Labor Market Volatility and Macroeconomic Shocks*

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PRELIMINARY AND INCOMPLETE

Abstract

In this paper, the labor market volatility puzzle (Shimer, 2005) has been reinvestigated. In recent years researchers have provided numerous plausible explanations; however, they usually suffer from two major caveats. Firstly, most of these studies have focused only on the contribution of technology shocks. Secondly, less attention has been paid to the time-varying properties of the macroeconomy and the shocks over business cycles. In this paper, we address these two caveats. We first estimate a time-varying parameter VAR (TVP-VAR) with stochastic volatility. We include the time series of GDP growth, inflation, real interest rate and vacancy rate and we provide reduced-form evidence on the time pattern of their volatilities.

Our structural identification strategy builds on a medium-scale DSGE model enriched with a search-and-matching framework and a large set of shocks. We combine long-run restrictions and model-implied sign restrictions to identify four structural shocks. We document that the variances and the impulse response functions of our variables exhibit considerable time variation. In the short run, the lion share of the variance of job creation is explained by cost-push and demand shocks, thus challenging the conventional practice of addressing the Shimer's puzzle under the assumption that technology shocks are the main driver of fluctuations in hiring. However, the importance of non-technology shocks for long-term volatility has dramatically dropped from the mid-'80s onwards.

Keywords: Labor market volatility, search and matching, structural Time Varying Parameters VAR, stochastic volatility, Bayesian estimation, Long-run restrictions, Sign restrictions, technology shocks.

JEL classification: C11, C32, E24, E32.

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1 Introduction

Since Shimer's influential work, it has been widely recognized that the high volatility displayed by labor market data cannot be easily replicated by conventional search and matching frameworks. Following his observation, researchers in recent years have provided numerous plausible solutions to explain this empirical finding. However, many of those proposals usually suffer from two major caveats. The first caveat is the widespread tendency in considering technology shocks as the unique driving force of economic fluctuations. Moreover, many studies focus on the magnitude of the conditional elasticity of vacancies to a given shock, while neglecting to verify that the sign is consistent with the empirical observations. The second caveat is that less attention has been paid to the time-varying features of the labor market and the time-varying volatilities of the underlying shocks. In this work we contribute to the literature by addressing these two caveats. We do that by combining an empirical methodology which allows us to study the time-varying properties of the data and a theoretical setup to guide our identification strategy.

Shimer (2005) observed that the conventional search and matching models cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to a shock of plausible magnitude. He documented that in the United States the volatility of the vacancy-unemployment ratio is almost 20 times as large as the volatility of average labor productivity [1].

Proposals revolve around three major conceptual points. The first one is based on real wage rigidity, which prevents full wage renegotiation (Hall, 2005; Gertler and Trigari, 2009). Another concurrent explanation hinges on a different calibration strategy, which assigns a higher value to non-market activities (Mortensen and Nagypál, 2007). This implies a small size of accounting profits which become more elastic to changes in productivity. Finally, Pissarides (2009) showed that the introduction of a fixed component in hiring cost fosters firms' response to productivity shocks.

These proposals usually suffer from two major shortcomings. First, most of the above mentioned contributions and many others in the field assume that the main or unique source of fluctuations is represented by technology shocks. Mortensen and Nagypál (2007) and Barnichon (2010; 2012) cast doubts on this approach. They highlight the possibility of non-technology shocks contributing to the observed volatility[2].

The second point we want to address in this paper regards the time-varying properties of the labor market. It has been well documented that macroeconomic shocks have time-varying volatilities (Primiceri, 2005; Justiniano and Primiceri, 2008). However the impact of such time-varying volatilities on the labor market has not been investigated thoroughly. To the best of our knowledge, only Barnichon (2010) and Benati and Lubik (2014) perform a similar exercise. Barnichon (2010) studies the correlation between unemployment and labor productivity and finds a substantial change in the mid-80s. Benati

[1] This observation has been validated for other countries such as Japan (see Esteban-Pretel, Nakajima and Ryuichi (2011)) and by Amaral and Tasci (2013) for the OECD countries. The ubiquitous presence of this puzzle has led to a great amount of research on the topic, which is not possible to cover here.

[2] Ravn and Simonelli (2007) provide some evidence on the importance of monetary policy shock in explaining the volatility of the labor market in the context of a large, constant SVAR. Moreover, while the magnitude of the elasticity of labor market variables to different types of shocks has gained lot of attention,. Barnichon (2010; 2012) and Balleer (2012) insist on the importance of studying the sign of the elasticity conditional on the type of shock. Importantly, they find that technology shocks generally determine a rise in unemployment, contrary to what implied by standard DMP models.

and Lubik (2014) investigate the time-varying properties of the Beveridge curve. Authors document that evidence point towards similarities and differences between the Great Recession and Volcker disinflation.

We contribute to the ongoing literature by addressing the two mentioned shortcomings. First, we estimate a time-varying parameter VAR (hereafter TVP-VAR) using GDP growth, inflation, real interest rate and vacancy rate and we provide reduced-form evidence on the job creation's time-varying volatility. Second, we provide a structural interpretation by building a DSGE model enriched by a search-and-matching framework and a large set of shocks. The framework we have adopted allows us to overcome some limitations of previous works; however, we maintain tractability and comparability by abstracting from capital accumulation and labor force participation. Our identification structure is robust to a wide range of parameterizations; moreover, we have also conducted an identification test to make sure that shocks are identified given the choice of our time series. We are thus able to study the impact of numerous shocks in a more realistic environment which features price stickiness and real wage rigidity *à la* Hall (2005). We then combine long-run restrictions on technology shocks and model-implied short-run sign restrictions to structurally identify the TVP-VAR parameters. This allows us to disentangle the contribution of each shocks to the labor market volatility throughout business cycles.

2 Methodology

We follow the methodology presented in Benati and Lubik (2012; 2014). We specify a TVP-VAR of order k. $Y_t = [\Delta y_t, \pi_t, r_t, v_t]$ is the vector which collects the time series of interest, where Δy_t is the real GDP growth computed as log difference of real GDP, π_t is inflation computed as the log difference of GDP deflator, r_t is the real interest rate computed as the difference between the 3 months Treasury bill rate and inflation and v_t is the vacancy rate, that is the composite Help-Wanted-Index (HWI) calculated by Barnichon (2010) and normalized by the size of the labor force[3]. The TVP-VAR(k) takes the following form:

$$Y_t = B_{0,t} + B_{1,t}Y_{t-1} + ... + B_{k,t}Y_{t-k} + \epsilon_t \equiv X_t^{'}\theta_t + \epsilon_t \tag{1}$$

$$X'(t) = I_n \otimes \left[1, Y_{t-1}', ...Y_{t-k}'\right]$$

where $\otimes$ is the Kronecker product and I_n is the identity matrix of dimension n.

As it is customary in the VAR literature we set the lag order $k = 2$ (Benati and Mumtaz, 2007; Primiceri, 2005). We then collect the VAR's time-varying coefficients at time t - that is, the elements of the matrices $B_{0,t}$, $B_{1,t}, ... B_{k,t}$ - in the vector θ_t and we postulate that they evolve according to:

$$p\left(\theta_t|\theta_{t-1}, Q\right) = I\left(\theta_t\right) f\left(\theta_t|\theta_{t-1}, Q\right) \tag{2}$$

[3] The 3 months Treasury bill rate is preferred to the federal funds rate because it is available for a longer period of time. Moreover, we transform it to quarterly frequency to make it consistent with our inflation measure. We prefer to include the real interest rate rather than the nominal one to ensure stationarity in the VAR.

with $I(\theta_t)$ being an indicator function that rejects the unstable draws, thus enforcing stationarity on the VAR. Following Primiceri (2005), $f(\theta_t|\theta_{t-1}, Q)$ is given by:

$$\theta_t = \theta_{t-1} + \eta_t \tag{3}$$

with η_t following a normal distribution of mean zero and variance-covariance matrix Q. The VAR's reduced-form innovations (ϵ_t) are assumed to be zero-mean and normally distributed. We factor the time-varying covariance matrix Ω_t as:

$$\mathrm{Var}(\epsilon_t) \equiv \Omega_t = A_t^{-1} H_t \left(A_t^{-1}\right)^{'} \tag{4}$$

where the matrices H_t and A_t are defined as follows:

$$H_t = \begin{bmatrix} h_{1,t} & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 \\ 0 & 0 & 0 & h_{4,t} \end{bmatrix} \quad A_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 \end{bmatrix} \tag{5}$$

The $h_{i,t}$ are assumed to evolve as geometric random walks :

$$\ln(h_{i,t}) = \ln(h_{i,t-1}) + \nu_{i,t} \tag{6}$$

As in Primiceri (2005) we can postulate that the non-zero and non-unity elements of the matrix A_t collected in the vector $\alpha_t = [\alpha_{21,t}, \alpha_{31,t}, ..., \alpha_{41,t}]^{'}$ evolve as driftless random walks

$$\alpha_t = \alpha_{t-1} + \tau_t \tag{7}$$

We assume the vector of innovations $\left[u_t^{'}, \eta_t^{'}, \tau_t^{'}, \nu_t^{'}\right]^{'}$ to be distributed as :

$$\begin{bmatrix} u_t \\ \eta_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N(0, V), \text{ with } V = \begin{bmatrix} I_4 & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & Z \end{bmatrix} \text{ and } Z = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \tag{8}$$

Since the model is heavily parameterized, we impose block-diagonal structure on V for parsimony. Moreover, allowing for a completely generic correlation structure among different sources of uncertainty would preclude any structural interpretation of the innovations. Finally, as an additional simplifying assumption, we adopt a block-diagonal structure for S :

$$S \equiv \mathrm{Var}(\tau_t) = \mathrm{Var}(\tau_t) = \begin{bmatrix} S_1 & 0_{1\times 2} & 0_{1\times 3} \\ 0_{2\times 1} & S_2 & 0_{2\times 3} \\ 0_{3\times 1} & 0_{2\times 2} & S_3 \end{bmatrix} \tag{9}$$

with $S_1 \equiv \mathrm{Var}(\tau_{21,t})$, $S_2 \equiv \mathrm{Var}\left([\tau_{31,t}, \tau_{32,t}]^{'}\right)$ and $S_3 \equiv \mathrm{Var}\left([\tau_{41,t}, \tau_{42,t}, \tau_{43,t}]^{'}\right)$. This implies that the non-zero and non-unity elements of A_t which belongs to different rows evolve independently. This assumption drastically simplifies inference, since it allows one to perform Gibbs sampling on the non-zero and non-unity elements of A_t equation by equation. The details of the algorithm are relegated to the Appendix.

3 Reduced-form Evidence

The reduced-form evidences are informative in itself. Figure 1 represents the original data together with the time-varying estimates of the states. The estimation tracks very well the pattern of the data. In what follows, the blue solid line represents the median of a given object among the 10,000 draws from the posterior distribution. Lower and upper red lines represent the 16th and 84th percentiles, respectively. The fourth panel shows the high volatility of job creation over the past four decades: this also affects the dispersion of the draws of the states, as highlighted by the wider red bands.

Figures 2 and 3 provide evidences of time variation in the VAR coefficients as well as in the volatility of the shocks. In figure 2 the coefficients which display substantial time variation are the ones corresponding to the vacancy equation. This can be interpreted as a first set of mild evidences that the response of labor market to the macroeconomy has changed over time. Figure 3 plots $\log\det(\Omega_t)$, where Ω_t is the variance-covariance matrix of the reduced-form VAR residuals. Following Cogley and Sargent (2005) we interpret this as the total amount of uncertainty hitting the economy at each point in time. Our findings are remarkably similar to those provided by Cogley and Sargent (2005) and Benati and Mumtaz (2007). The variance exhibits a substantial increase from 1965 to 1981; then, it decreases during the Great Moderation period and exhibits two small peaks around 2001 and the recent crisis.

4 Model

The benchmark model combines features of an otherwise standard RBC setting with labor market search frictions *à la* Mortensen-Pissarides. Time is discrete. The economy is populated by households, firms and policy authorities. Households consume, invest in the bond market and supply labor. We distinguish between wholesale and retail firms. Wholesale firms employ labor to produce a homogeneous good sold to retailers in a perfect competitive market. Workers are recruited on a frictional labor market. We consider both a flexible and a sticky wage setting. In the first case, wages are the outcome of a Nash-bargaining process. In the second case we introduce real wage stickiness *à la* Hall (2005). Retailers own a technology which allows them to differentiate the goods without any cost. The differentiated good is then sold to the households under monopolistic competition. As for policy, the monetary authority is in charge of setting the nominal interest rate following a standard Taylor rule, while the central government collects lump-sum taxes to finance public expenditure and unemployment benefits. The model is non-stationary for the presence of a unit root in the technological process. In addition, the model is enriched with a large set of transitory shocks.

4.1 Households

The economy is populated by a continuum of identical households of mass 1. They consume a composite good C_t which incorporates all goods produced by the retailers. They hold bonds and supply labor. Since in any period workers are either employed or unemployed (i.e. matched or unmatched), an income distributional problem may arise. As in Merz (1995), we assume that households pool consumption and they behave like a big family which fully insures each member against unemployment[4]:

$$\max\ \mathbb{E}_0 \sum_{t=0}^{\infty} \exp(\varepsilon_t^{\beta})\beta^t \left(\ln C_t - \psi \exp(\varepsilon_t^{\psi}) \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right)$$

$$\text{subject to } C_t + \frac{B_t}{R_t P_t} = \frac{B_{t-1}}{P_t} + \frac{w_t}{P_t} N_t + \frac{b_t}{P_t} U_t + \Pi_t - T_t$$

where σ_n is the inverse of Frisch elasticity[5]. ε_t^{β} is a shock to the discount rate which we interpret as a non-policy-demand shock. ε_t^{ψ} accounts for a potential shift in the desutility of labor and can be thus labeled as preference shock.

Households can allocate their income between consumption and nominal bonds, which pay the nominal (gross) interest rate R_t. In addition, households supply labor: the labor income is represented by the real wage paid to the household's members who are employed during the period (N_t). Unemployed workers receive benefits b_t from the government. Public expenditure and unemployment benefits are financed by lump-sum taxes T_t. Finally, households own firms, whose profits are denoted as Π_t. C_t is the Dixit-Stiglitz aggregator

$$C_t = \left(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{1-\epsilon}}$$

where ϵ is the demand elasticity.

The first order conditions are the following:

$$\frac{1}{C_t} = \lambda_t \tag{10}$$

$$\mathbb{E}_t \left[Q_{t,t+1} \frac{P_t}{P_{t+1}} R_t \right] = 1 \tag{11}$$

where $Q_{t,t+k} = \beta^k \frac{\exp(\varepsilon_{t+k}^{\beta})\lambda_{t+k}}{\exp(\varepsilon_t^{\beta})\lambda_t}$ is the stochastic discount factor and λ_t is the marginal value of wealth. Moreover, the demand for variety i is

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} C_t$$

where $P_t = \left(\int_0^1 P_{it}^{\epsilon-1} di \right)^{\frac{1}{1-\epsilon}}$ is the aggregate retail price index.

[4]Because utility is seperable in consumption and labor, workers enjoy perfect consumption insurance. This brings the consumption level of all household's members, i.e. employed or unemployed, to the same level. The representative household finance consumption and other expenditures with taxes and wages recieved from the employed workers. The household requires employed workers to directly remit their wage to the household. Presumably the household's ability to require this reflects that it *perfectly* observes the action of its workers; moreover, it has the leverage over them to implement a full consumption insurance and to exercise the threat of witholding it in case of no-compliance. This assumption greatly simplifies the model since we do not need to track the income distribution of each houshold's member. However, the implicit consequence of this widely exercised assumption is to make unemployed workers to have a higher utility than the employed ones, since the unemployed has both perfect consumption and exert no effort. There is much evidence that unemployment generates desutility. For a discussion of this point refer to Christiano (2011).

[5] Notice that the log specification makes these preferences consistent with balanced growth.

Labor supply decisions must take into account the frictions characterizing the labor market, which are relegated in the subsequent sections. Notice that, with perfectly competitive labor markets, the following condition would hold:

$$\frac{w_t}{P_t} = \psi \exp(\varepsilon_t^{\psi}) C_t N_t^{\sigma_N} = MRS_t \tag{12}$$

The above equation states that absent any friction, the wage adjustment mechanism guarantees there would be no voluntary unemployment and households supply labor by equating the wage to the intratemporal marginal rate of substitution.

4.2 Labor Market

Labor market clearing is prevented by search and matching frictions *à la* Mortensen and Pissarides (1994). Demand and supply conditions (number of vacancies posted and job-seekers, respectively) and labor market characteristics (matching efficiency) jointly determine the employment level.

Firms must post vacancies on the labor market in order to hire workers, incurring the real cost k_t^f[6]. The realized number of matches is the outcome of a Cobb-Douglas technology, which depends on the number of vacancies V_t and searchers U_t^0: $M_t\left(V_t, U_t^0\right) = \exp(\varepsilon_t^{\xi}) \xi V_t^{\eta} \left(U_t^0\right)^{1-\eta}$, where ε_t^{ξ} is a shock to the efficiency of the matching function. The probability that a firm matches with a worker is $p_t^f = \frac{M_t\left(V_t, U_t^0\right)}{V_t}$. The probability of being hired is then given by $q_t^w = \frac{M_t\left(V_t, U_t^0\right)}{U_t^0}$. The labor market tightness is defined as $\theta_t = \frac{V_t}{U_t^0}$. It is easy to show that p_t^f is a decreasing function of θ_t while q_t^w is an increasing function of it $\left(q_t^w = \theta_t p_t^f\left(\theta_t\right)\right)$.

In each period, timing is the following: i) a fraction of productive matches from the previous period get severed exogenously and separated workers enter the unemployment pool; ii) unemployed workers and firms search on the labor market and matches are formed; iii) shocks realize; iv) production occurs. Employment dynamic is thus given by:

$$N_t = \left(1 - s \exp(\varepsilon_t^s)\right) N_{t-1} + M_t \tag{13}$$

where s is the exogenous separation rate which is rendered time-varying by the shock ε_t^s. The first term in the right hand side of the above equation represents workers matched in the previous period who do not separate (surviving matches); the second term represents new matches realized at the beginning of the period before production occurs. The number of searchers is

$$U_t^0 = \frac{U_t}{1 - q_t^w} \tag{14}$$

where $U_t = 1 - N_t$ is current unemployment, which is defined after the matching process has taken place. Under this timing assumption, the matches become immediately productive.

4.3 Wholesale Firms

Wholesale firms employ labor to produce an homogenous good to be sold to retailers at price P^w. Because of the frictions in the labor market, these decisions potentially differ among firms, which we index by j.

[6] Because of the unit root in technology the vacancy cost is assumed to grow at the same rate of output.

In order to hire workers, firms must pay post vacancies on the labor market, by paying the fixed real cost k_t^f. The value of a vacancy for firm j is

$$J_t^V(j) = -k_t^f + p_t^f J_t^F(j) + \left(1 - p_t^f\right) \mathbb{E}_t \left(Q_{t,t+1} J_{t+1}^V(j)\right)$$

The above equation states that with probability p_t^f the firm fills the vacancy and gets the value of the match J_t^F and with a complementary probability the vacancy remains unfilled. Free entry implies:

$$J_t^F(j) = \frac{k_t^f}{p_t^f} \; \forall j \tag{15}$$

The value of a productive match is represented by the following equation:

$$\frac{k_t^f}{p_t^f} = \frac{1}{\mathcal{M}_t^p} MPN_t - \frac{w_t(j)}{P_t} + \mathbb{E}_t \left(Q_{t,t+1} \left(1 - s\exp(\varepsilon_{t+1}^s)\right) \frac{k_{t+1}^f}{p_{t+1}^f} \right) \tag{16}$$

where $\frac{1}{\mathcal{M}_t^p} = \frac{P_t^w}{P_t}$ is the relative price of the wholesale good in terms of the final good and $MPN_t = (1-\alpha)A_t N_t^{-\alpha}$ is the marginal productivity of labor.

The above equation states that firms keep posting vacancies until the real cost they bear (which depends on the fixed cost and the search spell) equates the current productivity gains and the savings on future vacancy costs. Search frictions distort firms' optimization condition on hiring. For future reference we can define the net hiring costs as the real cost of posting vacancies net the expected discounted saved cost:

$$D_t = \frac{k_t^f}{p_t^f} - \mathbb{E}_t \left[Q_{t,t+1} \left(1 - s\exp(\varepsilon_{t+1}^s)\right) \frac{k_{t+1}^f}{p_{t+1}^f} \right]$$

As for technology, we assume $A_t = A_t^T A_t^P$, where A_t^T denotes the transitory component and A_t^P is the permanent component.

$$\ln A_t^T = \rho_a \ln A_{t-1}^T + \sigma_{a^T} \varepsilon_t^{a^T}$$
$$\frac{A_t^P}{A_{t-1}^P} = \gamma_t^a$$
$$\ln \gamma_t^a = \ln \bar{\gamma}^a + \sigma_{a^P} \varepsilon_t^{a^P}$$

where $\varepsilon_t^{a^T}$ and $\varepsilon_t^{a^P}$ are standard normals.

4.4 Workers

Workers can be either employed or unemployed. We now characterize their value functions in both cases. The value function of a worker employed at firm j is:

$$J_t^w(j) = \frac{w_t(j)}{P_t} - MRS_t + \mathbb{E}_t \Big\{ Q_{t,t+1} \Big[\left(1 - s\exp(\varepsilon_{t+1}^s)\right) J_{t+1}^w(j) + s\exp(\varepsilon_{t+1}^s) \left(q_{t+1}^w J_{t+1}^w + (1 - q_{t+1}^w) J_{t+1}^u\right) \Big] \Big\}$$

where the second term is the marginal rate of intratemporal substitution, which expresses labor disutility in terms of consumption goods. The term in brackets is the continuation value; the worker will either stay with the firm with probability $1 - s\exp(\varepsilon^s_{t+1})$ or will get back to the unemplyment pool where he will be immediately available for a new job, that he will find with probability q^w_{t+1}. .J^u_t is the value of being unemployed, which is given by the following:

$$J^u_t = b_t + \mathbb{E}_t \left\{ Q_{t,t+1} \left[q^w_{t+1} J^w_{t+1} + (1 - q^w_{t+1}) J^u_{t+1} \right] \right\}$$

where $J^w_t = \int_0^1 \frac{M_t(j)}{M_t} J^w_t(j)\, dj$.

Remember that unemployment is defined after the matches of the current period have taken place. Unemployed agents can find a job in the following period with probability q^w_{t+1} or stay unemployed.

The surplus which accrues to a worker employed at firm j is thus given by:

$$S^w_t(j) = J^w_t(j) - J^u_t = \frac{w_t(j)}{P_t} - (MRS_t + b_t) + \mathbb{E}_t \left\{ Q_{t,t+1} \left(1 - s\exp(\varepsilon^s_{t+1})\right) \left(S^w_{t+1}(j) - q^w_{t+1} S^w_{t+1} \right) \right\}$$

where $S^w_t = \int_0^1 \frac{M_t(j)}{M_t} S^w_t(j)\, dj$ is the average surplus.

4.5 Nash Bargaining

4.5.1 Flexible wage setting

When wages can adjust in every period, they are set through Nash bargaining, thus implying the following relationship:

$$S^w_t(j) = \frac{\gamma}{1-\gamma} S^F_t(j)$$

where $S^w_t(j)$ is defined in above equation, γ is the worker's bargaining power and $S^F_t(j) = J^F_t(j)$ is the firm j's surplus. Eq. (15) implies that in equillibrium all firms offer the same wage. The index j will be thus omitted in what follows. The following real wage expression can be obtained after some mathematical manipulations:

$$\omega^N_t = MRS_t + b_t + \frac{\gamma}{1-\gamma} \left[\frac{k^f}{p^f_t} - \mathbb{E}_t \left(Q_{t,t+1} \left(1 - s\exp(\varepsilon^s_{t+1})\right) (1 - q^w_{t+1}) \frac{k^f}{p^f_{t+1}} \right) \right]$$

Where ω^N_t is the Nash-bargained real wage. The equation above shows that workers must be compensated for the desutility of working and for the foregone benefit (as in the competitive framework) but, as long as they have a positive bargaining power, they can also extract part of the firm's surplus (the term inside the brackets) [7].

[7] Notice that we do not consider entries and exits into the labor force. Including the extensive margin of labor supply may strongly affect the sign and the magnitude of the IRFs to different types of shocks. Given the lack of consensus in the literature, we prefer to leave this issue for future research. However, we check that the prevailing wage is i) always above the worker's reservation wage (i.e. the wage that makes the worker indifferent between employment and unemployment) ii) always above the "full participation wage" (i.e. the wage that makes the worker indifferent between unemployment and inactivity once imposing full participation) iii) always below the wage that gurantees a positive surplus to firms.

4.5.2 Sticky wage setting

Following Blanchard and Gali (2010) we introduce wage stickiness by imposing that the prevailing real wage in the economy is a geometric average of the Nash-bargained wage and the wage prevailing in normal times ($\bar{w}$):

$$\omega_t = \bar{\omega}^{\theta_w} \left(\omega_t^N\right)^{1-\theta_w}$$

In what follows we assume that $\bar{\omega}$ is the Nash bargained wage in steady state. This can be interpreted as a wage norm, in the sense of Hall (2005).

4.6 Retailers

The homogeneous wholesale good is sold to retail firms, which they differentiate it at no cost and sell it to households. We introduce price stickiness in the form of Calvo prices. Let θ_p be the probability of not reoptimizing prices in a given period. Retailers maximize their profits subject to the demand schedule for each individual good i:

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} \left(\frac{P_{it}^* - P_{t+k}^w}{P_{t+k}}\right) Y_{i,t+k|t}$$

$$\text{subject to } Y_{i,t+k|t} = \left(\frac{P_{it}^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

The price schedule turns out to be

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k} \left[\frac{P_t^*}{P_{t+k}} - \frac{\epsilon_t}{\epsilon_t - 1}\frac{P_{t-k}^w}{P_{t+k}}\right] = 0$$

where we have omitted the index i because in equilibrium all firms charge the same price. $\mathcal{M}_t^p = \frac{\epsilon_t}{\epsilon_t - 1}$ is the mark-up charged by retailers on the marginal cost when prices are perfectly flexible. The desired mark-up changes over time following an AR(1) process, leading to the presence of a cost-push shock in the linearized Philipps curve.

The aggregate price index follows the dynamic given by

$$P_t = \left[\theta_p P + (1-\theta_p)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{17}$$

4.7 Monetary Authority

The monetary authority sets the nominal interest rate following a standard Taylor rule:

$$R_t = R_{t-1}^{\rho_r} \left(\frac{1}{\beta}\left(\frac{Y_t}{\overline{\overline{Y}}}\right)^{\delta_y}\left(\frac{P_t}{P_{t-1}}\right)^{\delta_\pi}\right)^{1-\rho_r} \exp(\varepsilon_t^r) \tag{18}$$

where $\frac{1}{\beta}$ is the steady state value of the interest rate, ρ_r is the degree of the monetary policy inertia, δ_y and δ_π express the monetary policy reactions to output gap and inflation, respectively. ε_t^r is a contractionary monetary policy shock.

4.8 Fiscal Authority

The government raises lump-sum taxes T_t to finance public expenditure G_t and unemployment benefits.

$$G_t + b_t U_t = T_t$$

Both public expenditure and the unemployment benefits follow a random process:

$$\ln \tilde{G}_t = (1-\rho_g)\ln \bar{\tilde{G}} + \rho_g \ln \tilde{G}_{t-1} + \sigma_g \varepsilon_t^g$$

$$\ln \tilde{b}_t = (1-\rho_b)\ln \bar{\tilde{b}} + \rho_b \ln \tilde{b}_{t-1} + \sigma_b \varepsilon_t^b$$

where ε_t^g and ε_t^b are standard normals. The tilde denotes stationarized variables (i.e. the original variable divided by the permanent component of technology).

4.9 Closing the Model

The resource constraint implies

$$Y_t = C_t + G_t + k_t^f V_t \tag{19}$$

To summarize, the model is driven by one permanent shock to technology and other eight transitory shocks which all follow an AR(1) process with their corresponding persistences and volatilities. The transitory shocks can be classified as follows: temporary supply, demand-non-policy (shock to the households' discount factor), monetary policy, cost-push (shock to the elasticity of demand), public expenditure, unemployment benefits, matching efficiency and separation rate.

Because of the presence of unit root in technology, we detrend the non-stationary variables and we then linearize the model around the balanced growth path. The log-linearized model is reported in Appendix.

4.10 Calibration

We calibrate the model on the U.S. quarterly data and we mainly rely on estimates taken from the literature. β is 0.99, so that the annual steady state interest rate is around 4%. σ_n is taken as half. We assume a steady state unemployment of 5 percent, which corresponds to the average unemployment rate in our sample. We set both the job filling rate (p^f) and the job finding rate to 0.7. This implies and exogenous separation rate of 12%.

We impose that the Hosios efficiency condition holds: the elasticity of the matching function (η) equals the firms' bargaining power $(1-\gamma)$ at the value of 0.5. The total vacancy expenditure on GDP ($\mathcal{M}^p \frac{k^f V}{Y}$) is 0.2 percent, which implies that the unit hiring cost is almost 2% of the nominal wage in steady state. We calibrate α in order to obtain a labor share of 2/3. The price mark up is calibrated at 1.2. The baseline values for price and wage stickiness are both sets at 0.75, so that resets occur once a year on average.

We adopt a standard specification of the monetary policy rule, with quite high inertia ($\rho_r = 0.8$), and monetary policy reactions which respect the Taylor principle ($\delta_y = 0.5$ and $\delta_\pi = 1.5$). The variance of the monetary policy shock is calibrated at 0.0025, so that 1 standard deviation contractionary monetary policy shock raises the nominal interest rate by 25 bp. Since we are not interested in the quantitative performance of the model and evidence is scarce, we do not try to find a proper calibration for each shock process. We set all the persistences to 0.9 and standard deviations to 0.01 instead.

5 Structural Evidence

5.1 Identification Strategy

We identify four structural shocks, labeled as: permanent supply, monetary policy, demand non-policy and cost-push shocks. As in Gali and Gambetti (2009), permanent supply shocks are identified by imposing that they are the only ones which affect the level of output in the long run. This is consistent with the model, which features non-stationarity in technology. To clarify ideas, a time-varying VAR can be written in the following form [8].

$$Y_t = \mu_{0,t} + C_{t,\infty} u_t$$

where $C_{t,\infty} = C_0 \tilde{A}_{0,t}$ is the matrix of the cumulative IRFs and takes the following form:

$$C_{\infty,t} = \begin{bmatrix} C_{\infty,11} & 0 & 0 & 0 \\ . & & & . \\ . & & & . \\ C_{\infty,41} & .. & .. & C_{\infty,44} \end{bmatrix}$$

$C_{\infty,11}$is the cumulative long-run impact of the first shock on the first variable, in our case the supply shock on GDP growth. We impose zeros on all the elements of the first row except the first one. This implies that the supply shock is the only one which potentially has a permanent effect on output. No long-run restrictions are imposed on the other variables.

The other three structural shocks are identified by sign-restrictions on the impact matrix. These are fully derived from the model described above. For example, following an unexpected monetary policy shock, inflation falls. Therefore in the impact matrix, the reaction of inflation to an unexpected monetary policy shock is set negative. The full set of sign restrictions is reported in Table 1.

Since the structural identification is based on signs obtained from a model, it is necessary to make sure that our choice of time series is good enough to identify the underlying process (i.e. persistence and volatility) of those shocks. For example, suppose that the time series we include in the TVP-VAR fail to identify cost-push shocks. Then, the signs derived from the response of the model to the

[8]The MA representation permits us to write a VAR in the following form $Y_t = \mu_{0,t} + \sum \Psi_k \epsilon_{t-k}$, by using $\epsilon_t = A_{0,t} u_t$ the VAR can then be expressed in terms of the structural shocks

Table 1: The matrix of short-run restrictions

	Shocks		
Variable	Monetary Policy	Demand	Cost-push
Inflation	-	+	+
Real Interest Rate	+	+	?
Vacancy/Labor Force	+	+	-

cost-push shock are flawed, since the shock itself is not identified in the first place. To make sure that shocks are identified using the time series of our interest we follow Iskrev (2010) and Komunjer and Ng (2011). The method of Iskrev (2010) is based on calculating the unconditional moments of model and the method of Komunjer and Ng (2011) is based on the concept of minimality of systems borrowed from adaptive control theory. Results show strong degree of identification of shocks given the choice of our time series. Hence we can make sure the shocks are identified and the implied signs are not flawed. Details of the procedure are relegated to the Appendix.

5.2 Results

The TVP-SVAR analysis permits us to compute the time-varying IRFs for different time periods and forecast horizons. In Figure 7 the time-varying response of the vacancy is depicted. For the sake of a better comparison the responses have been standardized. The take away from the graph is that during the period of the Great Inflation, demand and cost-push shocks have an impact lower than average in a sense that demand shocks are less expansionary and more severe hiring cuts are associated with cost-push shocks. On the contrary, the monetary policy determine a higher than average response on impact with more contractionary long-term consequences. Demand and monetary policy shocks appear to have a larger impact from year 2000 onwards.

Figure 8 plots the response of the economy in four proposed dates, namely 1973Q1, 1985Q1, 1997Q1 and 2005Q1. Each of these dates is representative of the salient economic features of the decade. 1973Q1 represents the pre-Volcker era with high inflation, 1985Q1 captures the start of the financial deregulation, 1997Q1 the build up towards the dot com crash and 2005Q1 more financial deregulation. The choice of the specific quarter affects only slightly the quantitative results in terms of amplification and persistence and does not undermine the take-away message of the Figure, which is twofold: i) there is time variation in the strength of the reaction of job creation to business cycle fluctuations; ii) the time pattern of the elasticity of vacancies depends on the shock. This highlights the fact that the tendency of replicating the observed volatility in labor market computing an average over the whole sample period and considering only technology shock might not be a well-advised exercise. As already highlighted in the above discussion, during the '70s cost-push shocks are more recessionary and demand shocks less expansionary. In the same period, monetary policy shocks appear to have lower negative impact. The reverse is true for 2005: demand and monetary policy shocks affect vacancies more severely, while cost-push shocks have a smaller impact.

From the IRFs, we can compute the unconditional and conditional correlations, which are plotted in Figure 9 . They display the expected signs: vacancies are positively correlated to output and negatively correlated to the real interest rate, no matter the shock. The correlation between vacancies and inflation is more shock-dependent. Conditional on cost-push shocks it is negative and fairly constant throughout the sample period.

To compute the "median" FEVD, in each period we apply twice the Median Target (hereafter MT) criterion introduced by Fry and Pagan (2005). We make here a methodological point, which we think is sometimes overlooked in practice. As stressed by Fry and Pagan (2005), one should be careful in selecting median estimates for different objects of interest which may come from different models. We first consider each shock separately and find a measure of distance of each parameterization (i.e. draw of states from the posterior distribution and impact matrix rotation) to the one that would produce the median fraction of the variance explained by the shock. This is done by considering all forecast horizons jointly. Secondly, we apply again the MT criterion on the previous measures to take into account all shocks at the same time. The output of the this procedure is a "median" set of VAR parameters which corresponds to a unique draw from the posterior distribution of the states and to a unique impact matrix rotation. This is a sufficient condition to ensure orthogonality of the shocks. As a consequence, the variance of the data is fully accounted for and we can safely perform the decomposition.

Figure 10 plots the unconditional variance of the variables included in our model, as well as the contribution of each shock. We have plotted the 10 years ahead variance of the forecast error. In line with the evidence and the narratives on the Great Moderation, the unconditional variance spikes between the mid-70s and the beginning of the 80s for all the variables included in our econometric model. It then decreases to lower levels after marking the Great Moderation. The fourth panel shows the remarkable importance of non-technology shocks for the long-term variance of job creation. Despite a decrease in their contribution in the late 90s still it is comparable to that of the technology shock.

The median Forecast Error Variance Decomposition (FEVD) is reported in the Figure 11 [9]. Three observations can be made at first sight: i) the contributions of each shock to the variance of vacancies display considerable time variation; ii) the time patterns are different according to the forecast horizon; iii) technology shocks do not aways represent the lion share of the variance of vacancies (at least not for all periods and forecast horizons). The upper left panel of the Figure shows that the short term volatility is dominated by technology shocks, with a peak of 60% in 1976. However, starting from the beginning of the 80s this contribution has decreased to less than 15% on average. A third of the variance is explained by demand shocks. A fairly constant fraction of 30 percent is accounted for by cost-push shock.

Overall, this evidence draws a complex conclusion on the role of the technology shock in the labor market than the one widely accepted in the literature. Especially in recent years short term movements in vacancies are mainly driven by demand and cost-push shocks.

6 Conclusions

In this paper we have reinvestigated labor market volatility. We have addressed the importance of two caveats not generally covered by the existing literature: the contribution of non-technology shocks to the observed volatility of job creation and the time-varying properties of the labor market in connection

[9] To save space, we report only the results relative to vacancies. The others are available upon request.

to the macroeconomy. We have tried filling in the gap by performing a TVP-VAR analysis identified by long-run and short-run sign restrictions. The signs are derived from a NK DSGE model enriched with search and matching in the labor market. We can summarize the main findings regarding job creation as follow. Firstly, the responses to different shocks and the variance display considerable time variation. This time-variation is very eminent during the Great Inflation and to a lesser degree during the recent decade. Secondly, the Great Inflation period is characterized by stronger reaction to cost-push and monetary policy shocks and less expansionary demand shock; the opposite is true in the 2000s; iii) non-technology shocks represent a considerable share of the variance until the mid-'90s; since then their contribution to the long-term variance has declined but it is still considerable; iv) in the short term the contribution of non-technology shocks is still the dominant one (and even more so in the second part of the sample), with cost-push and demand shocks accounting for more than 60 % of the variance. No sharp conclusions can be drawn with respect to the response of vacancies to technology shocks identified with long-run restrictions. Results show the importance of taking into account the contribution of other shocks in understanding the observed volatility exhibited by the labor market.

References

Amaral, P. and Tasci, M., *The Cyclical Behavior of Equilibrium Unemployment and Vacancies Across OECD Countries*, 2013.

Balleer, A., 'New evidence, old puzzles: Technology shocks and labor market dynamics', *Quantitative Economics* 3 (2012).

Barnichon, R., 'Productivity and Unemployment over the Business Cycle', *Journal of Monetary Economics* (2010).

Barnichon, R., *The Shimer puzzle and the Endogeneity of Productivity*, 2012.

Benati, L. and Lubik, T.A., *Sales, Inventories, and Real Interest Rates: A Century of Stylized Facts*, 2012.

Benati, L. and Lubik, T.A., 'The Time-Varying Beveridge Curve', (Springer Series Monograph: Dynamic Modeling and Econometrics in Economics and Finance 17, 2014).

Benati, L. and Mumtaz, H., *U.S. Evolving Macroeconomic Dynamics: A Structural Investigation*, 2007.

Blanchard, O. and Gali, J., 'Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment', *American Economic Journal: Macroeconomics* 2 (2010).

Carter, C.K. and Kohn, R.P., 'On Gibbs Sampling for State Space Models', *Biometrika*, 81 (2004), p. 541âĂŤ553.

Christiano, Laurence, 'Comments on Unemployment in an Estimated New Keynesian Model', (2011).

Cogley, T. and Sargent, T.J., 'Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US', *Review of Economic Dynamics*, 8 (2005), p. 262âĂŞ302.

Esteban-Pretel, J., Nakajima, R. and Ryuichi, R., *Japan's Labor Market Cyclicality and the Volatility Puzzle*, 2011.

Fry, R. and Pagan, A., *Some Issues in Using VARs for Macroeconometric Research*, 2005.

Gali, J. and Gambetti, L., 'On the sources of the Great Moderation', *American Economic Journal: Macroeconomics*, 1 (2009), pp. 26–57.

Gertler, M. and Trigari, A., 'Unemployment Fluctuations with Staggered Nash Wage Bargaining', *Journal of Political Economy* 117 (2009).

Hall, R.E., 'Employment Fluctuations with Equilibrium Wage Stickiness', *American Economic Review*, 95 (2005), pp. 50–65.

Iskrev, N., 'Local Identification in DSGE Models', *Journal of Monetary Economics* 57 (2010).

Jacquier, E., Polson, N.G. and Rossi, P., 'Bayesian Analysis of Stochastic Volatility Models', *Journal of Business and Economic Statistic*, 12 (1994), p. 371âĂŤ418.

Justiniano, A. and Primiceri, G.E., 'The Time-Varying Volatility of Macroeconomic Fluctuations', *American Economic Review*, 98 (2008), pp. 604–41.

Komunjer, I. and Ng, S., 'Dynamic Identification of Dynamic Stochastic General Equilibrium Models', *Econometrica*, 79 (2011), p. 1995âĂŞ2032.

Merz, M., 'Search in the Labor Market and the Real Business Cycle', *Journal of Monetary Economics*, 36 (1995), pp. 269–300.

Mortensen, D.T. and Nagypál, E., 'More on Unemployment and Vacancy Fluctuations', *Review of Economic Dynamics*, 10 (2007), pp. 327–347.

Mortensen, D.T and Pissarides, C.H., 'Job Creation and Job Destruction in the Theory of Unemployment', *Review of Economic Studies*, 61 (1994), pp. 397–415.

Pissarides, C.A., 'The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?' *Econometrica*, 77 (2009), pp. 1339–1369.

Primiceri, G.E., 'Time Varying Structural Vector Autoregressions and Monetary Policy', *The Review of Economic Studies*, 72 (2005), pp. 821–852.

Ravn, M.O. and Simonelli, S., 'Labor Market Dynamics and the Business Cycle: Structural Evidence for the United States', *The Scandinavian Journal of Economics*, 109 (2007), pp. 743–777.

Shimer, R., 'The Cyclical Behavior of Equilibrium Unemployment and Vacancies', *American Economic Review*, 95 (2005), pp. 25–49.

Appendix

A) Log-Linearized Model

- Marginal utility of wealth (eq. 10):$\hat{\lambda}_t = -\sigma_c \hat{c}_t$.
- Dynamic IS:

$$\hat{c}_t = E_t(\hat{c}_{t+1}) - \frac{1}{\sigma_c}[\ln\beta + r_t - E_t(\pi^p_{t+1})] + \varepsilon^{IS}_t$$

where $\varepsilon^{IS}_t = -\frac{1}{\sigma_c}\varepsilon^{\beta}_t$ is the demand shock.

- Marginal rate of substitution (eq. 12): $\hat{mrs}_t = \sigma_c \hat{c}_t + \sigma_N \hat{l}_t + \varepsilon^{\psi}_t$.
- Employment (eq. 13): $\hat{n}_t = (1-s)\hat{n}_{t-1} + s(\hat{m}_t - \varepsilon^s_t)$.
- Searchers (eq. 14): $\hat{u}^0_t = \hat{u}_t + \frac{\bar{q^w}}{1-\bar{q^w}}\hat{q}^w_t$.
- Hirings: $\hat{m}_t = \varepsilon^{\xi}_t + \eta\hat{v}_t + (1-\eta)\hat{u}^0_t$.
- Job finding rate:$\hat{q}^w_t = \hat{m}_t - \hat{u}^0_t$.
- Job filling rate: $\hat{p}^f_t = \hat{m}_t - \hat{v}_t$.
- Net hiring costs:

$$\hat{b}_t = \frac{1}{1-\beta(1-s)}(-\hat{p}^f_t) + \frac{\beta(1-s)}{1-\beta(1-s)}E_t\left[\hat{p}^f_{t+1} + \hat{r}_t - E_t\pi^p_{t+1} - \epsilon^{IS}_t\right]$$

  where $B_t = \frac{k^f}{p^f_t} - \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(1 - s\exp(\varepsilon^s_{t+1})\right)\frac{k^f}{p^f_{t+1}}\right]$.

- Labor force: $\hat{f}_t = \frac{\bar{N}}{\bar{N}+\bar{U}}\hat{n}_t + \frac{\bar{U}}{\bar{N}+\bar{U}}\hat{u}_t$.
- Marginal productivity of labor: $\hat{mpn}_t = a_t - \alpha\hat{n}_t$.
- Production function: $\hat{y}_t = a_t + (1-\alpha)\hat{n}_t$.
- Wage inflation equation:

$$\pi^w_t = \beta(1-s)\theta_w E_t(\pi^w_{t+1}) + \lambda_w(\hat{\omega}^{tar}_t - \hat{\omega}_t)$$

$$\hat{\omega}^{tar}_t = \Upsilon \hat{mrs}_t + (1-\Upsilon)\left(\hat{mpn}_t - \hat{\mu}^p_t\right)$$

$$\hat{\omega}_t = \hat{\omega}_{t-1} + \pi^w_t - \pi^p_t$$

- where $\lambda_w = \frac{(1-\theta_w)[1-\beta(1-s)\theta_w]}{\theta_w(1-\gamma)}$ and $\Upsilon = \frac{(1-\gamma)\bar{MRS}}{(1-\gamma)\bar{MRS}+\gamma\bar{MPN}/\bar{\mathcal{M}}^p}$.

- Labor participation:

$$\hat{mrs}_t = \frac{1}{1-\bar{q}^w}\hat{q}^w_t - \hat{p}^f_t - \Xi\pi^w_t$$

- where $\Xi = \frac{\bar{W}/\bar{P}}{\kappa^f/\bar{p}^f}\frac{\gamma}{1-\gamma}\frac{\theta}{(1-\theta)(1-\beta\theta_w(1-s))}$.
- Taylor rule (eq. 18):$r_t = \rho_r r_{t-1} + (1-\rho_r)\left[-\ln\beta + \delta_y\hat{y}_t + \delta_\pi\pi_t\right] + \varepsilon^r_t$.
- Aggreagate resource constraint (eq.): $\hat{y}_t = \frac{\bar{C}}{\bar{C}+\kappa^f\bar{V}}\hat{c}_t + \frac{\kappa^f\bar{V}}{\bar{C}+\kappa^f\bar{V}}\hat{v}_t$.
- Price markup/job creating condition: $\hat{\mu}^p_t = \hat{mpn}_t - \left[(1-\Phi)\hat{\omega}_t + \Phi\hat{b}_t\right]$, where $\Phi = \frac{\bar{B}}{\bar{W}/\bar{P}+\bar{B}}$.

$$\hat{n}_t = \frac{1}{\alpha}\left\{a_t - \hat{\mu^p}_t - (1-\Phi)\hat{\omega}_t - \frac{\mathbf{\Phi}}{1-\beta(1-s)}\left[-\hat{p}^f_t + \beta(1-s)\left(E_t\hat{p}^f_{t+1} + \hat{r}_t - E_t\pi_{t+1} - \epsilon^{IS}_t\right)\right]\right\}$$

- Price inflation:

$$\pi^p_t = \beta E_t\pi^p_{t+1} - \lambda_p\hat{\mu}^p_t$$

where $\lambda_p = \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}$.

B) Details of Markov-Chain Monte Carlo (MCMC) Procedure

This section describes our choices for the priors and the MCMC algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data. The choice of the priors largely builds on Benati and Lubik (2012).

Priors

For the sake of simplicity, the prior distribution for the initial values of the states - θ_0 and h_0 - which we postulate all to be normal, are assumed to be independent both from each other and from the distribution of the hyperparameters. In order to calibrate the prior distributions for θ_0 and h_0 we estimate a time-invariant version of Equation 1 based on the first 10 years of data :

$$\theta_0 \sim N\left(\hat{\theta}_{OLS}, 4\widehat{V}\left(\hat{\theta}_{OLS}\right)\right)$$

where $\widehat{V}\left(\hat{\theta}_{OLS}\right)$ is the estimated asymptotic variance of $\hat{\theta}_{OLS}$. As for h_0 we proceed as follows. Let Σ_{OLS} be the estimated covariance matrix of ϵ_t from the time-invariant VAR and let C be its lower-triangular Cholesky factor, i.e.$CC' = \widehat{\Sigma}_{OLS}$. We set

$$\ln h_0 \sim N\left(\ln\mu_0, 10 \times I_N\right)$$

where μ_0 is a vector collecting the logarithms of the squared elements on the diagonal of C. As stressed by Cogley and Sargent (2005) "a variance of 10 is huge on a natural log scale, making this weakly informative" for h_0. Turning to the hyperparameters, we make the following, standard assumptions. The matrix Q is postulated to follow an inverted Wishart distribution

$$Q \sim IW\left(\overline{Q}^{-1}, T_0\right)$$

with prior degrees of freedom T_0 and scale matrix $T_0\overline{Q}$. In order to minimize the impact of the prior, thus maximizing the influence of sample information, we set T_0 equal to the minimum value allowed, the length of θ_t plus one. As for $\overline{Q}$ we calibrate it as $\overline{Q} = \gamma\Sigma_{OLS}$ setting $\gamma = 1 \times 10^{-4}$ which is slightly more "conservative" (in the sense of allowing for less random-walk drift) than the value of 3.5×10^{-4} used by Cogley and Sargent (2005). We assume independent inverse-Wishart distributions also for the blocks of S:

$$\begin{aligned}
S_1 &\sim IW\left(\bar{S}_1^{-1}, 2\right), \qquad & \bar{S}_1 &= 0.001 * |\hat{\alpha}_{2,1}| \\
S_2 &\sim IW\left(\bar{S}_2^{-1}, 3\right), \qquad & \bar{S}_2 &= 0.001 * \text{diag}[|\hat{\alpha}_{3,1}|, |\hat{\alpha}_{3,2}|] \\
S_3 &\sim IW\left(\bar{S}_3^{-1}, 4\right), \qquad & \bar{S}_3 &= 0.001 * \text{diag}[|\hat{\alpha}_{4,1}|, |\hat{\alpha}_{4,2}|, |\hat{\alpha}_{4,3}|]
\end{aligned}$$

where $\text{diag}(x_1, ..., x_n)$ is a diagonal matrix of order n with elements x_i's on the main diagonal and $|\hat{\alpha}_{i,i}|$ is the i, i element of the correlation matrix of the VAR shocks derived from $\widehat{\Sigma}_{OLS}$. As for α we assume: $f(\alpha) = N(\hat{\alpha}, 10 * |\hat{\alpha}|)$.

Finally as for the variance of the stochastic volatility innovations, we follow Cogley and Sargent (2005) and we postulate an inverse-Gamma distributions for $\sigma_i^2 \equiv \text{Var}(\nu_{i,t})$:

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

Simulating the Posterior Distribution

We simulate the posterior distribution of the hyperparameters and the states conditional on the data via the following MCMC algorithm, which combines procedures found in Cogley and Sargent (2005) and Primiceri (2005). In what follows x^t denotes the entire history of the vector x up to time t i.e. $x^t = \left[x_1', x_2', ..., x_t'\right]'$, while T is the sample length.

- *Drawing the elements of θ_t*: Conditional on Y^T, α and H^T the observation equation 1 is linear, with Gaussian innovations and a known covariance matrix. Following Carter and Kohn (2004) the density $p\left(\theta^T | Y^T, \alpha, H^T\right)$ can be factored as

$$p\left(\theta^T | Y^T, \alpha, H^T\right) = p\left(\theta_T | Y^T, \alpha, H^T\right) \prod_{t=1}^{T-1} p\left(\theta_t | \theta_{t+1}, Y^T, \alpha, H^T\right)$$

Conditional on α and H^T, the standard Kalman filter recursions nail down the first element on the right hand side: $p\left(\theta_T | Y^T, \alpha, H^T\right) = N(\theta_T, P_T)$, with P_T being the precision matrix of θ_T produced by the Kalman filter. The remaining elements in the factorization can then be computed via the backward recursion algorithm found, e.g. in Cogley and Sargent (2005). Given the conditional normality of θ_t we have:

$$\theta_{t|t+1} = \theta_{t|t} + P_{t|t} P_{t+1|t}^{-1} (\theta_{t+1} - \theta_t)$$

$$P_{t|t+1} = P_{t|t} + P_{t|t} P_{t+1|t}^{-1} P_{t|t}$$

which provides for each t from $T-1$ to 1 the remaining elements in equation 1, $p\left(\theta_{t|t+1} | Y^T, \alpha, H^T\right) = N\left(\theta_{t|t+1}, P_{t|t+1}\right)$. Specifically the backward recursion starts with a draw from $N(\theta_T, P_T)$, call it $\widetilde{\theta}_T$. Conditional on $\widetilde{\theta}_T$, the Kalman formulation above gives us $\theta_{T-1|T}$ and $P_{T-1|T}$ thus allowing us to draw $\widetilde{\theta}_{T-1}$ from $N\left(\theta_{T-1|T}, P_{T-1|T}\right)$ and so on till $t = 1$.

- *Drawing the innovation variance for VAR parameters* (Q): Conditional on a realization for θ^T, the VAR parameter innovations (η_t's) are observable. Under the linear transition law, η_t is i.i.d. normal. Given an inverse-Wishart prior and a normal likelihood, the posterior is inverse-Wishart.

- *Drawing the innovation variances for* α_t (S_1, S_2, S_3): Conditional on the vector of covariance parameters α, the innovations τ_t's are observables and follow a normal distribution. Given the inverse Wishart prior on the innovation variance-covariance matrices S_1, S_2, S_3 the posterior follows an inverse Wishart distribution as well.

- *Drawing the covariance parameters* (α_t): Conditional on Y^T and θ^T, the VAR residuals $\epsilon_t = Y_t - X_t^{'}\theta_t$ are observable, satisfying $A\epsilon_t = u_t$, with u_t being a vector of orthogonalized residuals with known time-varying variance H_t. Following Primiceri (2005), we interpret $A\epsilon_t = u_t$ as the observation equation and Equation 7 as the unobserved state equation. Then we apply the Carter and Kohn (2004)'s algorithm as in point a) to obtain a draw of α. Given the block-diagonal structure of S, the algorithm can be applied equation by equation.

- *Drawing the standard deviation of volatility innovations* (σ_i's): Conditional on a specific time path of $\log(h_t)$, the innovations to the logs of the stochastic volatilities (v_{it}'s) are directly observable. The v_{it}'s are i.i.d. normal with mean zero and variance σ_i^2. Assuming an inverse-gamma prior for σ_i, $i = 1, ..., 4$, the posterior is also inverse gamma.

- *Drawing the stochastic volatilities* (h_{it}'s): Since we assume that the stochastic volatilities evolve independently, we can sample them on a univariate basis by applying the algorithm of Jacquier, Polson and Rossi (1994) element by element.

Summing up, the MCMC algorithm simulates the posterior distribution of the states and the hyperparameters, conditional on the data. We use a burn-in period of 50,000 iterations to ensure convergence to the ergodic distribution. We then perform other 100,000 iterations sampling every 10th draw in order to reduce the autocorrelation across draws. What we are after are 10,000 draws from the ergodic distributions over which we calculate the statistics reported in the main text.

C) Identification à la Iskrev (2010)

Iskrev (2010) proposes the following strategy for local identification. A DSGE model can be respresented by the following g linear equations, where θ is the vector of deep parameters of the model and z_t is the vector of variables,

$$\mathbb{E}_t \left(g \left(z_t, z_{t-1}, z_{t+1}, u_t \right) | \theta \right) = 0$$

$$\Gamma_0 \left(\theta \right) z_t = \Gamma_1 \left(\theta \right) \mathbb{E}_t z_{t+1} + \Gamma_2 \left(\theta \right) \mathbb{E}_t z_{t-1} + \Gamma_3 \left(\theta \right) u_t$$

Assuming a unique solution which exists to the above system of equations, it takes the shape of the folloiwng form

$$z_t = A \left(\theta \right) z_{t-1} + B \left(\theta \right) u_t$$

Some of the variables are not observed, the above system is hence augmented by the measurement equations in the following form:

$$x_t = C z_t$$

The unconditional first and second moment of the model is then

$$\mathbb{E}_t x_t = \mu_x$$

$$\mathbb{E}_t x_{t+i} x_t = \Sigma_x \left(i \right)$$

where $\Sigma_x \left(i \right) = \begin{cases} C \Sigma_x \left(0 \right) C' & i = 0 \\ C A^i \Sigma_x \left(0 \right) C' & i > 0 \end{cases}$ and $\Sigma_x \left(0 \right)$ solves the matrix equation

$$\Sigma_x \left(0 \right) = A \Sigma_x \left(0 \right) A' + \Omega$$

where $\Omega = B \left(\theta \right) B' \left(\theta \right)$. Define the unconditional second moment for T observations as

$$\mathbb{E}_t X_T X_T' = \Sigma_T$$

Then the identification strategy is to check that the following matrix is the same given all parameters, $\sigma_T = \begin{bmatrix} \text{vec} \left(\Sigma_x \left(0 \right) \right)' & \text{vec} \left(\Sigma_x \left(1 \right) \right)' & \ldots & \text{vec} \left(\Sigma_x \left(T-1 \right) \right)' \end{bmatrix}'$,

$$\sigma_T \left(\theta \right) = \sigma_T \left(\theta_0 \right) \Longleftrightarrow \theta = \theta_0$$

The above global identification may never be achieved hence it is well advised to find the conditions resulting into local identification. The theorem states if $\sigma_T \left(\theta \right)$ is continously differentiable and let θ_0 be a regular point of the Jacobian matrix $J \left(T \right) = \frac{\partial \sigma_T}{\partial \theta'}$ then θ_0 is locally identifiable if the Jacobian has full column rank at the regular point.

A regular point is a point which there exists an open neighborhood where the rank of the matrix remains constant. This result is due to a classical work by Rothernberg (1971). A necessary condition for identification is that the number of deep parameters does not exceed the number of unique parameters in the utilized moment. If θ_j cannot be identified then Jacobian evaluated at that point is zero for all period (i.e. T). A corollary to the theorem is to look at the rank condition of $J_2(T) = \frac{\partial \tau}{\partial \theta'}$ where $\tau = \left[\text{vec}(A)' \ \ \text{vec}(C)' \ \ \text{vech}(\Omega)'\right]$.

We have conducted the exercise using our time series (in our case, GDP growth, normalized vacancy by labor force, real interest rate and inflation) and three variables (interest rate, inflation and vacancy). Results show strong degree of identification. Hence we can make sure the shocks are identified and the implied signs are not flawed.

Figure 1: Original data and median estimates of the states

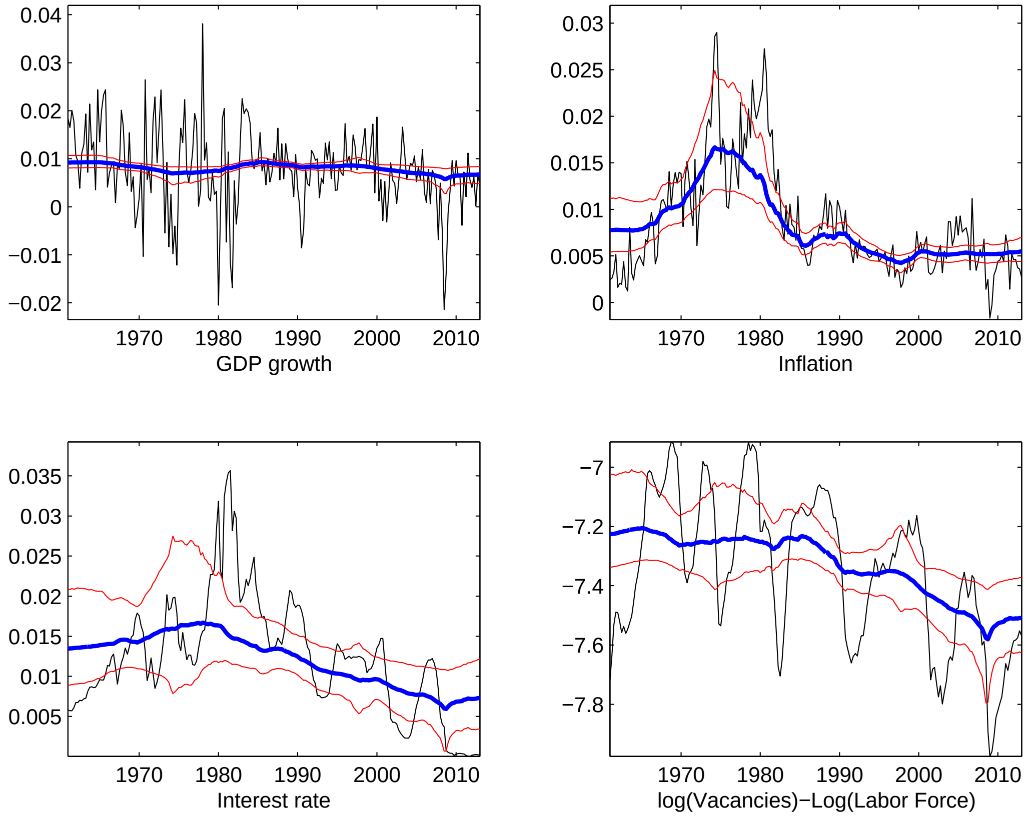

Figure 2: Median time-varying coefficients of the reduced-form VAR

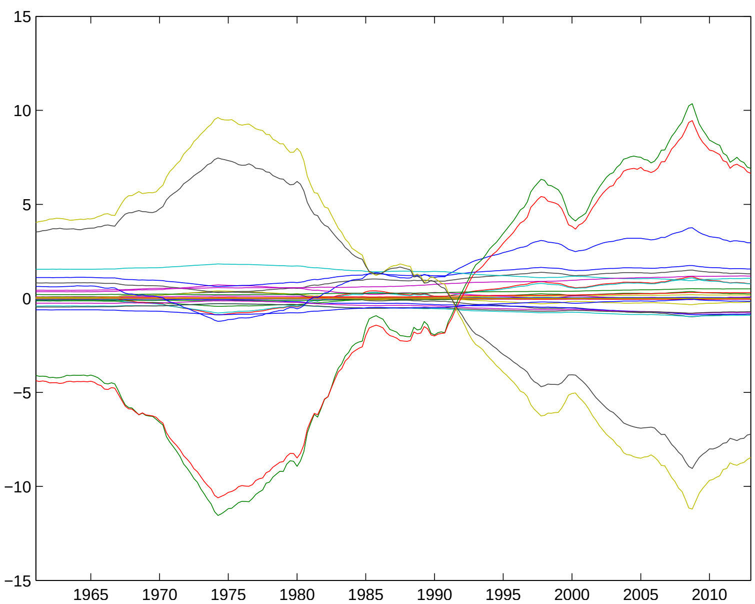

Figure 3: Time-varying total prediction variance (Bps)

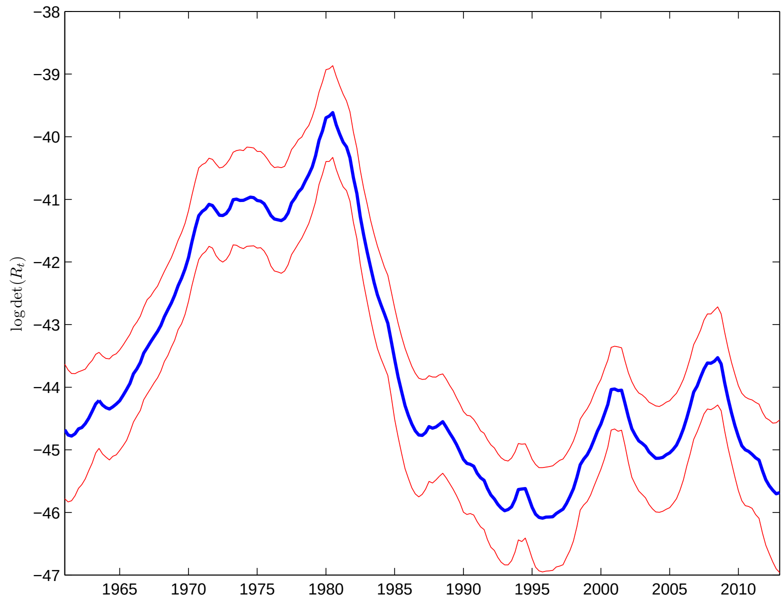

Figure 4: Time-varying volatility of reduced-form shocks (Bps)

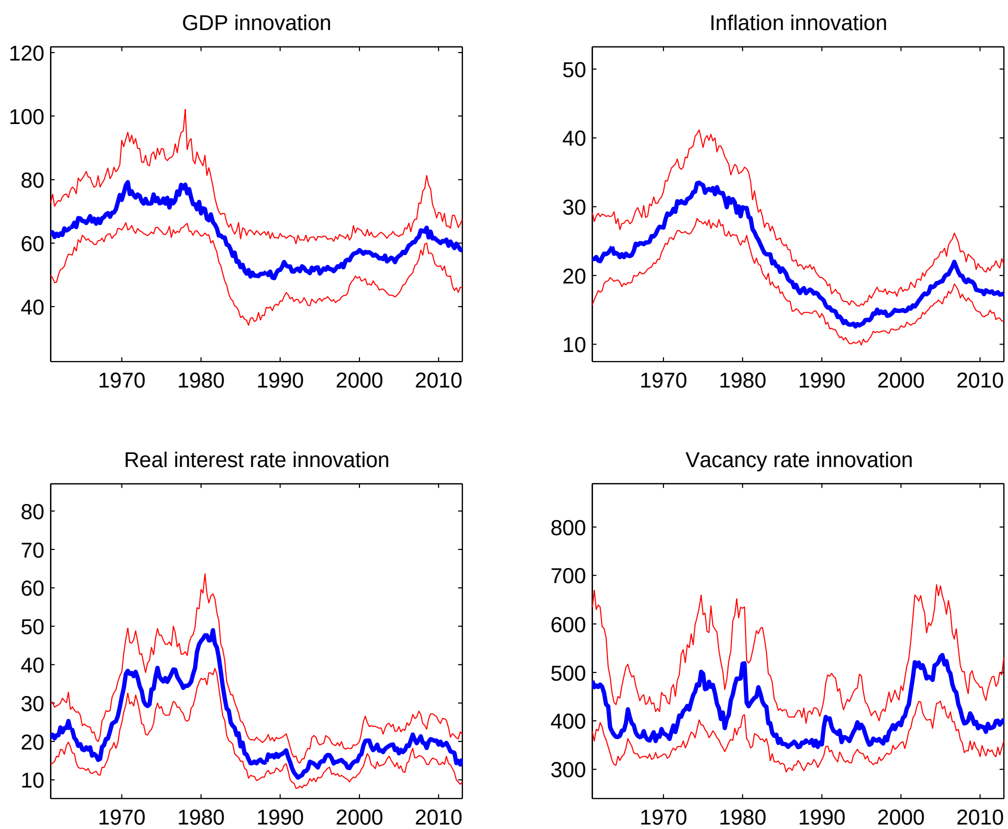

Figure 5: Time-avrying volatility of the reduced-form shocks relative to that of the GDP (Bps)

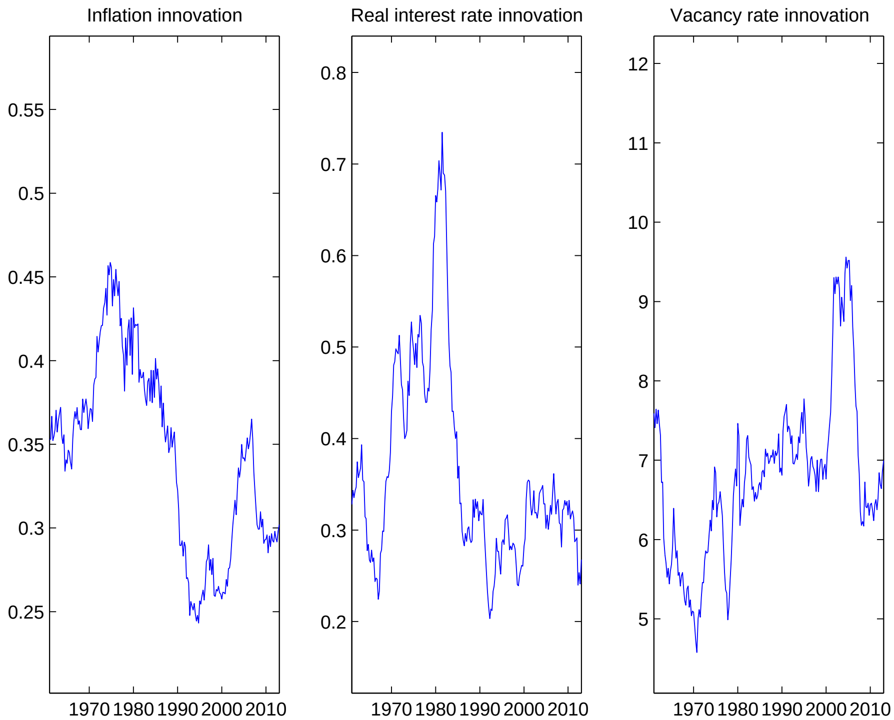

Figure 6: Time-varying correlations of the reduced-form shocks (Bps)

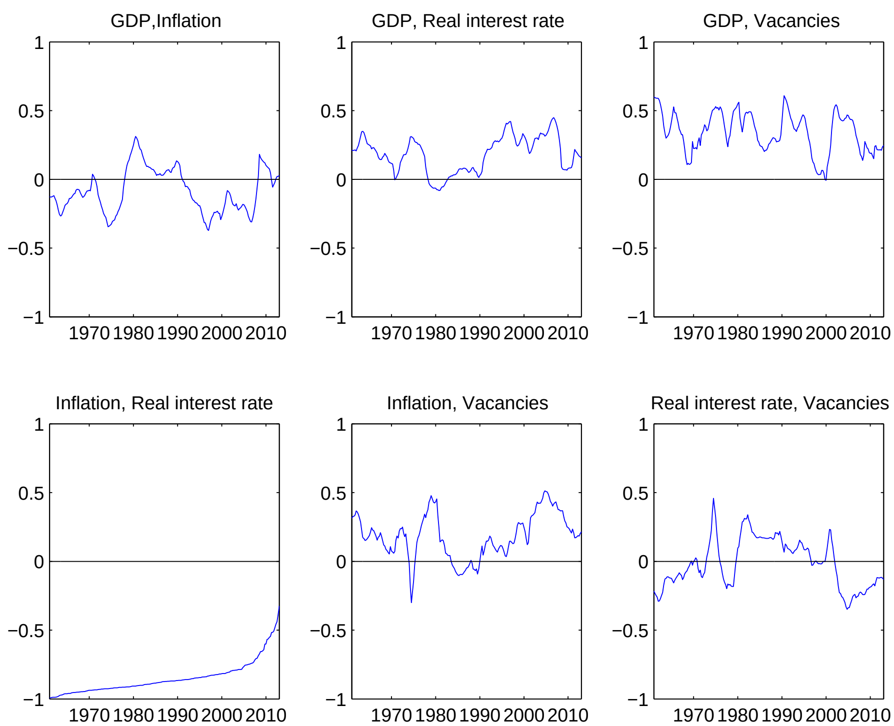

Figure 7: Vacancies: standardized time-varying response at different forecast horizons (Bps)

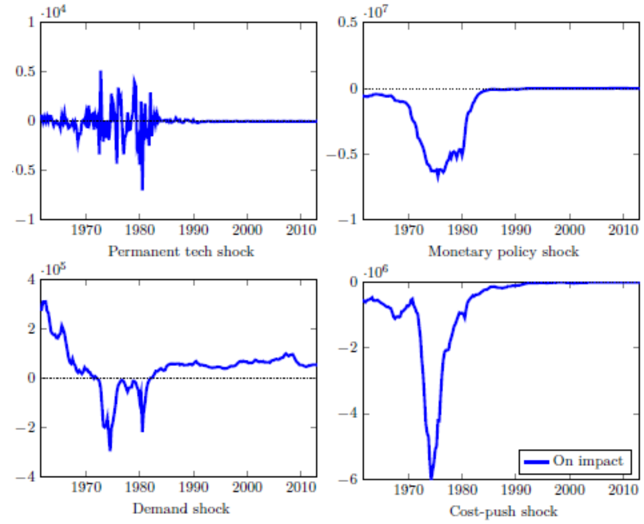

Figure 8: Vacancies: time varying Impulse Response Functions (Bps)

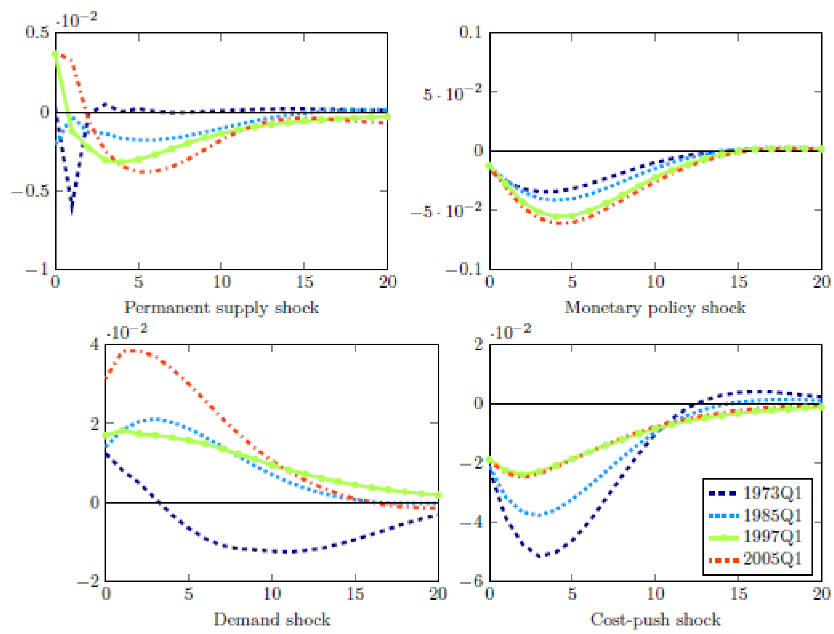

Figure 9: Vacancies time-varying conditional and unconditional correlations

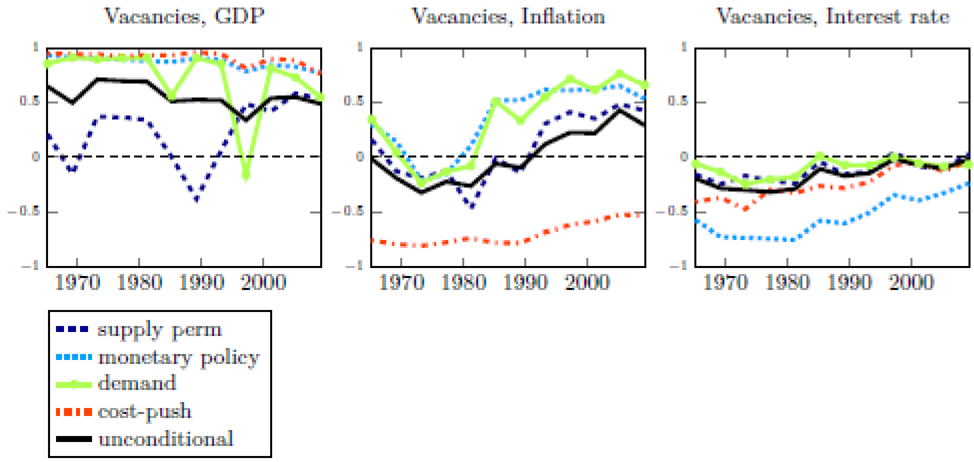

Figure 10: Time-varying conditional and unconditional variance

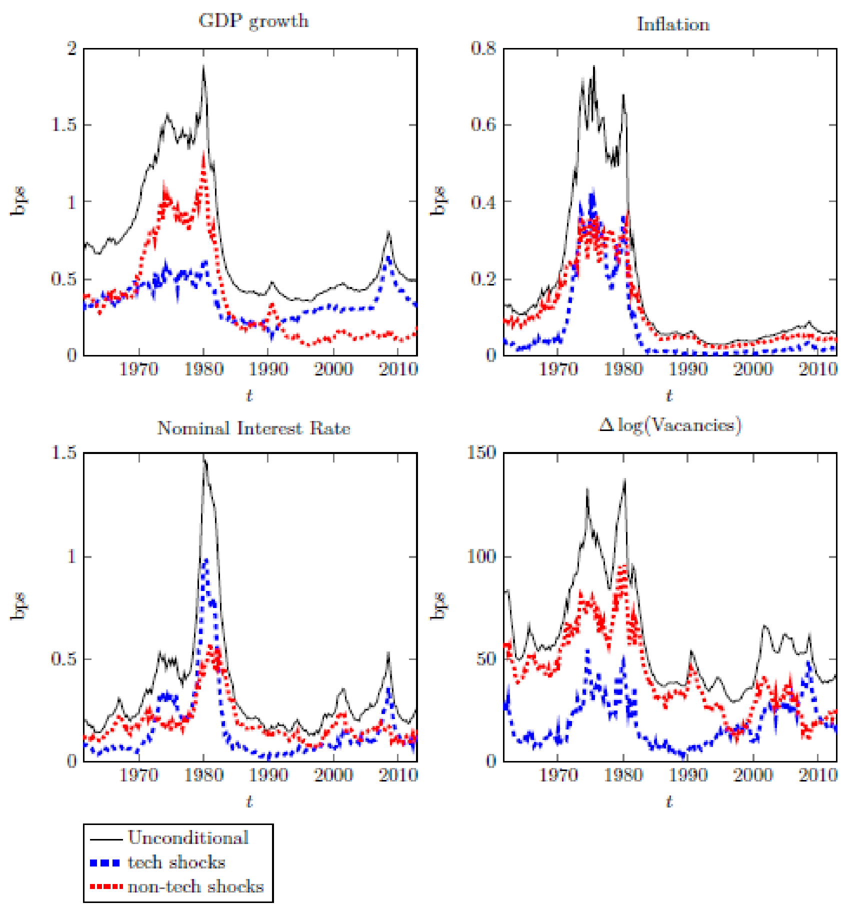

Figure 11: Vacancies Forecast Error Variance Decomposition

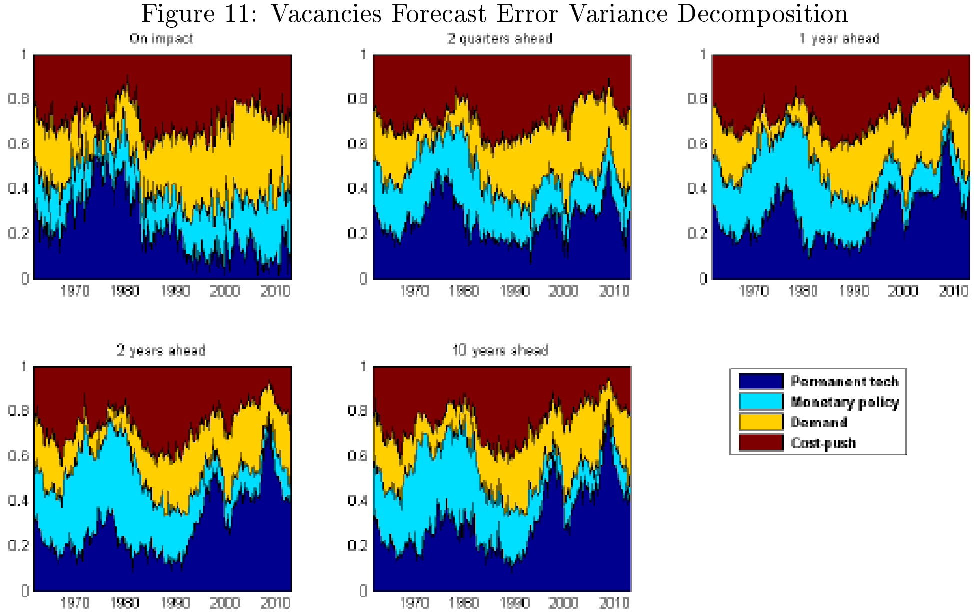

Figure 12: Strength of identification of shocks given the time series