Incentives to Default in Economies with an Informal Sector and Procyclical Fiscal Policies*

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Abstract

Emerging markets are often characterized by pro-cyclical fiscal policies, large informal sector and high default risk. However, until now, the literature has ignored the existence of the informal sector and its relevance to explain the high sovereign bonds interest rate spreads faced by these countries. This paper closes this gap by using a small open economy sovereign default model with two sectors and endogenous taxes to show that, in a country that follows pro-cyclical fiscal policies, an exogenous increase in the incentives to produce in the informal sector, increase the size of this sector, reducing the total level of output in the economy. This decrease in output leads to an increase in default risk and, consequently, on sovereign bonds interest rate spreads, a phenomenon that is even larger when the economy is in a recession.

Keywords: Fiscal Policy, Informal Economy, Sovereign Default
JEL Classification Numbers: E26, E62, F34, F41 H26

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1 Introduction

Emerging economies are often characterized by following pro-cyclical fiscal policies, defined by higher government spending and lower tax rates in good times than in bad times, by having a large informal sector and by countercyclical sovereign interest rate spreads and default risk. However, until now, the literature has ignored the existence of the informal sector and its relevance, in interaction with pro-cyclical fiscal policies, to explain the higher level of spreads faced by these countries when comparing to developed countries.

This paper explains why a country faces higher spreads and default risk if it follows pro-cyclical fiscal policies and there are low penalties for those who produce in the informal sector than if it does not follow such policies and/or the penalties are higher. Consider a recession period characterized by a low level of output. If, in addition, the government follows a pro-cyclical fiscal policy translated into countercyclical tax rates and there are low penalties for people operating in the informal sector, the increase in tax rates creates an additional incentive to shift production to that sector. This reallocation of production, specially in a recession, leads to an even lower level of output produced in the economy, reducing the incentives to repay existing debt and increasing the sovereign bonds interest rate spreads these countries are going to face. Therefore, both the pro-cyclicality of fiscal policies as well as the existence of a large informal sector help explain the high spreads existing in emerging economies.

In fact, the data corroborates this intuition as it shows that the size of the informal sector in a country has a positive impact over spread, meaning that, even after controlling for other possible causes of spread, a country with a high informal sector is expected to have a higher spread than a country with a lower informal sector and this impact is even higher if the country is in a recession. It is also possible to observe in the data that the size of the informal sector has a negative impact over total output after controlling for the country's debt level. Specifically, a high informal sector reduces the total output of the country and, also in here, this effect is higher if the economy is in a recession. These two observed facts show the relevance of the informal sector for a country's spread and output and are the ones this paper replicates.

To do so, the sovereign default framework of Eaton and Gersovitz (1981) is extended by introducing an informal sector and endogenous tax rates. The benchmark model features a small open economy with three domestic agents: firms, households and government and foreign lenders, two productive sectors: formal and informal and one good. Firms may produce their good in the more productive formal sector[1] where they have to pay taxes over revenues or in the less productive informal sector, but where the payment of taxes only happens if the firm is discovered by the government in which case besides the taxes the firm also has to pay a penalty surcharge. The households have preferences over consumption and leisure and may choose whether to allocate their work to the formal or the informal sector. The government issues one-period non-contingent bonds in the international market and in each period, decides whether to repay or default on its debt. Besides the default decision, the government also chooses the tax rate it is going to apply to firms in order to finance its public spending. If it decides to default, the country faces a productivity loss and is excluded from the international markets for a certain period of time.

In this model, if the economy is hit by a low productivity shock, the government optimally chooses

[1]The difference in productivities between the two sectors is documented in Porta and Shleifer (2008).

to increase tax rates in order to finance its fixed level of government consumption. The increase in tax rates creates an incentive for firms to shift their production to the less productive informal sector, but where the payment of taxes is not imposed. When the probability of being found producing in the informal sector and the penalties associated with it are low, the size of this sector is higher, reducing total production by more than if only the productivity shock had taken place and there were no taxes or informal sector. Consequently, the government finds it attractive to default even for low debt amounts, increasing the default risk and the sovereign bond interest rate spread.

I calibrate this model for Argentina letting the size of the informal sector be endogenously determined. On average, the model has an informal sector of 23% of official GDP as observed in the data. Due to the fixed government consumption, the economy exhibits countercyclical tax rates as in the data, which contribute to an increase of the informal sector in recessions. The interaction of countercyclical tax rates and consequent countercyclical size of the informal sector lead to lower output produced when the informal sector is larger, a fact that is exacerbated when productivity is low, as observed in the data. Consequently, a large informal sector associated with lower output increase the incentives of the government to default even for low amounts of debt, replicating the high default risk and interest rates on government bonds that is observed for emerging countries. In what respects the statistical moments, the model generally underestimates them relatively to those observed in the data, but still shows a high volatility of consumption relative to output, a strong positive correlation between these two variables and a negative correlation between output and both trade balance and interest rate spreads.

The remaining of this paper is structured as follows: section 2 establishes the relation between this paper and the existing literature, section 3 presents the data sources and empirical facts observed, section 4 introduces the model, section 5 calibrates it and analyses its main results and section 6 concludes.

2 Literature Review

As mentioned in the introduction, this paper closes an existing gap of the literature where the existence of an informal sector is ignored when analyzing a country's default risk, specially if the country follows a pro-cyclical fiscal policy. Therefore, it is related to the following three streams of the literature: fiscal policy, informal economy and sovereign default.

The pro-cyclicality of fiscal policies in developing countries is a fact well documented in the literature. Gavin and Perotti (1997) showed that this pro-cyclicality was followed by Latin American countries, Talvi and Vegh (2005) extended that analysis and conclusions to developing countries in general, while Kaminsky et al. (2005) collected evidence by income groups verifying this pro-cyclicality for low and middle income countries. Furthermore, the fact that these pro-cyclical fiscal policies often correspond to countercyclical tax rates has been showed, for example, by Talvi and Vegh (2005) for Mexico and Argentina and by Rial and Vicente (2008) for Uruguay. In Argentina, tax rates were low during the economic boom of 1991-1994 and 1996, but were high in 1995 and 2001 when the economy fell into a recession. In Mexico, tax rates were increased in the recession of 1995 and in Uruguay they were dramatically increased in 2002-2003 in the middle of a debt crisis.

There are several papers proposing explanations for this pro-cyclicality of fiscal policies in developing countries, including weak political institutions (Calderón et al. (2012)), incomplete markets and borrowing constraints. However, it is not the focus of this paper to explain why such pro-cyclical policies are followed, instead, this feature of developing countries is used together with the dimension of the informal sector to explain default risk and changes in sovereign bonds interest rate spread.

This paper is also related to the literature on the existence and size of the informal economy. The informal economy, according to Schneider (2005), includes all market-based legal production of goods and services that are deliberately concealed from public authorities to avoid payment of income, value added or other taxes; to avoid payment of social security contributions; to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and to avoid complying with certain administrative procedures, such as completing statistical questionnaires or other administrative forms. However, illegal activities and informal household economy are not included in this definition of informal economy.

Due to its definition and hidden nature, the size of the informal sector cannot be observed directly from the data and has to be estimated. Different methods have been developed over time to estimate its size. Enste and Schneider (2000) provide a summary of those various methods comparing their estimates and propose some causes and consequences of the existence of an informal sector. Among the various studies and estimates for the size of the informal sector as a share of official GDP around the world, Schneider (2005) reports it to be 17% in OECD countries, 38% in transition economies and 41% in developing countries between 1999 and 2000, supporting the fact mentioned before that emerging markets have a larger informal sector than developed countries. From the possible causes for its existence I want to stress the level of the tax rates and the quality of the country's institutions, both reported by Enste and Schneider (2000) and which are the ones in focus on this paper. Regarding the quality of the country's institutions Kuehn (2014) established a negative correlation between them, measured by the Worldwide Governance Indicators (WGI), and the size of the informal economy, supporting the fact that a country with high institutional quality has a lower informal economy.

Several papers have used the existence of an informal sector to explain diversified facts. Cerda and Saravia (2013) study how optimal taxation decisions change if firms have different productivities and can choose to produce in a formal sector where taxes are levied or in an informal sector where the government cannot do so. Busato and Chiarini (2004) state that the introduction of an informal sector improves the fit of the model to the data, especially in labor market dimensions, allowing households to smooth income through a reallocation of labor between sectors. However, to my knowledge, there is no paper showing the relation and impact that a large informal sector has on default risk and sovereign bonds interest rate spread, a relation that is observed in the data and that this paper addresses.

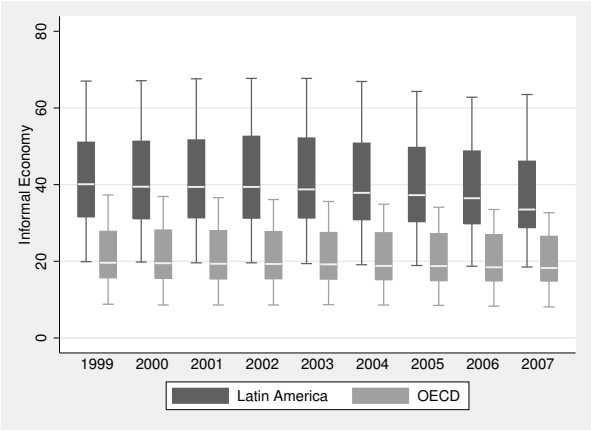
Last, this paper is an extension of the sovereign-default framework of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), among others. This paper presents a model with countercyclical interest rate spreads (a fact documented by Neumeyer and Perri (2005) and Uribe and Yue (2006)) and higher incentives to default in recessions than in expansions (as compiled in the survey from Peter (2002) over different econometric studies on the probability of sovereign default). Specifically, it introduces an informal sector in the sovereign default model with pro-cyclical fiscal policies of Cuadra et al. (2010) to help explain the facts observed in the data.

Therefore, by acknowledging the existence of an informal sector and showing how that fact, together with pro-cyclical fiscal policies, may explain why emerging markets with large informal sectors face higher default risk, spreads and lower output, specially in recessions, than developed countries, this model closes a gap existing in the literature.

3 Empirical Analysis

Using estimates from Schneider et al. (2010) and Schneider (2011) for the size of the informal economy as a fraction of official GDP for OECD and Latin American countries, Figure 1 reports the average size of this sector from 1999 to 2007. As it is possible to observe, Latin American countries have a larger informal sector than OECD countries, a fact that is related (among others) to the quality of the country's institutions as showed in Figure 2 and was documented by Kuehn (2014). In Figure 2, the quality of a country's institutions is measured by the Rule of Law[2] indicator one of the six WGI[3], however, the same correlation applies if any of the other WGI is used.

Figure 1: Size of the Informal Sector as a Fraction of Official GDP



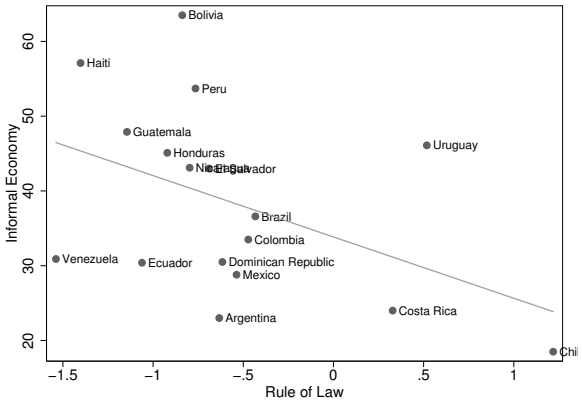
Source: Schneider et al. (2010) and Schneider (2011)

This high dimension of the informal economy in Latin American countries becomes more important when looking at the implications that the informal economy has on government bonds interest rate spreads and output. These relations were analyzed only for periods when the countries were not considered to be in default according to both Reinhart and Rogoff (2009) and Benjamin and Wright (2009) as it is considered that, if a country is in a situation of default, its spread is not as informative.

[2]"The Rule of Law captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence" (Kaufmann et al. (2011)).

[3]The Worldwide Governance Indicators are: Control of Corruption, Government Effectiveness, Political Stability and Absence of Violence, Regulatory Quality, Rule of Law and Voice and Accountability.

Figure 2: **Correlation between Informal Economy (% of GDP) and Rule of Law in Latin American countries for 2007**



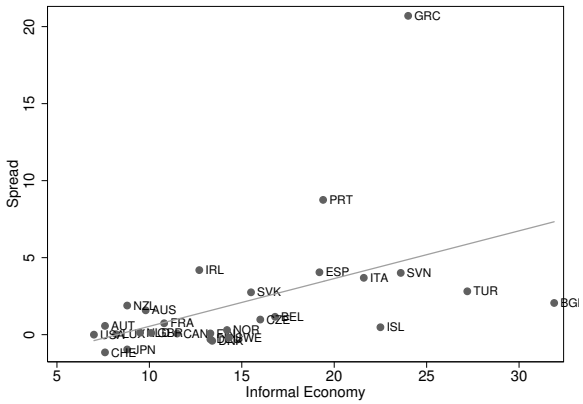
Source: Schneider et al. (2010) and WGI

Different data sources were used in this analyses. Spreads were measured using EMBI+ government bond spreads from the Global Financial Data (GFD) Website when available. For OECD countries, if EMBI+ data was unavailable, an approximation of spread defined as the difference between long term interest rates in the country and that in the United States was used. Data on real GDP and debt as a share of GDP was obtained from the World Bank's World Development Indicators (WDI). Specifically, debt was defined as external debt stocks public and publicly guaranteed (% of GDP) when such variable was available, otherwise, central government debt (% of GDP) was used. Finally, data on both inflation and deficit as a share of GDP was obtained from the World Economic Outlook (WEO) from the IMF, where inflation is defined as a percentage change in average consumer prices and deficit is the difference between general government total expenditure and general government revenue, both as a share of GDP.

Given this data, Figure 3 shows the correlation in 2012 between the spread and the size of the informal sector. It is possible to observe that a larger informal economy is positively correlated with the spread, a relation that becomes clearer by looking at Table 1.

Table 1 presents the results of a country and time fixed effects regression of spread on the size of the informal economy, on a dummy variable crisis which equals 1 when the cyclical component of log(GDP) is negative reflecting that the country is in a recession, an interaction term between this dummy variable and the size of the informal sector, on debt, deficit, inflation and on the cyclical component of GDP for 59 countries. It is clear from it that a larger informal sector leads to a higher spread and that this effect of the informal economy on spread is increased if the country is in a recession period. This result is one of the main focus of this paper and what the model proposed in here aims to explain. As mentioned in the introduction, in a country where the informal sector is large firms have incentives to shift their production to this sector, incentives that increase if the country follows a pro-cyclical fiscal policy, increasing the tax rates in recession periods and, consequently, accentuating the incentives to default on existing debt.

Figure 3: **Correlation between Sovereign Bonds Interest Rate Spread and the Size of the Informal Economy (% of GDP) in 2012**



Source: Schneider et al. (2010), Schneider (2011), GFD and OECD

Table 1: **Fixed Effects Regression of Spread**

	Spread
Informal Economy (% of GDP)	0.18**
	(0.07)
Dummy Crisis	-0.52
	(0.34)
Informal Economy x Crisis	0.02***
	(0.01)
Debt (% of GDP)	0.02**
	(0.01)
Deficit (% of GDP)	0.06**
	(0.03)
Inflation (% change)	0.11*
	(0.02)
Log(GDP) (cyclical component)	-12.14***
	(6.25)

* 1% significant level, ** 5% significant level, *** 10% significant level

Source: Schneider et al. (2010), Schneider (2011), WDI and WEO

Table 1 also reports a positive and significant correlation between debt and spread and a negative correlation between the cyclical component of GDP and the dependent variable. The correlation with debt may be understood by the fact that a higher debt level implies a higher difficulty in repaying existing debt and, consequently, a higher probability of default. Therefore, lenders will require the payment of a higher interest rate to lend money to those countries, increasing the spread. In the other hand, the negative correlation with the cyclical component of GDP reflects the documented fact that when the economy is in expansion, countries are less likely to default on their debt and, consequently, lenders will charge a lower interest rate than when the country is in a recession.

It is also of interest to see the impact of the informal sector on output. In that sense, the results of a country and time fixed effects regression of the cyclical component of GDP on the size of the informal economy, a lagged dummy for crisis, an interaction term between the first two variables and debt are presented in Table 2, for 146 countries. In order to avoid a simultaneity problem, a dummy indicating whether the country was in a recession in the previous period was used instead of the previously defined dummy. It is possible to observe a negative correlation between the cyclical component of GDP and both the size of the informal economy and the interaction term of this size and the dummy variable. These negative correlations reflect the intuition that, controlling for the debt level, a country with a larger informal sector will have a lower output and that impact is even higher if the country was last period in a recession. The decrease in output may be explained by the fact that the informal sector is not considered in the official accounting of GDP and, therefore, if a large share of resources are allocated to this sector, less are used in the formal sector causing a decrease in official GDP.

Table 2: **Fixed Effects Regression of the Cyclical Component of log(GDP)**

	log(GDP)c
Informal Economy (% of GDP)	-0.0049*
	(0.0006)
Dummy Crisis_lag	-0.0015
	(0.0033)
Informal Economy x Crisis_lag	-0.0002**
	(0.0001)
Debt (% of GDP)	-0.0001*
	(0.0000)

* 1% significant level, ** 5% significant level,
Source: Schneider et al. (2010), Schneider (2011) and WDI

4 The Model

There is a small open economy with four agents: firms, households, government and foreign lenders, two sectors: formal and informal and one homogeneous good. The firm produces its final output by using two different technologies, one associated with the formal and the other with the informal sector.

Producing in the more productive formal sector implies the payment of taxes, while producing in the less productive (according to Porta and Shleifer (2008)) informal sector does not imply such payment. Nevertheless, in the spirit of Allingham and Sandmo (1972), the firm operating in the informal sector faces a non-negligible probability of being catch by the government, in which case it will have to pay not only the taxes it was evading, but also a penalty surcharge. Therefore, the dimension of the informal sector over time is determined by the tax rate chosen by the government and by the probability of being found producing in that sector and the associated penalty.

Households have preferences over consumption and leisure and have to decide how to allocate their work between the two productive sectors subject to a budget constraint. The government is the only agent in the economy that can borrow one period non-contingent bonds from the international markets.

Debt contracts are not enforceable since the government has the option to default on them. When it defaults it is temporarily excluded from credit markets and the economy suffers a productivity loss. Besides this borrowing decision, the government also decides the tax rate to apply over firm's total revenues in order to finance its public consumption. The foreign lenders charge a premium over the risk-free interest rate to account for the probability of not being paid back by the government.

4.1 Firms

There is a representative firm that, in each period, decides the quantity of its homogeneous good to produce in the formal and in the informal sector. Producing in the formal sector is more productive, but implies the payment of taxes over revenues, while the informal sector, although less productive, does not require such payment.

For period t, let y_t^f denote the output produced in the formal sector and y_t^i the output produced in the informal sector, such that total production is given by $y_t = y_t^f + y_t^i$.

Each sector uses labor as its only input and has decreasing returns to scale. Also, it is assumed that the productivity shock of the informal sector is a constant fraction $\nu \in (0,1)$ of the productivity in the formal sector. Therefore, the production functions are given by:

$$\begin{aligned} y_t^f &= A_t \left(l_t^f\right)^{\alpha} \\ y_t^i &= \nu A_t \left(l_t^i\right)^{\alpha} \end{aligned}$$

where α is the labor share, A_t is the stochastic productivity shock of the formal sector and l_t^j is the amount of work in each sector, $j \in \{f, i\}$.

Revenues from the formal sector, $q_t^f \left(1-\tau_t\right) y_t^f$, are taxed at the tax rate τ_t defined in each period by the government, where q_t^f denotes the price of the formal-sector-produced good. Firms do not pay taxes in informal sector revenues $q_t^i y_t^i$, where q_t^i is the price of the informal sector produced commodity. The firm, however, may be discovered operating in the informal sector with probability $p \in (0,1)$ and forced to pay the tax rate τ_t, increased by a penalty, $\gamma > 1$, applied to the standard tax rate. Both the probability of being caught and the penalty surcharge that would have to be paid are increasing functions of the parameter θ which represents the "institutional quality" level of the country and, consequently, the higher θ, the lower the informal sector in the country, independently of the tax rate chosen. Since the goods produced in the formal and in the informal sector are identical, in equilibrium they must have the same price. Therefore, $q_t^f = q_t^i = q_t$ and I normalize the price to unity without loss of generality.

In each period, the firm gets revenues of $\left(1-\tau_t\right) y_t^f$ from producing in the formal sector, while from the informal sector, with probability $(1-p)$ it receives y_t^i and with probability p it gets only $\left(1-\gamma\tau_t\right) y_t^i$, as it was discovered producing in the informal sector. Therefore, time-t total expected revenues are given by:

$$E\left(TR_t\right) = \left(1-\tau_t\right) y_t^f + \left(1-p\gamma\tau_t\right) y_t^i \tag{1}$$

where E is the mathematical expectations operator. From this expression, it is possible to conclude that, as long as $1 - p\gamma\tau_t > 0$ the firm has incentives to engage in the informal sector as the revenues from that sector are expected to be positive.

The labor market is perfectly competitive. The total cost of labor depends on the share of workers hired to the formal and to the informal sector. More precisely, I assume that the cost of labor in the formal sector is given by the nominal wage w_t^f, whereas the cost of labor hired in the informal sector is given by w_t^i. Accordingly, the total costs for the firm are defined as follows:

$$TC_t = w_t^f l_t^f + w_t^i l_t^i \tag{2}$$

Given equations (1) and (2), the optimal amount of final output produced by a firm is the solution to the maximizing profits problem:

$$\begin{array}{rll} & \max_{l_t^f, l_t^i} & E\left(TR_t\right) - TC_t \\ \text{s.t.} & y_t^f = & A_t \left(l_t^f\right)^{\alpha} \\ & y_t^i = & \nu A_t \left(l_t^i\right)^{\alpha} \end{array}$$

Firm's behavior is characterized by the following first order conditions:

$$w_t^f = \alpha\left(1-\tau_t\right) A_t \left(l_t^f\right)^{\alpha-1} \tag{3}$$

$$w_t^i = \alpha\left(1-p\gamma\tau_t\right) \nu A_t \left(l_t^i\right)^{\alpha-1} \tag{4}$$

Equation (3) describes the optimal demand for formal sector labor and equation (4) the optimal demand for labor in the informal sector.

The share of informal economy is given by $S_t = \frac{(1-p)y_t^i}{py_t^i + y_t^f}$ defined as the output produced in the informal sector that was not discovered by the government divided by the sum of the output produced in the formal sector and the output produced in the informal sector that was found by the government. This share corresponds to what is usually estimated to be the size of the informal economy as a share of GDP in the data, as GDP does not only consider formal output but also output from the informal sector that was discovered by the government.

4.2 Households

There is an infinitely lived representative household with preferences over sequences of consumption (c_t) and labor in the formal $\left(l_t^f\right)$ and informal $\left(l_t^i\right)$ sector given by:

$$U\left(c_t, l_t^f, l_t^i\right) = \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \eta \frac{\left(l_t^f + l_t^i\right)^{1+\zeta}}{1+\zeta} \right\} \tag{5}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ is the inverse of the inter-temporal elasticity of substitution, $\eta \geq 0$ is a preference parameter controlling for the disutility of working activities and $\zeta > 0$ denotes the inverse aggregate labor supply elasticity. Notice that the second term reflects the disutility of working in general, independently of the sector, allowing for perfect mobility across sectors.

The representative household faces the following budget constraint:

$$c_t = w_t^f l_t^f + w_t^i l_t^i \tag{6}$$

which states that the total amount the household can spend in consumption must be financed by the payment from his work $\left(w_t^f l_t^f + w_t^i l_t^i\right)$.

The utility maximization problem consists on choosing c_t, l_t^f and l_t^i that maximize the inter-temporal utility function (5) subject to the budget constraint (6) . An optimal allocation of consumption and labor supply to each of the sectors must satisfy the following conditions:

$$c_t^{-\sigma} = \lambda_t \tag{7}$$

$$\eta \left(l_t^f + l_t^i\right)^{\zeta} = \lambda_t w_t^f \tag{8}$$

$$\eta \left(l_t^f + l_t^i\right)^{\zeta} = \lambda_t w_t^i \tag{9}$$

where λ_t is the Lagrange multiplier for the budget constraint (6) . From the above equations it is possible to conclude that the wage in the formal and in the informal sector has to be the same such that the household is indifferent between working in either sector. Therefore, rearranging the above equations I have:

$$w_t^f = w_t^i \tag{10}$$

$$w_t = \frac{\eta \left(l_t^f + l_t^i\right)^{\zeta}}{c_t^{-\sigma}} \tag{11}$$

4.3 Government

Government is benevolent and its objective is to maximize the household's utility, subject to the firm's profit-maximization conditions, the government's budget constraint and the resources constraint. In each period the government has to choose the tax rate it is going to charge over revenues in order to finance its public consumption. It has also access to the international financial markets, where it can buy one-period non-contingent bonds $b^{'}$ at price $q\left(b^{'}, A\right)$ and can decide whether to repay or default on its debt. If it defaults, it is temporarily excluded from credit markets and the economy suffers a productivity loss. The bond price function $q\left(b^{'}, A\right)$ is endogenous to the government's incentives to default, and depends on the size of the bond $b^{'}$ and the aggregate productivity shock A. The purchase of a discount bond with negative face value for $b^{'}$ means that the government is borrowing $q\left(b^{'}, A\right) b^{'}$ units today and promises to pay, conditional on not defaulting, $b^{'}$ units of the good in the following

period and the opposite if $b^{'}$ has a positive face value, in which case the government is the lender.

The government's problem can be written in recursive form as:

$$V(b,A) = \max\left\{V^{c}(b,A), V^{d}(A)\right\}$$

where

$$\begin{aligned} V^{c}(b,A) &= \max_{c,l^{f},l^{i},b^{'},\tau} U\left(c,l^{f},l^{i}\right)+\beta\sum_{A^{'}}\pi\left(A^{'}|A\right)V\left(b^{'},A^{'}\right) \\ \text{s.t.} \quad g &= \tau A\left(l^{f}\right)^{\alpha}+p\gamma\tau\nu A\left(l^{i}\right)^{\alpha}+b-q\left(b^{'},A\right)b^{'} \\ c+g &= A\left(l^{f}\right)^{\alpha}+\nu A\left(l^{i}\right)^{\alpha}+b-q\left(b^{'},A\right)b^{'} \\ & \quad (3),\ (4),\ (10) \text{ and } (11) \end{aligned}$$

and

$$\begin{aligned} V^{d}(A) &= \max_{c,l^{f},l^{i},\tau} U\left(c,l^{f},l^{i}\right)+\beta\sum_{A^{'}}\pi\left(A^{'}|A\right)\left[\psi V\left(0,A^{'}\right)+(1-\psi)V^{d}\left(A^{'}\right)\right] \\ \text{s.t.} \quad g &= \tau h(A)\left(l^{f}\right)^{\alpha}+p\gamma\tau\nu h(A)\left(l^{i}\right)^{\alpha} \\ c+g &= h(A)\left(l^{f}\right)^{\alpha}+\nu h(A)\left(l^{i}\right)^{\alpha} \\ & \quad (3),\ (4),\ (10) \text{ and } (11) \end{aligned}$$

where $\psi \in (0,1)$ is the probability of going back to the market after default with zero debt and $h(A)$ is an increasing productivity loss function defined as:

$$h(A) = \begin{cases} A & \text{if } A \leq \hat{A} \\ \hat{A} & \text{if } A > \hat{A} \end{cases}$$

The government default policy can be characterized by default and repayment sets defined, respectively, as:

$$\begin{aligned} D(b) &= \left\{A \in \mathcal{A} : V^{c}(b,A) < V^{d}(A)\right\} \\ R(b) &= \left\{A \in \mathcal{A} : V^{c}(b,A) \geq V^{d}(A)\right\} \end{aligned}$$

The amount of government consumption is assumed to be fixed over time. Therefore, the equilibrium tax rate in each period has to satisfy the following condition:

$$\tau = \frac{g - b + q\left(b^{'}, A\right) b^{'}}{A\left(l^{f}\right)^{\alpha} + p\gamma\nu A\left(l^{i}\right)^{\alpha}} \tag{12}$$

4.4 Lenders

Foreign lenders have access to an international credit market where they can borrow or lend as much as needed at a constant interest rate $r > 0$. They have perfect information regarding the economy's productivity process as well as its level of output. Lenders are assumed to price defaultable bonds in a risk neutral manner such that in every contract offered they break even in expected value. In particular, every period, they choose loans $b^{'}$ to maximize expected profit π, taking prices as given:

$$\phi = q\left(b^{'}, A\right) b^{'} - \frac{1 - \delta\left(b^{'}, A\right)}{1 + r} b^{'}$$

where δ is the probability of default given by $\delta = \sum_{A^{'} \in D(b)} \pi\left(A^{'} | A\right)$. For positive levels of foreign asset holdings, $b^{'} \geq 0$, the probability of default is zero and, therefore, $q\left(b^{'}, A\right) = \frac{1}{1+r}$. For negative levels of foreign asset holdings, $b^{'} < 0$, the equilibrium price must account for the risk of default lenders face, such that the price of bonds must satisfy:

$$q\left(b^{'}, A\right) = \frac{1 - \delta\left(b^{'}, A\right)}{1 + r}$$

4.5 Timing

The timing of decisions is the following. After observing the productivity shock and taking as given its initial assets b the government decides whether to repay or default on its debt. If it defaults on its debt, it suffers a productivity loss and a positive probability of going back to the market in the following period without debt and it has to decide the tax rate τ to charge to firms in order to finance its public consumption. If the government decides to repay, then, taking as given the price of bonds function, it decides both the tax rate τ and the new amount of debt $b^{'}$, satisfying its budget constraint and the resources constraint. Then, lenders taking q as given choose $b^{'}$ and firms taking both the productivity shock and the tax rate as given choose their allocation of production between the two productive sectors. Finally, consumption c takes place.

4.6 Equilibrium

The recursive equilibrium for this economy is defined as:

1. a set of price functions for wages $w(b, A)$ and bonds $q\left(b^{'}, A\right)$
2. a set of policy functions for consumption $c(b, A)$, work in the formal sector $l^{f}(b, A)$, work in the informal sector $l^{i}(b, A)$, tax rate $\tau(b, A)$ and government's asset holdings $b^{'}(b, A)$
3. a set of value functions $V(b, A)$, $V^{c}(b, A)$ and $V^{d}(A)$

such that:

1. firms are maximizing profits at the prevailing prices and tax rates, i.e. (3) and (4) are satisfied
2. $c(b, A)$, $l^f(b, A)$, $l^i(b, A)$ solve the household's maximizing utility problem at the prevailing prices, i.e. (10) and (11) are satisfied
3. $\tau(b, A)$ and $b'(b, A)$ solve the government optimization problem at the given bond price function $q\left(b', A\right)$
4. bonds prices $q\left(b', A\right)$ reflect the government's default probability and are consistent with lender's expected zero profits

5 Quantitative Analysis

5.1 Calibration and Other Functional Forms

The model is solved numerically to evaluate the implications of different sizes of the informal sector on a country's default risk, spread and output when pro-cyclical fiscal policies are followed.

The values chosen are reported in Table 3. The stochastic process for productivity is estimated from the series of Argentina's GDP from 1980 to 2007. It is assumed to be a log-normal AR(1) process $\log\left(A_t^f\right) = \rho \log\left(A_{t-1}^f\right) + \varepsilon_t^f$, with $E\left[\varepsilon^f\right] = 0$ and $E\left[\left(\varepsilon^f\right)^2\right] = \sigma_f^2$. The estimated values are $\rho = 0.855$ and $\sigma_f = 0.0238$. The shock is then discretized into a 101-state Markov chain, using Tauchen (1986) method.

Table 3: **Parameters**

Productivity Process and Production Function		
Stochastic structure	$\rho = 0.855$	Argentina's GDP (1980-2007)
	$\sigma_f = 0.0238$	
Productivity of the Informal Sector	$\nu = 0.6$	Turnovsky and Basher (2009)
Productivity Loss	$\hat{A} = 0.969E(A)$	Arellano (2008)
Labor Share	$\alpha = 0.64$	Standard value
Utility Function Parameters		
Risk Aversion	$\sigma = 2$	Standard Value
Disutility of working	$\eta = 45$	Calibrated to match 1/3 of working time
Inverse of Labor Supply Elasticity	$\zeta = 2$	
Other Parameters		
Probability of reentering the market	$\psi = 0.282$	Arellano (2008)
Risk-free Interest Rate	$r = 1.7\%$	US 5-year bond quarterly yield (1980-2007)
Government Consumption	$g = 0.13$	Government consumption (% of GDP)
Institutional Quality	$\theta = 37.4$	Rule of Law estimate (0-100 scale)
Discount Factor	$\beta = 0.9$	23% Informal Sector as a % of GDP

The productivity of the informal sector is assumed to be a constant share $\nu = 60\%$ of the productivity in the formal sector, in accordance with the parametrization used by Turnovsky and Basher (2009). The default costs threshold $\hat{A}$ is $0.969E(A)$ and the probability of reentering the market after default ψ is 0.282, both parameters are the same as used by Arellano (2008). I assume that the labor share in the production function of the two sectors is the same and equal to 0.64, a standard value.

The risk aversion parameter σ is standard and equal to 2 as well as the value for the inverse of labor supply elasticity ζ. The parameter on the disutility of work η was calibrated to 45 in order to match the fact that one-third of the available time is allocated to work in general, independently of being in the formal or in the informal sector. The risk-free interest-rate r is set to 1.7%, the average quarterly interest rate of a five-year US treasury bond between 1980 and 2007. The value for government consumption g was estimated from the quarterly average government consumption over GDP for Argentina between 1993 and 2007.

Specifically for this paper, the quality of the institutions should be interpreted as a factor that determines the probability of a firm operating in the informal sector be found and the penalty surcharge it is going to pay in such case. Therefore, a country with high institutional quality, in this paper, is a country that has a high probability of finding informal sector firms and charges them a higher penalty, creating disincentives to engage in such activities and establishing the negative correlation observed in the data. Therefore, to calibrate θ, the "institutional quality" parameter, the estimate for the Rule of Law indicator for Argentina in 2007 was used. Argentina was classified as -0.63 in a -2.5 to 2.5 scale, according to this indicator. Taking this value, I set the institutional quality parameter θ equal to 37.4 which is the corresponding value of the estimate above-mentioned converted to a 0 to 100 scale.

The discount factor parameter β as well as the probability function of being found producing in the informal sector p were defined in order to match the share of the informal sector as a percentage of official GDP in Argentina which was estimated by Schneider et al. (2010) to be 23% in 2007. Therefore, I set β to be 0.9 and define the probability function as $p = \frac{\theta}{125}$, an increasing function of θ.

Last, to define the function for the penalty surcharge the firm has to pay if found producing in the informal sector, γ, I took into consideration the findings of Bergman (2009). He observed that, in Argentina, only 1% of the criminal cases filled by Argentine Tax Administration between 1990 and 1995 were convicted, a share that is very different from that in the US (90%), Great Britain (93%) or even Chile (more than 80%). Besides this, in those cases that were solved favorably to the tax administration only a small share of the value claimed was received by the tax administration. This lack of enforcement and the low level of penalties that have effectively to be paid if a firm is found producing in the informal sector make the penalty in my model be also low. To derive the penalty formula I used data on OECD countries and extrapolated for the Argentine case. Specifically, Kuehn (2014) reported a penalty surcharge around 136% for the average of OECD countries, whose estimate for the Rule of Law indicator was 1.28 in 2007. Taking these two informations as given, I defined the function for the penalty surcharge as $\gamma = 1 + \frac{\theta}{210}$.

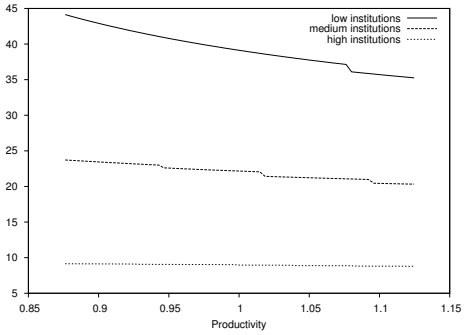
5.2 Results

This section starts by looking at the size and cyclicality of the informal sector and tax rates. Then it analyzes the policy functions of the model and how they change with different calibrations for the

parameter θ. Finally it looks briefly at the model's quantitative performance in comparison with the data.

Figure 4 shows how the size of the informal sector changes with the "institutional quality", θ, and the productivity level. As mentioned in the previous section, the benchmark calibration for θ is 37.4 corresponding to the Rule of Law estimate for Argentina in 2007, this section uses also a value of 11.6 corresponding to the estimate for Afghanistan and 75.6, the average estimate for OECD countries in the same year, in order to analyze how different probabilities of being discovered producing in the informal sector and penalties affect the size of this sector.

Figure 4: Size of the Informal Sector for Different Institutional Quality Levels



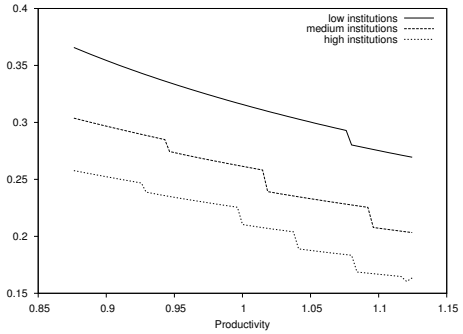
From the figure two main conclusions can be driven. First, independently of the value of θ, the size of the informal sector is countercyclical, a fact that is consistent with what was reported by Busato and Chiarini (2004) for developed countries and by Elgin (2012) using data from 152 countries. The counter-cyclicality of the informal sector can be easily understood in the context of this model. A country facing a fixed government spending needs to apply countercyclical tax rates in order to finance it. Consequently, in periods of low productivity, there is an increase in tax rates that creates an incentive for firms to shift their production from the formal to the informal sector in order to avoid the payment of taxes. Besides, the change in the size of the informal sector with productivity is going to be higher the lower the "institutional quality", θ, as in such situation the penalties of producing in the informal sector are lower.

Second, it is possible to observe that a high value for the parameter θ corresponding to better institutional quality and, consequently, a higher probability of being found and penalty to be paid, conducts to a smaller informal sector in the economy independently of the productivity level. On average, the size of the informal sector will be 15% for the OECD value, 23% in the benchmark case and 29% for the lowest value. Due to this high correlation between the institutional quality parameter and the size of the informal sector, in the remaining of this section, I will refer to countries with large or small informal sector, corresponding to countries where θ is low or high, respectively.

The other main feature of this model that determines not only the size of the informal sector but also the bond price schedules and which is observed in the data is the pro-cyclicality of fiscal policies, in special the counter-cyclicality of tax rates. Figure 5 shows that tax rates decrease as productivity

increases for a given debt level and that the tax rate level is higher the lower the "quality of the institutions". The counter-cyclicality of tax rates is explained by the constant government spending that must be financed by taxes. When productivity is low, output produced is also lower and, therefore, the government needs to impose a higher tax rate to capture all the funds it needs. Besides, if the "institutional quality" is low, the probability of being found producing in the informal sector and the penalty that has to be paid are also low, creating higher incentives to produce in this less productive informal sector, reducing the level of output produced in the formal sector for each productivity level. Given this lower production level, specially in the formal sector which is the sector paying the taxes, the government needs to charge a higher tax rate then if θ was higher and the informal sector lower.

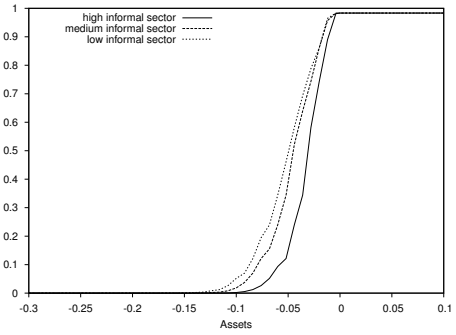
Figure 5: Tax Rates for Different Sizes of the Informal Sector and Constant Debt



Knowing how both the informal sector and the tax rates evolve for different productivity and "institutional quality" levels, I now turn to the analysis of the bond price functions. Figure 6 shows the bond price schedule as a function of assets b for three different sizes of the informal sector in the economy and a constant productivity level. Bond prices are lower for economies with a larger informal sector and are increasing functions of assets, meaning that, higher values of debt (lower assets) imply a higher interest rate and lower prices. The lower bond prices and higher interest rates for countries with high informal sector is due to the fact that one of the main determinants of the size of this sector, as was showed before, is the parameter θ, which is exogenous and fixed over time. Consequently, a country with a high informal sector today is expected to have a high informal sector also in the future and, therefore, a lower level of output for each productivity. This lower level of output makes it more attractive to default even for low amounts of debt, increasing the probability of default and decreasing the prices.

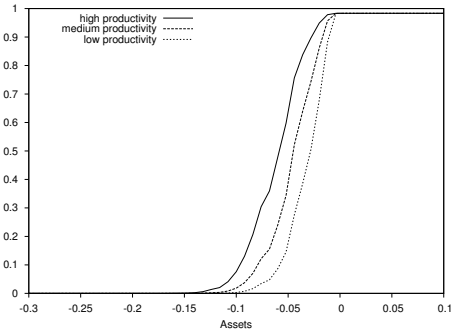
Figure 7 shows the bond price functions for different productivity levels. It is possible to observe that the higher the productivity level, the higher the price as the productivity shock is assumed to be persistent. A low level of productivity today is expected to conduct to a low level of productivity tomorrow and, therefore, similarly to what happens when the informal sector is high, the agent would like to default even in small amounts of debt. From the results of Figures 6 and 7 it is possible to

Figure 6: Price of Bonds Function for Different Sizes of the Informal Sector



conclude that, as in the data, controlling for debt level, the country faces a higher spread (lower bond prices) the larger its informal sector and the lower its productivity level and at a low productivity level its informal sector and spread will be even higher due to higher tax rates.

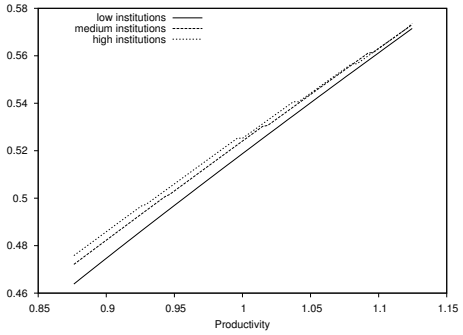
Figure 7: Bond Price Function for Different Productivity Levels



The last result from the data that this model can replicate is the relation between the size of the informal sector and total output produced. Figure 8 illustrates that, for a fixed debt level, the existence of a large informal sector (low θ) reduces output by more in a recession, when productivity is low than in an expansion. When productivity is low, the fact that θ is low leads to a decrease in output higher than when θ is high. This relation results from the interaction of productivity level, tax rates and "institutional quality" (θ). For low levels of productivity, given a fixed government consumption, taxes are higher. If, in addition, the penalties associated with producing in the informal sector are lower, more firms will prefer to produce in this sector which is less productive, reducing, consequently, the total amount of output produced.

By combining the observed facts from the data showing a negative correlation between the size of the informal economy and the quality of the institutions and a countercyclical fiscal policy in emerging markets, this model shows that a larger informal sector implies higher and countercyclical spreads

Figure 8: Total Output Produced for for Different Sizes of the Informal Sector and Constant Debt



and lower output, being the effect of the informal sector over output higher when productivity is low. Additionally, this model also shows, as documented in the data, that the informal sector is countercyclical.

I now do a brief discussion of the quantitative predictions of the baseline model in terms of matching the data. As said before, I did not calibrate the model to match any specific moments besides the share of the informal economy which is perfectly matched to the 23% estimated by Schneider et al. (2010) for Argentina in 2007. As Table 4 shows, the model delivers a higher volatility of consumption than of output as in the data, but both values as well as the volatility of interest rate spread are underestimated. On the other hand, the volatility of trade balance is very close to that in the data. Regarding the correlation between variables, as in the data, the correlation between consumption and output in the model is very high and positive, while the correlations between output and both trade balance and spread are negative and underestimated. Regarding debt the model cannot match its value. Last, from the model, the optimal tax rate given all the assumptions would be 30%, a value that would increase if the size of the informal sector was higher and vice-versa. This value is very close to the legal tax rate on corporate income in Argentina which has been 35% since 2006.

Table 4: Business Cycle Statistics

	Quarterly Data (1993 to 2007)			**Model**		
	std(x)	**corr(x,y)**	**corr(x,s)**	**std(x)**	**corr(x,y)**	**corr(x,s)**
Private Consumption	5.20	98.64	−75.51	3.83	92.44	−24.17
Government Consumption	2.55	67.65	−78.34	—	—	—
Trade Balance	1.72	−92.18	72.70	1.44	−64.13	24.35
Interest Rates Spread (s)	12.88	−71.62		12.63	−19.92	
Output (y)	4.51			2.33		
Other Statistics	**Informal Economy**		23.6%	**Default Rate (%)**		2.12
	Debt (% GDP)		2.80	**Average Tax Rate**		30%
	Formal labor		0.22			
	Informal Labor		0.11			

6 Conclusion

In this paper I analyzed how a country's incentives to default and sovereign bond interest rate spreads would change with the size of its informal sector, assuming the country follows pro-cyclical fiscal policies.

I showed that emerging markets are characterized by large informal sectors and pro-cyclical fiscal policies and that, the size of the informal sector has impact over the spreads and the total output of the economy. Specifically, from the data, it was possible to observe that an increase in the size of the informal sector would increase spread and decrease total output, an effect that was larger when the country was in a recession.

To address these facts of the data I proposed a simple model with four agents. Firms chose whether to produce in the more productive formal sector where they would have to pay taxes over revenues or in the less productive informal sector, but where the payment of taxes would only take place if they were found producing in that sector, in which case they would have to pay not only the taxes, but also a penalty surcharge. Households had preferences over consumption and leisure, being indifferent between allocating their work to the formal or the informal sector. Government was the only agent benefiting from access to the international credit market where it could borrow one-period non-contingent bonds and choose whether or not to default on its existing debt. Besides the default decision, government had to decide the tax rate to charge to firms in order to finance its fixed level of public consumption. Foreign lenders charged a premium over the risk-free interest rate to account for the probability of not being paid back by the government.

The model could replicate the facts observed in the data. First, due to the fixed level of government consumption, the tax rates chosen in each period by the government were countercyclical, that is, they were higher when productivity was low. Second, the interaction of these countercyclical tax rates and the parameter on the quality of the institutions, which determined the probability of being found producing in the informal sector and the penalty associated with that situation, led to a countercyclical informal sector, which was even higher when institutions were weak as in emerging markets. By combining these two features of emerging markets, namely, the countercyclical tax rates and a large informal sector, the model showed that spreads in these countries were higher and that total output was lower, specially if the country was in a recession.

A country with low penalties associated with operating in the informal sector provides high incentives for firms to shift their production to that sector. Associated with this, countercyclical tax rates accentuate these incentives in recessions, causing a reallocation of work and production from the more productive formal sector to the less productive informal sector, reducing the total level of output produced. Given this lower output, government's will be willing to default even for low amounts of debt, increasing their default risk and, consequently, their interest rates.

This paper closes the gap that existed in the literature about the relevance that a large informal sector had on a country's incentives to default, spread and output, when it follows pro-cyclical fiscal policies as emerging markets do. By decreasing the size of the informal sector, it is expected that these countries would face lower spreads and lower default rates, contributing to more stable international credit markets.

In future work, it would be interesting to endogenize the probability of being found producing in

the informal sector as well as the penalty incurred such that the government's could decide how much effort to allocate to these activities and which would be the results over default risk, spread and output. It would also be of interest to see how the results might change if taxes were imposed on workers or consumption instead of firms revenues.

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A Computational Algorithm

The following algorithm is used to solve the model:

1. Start with some guess for the parameters to be calibrated: β, ψ and $\hat{A}$, a discretized state space for assets consisting of a grid of 51 points equally spaced and a discretized state space of productivity shocks consisting of a grid of 101 points.

2. Make a guess for the value functions such that $V^d(A) = \frac{u((1+\nu)A,0,0)}{1-\beta}$ and $V^c(b,A) = \frac{u((1+\nu)A+b,0,0)}{1-\beta}$

3. Taking as given the value functions, compute the expected continuation values and the bond price functions by deriving the optimal default decisions and use them to solve simultaneously for the equilibrium wage, which is the solution to equations (3), (4), (10) and (11) and tax rate, which is the solution to equation (12).

4. Solve for equilibrium labor in the formal sector $l^f(b,A)$, labor in the informal sector $l^i(b,A)$, consumption $c(b,A)$ and asset holdings $b^{'}(b,A)$.

5. Evaluate whether the maximum absolute deviation between the new and previous value functions is lower than 10^{-5}. If it is, the solution has been found. If it is not, repeat the optimization exercise from point 3. using the new value functions as guess, until it converges.