Explaining structural changes towards and within the financial sector*

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Abstract

This paper presents a 3x3 general equilibrium model of an OLG-economy with technological uncertainty, heterogeneous agents and quasi-homothetic preferences to analyze structural change between the real and the financial sector as well as within the financial sector. Besides the consumption and investment good two types of financial services are produced. The three factors of production are: Capital, skilled and unskilled labor. The financial services are needed for transforming savings into future consumption possibilities. The financial market provides deposits and an incomplete set of securities. Payoffs of assets are determined by the future profitability of the technologies in which they are invested. We show the channels through which structural change and inequality reinforce each other and show how they simultaneously emerge from rising per-capita income, an increase in skill supply and technical change.
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1 Introduction

Financialization and inequality are two topics that stir up the public debate – among experts as well as outside the scientific community. Discussions about financialization have gained momentum by the financial crisis (Philippon and Reshef (2012, 2013), Greenwood and Scharfstein (2013)); the inequality debate was brought "in from the cold" (Atkinson, 1997) towards the end of the last century and has reached the center court recently with the Piketty book (Piketty, 2014). This paper argues that the two phenomena are genuinely related to each other. Structural change towards and within the financial sector, as observed over the last three decades, enhances inequality. And rising inequality fosters financialization.

We present our argument in a model that comprises the most basic tools provided by economics for analyzing sectoral structure and distribution. Financialization means two things: The weight of financial business relative to non-financial business increases and the type of financial business changes. From a macroeconomic perspective the first aspect can be summarized as structural change towards the financial sector: The financial sector expands relative to the production sector. We do not approach this question from a monetary or financial aspect like the nominal transaction volume of the financial relative to the real sector. Our perspective is a real economics one: The financial sector employs resources and generates income for the resources employed. That is, there must be some kind of output (service) that is produced, sold and purchased. The relevant measures are therefore employment and income or output shares; the essential component to be modeled are the production function of the financial sector and the demand function for financial services. For capturing the second aspect of financialization – the shift from conventional banking type activities to sophisticated modern finance – an appropriate model structure requires to have two separate sub-sectors within the financial sector which differ in their demand and production characteristics. In sum, we have therefore a three sector model – one production sector and two financial sub-sectors.

Inequality requires to have heterogeneous agents which differ in their endowments. In our model we have low-skilled and high-skilled workers. They are mobile between sectors and cost-minimal skill-intensities differ across sectors. As a consequence, the interaction between sectoral structure and inequality comes through the skill premium. The focus on inequality between low-skilled and high-skilled workers is on the one side motivated by the empirical fact that the rise in inequality over the last decades has been driven to a large extent by skill premia and skill composition, as the ample evidence from the skill-bias literature shows (for instance, Machin and Van Reenen (1998); Piketty and Saez (2003)). On the other side, we see it as a first important step which later might be complimented by elements which focus on the functional distribution of income between workers and capitalists. There is capital in our model; it must be. After all, financial markets have the purpose to transform, under risk, current resources into future production possibilities. This requires on the one side saving decisions and on the other side capital investment into revenue bearing inputs to future production. In our model, returns on capital are generated by two different types of technologies (robust and risky) which transform savings into future consumption possibilities.

Structural change can be caused by the supply side: Changing endowments or technical change. The huge literature on directed technical change, for instance, has emphasized this channel (Acemoglu (2002)). There is, however, also an important role for the demand side. Although often neglected, the works of Boppart (2014, 2015), Föllmi and Zweimüller (2008) or Ngai and Pissarides (2007), for instance, have convincingly shown that the demand side is essential. We account for demand side effects by assuming that agents have quasi-homothetic preferences of the Stone-Geary form. The specific finance aspect enters the demand side of our model through the following channel: Demand for financial services comes from the need to manage portfolios and to finance investments into profitable projects in a way that reflects the preferences of the agents who own the endowments of the economy.

In our model the finance industry correctly assesses risks and productivity of investment projects and earns no rents. This is against popular views; neither does it reflect a common view of the authors of this paper. Actually, there are many sources for imperfections in the financial sector. For instance, prices and payoffs of financial products many be distorted by neglected correlation (Studer, 2015) or insider knowledge and barriers to entry generate rents for financial intermediation. A salient example is the so called finance premium. There is convincing evidence that a finance premium exists (Philippon and Reshef (2007, 2012), Célérier and Vallée (2015), that is, the same type of labor earns more in a finance job than in other occupations. Nonetheless, from a methodological point of view we consider it as important to start with a benchmark model in which distortions are kept at a minimum. Given the firm basis of such a benchmark, one can then be bold in looking at the role of imperfections which certainly exists in reality in general and in the financial business in particular. Section 7 gives extensions which provide some ideas how distortions affect the comparative-static results of this paper.

The related literature in a more narrow sense is rich as far as the empirical side is concerned. In particular, Philippon and his co-authors did pioneering empirical work on financialization. On the theoretical side the situation is quite different. To our knowledge there are only two attempts to explain structural change towards finance in a general equilibrium framework. Philippon (2014) sketches in his notes a 2x2 model with a real and a financial sector both producing with capital and labor. Thus, neither structural change within the financial sector nor inequality are addressed. Moreover, there are two types of households - infinitely living saver households on the one side and households which live two periods and borrow when young. There is only one interest bearing asset. In contract, in this paper there is one type of household and savings can be invested in a portfolio of safe and risky assets. Moreover, unit costs and relative prices for intermediation services for the respective prices are endogenous in our model as well as the demand structure for the services. The second theoretical explanation of structural change towards finance is provided by Gennaioli et al. (2014). Like in Philippon (2014) a 2x2 framework is considered. The real sector produces with capital and labor, the financial sector consists of financial intermediation experts in whom investors trust. Therefore they are willing to pay them fees. Like in our set-up there is one type of households, which live two periods and save when they are young. Moreover, they also account for risky assets. The saving decision is exogenous - young households save the entire wage - and the portfolio choice is determined by mean-variance preferences. The main driver for structural change towards finance in their model is the idea that financial intermediation services are not only required for the financing of new capital but also for the preservation of the entire stock of capital accumulated over time. Since in a Solow type growth model the capital coefficient increases, the share of financial services in GDP increases, too. In our model, which focuses on comparative-static equilibrium effects of skills and endowments, technologies and preferences, no long-run accumulation effect is considered.

The structure of the paper is as follows. The next section outlines the formal structure of our 3x3 model and its building blocks. Section 3 analyzes the production equilibrium, Section 4 derives the demand for goods and financial services. Section 5 summarizes the effects of inequality on the sectoral structure of the economy. In Section 6 the general equilibrium is characterized and comparative-static effects are derived. Section 7 gives extensions which provide some ideas how distortions affect the comparative-static results of this paper. An alternative to the benchmark specification of the model is discussed and the robustness of the results is shown in Section 8. Section 9 confronts the theoretical results with empirical evidence from the U.S.. Moreover, a calibration exercise is provided. Main conclusions are summarized in the last section.

2 Model

2.1 Model set-up

We model a 3 sector, 3 factor economy. There is a production sector X and a finance sector Z with two sub-sectors Z_1 and Z_2. All sectors employ low-skilled and high-skilled workers. Produced goods are used for consumption and investment. For transforming savings into future consumption possibilities more or less risky technologies are available which use capital as input and deliver consumption goods as output in the next period. (As an extension we present a variant of the model, in which capital is used in the X sector to set up firms.) Financial services have the function to support the transformation of savings into future consumption possibilities. Services Z_1 are used for safe savings. Services Z_2 provide state-dependent instruments and are used for savings in securities with risky returns.

We consider a (static) two-period OLG economy. The future $t = 1$ is uncertain. It consists of a set Θ of distinguishable events and a set $\bar{\Theta}$ of events which are indistinguishable in $t = 0$. The future state space is $\left\{ \{\theta | \theta \in \Theta\}, \bar{\Theta} \right\}$. We have prob($\Theta$)=$\mu$ and prob($\theta|\Theta$)=π_θ with $\sum_{\theta \in \Theta} \pi_\theta = 1$.[1] For $\theta \in \Theta$, state-contingent investment possibilities are available which pay off if and only if state θ is realized. No state-contingent investment possibilities exist for $\bar{\Theta}$ which reflects "true uncertainty".

2.2 Saving decision and portfolio choice

There are N agents who live for two periods. They are endowed with a skill level and work as either high-skilled or low-skilled worker when young. The number of low-skilled workers is $\bar{L}$ and the number of high-skilled workers is $\bar{H}$. The efficiency units of labor provided by a high-skilled and a low-skilled agent are given by b_H and b_L, respectively. They are paid a wage per efficiency unit at rate, $w_l, l \in \{L, H\}$. Income $y^l = w_l b_l$ can be con-

[1] This structure is taken from Falkinger (2014).

sumed in $t = 0$ or be saved and transformed to tomorrow's consumption possibilities. Agents are assumed to have quasi-homothetic preferences of the Stone-Geary form: Beyond a subsistence level to be expend they spend income on the good produced in the X-sector. They have an instantaneous indirect utility function of the form $\log(e_t - \bar{e}_t)$ where e_t is the expenditure for good X consumption and $\bar{e}_t$ is the subsistence expenditure level in time t. Intertemporal preferences are assumed to be additive logarithmic with a discount factor δ.

The intertemporal problem consists of two parts: A saving decision and a portfolio choice. On the one hand, agents have to decide how much to expend on consumption, e_0, and how much to save, s. On the other hand, they have to put the saving in an appropriate portfolio of financial products. For this purpose they demand financial services. With the support of these services they decide how much of the saving is put into deposits, d, with a safe payoff r, and how much into risky state-contingent financial products (Arrow securities), f_θ, which pay off R_θ if state θ is realized and zero otherwise. We assume that the Arrow securities have the same expected payoff, Specifically, there exists $R > 0$ so that

$$R_\theta = \frac{R}{\pi_\theta}, \quad \theta \in \Theta. \tag{1}$$

For transforming one unit of deposit, n_1 units of financial services from sub-sector 1 are needed, and for transforming one unit of Arrow securities, n_2 units of financial services from sub-sector 2 are required. Therefore, given the portfolio choice, $\{d, f\}$, with $f = \sum_{\theta\in\Theta} f_\theta$, agents have to pay a fee $T = p_{z_1} n_1 d + p_{z_2} n_2 f$ to the financial sector, where p_{z_1} and p_{z_2} are the prices for financial services Z_1 and Z_2, respectively. Without loss of generality, we assume that $n_1 = n_2 = 1$.[2] Suppose the fee is charged in the first period and agents internalize the fee in their portfolio choice. The expected utility maximization problem of an agent l with income y^l is then given by:

$$\max_{s^l,\{f_\theta^l\}_{\theta\in\Theta},d^l} \mathbb{E}U = \log(e_0^l - \bar{e}_0) + \delta\left[\mu \sum_{\theta\in\Theta} \pi_\theta \log(e_\theta^l - \bar{e}_1) + (1-\mu)\log(e_{\bar{\Theta}}^l - \bar{e}_1)\right]$$

s.t.

$$e_0^l + (1 + p_{z_1})d^l + (1 + p_{z_2})(s^l - d^l) = y^l, \tag{2}$$

$$e_\theta^l = \begin{cases} R_\theta f_\theta^l + rd^l, \text{ if } \theta \in \Theta \\ rd^l, \text{ otherwise} \end{cases} \tag{3}$$

$$s^l = \sum_{\theta\in\Theta} f_\theta^l + d^l. \tag{4}$$

In Section 4 aggregate demand functions for goods and financial services are derived from this program.

2.3 Production of goods (X-sector)

Firms in the X-sector employ low-skilled and high-skilled labor as input factors in a linear homogeneous production function

$$X = G^x(H_X, L_X),$$

[2]Without this normalization the cost of financial services per unit of saving would be $\tilde{p}_{z_i} = p_{z_i} n_i$ rather than p_{z_i}. Thus, for an empirical interpretation it is important to keep in mind that in our analysis p_{z_i} is the price per unit of saving rather than per unit of service.

where H_X, L_X denote respective labor employment in the X-sector. There is perfect competition with zero-profit prices. This means:

$$p_x = c_x(w_H, w_L), \tag{5}$$

where $c_x(w_H, w_L)$ are the unit costs and w_H, w_L are the wage rates per efficiency units.

The goods price is taken as numéraire, $p_x = 1$. Revenue X is distributed to labor as follows:

$$W_x = w_L L_x + w_H H_x = G^x(L_x, H_x),$$

where W_x is total wage earned in the X-sector.

Capital is used in technologies which transform savings into future consumption possibilities. Two types of technologies are available: A robust technology, which transforms under any condition (i.e., in Θ and $\bar{\Theta}$) one unit of capital invested today into r units of output tomorrow; furthermore, for $\theta \in \Theta$, a set of risky technologies specialized to θ-contingent environments. One unit of capital invested in technology θ delivers R_θ units of output if state $\theta \in \Theta$ occurs tomorrow and zero otherwise. Deposits are invested in the robust technology; savings in securities are invested in the respective risky technologies. The smaller the measure π_θ of the state to which a risky technology is targeted, the more productive the capital invested in the technology. Equation (1) expresses this relationship between specialization advantage and risk.

In the extension in Section 7.5, we show that essentially the same payoff structure arises if X is produced under monopolistic competition and capital is needed to set up firms – by robust and risky set-up technologies, respectively. Asset returns are then generated by the operating profits of the firms the set up of which has been financed by the asset.

In almost all of the further analysis only the relative payoff between robust and specialized risky technologies matters. It is given by:

$$\rho \equiv \frac{r}{R}.$$

The only exception is the discounting of future subsistence expenditure $\frac{\bar{e_1}}{r}$, for which the level of the return on the robust technology matters.

2.4 Production of financial services (Z-sectors)

The financial sector Z consists of two sub-sectors, Z_1 and Z_2. They provide financial services for transforming savings through safe and risky assets into future consumption possibilities. (The assets are invested in the robust and risky technologies, and households get the generated revenue as return on their investment.) Z_i, $i \in \{1, 2\}$, is produced with a linear homogeneous production function $G^{z_i}(.)$:

$$Z_i = G^{z_i}(H_{z_i}, L_{z_i}), \quad i \in \{1, 2\} \tag{6}$$

where H_{z_i}, L_{z_i} denote the volumes of resources (skill employments) in the Z_i-sector.

We assume perfect competition in the Z-sectors and have therefore zero-profit prices

$$p_{z_i} = c_{z_i}(w_H, w_L), \quad i \in \{1, 2\} \tag{7}$$

where $c_{z_i}(w_H, w_L)$ are the unit costs.

Revenue $p_{z_i} Z_i,\ i \in \{1, 2\}$, is distributed to labor

$$W_{z_i} = w_L L_{z_i} + w_H H_{z_i} = p_{z_i} G^{z_i}(H_{z_i}, L_{z_i}), \quad i \in \{1, 2\}$$

where W_{z_i} is total labor income earned in the Z_i-sector.

3 Production equilibrium and supply of goods and financial services

At the production side, the essential feature we want to address is variation in skill intensities. For an explicit comparative-static analysis we take production functions of the Cobb-Douglas form.

Let, for $j \in \{x, z_1, z_2\}$, G^j have Cobb-Douglas form

$$G^j(L_j, H_j) = A_j L_j^{1-\alpha_j} H_j^{\alpha_j},$$

where A_j is total factor productivity and α_j is the factor share of high-skilled workers in sector j. Then

$$a_j^L = \frac{1}{A_j \kappa_j^{\alpha_j}}, \quad a_j^H = \frac{\kappa_j^{1-\alpha_j}}{A_j}, \tag{8}$$

are the input coefficients, and cost-minimizing skill-intensities $\kappa_j \equiv a_j^H / a_j^L$ are given by

$$\kappa_j(\omega) = \frac{\gamma_j}{\omega}, \quad \gamma_j \equiv \frac{\alpha_j}{1 - \alpha_j}, \tag{9}$$

where $\omega \equiv w_H / w_L$ is the relative wage for one efficiency unit of skilled labor to unskilled labor, which reflects the skill premium (per efficiency unit).

3.1 Wages and prices

We have for variable unit costs in sector j:

$$c_j(w_H, w_L) = \frac{w_L^{1-\alpha_j} w_H^{\alpha_j}}{A_j \Gamma_j}, \ \Gamma_j \equiv \alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}. \tag{10}$$

Using (10) in the zero-profit price equation (5), and using $p_x = 1$, we obtain

$$w_L = A_x \Gamma_x \omega^{-\alpha_x}, \tag{11}$$

and from (7), for $i \in \{1, 2\}$,

$$p_{z_i} = \frac{A_x}{A_{z_i}} \frac{\Gamma_x}{\Gamma_{z_i}} \omega^{\alpha_{z_i} - \alpha_x}. \tag{12}$$

In sum, prices for financial services are related to the skill premium in the following way:

Fact 1. *The price of financial services Z_i, p_{z_i}, is an increasing function of ω if $\alpha_{z_i} > \alpha_x$. If $\alpha_{z_i} = \alpha_x$, then p_{z_i} is invariant with respect to ω. Moreover, $\alpha_{z_i} > \alpha_x$ ($\alpha_{z_i} = \alpha_x$) is equivalent to $\kappa_{z_i} > \kappa_x$ ($\kappa_{z_i} = \kappa_x$).*

As known from the Stolper-Samuelson theorem, this fact holds quite generally and is not an artifact of the Cobb-Douglas specification.

In the further analysis we make the following assumption about the factor intensity ranking of the three sectors.

Assumption 1. *$\alpha_{z_2} \geq \alpha_{z_1}$ and $\alpha_{z_1} \geq \alpha_x$ with at least one inequality holding strictly.*

In Section 9 we provide evidence on the sectoral skill intensities. The magnitude of total factor productivities depends on the unit in which financial services are measured. Since financial services are measured in units of savings, $A_x < A_{z_1} \leq A_{z_2}$ is a plausible restriction on total factor productivities. Analytically no such restriction is required for the results.

3.2 Resource constraints

Let total labor endowment in efficiency units be given by

$$L = b_L \bar{L}, \quad H = b_H \bar{H},$$

where $\bar{L}, \bar{H}$ denote the number of low-skilled and high-skilled workers and b_L, b_H are the efficiency units of labor supplied by the respective workers. The "skill richness" of the total labor force is

$$k \equiv \frac{b_H \bar{H}}{b_L \bar{L}}.$$

The aggregate resource constraints are:

$$\begin{aligned} a_x^L X + a_{z_1}^L Z_1 + a_{z_2}^L Z_2 &= b_L \bar{L}, \\ a_x^H X + a_{z_1}^H Z_1 + a_{z_2}^H Z_2 &= b_H \bar{H}. \end{aligned} \tag{13}$$

with a_j^l, $j \in \{x, z_1, z_2\}$, $l \in \{H, L\}$ being functions of the skill premium ω defined in (8).

We focus first on the allocation within the financial sector. Let total employment (in efficiency units) in the financial sector be given by L_z and H_z, respectively. Suppose $\alpha_{z_2} > \alpha_{z_1}$. Then the resource constraints $a_{z_1}^L Z_1 + a_{z_2}^L Z_2 = L_z$ and $a_{z_1}^H Z_1 + a_{z_2}^H Z_2 = H_z$ solve to:

$$Z_1 = \frac{L_z(\kappa_{z_2} - k_z)}{a_{z_1}^L(\kappa_{z_2} - \kappa_{z_1})}, \quad Z_2 = \frac{L_z(k_z - \kappa_{z_1})}{a_{z_2}^L(\kappa_{z_2} - \kappa_{z_1})}, \tag{14}$$

where $k_z \equiv \frac{H_z}{L_z}$ is the "skill richness" of the labor force in the financial sector. This implies for the supply structure within the financial sector:

$$\frac{Z_2}{Z_1} = \frac{a_{z_1}^L}{a_{z_2}^L} \frac{k_z - \kappa_{z_1}}{\kappa_{z_2} - k_z} \equiv \chi(\underset{+}{\omega}, \underset{+}{k_z}) \tag{15}$$

The following result on within sector structural change follows immediately.

Proposition 1. *If $\alpha_{z_2} > \alpha_{z_1}$, for a given level of employment in the financial sector, an increase in the skill premium as well as a rise in the skill richness of labor employed in the financial sector shift the supply structure from traditional financial services Z_1 to new financial services Z_2.*

Proof. According to (9), $\kappa_{z_2} > \kappa_{z_1}$ if $\alpha_{z_2} > \alpha_{z_1}$. For $\kappa_{z_2} > \kappa_{z_1}$, $\frac{\partial \chi}{\partial \omega} > 0$ and $\frac{\partial \chi}{\partial k_z} > 0$, as known from the Rybczynski analysis. □

Moreover, for a given level of the skill richness, k_z, of labor employed in the financial sector, system (13) can be written in the form

$$\begin{aligned} a_x^L X + L_z &= b_L \bar{L} \\ a_x^H X + k_z L_z &= b_H \bar{H} \end{aligned} \tag{16}$$

which leads to the following result.

Fact 2. *For a given level of skill richness in the financial sector, we have*

$$\frac{L_z}{L_x} = \frac{k - \kappa_x}{k_z - k}. \tag{17}$$

Proof. System (16) solves to $L_x = b_L \bar{L} \frac{k_z - k}{k_z - \kappa_x}$, $L_z = b_L \bar{L} \frac{k - \kappa_x}{k_z - \kappa_x}$. Assumption 1 implies $k_z > k > k_x$. □

Thus, for a given skill premium ω (so that κ_x is fixed) and a given skill richness k_z in the financial sector, employment in the financial sector is ceteris paribus higher in an economy with a large share of skilled labor k.

In a general equilibrium, however, employment in the financial sector is determined simultaneously with the allocation of resources to the goods sector.

4 Income distribution and aggregate demand

The demand for financial services comes from the need of agents to transform current savings into future income. For this purpose the asset-holding agents require financial products and expert services from the financial sector which support them by choosing and managing a portfolio of deposits and securities appropriate for the agents' preferences. For given prices p_{z_1} and p_{z_2} of the relevant financial services, the optimal portfolio is derived (see the Appendix A).

The program max $\mathbb{E}U$ subject to (2)-(4) is only well-defined if $e_0 > \bar{e_0}$ and $e_1 > \bar{e_1}$. This requires that

$$y^l = b_l w_l > \bar{y} \equiv \bar{e}_0 + (1 + p_{z_1}) \frac{\bar{e_1}}{r}, \quad l \in \{L, H\} . \tag{18}$$

$\bar{y}$ denotes the present value of subsistence expenditures in units of today's final output.

Assuming $y^H \geq y^L$, which is equivalently $\omega \geq b_L/b_H$, $y^L \geq \bar{y}$ is sufficient for (18). The following fact gives a necessary and sufficient condition for

$y^L > \bar{y}$. The sign below the parameters shows the sign of the respective partial derivatives.

Fact 3. *There exists a threshold* ω_L^+, *so that* $y^L > \bar{y}$, *if and only if* $\omega < \omega_L^+(\underset{+}{A_x}, \underset{+}{A_{z_1}}, \underset{+}{b_L}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}})$.

Proof. Appendix C. □

Savings in securities is positive if and only if the following condition holds: $\mu R\left(1+p_{z_1}\right) > \left(1+p_{z_2}\right) r$. The condition can be rewritten in the form

$$\mu > p\rho, \quad p \equiv \frac{1+p_{z_2}}{1+p_{z_1}}, \quad \rho \equiv \frac{r}{R}. \tag{19}$$

$p\rho$ is the relative net payoff (i.e., after correction for costs in terms of prices) of savings in safe assets compared to savings in risky assets. If the condition is violated, the expected net payoff of risky investments is lower than the net payoff of risk-free investments and all savings are in deposits.

In the next subsection we analyze individual saving and expenditure behavior. Subsection 4.2 deals with aggregate demand.

4.1 Individual saving and expenditure behavior

As is derived in Appendix A, under the assumption that inequalities (18) and (19) are satisfied, individual savings in deposits and securities are given by

$$d^l = s_d \frac{\delta}{1+\delta} \frac{y^l - \bar{y}}{1+p_{z_1}} + \frac{\bar{e_1}}{r}, \quad l = \left\{L, H\right\}, \tag{20}$$

and

$$f^l = s_f \frac{\delta}{1+\delta} \frac{y^l - \bar{y}}{1+p_{z_2}}, \quad f^l_\theta = \pi_\theta f^l, \quad \theta \in \Theta, \quad l = \{L, H\}, \tag{21}$$

respectively, with

$$s_d = \frac{1-\mu}{1-p\rho}, \quad s_f = \frac{\mu - p\rho}{1-p\rho}. \tag{22}$$

Apart from the savings for future subsistence expenditure $\frac{\bar{e_1}}{r}$ in form of deposits, the saving level is proportional to the supernumerary budget $y^l - \bar{y}$. In real terms, the value of the supernumerary budget, which is relevant as a basis for saving, depends on the price of the financial service charged on the particular form of savings – p_{z_1} for deposits and p_{z_2} for securities. The split of the savings on safe and risky assets is given by the marginal propensities to save in deposits, s_d, and in securities, s_f, respectively. The propensity of safe investment increases in the relative net payoff of safe assets, $p\rho$, and declines with the measure μ of states covered by securities. The propensity of risky investments reacts in the opposite direction.[3] In sum, the two propensities add up to one so that total savings, $s^l = d^l + f^l$, is given by:

$$s^l = \frac{\delta}{1+\delta} \frac{y^l - \bar{y}}{1+p_{z_1}} (s_d + \frac{s_f}{p}) + \frac{\bar{e_1}}{r} \tag{23}$$

If savings in securities is more costly than savings in deposits, s_f is discounted by the fee differential p.[4]

[3]For $\bar{e_0} = \bar{e_1} = 0$ and $p_{z_1} = p_{z_2} = 0$, we have $s_d = \frac{1-\mu}{1-\rho}$ and $s_f = \frac{\mu-\rho}{1-\rho}$. Defining $\bar{R} = \frac{R}{\mu}$ and $\bar{\rho} = \frac{r}{\bar{R}}$, we can rewrite the two terms in the form $s_d = \frac{\bar{R}(1-\mu)}{\bar{R}-r/\mu}$ and $s_f = \frac{\mu\bar{R}-r/\mu}{\bar{R}-r/\mu}$. Thus, with Cobb-Douglas preferences and zero financial intermediation cost, the portfolio choice coincides with the one in Acemoglu and Zilibotti (1997) where the conditional expectation $\bar{R}$ of the productivity of risky technologies is used rather than the unconditional expectation R.

[4]If inequality (19) is violated, then savings in securities is unattractive in the first place and we have a corner solution with $s_f = 0$ and $s_d = s = \frac{\delta}{1+\delta}\frac{y-\bar{y}}{1+p_{z_1}} + \frac{\bar{e_1}}{r}$.

In contrast to net savings, gross savings include the fee to be paid for the financial services consumed in support for the transformation of savings into future income. Adding up $(1+p_{z_1})d^l + (1+p_{z_2})f^l$, we have

$$s^l + t^l = \frac{\delta}{1+\delta}(y^l - \bar{y}) + \frac{(1+p_{z_1})\bar{e_1}}{r}, \tag{24}$$

where $t^l = p_{z_1}d^l + p_{z_2}f^l$ denotes the total fee paid by agent l.

Current expenditures $e_0^l = y^l - (s^l + t^l)$ are thus:

$$e_0^l = \frac{1}{1+\delta}(y^l - \bar{y}) + \bar{e_0}. \tag{25}$$

For the discussion of structural change within the financial sector on the demand side, the question how the portfolio structure reacts to income is of particular importance.[5] According to (20) and (21), richer agents invest a larger share of their saving in risky assets than the relatively poorer ones. The reason is that the provision for future subsistence expenditure by safe investments has diminishing weight if people become richer. This means that saving in deposits have the character of a "necessity" and saving in risky securities are a "luxury". The following fact summarizes this important implication of our model.

Fact 4. *If* $\bar{e_1} > 0$, *then* $\frac{\partial(f/d)}{\partial y} > 0$.

Proof. Follows immediately from (20) and (21). □

[5] Boppart (2015) analyzes the skill-content of the consumption basket of different income groups. With rising income, a household's demand shifts towards skill-intensive sectors (including financial services; also shown by Suellow (2015) in detail).

4.2 Aggregate demand for goods and financial services

Saving and expenditure behavior follow affine-linear functions. Therefore, aggregate behavior depend on two things: The level of aggregate income and the number of people over which the income is distributed. The latter comes in through the fact that subsistence requirements are bound to the existence of an agent, independent of her or his income.

Aggregating the two pools of agents, we have

$$N = \bar{L} + \bar{H}$$

for the size of the population and

$$W = w_L b_L \bar{L} + w_H b_H \bar{H}$$

for the level of aggregate income. In view of (11), the latter amounts to

$$W = A_x \Gamma_x b_L \bar{L} \omega^{-\alpha_x} (1 + \omega k). \tag{26}$$

The following fact shows that aggregate income, measured in units of X, is an increasing function of the skill premium (remember $\omega = w_H/w_L$).

Fact 5. *Under Assumption 1, W is increasing in ω. Specifically, we have*

$$\frac{\partial W}{\partial \omega} = A_w \omega^{-\alpha_x} (1 - \alpha_x)(k - \kappa_x) > 0, \tag{27}$$

where $A_w \equiv A_x \Gamma_x b_L \bar{L}$.

Proof. According to (26),

$$\begin{aligned} \frac{\partial W}{\partial \omega} &= A_w \omega^{-\alpha_x} \left[-\frac{\alpha_x}{\omega}(1 + \omega k) + k \right] \\ &= A_w \omega^{-\alpha_x} \left[-\frac{\alpha_x}{\omega} + (1 - \alpha_x) k \right] = A_w \omega^{-\alpha_x} (1 - \alpha_x) \left[k - \frac{\alpha_x}{1 - \alpha_x} \frac{w_L}{w_H} \right]. \end{aligned}$$

According to (9),

$$\frac{\alpha_x}{1-\alpha_x} = \frac{w_H a_x^H}{w_L a_x^L}.$$

Thus, the square-bracketed term reduces to $k - \kappa_x$, which is positive if Assumption 1 holds. □

Financial services provision is more skill intensive than goods production, at least on average. Therefore, in terms of goods, the wages rise. A different matter is the impact of the skill premium on the purchasing power for financial services, the price of which rises with the skill premium.

Aggregating individual investments in deposits, given by (20), we obtain

$$D = \left(s_d \frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1+p_{z_1}} + \frac{\bar{e_1}}{r} \right) N, \tag{28}$$

where $\bar{w} \equiv \frac{W}{N}$ denotes average income. In an analogous way, we have from (21):

$$F = s_f \frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1+p_{z_2}} N, \quad F_\theta = \pi_\theta F \tag{29}$$

for aggregate investments in securities. Aggregate savings are

$$S = \left[\frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1+p_{z_1}} \left(s_d + \frac{s_f}{p} \right) + \frac{\bar{e_1}}{r} \right] N \tag{30}$$

and aggregate current expenditures are

$$E_0 = \left[\frac{1}{1+\delta} (\bar{w} - \bar{y}) + \bar{e_0} \right] N. \tag{31}$$

5 The effect of the skill premium on the sectoral structure

In a general equilibrium, sectoral structure and skill premium are determined simultaneously. As an intermediate step we characterize the sectoral structure as a function of the skill premium and exogenous parameters, keeping in mind that in the end the skill premium depends on exogenous parameters too. Not all possible combinations of skill premia and parameters are of interest, but only those which are reasonable candidates for a general equilibrium in which both financial sectors are viable, the subsistence of all agents is feasible and a positive skill premium results. The following paragraphs characterize the set of parameter configurations which guarantee these equilibrium properties.

Assumption 1 that financial service provision is more skill intensive than goods production ($\kappa_x < k < \kappa_z$) is equivalent to $\frac{\gamma_x}{k} < \omega < \frac{\gamma_z}{k}$ as we know from (9). At $\omega_{min} \equiv \frac{\gamma_x}{k}$ the Z-sector vanishes and beyond $\omega_{max} \equiv \frac{\gamma_z}{k}$ there would be no longer an X-sector. Hence, we consider the range $\omega \in (\omega_{min}, \omega_{max})$ in our search for the equilibrium skill premium.

Moreover, according to Fact 3, $\omega < \omega_L^+(\underset{+}{A_x}, \underset{+}{A_z}, \underset{+}{b_L}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}})$ is required for guaranteeing subsistence for low-skilled agents. $\omega_L^+ \geq \omega_{max}$ holds if A_x, A_z, b_L or k are large enough (for given $\bar{e_0}, \frac{\bar{e_1}}{r}$), or $\bar{e_0}$ and $\frac{\bar{e_1}}{r}$ are not too high (for given A_x, A_z, b_L, k). If $\omega_L^+ < \omega_{max}$, only range $\omega \in (\omega_{min}, \omega_L^+)$ is feasible.

Finally, $\omega \geq b_L/b_H$ is required for $y^H \geq y^L$. This is guaranteed if $\omega_{min} \geq$

b_L/b_H, which is equivalent to

$$\gamma_x \geq \frac{\bar{H}}{\bar{L}}.$$

In terms of exogenous fundamentals, the requirements mean that we restrict the possible combinations of exogenous model parameters

$$\xi = \left\{ A_x, A_{z_1}, A_{z_2}, \alpha_x, \alpha_{z_1}, \alpha_{z_2}, b_L, b_H, \bar{H}, \bar{L}, k, \bar{e_0}, \frac{\bar{e_1}}{r}, \rho \right\}$$

to the following set:

$$\Xi_0 = \left\{ \xi \,\middle|\, \frac{\bar{H}}{\bar{L}} \leq \gamma_x, \frac{\gamma_x}{k} < \tilde{\omega}_{max} \right\}, \tag{32}$$

where $\tilde{\omega}_{max} \equiv \min \left\{ \omega_{max}, \omega_L^+(A_x, A_{z_1}, b_L, \bar{e_0}, \frac{\bar{e_1}}{r}) \right\}$.

In general, the interaction of allocation of resources between the X-sector and the Z-sector, on the one hand, and the allocation within the Z-sector on Z_1 and Z_2, on the other hand, are hard to disentangle in an economically transparent way. For qualitative robust insights on important channels we have to reduce complexity on either the demand or the supply side. In the benchmark analysis, we shut down relative price effects within the financial sector by assuming identical technologies for Z_1 and Z_2 (i.e., $\alpha_{z_1} = \alpha_{z_2} \equiv \alpha_z$ and $A_{z_1} = A_{z_2} \equiv A_z$).[6] This allows us to put focus on the income effects. Moreover, the assumption is motivated by Swiss evidence which suggests that skill intensities in the finance sector are higher than in production but of similar magnitude within the financial sector. US data, however, suggests

[6]Without normalization $n_1 = n_2 = 1$, the assumption would read $\frac{A_{z_1}}{n_1} = \frac{A_{z_2}}{n_2}$. That is the provision of financial services per unit of saving must be equal in the two sub-sectors. For instance, new financial services may be provided more productively than traditional services, but then at the same time more units of services are needed to transform a unit of saving into future payoff by complex rather than simple financial products.

that skill intensity in traditional finance is close to the intensity in production. So we consider in Section 8 also the case $\alpha_{z_2} > \alpha_{z_1} = \alpha_x$. Moreover, in a quantitative implementation of the model we provide numerical illustrations for $\alpha_{z_2} > \alpha_{z_1} > \alpha_x$ as a robustness check.

The purpose of this paper is to explain two types of structural change – the one between the goods sector and the financial sector and the other one within the financial sector. We analyze first the impact of an increase in the skill premium on structural change within the financial sector.

5.1 Within change

Value added of sub-sector $Z_i, i = \{1, 2\}$, is equal to aggregate expenditure on the produced services. According to (28) and (29), aggregate expenditures for financial services have the following structure:

$$\frac{p_{z_2} F}{p_{z_1} D} = \frac{s_f \zeta \bar{\eta}}{s_d \bar{\eta} + \frac{1+\delta}{\delta} \frac{\bar{e_1}}{r}} \equiv \Phi(\underset{-}{s_d}, \underset{+}{s_f}, \underset{-}{\frac{\bar{e_1}}{r}}, \underset{+}{\zeta(\omega)}, \underset{+}{\bar{\eta}(\omega)}) \tag{33}$$

where s_d, s_f are defined in (22), $\zeta(\omega) \equiv \frac{p_{z_2}}{p_{z_1}} \frac{1+p_{z_1}}{1+p_{z_2}}$ and $\bar{\eta}(\omega) \equiv \frac{\bar{w}-\bar{y}}{1+p_{z_1}}$. $\zeta(\omega)$ expresses relative price effects on the structure of expenditures for financial services. Since $p_{z_1} = p_{z_2} = p_z$ in the benchmark case, $\zeta(\omega)$ reduces to one. $\bar{\eta}(\omega)$ is the average supernumerary income weighted by the cost of future subsistence. It captures the income effects on within structural change. If $\bar{e_1} = 0$, there is no income effect on the demand structure for financial services. For $\bar{e_1} > 0$, the impact of the skill premium on the value-added share Φ of sector Z_2 compared to Z_1 depends in the benchmark only on the shape of $\bar{\eta}(\omega)$. The following lemma characterizes the properties of $\bar{\eta}(\omega)$.

Lemma 1. *a) If* $\xi \in \Xi_1 \equiv \Xi_0 \cap \{\xi | \alpha_x + \alpha_z > 1\}$*, then there exists a threshold* $\underline{\omega}(\underset{?}{A_x}, \underset{?}{A_z}, \underset{-}{k}, \underset{-}{\bar{e_0}}, \underset{?}{\frac{b_L \bar{L}}{N}})$ *with* $\frac{\partial \bar{\eta}}{\partial \omega}|_{\omega=\underline{\omega}} = 0$ *so that:*

$$\frac{\partial \bar{\eta}}{\partial \omega} < 0 \text{ for } \omega < \underline{\omega},$$
$$\frac{\partial \bar{\eta}}{\partial \omega} > 0 \text{ for } \omega > \underline{\omega}.$$

Especially, define $\Xi_D^1 \equiv \{\xi | \underline{\omega} > \omega_{min}\}$ *and* $\Xi_D^2 \equiv \{\xi | \underline{\omega} < \tilde{\omega}_{max}\}$*. If* $\xi \notin \Xi_D^1$*, then* $\frac{\partial \bar{\eta}}{\partial \omega} > 0$ *for all* $\omega \in (\omega_{min}, \tilde{\omega}_{max})$*. If* $\xi \notin \Xi_D^2$*, then* $\frac{\partial \bar{\eta}}{\partial \omega} < 0$ *for all* $\omega \in (\omega_{min}, \tilde{\omega}_{max})$*.*

b) For the comparative static analysis we have:

$$\bar{\eta}\left(\omega \left| \underset{+}{A_x}, \underset{+}{A_z}, \underset{+}{k}, \underset{+}{\frac{b_L \bar{L}}{N}}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}}\right.\right)$$

Proof. Appendix C. □

While the impacts of saving propensities s_d and s_f on the within structure are straightforward, the role of the skill premium is in general ambiguous. On the one hand, a higher ω raises the average wage. On the other hand, the prices of financial services are increasing, which has a negative effect on the purchasing power. According to Lemma 1, the first effect dominates if the skill premium is sufficiently high. The income effect on the financial structure vanishes if $\bar{e_1} = 0$. In addition to the income effect, there may be a substitution effect through a change in relative prices (see discussion in Section 8). This effect vanishes if the two financial sectors produce with the same technology.

In sum, we have the following partial results about within structural change in the finance sector.

Proposition 2. *a) For a given* $\xi \in \Xi_D \equiv \Xi_1 \cap \Xi_D^1 \cap \Xi_D^2$*, we have: If* $\bar{e_1} > 0$*, a rise in the skill premium leads to structural change from subsector* Z_1 *to subsector* Z_2 *(in terms of value-added) at high levels of the skill premium* ($\omega > \underline{\omega}$) *and to structural change from* Z_2 *to* Z_1 *at low levels of skill premium.*

b) For a given skill premium, a rise of $s_f, A_x, A_z, k, \frac{b_L \bar{L}}{N}$ *or a decline of* $\bar{e}_0, \frac{\bar{e_1}}{r}$ *lead to structural change from* Z_1 *to* Z_2*.*

Proof. (33) and Lemma 1. □

The proposition describes only a partial effect. For a full comparative-static equilibrium analysis, we have to combine the direct effects of exogenous fundamentals with their indirect effects through the equilibrium skill premium. We come back on the total effects in Section 6.4.

5.2 Between change

For $\alpha_{z_1} = \alpha_{z_2} = \alpha_z$ and $A_{z_1} = A_{z_2} = A_z$, aggregate supply of financial services reduces to:

$$Z(= Z_1 + Z_2) = A_z L_z \kappa_z^{\alpha}.$$

The allocation between the X-and the Z-sector is then determined by the resource constraints:

$$a_x^L X + a_z^L Z = b_L \bar{L},$$
$$a_x^H X + a_z^H Z = b_H \bar{H}.$$

In an analogous way to (14), we get from this as solution:

$$X = \frac{b_L \bar{L}}{a_x^L} \frac{\kappa_z - k}{\kappa_z - \kappa_x}, \quad Z = \frac{b_L \bar{L}}{a_z^L} \frac{k - \kappa_x}{\kappa_z - \kappa_x}. \tag{34}$$

Substituting $a_j^L = \frac{1}{A_j \kappa_j^{\alpha_j}}$, we have:

$$\frac{Z}{X} = \frac{A_z}{A_x} \phi(\underset{+}{\omega}, \underset{+}{k}), \quad \phi(\omega, k) \equiv \frac{\kappa_z^{\alpha_z}}{\kappa_x^{\alpha_x}} \frac{k - \kappa_x}{\kappa_z - k}. \tag{35}$$

This gives us the following result for the comparative-static effects on the supply structure. [7]

Proposition 3. *An increase in the skill premium shifts the supply structure from goods production to financial services provision. An increase in the high skilled labor share* (k) *or biased technical change in favor of financial services (so that total factor productivity* A_z *rises relative to* A_x*) have the same effect.*

Proof. The signs of the respective partial derivatives in (35) follow from $\kappa_z > \kappa_x$ and the Rybczynski analysis. □

The proposition characterizes the supply structure as a function of exogenous fundamentals and the skill premium. The supply structure interacts

[7] Note that (35) characterizes the supply structure in terms of labor resources employed. If capital is used as set-up capital as in the extended model in Section 7.5, then X is indeed the total size of final output in the goods sector. In the baseline model considered here there is in addition the output generated for old age consumption by past capital investments. Thus, the total size of goods transactions becomes $\bar{X} \equiv X + rD + \mu RF$ with $X = E_0 + S = E_0 + D + F$ and thus the between structural change ratio is $\bar{\psi} \equiv \frac{p_z D + p_z F}{\bar{X}} = \frac{p_z D + p_z F}{X + rD + \mu RF}$ with D, F, E_0 and S from (28)-(31). It is, ceteris paribus, increasing in ω if $(S' E_0 - S E_0' - (\mu R - r)(DF' - FD')) > 0$ where D', F', S' and E_0' are the respective derivatives with respect to ω. This means, if the between change $(S' E_0 - S E_0')$ is larger than within change $(DF' - FD')$ multiplied with the return difference $(\mu R - r)$.

with demand, which depends on aggregate income and prices and thus also reacts to the skill premium. To close the analysis, we have to determine the equilibrium skill premium. Section 6.3 will then summarize the general equilibrium effect of the skill premium on the between sectoral structure.

6 General equilibrium

Aggregate demand in the X-sector is composed of consumer goods demand, E_0, and investment goods demand, $S = D + F$. On top of it, old agents consume the output generated by the capital they invested in the period before.

Aggregating the individual budget constraints (2), we obtain:

$$E_0 + D + F + p_{z_1} D + p_{z_2} F = W. \tag{36}$$

$W = W_x + W_z$, where $W_x = X$ and $W_z = p_{z_1} G^{z_1}(H_{z_1}, L_{z_1}) + p_{z_2} G^{z_2}(H_{z_2}, L_{z_2})$. If the Z_1 and Z_2-markets are cleared, we have $G^{z_1}(H_{z_1}, L_{z_1}) = D$ and $G^{z_2}(H_{z_2}, L_{z_2}) = F$ so that (36) reduces to

$$E_0 + D + F = X.$$

Thus, the goods market is automatically cleared if the markets for financial services are cleared.

Aggregate demand for financial services comes from savings in deposits D and savings in securities F. Adding up (28) and (29), we have for aggregate demand in the Z-sector

$$Z^D = \left(\frac{\delta}{1+\delta}\frac{\bar{w}-\bar{y}}{1+p_z} + \frac{\bar{e_1}}{r}\right) N.^8 \tag{37}$$

From (34) we know that aggregate Z-supply in a production equilibrium is

$$Z^S = A_z b_L \bar{L} \kappa_z^{\alpha_z} \frac{k-\kappa_x}{\kappa_z-\kappa_x} \tag{38}$$

where $a_z^L = \frac{1}{A_z \kappa_z^{\alpha_z}}$ was used.

6.1 Existence, uniqueness and stability of equilibrium

Both market sides are functions of ω (which works through $\bar{w}$ and p_z on the demand side and through skill intensities κ_x, κ_z on the supply side). For a stable equilibrium, the condition

$$\frac{dZ^D}{d\omega} < \frac{dZ^S}{d\omega} \tag{39}$$

is required at the market clearing ω-value. (Since p_z is increasing in ω, inequality (39) guarantees that a rise in price p_z goes hand in hand with a reduction of excess demand and a fall in the price reduces excess supply). Before turning to the determination of the equilibrium we have to inspect the range of feasible ω more closely.

The supply function is characterized by the following fact.

[8] Without normalization we would have $\tilde{Z^D} = n_1 Z^D$ and the market clearing condition for traditional financial services would read: $Z^D = \frac{A_{z_1}}{n_1} b_L \bar{L} \kappa_z^{\alpha_z} \frac{k-\kappa_x}{\kappa_z-\kappa_x} = \tilde{A}_z b_L \bar{L} \kappa_z^{\alpha_z} \frac{k-\kappa_x}{\kappa_z-\kappa_x}$ for $\frac{A_{z_1}}{n_1} = \frac{A_{z_2}}{n_2} = \tilde{A}_z$. Thus, for an empirical interpretation it is important to notice that A_z is "productivity" per unit of saving rather than per unit of service.

Fact 6. *Z^S is an increasing strictly concave function of ω starting at $\lim_{\omega \to \omega_{min}} Z^S = 0$ and bounded above at ω_{max} by $A_z b_L \bar{L} k^{\alpha_z}$. More specifically,*

$$Z^S = A_z b_L \bar{L} \frac{\gamma_z^{\alpha_z}}{\gamma_z - \gamma_x} g(\underset{+}{\omega}, \underset{+}{k}), \quad g(\omega, k) = \omega^{-\alpha_z}(k\omega - \gamma_x). \tag{40}$$

Proof. Appendix C. □

For the demand side the following fact applies.

Fact 7. *Aggregate demand for financial services is given by:*

$$Z^D = \left[\frac{\delta}{1+\delta} \bar{\eta} \left(\omega \,\middle|\, \underset{+}{A_x}, \underset{+}{A_z}, \underset{+}{k}, \underset{+}{\frac{b_L \bar{L}}{N}}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}} \right) + \frac{\bar{e_1}}{r} \right] N,$$

where $\bar{\eta}$ was discussed in Lemma 1. For all $\xi \in \Xi_1$, Z^D is defined and positive on $\omega \in (\omega_{min}, \tilde{\omega}_{max})$. Moreover, it is either U-shaped in ω (for $\xi \in \Xi_D$), increasing in ω (for $\xi \in \Xi_D - \Xi_D^1$) or declining in ω (for $\xi \in \Xi_D - \Xi_D^2$).

Proof. Equation (37) and Lemma 1. □

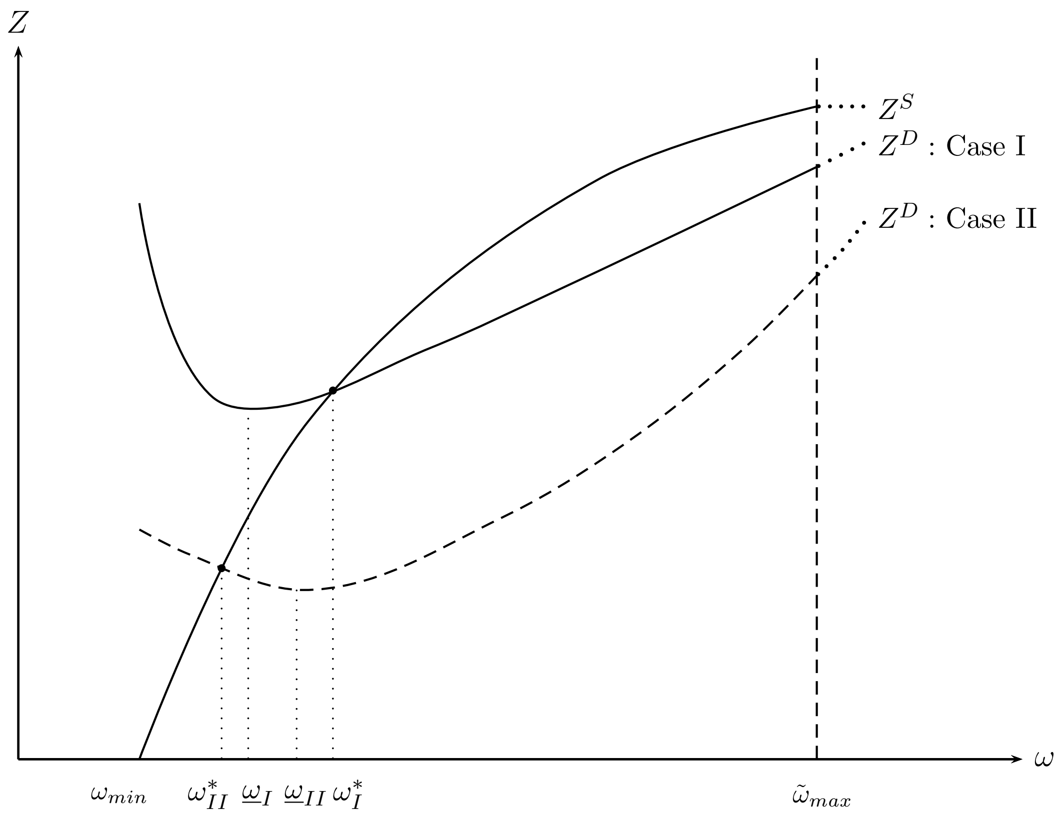

Figure 1: Equilibrium in the financial service sector.

Figure 1 shows in the (ω, Z)-space the supply and demand curves under the assumption that

$$Z^D(\tilde{\omega}_{max}) < Z^S(\tilde{\omega}_{max}), \tag{41}$$

where $\tilde{\omega}_{max}$ is defined in (32) and with exogenous parameters $\xi \in \Xi_D$. [9]

If inequality (41) holds, then the market clearing condition $Z^D(\omega) = Z^S(\omega)$ has a unique solution ω^* within $(\omega_{min}, \tilde{\omega}_{max})$. Moreover, stability condition

[9] If $\tilde{\omega}_{max} = \omega_L^+$, then $Z^D(\tilde{\omega})$ is to be read as $Z^D(\omega) < Z^S(\omega)$ for all $\omega < \omega_L^+ - \epsilon$, with ϵ arbitrarily small.

(39) is fulfilled at ω^*. This establishes the following proposition.

Proposition 4. *Define* $\Xi_E = \Xi_1 \cap \left\{\xi | Z^D(\tilde{\omega}_{max}) < Z^S(\tilde{\omega}_{max})\right\}$*. For all* $\xi \in \Xi_E$*, there exists a unique stable equilibrium.*

Proof. Continuity of Z^D on $\omega \in (\omega_{min}, \tilde{\omega}_{max})$ and properties of the shape of Z^D established in Fact 7. □

6.2 Equilibrium skill premium

For the comparative-static equilibrium analysis, we have to look at the excess demand function $Z^D - Z^S$. Because of stability condition $\frac{\partial(Z^D - Z^S)}{\partial \omega} < 0$, we know that for any exogenous change of a component φ of $\xi \in \Xi_E$

$$sign \frac{\partial \omega^*}{\partial \varphi} = sign \frac{\partial (Z^D - Z^S)}{\partial \varphi} |_{Z^D = Z^S}.$$

To get an explicit expression for $Z^D - Z^S$ we can derive for Z^D the following: Using (26) and (12), we have

$$\frac{\bar{w}N}{1 + p_z} = A_x b_L \bar{L} D_1(\omega | \underset{+}{\frac{A_z}{A_x}}, \underset{+}{k}), \tag{42}$$

where $D_1 \equiv \frac{\Gamma_x(1+\omega k)}{\omega^{\alpha_x} + \frac{A_x \Gamma_x}{A_z \Gamma_z} \omega^{\alpha_z}}$ and the signs below parameters in (42) express the signs of their impact on D_1.

Moreover, substituting (12) for p_{z_1} in (18) we can write the term $\frac{\delta}{1+\delta} \frac{\bar{y}}{1+p_z} - \frac{\bar{e_1}}{r}$ in the form:

$$D_0(\omega | \underset{+}{\frac{A_z}{A_x}}, \underset{+}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}}) = \frac{1}{1+\delta} \left[\frac{\delta \bar{e_0}}{1 + \frac{A_x \Gamma_x}{A_z \Gamma_z} \omega^{\alpha_z - \alpha_x}} - \frac{\bar{e_1}}{r} \right]. \tag{43}$$

The sign of the square-bracketed term is positive if the present subsistence expenditure $\bar{e_0}$ dominates the future subsistence expenditure $\bar{e_1}$. It is negative if $\bar{e_1}$ dominates $\bar{e_0}$. For the economic interpretation of the relevant notion of dominance it is useful to recall $\frac{A_x \Gamma_x}{A_z \Gamma_z} \omega^{\alpha_z - \alpha_x} = p_z$. Thus $D_0(\omega | \frac{A_z}{A_x}, \bar{e_0}, \frac{\bar{e_1}}{r}) > 0$ $(=, < 0)$ if and only if

$$\frac{\delta \bar{e_0}}{1 + p_z} > \frac{\bar{e_1}}{r} \; (=, < \frac{\bar{e_1}}{r}, \text{resp.}). \tag{44}$$

Using D_0 and (42) in (37) and combining the result with (40), we conclude that $Z^D - Z^S$ is equal to the term

$$A_x b_L \bar{L} \left[\frac{\delta}{1+\delta} D_1(\omega | \underset{+}{\frac{A_z}{A_x}}, \underset{+}{k}) - \frac{N}{A_x b_L \bar{L}} D_0(\omega | \underset{+}{\frac{A_z}{A_x}}, \underset{+}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}}) - \frac{A_z}{A_x} \frac{\gamma_z^{\alpha_z}}{\gamma_z - \gamma_x} g(\omega, \underset{+}{k}) \right]. \tag{45}$$

Hence, $\bar{e_1}$ has a positive impact on $Z^D - Z^S$ and thus on ω^*; $\bar{e_0}$ has a negative impact. $\frac{A_z}{A_x}$ and k have opposing effects so that their impacts cannot be signed unambiguously by inspection of (45).

The most interesting question is how technical change affects the equilibrium skill premium. For this we have to look at the impact of $\frac{A_x b_L \bar{L}}{N}$ on $Z^D - Z^S$. (Since $\frac{A_z}{A_x}$ has an ambiguous effect, we only consider uniform progress across sectors, that is, total factor productivity A_z rises pari passu with A_x.) The answer depends on condition (44). If $\frac{\delta \bar{e_0}}{1+p_z} > \frac{\bar{e_1}}{r}$, D_0 is positive and ω^* increases if $\frac{A_x b_L \bar{L}}{N}$ rises. If $\frac{\delta \bar{e_0}}{1+p_z} < \frac{\bar{e_1}}{r}$, then $-D_0$ is positive and ω^* declines if $\frac{A_x b_L \bar{L}}{N}$ increases.

In sum, we have the following partial effects of the parameters on the equilibrium skill premium:[10]

$$\omega^*(\underset{?}{\frac{A_z}{A_x}}, \underset{-}{\bar{e_0}}, \underset{+}{\frac{\bar{e_1}}{r}}, \underset{?}{k}, \underset{+/-}{\frac{A_x b_L \bar{L}}{N}}), \tag{46}$$

where the impact of $\frac{A_x b_L \bar{L}}{N}$ depends on the cases discussed above.

All addressed effects refer to the partial derivatives, that is, they hold under the condition that other parameters do not change simultaneously. In particular, for the effect of $\frac{b_L \bar{L}}{N}$ on ω^*, skill richness $k = \frac{b_H \bar{H}}{b_L \bar{L}}$ is held constant in the comparison. This requires a careful interpretation of the described effect of $\frac{b_L \bar{L}}{N}$. The following fact provides an economically meaningful description of the variations which are consistent with a constant k and a rise in $\frac{b_L \bar{L}}{N}$.

Fact 8. *A rise in $\frac{b_L \bar{L}}{N}$ is consistent with a constant k if there is:*

a) Uniform factor-augmenting technical progress, raising b_L pari passu with b_H.

b) A shift in labor supply from unskilled to skilled labor accompanied by unskilled labor biased technical progress (counterbalancing the increase of $\frac{\bar{H}}{\bar{L}}$ by a decline of $\frac{b_H}{b_L}$).

Proof. Use $N = \bar{L} + \bar{H}$ for $\frac{N}{b_L \bar{L}} = \frac{1+\frac{\bar{H}}{\bar{L}}}{b_L}$. Hence, $k = \frac{b_H \bar{H}}{b_L \bar{L}}$ remains constant under a decrease in $\frac{N}{b_L \bar{L}}$ if either b_L and b_H rise proportionally or $\frac{b_L}{b_H}$ grow proportionally to $\frac{\bar{H}}{\bar{L}}$. □

[10] The sign below the parameters represents the partial derivatives. The combination +/− is used for pointing to case-dependent impacts. A question mark means that the impact of the respective parameter cannot be signed without further investigation.

With these clarification the following proposition summarizes the comparative static equilibrium results.

Proposition 5. *a) In a stable equilibrium, the equilibrium skill premium is high, if future subsistence expenditure ($\bar{e}_1$) is high, and low if present subsistence expenditure ($\bar{e}_0$) is high.*

b) Uniform productivity growth across sectors (raising A_x and A_z proportionally) or uniform factor-augmenting technical progress (raising b_L and b_H proportionally) have a positive effect on the equilibrium skill premium if the present subsistence expenditure dominates the future subsistence expenditure; if the future subsistence expenditure dominates the skill premium declines.

c) A shift of labor supply from unskilled to skilled work accompanied by unskilled labor biased technical progress has the same effects on the equilibrium skill premium as factor augmenting progress that is uniform.

Proof. Fact 8 and main text. □

6.3 Structural change between production and financial service sectors

Combining the results of subsections 6.2 and 5.2, we obtain the following results for the structural change between production and financial services sector in equilibrium:[11]

Proposition 6. *For all $\xi \in \Xi_E$, at given $\frac{A_z}{A_x}$, k, any change in other exogenous fundamental which raises (lowers) the skill premium leads to structural change from X to Z (Z to X, respectively).*

Proof. Equation (35). Since p_z rises with ω, the rise of ϕ immediately implies that $\frac{p_z Z}{X}$ rises too. □

6.4 Structural change within the financial sector

Finally, for structural change within the financial sector, we have the following results in equilibrium:

Proposition 7. *Let $\underline{\omega}$ be the threshold defined in Lemma 1 and parameters fulfill $\xi \in \Xi_E$. Then, for $\bar{e_1} > 0$, the following comparative static results hold for structural change within the financial sector:*

a) At high levels of the skill premium ($\omega^* > \underline{\omega}$), a fall of $\bar{e_0}$ leads to a shift from Z_1 to Z_2. In addition, if present subsistence expenditure dominates future subsistence expenditure, uniform productivity growth across sectors which increases A_x and A_z proportionally (i.e., keeping $\frac{A_x}{A_z}$ unchanged) and an increase in $\frac{b_L \bar{L}}{N}$ changes the structure within the financial sector from Z_1 towards Z_2. According to Proposition 5 and 6, these changes induce an increase in the inequality level ω^*, accompanied

[11] Proposition 6 talks about the structural change between X and Z with respect to labor compensation without considering the effect of capital return of the size of the goods sector. See also footnote 7.

by a simultaneous structural change from the goods to the financial service sector.

b) *At low levels of the skill premium ($\omega^* < \underline{\omega}$), a fall of $\bar{e_1}$ leads to a shift from Z_1 to Z_2. In addition, if future subsistence expenditure dominates present subsistence expenditure, uniform productivity growth across sectors which increases A_x and A_z proportionally (i.e., keeping $\frac{A_x}{A_z}$ unchanged) and an increase in $\frac{b_L \bar{L}}{N}$ changes the structure within the financial sector from Z_1 towards Z_2. However, according to Proposition 5 and 6, these changes correspond to a decrease in the inequality level ω^*, accompanied by a simultaneous a structural change from the financial service to the goods sector.*

Proof. Using (33), (46), and Lemma 1, we have

$$\frac{F}{D} = \Phi\left\{ \underset{-}{s_d}, \underset{+}{s_f}, \underset{-}{\frac{\bar{e_1}}{r}}, \underset{+}{\bar{\eta}} \left[\underset{+/-}{\omega^*} \left(\underset{?}{\frac{A_z}{A_x}}, \underset{-}{\bar{e_0}}, \underset{+}{\frac{\bar{e_1}}{r}}, \underset{?}{k}, \underset{+/-}{\frac{A_x b_L \bar{L}}{N}} \right), \underset{+}{A_x}, \underset{+}{A_z}, \underset{+}{k}, \underset{+}{\frac{b_L \bar{L}}{N}}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}} \right] \right\},$$

where the signs below the parameters show the sign of the partial derivative by the respective functions, $\Phi\{\cdot\}$, $\bar{\eta}[\cdot]$ and $\omega^*(\cdot)$. The results follow by applying Lemma 1 and Proposition 5 and 6.

□

Proposition 5-7 show that our model can identify demand and supply channels which are able to drive inequality and the two-fold structural change simultaneously. It is worth noting that for $\bar{e_1} = 0$ there is no income effect on the portfolio structure so that the channel between skill premium and financial structure is shut down. Since in the benchmark considered here relative price effects within the financial sector were shut down too, there is no within sectoral change in finance if $\bar{e}_1 = 0$. This will change in the model variant with different technologies for Z_1 and Z_2 considered in Section 8.

7 Extensions

In this section we show how the presented model can be adapted to account for important features of the financial sector that were neglected in the benchmark analysis. Five extensions are considered: Fixed costs in the financial sector, rents in the financial sector, distorted portfolio choices of households, participation constraints in the finance subsector Z_2 and set-up capital for firms.

7.1 Fixed costs in the financial sector

Suppose that financial services are provided by banks. A bank b, serving N_b clients, needs $K_b = f_B N_b$ units of goods to set up the capacity to serve them. We assume that the fixed cost K_b is financed by a lump-sum fee

$$\tau = f_B$$

imposed on the clients. That is, bank size and number of banks affect neither aggregate fixed costs

$$K_B = f_B N$$

nor the households' budget constraint. In the latter, y^l reduces to $y^l - \tau$ so that the supernumerary budget becomes $y^l - \bar{y}_+$, with $\bar{y}_+ = \bar{y} + \tau =$

$\bar{e_0} + f_B + (1 + p_{z_1})\bar{e_1}/r$.

Hence, fixed cost f_B has the same comparative-static effects on household choices as an increase in subsistence expenditure $\bar{e_0}$. For the X-market this means, on the one hand, the absorption of X by households' consumption and investment is reduced by $K_B = f_B N$. On the other hand, K_B is spent by banks to set up the capacity to serve their clients. In sum, we have

$$E_0 - f_B N + D + F + K_B = X$$

for the goods market clearing, which reduces to the benchmark condition

$$E_0 + D + F = X$$

since $f_B N = K_B$. Hence, goods markets are cleared whenever the Z-markets are cleared.

In the markets for financial services, demand is reduced by the fact that $\bar{w} - \bar{y}_+$ rather than $\bar{w} - \bar{y}$ is now the relevant supernumerary income. The supply side remains unaffected. In equilibrium, the implications of fixed costs can be derived by looking in the benchmark model at the effect of a rise of $\bar{e_0}$ to $\bar{e_0} + f_B$.

Proposition 8. *A rise in fixed costs f_B has the following effects:*

a) The skill premium goes down.

b) The between sectoral structure shifts from Z to X.

c) The within sectoral structure shifts from Z_2 to Z_1 at high levels of the skill premium ($\omega^* > \underline{\omega}$). At low levels of the skill premium ($\omega^* < \underline{\omega}$) the effect is ambiguous.

Proof. Comparative-static results for $\bar{e_0}$ in Proposition 5, 6 and 7. □

7.2 Rents in the financial sector

Suppose that a club of agents in the finance sector has the power to extract rents from financial service provision.[12] One may think of rentiers who have unearned property rights or an elite subgroup of employees in the financial sector. We make two crucial assumptions: First, whoever are the rent extracting agents, they spend the rent like other agents. Thus, the redistribution of rents has no income effect on aggregate demand. (Total subsistence requirements and aggregate supernumerary income remain unchanged). Second, nobody can enter the club from outside so that the rent does not affect labor allocation.

In the presented model, two instruments can be used to extract rents. If there are fixed costs as in extension 7.1, the fee τ can be distorted to

$$\tilde{\tau} > f_B.$$

In this case, aggregate rents $(\tilde{\tau} - f_B)N$ are lump-sum redistributed. Everybody pays $\tilde{\tau}$ and an elite N_0 receives the rent. Thus, average supernumerary income becomes

$$\bar{w} - \bar{y} - \tilde{\tau} + \frac{N_0}{N}\frac{(\tilde{\tau} - f_B)N}{N_0} = \bar{w} - \bar{y} - f_B.$$

The rent has no aggregate effects in this case.

[12] As pointed out in the introduction, there is robust evidence that indeed a substantial finance premium exists. Like with any other rents it is hard to reconcile such rents with free entry equilibria. This paper deals with the consequences of rents, not with possible explanations why they exist.

An alternative instrument of rent extraction would be to charge a markup on unit cost prices in the financial sector so that households have to pay

$$\tilde{p}_{z_i} = p_{z_i}(1+o)$$

for financial services.

Using (12), we have

$$\tilde{p}_{z_i} = (1+o)\frac{A_x}{A_{z_i}}\frac{\Gamma_x}{\Gamma_{z_i}}\omega^{\alpha_{z_i}-\alpha_x}.$$

The rent o decreases D_1 in (42) to $\frac{A_x\Gamma_x(1+\omega k)}{\omega^{\alpha_x}+\frac{(1+o)A_x\Gamma_x}{A_z\Gamma_z}\omega^{\alpha_z}}$ and decreases D_0 in (43) to $\frac{1}{1+\delta}\left[\frac{\delta\bar{e_0}}{1+(1+o)\frac{A_x\Gamma_x}{A_z\Gamma_z}\omega^{\alpha_z-\alpha_x}} - \frac{\bar{e_1}}{r}\right]$. Hence, o has an ambiguous impact on $Z_D - Z_S$ and thus on ω^*.

Proposition 9. *Rents in the financial sector have the following effects:*

a) If rents are extracted by additional fixed costs, these do not affect equilibrium values.

b) If rents are extracted by a markup on financial service prices they affect all equilibrium values in a generally ambiguous way.

Proof. Main text. □

7.3 Distorted portfolio choice

Several empirical studies have pointed out that people get confused in dealing with complex financial markets (see Célérier and Vallée (2014) and the literature discussed there). In our model, the complex part that households have to solve is the choice of the portfolio of the securities. The choice may be based on a wrong assessment of relative risks and returns of different securities. In this case, we have distortion within Z_2 and consumption levels planned for the future may be deceived by actual payoffs.[13] As our study focuses on structural change between X and Z as well as between Z_1 and Z_2, we do not consider such distortions here. Rather we focus on distortions coming from misperception of the opportunities to save by securities investment rather than in deposits.

In particular, people may have wrong beliefs $\tilde{\mu}$ about the measure of future environments covered by state-contingent securities, relative to the non-covered part of possible future events. They may also misjudge the relative payoff of deposits compared to the payoffs of securities and base their decisions on a distorted $\tilde{\rho}$. Such distortions affect the propensities to save in deposits and in securities. For instance, if agents are euphoric about investments in securities and believe that $\tilde{\mu} > \mu$ or $\tilde{\rho} < \rho$, then s_f rises while s_d declines. The total propensity to save, however, does not change in the benchmark model with $p_{z_1} = p_{z_2}$.[14] Therefore, the only consequence of $\tilde{\mu} > \mu$ or $\tilde{\rho} < \rho$ is sectoral change within the financial sector. According to (33), Φ rises.

Proposition 10. *Euphoric beliefs about measure or performance of state-contingent financial instruments lead to within sectoral change from Z_1 to Z_2. Equilibrium skill premium and (X, Z)-structure are not affected.*

[13] Falkinger (2014) focuses on such distortions in a one sector economy.

[14] For $p_{z_1} \neq p_{z_2}$, however, we would have $s_d + \frac{s_f}{p}$ for the marginal propensity to save, as shown by (23). Thus, μ and ρ impact also on Z^D and therefore on ω and all other equilibrium outcomes.

Proof. Equation (33). □

7.4 Participation constraints

Suppose that a fixed fee τ is charged only to agents who invest in securities. Moreover, assume that

$$y^L > \bar{y} > y^L - \tau,$$
$$y^H > y^H - \tau > \bar{y}.$$

Then low-skilled agents do not participate in the securities market, while high-skilled agents do. According to equation (B.4) in Appendix B, we have for $l = L$:

$$s^L = d^L = \frac{\delta}{1+\delta}\frac{y^L - \bar{y}}{1+p_z} + \frac{\bar{e}_1}{r}.$$

For $l = H$, saving behavior is given by (20) and (21) with $\bar{y}_+ = \bar{y} + \tau$.

This gives us the following aggregate saving levels:

$$\begin{aligned} D &= \frac{\delta}{1+\delta}\frac{1}{1+p_z}\left[(y^L - \bar{y})\bar{L} + s_d(y^H - \bar{y}_+)\bar{H}\right] + \frac{\bar{e}_1}{r}N \\ &= \frac{\delta}{1+\delta}\frac{1}{1+p_z}\left[(\bar{w} - \bar{y})N - s_f(y^H - \bar{y})\bar{H} - s_d\tau\bar{H}\right] + \frac{\bar{e}_1}{r}N, \\ F &= s_f\frac{\delta}{1+\delta}\frac{\bar{H}}{1+p_z}(y^H - \bar{y}_+) \end{aligned}$$

and

$$S = \left(\frac{\delta}{1+\delta}\frac{\bar{w} - \bar{y} - \tau\frac{\bar{H}}{N}}{1+p_z} + \frac{\bar{e}_1}{r}\right)N.$$

Comparing S with Z^D in (37), we see the fee τ, combined with the participation constraint, impacts on Z^D and thus on the skill premium and the

(X, Z)-structure like an increase of $\bar{e_0}$ to

$$\tilde{e_0} = \bar{e_0} + \tau \frac{\bar{H}}{N}.$$

This is only the case if τF is absorbed by real fixed cost requirements as discussed in Section 7.1. If τF is a rent which is redistributed back to high-skilled agents, as in Section 7.2, we have $(y^H - \bar{y} - \tau)\bar{H} + \tau \bar{H} = y^H - \bar{y}$ instead of $y^H - \bar{y}_+$ so that

$$\begin{aligned}
D &= \frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1 + p_z} N \left(1 - s_f \beta_H\right) + \frac{\bar{e_1}}{r} N \\
F &= s_f \frac{\delta}{1+\delta} \frac{\bar{H}}{1 + p_z} (y^H - \bar{y}) \\
S &= \left(\frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1 + p_z} + \frac{\bar{e_1}}{r} \right) N
\end{aligned}$$

with $\beta_H \equiv \frac{y^H - \bar{y}}{\bar{w} - \bar{y}} \frac{\bar{H}}{N}$ denoting the income share of high-skilled agents. For the high-skilled nothing changes, but the low-skilled are only saving through D. This means that compared to the benchmark we have an increase in D and a decrease in F. $Z^D = S$ coincides with the expression in (30) so that equilibrium skill premium and (X, Z)-structure are not changed compared to the baseline.[15]

For the within sectoral structure in the Z-sector, we have in the benchmark case with $p_{z_1} = p_{z_2} = p_z$:

$$\begin{aligned}
\frac{F}{D} &= \frac{s_f \beta_H}{1 - s_f \beta_H + \frac{1+\delta}{\delta} \frac{1+p_z}{\bar{w} - \bar{y}} \frac{\bar{e_1}}{r}} \\
&= \frac{s_f \beta_H \bar{\eta}}{s_d \bar{\eta} + s_f (1 - \beta_H) \bar{\eta} + \frac{1+\delta}{\delta} \frac{\bar{e_1}}{r}} \equiv \tilde{\Phi}
\end{aligned}$$

[15] For $p_{z_1} \neq p_{z_2}$, however, the change in Z_2^D would also affect ω and all other equilibrium outcomes.

Comparing this with (33), we conclude that $\tilde{\Phi} < \Phi$ because $s_f\beta_H < s_f$, $s_f(1-\beta_H) > 0$.

In sum, we have the following result.

Proposition 11. *Other things equal, a pure participation constraint (with no real resource cost corellate) in securities markets, leads to within structural change from Z_2 to Z_1, but has no effects on the equilibrium skill premium and the (X,Z)-structure in equilibrium.*

Proof. Main text. □

7.5 Set-up capital for firms

In the baseline model invested capital is transformed by linear technologies, using capital as the only input, into future outcome. The extension in this section shows that the baseline can be seen as kind of reduced form of a richer model, in which capital is needed to set up firms. We assume now that firms in the X-sector use capital to set up technology G^x which then produces by employing low-skilled and high-skilled labor. Each established firm $\nu \in \{1,...,M\}$ produces a variety $x_\nu = G^x(L_{x_\nu}, H_{x_\nu})$ under monopolistic competition with free entry. Consumers spend the supernumerary income $e_t - \bar{e}_t$ according to a CES-utility function with substitution elasticity $\sigma > 1$ symmetrically over the variants x_ν in the X-sector, which implies an instantaneous indirect utility function of the form $\log(e_t - \bar{e}_t)$ (see Section 7.5.1) like before. So saving decision and portfolio choice remain the same as in the baseline model. Firms have positive operating profits which are distributed as payoff to the investors (see Section 7.5.2).

7.5.1 Consumer problem

Let the instantaneous utility of households be given by $u = \left[\sum_{\nu=1}^{M} x_\nu^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, $\sigma > 1$. Then, prices are determined by a constant markup on unit cost of production

$$p_\nu = \frac{\sigma}{\sigma - 1} c(w_H, w_L), \tag{47}$$

where $c(w_H, w_L)$ are the unit costs (as derived in section 2.3) and w_H, w_L are factor prices. Moreover, demand for variety x_ν of a household that spends "supernumerary budget" $e - \bar{e}$ is

$$x_\nu = (e - \bar{e}) \frac{p_\nu^{-\sigma}}{P^{1-\sigma}}, \quad P \equiv \left[\sum_{\nu=1}^{M} p_\nu^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

Since product variants use identical production technologies, their unit cost and prices are identical, too. Thus, x_ν reduces to $x = \frac{e-\bar{e}}{p_\nu M}$. Using this in u, we obtain for the instantaneous indirect utility $u = \frac{e-\bar{e}}{P}$. We set the price as numéraire (i.e., $p_\nu = 1$) such that variety effect $P = M^{\frac{1}{1-\sigma}}$ is a constant. Due to the log specification, this variety effect, though affecting the level of utility, does not matter for the intertemporal decision.[16] Thus, $\max \mathbb{E} log(u) = \max \mathbb{E} log(e_t - \bar{e}_t)$, which is identical to the intertemporal problem in Section 2.2.

[16]Note that $log \frac{e-\bar{e}}{P} = log(e - \bar{e}) - log P$ so that the P-levels add to $\mathbb{E}U$ a constant.

7.5.2 Firm entry and production in the X-sector

There are two types of set-up technologies, which induce capital demand of firms: A robust set-up technology which requires c_0 units of capital. Firms set up by the robust technology will be producing tomorrow under any condition (i.e., in Θ and $\bar{\Theta}$). Furthermore, there are risky set-up technologies with set-up input c_θ, which are only effective if state $\theta \in \Theta$ occurs. Otherwise, their set-up fails.

Assumption 2.

$$c_\theta = \pi_\theta c_1, \ \textit{where } c_1 < c_0.$$

Assumption 2 states that set-up capital required for a robust technology is larger than those of risky set-up technologies. Moreover, the smaller the measure π_θ of the state under which a set-up technology works, the lower the required set-up capital.[17] Robust set-up technologies are financed by loans, whereas the risky set-up techniques are financed by state-contingent securities.

Let K_0 be the aggregate set-up capital for robust technologies and denote by $K_\theta, \theta \in \Theta$ the aggregate set-up capital for specialized risky technologies. Then the number of firms which can be set up is $M_0 = \frac{K_0}{c_0}$ and $M_\theta = \frac{K_\theta}{c_\theta}$,

[17] See Falkinger (2014) for a more detailed discussion of the relationship between specialization and risk. There, technologies are more productive the more narrowly they are targeted to a specific environment. At the same time, they are more risky because the realization of the specific environment is less likely. Here this idea is applied to set-up costs rather than productivity.

respectively. In a closed economy, capital markets are cleared if

$$K_0 = D, \quad K_\theta = F_\theta = \pi_\theta F.$$

Hence, we have for to total number of firms

$$M = \begin{cases} \frac{D}{c_0} + \frac{F}{c_1} \equiv M_\Theta, & \text{if } \theta \in \Theta, \\ \frac{D}{c_0} \equiv M_{\bar{\Theta}} & \text{otherwise.} \end{cases}$$

After firms being set up their operating profits earned under mark-up prices (47) are

$$\Pi = (p_x - c)X = \frac{X}{\sigma},$$

where $p_x = 1$, which implies $c = \frac{\sigma-1}{\sigma}$, has been used. Since firms are symmetric, aggregated operating profits are distributed uniformly across firms so that operating profits per firm is:

$$\frac{\Pi_m}{M_m} = \frac{X}{\sigma M_m}, \; m \in \{\Theta, \bar{\Theta}\}.$$

The returns on one unit of set up capital are therefore

$$r_m = \frac{X}{c_0 \sigma M_m}, \; m \in \{\Theta, \bar{\Theta}\}$$
$$R_\theta = \frac{X}{c_\theta \sigma M_\Theta}, \; R = \frac{X}{c_1 \sigma M_m}$$

for safe and risky investments, respectively. ($\pi_\theta R_\theta$ reduces to R because of assumption $c_\theta = \pi_\theta c_1$.) Since the number of firms is different in Θ and $\bar{\Theta}$, aggregate operating profits have to be shared among more or less firms so that the return on robust investments is m-dependent. The relative rate of return, $\frac{r_\Theta}{R_\theta}$, however, is uniquely determined by the relative set-up requirements of specialized risky technologies compared to the robust technology. We have:

$$\rho = \frac{c_1}{c_0}$$

For the portfolio choice derived in Section 4 almost only the relative rate ρ matters. The exception is $\frac{\bar{e}_1}{r_m}$, since future subsistence can only be financed by deposits.[18] This means, we have to restrict the analysis of the paper to $\bar{e}_1 = 0$, or we reconcile the fluctuation of the earnings of robust firms with a safe return on deposits by assuming that firms hold buffers and distribute the expected profit per firm $\bar{\pi} \equiv [\frac{\mu}{M_\Theta} + \frac{1-\mu}{M_{\bar{\Theta}}}]\frac{X}{\sigma}$ to the investors.

For the general equilibrium analysis, a further caveat is in order. Under the presented extension, return r (even if smoothed by the buffer) is endogenous. It depends on M and X which are determined by saving behavior and resource allocation, respectively. Thus, in the general equilibrium, a further feedback loop is to be considered. We did not account for such feedbacks in Section 6, since in the baseline return r is exogenously given by the constant productivity of capital. For $\bar{e}_1 = 0$, however, the presented analysis remains fully valid also with set-up capital of firms since r matters only through the term $\frac{\bar{e}_1}{r}$. However, what one loses by setting $\bar{e}_1 = 0$ is the income effect on structural change within the financial sector.

[18]Formally the derivation of the portfolio choice presented in the appendix has to be adapted to account for m-dependent pay-offs in the budget constraints. For $\bar{e}_1 = 0$, return $r_{\bar{\Theta}}$ becomes irrelevant under the logarithm specification and the analysis remains valid – with $\rho = \frac{r_\Theta}{R_\Theta}$.

8 Robustness

As an alternative to the benchmark case, where $\alpha_{z_1} = \alpha_{z_2}$ was assumed, we consider now the case $\alpha_x = \alpha_{z_1} < \alpha_{z_2}$. In this case,

$$p_{z_1} = \frac{A_x}{A_{z_1}}$$

and $\bar{y} = \bar{e_0} + \frac{(1+\frac{A_x}{A_{z_1}})\bar{e_1}}{r}$ is a constant.

Moreover, the terms $a_x^l X$ and $a_{z_1}^l Z_1$, $l \in \{H, L\}$ in system (13) reduce to

$$X^+ \frac{A_x}{\kappa_x^{\alpha_x}} \text{ and } X^+ A_x \kappa_x^{(1-\alpha_x)},\ X^+ \equiv X + \frac{A_x}{A_{z_1}} Z_1,$$

respectively. Using this when solving (13), we obtain

$$X^+ = \frac{b_L \bar{L}}{a_x^L} \frac{\kappa_{z_2} - k}{\kappa_{z_2} - \kappa_x},\ Z_2 = \frac{b_L \bar{L}}{a_{z_2}^L} \frac{k - \kappa_x}{\kappa_{z_2} - k} \tag{48}$$

and

$$\frac{Z_2}{X^+} = \frac{A_{z_2}}{A_x} \tilde{\gamma}(\underset{+}{\omega}, \underset{+}{k}),\ \tilde{\gamma}(\omega, k) \equiv \frac{\kappa_{z_2}^{\alpha_{z_2}}}{\kappa_x^{\alpha_x}} \frac{k - \kappa_x}{\kappa_{z_2} - \kappa_x}, \tag{49}$$

where the signs for the partial derivatives of $\tilde{\gamma}$ follow from the Rybczynski analysis.

Substituting $A_{z_2} \kappa_{z_2}^{\alpha_{z_2}}$ for $\frac{1}{a_{z_2}^L}$ in the second equation of (48) and using (9), we have for the Z_2-supply:

$$Z_2^S = A_{z_2} b_L \bar{L} \frac{\gamma_{z_2}^{\alpha_{z_2}}}{\gamma_{z_2} - \gamma_x} g(\underset{+}{\omega}, \underset{+}{k}),\ g(\omega, k) \equiv \omega^{-\alpha_{z_2}} (k\omega - \gamma_x). \tag{50}$$

This coincides with (40) – with Z_2 instead of Z – so that Fact 6 remains valid under the alternative specification and applies to Z_2-supply.

Z_2-demand is given by

$$Z_2^D = F = s_f \frac{\delta}{1+\delta} \frac{\bar{w} - \bar{y}}{1 + p_{z_2}} N = \frac{\mu - \rho p}{1 - \rho p} \frac{\delta}{1+\delta} \tilde{\eta} N \tag{51}$$

with $\tilde{\eta} \equiv \frac{\bar{w}-\bar{y}}{1+p_{z_2}}$ and $p = \frac{1+p_{z_2}}{1+p_{z_1}}$. In an analogous way to Lemma 1 and Fact 7, one establishes that the income effect (i.e., $\tilde{\eta}$-part in Z_2^D) has an U-shaped form.[19] Further, s_f – which captures the substitution effect between D and F in some way – is decreasing in ω since according to (12) $\frac{\partial p}{\partial \omega} > 0$. The total effect of ω on Z_2^D depends on the relative importance of the two effects: Numerical simulation shows that the substitution effect is large if the price p_{z_2} is high and the income effect is stronger if subsistence expenditures are larger. For a high level of price p_{z_2} (based on (12) this means, for example, a low A_{z_2}) and low subsistence levels (such that $\bar{y}$ is close to zero) the substitution effect dominates. In this case $\frac{\partial Z_2^D}{\partial \omega} < 0$. However, for low levels of price p_{z_2} and large subsistence levels the income effect dominates and the U-shape form (of Z_2^D instead of Z) from the benchmark prevails. For this case, (50) and (51) give us the same picture as in Figure 1. Proposition 4 remains unchanged in both cases.

For Proposition 5, we have to write the excess demand function $Z_2^D - Z_2^S$ explicitly in terms of parameters. Using $W = b_L \bar{L} A_x \Gamma_x \omega^{-\alpha_x}(1+\omega k)$, $p_{z_2} = \frac{A_x \Gamma_x}{A_{z_2}\Gamma_{z_2}} \omega^{\alpha_{z_2}-\alpha_x}$ and (50), we have $Z_2^D - Z_2^S = 0$ being equivalent to

$$
\begin{aligned}
A_x b_L \bar{L} \Bigg[& \frac{\mu - \rho \frac{1+\frac{A_x\Gamma_x}{A_{z_2}\Gamma_{z_2}}\omega^{\alpha_{z_2}-\alpha_x}}{1+p_{z_1}}}{1 - \rho \frac{1+\frac{A_x\Gamma_x}{A_{z_2}\Gamma_{z_2}}\omega^{\alpha_{z_2}-\alpha_x}}{1+p_{z_1}}} \frac{\delta}{1+\delta} \frac{\Gamma_x \omega^{-\alpha_x}(1+\omega k) - \frac{N}{b_L \bar{L} A_x}\bar{y}}{1+\frac{A_x\Gamma_x}{A_{z_2}\Gamma_{z_2}}\omega^{\alpha_{z_2}-\alpha_x}} \\
& - \frac{A_{z_2}}{A_x} \frac{\gamma_{z_2}^{\alpha_{z_2}}}{\gamma_{z_2}-\gamma_x} \omega^{-\alpha_{z_2}} (k\omega - \gamma_x) \Bigg] \\
\equiv D & \left[\omega \middle| \underset{+}{\mu}, \underset{-}{\rho}, \underset{+}{\frac{\delta}{1+\delta}}, \underset{?}{\frac{A_{z_2}}{A_x}}, \underset{?}{k}, \underset{-}{\frac{N}{b_L \bar{L} A_x}}, \underset{-}{\bar{y}} \right] = 0
\end{aligned}
$$

[19]The only thing that changes is that now we have $\frac{\bar{y}}{1+p_{z_2}}$ with $\bar{y}$ constant instead of $\frac{\bar{y}}{1+p_{z_1}} = \frac{\bar{e_0}}{1+p_{z_1}} - \frac{\bar{e_1}}{r}$. Thus, apart from Z_2 instead of Z_1 in the modified proof we have $\bar{y}$ instead of $\bar{e_0}$ and no negative term $-\frac{\bar{e_1}}{r}$.

Hence, a decline in subsistence requirements $\bar{y}$ have unambiguously a positive impact on the equilibrium skill premium - regardless of whether the decline in $\bar{y}$ is caused by a decline in $\bar{e_0}$ or $\bar{e_1}$. All other b) and c) comparative-static effects are the same as stated in Proposition 5 for the case $D_0 > 0$. As a consequence of (49), also Proposition 6 remains valid.

Finally, the ratio of value-added in financial subsector Z_2 to value-added in subsector Z_1 is as in (33)

$$\frac{p_{z_2}F}{p_{z_1}D} = \frac{s_f\bar{\eta}}{s_d\bar{\eta} + \frac{1+\delta}{\delta}\frac{\bar{e_1}}{r}} \frac{p_{z_2}}{1+p_{z_2}} \frac{1+p_{z_1}}{p_{z_1}}. \tag{52}$$

Since p_{z_1} and $\bar{y}$ are constant, $\frac{\partial \bar{w}}{\partial \omega} > 0$ immediately implies $\frac{\partial \bar{\eta}}{\partial \omega} < 0$. Hence, the income effect unambiguously leads to structural change from Z_1 to Z_2 if the skill premium rises. For the relative price effect, we only have to consider $\frac{p_{z_2}}{1+p_{z_2}}$ because $\frac{1+p_{z_1}}{p_{z_1}}$ is constant. Since $\frac{\partial p_{z_2}}{\partial \omega} > 0$, it follows immediately that the relative price effect also pushes towards Z_2 if the skill premium rises. From the substitution effect we have $\frac{\partial s_f}{\partial \omega} < 0$ and $\frac{\partial s_d}{\partial \omega} > 0$, which drive the within finance sectoral structure from Z_2 towards Z_1. Thus, under the alternative specification, the additional substitution effect makes it more difficult to model the within structural change from Z_1 to Z_2 than it is in the benchmark. For high levels of price p_{z_2} and low subsistence expenditures the substitution effect dominates. Then, the presented model cannot predict a co-movement of ω and the within structural change from Z_1 to Z_2. In the other case, however, Proposition (7) applies.

9 Empirical evidence and numerical exercises

In this section we first provide empirical evidence on the two-fold structural change and on wage inequality and then we carry out numerical exercises to show how our model can (qualitatively) replicate the observed trends.

9.1 Empirics

9.1.1 Data

We use data from the March Current Population Survey (CPS) for the years 1980-2014. This data set allows us to split the sampled population (weighted with the sampling weight) into our three sectors and two skill levels: The X-sector consists of all sectors of the U.S. economy except finance. The finance sector is finance and insurance without real estate.[20] "Traditional finance" Z_1 includes savings institutions, including credit unions, non-depository credit and related activities and insurance carriers and related activities. "New finance" Z_2 is securities, commodities, funds, trusts, and other financial investments. We define a worker (if working) to be high-skilled if she/he holds a college degree or more. Then, $\bar{H}_j$ is the number of high-skilled workers in sector $j \in \{x, z_1, z_2\}$ and $\bar{L}_j$ is the number of low-skilled workers in sector $j \in \{x, z_1, z_2\}$. For the three sectors conditioned on the two skill levels we calculate average yearly hours worked (i.e., h_j^l, $j \in \{x, z_1, z_2\}$, $l \in \{H, L\}$) and the respective average hourly wages (i.e., w_j^l, $j \in \{x, z_1, z_2\}$, $l \in \{H, L\}$).

[20] This corresponds to the standard classification as in Philippon and Reshef (2007, 2012).

In our data analysis we use "actual" and "normalized" numbers for employment and wage levels. The "actual" numbers use the observed sector- and skill-specific average yearly hours worked and the respective average hourly wage. The "normalized" numbers are calculated all with the same basis of hours worked and hourly wage (i.e., the ones from the X-sector).[21] The sectoral structure-figures below show black and gray lines: The gray lines correspond to the "actual" numbers. The black lines correspond to the "normalized" ones. The normalization allows us to separate the effects we can identify in the theoretical, frictionless model from two frictions observed in reality: (i) Low- and high-skilled Z-workers work more hours per year than low- and high-skilled X-workers. More precisely, for the U.S. over the last decades one sees that on average a Z-worker has worked about 9% more than a X-worker. (ii) There is the finance premium on hourly wages for low- and high-skilled Z-workers.[22] CPS data show that the finance premium increased over time and differs for the two sub-sectors: In Z_1 workers earn about 15% more than in the X-sector, in Z_2 it is even 50%.

9.1.2 Empirical trends

As is described in the introduction and picked up in the model, financialization has several aspects: On the one hand, the weight of the financial sector relative to non-financial business has increased; this is structural change to-

[21]Since the skill premium is approximately identical in all three sectors in the U.S. the skill intensities in the sectors need not be "normalized". They already correspond to the frictionless numbers.

[22]See Célérier and Vallée (2015) or Philippon and Reshef (2007, 2012) for a detailed empirical discussion of the finance premium.

wards finance. On the other hand, the type of financial products and services has changed; this is structural change within finance. The next two figures show the two-fold structural change.

Figure 2 shows the ratio of the total finance sector (Z-sectors) compared to the non-finance economy (X-sector) for the U.S. based on the CPS data. On the one hand, the figure shows that finance has attracted new employment. The employment ratio (in terms of total hours worked) of the financial sector, defined by $\psi^{E}_{actual} \equiv \frac{h^H_{z_1}\bar{H}_{z_1}+h^H_{z_2}\bar{H}_{z_2}+h^L_{z_1}\bar{L}_{z_1}+h^L_{z_2}\bar{L}_{z_2}}{h^H_x\bar{H}_x+h^L_x\bar{L}_x}$, increased from 4.8% in 1980 to 5.6% in 2014. The respective "normalized" ratio $\psi^{E}_{normalized} \equiv \frac{h^H_x\bar{H}_{z_1}+h^H_x\bar{H}_{z_2}+h^L_x\bar{L}_{z_1}+h^L_x\bar{L}_{z_2}}{h^H_x\bar{H}_x+h^L_x\bar{L}_x}$ rose from 4.6% in 1980 to 5.1% in 2014. On the other hand, the figure illustrates the structural change towards the financial sector in terms of a growing wage sum ratio of finance. The wage sum ratio of the financial sector, defined as $\psi^{W}_{actual} \equiv \frac{w^H_{z_1}h^H_{z_1}\bar{H}_{z_1}+w^H_{z_2}h^H_{z_2}\bar{H}_{z_2}+w^L_{z_1}h^L_{z_1}\bar{L}_{z_1}+w^L_{z_2}h^L_{z_2}\bar{L}_{z_2}}{w^H_x h^H_x\bar{H}_x+w^L_x h^L_x\bar{L}_x}$, increased by 55% from about 5.1% in 1980 to 7.9% in 2014. The respective "normalized" ratio $\psi^{W}_{normalized} \equiv \frac{w^H_x h^H_x\bar{H}_{z_1}+w^H_x h^H_x\bar{H}_{z_2}+w^L_x h^L_x\bar{L}_{z_1}+w^L_x h^L_x\bar{L}_{z_2}}{w^H_x h^H_x\bar{H}_x+w^L_x h^L_x\bar{L}_x}$ rose by 23% from 4.7% in 1980 to 5.8% in 2014. Note, the difference of the sectoral structure between the employment E and the wage sum W ratio is the result of different skill-intensities in the different sectors. By comparing the "normalized" black with the "actual" gray lines one sees a large difference between the two ratios of the wage sum: Half of the increase in the ratio of the wage sum is the result of the frictions (i) and (ii). Yet, as the black line shows, even if one controls for the two frictions there is still the structural change towards finance. Comparison of the two black lines shows that the difference between the employment and the wage sum ratio increased over time.

Figure 2: Employment and wage sum ratio of the financial sector

Notes: ψ^E measures the employment ratio (in terms of total hours worked) of finance (including insurance) compared to the rest of the U.S. economy. ψ^W measures the ratio of the total wage sum in finance vs. the rest of the U.S. economy. "Actual" uses the observed sector-specific hours worked and hourly wages (for low-and high-skilled), whereas "normalized" uses the X-sector hours worked and hourly wages (for low-and high-skilled). Data from 1980-2014. Source: March CPS.

We observe a similar pattern for the within finance sectoral structure by splitting total finance up into sub-sectors Z_1 and Z_2. Figure 3 shows the employment and the wage sum ratios of finance sub-sector Z_2 compared to the sub-sector Z_1 for the U.S. since the eighties based on the CPS data set. "New finance" (sub-sector Z_2) grew strongly independent of the measure we use: The within employment ratio (in terms of total hours worked) of finance sub-sector Z_2, $\Phi^E_{actual} \equiv \frac{h^H_{z_2}\bar{H}_{z_2}+h^L_{z_2}\bar{L}_{z_2}}{h^H_{z_1}\bar{H}_{z_1}+h^L_{z_1}\bar{L}_{z_1}}$, more than doubled from about 8.8% in 1980 to 19.6% in 2014. The respective "normalized" ratio $\Phi^E_{normalized} \equiv$

$\frac{h_x^H \bar{H}_{z_2}+h_x^L \bar{L}_{z_2}}{h_x^H \bar{H}_{z_1}+h_x^L \bar{L}_{z_1}}$ is very similar with a rise from 8.6% in 1980 to 19.6% in 2014. The within finance wage sum ratio, defined by $\Phi^W_{actual} \equiv \frac{w_{z_2}^H h_{z_2}^H \bar{H}_{z_2}+w_{z_2}^L h_{z_2}^L \bar{L}_{z_2}}{w_{z_1}^H h_{z_1}^H \bar{H}_{z_1}+w_{z_1}^L h_{z_1}^L \bar{L}_{z_1}}$, increased dramatically from 11% in 1977 to 29.5% in 2012 peaking in 2009 at 40.2%. The respective "normalized" ratio $\Phi^W_{normalized} \equiv \frac{w_x^H h_x^H \bar{H}_{z_2}+w_x^L h_x^L \bar{L}_{z_2}}{w_x^H h_x^H \bar{H}_{z_1}+w_x^L h_x^L \bar{L}_{z_1}}$ rose from 9.3% in 1980 to 22.8% in 2014 with a peak in 2009 of 29.9% . Hence, about two-thirds of the actual rise in the wage ratio of "new finance" cannot be assigned to frictions: They are also observed in the "normalized" data. The rest of the rise comes from friction (ii), which is particularly strong in the finance sub-sector Z_2.

Figure 3: Employment and wage sum ratio of the financial sector

Notes: Φ^E measures the employment ratio (in terms of total hours worked) of "new finance" compared to "traditional finance". Φ^W measures the ratio of the total wage sum in "new finance" vs. "traditional finance". "Actual" uses the sector-specific hours worked and hourly wages (for low-and high-skilled), whereas "normalized" uses the X-sector hours worked and hourly wages (for low-and high-skilled). Data from 1980-2014. Source: March CPS.

As argued in the introduction financialization with the two-fold structural change and inequality are two closely related topics. Figure 4 shows the development of the "normalized" skill premium calculated by $\omega = \frac{w_x^H}{w_x^L}$ for the U.S. since 1980, based on the CPS data. It increased from 1.46 to 1.91 in 2014.[23] The time trend in ω illustrates that wage inequality increased over time. Nowadays high-skilled workers earn nearly double as much as low-skilled workers per hour. If one accounts in addition for the fact that high-skilled workers work more hours the income inequality is even larger (e.g., 2.15 in 2014).

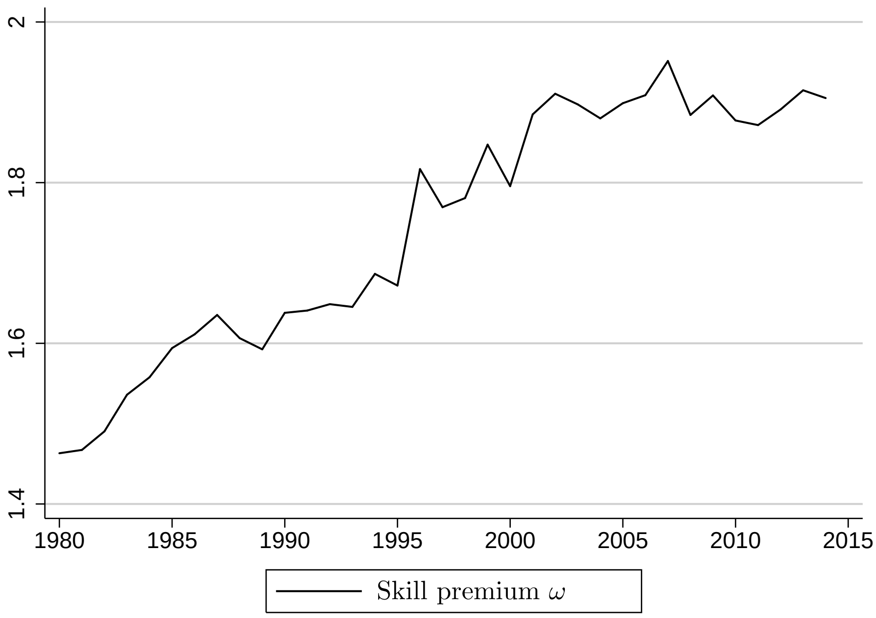

Figure 4: Skill premium

Notes: ω measures the skill premium (i.e., hourly wage of high-skilled divided by hourly-wage of low-skilled). Data from 1980-2014. Source: March CPS.

[23]Interestingly, the skill premium in the U.S. is about the same in the three sectors because the relative finance premium is the same for low- and high-skilled workers.

9.2 Numerics

In this section we calibrate our theoretical model and use it for several numerical exercises, which illustrate possible drivers of the empirical developments presented in Figures 2-4.

9.2.1 Calibration

We calibrate our model such that it fits the data for the average year from the time range 1980-2014. For labor endowments, output elasticities, technology in the X-sector, subsistence levels (calculated based on the poverty thresholds) and the safe return exogenous values from data are used as summarized in Table 1.

We scale the economy to one week (i.e., the yearly hours worked h^l, $l \in \{H, L\}$ and the poverty thresholds PT are divided by 52). We assume each worker must cover half of a two-people households' poverty threshold (i.e., division of the poverty thresholds PT by two). Further, we account for the fact that the ratio of working-time to retirement is $LEratio = 4.06$ (i.e., we divide the poverty thresholds PT^{65} further by 4.06). Hence, $\bar{e}_0 = PT_{65}/2/52$ and $\bar{e}_1 = PT^{65}/2/52/4.06$. The safe return $r = 1 + r^f$ corresponds to the Feds fund rate and the risky return is $R = r + 0.04$ such that the risk premium is four percentage points. We define the efficiency units from the model as $b_l \equiv \tilde{b}_l h^l$, $l \in \{H, L\}$ where $\tilde{b}_l$ are the unobservable efficiency units in the data and h^l are hours worked. We set $\tilde{b}_l = 1$, $l \in \{H, L\}$.

When solving the model numerically and calibrating it, we target the wage

Table 1: Exogenous parameters

Parameter	Data	Source	Description
$\bar{L}$	102.5m	CPS	# Low-skilled employees
$\bar{H}$	36.2m	CPS	# High-skilled employees
h^L	1717.8	CPS	Yearly hours of low-skilled
h^H	2009.2	CPS	Yearly hours of high-skilled
α_x	0.40	CPS	Output ela. of high-skilled in X
α_{z_1}	0.50	CPS	Output ela. of high-skilled in Z_1
α_{z_2}	0.75	CPS	Output ela. of high-skilled in Z_2
A_x	40.98	CPS	Technology level in X
PT_{65}	\$ 10,813	U.S. Dep. of commerce	Poverty threshold <65
PT^{65}	\$ 9,722	U.S. Dep. of commerce	Poverty threshold >65
$LEratio$	4.06	LE from World Bank	Old-age ratio
r^f	5.18	St.Louis Fed	Riskfree return (Feds fund rate)

Notes: $\alpha_j = \frac{\kappa_j \omega_j}{1+\kappa_j \omega_j}$ with $\kappa_j = \frac{h_j^H \bar{H}_j}{h_j^L \bar{L}_j}$ and $\omega_j = \frac{w_j^H}{w_j^L}$, $j \in \{x, z_1, z_2\}$, $h^H = h_x^H$ and $h^L = h_x^L$. $A_x = \frac{w_x^L}{\Gamma_x \omega_x^{-\alpha_x}}$ with $\Gamma_x = \alpha_x^{\alpha_x}(1-\alpha_x)^{1-\alpha_x}$. PT are the poverty thresholds of a two-people households with PT_{65} for householders younger than 65 and PT^{65} for older ones. $LEratio$ is the ratio of working-time to retirement: $(65-20)/(LE-65)$, where LE is life expectancy, 65 is the retirement age and 20 is assumed start of the working-life.

inequality and the "normalized" sectoral structure (in terms of employment) of the U.S. economy for the average year:[24] The targeting gives us (possible)

[24]For solving the model numerically, we use the demand functions in the goods and financial services markets to obtain the equilibrium values of X-, Z_1- and Z_2 as functions of ω (and exogenous parameters). Substituting these functions for X-, Z_1- and Z_2 in one of the labor market clearing conditions, we can solve for the equilibrium skill premium ω^*. (Then, at ω^*, the other labor market is also cleared.) From ω^* follow factor prices and prices, labor employments in the three sectors, outputs and the sectoral structure of the economy in a straightforward way.

values for the technology and preference parameters by internal calibration:[25] We internally calibrate $A_{z_1} = 52$, $A_{z_2} = 62$, $\mu = 0.93$ and $\delta = 0.074$. With this calibration we come close to the values of the targets as shown in Table 2:

Table 2: Targets

Parameter	Model	Data	Source	Description
ω^*	1.66	1.75	CPS	Skill premium
ψ^E	4.94%	4.95%	CPS	Between sectoral structure: Employment
Φ^E	16.69%	16.80%	CPS	Within sectoral structure: Employment

Notes: ω^* is the equilibrium skill premium. Employment is in terms of total hours worked.

The values of equilibrium skill premium ω^* and the sectoral structure ψ and Φ predicted by the model fit the targets fairly well. They deviate from the observed ones only by some percent.[26] From ω^* follow directly other equilibrium values: Hourly wages in our model are w^H = \$ 28.4 (data: \$ 29.1) and w^L = \$ 17.1 (data: \$ 16.6) and the resulting prices are $p_{z_1} = 0.85$ and $p_{z_2} = 0.71$.[27] Labor employments (total hours per week) are $H_x = 1305$m

[25] We internally calibrate to come as close as possible to the targeted values. More precisely, we search for the combination of A_{z_1}, A_{z_2}, μ and δ which minimizes the sum of the squared relative distances of the three model values from the corresponding data targets. Thereby, we allow only for values of δ which give us a saving rate of about 5%.

[26] Our average skill premium ω^* from the model does not exactly match the data because we are using the average α_j, $j \in \{x, z_1, z_2\}$, which are based on the year-specific ω^* and not the average ω^*.

[27] These could be interpreted that a households has to pay the unit costs of financial intermediation, estimated by Philippon (2015) to be 0.02, during all his/hers working years ($\sim$ 45-times).

(data: 1304m), $L_x = 3254$m (data: 3261m), $H_{z_1} = 73$m (data: 79m), $L_{z_1} = 121$m (data: 130m), $H_{z_2} = 21$m (data: 25m), $L_{z_2} = 11$m (data: 12m). Thus, these are very similar to the observed values in the CPS data. We get for skill intensities $\kappa_x = 0.4 < \kappa_{z_1} = 0.6 < \kappa_{z_2} = 1.9$ (data: $\kappa_x = 0.4 < \kappa_{z_1} = 0.6 < \kappa_{z_2} = 2$). Total yearly wage sums are $X = \$4.8 \cdot 10^{12}$ (data: $\$4.9 \cdot 10^{12}$), $Z_1 = \$\,0.2 \cdot 10^{12}$ (data: $\$\,0.2 \cdot 10^{12}$) and $Z_2 = \$\,0.04 \cdot 10^{12}$ (data: $\$\,0.05 \cdot 10^{12}$). We therefore have $\psi^W = 5.3$ (data: 5.4) and $\Phi^W = 19.1$ (data: 19.5). These numbers show, our model predicts the values of the U.S. economy fairly well.

9.2.2 Numerical exercises

Finally, we want to analyze how our model can predict the development of the structural change and the wage inequality over time – as a sequences of static equilibria. For this, we compare the predictions of the model for different scenarios with the empirics. Figure 2-4 summarize the development of our targets (i.e., skill premium ω and "normalized" sectoral structure, ψ^E, Φ^E) for the U.S. from 1980-2014 based on the CPS data, which we want to predict. Remember from the figures, we have the rise in the skill premium ω from 1.46 to 1.91 and the "normalized" two-fold structural change with respect to finance with ψ^E from 4.6% to 5.1% and Φ^E from 8.6% to 19.6%.

We now simulate a benchmark economy based on our model by using for each year the respective exogenous parameters (i.e., $\bar{H}$, $\bar{L}$, h^H, h^L, α_x, α_{z_1}, α_{z_1}, A_x, PT_{65}, PT^{65}, $LEratio$ and r^f for 1980-2014).[28] Further, we assume

[28] See Table 3 in Appendix D for the time series data from 1980-2014 for $\bar{H}$, $\bar{L}$, h^H, h^L, α_x, α_{z_1}, α_{z_1}, A_x, PT_{65}, PT^{65}, $LEratio$ and r^f. For R we use a constant risk premium of four percentage points.

uniform technological progress. This means, the productivity in the Z-sectors develops identical to the productivity in the X-sector: For the middle year 1997, which corresponds closest to the average year used in the calibration, we apply the calibrated parameters A_{z_1} and A_{z_2}. For all other years t we assume uniform technological progress with $A^t_{z_i} = g^t_z A^{1997}_{z_i}$, $g^t_{z_i} = g^t_x$ where $g^t_x = A^t_x / A^{1997}_x$ is given by the observed development of A_x. Furthermore, we apply the calibrated preference and certainty parameter δ and μ in all years. With this procedure, the simulated series of static equilibra generated by our model can qualitatively predict the rise in wage inequality and two-fold structural change.[29] More precisely, we predict a rise in the skill premium ω from 1.45 to 1.93 and the two-fold structural change with respect to finance with ψ^E from 4.1% to 6.2% and Φ^E from 8.2% in 1982 to 14.35%. These numbers show that our model overestimates the between structural change and underestimates the within structural change a little. Overall, the development simulated by our model matches data fairly well.

Further, we want to understand how biased technical change affects our predictions: First, if we allow for finance sectors-biased technical change such that Z-technology grows by about 1.5% on average (compared to X-technology which growth by around 1% on average), this decreases the overestimation of the between structural change and would fit data better.[30] Second, if we instead shut down the effect of factor-biased technical change (represented by the changing α_x, α_{z_1} and α_{z_2} over time), we cannot predict

[29]See Figure 5 in Appendix E. Note that for some years we have missing values because the viability constraint in (18) or (19) are violated and our model does not apply.

[30]To achieve an average growth rate of 1.5% we use $1.08 g^t_x = g^t_{z_i}$ for $t > 1997$ and $g^t_x / 1.08 = g^t_{z_i}$ for $t < 1997$. This simulation is shown in Figure 6 in Appendix E.

the increase in the skill premium anymore.[31] This shows, (sector and factor) biased technical change improve the prediction power of our model. Finally, we analyze whether a model without quasi-homothetic preferences is able to predict the increase in the skill premium and the two-fold structural change simultaneously. We find, if we neglect the subsistence levels with $\bar{e_0} = \bar{e_1} = 0$ the model cannot explain the between structural change with an increase of the ratio of the finance sector anymore.[32] Hence, channels from the supply (i.e., technological progress and directed technical change) and the demand side (i.e., income effects) – all used in our benchmark economy – are needed to generate inequality and the two-fold structural change with respect to finance.[33]

10 Conclusion

We propose a 3x3 model of production and financial services, which helps to explain the two-fold structural change towards and within the financial sector. Thereby, we account for demand side effects by using quasi-homothetic preferences of the Stone-Geary form and supply side effects by accounting for different skill-intensities in production of goods and financial services.

[31]See Figure 7 in Appendix E.

[32]Since we assume a different underlying utility function (i.e., log-preferences instead of Stone-Geary) we calibrate the model again with the same procedure as above. We get $A_{z_1} = 48$, $A_{z_2} = 55$, $\mu = 92$ and $\delta = 0.12$. The predicted development of our targets for the simulation without subsistence levels is shown in Figure 8 in Appendix E.

[33]If we would allow for an increase in the certainty measure μ (i.e., market completion over time) or a change in preferences with a rising δ over time, this would foster within and between structural change, respectively.

In the theoretical model we can derive analytically qualitative comparative-static equilibrium results which identify exogenous changes which lead to a rise in inequality, an increase in the share of the financial sector and a within finance shift towards “new finance”. The general qualitative results are illustrated quantitatively by calibrating the model to U.S. data from 1980-2014. The illustrations show that our model can explain the developments observed in that period fairly well. Hence, the paper contributes to the understanding of two important topics of recent public discussion - financialization and inequality - and highlights that the rising inequality and the (two-fold) structural change towards and within the financial sector may stem from a common set of supply and demand factors.

A Portfolio Choice

Agent index l is skipped in the appendix. If financial intermediaries take ex-ante a fee in the form $T = p_{z_1}d + p_{z_2}(s-d)$, the expected utility maximization problem is given by:

$$\max_{s,\{f_\theta\}_{\theta\in\Theta},d} \mathbb{E}U = \log(e_0 - \bar{e}_0) + \delta\left[\mu\sum_{\theta\in\Theta}\pi_\theta\log(e_\theta - \bar{e}_1) + (1-\mu)\log(e_{\bar{\Theta}} - \bar{e}_1)\right]$$

s.t.

$$e_0 + (1+p_{z_2})s + (p_{z_1} - p_{z_2})d = y, \tag{A.1}$$

$$e_\theta = \begin{cases} R_\theta f_\theta + rd, \text{ if } \theta \in \Theta \\ rd, \text{ otherwise} \end{cases} \tag{A.2}$$

$$s = \sum_{\theta\in\Theta} f_\theta + d. \tag{A.3}$$

Denoting by λ the Lagrange multiplier for constraint (A.3) the first-order conditions of the households' expected utility maximization problem give:

$$\frac{\partial\mathcal{L}}{\partial s} = -\frac{1+p_{z_2}}{e_0 - \bar{e}_0} + \lambda = 0, \tag{A.4}$$

$$\frac{\partial\mathcal{L}}{\partial f_\theta} = \delta\mu\pi_\theta\frac{R_\theta}{e_\theta - \bar{e}_1} - \lambda = 0, \tag{A.5}$$

$$\frac{\partial\mathcal{L}}{\partial d} = -\frac{p_{z_1} - p_{z_2}}{e_0 - \bar{e}_0} + \delta\left[\mu\sum_{\theta\in\Theta}\pi_\theta\frac{r}{e_\theta - \bar{e}_1} + (1-\mu)\frac{r}{rd - \bar{e}_1}\right] - \lambda = 0, \tag{A.6}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = s - \sum_{\theta\in\Theta} f_\theta - d = 0. \tag{A.7}$$

Using (A.4), (A.5) and (A.6), we have

$$d = \frac{\delta(1-\mu)}{\lambda\left(\frac{1+p_{z_1}}{1+p_{z_2}} - r/R\right)} + \frac{\bar{e}_1}{r}. \tag{A.8}$$

where $R = \pi_\theta R_\theta$. From (A.2), (A.5) and (A.7), we have

$$s = \frac{\delta\mu}{\lambda} + (1 - r/R)d + \frac{1}{R}\bar{e}_1. \tag{A.9}$$

In the end we have

$$\begin{aligned} d &= \frac{\delta(1-\mu)}{(1+\delta)P}(y - \bar{e}_0) + \frac{(1+\mu\delta)(1+p_{z_1}) - (1+\delta)(1+p_{z_2})r/R}{r(1+\delta)P}\bar{e}_1 \\ &= \frac{1-\mu}{1-p\rho}\frac{\delta}{1+\delta}\frac{y-\bar{y}}{1+p_{z_1}} + \frac{\bar{e}_1}{r}, \end{aligned} \tag{A.10}$$

where $P \equiv (1+p_{z_1})(1-p\rho)$, $p \equiv \frac{1+p_{z_2}}{1+p_{z_1}}$, $\rho \equiv \frac{r}{R}$ and $\bar{y} \equiv \bar{e}_0 + \frac{\bar{e}_1(1+p_{z_1})}{r}$.

Combining (A.10) with (A.8) and solving for λ, we obtain

$$\frac{1}{\lambda} = \frac{y - \bar{y}}{(1+\delta)(1+p_{z_2})} \tag{*}$$

Using this and (A.10) in (A.9), we have

$$\begin{aligned} s &= \frac{\delta}{(1+\delta)}\frac{y-\bar{y}}{1+p_{z_2}}\left[\mu + (1-\rho)\frac{p(1-\mu)}{1-p\rho}\right] + (1-\rho)\frac{\bar{e}_1}{r} + \frac{\bar{e}_1}{R} \\ &= \frac{\delta}{1+\delta}\frac{y-\bar{y}}{1+p_{z_2}}\frac{\mu - p\rho + p(1-\mu)}{1-p\rho} + \frac{\bar{e}_1}{r}, \end{aligned}$$

which can be rewritten in the form

$$s = \frac{\delta}{1+\delta}\frac{y-\bar{y}}{1+p_{z_2}}\left[1 + \frac{(p_{z_2} - p_{z_1})(1-\mu)}{(1+p_{z_1})(1-p\rho)}\right] + \frac{\bar{e}_1}{r}, \tag{A.11}$$

where $p - 1 = \frac{p_{z_2} - p_{z_1}}{1+p_{z_1}}$ has been used.

Finally, (A.7), (A.10) and (A.11) give us

$$f \equiv \sum_{\theta\in\Theta} f_\theta = \frac{\mu - p\rho}{1-p\rho}\frac{\delta}{1+\delta}\frac{y-\bar{y}}{1+p_{z_2}} \tag{A.12}$$

and from (A.1) we conclude

$$\begin{aligned} y - e_0 &= (1+p_{z_1})d + (1+p_{z_2})f \\ &= \frac{\delta}{1+\delta}(y - \bar{y}) + \frac{(1+p_{z_1})\bar{e}_1}{r}. \end{aligned} \tag{A.13}$$

For the allocation of f on $f_\theta, \theta \in \Theta$, we combine (A.2) with (A.5) to get

$$\begin{aligned} f_\theta &= \pi_\theta \left[\frac{\delta\mu}{\lambda} + \frac{\bar{e_1} - rd}{R} \right] \\ &= \pi_\theta \frac{\delta}{1+\delta} \frac{y - \bar{y}}{1 + p_{z_2}} \left[\mu - \rho \frac{1-\mu}{1-p\rho} p \right] = \pi_\theta f, \end{aligned}$$

where (A.10) and (*) have been used for the second equation.

B Corner solutions for securities demand

To account for the non-negativity constraint $f_\theta \geq 0$ we have to add $\sum_{\theta \in \Theta} \psi_\theta f_\theta$ to the Lagrange function for max EU – with $\psi_\theta \geq 0$ denoting the Lagrange multiplier for $f_\theta \geq 0$. Then, the first order condition for f_θ changes to

$$\delta \mu \pi_\theta \frac{R_\theta}{e_\theta - \bar{e}_1} - \lambda + \psi_\theta = 0 \tag{B.1}$$

with $\psi_\theta f_\theta \leq 0$.

Suppose that $f_\theta = 0$ for all θ. Then $s = d$ and

$$\begin{aligned} e_0 - \bar{e}_0 &= y - \bar{e}_0 - (1 + p_{z_1}) d \\ e_\theta - \bar{e}_1 &= rd - \bar{e}_1 \end{aligned} \tag{B.2}$$

and the first-order conditions

$$\begin{aligned} &(s) \qquad \lambda = \frac{1 + p_{z_2}}{e_0 - \bar{e}_0} \\ &(d) \qquad \delta \left[\mu \sum_{\theta \in S} \pi_\theta \frac{r}{e_\theta - \bar{e}_1} + (1-\mu) \frac{r}{rd - \bar{e}_1} \right] = \lambda + \frac{p_{z_1} - p_{z_2}}{e_0 - \bar{e}_0} \end{aligned} \tag{B.3}$$

reduce to

$$\delta \frac{r}{rd - \bar{e}_1} = \frac{1 + p_{z_1}}{e_0 - \bar{e}_0}.$$

With (B.2) this solves to

$$d = \frac{1}{1+\delta}\left[\frac{\delta(y-\bar{e}_0)}{1+p_{z_1}} + \frac{\bar{e}_1}{r}\right]. \tag{B.4}$$

Substituting the solution into (B.2) gives us

$$\begin{aligned} e_0 - \bar{e}_0 &= \frac{1}{1+\delta}\left[y - \bar{e}_0 - \frac{(1+p_{z_1})\bar{e}_1}{r}\right] \\ e_\theta - \bar{e}_1 &= \frac{\delta r}{(1+\delta)}\left[\frac{y-\bar{e}_0}{1+p_{z_1}} - \frac{\bar{e}_1}{r}\right]. \end{aligned} \tag{B.5}$$

Using this in (B.1) we obtain: $\psi_\theta \geq 0$ if and only if

$$\mu\pi_\theta R_\theta \leq \frac{1+p_{z_2}}{1+p_{z_1}} r \tag{B.6}$$

where $\lambda = \frac{1+p_{z_2}}{e_0-\bar{e}_0}$ has been used from (B.3).

Since $\pi_\theta R_\theta = R$, (B.6) reduces to

$$\frac{1+p_{z_1}}{1+p_{z_2}}\mu R \leq r,$$

which is equivalent to $R\mu(1+p_{z_1}) \leq (1+p_{z_2})r$.

Hence non-negativity $f_\theta > 0$, $\theta \in \Theta$, requires

$$R\mu(1+p_{z_1}) > (1+p_{z_2})r. \tag{B.7}$$

C Further proofs

Proof of Fact 3. With (11) and (12) the condition $y^L = b_L w_L > \bar{y} = \bar{e}_0 + \frac{(1+p_z)\bar{e}_1}{r}$ takes the form

$$A_x\Gamma_x\omega^{-\alpha_x}\left[b_L - \frac{\bar{e}_1}{rA_{z_1}\Gamma_{z_1}}\omega^{\alpha_{z_1}}\right] > \bar{e}_0 + \frac{\bar{e}_1}{r}.$$

The left side of the equation declines in ω. Thus $y^L > \bar{y}$ requires

$$\omega < \omega_L^+\left(\underset{+}{A_x}, \underset{+}{A_{z_1}}, \underset{+}{b_L}, \underset{-}{\bar{e}_0}, \underset{-}{\frac{\bar{e}_1}{r}}\right),$$

where ω_L^+ is determined by the equation:

$$b_L = (\bar{e}_0 + \frac{\bar{e}_1}{r})\frac{\omega^{\alpha_x}}{A_x\Gamma_x} + \frac{\bar{e}_1}{r}\frac{\omega^{\alpha_{z_1}}}{A_{z_1}\Gamma_{z_1}}.$$

□

Proof of Lemma 1. a) Let $B_1 \equiv A_x\Gamma_x\frac{b_L\bar{L}}{N}$ and $B_2 \equiv \frac{A_x\Gamma_x}{A_z\Gamma_z}$. Using (26) and (12), we have

$$\bar{w} = B_1\omega^{-\alpha_x}(1+\omega k), \quad p_z = B_2\omega^{\alpha_z-\alpha_x}.$$

Then $\bar{\eta}$ can be reformulated as

$$\bar{\eta} = \frac{\bar{w}-\bar{y}}{1+p_z} = \frac{B_1\omega^{-\alpha_x}(1+\omega k)-\bar{e}_0}{1+B_2\omega^{\alpha_z-\alpha_x}} - \frac{\bar{e}_1}{r},$$

where (18) is used to substitute $\bar{y}$. Taking limits to the expression, we have:

$$\lim_{\omega\to 0^+}\bar{\eta} = \lim_{\omega\to 0^+}\frac{B_1\omega^{-\alpha_x}(1+\omega k)-\bar{e}_0}{1+B_2\omega^{\alpha_z-\alpha_x}} - \frac{\bar{e}_1}{r} = +\infty.$$

$$\lim_{\omega\to+\infty}\bar{\eta} = \lim_{\omega\to+\infty}\frac{B_1(1+\omega k)-\bar{e}_0\omega^{\alpha_x}}{\omega^{\alpha_x}+B_2\omega^{\alpha_z}} - \frac{\bar{e}_1}{r} = \lim_{\omega\to+\infty}\frac{B_1k-\alpha_x\bar{e}_0\omega^{\alpha_x-1}}{\alpha_x\omega^{\alpha_x-1}+\alpha_zB_2\omega^{\alpha_z-1}} - \frac{\bar{e}_1}{r} = +\infty,$$

where the L'Hospital's rule is applied in the second equality.

To get the shape of $\bar{\eta}$, first notice that

$$sign\frac{\partial\bar{\eta}(\omega)}{\partial\omega} = sign\frac{\partial G(\omega)}{\partial\omega},$$

where $G(\omega) \equiv \frac{B_1(1+\omega k)-\bar{e}_0\omega^{\alpha_x}}{\omega^{\alpha_x}+B_2\omega^{\alpha_z}}$. Differentiating $G(\omega)$ we have

$$\frac{\partial G(\omega)}{\partial\omega} = \frac{\mathcal{L}(\omega)}{(\omega^{\alpha_x}+B_2\omega^{\alpha_z})^2},$$

where

$$\mathcal{L}(\omega) = B_1\omega^{\alpha_x}\left[k(1-\alpha_x) - \frac{\alpha_x}{\omega}\right] - B_1B_2\omega^{\alpha_z}\left[-k(1-\alpha_z) + \frac{\alpha_z}{\omega}\right] + \bar{e_0}B_2(\alpha_z - \alpha_x)\omega^{\alpha_x+\alpha_z-1}$$

From the expression of $\frac{\partial G(\omega)}{\partial \omega}$, we have $\frac{\partial G(\omega)}{\partial \omega} > 0$ if and only if $\mathcal{L}(\omega) > 0$. Notice that if $\alpha_x + \alpha_z > 1$, the $\mathcal{L}(\omega)$ is an increasing function in ω. By eye inspection we know

$$\lim_{\omega \to 0^+} \mathcal{L} = -\infty, \qquad \lim_{\omega \to +\infty} \mathcal{L} = +\infty.$$

Let $\mathcal{L}(\underline{\omega}) = 0$. Therefore, $\frac{\partial \bar{\eta}(\omega)}{\partial \omega} \gtreqless 0$ if and only if $\omega \gtreqless \underline{\omega}$.

b)

$$\bar{\eta} = \frac{A_x\Gamma_x\frac{b_L\bar{L}}{N}\omega^{-\alpha_x}(1+\omega k) - \bar{e_0}}{1 + \frac{A_x\Gamma_x}{A_z\Gamma_z}\omega^{\alpha_z-\alpha_x}} - \frac{\bar{e_1}}{r}.$$

By eye inspection we know:

$$\bar{\eta}\left(\omega \,\middle|\, \underset{+}{A_x}, \underset{+}{A_z}, \underset{+}{k}, \underset{+}{\frac{b_L\bar{L}}{N}}, \underset{-}{\bar{e_0}}, \underset{-}{\frac{\bar{e_1}}{r}}\right)$$

□

Proof of Fact 6. According to (38), $Z^S = A_z b_L \bar{L}\frac{\gamma_z^{\alpha_z}}{\gamma_z-\gamma_x}\omega^{-\alpha_z}(k\omega - \gamma_x)$, where $\kappa_j = \frac{\gamma_j}{\omega}$ has been used from (9).

We have $\frac{\partial \omega^{-\alpha_z}(k\omega-\gamma_x)}{\partial \omega} = \omega^{-\alpha_z}\left[(1-\alpha_z)k + \frac{\alpha_z\gamma_x}{\omega}\right]$. This term is positive and decreasing in ω. □

D Data

Table 3 shows the time series of the exogeneous parameters.

Table 3: Time series of exogenous parameters

Year	$\bar{L}$	$\bar{H}$	h^L	h^H	α_x	α_{z_1}	α_{z_1}	A_x	PT_{65}	PT^{65}	$LEratio$	r
1980	79.2	18.4	1743.5	2022.0	0.28	0.35	0.59	33.05	5537	4983	5.20	13.36
1981	80.8	19.3	1728.1	2017.6	0.29	0.35	0.60	32.70	6111	5498	5.00	16.38
1982	78.8	20.3	1727.4	1998.7	0.30	0.38	0.71	33.28	6487	5836	4.81	12.26
1983	93.8	23.3	1567.2	1934.2	0.32	0.40	0.66	33.27	6697	6023	4.76	9.09
1984	94.4	24.0	1585.5	1956.0	0.32	0.38	0.62	33.41	6983	6282	4.71	10.23
1985	97.2	24.9	1614.9	1987.7	0.33	0.40	0.61	34.00	7231	6503	4.71	8.10
1986	98.8	25.7	1629.6	1993.8	0.33	0.42	0.68	34.70	7372	6630	4.68	6.81
1987	100.0	26.7	1643.5	2002.2	0.34	0.42	0.63	35.73	7641	6872	4.61	6.66
1988	101.1	27.7	1662.7	2007.6	0.34	0.40	0.68	35.54	7958	7157	4.61	7.57
1989	88.4	27.5	1786.3	2064.5	0.36	0.41	0.71	37.42	8343	7501	4.49	9.22
1990	103.6	30.1	1692.0	2002.9	0.35	0.45	0.71	36.30	8794	7905	4.41	8.10
1991	103.9	30.4	1689.5	1996.4	0.36	0.46	0.70	35.70	9165	8241	4.34	5.69
1992	103.4	30.6	1683.3	1993.1	0.36	0.45	0.66	35.89	9443	8487	4.23	3.52
1993	102.9	31.6	1683.9	2016.4	0.37	0.46	0.71	36.33	9728	8740	4.32	3.02
1994	104.7	32.4	1689.4	2018.8	0.38	0.47	0.72	36.44	9976	8967	4.26	4.20
1995	105.2	33.9	1702.1	2033.8	0.38	0.50	0.71	37.27	10259	9219	4.24	5.84
1996	105.8	34.6	1728.8	2022.2	0.40	0.48	0.73	40.94	10564	9491	4.09	5.30
1997	107.6	35.3	1725.8	2040.6	0.40	0.48	0.74	41.19	10805	9712	3.94	5.46

Table 3: Time series of exogenous parameters (continued)

Year	$\bar{L}$	$\bar{H}$	h^L	h^H	α_x	α_{z_1}	α_{z_1}	A_x	PT_{65}	PT^{65}	$LEratio$	r
1998	107.9	36.7	1737.2	2040.3	0.41	0.50	0.82	41.69	10972	9862	3.89	5.35
1999	108.3	37.9	1756.8	2051.2	0.42	0.54	0.71	43.35	11213	10075	3.89	4.97
2000	110.0	39.0	1761.8	2056.9	0.42	0.54	0.78	43.21	11589	10418	3.87	6.24
2001	110.1	40.0	1774.7	2052.4	0.43	0.51	0.75	45.97	11920	10715	3.83	3.89
2002	109.9	41.7	1767.5	2027.9	0.45	0.53	0.80	47.06	12110	10885	3.80	1.67
2003	109.6	42.4	1759.2	2025.1	0.45	0.55	0.78	47.36	12384	11133	3.75	1.13
2004	108.7	43.3	1759.5	2011.9	0.45	0.54	0.85	47.12	12714	11430	3.65	1.35
2005	109.7	43.8	1775.8	2007.9	0.45	0.56	0.81	46.81	13145	11815	3.65	3.21
2006	110.7	44.8	1785.3	2011.7	0.46	0.57	0.86	46.65	13569	12201	3.57	4.96
2007	110.9	46.8	1794.6	2020.8	0.47	0.60	0.83	47.32	13954	12550	3.50	5.02
2008	110.4	48.5	1789.8	2016.3	0.47	0.60	0.85	46.80	14489	13030	3.48	1.93
2009	109.7	48.9	1745.9	1993.4	0.48	0.61	0.85	47.11	14439	12982	3.44	0.16
2010	106.0	49.0	1705.5	1971.3	0.49	0.62	0.87	48.20	14676	13194	3.32	0.18
2011	103.5	49.9	1713.4	1976.2	0.50	0.59	0.86	47.96	15139	13609	3.30	0.10
2012	103.6	51.0	1729.7	1985.2	0.51	0.62	0.87	48.09	15450	13892	3.27	0.14
2013	104.6	52.8	1732.1	1984.2	0.51	0.66	0.88	48.04	15679	14095	3.25	0.11
2014	104.5	53.7	1752.5	1980.1	0.51	0.63	0.83	48.43	15933	14324	3.21	0.09

Notes: See notes of Table 1. $\bar{L}$ and $\bar{H}$ in million. PT_{65} and PT^{65} in \$.

E Numerical exercise: Developments

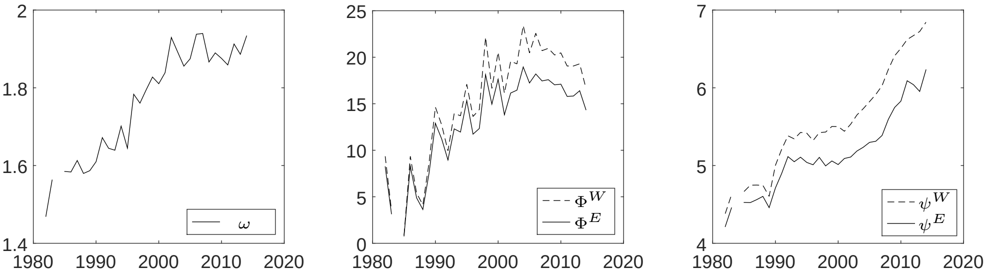

Figure 5: Model development

Notes: Simulation of skill premium ω and the sectoral structure Φ^W, Φ^E, ψ^W and ψ^W based on the time series changes in the exogenous parameters (i.e., $\bar{H}$, $\bar{L}$, h^H, h^L, α_x, α_{z_1}, α_{z_1}, A_x, PT_{65}, PT^{65}, $LEratio$ and r^f) for 1980-2014 and calibrated A_{z_1} and A_{z_2} (with uniform technical change with growth $g_z^t = g_x^t$), μ and δ. For the years with missing values the viability constraint in (18) or (19) were violated and our model does not apply.

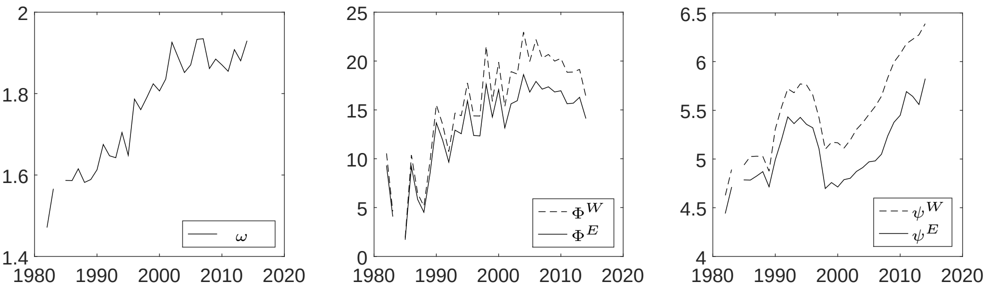

Figure 6: Model development with sector-biased technical change

Notes: Simulation of skill premium ω and the sectoral structure Φ^W, Φ^E, ψ^W and ψ^W based on the time series changes in the exogenous parameters (i.e., $\bar{H}$, $\bar{L}$, h^H, h^L, α_x, α_{z_1}, α_{z_1}, A_x, PT_{65}, PT^{65}, $LEratio$ and r^f) for 1980-2014 and calibrated A_{z_1} and A_{z_2} (with sector-biased technical change with growth $1.08g_x^t = g_{z_i}^t$ for $t > 1997$ and $g_x^t/1.08 = g_{z_i}^t$ for $t < 1997$), μ and δ. For the years with missing values the viability constraint in (18) or (19) were violated and our model does not apply.

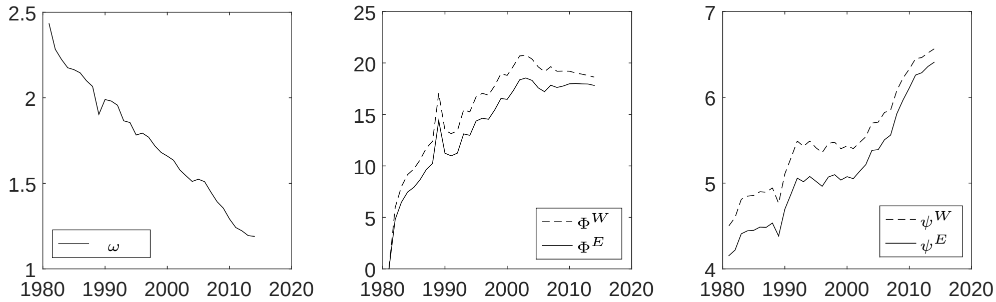

Figure 7: Model development without factor-biased technical change

Notes: Simulation of skill premium ω and the sectoral structure Φ^W, Φ^E, ψ^W and ψ^W based on the time series changes in exogenous parameters $\bar{H}$, $\bar{L}$, h^H, h^L, A_x, PT_{65}, PT^{65}, $LEratio$ and r^f for 1980-2014, fixed α_x, α_{z_1}, α_{z_1} and calibrated A_{z_1} and A_{z_2} (with uniform technical change with growth $g_z^t = g_x^t$), μ and δ. For the year with missing values the viability constraint in (18) or (19) were violated and our model does not apply.

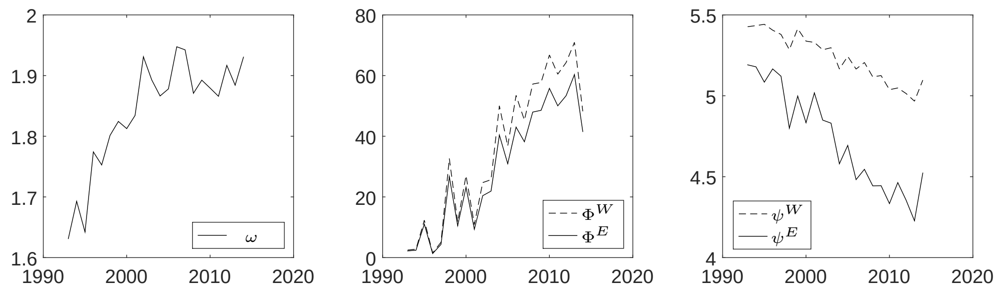

Figure 8: Model development without subsistence levels

Notes: Simulation of skill premium ω and the sectoral structure Φ^W, Φ^E, ψ^W and ψ^W based on the time series changes in the exogenous parameters (i.e., $\bar{H}$, $\bar{L}$, h^H, h^L, α_x, α_{z_1}, α_{z_1}, A_x and r^f) for 1980-2014, without subsistence levels (i.e., log-preferences) and calibrated A_{z_1} and A_{z_2} (with sector-biased technical change with growth $g_z^t = g_x^t$), μ and δ. For the years with missing values the viability constraint in (18) or (19) were violated and our model does not apply.

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