to work. On the other hand the fact that the mediator has the possibility to privately collect information about the player’s types allows her to use this information strategically to threaten parties with a higher outside option in order to force them into acceptance of the mediation result.

In this paper we examine the role of a mediator in conflict resolution. We model litigation (the outside option) as an all-pay contest (or a particular form of the all-pay auction). To make our analysis interesting we focus on a priori not legally clear conflicts, that is conflicts in which the court has no ability to judge upon publicly available evidence, but must rely on the evidence provided by the parties. As a consequence, the court follows the party that provided the most convincing evidence. However, the cost of evidence production is not necessarily the same between the parties and is private information. This resembles both the quality of the lawyer[1] as well as the availability of evidence to prove ones point.[2] Prior to litigation both parties may engage in one round of simultaneous mediation in which they disclose their information to an uninformed third party (the mediator). However, we allow

[1]A good lawyer might be able to produce the same piece of evidence in shorter time, since she is faster in understanding what the most convincing evidence is.

[2]This captures looking for witnesses to testify, understanding and interpreting documents and so on.

each party to publicly not participate.[3] After having collected information, the mediator processes this information according to an ex-ante publicly known protocol and proposes a sharing rule of the disputable prize.[4] Both parties can then, again simultaneously, decide whether to accept or reject the mediators share. Upon acceptance the conflict is "resolved" in a sense that it is replaced with a legally binding contract. If at least one of the parties denies the contract, mediation is considered to have failed and the case carries on to litigation.

While in many symmetric settings an equal sharing rule is always Pareto-superior to playing the contest, this is no longer true if players are asymmetric and the spread between low and high cost bidders increases. With this, mediation becomes more interesting since the mediator not only needs to take care of the proposed rule, but also needs to govern beliefs of the opponents post-mediation as well. Further, it is not possible for the mediator to avoid litigation at all times. This suggests, perhaps contrary to conventional wisdom, that mediation should only fail, if cost uncertainties are large, where as in ex-ante "tight" cases, mediation is more likely to succeed.

In addition we contribute to the question of how to optimally deal with deviators. In our setup it is, in addition to sending a message to the mediator, also possible to "not show up for mediation" (i.e. publicly and credibly announce not to have said anything). If the mediator had no ability to react to this, there is the need to secure the potential deviator a certain payoff (i.e. there is a participation constraint coming from the underlying game). If, however, the deviator has no possibility to commit not to listen to information he receives, we show that the mediator has a possibility to punish the deviator by strategically releasing information about the non-deviator's type. This way, the mediator can reduce the participation constraint of the players. In case of legal mediation it is realistic for a contestant to not show up for mediation and thereby publicly announce not to talk. However, committing to ignoring information given about the opponent (or the case) is much harder to sustain. In that sense we also depart from the classical framework set up by Myerson (1982) in that we do not force all players to send some message (at least a babbling one) to the mediator.

Finally, we believe our model is flexible enough to address other issues in which contests play a role, such as international relations (games of war and peace), strikes, political lobbying and patent races and standard setting organizations.

The remainder of the paper is structured as follows: Section 2 relates this work to the pre-existing literature on litigation, all-pay contest as well as mediation in general. Section 3 illustrates the main findings with help of a specific example, thereafter Section 4 lays out the general modelling assumptions. Section 5 analysis mediators optimal behaviour before we discuss the results in Section 6.

[3]Mediation in court is usually done at a pre-arranged date. If one of the parties does not show up to this date it is observed by the other party.

[4]One could also think that the mediator proposes a probability rule with which the prize is assigned to one or the other party.

2. Related Literature

To best of our knowledge the economic literature on litigation, trials and ADR is surprisingly small. While economists tend to model courts as an exogenous force, as for example Schweizer (1989) and Spier (1992), there are some exemptions such as Klemperer (2000) and Baye, Kovenock, and Vries (2005) who model litigation trials as a form of an all-pay auction similar to us. While Klemperer (2000) discusses litigation only as an application to the general all pay auction model, Baye, Kovenock, and Vries (2005) explicitly model the process of litigation. Similar to them we also assume that that the *best case wins* and a monotonic legal production function. The main differences however to our approach lie in the objective as well as in the modelling of the alternative to lawsuits. Baye, Kovenock, and Vries (2005) want to study how a change in the auction format (or litigation environment in their terminology) effect expenditures and outcomes while our objective is to model a communication device that leads to avoidance of the otherwise fixed trial.

The underlying all-pay contest (or auction) as a default game is essentially a limiting case of a Tullock-contest (Tullock, 1980). Following Tullock, there has been a large literature studying contests and all-pay auctions both under complete[5] and incomplete[6] information. An important aspect in the analysis about to be carried out in this article is the ex-ante asymmetry of players. In this respect we provide further evidence for the claim made by Szech (2011) that ex-ante symmetry of participants is not at all an innocuous assumption.

A recent strand of literature considers all-pay auctions that include reserve prices, head starts, or handicaps. This literature includes papers by Kirkegaard (2012), Pavlov (2013), Seel (2014), and Siegel (2014b). In our model reserve prices and head starts not only have an intuitive interpretation, but change in addition the possibilities that a mediator has. This underlines the result made by the above mentioned papers that head starts and reserve prices are fundamentally different in their effect compared to shifts in costs or valuations. Since mediation in its purest sense essentially is a communication device we also relate to Pavlov (2013) in this direction. He shows, that in all-pay auctions without reserve price, there is no communication equilibrium that is not outcome equivalent to the Nash Equilibrium. However, once reserve prices are introduced, this property is no longer true.

While our model, too, gets more interesting if reserve prices (or head starts) are introduced, we allow our mediator to have more power than only serving as a communication device. In addition our mediator has the ability to propose a contract to both parties which they may voluntarily sign and which avoids the play of the default game. We borrow this structure from Hörner, Morelli, and Squintani (2011) which probably is the closet paper to ours. Hörner, Morelli, and Squintani (2011) also derive the effect present in our model, that the mediator can improve upon unmediated communication by not completely revealing the truth, but by sending an incomplete (private) signal. The main difference however is, that in our model the likelihood of winning the conflict depends not only on your type within the conflict, but also on your actions within the conflict game. In addition, looking for a perfect Bayesian Nash Equilibrium, beliefs matter also in the

[5]Prominent papers are, e.g. Baye, Kovenock, and Vries (1996) and Siegel (2009, 2010).
[6]Examples include Amann and Leininger (1996), Szech (2011), and Siegel (2014a).

underlying game post-mediation. We show that this effect can partially be exploited by the mediator.

The possibility to go back to the default game at basically any point in the game puts our mediation mechanism into the class of "veto constraint mechanism". These have been studied, e.g. by Cramton and Palfrey (1995), Dequiedt (2006), and Celik and Peters (2013). The difference to Cramton and Palfrey (1995) is that instead of choosing the appropriate off-equilibrium belief to sustain a ratifiable mechanism we fix off-path "priors", that is the beliefs each party holds once we moved off-path, and employ Bayes' rule from there on.[7] This allows us to study a different channel namely off-path news arrival. This channel can be used by the mediator in order to change the deviators belief even if beliefs off-path are not as pessimistic as they would need to be to ratify the mechanism. The technique that we use for this is similar to Dequiedt (2006) and Celik and Peters (2011, 2013). However, signal realization does not effect incentive compatibility constraints, as it does in Celik and Peters (2013), since on-path communication with the mediator is disconnected from off-path signalling. Further, all of the above use mechanisms that have full commitment power (once the mechanism is unanimously accepted) on the players side. Although our proposed mechanism, or game form, is valid under the full commitment assumption, we equip the mediator with a weaker commitment power because of consistency with the considered application.[8]

Finally, our paper also relates to the more theoretic work on correlated equilibria under incomplete information. After collecting the information of both parties, the mechanism has in fact the ability to pick any Bayes' Correlated Equilibrium (BCE) as defined in Bergemann and Morris (2013). Our notion of punishment is essentially a form of a one-sided belief-invariant communication equilibrium as introduced by Forges (2006). The possibility to use correlated private signals in the punishment phase is similar in style to Rubinstein (1989) which relates back to the seminal assessment of higher order beliefs by Mertens and Zamir (1985).

3. Illustrative Example

Before turning to more general results, we make use of a fairly simple example to illustrate the main idea.

Consider two parties, player 1 ('she') and player 2 ('he') involved in a legal argument. Both believe they have developed a certain product which the other party is now using without paying any royalties. Winning the trial implies a value to each firm of 1 which is common knowledge among the contestants. In order to support their claims, both parties can produce evidence on their development of the product and their respective patent rights. The marginal cost to produce supporting evidence of player 1 are her private information and are drawn from a commonly known distribution that gives $c_l = \frac{1}{2}$ with probability $p_1^0 = \frac{2}{3}$ and $c_h = 5$ with the remaining probability. The marginal cost of player 2 are similar only that he gets c_l with probability $p_2^0 = \frac{1}{3}$ and c_h with the remaining

[7] Which basically constitutes the definition of Perfect Bayesian Equilibrium as given in Fudenberg and Tirole (1988).

[8] Furthermore, in contrast to Celik and Peters (2013) our proposed game form, or mechanism, satisfies inscrutability (see Myerson (1983)).

probability. Both parties need to produce a minimum amount $r = \frac{1}{7}$ in order to prove that the case is indeed relevant and a judge is actually needed. If only player 1 produces evidence above the threshold and player 2 does not, she gets the good with certainty. If none provides enough evidence there are no claims to be enforced and both end-up with no royalties. Once they are above the threshold, the court is going to award the player that produces the highest amount of evidence with the good and this player is going to earn the good.

Dealing with the legal issue is costly to both firms, and we assume these cost do not produce any value to any player except from increasing their likelihood to win the lawsuit.

A feature of all-pay contests is that in equilibrium there always is at least one type of player that mixes over a continuum of actions. Further each equilibrium is always outcome equivalent to any other equilibrium. In this particular example we get that both players do not participate in any trial if both of them have a high cost of producing evidence. If player 1 has low cost, she produces only the minimal amount r with probability $\frac{1}{2}$ and with the remaining probability mass she uniformly produces evidence on the interval $(r, r + \frac{2}{3}]$. Player 2, low-cost type, on the other hand uses all his probability mass to produce evidence on $(r, r + \frac{2}{3}]$. This gives both players an expected utility of the trial of $\frac{25}{42}$ given they are the low-cost type and 0 utility if they are the high-cost type.

Consider now a mediator ('it') that wants to maximize total welfare and therefore avoid the costly litigation game. Suppose further that the mediator has the ability to propose a contract that both parties may sign which decides upon the share of the prize to be divided among them.[9] As a quite natural assumption in our environment we further assume that contracts can only be signed at an interim stage, that is when players already know their type. By this it is straightforward to detect, that there is no pooling contract such that all player types would sign the contract.

Separation however is not straight forward as the high-cost type always has an incentive to mimic the low-cost type. However, even in the absence of utility transfer,[10] the mediator has still room to screen. In fact a mediation protocol that strategically uses the thread to impose litigation on the parties can help to improve the situation since it can also change the expected payoff if the case indeed is referred to court, as the beliefs about the opponents might be different from the priors. Since the effort put into the trial depends on the expected type of the opponent, the mediator can use this to both create incentive compatible type reports and to lower the outside option of participants.

To model the first point, consider an expected share $x_1(c_1, c_2)$ given to player 1[11] and a probability to enter litigation $\gamma(c_1, c_2)$ which is also conditional on the players reports.

The timing is as follows: The mediator announces and commits to the mediation structure.

Both players privately submit reports to the mediator who then announces the result of mediation to each party (litigation/ no litigation). Instead of submitting a report each player can also choose not to submit a report and has the ability to prove to the other

[9] A share of $\frac{1}{2}$ to each player would e.g. mean that they get to split any royalty payments between each other.

[10] Utility transfers are, given that parties differ only in the way they are able to produce evidence in case of litigation, of not much help here anyways.

[11] Money burning is of no use here which is why we assume the remainder to be given to player 2.

party that no cost that no report was submitted. If both agreed on mediation and the mediator announces no litigation, the price is divided according to the sharing rule. In all other cases litigation is carried out

In the present example a mediator could for example announce the the following protocol:

$$
\begin{aligned}
x_1(c_l, c_l) &= \frac{2}{5} = 0.2 & \gamma(c_l, c_l) &= \frac{1}{4} = 0.25 \\
x_1(c_l, c_h) &= \frac{47}{70} \approx 0.671 & \gamma(c_l, c_h) &= \frac{5}{16} \approx 0.313 \\
x_1(c_h, c_l) &= \frac{69}{70} \approx 0.986 & \gamma(c_h, c_l) &= \frac{7}{8} = 0.875 \\
x_1(c_h, c_h) &= \frac{11}{20} = 0.55 & \gamma(c_h, c_h) &= 0.
\end{aligned}
$$

This protocol is both incentive compatible and satisfies the participation constraint of both parties, assuming passive beliefs on the off-path event that mediation is rejected in the first place. With this protocol, the mediator can reduces the probability of litigation to only $\frac{7}{24}$ (roughly 29.2%) of the cases. This is the minimum amount of litigation needed in order to force parties into both participating in mediation and to report truthfully. Since direct payments are not allowed, the mediator "pays" the parties by sending them into litigation from time to time which yields them a higher expected return conditional on being the low type. This helps to ensure truth-telling and participation.

The above mentioned protocol however neglects a second instrument the mediator has. The mediator can use whatever information it possesses and release it in a perturbed way. This perturbation can be used by the mediator to punish a player through the information it has acquired, in case he refused to participate in the mediation mechanism. If it does so, again, using a predefined protocol, it can only use mean preserving spreads to release credible signals. In other words, those signals need to be Bayes' plausible in the sense of Kamenica and Gentzkow (2011) with respect to the prior. Given such a signal it is interim optimal to update according to the signals realization, at least if not all players can commit ex-ante to not follow the signal. In the present example one such signal could be a binary signal with realizations l and h that realize conditional on the information the mediator has about player 2. The following could be such a protocol:

$$
\begin{aligned}
P(l|2 = c_l) &= 1 & P(h|2 = c_l) &= 0 \\
P(l|2 = c_h) &= \frac{3}{4} & P(h|2 = c_h) &= \frac{1}{4}
\end{aligned}
$$

Posteriors about player 2 change, of course, with this signal structure, since after each realization h, player 1 is certain about player 2's type (c_h), while the signal l leads to player 1 believing that player 2 is of type c_l with probability $\frac{2}{3}$. This change of beliefs unsurprisingly changes equilibrium effort behaviour and leads (in expectations) to a reduction of player 1's expected utility from litigation. After realisation of h player 1 needs to invest very little in order to win with the contest high probability, while after realisation l she needs to invest a lot. This changes her expected outcome after each realisation.

A necessary condition of this punishment by the mediator is, that the utility function is strictly concave in the neighbourhood of at least one point given the parameters (in this case anything but p_2). It turns out that here, there is exactly one point around which the function is strictly concave. As the prior is not at the extremes and any other point has a linear neighbourhood, we can employ punishment here and thus reduce the outside option of player 1, i.e. relax her participation constraint.

Not surprisingly, this reduction allows the mediator to alter the mediation protocol given above to the following (which is not feasible without the signalling protocol):

$$
\begin{aligned}
x_1(c_l, c_l) &= \frac{2}{5} \approx 0.20 & \gamma(c_l, c_l) &= \frac{37}{160} \approx 0.231 \\
x_1(c_l, c_h) &= \frac{20497}{31850} \approx 0.644 & \gamma(c_l, c_h) &= \frac{37}{128} \approx 0.289 \\
x_1(c_h, c_l) &= \frac{3249}{4270} \approx 0.761 & \gamma(c_h, c_l) &= \frac{259}{320} \approx 0.809 \\
x_1(c_h, c_h) &= \frac{431}{800} \approx 0.539 & \gamma(c_h, c_h) &= 0.
\end{aligned}
$$

The above protocol reduces the probability of on path litigation further to $\frac{259}{960}$ (about 27.0%) of the cases and the sum of expected utilities rises to $\frac{1933}{2010}$ which is compared to the unmediated case an increase of over 82%.

One conclusion we want to draw from this example is that the ability to propose a sharing of (or a lottery over) the prize in all-pay contests has a tremendous effect on the reduction of costly effort and player's utilities. Further, we can see that informational punishment might help to reduce the probability of contest even further. While this example illustrates the main idea of this paper, in what follows we are analysing a more general model of an all-pay contest in order to understand the role our type of mediator has. In addition, we derive the optimal mediation protocol and asses the question when and to what extend informational punishment is helpful in this process.

4. The Model

4.1. Underlying Default Game

The underlying default game of our model is an all-pay contest as formalized by Siegel (2009) and Szech (2011).[12] There are two contestants $i = 1, 2$ who have a commonly

[12] While Szech (2011) calls our model all-pay auction, most of the literature differentiates between contests and auctions. In contests the good is assumed to have a common and known value (usually normalized to 1) and possibly asymmetric costs. In auctions the costs are assumed to be known and simply money and there are possibly asymmetric valuations across bidders. While in the baseline specification it makes no difference at all, the process changes once we introduce a possible mechanism to the setup.

known valuation normalized to 1 of winning. Both players simultaneously decide on a score s_i and the player with the highest score wins. Obtaining a score is costly. Marginal cost functions are constant and either low, $c_l > 0$, with probability $p_i^0 \in (0,1)$, or high $c_h > c_l$ with probability $1 - p_i^0$. In addition there is a minimum score $r \geq 0$ such that given both players bid below r no one is assigned the good and all players receive payoff 0.[13,14] All but the realization of the cost draw, which is private to each player, is common knowledge.

Without loss of generality we restrict attention in this paper to cases in which $p_1^0 \geq p_2^0$, i.e. cases in which it is more likely that player 1 has low marginal cost of effort.

4.2. Mediation

We model the mediator as an outside third party possessing no private information that announces a protocol $\mathcal{X}$ and a signal structure $\mathcal{S}$ prior to the game and has the ability to commit to it. Each party has then the possibility to send a message out of their message space $\mathcal{M}$ to the mediator who then processes this information according to $\mathcal{X}$ and $\mathcal{S}$.

As the revelation principal applies here, we model $\mathcal{X}$ as a triple (G, X_1, X_2) with

$$X_i = \begin{pmatrix} x_i(l,l) & x_i(l,h) \\ x_i(h,l) & x_i(h,h) \end{pmatrix},$$

and

$$G = \begin{pmatrix} \gamma(l,l) & \gamma(l,h) \\ \gamma(h,l) & \gamma(h,h) \end{pmatrix},$$

where $x_i(m_1, m_2)$ denotes the share assigned to player i after the message profile $M = (m_1, m_2)$[15] and $\gamma(m_1, m_2)$ denotes the probability of the contest being played after the message profile M. That is, formally the mediator is a mechanism designer lacking the instrument of transfer payments who is not able to enforce actions in the default game. Instead it can elicit parties private information through the decision whether to allocate the prize by means of proposing shares or by the play of the contest. In the latter instance the third party takes the role of a mediator in the sense of Myerson (1982) and recommends actions to each party.

We believe that in light of our application cost heterogeneity is the more plausible assumption.

[13]In our application this captures the initial cost of attending the court meeting at all. We assume that some minimum effort is necessary to even bring the case to court and to show that you have relevant concerns in order to make an actual lawsuit necessary. The effort to make this case might again be different depending on you being a low or a high-cost type. We assume this for the ease of notation, since very little would change if we assumed instead of some minimum score a (possibly type specific) entry fee K_i and no minimum score. In fact with $K_i = rc_i$ we deal with a special case of this setting here.

[14]Another way to model litigation would also be to have someone being the default owner of the prize who if not told different by the court has the ability to keep it. This would be resembled in what is called a "head start" in the contest literature and can, at the cost of additional notation, be used instead of the minimum score.

[15]For the ease of notation we assume without loss of generality that the message k is assigned to the meaning "I am type c_k".

Ex-ante, the mediator thus commits to a signal structure $\mathcal{S}$ as a mapping from message profiles into a distribution $\Sigma = (\sigma_1, \sigma_2)$ where σ_i is the strategy the mediator would recommend player i to play in the default game if it was played.

To capture the possibility to withdraw from the mediation process until the final agreement we assume interim individual rationality given signal structure $\mathcal{S}$.[16]

By the revelation principle we can impose without loss of generality that each Σ is consistent with the messages and constitutes a Nash Equilibrium for a given tupel of belief-hierarchies (π_1, π_2). The distribution of these belief-hierarchies is Bayes' plausible, i.e. it is a mean preserving spread with respect to some initial belief-hierarchy (π_1^0, π_2^0). This hierarchy is endogenously determined by Bayes' rationality, parties common knowledge of the protocol and their common knowledge that the contest is played.(As we still need to show) it is without loss of generality to identify the initial belief-hierarchy with its (possibly) type dependent, level 1 beliefs $((p_1, p_2), (p_1, p_2))$.[17]

Before turning to the analysis of the game it is useful to define a set of auxiliary variables.

First, let the vector of the ex-ante beliefs player i holds be $\rho_i^0 = (p_{-i}^0, 1 - p_{-i}^0)$.

Then we may define the interim, type and player dependent probability to enter litigation given G and assuming truthful behaviour as

$$\gamma_1 = \begin{pmatrix} \gamma_1(l) \\ \gamma_1(h) \end{pmatrix}^T := \begin{pmatrix} p_2^0\gamma(l,l) + (1-p_2^0)\gamma(l,h) \\ p_2^0\gamma(h,l) + (1-p_2^0)\gamma(h,h) \end{pmatrix}^T = \rho_1^0 G,$$

and

$$\gamma_2 = \begin{pmatrix} \gamma_2(l) \\ \gamma_2(h) \end{pmatrix} := \begin{pmatrix} p_1^0\gamma(l,l) + (1-p_1^0)\gamma(h,l) \\ p_1^0\gamma(l,h) + (1-p_1^0)\gamma(h,h) \end{pmatrix} = G(\rho_2^0)^T.$$

As G is not necessarily symmetric, it is also necessary to define the subjective beliefs player's hold about the opponent if the contest is called, but *before* the contest is played.

We do this by defining the vector $\rho_i(l_{-i}) = (p(l_{-i}|l_i), p(l_{-i}|h_i))$ to be the vector of probabilities that player $-i$ is of type c_l for given a report by player i. Te 2x2 matrix $P(l) = (\rho_1, \rho_2)^T$ can be considered as an *information structure* of the contest in a sense, that it describes the beliefs players hold whenever the mediator refers the players back to the contest.[18]

[16]This allows the mechanism to propose a contract that only pins down *expected* shares of the parties. If we were to impose ex-post individual rationality given signal structure $\mathcal{S}$, we would constraint the mediator further to a contract that pins down *actual* shares of the good. This can easily be done.

[17]Note that in any implementation the mediator could always send (a vector of) binary signal realizations privately to each player. With such a signal structure we could drop the Nash assumption. However, in order to make our model flexible to any type of higher order belief structure imposed by the mediator (as long as it is Bayes' plausible) the recommended strategy (together with the Nash assumption and Bayes' plausibility) as an outcome of the signal structure can be used without loss of generality. This way we can also capture non-common priors as a result of the signal sent by the mechanism.

[18]Whenever it is more convenient to write $P(h)$ or $p(h_{-i}|l_i)$ we will do so noticing of course that those are redundant formulations.

Notice that the information structure $P(l)$ is in fact entirely defined by choosing G as all $p(\cdot|\cdot)$ have a form such as

$$p(l_2|m_1) = \frac{p_2^0\gamma(m,l)}{\gamma_1(m)}. \tag{1}$$

It is worth noting that $P(l)$ is homogeneous of degree 0 with respect to G as can easily be seen by inspecting Equation (1). Furthermore, γ_i is homogenous of degree 1 with respect to G.

Next, if $X \circ (1-G)$ denotes the Hadamard product of X and $(1-G)$ such that

$$X_1 \circ G = \begin{pmatrix} x_1(l,l)(1-\gamma(l,l)) & x_1(l,h)(1-\gamma(l,h)) \\ x_1(h,l)(1-\gamma(h,l)) & x_1(h,h)(1-\gamma(h,h)) \end{pmatrix}^T,$$

we can define the (unconditional) expected share of player i, z_i, in the same fashion as γ_i, only using $X_i \circ (1-G)$ instead of G only, that is

$$z_1 = \begin{pmatrix} z_1(l) \\ z_1(h) \end{pmatrix}^T := \rho_1^0(X_1 \circ (1-G))$$

and

$$z_2 = \begin{pmatrix} z_2(l) \\ z_2(h) \end{pmatrix} := (X_2 \circ (1-G))(\rho_2^0)^T.$$

Thus the expected share conditional on litigation being called off by the mediator after report k is given by

$$x_i(k) := \frac{z_i(k)}{1-\gamma_i(k)}.$$

To close this section, we define the payoff functions both if the contest is called as well as the overall interim payoff.

By $U_i(k|m)$ we denote player i's expected paoyff if she is type c_k and sends message m, assuming the opponent reports truthfully. Note that this utility entirely depends on the information structure $P(l)$ which pins down beliefs and therefore strategies.

Player i's interim payoff $\Pi_i(k,m)$, induced by the mediation mechanism is defined as

$$\Pi_i(k,m) := z_i(m) + \gamma_i(m)U_i(k|m).$$

Whenever it is clear from the context that $k = m$, we may suppress the report in the payoff function for the sake of readability.

4.3. Timing of the Game

First, the mediator commits to $(\mathcal{S}, \mathcal{X})$ and players learn their type privately. Then, players simultaneously decide whether to participate in the mediation mechanism, or not. To participate a player sends a message to the mediator.

In the third stage players learn whether or not both of them sent a message to the mediator, receive their strategy recommendations from the mechanism, and in case the contest is played simultaneously decide on their score. The winner earns the good. In case of no contest, payoffs realize immediately according to the shares each party receives.

We are looking for a perfect Baysian Nash Equilibrium and assume passive beliefs of the non-deviator in the first off-path node.[19]

5. The Analysis

5.1. Useful results from the literature

Before the analysis of our problem, it appears useful to repeat one definition and two results stated by Siegel (2014a) in the notation of our setup.

The results are useful in the analysis of the contest subgames. In addition they help to shape an intuition about the mechanics at work in a contest. As the proof is, apart from notation and the fact that we allow for a positive reserve bid, identical to that in Siegel (2014a), we omit proving the results here and state them as corollaries to the respective lemmas in Siegel (2014a).

First, notice the following definition

Definition 1. An equilibrium is called **monotonic** if it holds that if b_l is part of the optimal bidding strategy of player i, type c_l, and b_h is part of the optimal bidding strategy of player i, type c_h, then $b_h \leq b_l$.

Further, Lemma 1 and Lemma 2 of Siegel (2014a) are of great use, and their translation to our setup is stated as a corollary

Corollary 1 (to lemma 1 in Siegel (2014a))**.** *In any equilibrium with reserve bid r, (i) there is no bid at which both players have an atom, (ii) if a positive bid is not a best response for some player for any of his types, then no weakly higher bid is a best response for either player for any type, and (iii) each player has at least one type for which either $r, 0$ or bids arbitrarily close to r are best responses.*

Corollary 2 (to lemma 2 in Siegel (2014a))**.** *In a monotonic equilibrium, it must hold that if player i, type c_h places a positive bid, then the supremum of possible equilibrium bids of player i, type c_h, equals the infimum of of the very same player i, type c_l, that is* $\sup(BR_i(c_h)) > 0 \implies \sup(BR_i(c_h)) = \inf(BR_i(c_l)) \quad \forall i$.

[19] This assumption is for simplicity; only. It is straightforward to fix beliefs to basically anything else. The only thing we really need is that the equilibrium to be played off-path is known to all players including the mediator.

Allowing a positive reserve bid r, the power of the two lemmas is slightly reduced, but only in results not relevant for our setup. What remains is the two corollaries stated here. Intuition for both is straightforward. Equilibria can be constructed by noticing that the highest player need to have the same maximum bid, otherwise one could profitably deviate downwards. Further, types must try to earn the good as cheap as possible (i.e bidding r) because otherwise the sure loser always has a profitable deviation to 0 which in turn leads to a downward deviation of the "smallest winner". Finally, "wholes" in bidding cannot occur, as this again, would allow for a profitable downward deviation for the bidder that bids the lowest amount above the "whole".

Finally, the literature usually assumes some sort of monotonicity condition in order to guarantee a monotonic equilibrium. In our setup the monotonicity condition is given by

$$\frac{p(l_{-i}|l_i)}{p(l_{-i}|h_i)} \geq \frac{c_l}{c_h}. \tag{M}$$

5.2. Equilibrium Characterization of the Default Game

Before considering mediation, we characterize the equilibrium result in the all-pay contest under different information setups.

For the moment, we abstract from interdependent beliefs and assume the all-pay auction is played with independent priors.

Let $\mathcal{I}$ denote the set of all possible combinations (p_1, p_2) such that $1 \geq p_1 \geq p_2 \geq 0$, i.e. all possible information structures of the default game under priors.

Further, let

$$\begin{aligned}
\mathcal{I}_0 &:= \left\{[0,1]^2 | p_2 > 1 - rc_l\right\}, \\
\mathcal{I}_A &:= \left\{[0,1]^2 | 1 - rc_l \geq p_2 \geq 1 - rc_h\right\}, \\
\mathcal{I}_B &:= \left\{[0,1]^2 | p_1 \geq 1 - rc_h > p_2\right\}, \\
\mathcal{I}_C &:= \left\{[0,1]^2 | 1 - rc_h > p_1 \geq p_2\right\}.
\end{aligned}$$

Hence $\mathcal{I} = \mathcal{I}_0 \cup \mathcal{I}_A \cup \mathcal{I}_B \cup \mathcal{I}_C$.

Figure 1 depicts the different areas. Region 0 (in light grey) depicts the area in which all players earn utility 0 in expectations as it is too likely to meet another low-cost type in the contest.

Region A (in blue) is the region in which both players have a relatively high probability to be the low-cost type, which forces the high-cost types to stay out. However, there is a sufficiently high probability that at least player 2 is of high cost, such that low-cost types need not to bid to aggressively.

Region B (in dark grey) then is an area in which the probability of low-cost types is sufficiently small for player 2, such that player 1 can also earn a positive expected payoff, even if she is the high-cost type.

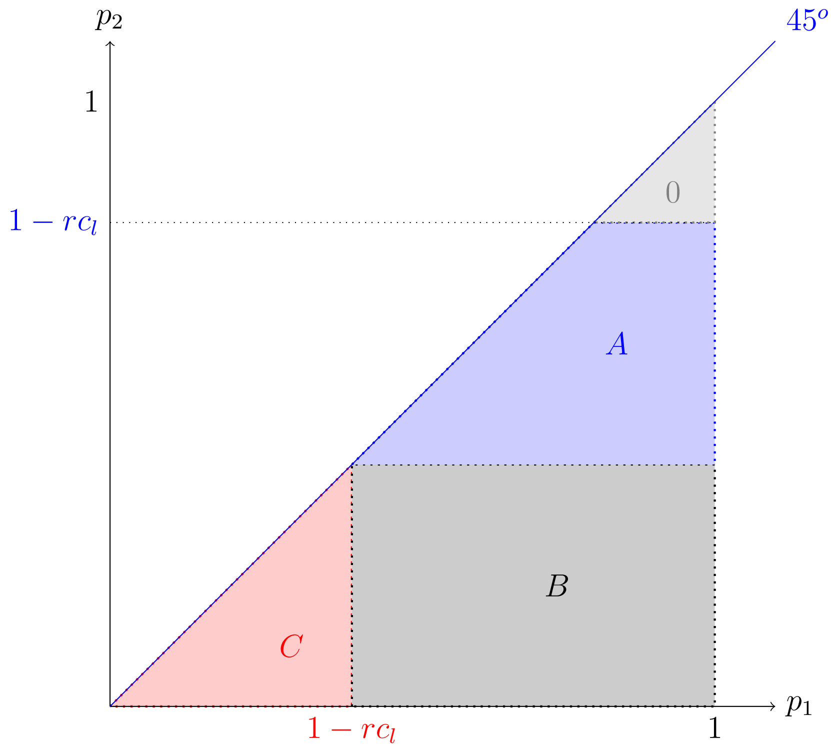

Figure 1: *Partitioning the information set, given $p_1 \geq p_2$.*

Finally Region C (in red) is the area in which both players have a sufficiently high probability to be the high-cost type. Thus, bidding is less aggressive among low-cost types, which allows high-cost types to increase their bids compared to the other regions.

Lemma 1. *Consider an all-pay contest with minimum score r, and an environment in which player i has marginal bidding cost $c_l < c_h$ with probability p_i. Let $\Delta_i := \frac{1-p_i}{c_h}$ and assume the commonly known information set I lies in $\mathcal{I}$. Then, the expected interim utilities can conditional on I be expressed as follows:*

If $I \in \mathcal{I}_0$, *the expected interim utilities of each player and type are* 0.

If $I \in \mathcal{I}_A$, *the expected interim utilities of each player and type are given by*

$$\begin{aligned} U_1(l|I \in \mathcal{I}_A) = U_2(l|I \in \mathcal{I}_A) &= 1 - rc_l - p_2 \\ U_1(h|I \in \mathcal{I}_A) = U_2(h|I \in \mathcal{I}_A) &= 0. \end{aligned}$$

If $I \in \mathcal{I}_B$, *the expected interim utilities of each player and type are given by*

$$\begin{aligned} U_1(l|I \in \mathcal{I}_B) = U_2(l|I \in \mathcal{I}_B) &= \Delta_2(c_h - c_l) \\ U_1(h|I \in \mathcal{I}_B) &= (\Delta_2 - r)(h - c_l) \\ U_2(h|I \in \mathcal{I}_B) &= 0. \end{aligned}$$

If $I \in \mathcal{I}_C$, *the expected interim utilities of each player and type are given by*

$$U_1(l|I \in \mathcal{I}_C) = U_2(l|I \in \mathcal{I}_C) = \Delta_2(h - c_l)$$
$$U_1(h|I \in \mathcal{I}_C) = (\Delta_2 - \Delta_1)(c_h - c_l)$$
$$U_2(h|I \in \mathcal{I}_C) = 0.$$

The proof of the lemma can, as all others not in the text, be found in the appendix. Whenever we talk about a contest with independent priors in what follows we mean a game to which the lemma applies, unless otherwise noted. The result itself follows from the observation that all-pay contests generically only have a mixed strategy equilibrium. As marginal cost are constant and types are discrete, players usually are indifferent on some interval, conditional on the opponents strategy. Since players are heterogeneous and types are private information, players need to account for the opponent to be any of the two types. Thus, they need to play such that either of them is indifferent in order to sustain the equilibrium. This leads to a set of at most two intervals on which players mix uniformly. By the minimum required score, the only mass points players can set are on the minimum score r (to only win if the other player stays out of the contest) or 0 in order to obtain 0 utility. Further, it cannot be the case that everyone receives a positive payoff, as that would lead to an incentive of at least one player type to increase their respective score marginally.[20]

With help of Lemma 1 it is straight forward to determine the outcome of a contest. Notice, that, given $p_1 \geq p_2$, p_1 does not enter the utility function of any type of any player, with the exception of player 1, type c_h, in information set $I \in \mathcal{I}_C$. As mediation can only alter the information structure in the contest whenever it is played both on and off-path, this lemma proves to be useful as it delivers us a closed form solution that is a function of only one of the (interim) beliefs about the other type under the common priors.

5.3. Informational Punishment

To understand our assumption of rejecting mediation publicly, notice first the following: There always exists the possibility to not publicly reject mediation by just sending an off-path message to the mediator. However, this is different from public refusal as the opponent does not know about the off-path message. Thus, the mediator can deter such a refusal by committing to interpret the off-path message to be any worst type message. As the revelation principle applies to our setting it is not possible to profit from such an off-path message given the direct mechanism is designed optimally, as already shown in Myerson (1982).

Public refusal however is not covered in Myerson (1982). The main difference, of course, is that the opponent learns about the rejection of the mechanism. As Cramton and Palfrey (1995) argue, one can in certain games construct equilibria in which "learning from rejection" takes place, that is having the "right" beliefs about the deviator's type deters this type of public rejection. Cramton and Palfrey (1995) call such mechanisms "ratifiable". Given, however, that all this happens off-path it is rather unlikely that the game to be played off-path that makes the mechanism ratifiable, is the only sustainable

[20] Compare also Section 5.1 on this.

off-path game. The additional channel we use in this paper is therefore a different one. We fix off-path beliefs to be passive.

In our setting public refusal can be interpreted as a form of a participation constraint, as any mediation protocol that does not pay (in expectations) as much as the player expects to earn in case of public rejection is not sustainable as an equilibrium. However, if we assume such a rejection is not possible prior to the game, than it is not necessarily the case that the value of this participation constraint corresponds to each types payoff in the default game (i.e. the contest under priors p_i^0), since the mediator posses information on the non-deviator that was expecting to play equilibrium.

In a way similar to Celik and Peters (2013) the mediator can use this information and we assume it has the ability to publicly reveal (parts of) it before the contest is played. If it does so, it can only implement Bayes' plausible signals as in Kamenica and Gentzkow (2011), Bergemann and Morris (2013), and Celik and Peters (2013). That is we allow the mediator to pre-commit to a signal structure in the case of public refusal that transforms the information the mediator has into a public signal which constitutes a mean-preserving spread of the actual value. Given such a commitment, none of the players have an incentive to ignore this information.[21] Under certain assumptions on parameters, this strategic information release by the mediator, punishes some types of players at the interim stage, that is it reduces their interim expected payoff from deviation, as they receive informational punishment as described above when deviating.

Before turning to some general results on informational punishment, some remarks on the assumption of public refusal are in order. In case of litigation, courts often call for mediation. That is, although there is no commitment imposed on the players to tell the truth or to even tell anything, they are requested to participate in a round of mediation. While announcements to not attend mediation *before* reports are sent can be considered as cheap talk (all parties would announce this in order to signal low marginal cost), not having sent a report can easily be verified ex-post by the deviator. Another possible way to understand this assumpitio is to imagine a mediation processes involving a stage in which both parties need to sign a contract to not go to court (in case mediation is successful in preventing court appeal). If such a contract is not signed by one of the players this is learned by the other. As the information of the two parties is the same at this stage, this is equivalent to public refusal in the reporting stage, given the deviator babbled in the report stage.

Whether informational punishment is possible in this setup depends on the shape of the utility function of players. Given passive beliefs on the deviating player i, that is $p_i = p_i^0$, the only variable that changes compared to the default game is p_{-i}. As the mediator only holds information about the realization of c_i, it has a possibility to influence p_{-i} by revealing information. As extensively argued in Kamenica and Gentzkow (2011), the sign of the effect of Bayes' plausible signals on some function, depends on the curvature of this function with respect to the belief about the variable the signal is about. The result is a direct consequence of Jensen's inequality. As we want to punish (at least) one player, this is only possible if the function is concave around the prior, which, again, follows by Jensen's inequality.

[21] Of course, the mediator could also send private signals to the parties. For the moment however, we abstract from this possibility and assume that the mediator only publicly announces the signal.

In what follows denote $V_i(c_i)$ the expected payoff of player i, refusing to participate in the mechanism.

Proposition 1. *Consider the contest as described above. Then, player 1 can be punished if and only if the game under priors is played under information set $I \in \mathcal{I}_A \cup \mathcal{I}_B$ and $p_2^0 \in (0, p_1)$. Player 1, type c_l, receives at least utility*

$$V_1(l) = \left(1 - \frac{p_2^0}{p_1^0}\right) \frac{c_h - c_l}{c_h} + \frac{p_2^0}{p_1^0}(1 - c_l r - p_1^0)$$

under punishment.

Further player 2 cannot be punished.

The statement primarily stems from the fact that making the ex-ante weaker player (i.e. player 2) stronger by increasing his likelihood to be of low cost, has a non-constant effect on the stronger player. While at first, i.e. for small p_2^0, both high-cost type players actively participate in the contest and thus the low-cost type players need not to react as strong as they would if they were certain only to play against another low-cost type player. They are covered by the "possibility" of themselves being the latent type which in turn makes their opponent bid less aggressively. However, once p_2^0 is large enough, high-cost types evaluate the risk of playing against a low-cost type (and thus losing) as quite high and decide not to participate in the contest. As a consequence this makes low-cost types react stronger to changes in p_2^0. The mediator can use this effect to punish the strong player in making her either sure not to play against a high-cost type or signal her that the contest is ex-ante as symmetric as possible and therefore as close as possible inducing strong incentives to invest in her score.

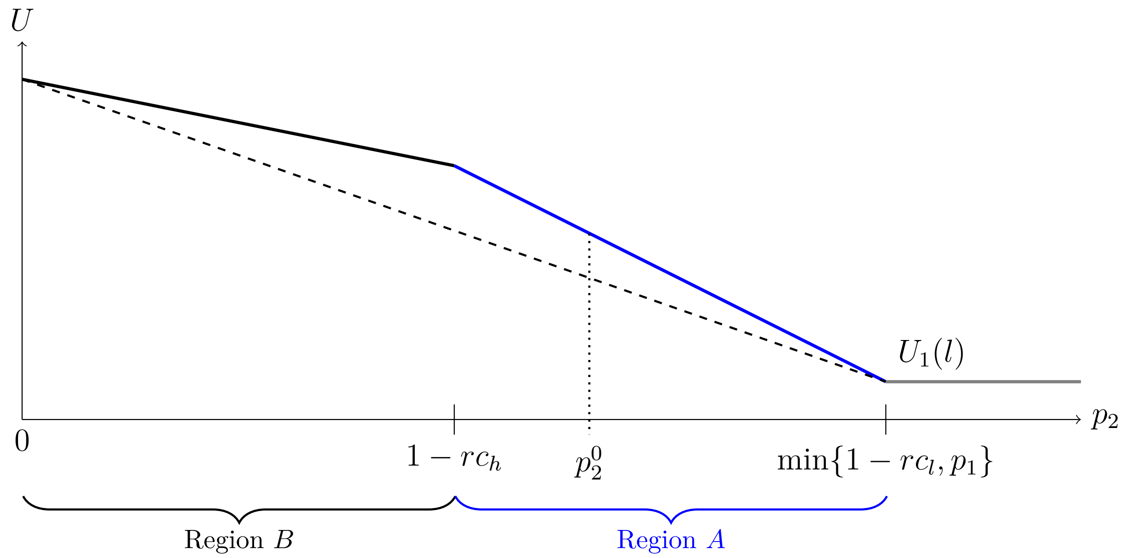

Figure 2: *Utility of the low cost type of player 1 as a function of p_2 given that $p_1 > 1 - rc_h$. p_2^0 depicts the ex-ante probability that Player 2 is the low cost type. The dashed line represents the minimum utility player 1 receives, if punished and is constructed by a mean preserving spread over p_2^0 using the whole region over which the function is strictly concave.*

Figure 2 depicts such a situation and shows where the discontinuity in the derivative of player 1's utility function allows for punishment if we start in region A or B. Note that for this graph to be valid, the ex-ante probability of player 1 being the low-cost type must be higher than $1 - rc_h$ otherwise we would enter region C. The function is linear in p_2^0 within each region, but has a kink when we move from region B to A which can be exploited. Once p_2^0 becomes to large, however, the utility is independent of p_2^0 because it is either 0 (if $p_2^0 \geq 1 - rc_l$) or constant and positive (if $p_2^0 \geq p_1$). Thus, the function is (strictly) convex around this point.

To exclude trivial cases in which mediation can (given it can use informational punishment) always avoid playing the contest on path completely we make the following assumption.[22]

Assumption 1. Under the thread of optimal punishment, the interim expected utilities of the low-cost types facing a mediator add up to more than one, i.e. $V_1(l) + V_2(l) > 1$

The assumption is innocuous in the sense that we only exclude cases in which mediation is "as successful as it gets."

Two results that directly pop out of Assumption 1 are summarized in the following lemma

Lemma 2. *Assumption 1 implies that $c_h > 2c_l$ and $p_2^0 < \frac{1}{2}$. Further, if the default game under priors is in Region A, it additionally holds that $c_h > \frac{1}{2r} + c_l$.*

5.4. Mediation

We start our analysis by characterizing the optimal mediation protocol under the assumption that the conflicting parties can fully commit to the mediation mechanism, once agreed to participate by submitting a type report.

We consider the case in which the mediator has no possibility to reveal signals to the players, whenever the mediation is unanimously accepted. That is, in the event the mediator refers the players to the contest, the latter game is played under the commonly known beliefs $P(l)$.

If the objective of the mediator is to minimize the occurrence of a disagreement, the problem takes the following form:

$$\min_{\mathcal{X}} \rho_1^0 G (\rho_1^0)^T \tag{P1}$$

[22] Note that there are scenarios in which without informational punishment there is no ability for the mediator to achieve first best, i.e. no contest at all, but with punishment there is in fact.

$$\Pi_1(l) = z_1(c_l) + \gamma_1(c_l)U_1(l|c_l) \geq V_1(l) \qquad (PC_1)$$
$$\Pi_2(l) = z_2(c_l) + \gamma_2(c_l)U_2(l|c_l) \geq V_2(l) \qquad (PC_2)$$
$$\Pi_1(h,h) = z_1(h) + \gamma_1(h)U_1(h|h) \geq z_1(l) + \gamma_1(l)U_1(h|l) = \Pi_1(h,l) \qquad (IC_1^+)$$
$$\Pi_1(l,l) = z_1(l) + \gamma_1(l)U_1(l|l) \geq z_1(h) + \gamma_1(h)U_1(l|h) = \Pi_1(l,h) \qquad (IC_1^-)$$
$$\Pi_2(h,h) = z_2(h) + \gamma_2(h)U_2(h|h) \geq z_2(l) + \gamma_2(l)U_2(h|l) = \Pi_2(h,l) \qquad (IC_2^+)$$
$$\Pi_2(l,l) = z_2(l) + \gamma_2(l)U_2(l|l) \geq z_2(h) + \gamma_2(h)U_2(l|h) = \Pi_2(l,h) \qquad (IC_2^-)$$

Mediator's choice variables are an allocation of feasible shares and contest probabilities. Problem (P1) is subject to players upward adjacent interim incentive compatibility constraints Equations (IC_1^+) and (IC_2^+) and downward adjacent incentive compatibility constraints Equations (IC_1^-) and (IC_2^-). Moreover, participation constraint Equations (PC_1) and (PC_2) are imposed at the interim level.

Without mediation, low-cost types have a comfortable standing in comparison to high-cost types. In order to agree to the mediation protocol, the former types must be compensated in terms of shares and payoff induced by the on path play of the contest. In contrast, high-cost types receive rather low payoff from not participating in the mediation mechanism. However, when mediation takes place, high-cost types are able to jeopardize low-cost types. Whereas in any optimal mediation mechanism high-cost type's individually rationality constraint is trivially satisfied, incentive compatibility restricts the allocation dedicated to low-cost types.

By allotting the shares equally between types of a given player, the mediator is able to avoid the occurrence of the contest in the event that low-cost type's outside option is rather moderate. If, however, this allocation does not satisfy low-cost type's participation constraint, the mediator desires to screen player's private information. The mediator can elicit this information by referring players back to the contest sometimes, since this referral induces a player and type dependent payoff. Given Assumption 1, we are in an environment in which the mediator desires to impose some inefficiencies to screen at least one player's type. As a consequence, the participation constraint of at least one low-cost type is binding.

There is some weak analogy to standard problems of allocating a good in a quasi linear environment. Instead of money, the non negative share takes the role of the numeraire good. The payoff induced by the on path play of the contest can be thought of as the valuation of the good. As a consequence of this analogy, at least one player's low-cost type participation constraints is binding. If the payoff induced by the mediation mechanism leaves a low-cost type of a given player with strictly more than her outside option, the mediator is able to decrease the shares of this player, as long they are strictly positive. The imposed slack on resource feasibility can be used to decrease the probability that the contest occurs. Moreover, as at least one player's low-cost type participation constraints is binding, the same is true for high-cost types' incentive compatibility constraints. If a high-cost type strictly prefers not to jeopardize a low-cost type, the mediator is able to increase the share, and in turn the payoff, of any low-cost type. As implication, the contest's occurrence probability can be decreased. The next lemma summarizes these observations.

Lemma 3. *Suppose Assumption 1 holds. If $\mathcal{X}$ is a solution to problem $(P1)$, then $\Pi_i(h,h) = \Pi_i(h,l)$ $\forall i$. Moreover, $\Pi_i(l) = V_i(l)$ for at least one $i \in \{1,2\}$. If $z_i(l) > 0$ $\forall i$ or $z_i(l) = 0$ $\forall i$, then $\Pi_i(l) = V_i(l)$ $\forall i,$. If $z_{-i}(l) = 0$ and $z_i(l) > 0$, then $\Pi_i(l) = V_i(l)$. Moreover, $x_1(c_1,c_2) + x_2(c_1,c_2) = 1$.*

In light of the first assertion of Lemma 3, the fact that a solution to problem $(P1)$ requires shares to sum to one is a consequence form imposing the participation constraints at the interim level.

Problem $(P1)$ does not exhaust in the analogy of standard good allocation problems.

A player's payoff induced by the on-path play of the contest is not exogenously given. Instead it depends on how the contest is played, i.e. the chosen randomization over bids, which in turn depends on the player's information when deciding about the randomization. This information results from mediator's protocol, in particular from the implemented γ_i. As a consequence, player's payoff received from the contest depends on her report. By the features of an all pay auction, the following statement holds true for any (monotonic) information structure:

Lemma 4. *Fix a feasible $\mathcal{X}$. The following is true:*

$$U_{-i}(l|h) = U_i(l|l) = U_i(l|h) = U_{-i}(l|l) > \max\{U_i(h|l), U_i(h|h)\}.$$

Suppose furthermore that $\gamma(h,h)$ is sufficiently small, such that $U_i(h|h) = 0$. Then

$$U_i(h|l) \geq U_i(h|h).$$

According to Lemma 4 low-cost type's utility does not depend on the report. In contrast, a high-cost type might prefer to play the contest under the information structure which results from reporting to be the low-cost type. Moreover, a low-cost type strictly receives higher payoff from the play of the contest, than a high-cost type.

One might interpret the type (and report) dependent payoffs as screening parameter of the model. To screen players types, the payoff difference $U_i(k|m) - U_i(k|m)$ in case of submitting the same report is relevant. This difference depends on the information structure, that is on the relative fractions of G and thus $P(l)$.

Using the constraint Equations (IC_1^+) to (IC_2^-) together with the optimality conditions (Lemma 3) and the fact that low-cost on path contest utilities need to be the same (Lemma 4) one can rewrite the incentive compatibility constraints for player i as

$$(\gamma_i(l) - \gamma_i(h))U_i(l|l) \geq z(h) - z(l) = \gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h) \tag{2}$$

Both the very left as well as the very right expression are homogeneous of degree 1 as $U(\cdot|\cdot)$ depends only on the information structure and $\gamma_i(\cdot)$ is homogeneous of degree 1. Thus, incentive compatibility is completely pinned down by the *relative* contest probabilities, as scaling G has no effect on incentive compatibility.

As a consequence, we can describe an optimal solution to problem $(P1)$, by characterizing the aligned optimal information structure. Ignoring for the moment the costs an information structure implies on designer's objective, which depend on the ex ante probabilities ρ_i^0, an optimal information structure is subject to the following considerations:

To assure low costs types participation, given high-cost types incentive to jeopardize the former, a large screening parameter $U_i(l|l) - U_i(h|l)$ is desired. Using the constraints (IC_i^+) together with the optimality conditions (Lemma 3), low-cost types payoff obeys the following representation:

$$\Pi_i(l) = \gamma_i(l)(U_i(l|l) - U_i(h|l)) + z_i(h) + \gamma_i(h)U_i(h|h) \tag{3}$$

Indeed, from type high's binding incentive compatibility constraint, it follows that the higher the screening parameter, the lower $z_i(h) - z_i(l)$, and therefore, the higher $z_i(l)$.

In addition, a low $\gamma_i(h)U_i(h|h)$ is desired. Increasing $U_i(h|h)$ comes at the cost of increasing $\gamma(h, h)$. Increasing the likelihood of the event that the prize is allocated between high types in costly litigation, leaves a high-cost type with lower payoff than allocating the prize without litigation. That is, via feasibility, the lower $\gamma(h, h)$, the higher $z_i(h)$ can be chosen.

The last consideration suggests that an optimal information structure should feature a comparably low $\gamma(h, h)$ and thus a high $p(l_i|h_{-i})$. Under a small choice of $\gamma(h, h)$, one can distinguish information structures according to three cases. Suppose that $p(l_i|l_{-i}) \geq p(l_{-i}|l_i)$, then:

$$\begin{aligned}
\tilde{\mathcal{A}} &:= \{p(l_{-i}|l_i) \geq 1 - rc_h\}\,, \\
\tilde{\mathcal{B}} &:= \{1 - rc_h > p(l_{-i}|l_i) \geq (1 - rc_h)p(l_i|l_{-i})\}\,, \\
\tilde{\mathcal{C}} &:= \{(1 - rc_h)p(l_i|l_{-i}) \geq p(l_{-i}|l_i)\}\,.
\end{aligned}$$

When confining our attention to information structures that induce a monotonic equilibrium, i.e. information structures that satisfy condition (M), that is $p(l_{-i}|l_i)/p(l_{-i}|h_i) > c_l/c_h$, there are these three regions in which the equilibrium can fall into. The next lemma which follows from the calculation of the monotonic equilibrium describes how the difference $U_i(l|l) - U_i(h|l)$ changes across the regions. A more detailed version of the lemma can be found in Appendix A.6.

Lemma 5. *Suppose $p(l_i|l_{-i}) \geq p(l_{-i}|l_i)$ and let*

$$\Delta := \frac{1}{ch}(\left(1 - \frac{p(l_i|h_{-i})}{p(l_i|l_{-i})}p(l_{-i}|l_i)\right).$$

Further, assume an monotonic equilibria exists, then

If *$I \in \tilde{\mathcal{A}}$, the expected deviation payoffs of each player's high type are given by*

$$U_i(h|l) = U_{-i}(h|l) = 0.$$

The screening parameter is given by

$$U_i(l|l) - U_{-i}(h|l) = 1 - p(l_{-i}|l_i) - rc_l < r(c_h - c_l)$$
$$U_{-i}(l|l) - U_{-i}(h|l) = 1 - p(l_{-i}|l_i) - rc_l < r(c_h - c_l).$$

If $I \in \tilde{\mathcal{B}}$***,*** *the expected deviation payoffs of each player's high type are given by*

$$U_{-i}(h|l) = U_i(h|l) = U_i(l|l) - r(c_h - c_l) > 0.$$

The screening parameter is given by

$$U_i(l|l) - U_i(h|l) = r(c_h - c_l)$$
$$U_{-i}(l|l) - U_{-i}(h|l) = r(c_h - c_l).$$

If $I \in \tilde{\mathcal{C}}$***,*** *the expected deviation payoffs of each player's high type are given by*

$$U_i(h|l) = U_i(l|l) - r(c_h - c_l) > 0.$$
$$U_{-i}(l|l) - r(c_h - c_l) > U_{-i}(h|l) = 1 - p(l_{-i}|l_i) - (\Delta - r)\frac{p(l_i|l_{-i})}{p(l_i|h_{-i})}c_h - rc_h > 0$$

The screening parameter is given by

$$U_i(l|l) - U_i(h|l) = r(c_h - c_l)$$
$$U_{-i}(l|l) - U_{-i}(h|l) > r(c_h - c_l).$$

Note, although two different information structures might give rise to the same screening parameter $U_i(l|l) - U_i(h|l)$, they still differ in terms of the implied γ_i.

As can be seen from Lemma 5, $U_i(h|l)$ might be positive if $p(l_i|h_{-i})$ is sufficiently high. Clearly, if $p(l_i|h_{-i})$ is close to one, it follows that a low-cost type rationally expects to play the contest against a high-cost type with larger probability than a high-cost type expects to face a high-cost type as opponent. Thus, the latter prefers to play the contest under the (rational) beliefs following a low-cost report, than under truthful reporting. In other words, the mediator faces a potential trade off, as both $U_i(l|l)$ and $U_i(h|l)$ rise. This observation is summarized in the next two lemmas:

Lemma 6. *Consider a mediation protocol $\mathcal{X}$ that yields an on-path monotonic equilibrium. Assuming the on path utility of player i, c_h, $U_i(h|h)$, as well as the probability of meeting a low-cost type in the contest, $p(l_{-i}|l_i)$, is strictly positive. the following two statements are equivalent.*

- $U_i(h|l) > U_i(h|h)$ *and*
- $p(l_{-i}|l_i) < p(l_{-i}|h_i)$

Lemma 7. *Consider a mediation protocol $\mathcal{X}$ that yields an on-path monotonic equilibrium. Then, $p(l_{-i}|l_i) < p(l_{-i}|h_i)$ is a necessary condition for $U_i(h|l) > U_i(h|h)$.*

If $\gamma(h,h)$ is sufficiently small, Lemma 6 and Lemma 7 imply $U_i(h|l) \geq U_i(h|h)$. By Lemma 4 we know in addition that $U_i(l|l) = U_i(l|h)$. This single crossing condition property requires an incentive compatible solution to problem $(P1)$ to satisfy $\gamma_i(l) \geq \gamma_i(h)$.

Lemma 8. *Suppose $\mathcal{X}$ is a solution to problem $(P1)$ such that $U_i(h|l) \geq U_i(h|h)$. Then $\gamma_i(l) \geq \gamma_i(h)$. Moreover, $z_i(h) \geq z_i(l)$, with equality if $\gamma_i(l) = \gamma_i(h)$.*

The last assertion of Lemma 8 follows from low cost's binding participation constraints, i.e. Lemma 3.

An additional limit to the choice of G is introduced by the following lemma

Lemma 9. *If $\mathcal{X}$ is an optimal mediation protocol, then (any of) the on-path contest utility of player i, type c_l, i.e. $U_i(l|l)$, must be at least as large as the highest payoff received in the default game under priors.*

The intuition is straightforward and follows directly from homogeneity of $U(\cdot|\cdot)$ in $p(\cdot|\cdot)$. If $U_i(l|l)$ is lower than the payoff the player receives in the default game, we would always want to scale down any feasible solution by multiplying G with $a \longrightarrow 0$. This imposes slack on the participation constraints and cannot be optimal. Instead the optimal information structure must be such that $U_i(l|l)$ is large enough, or in other words, there always is a desire to lower $\gamma(h,h)$ somewhat.

Up to this point we were ignorant about the direct effects of an information structure on the designer's objective in a sense that we treated all elements of G equally. This is however not true, as we know from Lemma 2 that $p_2^0 < 0.5$ and the situation is therefore not entirely symmetric. In fact Equation (P1) depends besides G also on the ex-ante probabilities ρ_i^0 which provide a weighting of the different elements in G. As we assumed $p_2^0 < p_1^0$ we know that for example that

$$(1-p_2^0)p_1^0\gamma(l,h) > \max\{(1-p_1^0)p_2^0\gamma(h,l), \;\; p_1^0 p_2^0 \gamma(l,l)\}.$$

This means the mediator always prefers the reduction of $\gamma(l,h)$ by one increment to that of $\gamma(l,l)$ or $\gamma(h,l)$.

Overall it is ρ_i^0, that pins down the optimal information structure $P(l)$. If the optimal information structure was such that the low-cost incentive compatibility constraints are non-binding for both players, then $\gamma(h,h) = 0$ is true for this information structure as well. The reason is, that in any environment in which $\gamma(h,h) > 0$ and the low-cost incentive compatibility is not binding, reducing $\gamma(h,h)$ down to 0 comes at no cost on the constraints and is to the benefit of the objective. The more interesting case is, if some player's low-cost types incentive constraint is binding. In such a situation, the effect on low type's deviation payoff is important, as this imposes restrictions on γ_i by Lemma 8.

Making use of Lemma 4, if for player i both (IC_i^-) and (IC_i^+) bind, we can rewrite Equation (2) as:

$$\gamma_i(l)(U_i(l|l) - U_i(h|l)) = \gamma_i(h)U_i(l|h) = \gamma_i(h)U_i(l|l) \tag{4}$$

Equation (4) allows for a positive local effect of having a small but positive $\gamma(h,h)$. To see this, note that a small increase implies a high-cost player expecting to play the contest more often against another high-cost type and hence bidding more aggressively. If the optimal information structure, given $\gamma(h,h) = 0$ lies in region $\tilde{\mathcal{C}}$, this has an effect on $U_i(l|l)$ and $U_i(h|l)$. Both functions decrease by the same amount for small increases of $\gamma(h,h)$. As a result the left hand side of Equation (4) remains constant. If in addition $\gamma_i(h)U_i(l|l)$ decreases, a small increase $\gamma(h,h)$ might be desired in terms of the slack it implies on a low-cost types incentive compatibility constraint.

However, if the optimal information structure given $\gamma(h,h) = 0$ is in $\tilde{\mathcal{A}}$ or $\tilde{\mathcal{B}}$, it is also optimal without the restriction on $\gamma(h,h)$. The reason for this is that the $U(l|l)$ is not depending on $\gamma(h,h)$ in such a case.

6. Discussion of the Results

We have modeled mediation in court as an uninformed third party that has the ability to offer contestants shares of the good if they decide to settle the dispute outside the court. Contestants always have the ability to refuse mediation and force a court decision. Trials, however, are modeled as costly all-pay contests. Contestants posses private information about their constant marginal cost of evidence production and are heterogeneous with respect to the distribution their costs are drawn from.

As contestants are only heterogeneous in events that end up in front courts, they might refuse to participate in mediation if they are not paid a high enough rent by the mediator. This participation constraint, the mediator faces can in the first place be mitigated by the instrument of informational punishment. That is, if the parties cannot commit to ignore information sent by the mediator, the mediator can use the kink in the strong players utility function to reduce her outside option by threatening her to send her distorted information about her opponent. While it is interim rational to listen to the information provided by the mediator, the low-cost type can suffer ex-ante. We provide conditions under which informational punishment is possible and quantify the magnitude with which the ex-ante strongest player is punished. We show further, that punishment of the ex-ante weaker player is not possible. As participation constraints bind under optimal mediation this delivers our first result namely that the ex-ante strongest player might be strictly worse off if our type of mediation is institutionalized as she is the only one eligible to informational punishment.

Second, we characterize several aspects of optimal mediation in cases in which the mediator can not implement the pooling solution, i.e. a solution in which each player and type receives half the share. A necessary condition for such a case is both that the spread between low and high cost of evidence production $c_h - c_l$ is big enough, as well as that the high-cost type is sufficiently likely for the weaker player, that is $p_2 < 0.5$.

We show that the high-cost type's incentive constraint as well as the participation constraint of the low-cost type is binding at the optimum. Further we distinguish between the *information structure* which describes only the relative relation within the on-path litigation probability matrix G and the *scale* of the information structure which describes the absolute value of those variables.

We show that incentive compatibility is entirely pinned down by the information structure, as it is driven by the difference between on and off-path contest utilities. As low-cost types receive the same utility on and off path, we can show that the choice of the information structure is heavily influence by the nature of $U_i(l|l) - U_i(h|l)$ that is the difference between the contest utility of the low-cost type and the contest utility of the high-cost type pretending to be a low-cost type.

We strongly conjecture that the optimal mediation mechanism involves the probability of litigation given to high-cost reports, $\gamma(h, h) = 0$. However so far we are only able to show that an information structure involving $\gamma(h, h) = 0$ constitutes a local optimum if the mediator offers an optimal protocol in which the information set is such that the probability of meeting a low-cost type when being a low-cost type is sufficiently high, that is if either the two subjective probabilities are close or the lower of the two is larger than the threshold value $1 - rc_h$. If these probabilities are too different between the two, that is if priors differ too much for low-cost types no clear statement can yet be made. Interestingly the screening parameter $U_i(l|l) - U_i(h|l)$ exhibits its largest value in this area indicating a trade-off between a slight increase of $\gamma(h, h)$ (which is costly to the objective) and an decrease in $U_i(l|l) - U_i(h|l)$ (which is costly for the screening purpose). We conjecture the solution to this trade-off lies within the relative cost of the elements of G, but are not yet able to prove a definite result.

Moreover, so far we have been looking at monotonic information structures only. Although we strongly conjecture that an optimal information structure satisfies monotonicity, we have not proven this claim yet.

Our findings are in line with the idea of "alternative dispute resolution", that is to mitigate legal expenditures. We can show that the amount of court cases drops if mediation is in place. Second, we show that the information the mediator gathers along the process is something it should use and communicate to use it, in order to reduce the amount of cases brought to court even further. Finally, we argue that ADR only works as a supplement to litigation and cannot replace it completely if the parties only differ in their cost of evidence production. The mediator's only screening tool is to "fail" ADR sometimes and refer the parties back to court.

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A. Proofs

A.1. Proof of Lemma 1

We prove here a larger lemma that also pins down the strategies of the players and is therefore stated here before the proof

Consider an all-pay contest with minimum score r, and an environment in which player i has marginal bidding cost $c_l < c_h$ with probability p_i. Let $\Delta_i := \frac{1-p_i}{c_h}$ and assume the commonly known information set I lies in $\mathcal{I}$. Then, the equilibrium takes the following form conditional on I:

Lemma 10. ***If*** *$I \in \mathcal{I}_0$,*

- *Player 1 and 2, type c_h, stay out of the auction,*
- *Player 1, type c_l, uniformly mixes on $(r, \frac{1}{c_l}]$ with density $\frac{c_l}{p_1}$ and bids r with probability $1 - \frac{1+rc_l}{p_1}$*
- *Player 2, type c_l, uniformly mixes on $(r, \frac{1}{c_l}]$ with density $\frac{c_l}{p_2}$ and stays out with probability $1 - \frac{1+rc_l}{p_2}$.*

The expected interim utilities of each player and type are 0.

If *$I \in \mathcal{I}_A$,*

- *Player 1 and 2, type c_h, stay out of the auction,*
- *Player 1, type c_l, uniformly mixes on $(r, \frac{p_2}{c_l} + r]$ with density $\frac{c_l}{p_1}$ and bids r with probability $1 - \frac{p_2}{p_1}$*
- *Player 2, type c_l, uniformly mixes on $(r, \frac{p_2}{c_l} + r]$ with density $\frac{c_l}{p_2}$.*

The expected interim utilities of each player and type are given by

$$U_1(l|I \in \mathcal{I}_A) = U_2(l|I \in \mathcal{I}_A) = 1 - rc_l - p_2$$
$$U_1(h|I \in \mathcal{I}_A) = U_2(h|I \in \mathcal{I}_A) = 0.$$

If *$I \in \mathcal{I}_B$,*

- *Player 1, type c_h, bids r*
- *Player 1, type c_l, uniformly mixes on $(r, \Delta_2]$ with density $\frac{c_h}{p_1}$, on $(\Delta_2, \Delta_2 + \frac{p_2}{c_l}]$ with density $\frac{c_l}{p_1}$ and bids r with probability $1 - \frac{1-rc_h}{p_1}$.*
- *Player 2, type c_h, uniformly mixes on $(r, \Delta_2]$ with density $\frac{c_l}{1-p_2}$ and stays out with probability $1 - \frac{c_l}{c_h}\left(1 - \frac{r}{\Delta_2}\right)$.*
- *Player 2, type c_l, uniformly mixes on $(\Delta_2, \Delta_2 + \frac{p_2}{c_l}]$ with density $\frac{c_l}{p_2}$.*

The expected interim utilities of each player and type are given by

$$U_1(l|I \in \mathcal{I}_B) = U_2(l|I \in \mathcal{I}_B) = \Delta_2(c_h - c_l)$$
$$U_1(h|I \in \mathcal{I}_B) = (\Delta_2 - r)(h - c_l)$$
$$U_2(h|I \in \mathcal{I}_B) = 0.$$

If $I \in \mathcal{I}_C$,

- *Player 1, type c_h, uniformly mixes on $(r, \Delta_1]$ with density $\frac{1}{\Delta_1}$ and bids r with probability $\frac{r}{\Delta_1}$*
- *Player 1, type c_l, uniformly mixes on $(\Delta_1, \Delta_2]$ with density $\frac{c_h}{p_i}$, on $(\Delta_2, \Delta_2 + \frac{p_2}{c_l}]$ with density $\frac{c_l}{p_1}$.*
- *Player 2, type c_h, uniformly mixes on $(r, \Delta_1]$ with density $\frac{1}{\Delta_2}$ on $(\Delta_1, \Delta_2]$ with density $\frac{c_l}{1-p_2}$ and stays out with probability $(\Delta_2 - \Delta_1)\frac{c_h - c_l}{(1-p_2)} + \frac{r}{\Delta_2}$.*
- *Player 2 type c_l, uniformly mixes on $(\Delta_2, \Delta_2 + \frac{p_2}{c_l}]$ with density $\frac{c_l}{p_2}$.*

The expected interim utilities of each player and type are given by

$$U_1(l|I \in \mathcal{I}_C) = U_2(l|I \in \mathcal{I}_C) = \Delta_2(h - c_l)$$
$$U_1(h|I \in \mathcal{I}_C) = (\Delta_2 - \Delta_1)(c_h - c_l)$$
$$U_2(h|I \in \mathcal{I}_C) = 0.$$

Proof. The equilibrium construction in each case follows essentially that of Siegel (2014a).

By proposition 2 in Siegel (2014a) it is without loss of generality (in terms of the outcome) to restrict ourselves to constructing one equilibrium as all equilibria are payoff equivalent.

Given the strategies of her opponent σ_2 (and the information structure I), player 1 , type c_l, chooses a score e that satisfies:

$$Pr'(e_1 > e_2|\sigma_2, I) - c = 0$$

And vice versa for player 2

Given this, strategies satisfy the local optimality condition for any information structure by construction.

Thus, what is left to prove is global optimality. This is done case by case:

Case 1: $I \in \mathcal{I}_A$

Global optimality follows from $p_1 \geq p_2 \geq 1 - rc_h$, :

If player 1, type c_h bids r, she receives payoff $1 - p_2 - rc_h < 0$. Similarly, if player 2, type c_h bids r, she receives payoff $1 - p_1 - rc_h < 0$.

Player 2, type c_l receives payoff $(1 - p_1) + (p_1 - p_2) - rc_l$ from bidding arbitrarily above r, which is the same when bidding until the top of the specified interval.

Case 2: $I \in \mathcal{I}_B$

Global optimality follows $p_1 \geq 1 - rc_h > p_2$:

If player 1, type c_h bids r, she receives payoff

$$\begin{aligned} U_1(h|I \in \mathcal{I}_B) = (1-p_2)\frac{(c_h - c_l)(1-p_2) + rc_hc_l}{c_h(1-p_2)} - rc_h &= \\ \frac{(c_h - c_l)(1-p_2) + rc_hc_l - rc_hc_h}{c_h} &= \\ \frac{(c_h - c_l)(1-p_2) - rc_h(c_h - c_l)}{c_h} &= \\ (c_h - c_l)(\Delta_2 - r) \end{aligned}$$

which is larger than 0. Bidding above $r+\epsilon$ instead of r increases player 1's probability to win by $(1-p_2)\frac{c_l}{1-p_2}\epsilon$ at the cost of $c_h\epsilon$, which is negative if $c_h > c_l$.

By construction, player 2, type c_h is indifferent between bidding arbitrarily larger than r and zero, since any score $k \in (r, \Delta_1)$ yields utility

$$\begin{aligned} U(k) &= (1-p_1) + p_1\left(\left(1 - \frac{1-rc_h}{p_1}\right) + (k-r)\frac{c_h}{p_1}\right) - kc_h \\ &= (1-p_1) + p_1 - (1-rc_h) + kc_h - rc_h - kc_h = 0 \end{aligned}$$

Player 1, type c_l receives payoff

$$(1-p_2)\frac{(c_h - c_l)(1-p_2) + rc_hc_l}{c_h(1-p_2)} - rc_l = \Delta_2(c_h - c_l)$$

from bidding r, which is the same when bidding until the top of the specified interval.

Player 2, type c_l receives payoff

$$(1-p_1) + p_1(1 - \frac{p_{-i}}{p_1}) - (\Delta_2)c_l = \Delta_2(c_h - c_l)$$

from bidding the lower bound of the specified interval. This is the same payoff he receives when bidding the upper bound of the specified interval.

Case 3: $I^i \in \mathcal{I}_C$

Global optimality follows $1 - rc_h > p_1 \geq p_2$:

If player i, type c_h bids r, she receives payoff

$$U_2(h|I \in \mathcal{I}_C) = (1-p_2)\frac{(c_h - c_l)(p_1 - p_3) + rc_h^2}{c_h(1-p_2)} - rc_h = (\Delta_2 - \Delta_1)(c_h - c_l)$$

which is larger than 0.

By construction, player 2, type c_h is indifferent between bidding arbitrarily larger than r and zero:

$$(1-p_1)\frac{rc_h}{1-p_1} - rc_h = 0$$

Player 1, type c_l receives payoff

$$(1-p_2)(1-(\frac{p_1-p_2}{c_h}\frac{c_l}{1-p_1})) - (\Delta_1)c_l = \Delta_2(c_h - c_l)$$

from bidding Δ_1, which is the same when bidding until the top of her specified interval.

Player 2, type c_l receives payoff

$$(1-p_1) + p_1(1-\frac{p_2}{c_l}\frac{c_l}{p_1}) - (\Delta_1 + \frac{p_1-p_2}{c_h})c_l = \Delta_2(c_h - c_l) \tag{5}$$

from bidding the lower bound of the specified interval. This is the same payoff she receives when bidding the upper bound of the specified interval.

□

A.2. Proof of Proposition 1

Proof. Fix some p_1^0 and notice as the information is unidimensional (and so is the image of the utility functions) it suffices to consider binary signals.

To proof the if part of the first claim, assume I is in $\mathcal{I}_A \cup \mathcal{I}_B$ and $p_1 > p_2 > 0$. Notice that given p_1^0 we are in region $\mathcal{I}_B$ for any $p_2 \in [0, 1-rc_h)$ and continue in region $\mathcal{I}_A$ as $p_2 \in [1-rc_h, p_1)$.

Thus, using Lemma 1

$$\frac{dU_1(l|I \in \mathcal{I}_B)}{dp_2} = -\frac{c_h - c_l}{c_h}$$
$$\frac{dU_1(l|I \in \mathcal{I}_A)}{dp_2} = -1.$$

as $\frac{c_h - c_l}{c_h} < 1$ by construction, the utility function is strictly concave around $1-rc_h$. Given this strict concavity and Jensen's inequality it is possible to construct a mean preserving spread such that $E[U_1(\cdot)] < U_1(E[\cdot])$ if we pick the posterior after one signal realisation to be to the left of $1-rc_h$ and the other one to the right provided we are not on the boundaries under the prior.

As for the only if part we can first exclude the case in which $p_2^0 = 0$, as there is no uncertainty about c_2 in the system in the first place. Second, suppose we are in $\mathcal{I}_C$. Keeping p_2 below p_1^0 is of no help as

$$\frac{dU_1(l|I \in \mathcal{I}_C)}{dp_2} = -\frac{c_h - c_l}{c_h}$$

and thus utility is linear within $\mathcal{I}_C$. Allowing p_2 to be larger p_1^0 for some signal realisation, however, can, by $p_1^0 < 1 - rc_h$, only put as in a region $\mathcal{I}_B^2$ which is similar to $\mathcal{I}_B$ only that $p_2 > p_1$. For any $p_2 > p_1^0$ the utility does not depend on p_2 any more and the derivative is thus, 0. Hence, the utility function is convex in p_2 around p_1^0 and no punishment is possible. As the utility is constant in p_2 for all $p_2 \geq p_1^0$, there is also no punishment possible at $p_2 = p_1^0$. The last region to check would be $\mathcal{I}_0$, but then the marginal utility is either constant in p_2 and the function is convex at the boundary to region A. Thus, no punishment can be achieved.

The minimum punishment utility $V_1(l)$ can be calculated by observing through *Lemma* 1 and the previous argument that the smallest utility of type l is obtained whenever $p_2 = p_1^0$. The largest utility is, by the same argument obtained for $p_2 = 0$. Thus, a signal structure that either releases a realization that leads to $p_2 = 0$ or a realization that leads to $p_2 = p_1^0$ is always going to lead to an before-signal-realization (but still interim) expected utility that is worse than any other combination, as it always lies on the convex hull of $U_1(p_2)$.

Finally, punishment for player 2, c_l (i.e. fixing p_2^0) is not possible as long as $p_1 > p_2^0$ as marginal utility is 0 for all regimes. Moving into region $p_1 < p_2$ and thus $\mathcal{I}_C^2$ is of no help either, as the function is convex around $\mathcal{I}_C$. Player 2, type c_h receives utility 0 in all regions and can therefore not be punished either. □

A.3. Proof of Lemma 2

Proof. Consider first a default game in region B or C as described in **??**. Then $V_2 = (1 - p_2)(c_h - c_l)$. By Assumption 1 $V_1 + V_2 > 1$ and together with the result from Proposition 1, that is $V_2 \geq V_1$, it must hold that $V_2 > \frac{1}{2}$. But then

$$\begin{aligned} & \frac{1}{2} < \frac{1-p_2}{c_h}(c_h - c_l) \\ \Leftrightarrow \quad & \frac{1}{2}c_h < (1-p_2)(c_h - c_l) \\ \Leftrightarrow \quad & p_2 < \frac{c_h - 2c_l}{2(c_h - c_l)}, \end{aligned}$$

which, as $0 \leq p_2 \leq 1$ can only be true if $c_h > 2c_l$ and more over since

$$p_2 < \frac{c_h - 2c_l}{2(c_h - c_l)} < \frac{c_h - c_l}{2(c_h - c_l)} = \frac{1}{2}$$

it also needs to hold that $p_2 < 1 - rc_h$.

Next consider prior beliefs such that we indeed find ourselves in region A. Recall from **??** as depicted in Figure 2 that region A involves a higher p_2 than region B for any given environment. Further notice since the slope of the utility as a function of p_2 is steeper in region A, that the hypothetical region B-like payoff, that is if we were just to continue the payoff function the low-cost type has in region B to region A, this payoff function would always lie above the actual payoff in region A. As we did not use the conditions on the primitives that guarantees we are in region B, we may use the hypothetical B-like payoff as an upper bound for region A and can claim, that the necessary conditions indeed carry over to region A.

Furthermore we know that since $1 - rc_h \leq p_2$ that

$$\frac{1}{2} < 1 - p_2 - rc_l \leq r(c_h - c_l) \tag{6}$$

and thus $c_h > \frac{1}{2r} + c_l$. □

A.4. Proof of Lemma 3

Proof. We first show, that it is without loss of generality to assume that (IC_i^+) is satisfied with equality at the optimum:

Suppose to the contrary that for at least one player i the following is true:

$$\Pi_i(h,h) > \Pi_i(h,l)$$

That is

$$z_i(h) - z_i(l) > \gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)$$

We can decrease $z_i(h)$ such that the above equation is satisfied with equality. This decrease does not alter the feasibility constraint, but - if anything - eases (IC_i^-).

Next, we want to show that player i's low cost types participation constraints binds at the optimum, if $z_i(l) > 0$ and the optimum is such that the contest occurs with strictly positive probability.

Afterwards, we verify that if an optimal solution is such that $z_i(l) = 0$ for $i = 1, 2$, then at least participation constraints binds.

Suppose to the contrary that for at least one player i the following is true:

$$\Pi_i(l,l) > V_i(l)$$

That is, the hypothetical optimum is such that

$$\Pi_1(l,l) = \gamma_1(l)U_1(l|l) + z_1(l) > V_1(l)$$

$$\Pi_2(l,l) = \gamma_2(l)U_2(l|l) + z_2(l) \geq V_2(l)$$

Moreover, using that (IC_i^-) is satisfied with equality, we know:

$$z_i(h) - z_i(l) = \gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(h|h) \tag{7}$$

To derive a contradiction, we argue that there exists a different allocation which implies a lower probability of the contest being played.

We first decrease $z_1(l)$ and $z_1(h)$ uniformly by a small amount, such that $\Pi_1(l,l) > V_1(l)$ is preserved. This decrease does not alter player i's incentive compatibility constraints.

The imposed slack of the decrease of $z_1(l), z_1(h)$ on the feasibility constraint, allows us to increase $z_2(l)$ and $z_2(h)$ uniformly by an small positive amount. This increase does not alter player -i's incentive compatibility constraints and implies $\Pi_2(l,l) > V_2(l)$.

Note, as the information structure is homogeneous in G, we can scale down G without changing the information structure.

We scale G by $\alpha < 1$, but arbitrary close to 1. As consequence, the probability with which the contest occurs decreases.

We show the existence of shares $\tilde{z}_i(\cdot)$, such that incentive compatibility and participation constraints are satisfied.

For any player i, we have to distinguished two possible cases:

- Case 1: $(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) > 0$
- Case 2: $(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) < 0$

Consider first case 1.

We hold fix $z_i(l)$, i.e. $\tilde{z}_i(l) = z_i(l)$ and choose

$$\tilde{z}_i(h) = \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) + z_i(l) \tag{8}$$

First note, for any $\alpha > 0$, (IC_i^-) and (IC_i^+) imply that an allocation is incentive compatibility if and only if it satisfies:

$$\alpha(\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) + z_i(l) \geq \tilde{z}_i(h) \geq \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) + z_i(l)$$

That is, $\tilde{z}_i(h) - z_i(l)$ must satisfy:

$$\begin{aligned}
\alpha(\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) \geq \tilde{z}_i(h) - z_i(l) &\geq \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) \\
\iff \alpha(\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) &\geq \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) \\
\iff (\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) &\geq (\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)),
\end{aligned}$$

where the second equation follows from the choice of $\tilde{z}_i(\cdot)$ and the last equation follows from the assumption that we started with an incentive compatible allocation.

Moreover, from Equation (7) and Equation (8)

$$z_i(h) - \tilde{z}_i(h) = (1-\alpha)(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h) > 0,$$

as $\alpha < 1$ and $(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) > 0$ by assumption. Because the occurrence of war decreased, the choice of $\tilde{z}_i(h), \tilde{z}_i(l) = z_i(l)$ is feasible.

Moreover, by the choice of $\tilde{z}_i(l)$, low cost's payoff reads:

$$\alpha U_i(l|l) + z_i(l) > V_i(l),$$

where the last inequality follows from the choice of α and the hypothesis that player i's participation constraint was satisfied with strict inequality.

Now consider case 2.

We hold fix $z_i(h)$, i.e. $\tilde{z}_i(h) = z_i(h)$ and choose

$$\tilde{z}_i(l) = -\alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) + z_i(h) \tag{9}$$

Similar to case 1, $z_i(h) - \tilde{z}_i(l)$ must satisfy:

$$\begin{aligned} \alpha(\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) \geq z_i(h) - \tilde{z}_i(l) \geq \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) \\ \iff \alpha(\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) \geq \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) \\ \iff (\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h)) \geq (\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)), \end{aligned}$$

where the second equation follows from the choice of $\tilde{z}_i(\cdot)$ and the last equation follows from the assumption that we started with an incentive compatible allocation.

From Equation (7) and Equation (9)

$$z_i(l) - \tilde{z}_i(l) = -(1-\alpha)\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h))) > 0,$$

as $\alpha < 1$ and $(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) < 0$ by assumption. Because the occurrence of war decreased, the choice of $\tilde{z}_i(h) = z_i(h), \tilde{z}_i(l)$ is feasible.

Moreover, by the choice of $\tilde{z}_i(l)$, low cost's payoff reads:

$$\alpha U_i(l|l) - \alpha(\gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)) + z_i(h) > V_i(l),$$

where the last inequality holds by the choice of α, Equation (7) and the assumption that player i's participation constraint was satisfied with strict inequality, i.e.:

$$\Pi_i(l) = \gamma_i(l)U_i(l|l) - \gamma_i(l)U_i(h|l) + \gamma_i(h)U_i(h|h) + z_i(h) > V_i(l)$$

Finally, we want to show that for at least one player i, (PC_i) must bind in optimum.

Given the above argument, we can assume without loss of generality that $z_i(l) = 0$ for $i = 1, 2$. Hence:

$$\Pi_i(l,l) = \gamma_i(l)U_i(l|l) > V_i(l)$$

Making use of the homogeneity of G, there exists $\alpha < 1$, but arbitrary close to 1, such that the information structure αG implies low-cost type's the payoff:

$$\alpha\gamma_i(l)U_i(l|l) > V_i(l),$$

and satisfies incentive compatibility. □

A.5. Proof of Lemma 4

Proof. Let us first prove $U_i(l|l) = U_i(l|h)$.

Suppose that low-cost type player i reports c_h. In case the contest occurs, -i type c_l's probability assessment on the types of player i are common knowledge. Hence, also player -i type c_l's' strategy is common knowledge. By submitting an arbitrary higher bid than -i type low cost's largest equilibrium bid, player i type low wins the prize with certainty. Since player i high-cost type plays a mixed strategy and has the same probability assessment on the types of player -i as low-cost type i, the described action is indeed optimal. Moreover, since a bid which makes -i type low winning the contest with certainty, is in the support of its equilibrium strategy, it follows that $U_{-i}(l|l) = U_i(l|h)$.

Now consider type high of player i. Suppose that $\gamma(h,h)$ is sufficiently small, such that $p(h_{-i}|h_i) \leq 1 - rc_h$. In this instance, player i type high receives zero payoff from playing the contest on path. As direct consequence $U_i(h|l) \geq U_i(h|h)$. □

A.6. Proof of Lemma 5

As in Appendix A.1 we prove a lemma that contains the statement from Lemma 5 instead.

Lemma 11. *Assume that $\gamma(h,h) = 0$, i.e. $p(l_i|h_{-i}) = 1$ and that monotonicity is satisfied, i.e. $p(l_i|h_{-i})\frac{c_l}{p(l_i|l_{-i})} \leq c_h$. Let $\Delta := \frac{1-p(l_i|h_{-i})\frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}}{c_h}$ and assume the commonly known information set I lies in $\mathcal{I}$. Then, the equilibrium takes the following form conditional on I:*

If *$I \in \tilde{\mathcal{C}}$*

- *Player i, type c_h, stays out of the auction,*
- *Player i, type c_l, uniformly mixes on $(r, \Delta]$ with density $\frac{c_h}{p(l_i|h_{-i})}$, and on $(\Delta, \frac{p(l_{-i}|l_i)}{c_l} + \Delta]$ with density $\frac{c_l}{p(l_i|l_{-i})}$ and bids r with probability $1 - (\Delta - r)\frac{c_h}{p(l_i|h_{-i})} - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}$*
- *Player -i, type c_h, uniformly mixes on $(r, \Delta]$ with density $\frac{c_l}{p(h_{-i}|l_i)}$ and stays out with probability $1 - (\Delta - r)\frac{c_l}{p(h_{-i}|l_i)}$*
- *Player 2, type c_l, uniformly mixes on $(\Delta, \frac{p(l_{-i}|l_i)}{c_l} + \Delta]$ with density $\frac{c_l}{p(l_{-i}|l_i)}$.*

The expected interim utilities of each player and type are given by

$$U_i(l|l) = U_{-i}(l|l) = 1 - p(l_{-i}|l_i) - \Delta c_l$$
$$U_i(h|h) = U_{-i}(h|h) = 0.$$

If $I \in \tilde{\mathcal{B}}, \tilde{\mathcal{A}}$,

- *Player i and -i, type* c_h*, stay out of the auction,*
- *Player i, type* c_l*, uniformly mixes on* $(r, \frac{p(l_{-i}|l_i)}{c_l} + r]$ *with density* $\frac{c_l}{p(l_i|l_{-i})}$ *and bids* r *with probability* $1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}$*,*
- *Player 2, type* c_l*, uniformly mixes on* $(r, \frac{p(l_{-i}|l_i)}{c_l} + r]$ *with density* $\frac{c_l}{p(l_{-i}|l_i)}$*.*

The expected interim utilities of each player and type are given by

$$U_i(l|l) = U_{-i}(l|l) = 1 - p(l_{-i}|l_i) - rc_l$$
$$U_i(h|c_h) = U_{-i}(h|h) = 0.$$

Proof. The equilibrium construction is similar to the one in Lemma 1. By construction, strategies satisfy the local optimality condition for any information structure. As we assume the existence of an monotonic equilibrium, it is left to prove that a type of a given player is indifferent between choosing the lowest and the highest bid from his mixing interval. This is done case by case:

Case 1: $I \in \tilde{\mathcal{C}}$

First note that $\Delta - r \geq 0$, as

$$1 - rc_h \geq p(l_i|h_{-i})\frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}$$

and $p(l_i|h_{-i}) = 1$.

If player i, type c_h bids r, she receives payoff $-rc_h < 0$. If player -i, type c_h bids r, she receives payoff

$$(1 - p(l_i|h_{-i})) + p(l_i|h_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})} - (\Delta - r)\frac{c_h}{p(l_i|h_{-i})}) - rc_h = 0.$$

If player -i, type c_h bids Δ, she receives payoff

$$(1 - p(l_i|h_{-i})) + p(l_i|h_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}) - \Delta c_h = 0.$$

If player i, type c_l bids r, she receives payoff

$$(1 - p(l_{-i}|l_i))(1 - (\Delta - r)\frac{c_l}{p(h_{-i}|l_i)}) - rc_l = 1 - p(l_{-i}|l_i) - \Delta c_l$$

If player i or -i, type c_l bids $\Delta + \frac{p(l_{-i}|l_i)}{c_l}$, she receives payoff $1 - p(l_{-i}|l_i) - \Delta c_l$. If player -i, type c_l bids Δ, she receives payoff

$$1 - p(l_i|l_{-i}) + p(l_i|l_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}) - \Delta c_l = 1 - p(l_{-i}|l_i) - \Delta c_l$$

.

Moreover, if player -i, type c_h reports to be type c_l and bids r, she receives payoff

$$(1 - p(l_i|l_{-i})) + p(l_i|l_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})} - (\Delta - r)\frac{c_h}{p(l_i|h_{-i})}) - rc_h$$

$$= 1 - p(l_{-i}|l_i) - (\Delta - r)\frac{p(l_i|l_{-i})}{p(l_i|h_{-i})}c_h - rc_h$$

Case 2: $I \in \tilde{\mathcal{B}}, \tilde{\mathcal{A}}$

If player i, type c_h bids r, she receives payoff $-rc_h < 0$. If player -i, type c_h bids r, she receives payoff

$$p(l_i|h_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}) - rc_h \leq 0.$$

If player i, type c_l bids r, she receives payoff

$$1 - p(l_{-i}|l_i) - rc_l.$$

If player -i, type c_l bids r, she receives payoff

$$(1 - p(l_i|l_{-i})) + p(l_i|l_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}) - rc_l = 1 - p(l_{-i}|l_i) - rc_l.$$

If player i or -i, type c_l bids $\frac{p(l_{-i}|l_i)}{c_l} + r$, she receives payoff

$$1 - p(l_{-i}|l_i) - rc_l.$$

Moreover, if player -i, type c_h reports to be type c_l and submits bid r, she receives payoff

$$1 - p(l_i|l_{-i}) + p(l_i|l_{-i})(1 - \frac{p(l_{-i}|l_i)}{p(l_i|l_{-i})}) - rc_h = 1 - rc_h - p(l_{-i}|l_i).$$

This quantity is positive if $\tilde{\mathcal{B}}$ applies, i.e. $p(l_{-i}|l_i) < 1 - rc_h$, and negative if $\tilde{\mathcal{A}}$ applies.

If player -i, type c_h reports to be type c_l and submits bid r, she receives payoff $(1 - p(l_{-i}|l_i)) - rc_h$

□

A.7. Proof of Lemma 6

To proof th lemma, help of an additional lemma is needed which is first provided

Lemma 12. *Consider a mediation protocol $\mathcal{X}$ that provides an on-path monotonic equilibrium such that $p(l_1|l_2) > p(l_2|l_1)$. Define*[23] $\underline{b}_{i,l} = \min\{b_i(c_l)\}$*. Consider now player i, type c_h who miss-reports it's type (i.e. files report c_l). Then the off-path best response never exceeds the on-path maximum bid.*

Proof. First, recognize that $\underline{b}_{1,l} < \underline{b}_{2,l}$ follows by construction from $p(l_1|l_2) > p(l_2|l_1)$ (proof is a direct application of the construction made in Siegel (2014a)).

Second, observe that player i, type c_h never wants to bid above $\underline{b}_{i,l}$ if deviating, as player i, type c_l is actually indifferent between $\underline{b}_{i,l}$ and $\overline{b}_{i,l} = max\{b_i(c_l)\}$ again by construction of the equilibrium.

Thus, it must hold that the utility of bid $\underline{b}_{i,l}$ given truthful (type-report) c_l is

$$U_i(b_i(c_l) = \underline{b}_{i,l}|c_l) = P(win|\underline{b}_{i,l}) - \underline{b}_{i,l}c_l = 1 - \overline{b}_i c_l = U_i(b_i(c_l) = \overline{b}_{i,l}|c_l).$$

But then, for type c_h, given (miss-)report c_l it holds that

$$\begin{aligned} U_i(b_i(c_h) = \underline{b}_{2,l}|c_l) &= P(win|\underline{b}_{i,l}) - \underline{b}_{i,l}c_h \\ &= P(win|\underline{b}_{i,l}) - \underline{b}_{i,l}c_l - \underline{b}_{i,l}(c_h - c_l) \\ &= 1 - \overline{b}_{i,l}c_l - \underline{b}_{i,l}(c_h - c_l) \\ &> 1 - \overline{b}_{i,l}c_l - \overline{b}_{i,l}(c_h - c_l) \\ &= U(b_2(c_h) = \overline{b}_{i,l}|c_l). \end{aligned}$$

Thus, he never chooses a bid larger than $\underline{b}_{i,l}$. □

Now we are ready to proof Lemma 6 itself

Proof. If the on-path utility of player i is indeed positive, she also makes a strictly positive bid on path. Further, recall from Lemma 12 that the off-path best response is never larger than the on-path maximum bid, given the on path bid is strictly positive. Now, take any bid $b > 0$ that is an off-path best response of player i and compare off-path and on-path utility

[23] Ignoring openness problems.

$$
\begin{aligned}
U_i(h|l) &\geq U_i(h|h) \\
p(h_{-i}|l_i)F^h_{b_{-i}}(b) + p(l_{-i}|l_i)F^l_{b_{-i}}(b) &> p(h_{-i}|h_i)F^h_{b_{-i}}(b) + p(l_{-i}|h_i)F^l_{b_{-i}}(b) \\
\Big(p(h_{-i}|l_i) - p(h_{-i}|h_i)\Big)F^h_{b_{-i}}(b) &> \Big(p(l_{-i}|h_i) - p(l_{-i}|l_i)\Big)F^l_{b_{-i}}(b) \\
\Big(p(l_{-i}|h_i) - p(l_{-i}|l_i)\Big)F^h_{b_{-i}}(b) &> \Big(p(l_{-i}|h_i) - p(l_{-i}|l_i)\Big)F^l_{b_{-i}}(b)
\end{aligned}
$$

As $F^h_{b_{-i}}(b) \geq F^l_{b_{-i}}(b)$ by the nature of a monotonic equilibrium. If $p(l_{-i}|l_i) > 0$ then any bid $\tilde{b}$ that is a best response must be (strictly) smaller than the low-cost types maximum bid $b < \bar{b}_{l,2}$. Thus $F^h_{b_{-i}}(b)(\tilde{b}) > F^l_{b_{-i}}(b)(\tilde{b})$ which concludes the proof. □

A.8. Proof of Lemma 7

Proof. By Lemma 6 the claim holds if both $U_i(h|h)$ and $p(l_{-i}|h_i) > 0$. If $U_i(h|h) = 0$ then $U_i(h|l) \geq U_i(h|h)$ follows trivially. However if $p(l_{-i}|l_i) \geq p(l_{-i}|h_i)$, then the probability of meeting a low-cost type in the contest is larger off-path then on-path. If the player experience 0 utility on path in expectations, she must have been either been indifferent between staying out and competing against a low-cost type or have strictly preferred to stay out. As the strategy of the low cost player doesn't change, but is more likely to be present, the players utility cannot increase.

Finally, consider the case in which $p(l_{-i}|l_i) = 0$. We only need to check that with $p(l_{-i}|h_i) = 0$ it holds that $U_i(h|l) \leq U_i(h|h)$. As in such a case the probability of meeting a high type is 1 no matter what the report is, the opponents high-cost type must play the same strategies in both scenarios. Thus, utilities must be the same. □

A.9. Proof of Lemma 8

Proof. Suppose $\mathcal{X}$ is a solution to problem (P1). By Lemma 3 we know that

$$\Pi_i(h,h) = z_i(h) + \gamma_i(h)U_i(h|h) = z_i(l) + \gamma_i(l)U_i(h|l) = \Pi_i(h,l)$$

$$\iff z_i(h) - z_i(l) = \gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)$$

By Equations (IC_1^-) and (IC_2^-)

$$\Pi_i(l,l) = z_i(l) + \gamma_i(l)U_i(l|l) \geq z_i(h) + \gamma_i(h)U_i(l|h) = \Pi_i(l,h)$$

$$\iff \gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h) \geq z_i(h) - z_i(l)$$

Hence,

$$\gamma_i(l)U_i(l|l) - \gamma_i(h)U_i(l|h) \geq \gamma_i(l)U_i(h|l) - \gamma_i(h)U_i(h|h)$$

$$\iff \gamma_i(l)(U_i(l|l) - U_i(h|l)) \geq \gamma_i(h)(U_i(l|h) - U_i(h|h))$$

Since $0 < U_i(l|l) - U_i(h|l) \leq U_i(l|h) - U_i(h|h)$ by Lemma 4 and $U_i(h|l) \geq U_i(h|h)$, it follows that $\gamma_i(l) \geq \gamma_i(h)$ and therefore $z_i(h) \geq z_i(l)$, with equality if $\gamma_i(h) = \gamma_i(l)$. □

A.10. Proof of Lemma 9

Proof. First notice that $U_i(l|l)$ is the same for any i by Lemma 4. Further notice that by Proposition 1, player 2 cannot be punished in the default game. Thus it is without loss of generality to concentrate on $U_2(l|l)$ and V_2.

Now suppose that $U_2(l|l) < V_2$, but $\mathcal{X}$ is an optimal solution. As we know from lemma 3, the participation constraint must be binding for player 2's low-cost type as $z_2(l) > 0$. It is given by

$$z_2(l) + \gamma_2(l)U_2(l|l) = V_2. \tag{PC}$$

Thus $x_2(l) = \frac{z_2(l)}{(1-\gamma_2(l))} > V_2$ for (PC) to hold. Incentive compatibility is pinned down by the information structure, and must hold as we are in an optimal solution. But, then we may multiply G with a scalar $\alpha < 1$ which keeps IC's fulfilled. But $x_2(l)$ can be held constant as well, as any additional instance in which the contest is not played on path under the new regime, the money on the table increases, as the contest can never payoff more than 1 in total to both contestants while without a contest exactly 1 is divided among the two. Thus, we can always rescale X such that the $x_i(k)$ remain constant even when scaling down G. But, as the objective decreases this dominates $\mathcal{X}$, which proves the claim. □